A Kernel-Based Approach to Gas Storage and Swing Contracts Valuations in Multiple Dimensions

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Abstract

We are expanding upon the original work from Hubbert & Mazières [6] where the Tensor of Radial Basis Functions (TBF) was first introduced and from Boogert and Mazières[1], where it was applied in gas storage valuation in two dimensions. We do so by introducing multi-factor price and multi-volume dimensions encountered in both gas storage and swing contracts, see Geman [5] for more details on these types of contracts. This is achieved by introducing new multivariate methods deriving from kernel-based regressions.

The kernels used here are Radial Basis Functions (RBF) and TBF. Both are 'dimension blind' hence the key to the multi-dimensional Least-Squares Monte Carlo (LSMC) method used to value these gas contracts. The LSMC method of interest here was first studied in the context of American option valuation, see Carrière, Longstaff & Schwartz and Tsitsiklis & Van Roy [3, 7, 9]. On the other hand the use of RBF is very new to gas storage and gas swing contracts, but the underlying mathematics and properties of RBF has been extensively studied, see for instance Buhmann, Schaback and Wendland [2, 8, 10]. TBF has very similar properties to RBF, but it is a lot more of a recent concept developed by Hubbert & Mazières[6]. Further, TBF kernels have significantly more potential than RBF kernels as, thanks to some tensor operator property, it makes the solution of linear system (and overdetermined system) a lot less computationally intensive than with RBF. Some of the mathematics relevant to the construction and analysis of TBF can be found in Cheney & Light[4].

The use of the above methods i.e., TBF and RBF, makes the valuation of multi-dimensional gas storage and gas swing contracts possible. Both contracts are investigated with regression methods based upon RBF and TBF kernels and performances are compared. The gas storage contract investigated here is a three dimensional problem with a two factors process and a volume dimension for the gas storage inventory. The Gas swing

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contract is more challenging as it is a four dimensional problem, where here again a two factors process is used along side two volume dimensions, keeping track of Annual Contract Quantity (ACQ) and Other Contract Quantity (OCQ) levels.

Finally it is found that the TBF based LSMC method consistently managed to extract more value, for both the gas storage and gas swing contract than the RBF and rolling intrinsic methods. In addition, the TBF based valuation method manages all of this for less computational time than required by the two other methods.

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