Electricity Options and Additional Information

Fred E. Benth, Richard Biegler-König and Rüdiger Kiesel

Abstract Electricity markets feature a non-storable underlying, which implies the break down of traditional cash-and-carry arguments as well as the well-known spot-forward relationship. We introduce the notion of information premium to describe the influence of future information - such as planned power plant maintenance - on the relationship between forward contracts and the spot market. In a recent paper we designed a statistical test to show the existence of the premia. Here, we examine how the presence of an information premium alters the prices of options on forwards. Also, we apply the technique of enlargement of filtrations to show how to calculate the premium specifically for certain types of information and delivery periods. Furthermore, we illustrate the results in various stylised examples.

1 Introduction

Since deregulation in the 1990s, electricity has been traded on exchanges in various regions such as Europe and the US. As an underlying, electricity is special in many ways, with market design having to take different technical and regulatory constraints into consideration. The most fundamental of the intrinsic properties of electricity is its non-storability. This has a huge impact on price behaviour, espe-

Fred E. Benth
CMA, University of Oslo, PO Box 1053 Blindern, 0316 Oslo, Norway, e-mail: fredb@math.uio.no

Richard Biegler-König
Chair for Energy Trading and Finance, University Duisburg-Essen, Universitätsstrasse 12, 45141 Essen, Germany, e-mail: richard.biegler-koenig@uni-due.de

Rüdiger Kiesel
Chair for Energy Trading and Finance, University Duisburg-Essen, Universitätsstrasse 12, 45141 Essen, Germany and CMA, University of Oslo, PO Box 1053 Blindern, 0316 Oslo, Norway, e-mail: ruediger.kiesel@uni-due.de

cially when it comes to the relation between spot and forward prices. Traditional theory utilises no-arbitrage and cash-and-carry arguments to derive the well-known spot-forward relationship

$$F(t,T) = e^{(r-y)(T-t)} S_t \tag{1}$$

where $F(t,T)$ is the forward price in t with maturity in T, S_t is the spot price, r is the interest rate and y reflects storage costs and convenience yield. In probabilistic terms this corresponds to the risk-neutral valuation formula

$$F(t,T) = \mathbb{E}^{\mathbb{Q}}[S_T | \mathscr{F}_t] \tag{2}$$

Here, $\mathbb{Q}$ is a pricing measure and $\mathscr{F}_t$ denotes the historical filtration, i.e. the filtration as generated by the past and present of the spot price process. This definition motivates the introduction of the so-called *risk premium*, i.e. the difference between the expectations under a pricing measure and the real-world measure. The risk premium is subject of intense research, discussed for example in Longstaff and Wang [28], Bessembinder and Lemmon [9], Lucia and Torró [34], Furiò and Meneu [18], Diko et al. [16] and Benth, Cartea, Kiesel [6].

Still, with electricity being non-storable this classical relationship collapses. As a stylised example we consider the announcement of planned maintenance of some utility in the future. This is very likely to induce higher forward prices around that time, whereas spot prices today will remain unchanged. In other words, we experience a situation in which movements in forward prices are not anticipated by spot prices. Mathematically, this means that a non-storable underlying results in an asymmetry between the historical filtration and what we will call the market filtration ($\mathscr{G}_t$).

This asymmetry leads to two different models in terms of a process and a filtration, namely $(S_t, \mathscr{F}_t)$ and $(S_t, \mathscr{G}_t)$. Looking at the spot isolated from the forward market we have the non-validity of the *Efficient Market Hypothesis* (i.e. not all available information is reflected) and thus $(S_t, \mathscr{F}_t)$ is the actual spot model with knowledge only of the evolution of the spot thus far. The forward market, on the other hand, is efficient (at least theoretically) and future information is taken into account. Hence, forward contracts are priced according to $(S_t, \mathscr{G}_t)$ in our framework.

In a first paper, Benth and Meyer-Brandis [7] described the inadequacy of the historical filtration when relating spot and forward prices on electricity markets. They propose to complement the filtration by specific pieces of future information and introduce a new pricing relationship. Referring to the *risk premium* they introduce the notion of the *information premium*. This is defined as the difference between the forward price under an enlarged market filtration and that under the historical filtration. For an arithmetic spot model they apply the theory of enlargement of filtrations (French: *grossissement de filtration*) to calculate the information premium in some cases. This theory was initiated by Itō [23] and developed by French mathematicians in the 1980s and mainly provides the decomposition of semi-martingales under an initially enlarged filtration.

Benth, Biegler-König and Kiesel present a thorough empirical investigation of the information premium in [5]. They prove its existence by a specially designed test and analyse in detail two market situations, one being the Moratorium discussed as a motivating example below, the other being the beginning of the second phase of the EU ETS in 2008 when additional costs for CO_2 were priced into the forwards while the spot remained unchanged. They find significant information premia for both scenarios, which match expectations in terms of size and shape.

To illustrate further, as mentioned above, we will consider a market situation that occurred on the German EEX in March 2011. On 11 March 2011 the Tōhoku earthquake and the consequent tsunami heavily damaged several nuclear power plants in Japan, in particular the one in Fukushima. Only three days later, on 14 March 2011, the German government reevaluated their nuclear policy and issued the so called ”Atom Moratorium”, by which the seven oldest plants (eight reactors with a capacity of more than eight GW) in Germany were to be shut down for three months. This measure was to allow for a new evaluation of the usage of nuclear power in Germany. Consequently, the market exhibited a sharp increase in forward prices while spot prices remained at their pre-Moratorium level. Considering the merit-order, this might, at first, sound surprising. Still, of the eight reactors, Brunsbüttel and Krümel (both in Schleswig-Holstein) had been offline for some time due to constant maintenance problems. Also, Biblis B (in Hessen) had gone into regular revision earlier. Hence, in effect, only around 4000 MW were switched off. At the same time, not only was more solar and wind electricity produced (rather accidently) but also Germany started importing cheap nuclear power from France (Germany actually exported four GW before and imported around two GW after the Moratorium). Summarising, there was no change of the price-setting technology. Obviously, on the demand side this was also due to the mild season. We refer to the report written by the *Bundesnetzagentur* to the federal ministry of economics and technology [1] for more details. Although the official end of the Moratorium was 15 June 2011, it was widely expected that the seven plants would stay offline even after that date, and indeed their permanent shut-down was decided on 31 May 2011.

The effect of the Moratorium and this future outlook was a sharp increase in forward prices, not only of those whose delivery fell into the three months of the Moratorium but also of those with a later delivery period.

As an example, when considering the evolution of the price of the forward with maturity in May 2011, we find that the Moratorium is the most striking date, exhibiting a huge increase in prices (i.e. a positive information premium). This forward had a mean price of 46.93 Euro before the Moratorium and a 57.83 Euro post-Moratorium mean. This corresponds to an increase of more than 10 Euro, i.e. almost 25%. Prices remained at a high level until the end of the delivery period.

On the other hand, the forward with delivery in July 2011 behaved differently. Again, there was a huge and sudden price increase following the Moratorium resulting in a higher price level all through April and May. Then, with the beginning of June the price returned to its pre-Moratorium level, i.e. the (positive) information premium was neutralised by other effects: another four reactors had gone offline in May and were only coming back online in the beginning of June (again, we refer to

[1]). Furthermore, demand was low because of the season. Still, with the final decision to shut down the seven old plants, political uncertainty was also removed and market participants began to better understand the new situation for summer 2012. This shows that the impact of future information can indeed change over time, i.e. the information premium is a function in time. For more details on the Moratorium we also refer to the recent paper [33] in which forward prices as well as fuel prices are examined empirically. The author can explain price paths of forwards and concludes that the market reacted efficiently to the new legislative framework.

Thus, summarising, one finds that forward prices reacted to some future information (or market sentiment) which was publicly available, but the spot did not.

In this article though, we want to extend the theoretical results of Benth and Meyer-Brandis [7]. In particular, we will examine how the information premium interacts with option prices. The necessary definitions and basic concepts will be introduced in Section 2. In Section 3 we will use a very simple Brownian spot model to examine the behaviour of option prices under additional information. This will be closely related to the literature on modelling insider trading, which will be discussed in Section 3.1. As a tool for pricing options, we will then provide formulae for the information premium. Here, we will consider more complex and more realistic situations than in [7]. In Section 5 we will illustrate our findings by presenting a number of stylised examples. Finally, Section 6 will conclude the article.

We remark that the approach taken in this article (and also in [7] or [5]) is that of modelling the spot using a reduced-form model. Our goal is to explore the relationship between spot and forward on electricity markets in general. Another branch of the literature introduces so-called fundamental or structural models. These take into consideration driving factors of electricity markets and deduce their prices from those factors. As an example, let us mention the paper by Aïd et al. [2]. Here, the authors model prices of fuels, demand and capacity and then deduce spot prices as the marginal production costs using the most expensive needed production technology. Coulon and Howison [14] and Burger et al. [12] follow a similar approach. Furthermore, it is worth mentioning the paper by Cartea et al. [13]. Here, the authors set up a spot price model in the usual (reduced-form) way but they also include a regime-switching part. Their switching parameter is deterministic and derived by comparing forecasted demand and forecasted available capacity. Thus, they incorporate specific future information into their spot model.

2 Preliminaries

In this section we will provide important definitions and first results about the information premium. We will consider forwards with a single delivery point rather than the more realistic delivery period. This is to ease notation. Once we start calculations (Section 3 and Section 4) everything can easily by adapted to delivery periods. In the second part we will introduce the theory of enlargement of filtrations, including the theorems and auxiliary results which will be used later.

2.1 The information premium

The classical spot-forward relationship has already been mentioned in the introduction. We define it formally again:

Definition 1. Classical Spot-Forward relationship. With $\mathbb{Q}$ a pricing (risk-neutral) measure[1], we have

$$F(t,T) = \mathbb{E}^{\mathbb{Q}}[S_T|\mathscr{F}_t] \tag{3}$$

where $F(t,T)$ denotes the time t-price of a forward maturing at T, $T \geq t \geq 0$, S_t is the spot price and $\mathbb{E}^{\mathbb{Q}}[\cdot|\mathscr{F}_t]$ is the conditional expectation under the historical filtration $\mathscr{F}_t = \sigma(S_u : u \leq t)$.

We can now compare the conditional expectation under the real-world measure with the conditional expectation under the pricing measure, and use the difference as an indicator for market sentiment.

Definition 2. Risk Premium. The risk premium is defined as

$$R^{\mathbb{Q}}(t,T) = \mathbb{E}^{\mathbb{Q}}[S_T|\mathscr{F}_t] - \mathbb{E}^{\mathbb{P}}[S_T|\mathscr{F}_t] \tag{4}$$

Note that observed forward prices are often used for the expression $\mathbb{E}^{\mathbb{Q}}[S_T|\mathscr{F}_t]$ (by assuming the correctness of equation (3)). After calculating expectations under $\mathbb{P}$ one then analyses the difference.

Since we want to study the impact of different information sets on forward prices, we introduce further filtrations finer than the historical filtration. We need a filtration which contains specified information on future spot prices and a slightly coarser filtration which contains some un-specified additional information. To be precise:

Definition 3. Filtrations. Let $\mathscr{H}_t$ be a filtration which includes the historical filtration as well as precise knowledge of the future value of the underlying at some time point T_Υ, i.e.

$$\mathscr{H}_t = \mathscr{F}_t \vee \sigma(S_{T_\Upsilon}) \tag{5}$$

Also, let $\mathscr{G}_t$ be a filtration that includes some information on the level of the future value of the underlying at time T_Υ. We will call this filtration the market filtration and we will assume that it represents the information available to market traders. This yields the relationship $\mathscr{F}_t \subseteq \mathscr{G}_t \subseteq \mathscr{H}_t$.

As an example of possible future information available to the market we might consider $\mathscr{G}_t = \mathscr{F}_t \vee \sigma(\mathbb{1}_{\{S_{T_\Upsilon} \geq K\}})$. For this threshold information we know the value

[1] We remark that the spot price of electricity is not a traded asset and thus its discounted value needs not be a martingale under the risk-neutral measure. Hence, all measures $\mathbb{Q}$ equivalent to the real-world measure $\mathbb{P}$ are possible candidates.

of the underlying at time T_Υ will be larger than some constant K but we do not know the precise value.

Having in mind our main examples (the Moratorium and the introduction of the CO_2 certificates) a more realistic approach than information given only in T_Υ would be additional information about the spot at a number of future time points. We remark that the calculations of Section 4 can also be conducted for this multiple information case. Still, in this report we will concentrate on the single information case to keep things as simple as possible.

Now we can define the information premium properly:

Definition 4. Information Premium. Let $\mathscr{G}_t$ be the market filtration with extra information at T_Υ. Then the information premium is defined as

$$I_{\mathscr{G}}^{\mathbb{Q}}(t,T;T_\Upsilon) = F_{\mathscr{G}}^{\mathbb{Q}}(t,T) - F_{\mathscr{F}}^{\mathbb{Q}}(t,T) \tag{6}$$

i.e. the difference between the forward prices under the market and the historical filtration.

In the following we will assume that all market participants work with the filtration $\mathscr{G}$. This implies that instead of assuming observed forward prices equal $\mathbb{E}^{\mathbb{Q}}[S_T|\mathscr{F}_t]$ forward prices are calculated by market participants as $\mathbb{E}^{\mathbb{Q}}[S_T|\mathscr{G}_t]$. In other words, we assume that under additional information traders price electricity forwards according to

$$F_{\mathscr{G}}^{\mathbb{Q}}(t,T) = F_{\mathscr{F}}^{\mathbb{Q}}(t,T) + I_{\mathscr{G}}^{\mathbb{Q}}(t,T;T_\Upsilon) \tag{7}$$

i.e. the traditional forward price adjusted by the information premium.

Lemma 1 provides the most important property of the information premium.

Lemma 1. ***Orthogonality of the information premium.*** *The information premium is the residual when projecting the forward price under $\mathscr{G}_t$ onto the space $L^2(\mathscr{F}_t;\mathbb{Q})$. In other words,*

$$\mathbb{E}^{\mathbb{Q}}[I_{\mathscr{G}}^{\mathbb{Q}}(t,T)|\mathscr{F}_t] = 0 \tag{8}$$

Proof. From Definition 4, the fact that $\mathscr{F}_t \subseteq \mathscr{G}_t$ and the tower property the result follows straightforwardly.

The consequence of this lemma is that the information premium cannot be attained by a measure change (the general approach in Financial Mathematics and the method used frequently to deduce the risk premium). For the difficulties this causes, particularly in empirical investigations, the reader is referred to [5].

2.2 Enlargement of filtration

Itō [23] initiated the theory of enlargement of filtration and provided a first theorem. Most results have since been proposed by French mathematicians, especially in the

1970s and 1980s, for example Jeulin, Yor or Jacod [24, 26, 27]. A comprehensive introduction is provided in Protter's book [32, Chapter VI] but also in Amendinger's thesis [3]. The most important application of the theory in finance is modelling stock markets with insider traders. We will discuss the corresponding literature in Section 3.1. Another application is default risk, discussed for example by Jeanblanc, Yor and Chesney in their recent book [25, Chapter 7].

Generally, using the notation from Definition 3, we want to know whether a $\mathscr{F}$-semimartingale remains a semimartingale under $\mathscr{G}$. If yes, we want to identify its martingale decomposition under $\mathscr{G}$.

The answer to the first question is yes if some conditions are satisfied (we refer to [32] for details). For the second question we are searching for a $\mathscr{G}$-measurable process $\mu_t^{\mathscr{G}}$ such that for an $\mathscr{F}$-martingale W and a $\mathscr{G}$-martingale ξ we have

$$\xi_t = W_t - \int_0^t \mu_s^{\mathscr{G}} ds \tag{9}$$

In the context of the information premium we will call $\mu_t^{\mathscr{G}}$ the information drift. Note that $\mu_t^{\mathscr{G}}$ is $\mathscr{G}_t$-adapted, so that we can attain it by changing measure under $\mathscr{G}$. Under $\mathscr{F}$ this is not possible (this is Lemma 1 stated differently). There are various ways to calculate the information drift: one can adapt Itō's first theorem to provide $\mu_t^{\mathscr{G}}$ for enlarging a Lévy process L_t by incomplete knowledge of its future value L_{T_Υ}, $t < T_\Upsilon$ (amongst others, this result is provided in [32] and [5]). Enlarging a Brownian motion by a more general random variable (usually called G) can be done using Yor's method and Jacod's criterion. This includes the important cases of enlargement by functionals of the Brownian motion (such as a spot price process). Finally, Imkeller uses Malliavin calculus and introduces another method for this case (as discussed in [20, 21]). We will show how to calculate the information drift in Section 4. Still, for Section 3, we will work with the general form $\mu_t^{\mathscr{G}}$.

3 Electricity Options

In this section we will examine the problems and modifications that arise when pricing options (on forwards) under the historical filtration $\mathscr{F}$ and the (enlarged) market filtration $\mathscr{G}$.

In order to find closed-form solutions we will consider a standard Gaussian Ornstein-Uhlenbeck process X_t as our spot price model, in other words $S_t = X_t$. This is the base signal only of the arithmetic spot price model used in [7, 5]. Additional information about this part of the spot model is suitable to analyse the two main examples mentioned in the introduction (especially when considering medium time horizons (i.e. less than six months) as in [5]). For the Moratorium $\mathscr{G}$ would include extra information from 14 March 2011 onwards. Furthermore, we remark that the following analysis would not change for $S_t = \Lambda_t + X_t$ with Λ_t being a seasonality function and indeed we will consider the (stylised) case $\Lambda_t = \mu$ (μ a constant) in Section 5.

Before we begin calculations, though, we will try to relate the existing results from the literature on insider trading to our electricity markets. In particular, we need to identify carefully traded assets as well as non-traded objects.

3.1 Assets and insider trading

In the context of modelling insider trading on stock exchanges, the technique of enlargement of filtrations has been applied in a variety of research papers: Examples are Karatzas and Pikovsky [30], Imkeller in [22], [21] and [20], Ankirchner [4], Amendinger [3], Biagini and Øksendal [10], Hu and Øksendal [19], Elliot, Geman and Korkie [17]. The general idea in all these publications is that the normal market trader's flow of information is modelled by the historical filtration whereas an insider's flow of information is given by an enlarged filtration. Most papers above also consider the utility of both types of investors rather than calculating specifically prices of contingent claims. The reason for this is a result (proven very generally in [3] for example) stating that for $\mathscr{F}_T$-measurable payoffs both investors assign the same value to options. Basically, enlarging the filtration changes drift terms and not volatilities. Thus, due to the $\mathscr{G}$-measurability of these drifts they will be removed by the insider's pricing measure. The resulting risk-free dynamics of the underlying will then be the same as those of the normal trader - which in consequence will give equal option prices. Apart from this mathematical reasoning this can also be justified economically. Stocks are conventional (and, in particular, storable) assets. It is well known from classical Financial Mathematics that (in a complete market setup, cf. [11]) normal derivatives can be perfectly replicated by the uninformed trader using only basic assets (for example using a delta-hedge). Thus, prices assigned by both types of investors must coincide as their hedge-portfolios do.

We are facing a different situation when the underlying is electricity. The spot is non-storable and thus not an asset in the classical sense as it is not tradeable. This poses a number of questions when trying to price forwards and options on forwards. For example, one might ask, whether the results from the literature can be translated, i.e. that options have identical prices under both filtrations. In the end, as the spot is not tradeable, one cannot follow the traditional argument and compare hedges. However, forwards are traded assets but then we now have two versions of the forward price, one under the historical and one under the market filtration. Hence, it is difficult to assign to each filtration one type of investor (as in the insider literature) and to consider both investors coexisting on the market. If the underlying is electricity we should think of the different objects as prices under different models rather than traded assets.

Summarising, our way to interpret the objects discussed previously is as follows: the informed and the uninformed traders calculate two sets of prices for themselves, depending on their best knowledge. Our analysis consequently ignores the question of how observed market forward prices are then amalgamated from these two individual sets of prices.

3.2 Vanilla Options on Forwards with delivery period

We want to price a plain vanilla call on a forward on electricity. The option expires in T and the forward has delivery period between T_1 and T_2. Furthermore, there is relevant additional future information in T_Υ. This setup is further illustrated in Figure 1 (note that T_Υ could be any time after T, though).

Fig. 1 The setup of the time axis. T is the maturity of the option, $[T_1, T_2]$ the forward delivery period and T_Υ the time of additional information.

As mentioned above, we will assume that the spot follows a standard Gaussian Ornstein-Uhlenbeck process with constant parameters. Hence, $S_t = X_t$ where, for $t < T$

$$X_T = e^{-\alpha(T-t)} X_t + \sigma \int_t^T e^{-\alpha(T-u)} dW_u \tag{10}$$

Here, W_t is a Brownian motion, α and σ are constant parameters. If we assume forward prices are settled financially at the end of the delivery period we can show (as for example in [8], p. 29) that the $(\mathscr{F}, \mathbb{P})$-forward price is given by

$$F^{\mathbb{P}}_{\mathscr{F}}(t, T_1, T_2) = \frac{1}{T_2 - T_1}\ \mathbb{E}^{\mathbb{P}}\left[\int_{T_1}^{T_2} S_u du | \mathscr{F}_t\right] \tag{11}$$

For the spot as in equation (10) this can be calculated to be equal to

$$F^{\mathbb{P}}_{\mathscr{F}}(t, T_1, T_2) = \tfrac{1}{T_2 - T_1}\ \bar{\alpha}(t, T_1, T_2) X(t) \tag{12}$$

where

$$\bar{\alpha}(t, T_1, T_2) = \begin{cases} -\frac{1}{\alpha}\ \left(e^{-\alpha(T_2-t)} - e^{-\alpha(T_1-t)}\right) & t \le T_1 \\ -\frac{1}{\alpha}\ \left(e^{-\alpha(T_2-t)} - 1\right) & t > T_1 \end{cases}$$

Now, we can calculate the forward dynamics:

$$dF^{\mathbb{P}}_{\mathscr{F}}(t, T_1, T_2) = \tfrac{1}{T_2 - T_1}\ \left(d\bar{\alpha}(t, T_1, T_2) X_t + \bar{\alpha}(t, T_1, T_2) dX_t\right)$$

The function $\bar{\alpha}(t, T_1, T_2)$ is deterministic and we have $t < T_1$. Thus

$$d\bar{\alpha}(t,T_1,T_2) = d(-\tfrac{1}{\alpha}(e^{-\alpha(T_2-t)} - e^{-\alpha(T_1-t)})) = \alpha\bar{\alpha}(t,T_1,T_2)dt$$

Hence,

$$\begin{aligned} dF^{\mathbb{P}}_{\mathscr{F}}(t,T_1,T_2) &= \tfrac{1}{T_2-T_1}\ \left(\bar{\alpha}(t,T_1,T_2)(-\alpha X_t dt + \sigma dW_t) + \alpha\bar{\alpha}(t,T_1,T_2)X_t dt\right) \\ &= \tfrac{1}{T_2-T_1}\ \sigma\bar{\alpha}(t,T_1,T_2)dW_t \end{aligned} \tag{13}$$

Now W_t is a $(\mathscr{F},\mathbb{P})$ Brownian motion and thus the forward price is already a martingale. We can integrate and get

$$F^{\mathbb{P}}_{\mathscr{F}}(T,T_1,T_2) = F^{\mathbb{P}}_{\mathscr{F}}(t,T_1,T_2) + \frac{1}{T_2-T_1}\ \sigma\int_t^T \bar{\alpha}(s,T_1,T_2)dW_s \tag{14}$$

The electricity market is incomplete and we can choose our risk-neutral pricing measure; for simplicity we will use $\mathbb{Q} = \mathbb{P}$.

Starting with formula (13), we rewrite the forward dynamics under the enlarged market filtration $\mathscr{G}$ in terms of the information drift $\mu^{\mathscr{G}}_t$ as given by equation (9):

$$\begin{aligned} dF^{\mathbb{P}}_{\mathscr{G}}(t,T_1,T_2) &= \tfrac{1}{T_2-T_1}\ \sigma\bar{\alpha}(t,T_1,T_2)\ d(\xi_t + \int_0^t \mu^{\mathscr{G}}_s ds) \\ &= \tfrac{1}{T_2-T_1}\ \left(\sigma\bar{\alpha}(t,T_1,T_2)d\xi_t + \sigma\bar{\alpha}(t,T_1,T_2)\mu^{\mathscr{G}}_t dt\right) \end{aligned} \tag{15}$$

So,

$$\begin{aligned} &F^{\mathbb{P}}_{\mathscr{G}}(T,T_1,T_2) \\ &\quad = F^{\mathbb{P}}_{\mathscr{G}}(t,T_1,T_2) + \frac{\sigma}{T_2-T_1}\left(\int_t^T \bar{\alpha}(s,T_1,T_2)d\xi_t + \int_t^T \bar{\alpha}(s,T_1,T_2)\mu^{\mathscr{G}}_s ds\right) \end{aligned} \tag{16}$$

This is, again, a $(\mathscr{G},\mathbb{P})$ semimartingale. The dt terms are $\mathscr{G}_t$-measurable; thus we can change measure to obtain martingale dynamics under $\mathscr{G}$ and a new measure $\tilde{\mathbb{P}}$. This connection was discovered by Protter in his note [31] and notation in the following will be similar to that used there. We define new processes

$$M_t = \int_0^t (-\mu^{\mathscr{G}}_s)d\xi_s$$
$$N_t = 1 + \int_0^t N_s dM_s$$

Thus, process N_t is an exponential martingale, and, one has the well-known solution

$$N_t = N_s \exp\left(-\int_s^t \tfrac{1}{2}(\mu^{\mathscr{G}}_u)^2 du - \int_s^t \mu^{\mathscr{G}}_u d\xi_u\right)$$

As N_t has expectation one (i.e. is in $L^1(\mathscr{G},\mathbb{P})$), we can now apply the Girsanov-Meyer theorem (see [32] or [15]) with $\frac{d\tilde{\mathbb{P}}}{d\mathbb{P}}|_{\mathscr{G}_t} = N_t$ or $\frac{d\mathbb{P}}{d\tilde{\mathbb{P}}}|_{\mathscr{G}_t} = N_t^{-1}$, respectively. The theorem states that the $\tilde{\mathbb{P}}$-decomposition of the Brownian motion ξ_t is

$$\xi_t = \left(\xi_t - \int_0^t \frac{1}{N_s}\, d < N, \xi >_s\right) + \int_0^t \frac{1}{N_s}\, d < N, \xi >_s$$

Calculating the integral yields

$$\begin{aligned}
\int_0^t \frac{1}{N_s}\, d < N, \xi >_s &= \int_0^t \frac{1}{N_s}\, d < \int_0^{\cdot} (-N_u \mu_u^{\mathscr{G}}) d\xi_u, \int_0^{\cdot} d\xi_u >_s \\
&= \int_0^t \frac{1}{N_s}\, d\left(\int_0^s (-N_u \mu_u^{\mathscr{G}}) du\right) \\
&= \int_0^t \frac{1}{N_s} (-N_s \mu_s^{\mathscr{G}}) ds \\
&= \int_0^t -\mu_s^{\mathscr{G}} ds
\end{aligned}$$

so that under $(\mathscr{G}, \tilde{\mathbb{P}})$ we have

$$\xi_t = \left(\xi_t + \int_0^t \mu_s^{\mathscr{G}} ds\right) - \int_0^t \mu_s^{\mathscr{G}} ds = W_t - \int_0^t \mu_s^{\mathscr{G}} ds$$

This means that the original $(\mathscr{F}, \mathbb{P})$-Brownian motion W_t is also a Brownian motion under $(\mathscr{G}, \tilde{\mathbb{P}})$, and consequently, rewriting equation (15), the forward dynamics under $(\mathscr{G}, \tilde{\mathbb{P}})$ are

$$\begin{aligned}
dF_{\mathscr{G}}^{\tilde{\mathbb{P}}}(t, T_1, T_2) &= \tfrac{1}{T_2 - T_1} \sigma \bar{\alpha}(t, T_1, T_2) d\left(W_t - \int_0^t \mu_s^{\mathscr{G}} ds\right) + \tfrac{1}{T_2 - T_1} \sigma \bar{\alpha}(t, T_1, T_2) \mu_t^{\mathscr{G}} dt \\
&= \tfrac{1}{T_2 - T_1}\, \sigma \bar{\alpha}(t, T_1, T_2) dW_t^{\mathscr{G}, \tilde{\mathbb{P}}} \qquad (17)
\end{aligned}$$

Hence, the forward price is a martingale. Integrating,

$$F_{\mathscr{G}}^{\tilde{\mathbb{P}}}(T, T_1, T_2) = F_{\mathscr{G}}^{\tilde{\mathbb{P}}}(t, T_1, T_2) + \frac{1}{T_2 - T_1}\, \sigma \int_t^T \bar{\alpha}(t, T_1, T_2) dW_s^{\mathscr{G}, \tilde{\mathbb{P}}} \qquad (18)$$

In order to price options we need the distribution of the forward price. For both filtrations, it is conditionally normally distributed. We calculate the first two moments under $(\mathscr{F}, \mathbb{P})$:

$$\mathbb{E}[F_{\mathscr{F}}^{\mathbb{P}}(T, T_1, T_2) | \mathscr{F}_t] = F_{\mathscr{F}}^{\mathbb{P}}(t, T_1, T_2) \qquad (19)$$

and using Itō's isometry for the variance,

$$
\begin{aligned}
Var(F_{\mathscr{F}}^{\mathbb{P}}(T,T_1,T_2)|\mathscr{F}_t) &= \frac{1}{(T_2-T_1)^2}\ \sigma^2 \int_t^T \bar{\alpha}^2(t,T_1,T_2)ds \\
&= \frac{\sigma^2}{(T_2-T_1)^2}\ \frac{1}{\alpha^2}\int_t^T (e^{-2\alpha(T_2-s)} - e^{-\alpha(T_2-s)}e^{-\alpha(T_1-s)} + e^{-2\alpha(T_1-s)})ds \\
&= \frac{\sigma^2}{(T_2-T_1)^2}\ \frac{1}{\alpha^2}\Big(\tfrac{1}{2\alpha}\Big(e^{-2\alpha(T_2-T)} - e^{-2\alpha(T_2-t)} + e^{-2\alpha(T_1-T)} - e^{-2\alpha(T_1-t)}\Big) \\
&\qquad\qquad -2\tfrac{1}{2\alpha}\Big(e^{-\alpha(T_2+T_1-2T)} - e^{-\alpha(T_2+T_1-2t)}\Big)\Big) \\
&= \frac{\sigma^2}{(T_2-T_1)^2}\ \frac{1}{\alpha^3}\Big(\tfrac{1}{2}\Big(e^{-2\alpha(T_2-T)} - e^{-2\alpha(T_2-t)} + e^{-2\alpha(T_1-T)} - e^{-2\alpha(T_1-t)}\Big) \\
&\qquad\qquad - \Big(e^{-\alpha(T_2+T_1-2T)} - e^{-\alpha(T_2+T_1-2t)}\Big)\Big) \\
&= \Sigma^2(t,T,T_1,T_2)
\end{aligned}
$$

Under $\mathscr{G}$ and corresponding pricing measure $\tilde{\mathbb{P}}$ the first moments are given by

$$
\begin{aligned}
\mathbb{E}[F_{\mathscr{G}}^{\tilde{\mathbb{P}}}(T,T_1,T_2)|\mathscr{G}_t] &= F_{\mathscr{G}}^{\tilde{\mathbb{P}}}(t,T_1,T_2) \\
Var(F_{\mathscr{G}}^{\tilde{\mathbb{P}}}(T,T_1,T_2)|\mathscr{G}_t) &= \Sigma^2(t,T,T_1,T_2)
\end{aligned} \tag{20}
$$

so that only start values are modified and variances remain unchanged. Now we have the ingredients to calculate options on futures under the two filtrations. The crucial difference between the insider literature and our analysis is that although we replicate the result that the underlying has the same dynamics under both filtrations we have different starting values in t. The trader using the historical filtration will price his or her option using the traditional forward price in t whereas the informed trader will include his or her future knowledge. Of course, this will have a huge impact on the risk valuation of these options.

3.2.1 Vanilla Call under $\mathscr{F}$ on an $\mathscr{F}$-forward

This is the standard case known from the literature. Our starting point is the risk-neutral-valuation-formula, and we will assume $r = 0$ in the following:

$$
C_{\mathscr{F}}(t,T,F_{\mathscr{F}}(T,T_1,T_2,K)) = \mathbb{E}^{\mathbb{Q}}[(F_{\mathscr{F}}(T,T_1,T_2)-K)^+|\mathscr{F}_t]
$$

Note that $\mathbb{Q} = \mathbb{P}$ because the forward is already a martingale under $\mathbb{P}$. Introducing an auxiliary function

$$
d_1^{\mathscr{F}} = \frac{F_{\mathscr{F}}^{\mathbb{P}}(t,T_1,T_2)-K}{\Sigma(t,T,T_1,T_2)} \tag{21}
$$

as well as a standard normal random variable Z, we rearrange equation (14)

$$\mathbb{E}[(F_{\mathscr{F}}^{\mathbb{P}}(T,T_1,T_2)-K)^+|\mathscr{F}_t] = \mathbb{E}[(F_{\mathscr{F}}^{\mathbb{P}}(t,T_1,T_2)-K+\Sigma(t,T,T_1,T_2)Z)^+|\mathscr{F}_t]$$

Hence, we are in the classical Bachelier setup.

Theorem 1. Option Price under the historical filtration. *The price at t of a Vanilla call option with maturity T and strike K under filtration $\mathscr{F}$ on an electricity forward priced under $\mathscr{F}$ with delivery period in $[T_1,T_2]$ is given by*

$$C_{\mathscr{F}}(t,T,F_{\mathscr{F}}(T,T_1,T_2,K)) = (F_{\mathscr{F}}^{\mathbb{P}}(t,T_1,T_2)-K)\Phi(d_1^{\mathscr{F}})+\Sigma\phi(d_1^{\mathscr{F}}) \tag{22}$$

where $\phi(\cdot),\Phi(\cdot)$ denote the standard-normal density and distribution and $d_1^{\mathscr{F}}$ is defined as in equation (21)*.*

Proof. Straightforward calculations.

Next, we will consider option prices as calculated by a trader taking additional future information into consideration.

3.2.2 Vanilla Call under $\mathscr{G}$ on a $\mathscr{G}$-forward

The risk-neutral valuation formula in this setting is

$$C_{\mathscr{G}}(t,T,F_{\mathscr{G}}(T,T_1,T_2),K) = \mathbb{E}^{\mathbb{Q}}[(F_{\mathscr{G}}(T,T_1,T_2)-K)^+|\mathscr{G}_t]$$

We found that the $\mathscr{G}$-Forward was a martingale under the measure $\tilde{\mathbb{P}}$, so this is our pricing measure. Again, the forward is conditionally normal with first moment given by (20) and second moment Σ as before. As in equation (21), we define

$$d_1^{\mathscr{G}} = \frac{F_{\mathscr{G}}^{\tilde{\mathbb{P}}}(t,T_1,T_2)-K}{\Sigma(t,T,T_1,T_2)}$$

We can then state

Theorem 2. Option Price under the market filtration. *The price at t of a Vanilla call option with maturity T and strike K under filtration $\mathscr{G}$ on an electricity forward priced under $\mathscr{G}$ with delivery period in $[T_1,T_2]$ is given by*

$$C_{\mathscr{G}}(t,T,F_{\mathscr{F}}(T,T_1,T_2,K)) = (F_{\mathscr{G}}^{\tilde{\mathbb{P}}}(t,T_1,T_2)-K)\Phi(d_1^{\mathscr{G}})+\Sigma\phi(d_1^{\mathscr{G}}) \tag{23}$$

where $\phi(\cdot),\Phi(\cdot)$ denote the standard-normal density and distribution and $d_2^{\mathscr{G}}$ is defined as in equation (23)*.*

Proof. As in Theorem 1.

We remark that the pricing formulae of Theorem 1 and Theorem 2 are identical except for the forward prices.

4 Calculating the Information Premium

In this section we will show how to calculate the information premium for our simple spot model in different situations. The additional information we consider first will be some knowledge about the value of the spot at some future time point T_Υ, i.e.

$$\mathscr{G}_t \subseteq \mathscr{H}_t = \mathscr{F}_t \vee \sigma(S_{T_\Upsilon}) = \mathscr{F}_t \vee \sigma(X_{T_\Upsilon})$$

We have trivially that

$$\mathscr{F}_t \vee \sigma(X_{T_\Upsilon}) = \mathscr{F}_t \vee \sigma\left(\int_t^{T_\Upsilon} e^{-\alpha(T_\Upsilon - s)} dW_s\right)$$

so we are enlarging by a normally distributed random variable. We will call

$$G = \int_0^{T_\Upsilon} e^{-\alpha(T_\Upsilon - s)} dW_s$$

Let further

$$\begin{aligned} m_t &= \int_0^t e^{-\alpha(T_\Upsilon - s)} dW_s \\ s_t &= Var(m_t) = \tfrac{1}{2\alpha}(e^{-2\alpha(T_\Upsilon - t)} - e^{-2\alpha T_\Upsilon}) \end{aligned}$$

and

$$\begin{aligned} P^G(dl) &= \mathbb{P}(G \in dl) = \tfrac{1}{\sqrt{2\pi s_{T_\Upsilon}^2}} \exp(-\tfrac{1}{2}\tfrac{l^2}{s_{T_\Upsilon}^2}) dl \\ P_t^G(dl) &= \mathbb{P}(G \in dl | \mathscr{F}_t) = \tfrac{1}{\sqrt{2\pi(s_{T_\Upsilon}^2 - s_t^2)}} \exp(-\tfrac{1}{2}\tfrac{(l-m_t)^2}{s_{T_\Upsilon}^2 - s_t^2}) dl \end{aligned}$$

Simplified, Jacod's criterion (see [24]) says that if $P_t^G(dl) = p_t(l)P^G(dl)$ for some $p_t(l)$ then the $\mathscr{G}$-decomposition of the $\mathscr{G}$-Brownian motion ξ_t is

$$\xi_t = W_t - \int_0^t \frac{d < p.(l), W >_s}{p_s(l)}$$

in other words

$$\mu_t^{\mathscr{G}} = \frac{d < p.(l), W >_t}{p_t(l)}$$

Calculating p_t in this case is cumbersome (it involves a lengthy application of Itō's theorem). Hence, we will use Imkeller's method. Imkeller proves that (under certain conditions, see [21])

$$\mu_t^{\mathscr{G}} = \frac{d\mathscr{D}_t P_t^G(\cdot, dl)}{dP_t^G(\cdot, dl)}(l)$$

where $\mathscr{D}$ denotes the Malliavin derivative. For more details on Malliavin calculus we refer to [29]. We can return to our example and calculate

$$\begin{aligned}
\mathscr{D}_t P_t^G &= \mathscr{D}_t\big(\tfrac{1}{\sqrt{2\pi(s_{T_\Upsilon}^2 - s_t^2)}} \exp(-\tfrac{1}{2}\tfrac{(l-m_t)^2}{s_{T_\Upsilon}^2 - s_t^2})\big) \\
&= P_t^G \mathscr{D}_t(-\tfrac{1}{2}\tfrac{(l-m_t)^2}{s_{T_\Upsilon}^2 - s_t^2}) \\
&= P_t^G \tfrac{l-m_t}{s_{T_\Upsilon}^2 - s_t^2} \mathscr{D}_t(m_t) \\
&= P_t^G \tfrac{l-m_t}{s_{T_\Upsilon}^2 - s_t^2} e^{-\alpha(T_\Upsilon - t)}
\end{aligned}$$

Here, we used the Malliavin chain rule and the fact that m_t is a simple Wiener polynomial. Dividing by P_t^G allows to write down the decomposition

$$\begin{aligned}
W_t &= \xi_t + \int_0^t \frac{l-m_s}{s_{T_\Upsilon}^2 - s_s^2} e^{-\alpha(T_\Upsilon - s)} ds = \xi_t + \int_0^t \frac{\int_s^{T_\Upsilon} e^{-\alpha(T_\Upsilon - u)} dW_u}{\frac{1}{2\alpha}(1 - e^{-2\alpha(T_\Upsilon - s)})} e^{-\alpha(T_\Upsilon - s)} ds \\
&= \xi_t + \int_0^t \left(\int_s^{T_\Upsilon} e^{\alpha u} dW_u\right) \underbrace{\frac{2\alpha e^{\alpha s}}{e^{2\alpha T_\Upsilon} - e^{2\alpha s}}}_{=a(s)} ds
\end{aligned} \tag{24}$$

Also, this can be written in terms of the process X_t

$$W_t = \xi_t + \int_0^t \tfrac{1}{\sigma} e^{\alpha T_\Upsilon}(X_{T_\Upsilon} - e^{-\alpha(T_\Upsilon - s)} X_s) \frac{2\alpha e^{\alpha s}}{e^{2\alpha T_\Upsilon} - e^{2\alpha s}} ds \tag{25}$$

Using this decomposition we can then calculate the information premium by substituting into the definition.

Theorem 3. The Information Premium. *Let $0 \le t \le T_1 < T_2 \le T_\Upsilon$. Then the information premium with delivery period is given by*

$$I_{\mathscr{G}}(t, T_1, T_2; T_\Upsilon) = \frac{1}{T_2 - T_1} \frac{1}{\alpha} \left(\frac{e^{2\alpha T_2} + e^{2\alpha t}}{e^{\alpha T_2}} - \frac{e^{2\alpha T_1} + e^{2\alpha t}}{e^{\alpha T_1}} \right) \frac{e^{\alpha T_\Upsilon} \mathbb{E}[X_{T_\Upsilon} | \mathscr{G}_t] - e^{\alpha t} X_t}{e^{2\alpha T_\Upsilon} - e^{2\alpha t}} \tag{26}$$

Proof. The information premium with delivery period is defined as

$$I_{\mathscr{G}}(t, T_1, T_2; T_\Upsilon) = F_{\mathscr{G}}(t, T_1, T_2) - F_{\mathscr{F}}(t, T_1, T_2)$$

Looking at formulae (10) and (11) we realise that terms including X_t cancel. The $\mathscr{F}$-expectation of the Itō integral is zero. Hence, we only have the $\mathscr{G}$-expectation of the Itō integral, and we substitute the decomposition as in equation (24) as follows:

$$
\begin{aligned}
I_{\mathscr{G}}(t,T_1,T_2;T_\Upsilon) &= \frac{1}{T_2-T_1}\,\mathbb{E}\left[\int_{T_1}^{T_2}\int_t^u \sigma e^{-\alpha(u-s)}dW_s du|\mathscr{G}_t\right]\\
&= \frac{\sigma}{T_2-T_1}\,\mathbb{E}\left[\int_{T_1}^{T_2}\int_t^u e^{-\alpha(u-s)}\left(a(s)\int_s^{T_\Upsilon} e^{\alpha v}dW_v\right)ds\,du|\mathscr{G}_t\right]\\
&= \frac{\sigma}{T_2-T_1}\,\int_{T_1}^{T_2}\int_t^u e^{-\alpha(u-s)}a(s)\mathbb{E}\left[\int_s^{T_\Upsilon} e^{\alpha v}dW_v|\mathscr{G}_t\right]ds\,du
\end{aligned}
$$

Now we apply Theorem A.1 with $f(u)=e^{\alpha u}$ and $g(s)=a(s)$. Solving the resulting integral equation yields

$$
\begin{aligned}
&I_{\mathscr{G}}(t,T_1,T_2;T_\Upsilon)\\
&\quad= \frac{\sigma}{T_2-T_1}\int_{T_1}^{T_2}\int_t^u e^{-\alpha(u-s)}a(s)\frac{e^{2\alpha T_\Upsilon}-e^{2\alpha s}}{e^{2\alpha T_\Upsilon}-e^{2\alpha t}}\mathbb{E}\left[\int_t^{T_\Upsilon} e^{\alpha v}dW_v|\mathscr{G}_t\right]ds\,du\\
&\quad= \frac{\sigma}{T_2-T_1}\,\frac{\mathbb{E}\left[\int_t^{T_\Upsilon} e^{\alpha v}dW_v|\mathscr{G}_t\right]}{e^{2\alpha T_\Upsilon}-e^{2\alpha t}}\int_{T_1}^{T_2}\int_t^u 2\alpha e^{-\alpha u}e^{2\alpha s}ds\,du\\
&\quad= \frac{1}{T_2-T_1}\,\frac{e^{\alpha T_\Upsilon}\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t]-e^{\alpha t}X_t}{e^{2\alpha T_\Upsilon}-e^{2\alpha t}}\int_{T_1}^{T_2} e^{\alpha u}-e^{2\alpha t}e^{-\alpha u}du
\end{aligned}
$$

and evaluating the last integral yields the result.

Using this formula, we can now find numerical values for the information premium and thus for option prices on the corresponding forwards.

So far, we have, for technical reasons, assumed that T_Υ was larger than T_2. Under the forward-pricing model $(S_t,\mathscr{G}_t)$ the whole evolution of the spot is changed and all forward prices are adjusted, without regard to whether the future information is located on the time axis before, during or after the maturity of the contract. Thus, it is perfectly sound to take into consideration estimated future information in terms of timing. Still, we now face a problem from a modelling perspective: for the CO_2 scenario mentioned in the introduction our model will result in a positive information premium for December 07 although the second phase of the EU ETS began on January 1. We remark, though, that this effect is negligible, in particular due to the mean-reversion rates observable on the market (we refer to [5] for a discussion) and even more so when considering longer delivery periods.

Technically, it is relatively easy to adapt the result of Theorem 3 to other orderings of time points.

Lemma 2. The Information Premium (information before delivery period). *For $0\le t<T_\Upsilon\le T_1<T_2$, i.e. extra information before the delivery period, the information premium is given by*

$$
I_{\mathscr{G}}(t,T_1,T_2;T_\Upsilon)=\frac{1}{T_2-T_1}\,\bar{\alpha}(T_\Upsilon,T_1,T_2)\left(\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t]-e^{-\alpha(T_\Upsilon-t)X_t}\right) \tag{27}
$$

Proof. One uses the definition and decomposes

$$
\begin{aligned}
I_{\mathscr{G}}(t,T_1,T_2;T_\Upsilon) &= \frac{1}{T_2-T_1}\left(\mathbb{E}\left[\int_{T_1}^{T_2} X_u du|\mathscr{G}_t\right] - \mathbb{E}\left[\int_{T_1}^{T_2} X_u du|\mathscr{F}_t\right]\right)\\
&= \frac{1}{T_2-T_1}\left(\mathbb{E}\left[\int_{T_1}^{T_2}\left(e^{-\beta(u-T_\Upsilon)}X_{T_\Upsilon} + \int_{T_\Upsilon}^{u} e^{-\alpha(u-s)}dW_s\right)du|\mathscr{G}_t\right] - \bar{\alpha}(t,T_1,T_2)X_t\right)
\end{aligned}
$$

Now, the filtrations satisfy $\mathscr{G}_t \subseteq \mathscr{F}_{T_\Upsilon}$ so

$$
\begin{aligned}
&I_{\mathscr{G}}(t,T_1,T_2;T_\Upsilon)\\
&= \frac{1}{T_2-T_1}\left(\mathbb{E}\left[\int_{T_1}^{T_2}\left(e^{-\beta(u-T_\Upsilon)}X_{T_\Upsilon} + \mathbb{E}\left[\int_{T_\Upsilon}^{u} e^{-\alpha(u-s)}dW_s|\mathscr{F}_{T_\upsilon}\right]\right)du|\mathscr{G}_t\right]\right.\\
&\qquad\qquad \left. - \bar{\alpha}(t,T_1,T_2)X_t\right)\\
&= \frac{1}{T_2-T_1}\left(\bar{\alpha}(T_\Upsilon,T_1,T_2)\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t] - \bar{\alpha}(t,T_1,T_2)X_t\right)\\
&= \frac{1}{T_2-T_1}\,\bar{\alpha}(T_\Upsilon,T_1,T_2)\left(\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t] - e^{-\alpha(T_\Upsilon-t)}X_t\right)
\end{aligned}
$$

which is exactly the claim.

The case for which the extra information is in between T_1 and T_2 is a mixed case of Theorem 3 and Lemma 2.

Lemma 3. The Information Premium (information during delivery period). *For $0 \le t \le T_1 < T_\Upsilon < T_2$, i.e. extra information during the delivery period, the information premium is given by*

$$
I_{\mathscr{G}}(t,T_1,T_2;T_\Upsilon) = \frac{1}{T_2-T_1}\left((T_\Upsilon-T_1)\underbrace{I_{\mathscr{G}}(t,T_1,T_\Upsilon;T_\Upsilon)}_{Theorem 3} + (T_2-T_\Upsilon)\underbrace{I_{\mathscr{G}}(t,T_\Upsilon,T_2;T_\Upsilon)}_{Lemma 2}\right) \tag{28}
$$

Proof. One uses the definition of the information premium and separates the two cases by splitting the integrals in the expectations.

We can recover the information premium without delivery period (denoted by $I_{\mathscr{G}}(t,T_1;T_\Upsilon)$, calculated in [7]) by taking limits:

Lemma 4. *In the situation of Theorem 3 we have*

$$
\lim_{T_2\to T_1} I_{\mathscr{G}}(t,T_1,T_2;T_\Upsilon) = I_{\mathscr{G}}(t,T_1;T_\Upsilon)
$$

Proof. We need to evaluate

$$
\begin{aligned}
&\lim_{T_2\to T_1} I_{\mathscr{G}}(t,T_1,T_2;T_\Upsilon)\\
&= \lim_{T_2\to T_1}\frac{1}{T_2-T_1}\,\frac{1}{\alpha}\left(\frac{e^{2\alpha T_2}+e^{2\alpha t}}{e^{\alpha T_2}} - \frac{e^{2\alpha T_1}+e^{2\alpha t}}{e^{\alpha T_1}}\right)\frac{e^{\alpha T_\Upsilon}\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t] - e^{\alpha t}X_t}{e^{2\alpha T_\Upsilon} - e^{2\alpha t}}
\end{aligned}
$$

We use L'Hospital's rule

$$\begin{aligned}
\dots &= \frac{e^{\alpha T_\Upsilon}\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t] - e^{\alpha t}X_t}{e^{2\alpha T_\Upsilon} - e^{2\alpha t}} \lim_{T_2 \to T_1} \frac{1}{\frac{\partial}{\partial T_2}(T_2 - T_1)} \frac{1}{\alpha} \frac{\partial}{\partial T_2} \frac{e^{2\alpha T_2} + e^{2\alpha t}}{e^{\alpha T_2}} \\
&= \frac{e^{\alpha T_\Upsilon}\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t] - e^{\alpha t}X_t}{e^{2\alpha T_\Upsilon} - e^{2\alpha t}} e^{-\alpha T_\Upsilon}\left(e^{2\alpha T_1} + e^{2\alpha t}\right) \\
&= I_{\mathscr{G}}(t, T_1; T_\Upsilon)
\end{aligned}$$

and this is the expression calculated in [7].

5 Discussion and stylised Examples

In this section we will present various stylised examples to illustrate the theory discussed so far. Firstly, we will assume that the spot satisfies $S_t = \mu + X_t$ (for some constant μ) and use the results of section Section 4 to analyse properties of the information premium. We will assume that market agents are given non-precise future spot information about X_{T_Υ}, i.e. we know the value of $\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t]$. Furthermore, we choose toy-parameters for α and σ which are similar to those fitted to market data. Also, for the time axis we follow the daily convention, meaning that for example $T_1 = 10, T_2 = 20$ denotes a delivery period of 20 days, starting from day ten.

Figure 2 illustrates the information premium for different values of this expectation and for moving T_Υ. Further parameters were set to $t = 0, T_1 = 20, T_2 = 30, X_0 = 0, \alpha = 0.2$ and $\sigma = 3.0$. Remembering for example formula (26) we see that with these values the sign of the premium depends only on $\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t]$, as expected. We observe a vanishing information premium for a T_Υ that is either far before the delivery or far after. If the extra information is a zero expectation of X_{T_Υ} then this does not constitute genuinely new information and the information premium becomes zero (green line). Generally, the value of the premium takes its maximum/minimum in the middle of the delivery period. This makes sense economically: depending on σ and α knowing the expected value at T_Υ gives a vague idea of spot values in the vicinity. Hence, the further away the beginning and the end of the delivery period are from T_Υ, the more of the interval around T_Υ will lie in the period. We can, for example, use the *half life* of the Ornstein-Uhlenbeck process (defined as $\frac{\ln 2}{\alpha}$) as an estimate of how many days are influenced by the additional information. For example, in the case of Figure 2 the half life is $\frac{\ln 2}{0.2} \approx 3.5$. The value of the information premium with knowledge about $T_\Upsilon = 25$ is around 3 (solid red line). We can calculate the area under the first two half lives by solving $30 = 1.5 \cdot 3.5 \cdot \mathbb{E}[X_{25}|\mathscr{G}_0]$. This gives $\mathbb{E}[X_{25}|\mathscr{G}_0] \approx 5.7$, the true value being 5.0.

The interaction between the values at X_t and X_{T_Υ} is illustrated in Figure 3. Here, the value of the information premium for different T_Υ and different α is plotted. We used the same parameters as above, except for $t = 10, X_t = 10$ and $\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t] = 5$. The two dotted lines are similar to the graphs in Figure 2. The absolute value of the

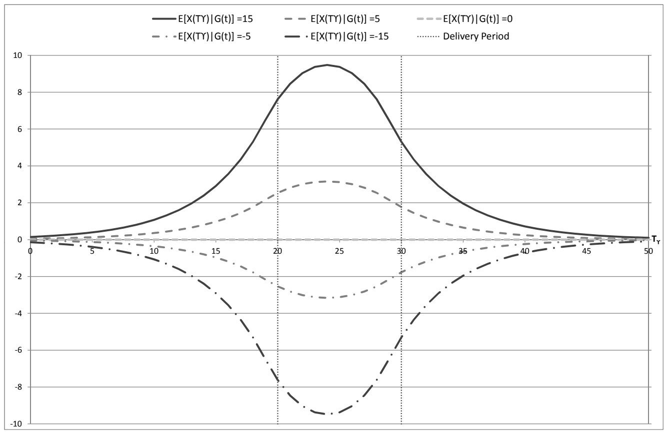

Fig. 2 The information premium over T_Υ for different values of $\mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_t]$. Other parameters are: $t = 0, T_1 = 20, T_2 = 30, X_0 = 0, \alpha = 0.2$ and $\sigma = 3.0$

premium in this case is small because a higher mean reversion implies a lesser value of information. We also see that the value of $X_{10} = 10$ does not play a role, again, due to large α. For medium values of α and small T_Υ ignorant (i.e. $\mathscr{F}$-) traders calculate a large forward price because they observe a large spot price today. The $\mathscr{G}$-traders though, know that in T_Υ (being slightly larger than t) the value will be smaller. Thus, the information premium is negative at first. Moving T_Υ further right we exhibit a change in sign of the premium. For very small α (for example the solid blue line) we also see that the impact of information lying outside of the delivery period is much bigger.

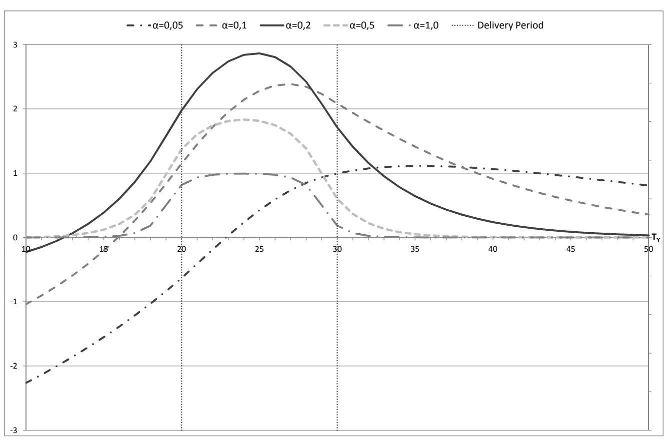

Fig. 3 The information premium over T_Υ for different values of α. Other parameters are: $t = 10, T_1 = 20, T_2 = 30, X_{10} = 10, \mathbb{E}[X_{T_\Upsilon}|\mathscr{G}_{10}] = 5$ and $\sigma = 3.0$

For different values of the volatility and the speed of mean reversion Figure 4 illustrates the value of an at-the-money European Call option under filtration $\mathscr{F}$ on the forward under $\mathscr{F}$, calculated as in theorem Theorem 1. Here, we assume $\mu = 30$. With most combinations of parameter values this option practically has zero value, the reason being the averaging effect of the delivery period (which has a length of ten days in this example). For a very low mean reversion and large volatility we find a positive value for this option.

Figure 5 shows the corresponding picture for the at-the-money option on the forward, both under the market filtration $\mathscr{G}$. The additional information is given at $T_\Upsilon = 25$ and gives the expected value of the Ornstein-Uhlenbeck process as $\mathbb{E}[X_{25}|\mathscr{G}_{10}] = 5$. Not surprisingly, the value of the option has a non-zero positive value for all combinations of α and σ. We observe the same effect as above, i.e. larger option prices for large volatility and small speed of mean reversion. Still, the increase in the option price is smoother than in the case of the historical filtration.

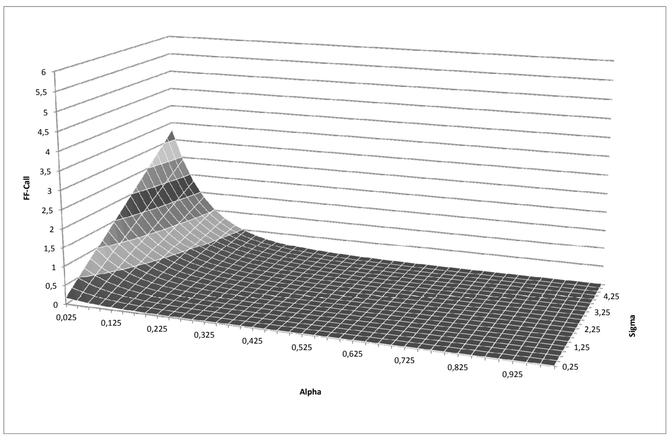

Fig. 4 Vanilla Call Price under $\mathscr{F}$ on a $\mathscr{F}$-forward. For different α and σ. Other parameters are: $t = 10, T_1 = 20, T_2 = 30, X_{10} = 0, \mu = 30$ and $K = 30$.

An example of an in-the-money option is given in Figure 6, where we assume again $\mu = 30$ and a strike of $K = 25$. This results in an almost flat price at level 5 as expected. Only for very small speeds of mean reversion and large volatility does the price increase. Figure 7 illustrates the in-the-money call under the market filtration with additional information that the Ornstein-Uhlenbeck process will be -5 in the middle of the delivery period. The price of the option is generally lower and decreasing with decreasing speed of mean reversion. This is due to the lesser significance of the future information for a higher degree of mean reversion. But the most striking feature is the fact that the option price increases again for very small α and large σ. In that case, the volatility of the spot price is no longer significantly dampened by the mean-reversion of X_t and higher volatility causes higher option prices. Hence, there are two forces effecting the option price for small α.

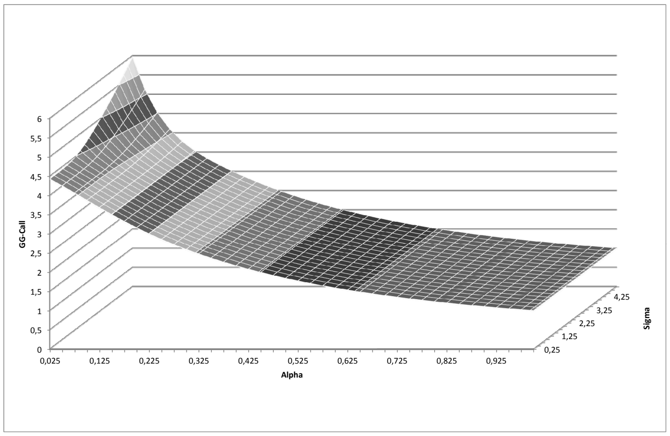

Fig. 5 Vanilla Call Price under $\mathscr{G}$ on a $\mathscr{G}$-forward. For different α and σ. Other parameters are: $t = 10, T_1 = 20, T_2 = 30, X_{10} = 0, \mathbb{E}[X_{25}|\mathscr{G}_{10}] = 5, \mu = 30$ and $K = 30$.

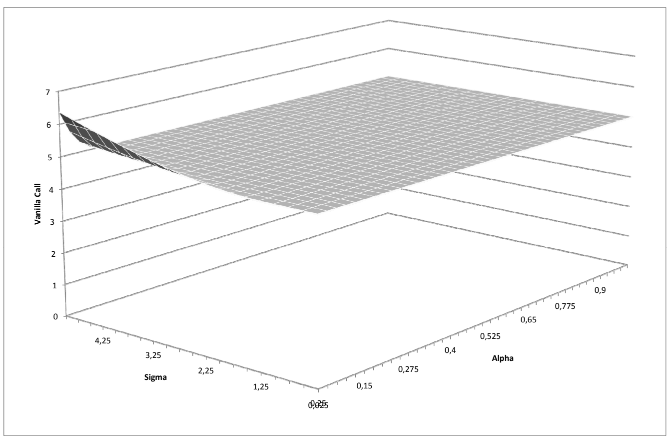

Fig. 6 Vanilla Call Price under $\mathscr{F}$ on a $\mathscr{F}$-forward. For different α and σ. Other parameters are: $t = 10, T_1 = 20, T_2 = 30, X_{10} = 0, \mu = 30$ and $K = 25$.

6 Conclusion

The special properties of electricity markets, especially that of the non-storability of the underlying commodity, lead to non-validity of the classical spot-forward relationship. This is the motivation for introducing the notion of the information premium as presented in [7]. This premium is defined as the difference between forward prices calculated under an enlarged market filtration and the traditional forward price under the historical filtration. Its very existence has recently been shown by means of a newly developed statistical test in [5].

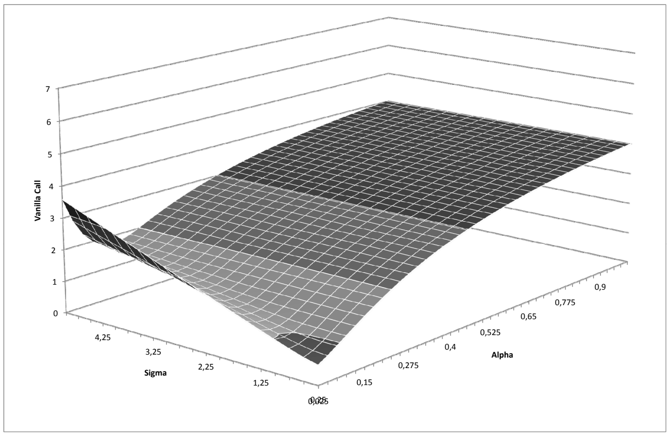

Fig. 7 Vanilla Call Price under $\mathscr{G}$ on a $\mathscr{G}$-forward. For different α and σ. Other parameters are: $t = 10, T_1 = 20, T_2 = 30, X_{10} = 0, \mathbb{E}[X_{25}|\mathscr{G}_{10}] = -5, \mu = 30$ and $K = 25$.

In this report we discussed the issue of how options can be priced in the presence of additional future information. Our starting point was the existing literature on modelling insider trading on stock markets. In a number of papers various authors have used the mathematical technique of the enlargement of filtrations to represent the extra knowledge an insider possesses. With a traditional underlying such as stocks they find that both – insider and ”normal” traders – assign the same value to contingent claims, the intuitive reason for this result being that both types of traders have access to a self-financing replicating portfolio. This is not the case for electricity markets. The underlying spot price is not a traded asset and thus, unlike the well established result, traders who take into consideration future information will come up with a different set of prices for the available financial products. In Section 3 we established formulae for vanilla call options on electricity forwards for both types of traders. To this end we utilised a very simple (Gaussian) spot model. In this case, we found that the price of an option under the market filtration is given by a Bachelier type of pricing formula, requiring only previously calculating the information premium.

Thus, in Section 4 we found explicit expressions for the information premium for forwards with delivery period and additional future information of a simple kind. We also provided the necessary established results from the enlargement of filtration. In particular we used Imkeller's method of applying Malliavin calculus.

Section 5 provided a number of stylised examples of the size and shape of the information premium and of options on futures. These matched our economic intuition.

In sum: we advocate taking relevant future information into consideration when examining energy markets. We also propose using the information premium as well as the traditional risk premium when describing the spot-forward relationship.

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Appendix

The following theorem will help us to calculate the information premium. In [7] it is proposition A.3.

Theorem A.1. *Let $L(t)$ be a Lévy process and $\mathscr{F}_t$ the historical filtration. Also, let $\mathscr{G}_t \subseteq \mathscr{H}_t = \mathscr{F}_t \vee \sigma(L(T_\Upsilon))$ be the enlarged filtration. Further, one assumes that the information drift is of the form*

$$\mu_s^{\mathscr{G}} = g(s)\mathbb{E}\left[\int_s^{T_\Upsilon} f(u)dL(u) \mid \mathscr{G}_s\right]$$

where g and f are continuous function on $[0,T_\Upsilon]$. Then one has the identity

$$\mathbb{E}\left[\int_s^{T_\Upsilon} f(u)dL(u) \mid \mathscr{G}_t\right] = \mathbb{E}\left[\int_t^{T_\Upsilon} f(u)dL(u) \mid \mathscr{G}_t\right] e^{-\int_t^s f(u)g(u)du}$$

for time points $t \le s \le T_\Upsilon$. This result is in particular true for L(t) being a simple Brownian motion.

Proof. Defining the auxiliary process Y_s as

$$Y(s) = \mathbb{E}\left[\int_s^{T_\Upsilon} f(u)dL(u) \mid \mathscr{G}_t\right]$$

gives rise to (by making use of the tower property)

$$\begin{aligned}
Y(s) &= Y(t) - \mathbb{E}\left[\int_t^s f(u)dL(u) \mid \mathscr{G}_t\right] \\
&= Y(t) - \mathbb{E}\left[\int_t^s f(u)\left(g(u)\mathbb{E}\left[\int_u^{T_\Upsilon} f(v)dL(v) \mid \mathscr{G}_u\right]\right)du \mid \mathscr{G}_t\right] \\
&= Y(t) - \int_t^s f(u)g(u)\mathbb{E}\left[\int_u^{T_\Upsilon} f(v)dL(v) \mid \mathscr{G}_t\right]du \\
&= Y(t) - \int_t^s f(u)g(u)Y(u)du
\end{aligned}$$

and the solution to this integral equation is

$$Y(s) = Y(t)e^{-\int_t^s f(u)g(u)du}$$

$$\mathbb{E}\left[\int_s^{T_T} f(u)dL(u) \mid \mathscr{G}_t\right] = \mathbb{E}\left[\int_t^{T_T} f(u)dL(u) \mid \mathscr{G}_t\right] e^{-\int_t^s f(u)g(u)du}$$

and this completes the proof.

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