Flexibility Premium in Marketable Permits*

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Abstract

We study the market for emission permits in the presence of reversible abatement measures characterized by delay in implementation. We assume that the new operating profits follow a one-dimensional geometric Brownian motion and that the company is risk-neutral. The optimal "investment" and "disinvestment" policy for reversible abatement options is evaluated under both instantaneous and Parisian criteria, nesting the model of Bar-Ilan and Strange (1996). By taking the difference between these two values at their respective optima, we derive an analytic solution of the premium for flexibility embedded in marketable permits. This extends the findings in Chao and Wilson (1993) and Zhao (2003). Numerical results are presented to illustrate the likely magnitude of the premium and how it is affected by uncertainty and delays in implementation.

Keywords: Emission Permits, Implementation Delay, Optimal Stopping, Reversible Investments.

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1 Introduction

In a pollution-constrained economy where polluting companies are subject to environmental regulations that cap their noxious emissions, each firm faces a basic choice from three main abatement alternatives: modify the production process which generates the emissions as a by-product; change the production technology; trade marketable permits.[1] This last option, also referred to as emissions trading, is a market-based measure which is currently very popular among policy makers. In a system for marketable permits, relevant companies exchange permits on the theory that trading creates economic incentives that encourage firms to minimize the costs of pollution control to society. The chief appeal of economic incentives as the regulatory device for achieving environmental standards is the potentially large cost-saving that they promise. We refer to Baumol and Oates (1988) for a complete discussion on market-based policy measures. There is now an extensive body of empirical studies that estimate the cost of achieving standards for environmental quality under existing command-and-control regulatory programs. These are typically programs under which an environmental authority prescribes the treatment procedures that are to be adopted by each source. The studies compare costs under command-and-control programs with those under a more cost-effective system of economic incentives. The results have been quite striking: they indicate that control costs under existing programs have often been several times the least-cost levels. We refer to Tietenberg (1985) for a survey on cost studies. The source of these large cost savings is the capacity of economic instruments to take advantage of large differential abatement costs across polluters. Various aspects of the emission permits market have been investigated by Misolek and Elder (1989), Hahn (1983), Bohi and Burtraw (1992) and Stavins (1995).

The aim of our paper is to develop a simple model for the permit price that can support decision-makers in relevant companies in identifying the optimal time for undertaking a reversible abatement measure, such as a modification of the production process, in the presence of implementation delays or trading permits. In other words, we evaluate the price level at which trading permits is a cheaper solution. Similar to Chao and Wilson (1993), the price of marketable permits is not simply equal to the marginal cost of abatement of pollution. In fact, we show that the price of marketable permits includes a premium representing the value of flexibility as compared to the alternative of (non-instantaneous) investments in abatement strategies that reduce the quantity of emissions. In particular, instead of limiting our analysis to irreversible and instantaneous commitments as in Chao and Wilson (1993) and Zhao (2003), we allow the company to undo the abatement measure at most one time. Further, we account for those realistic situations where a firm faces physical or technical constraints that allow the implementation of economic decisions only after a given time-interval.[2] In such situations, it is reasonable to undertake (respectively

[1] Niemeyer (1990) gives a more detailed list of abatement alternatives.

[2] This analysis differs from the "construction-lag" or "time-to-build" literature, where the lag refers to the time

undo) an abatement project only when market conditions remain favorable (respectively unfavorable). The Parisian structure defined in appendix B reflects such an optimal criterion. By solving the one-time reversible investment problem under both the instantaneous and Parisian criteria we obtain analytic solutions for optimal investment and disinvestment levels, nesting the model of Bar-Ilan and Strange (1996) which focuses on the effects of time-lags on irreversible investments. Relying on this result, we derive an analytic solution of the premium for the flexibility embedded in marketable permits, extending a result of Chao and Wilson (1993). Showing that an increase in the uncertainty of the underlying process hastens the decision to disinvest, we extend the results of Bar-Ilan and Strange (1996) to the disinvestment case. This theoretical result explains the different behavior of the premium for the flexibility of emission permits under both reversible and irreversible investment assumptions.

Xepapadeas (2001) is one of several authors who explores how instantaneous investments in abatement projects respond to environmental policies, such as marketable permits, under uncertainty and irreversible decision constraints. Xepapadeas considers a firm that at each instant of time maximizes its profit deciding about the abatement investments having optimally chosen the output production level. In this model, uncertainty is present in the form of a stochastic evolution of the output price or of the price of the marketable permits. Assuming these prices follow a geometric Brownian motion and applying dynamic control, Xepapadeas identifies a "barrier control" policy. For any given investment abatement level, the random price movements of the output product or of the marketable permit determine the optimal strategy. If such a price is above the barrier, then new investment in abatement projects is instantaneously undertaken; otherwise, the abatement level remains the same. In other words, Xepapadeas determines the continuation intervals and those intervals during which firms take irreversible and instantaneous investment decisions.

In line with Xepapadeas (2001), Zhao (2003) investigates the impact of cost-uncertainty of irreversible abatement projects on investment incentive. Following the literature on abatement investments such as Farzin et al. (1998) and Farzin and Kort (2000), Zhao develops an equilibrium model where companies are permit price takers and the dynamics of their abatement costs is stochastic. Assuming efficient permit trading, marginal abatement costs are equalized across all companies and, therefore, the equilibrium price of the permits is determined endogenously. The author derives the price of permits in analytic form by adopting a specific functional form for the investment cost function. Further, Zhao shows that the aggregate investment behavior of firms together with cost-uncertainty, determine the time-path of the price of emission permits.

between the decision to invest and the receipt of the project's first revenues (see Majd and Pindyck (1987) and Pindyck (1991, 1993)). In our case the implementation-lag measures a systematic delay that occurs before the decision to undertake, also called investment decision, or undo, also called disinvestment decision, the project effectively takes place, i.e. before the consequence of hitting the trigger price comes into play.

Aligning himself with the results in the real option literature on instantaneous investments,[3] Zhao proves that cost-uncertainty reduces the incentive to invest in irreversible projects, shifting the investment-barrier upward. Our findings in the instantaneous case confirm such a result: more uncertainty delays both investment and disinvestment decisions and thus generates more so-called inertia. Delays in implementation account for the presence of realistic technical constraints. When we introduce delays, conventional findings on the effect of the uncertainty on the investment and disinvestment are reversed. In particular, a higher volatility of the underlying process hastens both investment and disinvestment when delays force a firm to decide whether or not to undertake a decision in the near future. The reason is that in our model the option to undo the abatement project makes profit a convex function of the underlying stochastic. Therefore, in line with Bar-Ilan and Strange (1996) and in the spirit of the works of Abel (1983) and Caballero (1991), the higher the uncertainty, the higher the expected profit.

Chao and Wilson (1993) relax the assumption of perfect substitution among all abatement measures. As proved by Montgomery (1972), such a relationship holds in the absence of uncertainty about emission permit and technology costs. In particular, Chao and Wilson analyze the main pollution control measures available in the SO_2 market in U.S.: trading permits, scrubbing emissions and changing the production technology. For convenience we label these measure as short, medium and long-term abatement options. Since relevant companies face considerable uncertainty, they perceive modification of the production process and technology changes, i.e. medium and long-term abatement measures, as inferior substitutes for emission permits. As a matter of fact, technology changes are typically irreversible and expensive investments which last for decades cannot therefore be considered as equal alternatives.[4] In line with Montgomery (1972) and Rubin (1996), Chao and Wilson show that the equilibrium permit price is equal to the marginal cost of a scrubber that is installed instantaneously. This holds in a world of certainty where all state variables are fixed forever. In practice, the demand for permits varies in response to variations in the output market, as well as other developments in input costs and technologies for pollution control. In such a situation, medium and long-term measures are imperfect substitutes for emission permits implying that the permit price will sell at a higher level than the marginal cost of pollution-abatement. Characterizing the demand for permits by an exogenous stochastic process, Chao and Wilson show that permit prices that clear the market, i.e. the permits demand equals permits supply, can exceed the marginal cost of a scrubber by an amount called "option value". This premium measures the discounted expected value of the greater flexibility that emission permits provide, compared to the irreversible and instantaneous commitments required by investments in scrubbers. The sum of the marginal cost and the premium can then be correctly interpreted as a cap on the permit price. Such a value is binding only

[3]Pindyck (1991) provides a comprehensive survey of this literature.

[4]We refer to Zhao (2003) for more general discussions on irreversible abatement measures.

when the demand of emission permits is sufficiently large such that companies find it advantageous to install additional scrubbers. Though we do not model directly the demand for emission permits, we obtain analytically a premium for the flexibility of emission permits as in Chao and Wilson (1993) evaluating under which permit price conditions short and medium-term abatement measures are equivalent alternatives.

Chao and Wilson first, and Zhao some years later, framed their studies of marketable permits calling into question the principle of perfect substitution between permits and medium-term abatement measures. Assuming efficient permit trading, the two strategies must be comparable. Intuitively, trading permits and modifying production process are *equivalent* alternatives exclusively when both lead to equal pollution emissions reduction for the same total costs. In reality, the decision to undertake a reversible modification of the production process is typically characterized by a significant implementation lag which implicitly has a profound impact on the profitability of the economic decision undertaken. In fact, depending on the evolution of the new operating profits during the implementation lag, the investment or disinvestment abatement opportunity may partially lose its attractiveness. Therefore, any reversible modification of the production process, i.e. any so-called medium-term abatement measure, is perceived as an inferior substitute for marketable permits. Permits can be easily adapted to changing conditions whereas a production modification, though reversible, might be too costly if the output market demand falls over a short period of time. For these reasons, the price of marketable permits must include a premium that recognizes the value of flexibility. Moreover, the existence of implementation lags affects reversible decisions and increases further the flexibility which characterizes policy instruments like marketable permits. Chao and Wilson (1993) were the first to identify the flexibility embedded in the SO_2 emission permits in the U.S. market. However, as mentioned before, the authors concentrated their analysis on irreversible investments. It is our purpose in this paper to derive such a premium in the presence of reversible decisions and implementation delays. In doing so, we generalize the case of irreversible abatement opportunities. In determining the premium, we rely on the Parisian criterion introduced by Chesney et al. (1997) and upon the results of Gauthier (2002) and Chesney and Gauthier (2006).

2 Problem formulation: the Price for Flexibility in Marketable Permits

Consider a company which has an infinite time horizon and that, from time zero up to T, is subject to environmental regulations which impose a cap on the yearly amount of pollution emitted during the production process. To encourage firms to minimize the costs to society of pollution control, a system of marketable permits as described in Tietenberg (1985) is introduced.

The company maximizes its expected discounted pay-off flow. At any given time $(t, 0 \leq t \leq \infty)$ the firm yields an operating profit that depends on the instantaneous cash flow $h^B(Y_t)$, for some function h^B, where Y_t is a one-dimensional Itô process defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ and which drives the entire economy. Moreover, the company constantly emits α units of pollution at each instant $(t, 0 \leq t \leq \infty)$.[5] We denote by V_t^B the expected sum of the discounted operating profits from t to infinity without any environmental constraints:

$$V_t^B = \mathbb{E}_t\Big[\int_t^\infty e^{-\rho(u-t)} h^B(Y_u) du\Big], \tag{1}$$

where the discount rate ρ is constant and $\mathbb{E}_t[\cdot]$ stands for the conditional expectation $\mathbb{E}[\cdot|\mathcal{F}_t]$.
To fulfill environmental regulations the company can either undertake a modification of the production process at any time $(t, 0 \leq t \leq T)$ or purchase the necessary marketable permits at time T. In the model the decision to modify the production process is reversible but we assume it can be reversed only once. Running the plant after production modification yields a new operating profit S_t which follows a one-dimensional geometric Brownian motion,

$$\frac{dS_t}{S_t} = \mu dt + \sigma Z_t, \qquad S_0 = x, \tag{2}$$

where μ and σ are constants and $(Z_t, t \geq 0)$ is a Brownian motion defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$. We assume that undertaking such modification, the instantaneous emission rate boils down to zero.[6] We denote by V_t^S the expected sum of the new discounted operating profits from t to infinity:

$$V_t^S = \mathbb{E}_t\Big[\int_t^\infty e^{-\rho(u-t)} S_u du\Big], \tag{3}$$

and we impose $V_t^B > V_t^S$ for every $(t, 0 \leq t \leq \infty)$ such that the modification of the production process corresponds realistically to a loss in profits.

Lemma 2.1 *Assume that S_t follows a geometric Brownian motion and that $\rho > \mu$. Then V_t^S is a geometric Brownian motion and moreover*

$$V_t^S = \frac{S_t}{\rho - \mu}.$$

[5]In the paper we do not address questions such as how uncertainty affects the production (pollution) processes or the choice of emission abatement measures. We refer interested readers to the general discussions in Taschini (2008) and reference therein.

[6]The model can be extended to account for any given and fixed amount of instantaneous pollution reduction $\tilde{\alpha}$ where $\tilde{\alpha} \in [0, \alpha]$.

Proof. We have

$$V_t^S = \mathbb{E}_t\Big[\int_t^{\infty} e^{-\rho(u-t)} S_t e^{\mu(u-t)} e^{-\frac{\sigma^2}{2}(u-t)+\sigma(Z_u-Z_t)} du\Big]$$

where the last term is a martingale. Applying Fubini's theorem we get:

$$V_t^S = S_t \int_t^{\infty} e^{-\rho(u-t)} e^{\mu(u-t)} du = \frac{S_t}{\rho-\mu}. \blacksquare$$

Therefore, V_t^S satisfies the following stochastic differential equation:

$$\frac{dV_t^S}{V_t^S} = \mu dt + \sigma Z_t, \qquad V_0^S = \frac{S_0}{\rho-\mu}. \tag{4}$$

Alternatively, to fulfil environmental regulations, the company can rely on the market for permits and purchase at time T, an amount of permits equivalent to the pollution reduction obtained with the previous reversible abatement measure.[7] By standard no-arbitrage arguments, we claim that if the company undertakes a reversible and costly modification of the production process at time τ, then an equivalent reduction of pollution by trading permits, evaluated at time $(s, 0 \le s \le \tau)$, must corresponds to a purchase at time T of $\alpha \cdot (T-s)$ permits for a unit-price of P.

Lemma 2.2 *Purchasing permits at time T is equivalent to undertaking a reversible and costly modification of the production process at time τ if, and only if, the price of the permits at time $(s, 0 \le s \le \tau)$ is*

$$P = e^{\rho(T-s)} \cdot \frac{V_s^B - V_\tau^S}{\alpha(T-s)},$$

where V_s^B and V_τ^S are defined respectively in equation (1) and (3), and τ is a stopping time.

Proof. On the one hand, under environmental constraints, the strategy of undertaking a production modification at time τ corresponds to a total loss of operating profits equal to $V_s^B - V_\tau^S$, where s is the time of evaluation. On the other hand, the strategy of purchasing permits at time T corresponds to a cost of $e^{-\rho(T-s)} \cdot \alpha \cdot (T-s)P$ at time s. Equating the two different strategies, the expected result is yielded. The reverse implication follows in similar fashion. $\blacksquare$

Since we are in the presence of reversible abatement measures, we firstly attempt to determine when it is optimal for a company to undertake and to undo the modification of the production process. Such a choice corresponds to the well-known instantaneous investment and instantaneous

[7] In the paper we consider the case of a company in need of permits which is able to purchase all the permits it requires. Similar results can be obtained when the company undertakes the modification of the production process with the aim of freeing-up emission permits and selling them in the market. In this situation, the structure of the problem has a reverse sign and coincides with the previous one by means of a simple sign manipulation.

disinvestment decision problem. By investigating this problem we will determine the company's threshold for the price of the emission permits. Each time the price exceeds such a threshold, the company finds it advantageous to change the production process. Secondly, by solving the investment and disinvestment decision problem under the Parisian criterion, we determine the premium for the flexibility of the emission permits in analytic form.

As this is clearer subsequent to Theorem 3.1, we will concentrate our analysis on the process V_t^S. Assuming the company to be risk-neutral, and that the abatement decision can be reversed only once, the entire problem can be written as a discounted expectation of the new operating profits flow:

$$\mathbb{E}\Big[e^{-\rho\tau_I}(V_{\tau_I}^S - C_I)^+ + e^{-\rho\tau_D}(C_D - V_{\tau_D}^S)^+\Big],$$

where C_I represents the cost of undertaking the abatement measure, which we call investment cost.[8] Similarly, C_D represents the cost to undo such a decision, which we call disinvestment cost. We assume both C_I and C_D to be constant. Given a pre-specified level of the state process V_t^S has reached, τ_I (τ_D) represents the first instant after a time interval longer than a fixed amount of time (a so-called time-window) has passed. The time-window corresponds to the implementation delay,[9] whereas the pre-specified level is set at an optimal value: the optimal investment (disinvestment) threshold h_I^* (h_D^*). We assume that the time-window associated with the investment (disinvestment) is a fixed amount of time d_I (d_D). The decision-triggering criterion is the so-called Parisian stopping time which depends on the size of the excursions of the state variable over (below) the optimal thresholds.

In this simple model, the firm maximizes the current value of its new operating profits, namely it solves:

$$VS(V_0^S) = \max_{\tau_I<\tau_D} \mathbb{E}\Big[e^{-\rho\tau_I}(V_{\tau_I}^S - C_I)^+ \ 1_{\{\tau_I<\infty\}} + e^{-\rho\tau_D}(C_D - V_{\tau_D}^S)^+ \ 1_{\{\tau_D<\infty\}}\Big].$$

Because we are in the perpetual case, the investment (disinvestment) decision will occur at the first instant when V_t^S hits some constant optimal threshold h_I^* (h_D^*). Letting τ_I and τ_D be the stopping times corresponding to the Parisian criterion with time-windows d_I, d_D and levels h_I,

[8] In our model the investment action corresponds, for instance, to the decision to purchase a new input factor which will then substitute the one used a the moment the order is submitted. Another more generic situation corresponds to a case where the firm switches from a cheap-but-dirty production plant to an expensive-but-clean one.

[9] With respect to the example in the previous footnote, the implementation delay represents the time required to switch from one input factor to another. In the energy-production industry, for instance, such a time-interval is called "rump-up" time. This corresponds to the time needed for the unit to switch from one production regime (coal-burning) to a second one (gas-burning) and reach a certain operating efficiency of standard plant and output levels. Similarly, "rump-down" time is associated with the reverse switch.

h_D respectively, the present value of the investment and disinvestment decision problem becomes:

$$VS(V_0^S) = \max_{h_D \le h_I,\ V_0 \le h_I} \mathbb{E}_0 \Big[e^{-\rho \tau_I} (V_{\tau_I}^S - C_I)\ 1_{\{\tau_I < \infty\}} \ + \ e^{-\rho \tau_D} (C_D - V_{\tau_D}^S)\ 1_{\{\tau_D < \infty\}} \Big]. \tag{5}$$

The two measures of compliance are equivalent alternatives when a company is indifferent in undertaking at time τ an instantaneous modification of the production process or purchasing at time T an amount of permits equal to $\alpha \cdot (T - s)$ for a unit-price P. In other words, trading permits and production modification are *perfect substitutes* when $d_I = d_D = 0$ and Lemma 2.2 holds. However, the existence of physical or technical constraints that allow the implementation of the abatement decision only after a given time-interval makes marketable permits more flexible instruments. This implies that the premium measures the expected present value of the greater flexibility that emission permits provide, compared to the reversible commitments required by modifying the production process. In order to obtain the premium for the flexibility we have to solve (5) when $d_I > 0$ or $d_D > 0$ and compare it with the instantaneous case, i.e. $d_I = d_D = 0$.

3 Solution of the Problem: the Premium for Flexibility

In this section we solve the maximization problem (5), obtaining an analytic solution of the premium for the flexibility embedded in marketable permits. The first abatement strategy corresponds to the modification of the production process. When such a strategy can be implemented instantaneously, trading permits are equivalent to the pollution abatement measure. However, the presence of implementation delays, that we model by the Parisian criterion, make production modification an attractive alternative only if a company is sufficiently compensated. Therefore, the premium for the flexibility of emission permits is obtained by taking the difference between the value of the instantaneous investment and disinvestment decision problem and the Parisian investment and disinvestment decision problem, at their respective optima.

Following the literature on Parisian options, we translate the problem in terms of the drifted Brownian motion. We define:

$$V_t^S = V_0^S e^{\sigma X_t}, \quad \text{where } X_t = bt + Z_t, \ \text{ and } \ b = \frac{\mu - \frac{\sigma^2}{2}}{\sigma}. \tag{6}$$

and construct a new probability measure $\mathbb{P}^*$ under which X_t becomes a $\mathbb{P}^*$-Brownian motion,

$$\frac{d\mathbb{P}^*}{d\mathbb{P}}\Big|_{\mathcal{F}_t} = e^{\frac{b^2}{2}t - bX_t}. \tag{7}$$

Applying the Girsanov theorem, we change the probability measure in (5) and using the independence result from Revuz and Yor (1991) and Chesney et al. (1997), we obtain

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-(\rho+\frac{b^2}{2})\tau_I}\right] \cdot \mathbb{E}_{\mathbb{P}^*}\left[e^{bX_{\tau_I}}(V_0^S e^{\sigma X_{\tau_I}} - C_I)\right] \tag{8}$$

for the first term in the maximization problem. Similarly, the second term becomes

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-(\rho+\frac{b^2}{2})\tau_D}\right] \cdot \mathbb{E}_{\mathbb{P}^*}\left[e^{bX_{\tau_D}}(C_D - V_0^S e^{\sigma X_{\tau_D}})\right]. \tag{9}$$

The Laplace transform of the Parisian investment time under the measure $\mathbb{P}$ is computed first in Chesney et al. (1997). Using standard results of excursion theory and the main findings of Gauthier and Morellec (2000) and Gauthier (2002), in the Appendix C we calculate the moment generating function for the process X_t defined in (6), stopped at the Parisian investment and disinvestment times. This corresponds to the time when the new operating profits start and stop after the implementation delay. Finally, we evaluate the first hitting time of X_t which starts from the Parisian investment time. After that, and combining Proposition C.6 and Proposition C.7, we can re-write the maximization problem (5) as

$$\begin{aligned} VS(V_0^S) = \max_{h_D \leq h_I,\ V_0^S \leq h_I} & \left(\frac{V_0^S}{h_I}\right)^{\theta_1} \frac{\phi(b\sqrt{d_I})}{\phi(\sqrt{(2\rho+b^2)d_I})} \Big\{ h_I \frac{\phi(\sqrt{d_I}(\sigma+b))}{\phi(b\sqrt{d_I})} - C_I + \\ & + \left(\frac{h_I}{h_D}\right)^{\theta_2} \frac{\phi(-\sqrt{(2\rho+b^2)d_I})}{\phi(\sqrt{(2\rho+b^2)d_D})} \frac{\phi(-b\sqrt{d_D})}{\phi(b\sqrt{d_I})} \Big(C_D - h_D \frac{\phi(-(b+\sigma)\sqrt{d_D})}{\phi(-b\sqrt{d_D})} \Big) \Big\} \end{aligned} \tag{10}$$

where, to simplify the already complicated formula, we adopt the following notations:

$$\theta_1 = \frac{-b + \sqrt{2\rho + b^2}}{\sigma} \quad \text{and} \quad \theta_2 = \frac{-b - \sqrt{2\rho + b^2}}{\sigma}, \tag{11}$$

and ϕ is defined as:

$$\phi(z) = \mathbb{E}(\exp(zx)) = \int_0^\infty \exp(zx)\mathbb{P}_x(dx). \tag{12}$$

To obtain a solution, we first solve the unconstrained problem corresponding to (10) where we assume $V_0 \leq h_I$.Taking the partial derivative with respect to h_D and solving for the critical value, we obtain an explicit solution for the optimal disinvestment threshold h_D^*,

$$h_D^* = \frac{\phi(-b\sqrt{d_D})}{\phi(-(b+\sigma)\sqrt{d_D})} \frac{\theta_2 C_D}{\theta_2 - 1}. \tag{13}$$

This indicates the optimal level for undoing the modification of the production process undertaken at τ_I. When $d_D = 0$, h_D^* equals the well-known optimal instantaneous-disinvestment threshold, i.e. $h_{ND}^* = \frac{\theta_2 C_D}{\theta_2 - 1}$. Intuitively, h_D^* increases if the disinvestment fixed-costs C_D, the costs to change once again the input factor for instance, increases. This means that, similar to the instantaneous investment and disinvestment problem, the higher the disinvestment costs the sooner the firm wants to modify the production process. Furthermore, since ϕ is an increasing function, we obtain that $h^*_{ND} \leq h^*_D$, meaning that the firm has decided to change the production process earlier in the presence of an implementation delay.

Taking the partial derivative with respect to h_I and solving for the critical value, we obtain an implicit solution for h^*_I. In particular, $h^*_I = \max\{V_0, x^*\}$ where x^* solves the implicit equation

$$x = \frac{\theta_1 C_I}{\theta_1 - 1} \frac{\phi(b\sqrt{d_I})}{\phi((b+\sigma)\sqrt{d_I})} + \left(\frac{x}{h^*_D}\right)^{\theta_2} \frac{\theta_2 - \theta_1}{\theta_1 - 1} \frac{\phi(-\sqrt{(2\rho + b^2)d_I})}{\phi(\sqrt{(2\rho + b^2)d_D})} \frac{\phi(-b\sqrt{d_D})}{\phi((b+\sigma)\sqrt{d_I})} \frac{C_D}{1-\theta_2}. \tag{14}$$

When $d_I = d_D = 0$, h^*_I equals the implicit equation for the optimal instantaneous investment threshold h^*_{NI}, i.e. the implicit equation

$$x = \frac{\theta_1 C_I}{\theta_1 - 1} + \left(\frac{x}{h^*_{ND}}\right)^{\theta_2} \frac{\theta_2 - \theta_1}{\theta_1 - 1} \frac{C_D}{1 - \theta_2}. \tag{15}$$

Recall that the two abatement options are equivalent alternatives at time zero when $P = e^{\rho T} \cdot \frac{V_0^B - V_\tau^S}{\alpha T}$ and when they have the same implementation schedule, i.e. when $d_I = d_D = 0$. As observed before, in this last situation we recover the well-known case of the instantaneous investment and disinvestment problem and we obtain $h^*_I = h^*_{NI}$ and $h^*_D = h^*_{ND}$. Relying on this, we analytically quantify the flexibility premium which must be embedded in the price for marketable permits as follows:

Theorem 3.1 *Consider a company which has two pollution abatement measures at its disposal: trading permits or modifying the production process. The premium for the flexibility embedded in the price for marketable permits at time zero is:*

$$\theta = e^{\rho \cdot T} \cdot \frac{VS^*_I(V_0^S) - VS^*_D(V_0^S)}{\alpha \cdot T}, \tag{16}$$

where $VS^*_I(V_0^S)$ is the solution of the instantaneous investment and disinvestment problem (10), i.e. when $d_I = d_D = 0$, whereas $VS^*_D(V_0^S)$ is the solution of the maximization problem (10) under the Parisian criterion, i.e. when $d_D \geq 0$ and $d_I \geq 0$.

Proof. Let us consider a company at time zero which has two possible alternatives to abate emissions, as described before. When the modification of the production process can be implemented instantaneously, the firm will be indifferent between the two abatement measures for a price:

$$P_I = e^{\rho \cdot T} \cdot \frac{V_0^B - VS^*_I(V_0^S)}{\alpha \cdot T}, \tag{17}$$

by Lemma 2.2 and where $VS_I^*(V_0^S)$ is defined in (10). Similarly, when the firm faces implementation delays, it will be indifferent between the two abatement measures for a price:

$$P_D = e^{\rho \cdot T} \cdot \frac{V_0^B - VS_D^*(V_0^S)}{\alpha \cdot T}, \tag{18}$$

again applying Lemma 2.2. Let us now consider the firm as a permit seller: given the presence of implementation delay, the firm needs to be compensated for bearing such costs and will require a premium. Then, the result follows applying standard no-arbitrage conditions to $P_D = P_I + \theta$. ■

4 Model Results

We now investigate the sensitivity of the premium for flexibility θ and assess its likely magnitude. In doing so, we show that the model of Bar-Ilan and Strange (1996) is nested into ours. Further, we perform a numerical evaluation to illustrate that our one-time reversible investment framework extends the irreversible investment model of Chao and Wilson (1993) when $d_D \to \infty$. Recall that when the company undertakes a modification of the production process, it faces a loss of profit. Also, the existence of implementation delays makes such a modification less attractive to the company. In each of the following tables we report the value of θ for different pairs of time-windows where columns and rows correspond to the values of d_I and d_D respectively. Table 1 is our benchmark situation, i.e. where we set the discount factor $\rho = 0.13$, the drift rate $\mu = 0.05$, the volatility rate $\sigma = 0.40$, the disinvestment cost $C_D = 50$, the investment cost $C_I = 170$, the initial value for the process V_t^S equal to $V_0^S = 100$, $\alpha = 1$ and $T = 0.8$. Because trading permits and production modification are perfect substitutes if Lemma 2.2 holds, we expect the flexibility premium to be equal to 0 when $d_I = d_D = 0$ (upper-left corner of all tables). This implies, first, that the company is indifferent in undertaking an instantaneous modification of the production process or purchasing the needed amount of permits and, second, that the firm is not willing to spend more (or to receive less) than P_I.

d	0	2	4
0	0	0.0151	0.0285
3	3.5083	3.5253	3.5405
5	5.9422	5.9604	5.9767

Table 1: Premium benchmark case. The parameters we used are $\rho = 0.13; \mu = 0.05; \sigma = 0.40; C_D = 50; C_I = 170; V_0 = 100; \alpha = 1; T = 0.8$.

In what follows we discuss the impact on θ of a variation in the emission reduction, the time-windows and the investment/disinvestment costs. In addition, we investigate the effect of the uncertainty of the underlying process on investment and disinvestment. In line with Bar-Ilan and Strange (1996), we show that conventional findings are reversed.

We do not study the impact of T because this is trivially linked to the concept of time-value of the money.

(a) If the reduction of emissions obtained by modifying the production process is lower than α, then the cost per unit-reduction increases and the company requires a higher premium to undertake the abatement measure, as reported in Table 2.

d	0	2	4
0	0	0.0302	0.0570
3	7.0165	7.0506	7.0811
5	11.8844	11.9209	11.9535

Table 2: Premium for $\alpha = 0.5$, all other parameters being equal.

(b) The impact of a variation of the investment costs C_I on the project value is aligned with conventional findings in the literature: the lower (higher) the investment costs the higher (lower) the value of an instantaneous investment VS_I^*. For general discussions we refer to Majd and Pindyck (1987), Dixit and Pindyck (1994), and Dixit (1989). However, confirming the results of Bar-Ilan and Strange (1996), the investment value under the Parisian criterion VS_D^* does not increase (decrease) with the same magnitude. As a consequence, the profit-loss associated with the modification of the production process in presence of delays decreases (increases) at a lower rate with respect to the profit-loss associated with the instantaneous process modification. This corresponds to a higher premium for flexibility if C_I decreases (Table 3) and, conversely, a lower premium for flexibility if C_I increases (Table 4).

d	0	2	4
0	0	0.0204	0.0387
3	3.7197	3.7427	3.7633
5	6.3008	6.3253	6.3473

Table 3: Premium for $C_I = 150$, all other parameters being equal.

d	0	2	4
0	0	0.0115	0.0217
3	3.3299	3.3429	3.3545
5	5.6399	5.6538	5.6663

Table 4: Premium for $C_I = 190$, all other parameters being equal.

(c) The interpretation of the impact of C_D on θ is not as straightforward as in the case of C_I and requires a deeper analysis. However, we identify a decrease/increase in the premium when the variation of the project value in the instantaneous investment case is lower (higher) than the variation of the project value under the Parisian criterion. Formally, if disinvestment costs move to C'_D we obtain a premium θ' such that

$$P_D = P_I + \theta' \quad \text{where} \quad \theta' \quad \text{is} \begin{cases} > \theta & \text{if} \quad VS_I^*(C_D) \text{ - } VS_I^*(C'_D) < VS_D^*(C_D) \text{ - } VS_D^*(C'_D) \\ < \theta & \text{if} \quad VS_I^*(C_D) \text{ - } VS_I^*(C'_D) > VS_D^*(C_D) \text{ - } VS_D^*(C'_D) \end{cases}$$

d	0	2	4
0	0	0.0054	0.0101
3	3.5171	3.5233	3.5287
5	5.9565	5.9631	5.9690

Table 5: Premium for $C_D = 30$, all other parameters being equal.

d	0	2	4
0	0	0.0287	0.0545
3	3.4967	3.5287	3.5575
5	5.9240	5.9577	5.9883

Table 6: Premium for $C_D = 70$, all other parameters being equal.

(d) Since the premium measures the expected present value of the greater flexibility granted by marketable permits, it is unsurprising to observe that the shorter (longer) the time-windows the more (less) attractive is the modification of the production process to the company. The longer the investment or disinvestment implementation delay, the higher the premium required, as reported in Tables 7 and 8 respectively.

d	0	2	4
0	0	0.0151	0.0285
1	1.1198	1.1355	1.1496
2	2.3015	2.3179	2.3326
3	3.5083	3.5253	3.5405
5	5.9422	5.9604	5.9767
6	7.1531	7.1719	7.1887
8	9.5365	9.5564	9.5743

Table 7: Premium for $d_I = \{0, 1, 2, 3, 5, 6, 8\}$. All other parameters being equal.

d	0	0.5	1	2	4	8
0	0	0.0039	0.0077	0.0151	0.0285	0.0502
3	3.5083	3.5127	3.5170	3.5253	3.5405	3.5652
5	5.9422	5.9469	5.9515	5.9604	5.9767	6.0032

Table 8: Premium for $d_D = \{0, 0.5, 1, 2, 4, 8\}$. All other parameters being equal.

The presence of an imbalanced effect of the time-windows on the premium is of particular interest. In fact, the impact of d_D on θ is not as evident as the impact of d_I. This is due to the option to reverse the abatement decision. Because a firm's profits are a convex function of the stochastic underlying V_t^S and disinvestment is possible at a cost, a firm will invest at a lower level when the implementation delay forces it to decide in advance whether to enter a few periods ahead or not. Such asymmetry is observable also in Bar-Ilan and Strange (1996).

(e) Xepapadeas (2001) and Zhao (2003) show that cost-uncertainty reduces the incentive to invest in irreversible abatement measures, shifting the investment barrier upward, i.e. delaying the project. Their findings align with the literature on irreversible investments, see Pindyck (1988) for a comprehensive discussion. The intuition is that an increase in uncertainty raises the benefit of waiting but not its opportunity cost, i.e. the foregone profits during the period of inaction. Since in the models of Xepapadeas and Zhao a firm can undertake an abatement investment instantaneously, the opportunity cost of waiting is independent of uncertainty. Our model confirms such a result: an increase in uncertainty delays instantaneous irreversible investments, increasing the instantaneous investment threshold $h_{II}^* = \frac{\theta_1 C_I}{\theta_1 - 1}$, as observable in the first row of Table 9. As σ goes from 0.05 to 0.40, h_{II}^* rises from 282.90 to 527.16.

In the presence of implementation delays and options to reverse the project, the opportunity cost of waiting is no longer independent of uncertainty. As a result, conventional findings on the effect of the uncertainty of the underlying process on investment and disinvestment are reversed. See the last four columns of Table 9. For instance, when $d_I = d_D = 3$, h_I^* falls from 232.21 to 208.07, while h_D^* rises from 52.01 to 58.84. Since a firm can undo the abatement measure at a cost, the downside risk of the project is bounded. This makes operating profits a convex function of the stochastic underlying V_t^S, and the expected return of the abatement project rises with uncertainty. Therefore, in our model a higher volatility hastens both investment and disinvestment when delays force a firm to decide in advance whether or not to undertake a decision in the near future. The work of Bar-Ilan and Strange (1996) who study the effect of delays on irreversible investment is then nested into our model. Figure 1 shows graphically the effect of the volatility σ of the new operating profits V_t^S on the premium for the flexibility. The larger the uncertainty about V_t^S, the higher is the premium required by the company that undertakes the modification of the production process.

Finally, this theoretical result coincide with the findings of Abel (1983) and Caballero (1991). In their model they both assume adjustment of the investment capital to be costly, which make it less flexible than labor. Employing such an assumption and, contrary to Xepapadeas

		$d_I = d_D = 3$		$d_I = d_D = 5$	
σ	h^*_{II}	h^*_I	h^*_D	h^*_I	h^*_D
0.05	282.91	232.21	52.01	209.73	52.44
0.06	285.70	230.39	52.50	207.60	53.14
0.07	288.90	228.65	52.95	205.37	53.84
0.08	292.49	227.02	53.36	203.12	54.52
0.09	296.43	225.53	53.74	200.91	55.19
0.10	300.70	224.17	54.08	198.75	55.83
0.11	305.26	222.94	54.40	196.68	56.46
0.12	310.11	221.83	54.68	194.67	57.07
0.13	315.23	220.82	54.94	192.74	57.67
0.14	320.59	219.89	55.18	190.87	58.25
0.15	326.18	219.05	55.40	189.07	58.82
0.16	331.99	218.27	55.60	187.32	59.39
0.17	338.02	217.56	55.78	185.62	59.94
0.18	344.26	216.90	55.96	183.97	60.49
0.19	350.69	216.28	56.12	182.36	61.03
0.20	357.32	215.70	56.28	180.78	61.57
0.21	364.14	215.16	56.42	179.24	62.11
0.22	371.14	214.65	56.56	177.74	62.65
0.23	378.32	214.17	56.70	176.26	63.18
0.24	385.69	213.71	56.83	174.80	63.72
0.25	393.23	213.28	56.96	173.38	64.26
0.26	400.95	212.86	57.08	171.97	64.80
0.27	408.84	212.47	57.20	170.58	65.34
0.28	416.90	212.08	57.32	169.22	65.89
0.29	425.14	211.72	57.44	167.87	66.44
0.30	433.56	211.36	57.56	166.53	67.00
0.31	442.14	211.01	57.68	165.21	67.57
0.32	450.90	210.67	57.80	163.91	68.14
0.33	459.83	210.34	57.93	162.61	68.72
0.34	468.93	210.01	58.05	161.33	69.30
0.35	478.20	209.69	58.18	160.05	69.89
0.36	487.65	209.37	58.30	158.79	70.50
0.37	497.26	209.04	58.43	157.53	71.11
0.38	507.06	208.72	58.57	156.28	71.73
0.39	517.02	208.40	58.70	155.03	72.36
0.40	527.17	208.07	58.84	153.79	73.00

Table 9: Optimal instantaneous irreversible investment value h^*_{II}, and Parisian optimal investment h^*_I and disinvestment h^*_D values. The parameters we used are $\rho = 0.13; \mu = 0.05; C_D = 50; C_I = 170; V_0 = 100; \alpha = 1; T = 0.8$.

(2001) and Zhao (2003), adopting a constant return technology the authors show that the marginal revenue product of capital is a convex function of the underlying stochastic price. Relying on Jensen's inequality, this implies the expected return of the investment rises with uncertainty. Such an argument might suggests a reasonable explanation for the early over-investment in pollution control observed by Stavins (1998) in the U.S. market for SO_2. Though installing a scrubber is an irreversible investment, non-decreasing return to scale and costly capital adjustment might explain the positive correlation between uncertainty and investment.

(f) Chao and Wilson (1993) were the first to identify the flexibility embedded in marketable permits and to investigate it in the presence of irreversible investments. Here we extend their framework developing a one-time reversible investment model. Indeed, letting $d_D \to \infty$ we obtain the special case of an irreversible investment. More precisely, when $d_D \to \infty$ the level h_I^* converges to

$$h_{OI}^* = \frac{\theta_1 C_I}{\theta_1 - 1} \frac{\phi(b\sqrt{d_I})}{\phi((b+\sigma)\sqrt{d_I})},$$

where h_{OI}^* represents the optimal investment threshold for time-window d_I while disinvestment is not possible. Gauthier and Morellec (2000) were the first to obtain this result.

As expected and as is observable in the last picture of figure 1, the required premium for irreversible investments (θ_{OI}) is larger than that for those which are reversible (θ_D). This theoretical result has a practical relevance. Since abatement measures are not perfect substitutes and companies value the flexibility of trading permits - as described in section 2 - policy makers should be concerned about the degree of reversibility of all abatement alternatives available to companies when designing and implementing effective environmental regulations.

5 Conclusions

In a constrained economy where companies are subject to environmental regulations and are endowed with a marketable permits scheme, environmental economics literature claims that trading emission permits and physical abatement reduction are equal alternatives. However, this only holds in a deterministic setup. In reality, the presence of uncertainty and technical constraints like implementation delays, make these measures perceived as imperfect substitutes. Relaxing the assumption of perfect substitution among all abatement alternatives, we develop a model for one-time reversible investments where we assume that the new operating profits follow a one-dimensional geometric Brownian motion. The optimal "investment" and "disinvestment" problem is solved under both the instantaneous and the Parisian criteria, obtaining an analytic solution to the optimal "starting" and "stopping" levels and nesting the model of Bar-Ilan and Strange (1996). Relying on these results, we also derive an analytic solution of the premium for the flexibility embedded in marketable permits. We compare our findings with the instantaneous and irreversible investment case, and show that an increase in the length of the time-windows enhances the required premium whereas the absence of the reversibility-option increases the premium, extending a result of Chao and Wilson (1993).

An interesting direction for future research would be to investigate explicitly the price of emission permits assuming a tractable dynamics for the stochastic process V_t^B. Further, it would be interesting to test empirically the model in this paper parametrizing the process V_t^S to real data. This might simplify the interpretation of the impact of C_D on the premium.

Appendix

A Definitions and Results

The Brownian meander and the Parisian criterion are closely related. In the following, we define the Brownian meander and list some of its properties. We refer to Revuz and Yor (1991) for details about the Brownian meander. Then, we present the connection existing between the Brownian meander and the Parisian criterion.

Let $(Z_t, t \geq 0)$ be a standard Brownian motion on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. For each $t > 0$, we define the random variables

$$g_t = \sup\{s : s \leq t, Z_s = 0\}, \tag{19}$$

$$d_t = \inf\{s : s \geq t, Z_s = 0\}. \tag{20}$$

The interval (g_t, d_t) is called "interval of the Brownian excursion" which straddles time t. For u in this interval, $sgn(Z_t)$ remains constant. In particular, g_t represents the last time the Brownian motion crossed the level 0. It is known that g_t is not a stopping time for the Brownian filtration $(\mathcal{F}_t)_{t\geq 0}$, but for the slow Brownian filtration $(\mathcal{G}_t)_{t\geq 0}$, which is defined by $\mathcal{G}_t = \mathcal{F}_{g_t} \vee \sigma(sgn(Z_t))$. The slow Brownian filtration represents the information on the Brownian motion until its last zero plus the knowledge of its sign after this.

The Brownian meander process ending at t is defined as (see Revuz and Yor (1991))

$$m_u^{(t)} = \frac{1}{\sqrt{t - g_t}} |Z_{g_t + u(t-g_t)}|, \quad 0 \leq u \leq 1. \tag{21}$$

The process $m_u^{(t)}$ is the non-negative and normalized Brownian excursion which straddles time t and is independent of the σ-field $(\mathcal{G}_t)_{t\geq 0}$. When $u = 1$ and $t = 1$, we conveniently denote $m_1 = m_1^{(1)}$. The random variable m_1 will play a central role in the calculation of many other variables that will be introduced later on. The distribution of m_1 is known to be (see Revuz and Yor (1991))

$$\mathbb{P}(m_1 \in dx) = x \exp(-\frac{1}{2}x^2) 1_{x>0} dx,$$

and the moment generating function $\phi(z)$ is given by (see Revuz and Yor (1991))

$$\phi(z) = \mathbb{E}(\exp(zm_1)) = \int_0^\infty x \exp(zx - \frac{1}{2}x^2)dx. \tag{22}$$

We now look at the first instant when the Brownian motion spends d units of time consecutively above (resp. below) the level 0. For $d \geq 0$, we define the random variables

$$\begin{aligned} H_d^+ &= \inf\{t \geq 0 : t - g_t \geq d, \quad Z_t \geq 0\} \quad (23) \\ H_d^- &= \inf\{t \geq 0 : t - g_t \geq d, \quad Z_t \leq 0\} \quad (24) \end{aligned}$$

The variables H_d^+ and H_d^- are $\mathcal{G}_t$-stopping times and hence $\mathcal{F}_t$-stopping times (see Revuz and Yor (1991) for more details). From equation (21) we can easily deduce that the process

$$\left(\frac{1}{\sqrt{d}}|Z_{g_{H_d^+}+ud}|\right)_{u\leq 1} = \left(m_u^{(H_d^+)}\right)_{u\leq 1}$$

is a Brownian meander, independent of $\mathcal{G}_{g_{H_d^+}}$. In particular, $(1/\sqrt{d})Z_{H_d^+}$ is distributed as m_1 (see Revuz and Yor (1991)),

$$\mathbb{P}(Z_{H_d^+} \in dx) = \frac{x}{d}\exp(-\frac{x^2}{2d})1_{x>0}dx. \tag{25}$$

and the random variables H_d^+ and $Z_{H_d^+}$ are independent.

Similarly, $(1/\sqrt{d})Z_{H_d^-}$ is distributed as $-m_1$ (see Revuz and Yor (1991)),

$$\mathbb{P}(Z_{H_d^-} \in dx) = \frac{-x}{d}\exp(-\frac{x^2}{2d})1_{x<0}dx. \tag{26}$$

and the random variables H_d^- and $Z_{H_d^-}$ are independent.

The Laplace transform of H_d^+ was first calculated in Chesney et al. (1997). We present the result in the next theorem.

Theorem A.1 *Let H_d^+ be the stopping time defined in (23) and ϕ the moment generating function defined in equation (22). For any $\lambda > 0$,*

$$\mathbb{E}[\exp(-\lambda H_d^+)] = \frac{1}{\phi(\sqrt{2\lambda d})}. \tag{27}$$

The proof is based on the Azèma martingale, $\mu_t = sgn(Z_t)\sqrt{t - g_t}$ - a remarkable $(\mathcal{G}_t)$ martingale. The same results hold also when H_d^+ is replaced with H_d^-.

So far we only looked at the Brownian motion excursions above or below level 0. More generally, we can define for any $a \in \mathbb{R}$ and any continuous stochastic process X that

$$\begin{aligned} g_t^{X_0,a}(X) &= \sup\{s : s \le t, X_0 = X_0, X_t = a\}, & (28)\\ H^+_{(X_0,a),d}(X) &= \inf\{t \ge 0 : t - g_t^{X_0,a} \ge d, X_0 = X_0, \quad X_t \ge a\}, & (29)\\ H^-_{(X_0,a),d}(X) &= \inf\{t \ge 0 : t - g_t^{X_0,a} \ge d, X_0 = X_0, \quad X_t \le a\} & (30) \end{aligned}$$

Thus, $g_t^{X_0,a}(X)$ represents the last time the process X crossed level a. As for the Brownian motion case, $g_t^{X_0,a}(X)$ is not a stopping time for the Brownian filtration $(\mathcal{F}_t)_{t\ge 0}$, but for the slow Brownian filtration $(\mathcal{G}_t)_{t\ge 0}$. The random variables $H^+_{(X_0,a),d}(X)$ (resp. $H^-_{(X_0,a),d}(X)$) represent the first instant when the process X spends d units of time above (resp. below) the level a. The variables $H^+_{(X_0,a),d}(X)$ and $H^-_{(X_0,a),d}(X)$ are $\mathcal{G}_t$-stopping times and hence $\mathcal{F}_t$-stopping times. In the notation we use, we indicate the starting point of the process X, the level a and the length of time d. Although indicating the starting point seems unnecessary, it turns out to be extremely helpful in the context of the Parisian criterion.

Another relevant random variable is the first hitting time of level a, which we define below:

$$T_{X_0,a}(X) = \inf\{s : X_0 = X_0, X_s = a\}. \tag{31}$$

B Parisian Criterion

According to the notation introduced in Section 2, the investment stopping time τ_I which satisfies the Parisian criterion corresponds to $H^+_{(V_0,h_I),d_I}(V)$. In order to express the disinvestment stopping time τ_D in mathematical formulas, we need to extend the definition of $H^+_{(V_0,h_I),d_I}(V)$. Let τ be any stopping time, $a \in \mathbb{R}$, X a continuous stochastic process and $g_t^{X_0,a}(X)$ as defined in equation (28. Then

(a) the first instant after τ when the process X spends d units of time above (resp. below) level a is given by the stopping time $H^{+,\tau}_{(X_0,a),d}(X)$ (resp. $H^{-,\tau}_{(X_0,a),d}(X)$)

$$\begin{aligned} H^{+,\tau}_{(X_0,a),d}(X) &= \inf\{t \ge \tau : t - g_t^{X_0,a} \ge d, X_0 = X_0, \quad X_t \ge a\}, & (32)\\ H^{-,\tau}_{(X_0,a),d}(X) &= \inf\{t \ge \tau : t - g_t^{X_0,a} \ge d, X_0 = X_0, \quad X_t \le a\}; & (33) \end{aligned}$$

(b) the first hitting time after τ of level a is the stopping time $T^\tau_{X_0,a}(X)$

$$T^\tau_{X_0,a}(X) = \inf\{s \ge \tau : X_0 = X_0, X_s = a\}. \tag{34}$$

If X has the strong Markov property and τ is a finite stopping time, we have the following equalities in distribution $H^{+,\tau}_{(X_0,a),d}(X) = H^+_{(X_\tau,a),d}(X)$, $H^{-,\tau}_{(X_0,a),d}(X) = H^-_{(X_\tau,a),d}(X)$, and $T^\tau_{X_0,a}(X) = T_{X_\tau,a}(X)$. Now we can state the formulas for the stopping times τ_I and τ_D, which satisfy the Parisian criterion.

Proposition B.1 *Let τ_I and τ_D be the stopping times corresponding to the Parisian criterion with time windows d_I, d_D and levels h_I, h_D respectively. Then the following equalities hold*

$$\tau_I = H^{+}_{(V_0,h_I),d_I}(V), \tag{35}$$

$$\tau_D = H^{-,\tau_I}_{(V_0,h_D),d_D}(V). \tag{36}$$

Otherwise, in terms of the drifted Brownian motion, the Parisian stopping times are

$$\tau_I = H^{+}_{(V_0,h_I),d_I}(V) = H^{+}_{(l_0,l_I),d_I}(X), \quad \text{where } l_0 = 0, \ \text{ and } \ l_I = \frac{1}{\sigma}\log\left(\frac{h_I}{V_0}\right),$$

and

$$\tau_D = H^{-,\tau_I}_{(V_0,h_D),d_D}(V) = H^{-,\tau_I}_{(l_0,l_D),d_D}(X), \quad \text{where } l_0 = 0, \ \text{ and } \ l_D = \frac{1}{\sigma}\log\left(\frac{h_D}{V_0}\right).$$

Proposition B.2 *Let τ be any finite stopping time such that τ and V_τ are independent and assume $h_D \le V_\tau$ a.s. Then the following equality in distribution holds*

$$H^{-,\tau}_{(V_0,h_D),d_D}(V) = \tau + T_{V_\tau,h_D}(V) + H^{-}_{(h_D,h_D),d_D}(V),$$

and the terms of the sum are independent. A similar relationship holds for $H^{+,\tau}_{(V_0,h_I),d_I}(V)$ if we assume $V_\tau \le h_I$ a.s.

Proof. The strong Markov property and the continuity of the process V give us the equality. The independence follows from our hypothesis that τ and V_τ are independent. ∎

C Laplace Transforms and Moment Generating Functions

To obtain an explicit solution for the optimal disinvestment threshold and an implicit solution for the optimal investment threshold we need to calculate all terms that enter into the maximization problem. We first find the Laplace transform of the Parisian investment time under the measure $\mathbb{P}^*$ defined in (7).

Proposition C.1 *For any $\lambda > 0$, the following equality holds:*

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_I}\right] = \left(\frac{V_0}{h_I}\right)^{\frac{\sqrt{2\lambda}}{\sigma}} \frac{1}{\phi(\sqrt{2\lambda d_I})}.$$

Proof. From Proposition B.2 we have

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_I}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda T_{l_0,l_I}}\right]\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda H^+_{(l_I,l_I),d_I}(X)}\right]$$

using the corresponding Laplace transforms we obtain

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_I}\right] = e^{-(l_I-l_0)\sqrt{2\lambda}}\frac{1}{\phi(\sqrt{2\lambda d_I})} = \left(\frac{V_0}{h_I}\right)^{\frac{\sqrt{2\lambda}}{\sigma}}\frac{1}{\phi(\sqrt{2\lambda d_I})}. \blacksquare$$

In the next proposition we calculate the moment generating function for the process X_t defined in (6), stopped at the Parisian investment time.

Proposition C.2 *For any $\lambda \in \mathbb{R}$, the following equality holds,*

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda X_{\tau_I}}\right] = \left(\frac{h_I}{V_0}\right)^{-\frac{\lambda}{\sigma}}\phi(-\lambda\sqrt{d_I}).$$

Proof. Using the definition of X_{τ_I}

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda X_{\tau_I}}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda(l_I+m_1\sqrt{d_I})}\right].$$

Now, using the definition of l_I and ϕ, we obtain

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda X_{\tau_I}}\right] = e^{-\lambda l_I}\phi(-\lambda\sqrt{d_I}) = \left(\frac{h_I}{V_0}\right)^{-\frac{\lambda}{\sigma}}\phi(-\lambda\sqrt{d_I}). \blacksquare$$

In the following, we calculate the Laplace transform of the first hitting time of X, starting at the Parisian investment time.

Proposition C.3 *For any $\lambda > 0$, the following equality holds*

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda T_{(X_{\tau_I},l_D)}(X)}\right] = \left(\frac{h_D}{h_I}\right)^{\frac{\sqrt{2\lambda}}{\sigma}}\phi(-\sqrt{2\lambda d_I})$$

Proof. Conditioning, we write

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda T_{(X_{\tau_I},l_D)}(X)}\right] = \mathbb{E}_{\mathbb{P}^*}\left[\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda T_{(X_{\tau_I},l_D)}(X)}\middle|\mathcal{F}_{\tau_I}\right]\right]$$

since $X_{\tau_I} \geq l_D$ a.s., we can use the Laplace transform of the hitting time to obtain

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-(X_{\tau_I}-l_D)\sqrt{2\lambda}}\right].$$

Using the formulas for X_{τ_I} and l_d, we know that $X_{\tau_I} - l_d = \frac{1}{\sigma}\log\frac{V_{\tau_I}}{h_D}$ and hence we obtain

$$\mathbb{E}_{\mathbb{P}^*}\left[\left(\frac{V_{\tau_I}}{h_D}\right)^{-\frac{\sqrt{2\lambda}}{\sigma}}\right] = h_D^{\frac{\sqrt{2\lambda}}{\sigma}}\mathbb{E}_{\mathbb{P}^*}\left[V_{\tau_I}^{-\frac{\sqrt{2\lambda}}{\sigma}}\right] = \left(\frac{h_D}{V_0}\right)^{\frac{\sqrt{2\lambda}}{\sigma}}\mathbb{E}_{\mathbb{P}^*}\left[e^{-\sqrt{2\lambda}X_{\tau_I}}\right]$$

Applying Proposition C.2 now we arrive at our result. ■

Then, we find the Laplace transform of the Parisian disinvestment time under the measure $\mathbb{P}^*$ defined in (7).

Proposition C.4 *For any $\lambda > 0$, the following equality holds*

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_D}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_I}\right]\frac{\phi(-\sqrt{2\lambda d_I})}{\phi(\sqrt{2\lambda d_D})}\left(\frac{h_D}{h_I}\right)^{\frac{\sqrt{2\lambda}}{\sigma}}.$$

Proof. Using Proposition B.2, we can write

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_D}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda\tau_I}\right]\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda T_{(X_{\tau_I},l_D)}(X)}\right]\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda H^-_{(l_D,l_D),d_D}(X)}\right].$$

Now, using the corresponding Laplace transforms, we arrive at the desired result. ■

Again we calculate the moment generating function for the process X_t defined in (6), stopped at the Parisian disinvestment time.

Proposition C.5 *For any $\lambda \in \mathbb{R}$, the following equality holds*

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda X_{\tau_D}}\right] = \left(\frac{h_D}{V_0}\right)^{-\frac{\lambda}{\sigma}}\phi(\lambda\sqrt{d_D}).$$

Proof. Using the definition of X_{τ_D}

$$\mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda X_{\tau_D}}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-\lambda(l_D - m_1\sqrt{d_D})}\right].$$

Now, using the definition of l_D and ϕ, we obtain the desired result. ■

Finally, we are able to calculate the first term appearing in the maximization problem (5).

Proposition C.6 *The following equality holds*

$$\mathbb{E}_{\mathbb{P}}\left[e^{-\rho\tau_I}(V_{\tau_I} - C_I)1_{\{\tau_I<\infty\}}\right] = \mathbb{E}_{\mathbb{P}^*}\left[e^{-(\rho+\frac{b^2}{2})\tau_I}\right]\left(\frac{h_I}{V_0}\right)^{\frac{b}{\sigma}}\phi(b\sqrt{d_I})\times$$

$$\times\left\{h_I\frac{\phi(\sqrt{d_I}(\sigma+b))}{\phi(b\sqrt{d_I})} - C_I\right\}.$$

Proof. Using equation (8) and Proposition C.2, the left hand side in the above equality becomes

$$\mathbb{E}_{\mathbb{P}^*}\Big[e^{-(\rho+\frac{b^2}{2})\tau_I}\Big]\Big\{V_0\Big(\frac{h_I}{V_0}\Big)^{\frac{\sigma+b}{\sigma}}\phi(\sqrt{d_I}(\sigma+b))-C_I\Big(\frac{h_I}{V_0}\Big)^{\frac{b}{\sigma}}\phi(b\sqrt{d_I})\Big\}$$

and grouping the terms we arrive at the desired result. ■

Similarly, we calculate the second term appearing in the maximization problem (5).

Proposition C.7 *The following equality holds*

$$\mathbb{E}_{\mathbb{P}}\Big[e^{-\rho\tau_D}(C_D-V_{\tau_D})1_{\{\tau_D<\infty\}}\Big]=\mathbb{E}_{\mathbb{P}^*}\Big[e^{-(\rho+\frac{b^2}{2})\tau_I}\Big]\Big(\frac{h_I}{V_0}\Big)^{\frac{b}{\sigma}}\Big(\frac{h_I}{h_D}\Big)^{\frac{-b-\sqrt{2\rho+b^2}}{\sigma}}\times$$

$$\times\frac{\phi(-\sqrt{(2\rho+b^2)d_I})}{\phi(\sqrt{(2\rho+b^2)d_D})}\phi(-b\sqrt{d_D})\Big\{C_D-h_D\frac{\phi(-(b+\sigma)\sqrt{d_D})}{\phi(-b\sqrt{d_D})}\Big\}$$

Proof. Using equation (9), Propositions C.4 and Proposition C.5, the left hand side in the equality above becomes

$$\mathbb{E}_{\mathbb{P}^*}\Big[e^{-(\rho+\frac{b^2}{2})\tau_I}\Big]\Big(\frac{h_D}{h_I}\Big)^{\frac{\sqrt{2\rho+b^2}}{\sigma}}\frac{\phi(-\sqrt{(2\rho+b^2)d_I})}{\phi(\sqrt{(2\rho+b^2)d_D})}\Big\{C_D\Big(\frac{h_D}{V_0}\Big)^{\frac{b}{\sigma}}\phi(-b\sqrt{d_D})-$$

$$-V_0\Big(\frac{h_D}{V_0}\Big)^{\frac{\sigma+b}{\sigma}}\phi(-(b+\sigma)\sqrt{d_D})\Big\}$$

and now factoring out and grouping the terms we arrive at the desired result. ■

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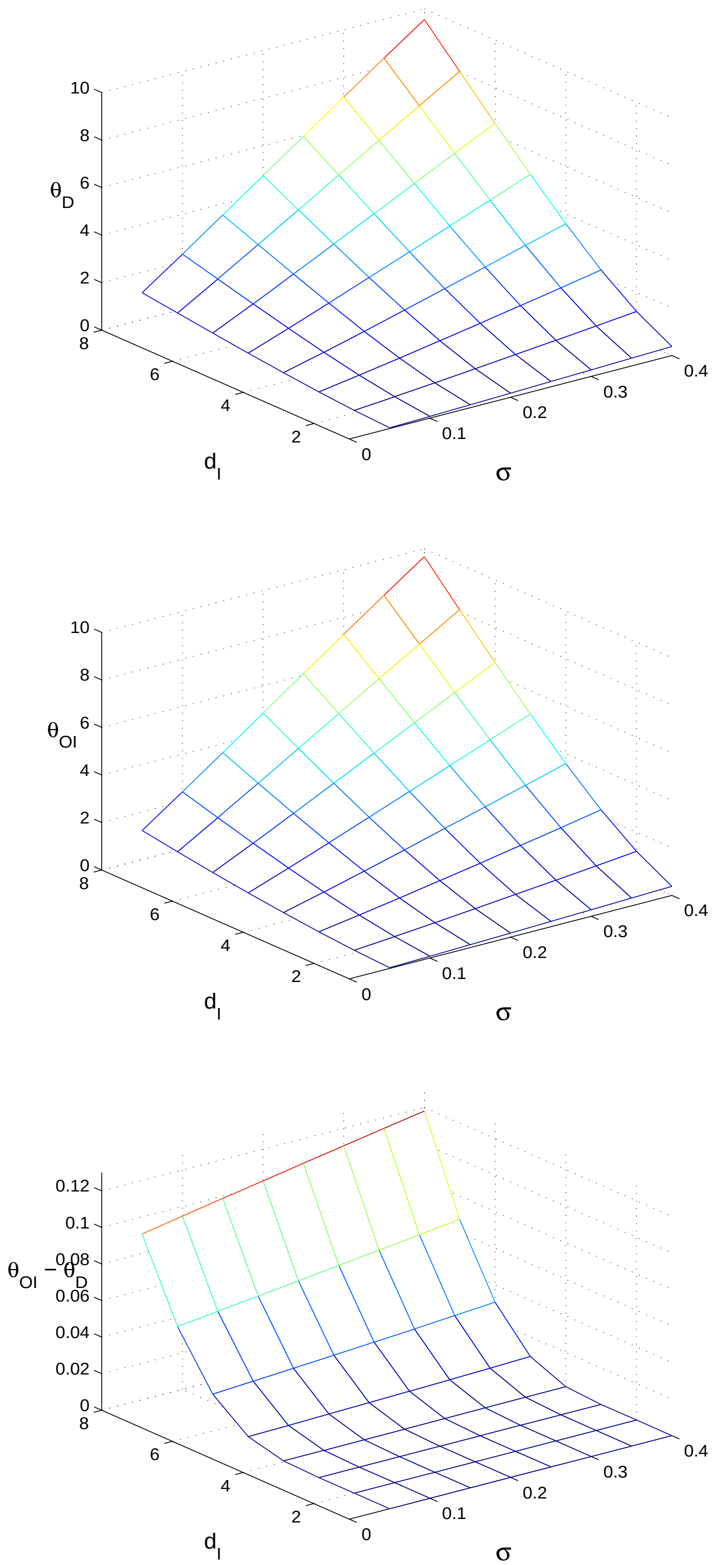

Figure 1: θ_D and θ_{OI} are the premia respectively for reversible (right picture) and irreversible investments (middle picture). Both are functions of the investment time-window d_I and the volatility rate σ. The left picture plots the difference between the two premia.