Sequential Investments and Capacity Choice

A real options approach

Michael Drud & Trine Krogh Boomsma

September 1, 2012

Abstract

Governments worldwide apply political targets to reduce the use of fossil fuels and greenhouse gas emissions. In order to do this, many large investments in the electricity system will likely be initiated in the future and it is highly relevant to analyze these potential energy investments.

In order to evaluate investments in energy technologies, the real options approach will be taken as an alternative to the traditional net present value rule. In contrast to the net present value rule, the real options approach allows us to explicitly take into account both uncertainties and flexibilities (the options) in the investments.

The aim of this paper is to consider sequential capacity investments in a single project, for instance an offshore wind site. The agent can invest an initial capacity level below a certain limit and any capacity that is not installed in the initial investment can be installed at a later time. The initial capacity choice determines the value of the profit flow that is received up until the next investment, and affects the value of the secondary investment option as well.

We find that even without learning effects, splitting the capacity choice problem into sequential investments will result in smaller but earlier initial investments. Furthermore we find that the option to make a secondary investment will raise the value of the investment option as a whole.

1 Introduction

Large-scale investments are typically split into smaller stages, in order to decrease the initial outlay of capital, reduce the exposure to unknown risks or to get knowledge from the first investment stages to improve results of the later stages. Stage-wise investment in energy technologies has for example been examined in Gollier et al (2005), while the investment-scale-decision has been discussed in Décamps, Mariotti and Villeneuve (2006). The combination of the two issues - a sequential capacity choice problem - in which the two decision affect one another, has, to the best of our knowledge, not been addressed.

We will take a real options approach to modeling the value of the options and solve as much as possible of the problem analytically. Where we are no longer able to find analytical solutions we will use a least-squares Monte Carlo simulation approach developed by Longstaff & Schwartz (2001), in order to find the optimal investment timing as well as capacity choices. We find that the inclusion of stages in the capacity choice problem will encourage the agent to initiate the investment earlier with a smaller initial investment compared to the results obtained if we only allow a single investment decision with a single capacity choice. The addition of stage-wise capacity decisions thus speeds up the initiation of large-scale projects, by starting the first stage of the project earlier than if the full project had to be initiated all at once. We also find that including this two-stage flexibility in the option greatly increases it's value.

The remainder of this paper is organized as follows: The first section contains a brief discussion of the real options analysis method as well as the least-squares Monte Carlo method used in this paper. We then determine the value of a project and the optimal capacity choice in the case of a single possible investment. We also find the value of holding the option to initiate this investment, as well as the investment trigger. In the following section we expand the model so the investments can occur sequentially, where the first capacity choice affects the value of the secondary investment opportunity. Then a numerical example follows in which we find results of the two models and compare investment timings as well as capacity choices. We conclude with a discussion of our results, their implications and some potential extensions to the research.

2 The Method

Real options valuation - or real options analysis - applies methods from option pricing to evaluate managerial budgeting decisions. A real option can be defined as the right - but not the obligation - to commence various business initiatives, for example beginning, expanding, abandoning or postponing an investment project. The term 'Real Options' was first used in 1977, four years after the notable paper by Black & Scholes (1973) and the method is thus tightly linked with the option valuation techniques developed by these authors. As a mathematical discipline real options has it's roots in financial engineering but has been expanded to cover decision making under uncertainty as a much broader field. It now provides an enhanced analytical framework within which to make decisions given uncertainty in one or more underlying variables. For a general text-book on real options refer to Dixit & Pindyck (1994).

There is a great number of papers where real options analysis is applied to the field of renewable energy, including Siddiqui and Fleten (2010) in which the deployment of competing energy technologies is compared, Gollier et al (2005) who model and examine the value of modularizing a nuclear power plant investment and Décamps, Mariotti and Villeneuve (2006) who study the scale decision of an investment under price uncertainty. Fleten, Maribu and Wangensteen (2007) examine investments in renewable power generation under uncertainty, including a case study of investment in wind power generation.

Developments in contingent claims analysis was started with the papers by Black & Scholes (1973) and Merton (1973), both of which form the basis of the analysis that is used in this paper. To facilitate the analysis we will need to make an identification and description of the underlying uncertainties. We will take a simple approach to modeling the uncertainty, and assume that the revenue stream of the project follows a geometric Brownian motion, as is common in the literature.

Since we are not able to fully solve the two-stage model analytically, we will use a numerical approach to obtain comparable results. More specifically we will use the least-squared Monte Carlo approach presented in Longstaff & Schwartz (2001) to produce the results we will then compare to those of the single-investment model. This method was developed to price financial options of the American type but can easily be applied to our problem.

3 The single-investment model

We begin by solving the problem of investment if the agent is only allowed to make a single investment, by choosing a capacity level $q \leq \bar{q}$. The capacity limit should be interpreted as a site-specific limit, reflecting geographic and spacial constraints, legal limitations, as well as budget constraint and any other limiting factors. Assume that the profit flow of the project is a concave and increasing function of capacity. Due to production losses such as wake effects, the profit flow is marginally decreasing with capacity: $\Pi(\pi; q) = a\pi q^b$, with $a > 0$ and $0 < b < 1$, where π is a measure of the general profitability, varying stochastically with prices, realized production, operational costs and other factors that impact the revenue stream of the project. Given a set capacity, the value of the offshore wind power project is the total expected discounted future profits over its lifetime, T:

$$V(\pi; q) = \mathbb{E}\Big[\int_0^T e^{-rt} a\pi(t) q^b dt \Big| \pi\Big],$$

where r is the rate used to discount future payoffs. Given that the revenue stream π follows a geometric Brownian motion with drift rate α and volatility σ:

$$d\pi(t) = \alpha\pi dt + \sigma\pi dZ,$$

the value of the wind power project can be found analytically and is, for a given capacity:

$$V(\pi; q) = \frac{a\pi q^b}{r-\alpha}\left(1 - e^{-(r-\alpha)T}\right) \equiv \gamma\pi q^b.$$

The optimal capacity of an investment can be determined by solving the simple maximization problem

$$\max_{q\le\bar{q}} \{V(\pi; q) - I(q)\} = \max_{q\le\bar{q}} \left\{\gamma\pi q^b - Aq - B\right\},$$

where we assume that investment costs is an affine and increasing function of capacity: $I(q) = Aq + B$, with $A, B > 0$. Since $V(\pi; q) - I(q)$ is a concave function of the capacity level, the unconstrained problem has a unique solution which can be found by solving $\frac{\partial}{\partial q}V(\pi; q) = I'(q)$. The solution is to choose

$$\tilde{q}(\pi) = \left(\frac{b\gamma\pi}{A}\right)^{\frac{1}{1-b}}. \tag{1}$$

If this is not a valid solution (i.e. if $\tilde{q}(\pi) > \bar{q}$), it is optimal to instead invest the full capacity $\bar{q}$. We thus have that

$$q^*(\pi) = \min\{\tilde{q}(\pi), \bar{q}\}.$$

Note that $\tilde{q}$ depends on the value of the revenue stream π at the time of investment, and so q^* does as well. Inserting the optimal capacity in the value function yields

$$V(\pi; q^*) - I(q^*) = D_i\left(\frac{b\gamma\pi}{A}\right)^{\eta_i} - E_i. \tag{2}$$

where $i = 1$ when $q^*(\pi) = \tilde{q}(\pi)$ and $i = 2$ when $q^*(\pi) = \bar{q}$, with

$$D_1 = \frac{A(1-b)}{b} > 0, \quad E_1 = B > 0, \quad \eta_1 = \frac{1}{1-b} > 1, \tag{3}$$

$$D_2 = \frac{A\bar{q}^b}{b} > 0, \quad E_2 = A\bar{q} + B > 0, \quad \eta_2 = 1. \tag{4}$$

The option to initiate the investment can be exercised at anytime, and must fulfill the following Bellman equation

$$U(\pi) = \max\left\{\max_{q\le\bar{q}} \{V(\pi; q) - I(q)\}, \tfrac{1}{1+rdt}\mathbb{E}\left[U(\pi + d\pi)\right]\right\}. \tag{5}$$

The partial differential equation in the continuation region (i.e. before the investment is made) can be found using Itô's Lemma and is:

$$\frac{1}{2}\sigma_V^2\pi^2U'' + \alpha_V\pi U' - rU = 0,$$

with boundary conditions

$$U(0) = 0, \;\; U(\pi^*) = V(\pi^*; q^*) - I(q^*), \;\; U'(\pi^*) = V'(\pi^*; q^*),$$

where π^* is the investment trigger (or threshold), such that the option is exercised and the investment initiated once the profit flow surpasses this level. The partial differential equation has an analytical solution of the form $U(\pi) = C_1 \pi^{\beta_1} + C_2 \pi^{\beta_2}$.

We can use this functional form to find a characteristic polynomial for the β-parameters:

$$\mathcal{Q}(\beta) = \frac{1}{2}\sigma^2 \beta(\beta - 1) + \alpha\beta - r \quad = \quad 0. \tag{6}$$

Using that $\mathcal{Q}(0) = -r < 0$ and $\mathcal{Q}(1) = \alpha - r < 0$ (by assumption), together with the fact that $\mathcal{Q}(\beta)$ is a parabola with a positive coefficient in front of the second order term, we conclude that one root must lie to the left of zero (i.e. be negative) and one root must lie to the right of 1 (i.e. be positive). In order to satisfy the first boundary condition, we only need the positive solution, and conclude that U must be of the form $U(\pi) = C\pi^\beta$, where $\beta > 1$ is the positive root of the polynomial above.

Using this functional form as well as the expression for V found earlier we can use the boundary conditions to find that

$$\begin{aligned} \pi^* &= \frac{A}{b\gamma} \left(\frac{E_i}{D_i} \frac{\beta}{\beta - \eta_i} \right)^{\frac{1}{\eta_i}} & (7) \\ C_i &= E_i \frac{\eta_i}{\beta - \eta_i} (\pi^*)^{-\beta}, & (8) \end{aligned}$$

In order to determine when we invest $\tilde{q}(\pi^*)$ and when we invest $\bar{q}$, we plug the results for π^* into the expression for $\tilde{q}$ to see that

$$i = 1 \quad \Longleftrightarrow \quad q^* = \tilde{q} \quad \Longleftrightarrow \quad \tilde{q} < \bar{q} \quad \Longleftrightarrow \quad \bar{q} > \frac{Bb}{A\left(1 - b - \frac{1}{\beta}\right)}, \tag{9}$$

and $i = 2$ if the opposite inequality holds.

4 Two sequential investments

Assume now that the investor has the possibility to make a secondary investment of $q_2 \leq \bar{q} - q_1$ at any time, after the initial investment has been made. The initial investment thus yields both a project value as well as a secondary option value; after the initial investment the agent is left with the choice between making the secondary investment or waiting for more information.

4.1 The secondary investment

When the second investment is initiated the capacity will increase from q_1 to $q_1 + q_2$, thus increasing the value of the project by $V_2(\pi) = \gamma\pi((q_1 + q_2)^b - q_1^b)$, while paying the investment costs $I_2(q_2) = Aq_2 + B_2$, again with $A, B_2 > 0$. The Bellman equation for the secondary option value thus becomes:

$$U_2(\pi; q_1) \quad = \quad \max\left\{ \max_{q_2 \leq \bar{q} - q_1} \left\{ \gamma\pi((q_1 + q_2)^b - q_1^b) - Aq_2 - B_2 \right\}, e^{-rdt}\mathbb{E}\left[U_2(\pi + d\pi; q_1)\right] \right\}.$$

Note that for any given q_1, the partial differential equation governing the option value in the continuation region is exactly the same as in the one-investment-problem of section 3. It thus also has the solution $U_2(\pi; q_1) = C\pi^\beta$. We will later see that the constant C will depend on the initial capacity choice q_1.

The optimal capacity choice is calculated by solving the first order condition as before and we find that

$$q_2^*(\pi) = \min\{\tilde{q}(\pi) - q_1, \bar{q} - q_1\} = \begin{cases} \left(\frac{\gamma\pi b}{A}\right)^{\frac{1}{1-b}} - q_1, & \text{for } \left(\frac{\gamma\pi b}{A}\right)^{\frac{1}{1-b}} \leq \bar{q} \\ \bar{q} - q_1 & \text{for } \left(\frac{\gamma\pi b}{A}\right)^{\frac{1}{1-b}} > \bar{q} \end{cases}$$

where $\tilde{q}(\pi)$ is defined in equation (1). Using these we find that the value of making the investment (using the optimal capacity choice found above) is

$$V_2(\pi, q_2^*; q_1) - I_2(q_2^*) = \begin{cases} A\left(\frac{1-b}{b}\right)\left(\frac{\gamma\pi b}{A}\right)^{\frac{1}{1-b}} - (\gamma\pi q_1^b - Aq_1) - B_2, & \text{for } \left(\frac{\gamma\pi b}{A}\right)^{\frac{1}{1-b}} \leq \bar{q} \\ \gamma\pi\bar{q}^b - (\gamma\pi q_1^b - Aq_1) - A\bar{q} - B_2 & \text{for } \left(\frac{\gamma\pi b}{A}\right)^{\frac{1}{1-b}} > \bar{q} \end{cases} \tag{10}$$

For either case we find that the option value can be expressed as

$$U_2(\pi, q_1) = \begin{cases} \left(\frac{\pi}{\pi_2^*(q_1)}\right)^\beta (V_2(\pi, q_2^*; q_1) - I_2(q_2^*)), & \text{for } \pi < \pi_2^*(q_1) \\ (V_2(\pi, q_2^*; q_1) - I_2(q_2^*)) & \text{for } \pi \geq \pi_2^*(q_1) \end{cases}$$

where $V_2(\pi, q_2^*; q_1) - I_2(q_2^*)$ is defined in equation (10). However, only when $\left(\frac{\gamma\pi_2^* b}{A}\right)^{\frac{1}{1-b}} > \bar{q}$ (so $q_2^*(\pi_2^*) = \bar{q} - q_1$) is it possible to find an analytical solution for the investment threshold $\pi_2^*(q_1)$ and fully solve the problem. In order to get results comparable to the single-investment model we will instead use a numerical simulation method.

4.2 The initial investment

When deciding the timing of the initial investment the gains of undertaking the investment is the project value V_1 as well as the secondary option value U_2. The Bellman equation that describes this decision problem is

$$\begin{aligned} U_1(\pi) &= \max\left\{\max_{q_1\le\bar{q}}\left\{V_1(\pi,q_1)-I_1(q_1)+U_2(\pi,q_1)\right\}, e^{-rdt}\mathbb{E}\left[U_1(\pi+d\pi)\right]\right\} \\ &= \max\left\{\max_{q_1\le\bar{q}}\left\{\gamma\pi q_1^b-Aq_1-B_1+U_2(\pi,q_1)\right\}, e^{-rdt}\mathbb{E}\left[U_1(\pi+d\pi)\right]\right\}. \end{aligned}$$

In order to continue we need to make use of a small lemma:

Lemma 1: We must have that $\pi_1^* < \pi_2^*$.
Proof: Assume the opposite, that $\pi_1^* \ge \pi_2^*$. This means that once the initial investment is made (at the threshold level π_1^*), it is optimal to initiate the secondary investment right away. Doing so will lead to a total invested capacity of q_1+q_2 at a cost of $A(q_1+q_2)+B_1+B_2$. Had the total capacity q_1+q_2 been invested in one go, the costs would have been $A(q_1+q_2)+B_1$ which would make the investor better off. So, the secondary investment must have a higher profit threshold than the initial one.

We can use Lemma 1 to conclude that $U_2(\pi,q_1)=\left(\frac{\pi}{\pi_2^*(q_1)}\right)^\beta\left(V_2(\pi,q_2^*;q_1)-I_2(q_2^*)\right)$ at $\pi=\pi_1^*$. The initial capacity choice problem can now be reduced to choosing the q_1 that maximizes

$$\gamma\pi q_1^b-Aq_1-B_1+\left(\frac{\pi}{\pi_2^*(q_1)}\right)^\beta\left(V_2(\pi,q_2^*;q_1)-I_2(q_2^*)\right),$$

where the form of the V_2-I_2 term depends on the parameters of the model (as seen in equation (10)), and where we are not always able to find an analytical expression for $\pi_2^*(q_1)$.

5 Results

We will use the least-squared Monte Carlo approach presented in Longstaff & Schwartz (2001) to generate results for the two models in order for us to properly compare them. We begin by simulating a number of paths for the underlying π, using a drift rate of $\alpha=0.01$ and a volatility of $\sigma=0.20$. We initiate the paths at a normalized $\pi_0=100$. To solve for the initial capacity choice and timing of the first investment, we discretize the capacity q_1 so as to take values in $\{0,40,80,\dots,\bar{q}=4000\}$. For every path and every time period we find the capacity that would yield the best payoff (consisting of both project value and expected value of the secondary option[1]). Using this optimal payoff of investing at any given time period, we work our way backwards through the paths: For any period the agent can either invest now (earning the payoff of the current optimal capacity) or choose to wait. In order to determine the value of waiting we use the least-squares Monte Carlo approach described in Longstaff & Schwartz (2001) - we use a second order polynomial to estimate the conditional value of waiting. Working backwards from the last time period to the first, we find the optimal time of investing

[1]the secondary option value is found using this exact same least-squares Monte Carlo method

for every simulated path, as well as the initial capacity, q_1 invested in each of them. The parameters used for the calculation of payoffs are $a = 0.08$, $b = 0.06$, $A = 3$ and $B = 300$. We simulate 10000 price paths of 10 periods[2] (years), with possible investments each period.

Using these numbers the value of the option (at time zero) increased from 4198 in the one-stage model to 5205 in the two-stage model, an increase in value of 24%. The value of considering sequential capacity choices and not just a single investment is thus immense and should definitely not be overlooked. We now take a closer look at the actual solutions of the two models and try to find how the extra flexibility of the two-stage model adds so much value to the investment option.

5.1 Investment timing

We will begin by examining and comparing the optimal investment timing of the two models. For each simulated path (note that the same set of simulations of the underlying is used for both models), the time of the investment is stored as part of the final cash-flow matrix (as described in Longstaff & Schwartz (2001)). In the one-stage model it is simply the time at which the investment is made, in the two-stage model we have only recorded the time that the initial investment is made. The distributions of the investment timing can be seen in the histograms in Figure 1:

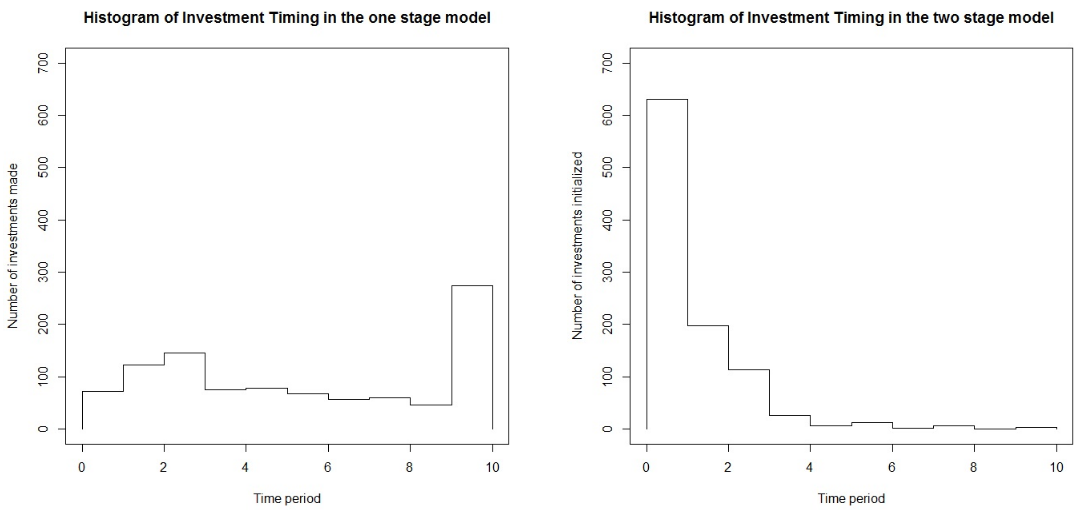

Figure 1: Distribution of the (first) investment timing in the two models.

[2]More time periods could have been chosen, but since options to invest in for example a wind farm site are not infinite, it was chosen to keep the time horizon relatively short.

We see right away that by allowing a second investment in the project, the entire distribution of the initial investment time has shifted to the left. That is, the agent is much more likely to invest early. The 'average investment initiation time' has decreased from 5.8 in the one-stage model to only 1.7 in the two-stage model. The reason for this much speedier investment is that when a secondary investment is allowed after the initial one, it is profitable to start the project at less-than-perfect conditions (measured in the value of the underlying π), since it will still be possible to expand should the conditions improve. In the one-stage model, however, this future expansion is not possible, and the agent will be more willing to wait for better conditions before making the final decision to invest.

5.2 Capacity invested

If we instead turn towards the capacity invested in the one-stage model and compare it to the capacity invested in the first stage of the two-stage model, we see that the choice is to invest much less when there is an opportunity to increase the installed capacity at a later date. In the one-stage model, the full capacity limit is installed as often as 40% of the time, whereas the initial investment of the two-stage model almost never exceeds half of the capacity limit. Two histograms of the chosen capacity levels of the two models can be found in Figure 2:

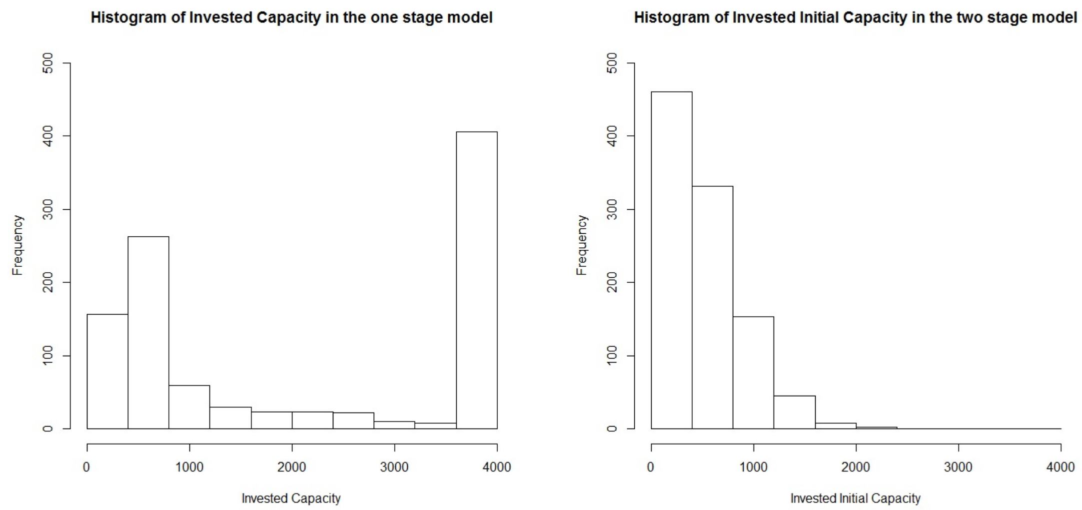

Figure 2: Distribution of the (initial) capacity choice in the two models.

So, not only will the agent invest earlier, the chosen capacity level will also initially be lower. The reason is of course as before, that the capacity can be increased at a later date. This means that a small initial capacity project can still take advantage of improving conditions (by increasing the capacity), a feature which does not exist in the one-stage model. The reason for this is that the profitability of the project is a concave function of capacity - i.e. the marginal value of capacity is decreasing, which means that the first capacity installed is also the most valuable. This suggests that the project should be initiated much faster, since the first small investment will be profitable at much lower values of the underlying π. And, if conditions improve, the size of the project can simply be increased at a later time.

6 Conclusions

The possibility of stage-wise capacity choices in a project will lead the investor to undertake investments sooner, but start off the project at a smaller scale. Including this possibility vastly increases the value of the investment option as a whole, in our numerical case the option value increased by 24%. Comparing the results of the two models we can see that in the one-stage model the agent waits for the value of the underlying to reach a level that will support a large investment, as this increases the expected value of the project. In the two-stage model, the agent can instead make a small investment in the project very early, start earning a payoff and once the value of the underlying reaches a high enough level, the large scale expansion is made. The fact that future payoffs are discounted (so, all else equal, a fast investment is worth more than a late investment) only serves to magnify this effect.

This means that whenever the profitability of a project is a concave function of installation size (for example large-scale wind sites which feature production losses due to wake effects), the investors should consider splitting the projects into smaller stages of investment, using the methods discussed in this paper to determine the size of each stage. Empirically we already see that most large-scale off shore wind farms are indeed being installed in smaller stages. Two examples include the German Innogy Nordsee 1-3: a three stage 1000 MW site in the North Sea of which the first 330 MW stage has just been approved, and the London Array: an offshore wind farm for which the first phase of 630 MW capacity is being constructed in the outer Thames Estuary.

Potential extensions to the research covered in this paper includes a study of the comparative statics, testing how changes in the different parameters will affect the results of the analysis. The authors would also like to acquire a proper case study to fit the model and test the results under a set of case-specific parameters. Finally it should be noted that the assumption that the underlying should follow a geometric Brownian motion is only needed to yield the analytical solutions, but a much wider selection of models can be used, including mean-reverting or jump processes, to simulate the paths of the underlying for the numerical analysis. Lastly the authors would also like to look at including learning effects in the model. We note that most of these extensions are straightforward to include.

References

[1] Black, F. & Scholes, M. *The Pricing of Options & Corporate Liabilities*, The Journal of Political Economy (May '73).

[2] Dixit, A.K. & R.S. Pindyck, *Investment under Uncertainty*, Princeton University Press, 1994, chapters 1-7, pp. 3-244.

[3] Décamps, J.P., Mariotti, T., & Villeneuve, S., *Irreversible Investment in Alternative Projects*, Economic Theory 28(2): 425-448 (2006).

[4] Fleten, S.-E., Maribu, K.M., & Wangensteen, I., *Optimal Investment Strategies in Decentralized Renewable Power Generation under Uncertainty*, Energy 32(5): 803-815 (2007).

[5] Gollier, C., Proult, D., Thais, F. & Walgenwitz, G. *Choice of nuclear power investments under price uncertainty: Valuing modularity*, Energy Economics, Elsevier, vol. 27(4), pages 667-685, July 2005.

[6] Longstaff, F. A., Schwartz, E. S., *Valuing American Options by Simulation: A Simple Least-Suares Approach*, The Review of Financial Studies, 14:113-147, 2001.

[7] Merton, R.C., *Theory of rational option pricing*, Bell Journal of Economics and Management Science, 4 (1), 141-183. Available in Merton, 1973.

[8] Siddiqui, A.S. & Fleten, S.-E., *How to Proceed with Competing Alternative Energy Technologies: a Real Options Analysis*, Energy Economics 32(4): 817-830 (2010).