Impulse Maximum Principle under Pure State Constraint: An Application to Optimal Capacity Expansion

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May 2012

The mechanisms underlying capacity expansion have been investigated for more than fifty years. The problem was introduced in the 1950's, and many papers contributed to a better understanding of how firms should invest in production capacity. Many approaches were used, the most popular including linear programming (beginning with [Massé and Gibrat1957]), mixed integer programming [Noonan and Giglio 1977] and dynamic programming (for instance [Petersen 1973]). Different assumptions were also made, in both deterministic and stochastic frameworks. Surprisingly, the capacity expansion problem was hardly ever formulated as an optimal control problem, as was done in [Park and al. 1984, Kort 1987], and solved using one of the maximum principles derived from the original work of Pontryagin's team at the Steklov Institute in the 1970's.

In this presentation, we would like to share some results of a work in progress. We formulate the capacity expansion problem as an optimal control problem with pure state constraint, and show some characteristics of the optimal solution with different assumptions over the investment cost function. An interesting case is obtained for concave investment costs (economies of scale), as they result in the non convexity of the Hamiltonian to be minimized. We show that the problem restricted to the space of continuous functions has no solution, but that a solution exists if we allow the state variable to be discontinuous. An impulse maximum principle as established by [Arutyunov and al. 2005] enables us to compute the optimal investment strategy, proving the results obtained by [Manne 1961].

Outlooks for the work are the introduction of uncertainty and endogenous pricing in an oligopolistic competition, switching the objective function from cost minimization to profit maximization. A parallel to real options, as introduced by [Dixit and Pindyck 1994], will be made. Potential applications for this work include the analysis of the investment dynamics in deregulated electricity markets.

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