A Fundamental Electricity Pricing Model with Forward-Looking Information*

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This version: October 2012

Abstract

In an initiative to increase overall transparency on key operational information, power transmission system operators publish an increasing amount of fundamental data, including forecasts of electricity demand and available capacity. We develop a fundamental model for electricity prices which lends itself well to integrating such forecasts, while retaining a focus on ease of implementation and tractability to allow for analytic derivatives pricing formulae. In an extensive futures pricing study, the pricing performance of our model is shown to further improve based on the inclusion of electricity demand and capacity forecasts, thus confirming the importance of forward-looking information for electricity pricing.

JEL classification: G12, G13, Q4, Q41

Keywords: Electricity Pricing, Fundamental Model, Derivatives Pricing, Forward-looking Information, Enlargement of Filtrations

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*We have greatly benefited from the helpful comments of Enrico De Giorgi and Karl Frauendorfer. We also thank Richard Frape from Marex Spectron and Richard Price from National Grid for providing data as well as explanations on the intricacies of the British electricity market.

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I Introduction

Following the liberalization of electricity markets in many countries, utility companies and other market participants have been facing an increasing need for new pricing models in order to accurately and timely evaluate spot and derivative electricity contracts that they aim to buy and sell as part of their business. However, the end of cost-based pricing and the transition towards a *de*-regulated market environment also gave rise to new financial risks, threatening to impose substantial losses especially for, e.g., sellers of electricity forward contracts. As such, the necessity to now optimize against the market for both standard electricity products as well as tailored contingent claims additionally required effective and integrated risk management strategies to be developed.

These developments have to be seen in the context of the unique behavior of electricity (spot) prices, which is primarily induced by the non-storability of this commodity: apart from hydropower with limited storage capabilities, an exact matching of electricity demand and supply is required at every point in time. The resulting price dynamics with their well-known *stylized facts* such as spikiness, mean-reversion, and seasonality, have extensively been analyzed in related literature,[1] yet still pose a challenge to both practitioners and researchers in terms of adequately modeling and forecasting their trajectories.

However, the non-storability of electricity has further implications on the price-formation mechanism. First, it is the instantaneous nature of electricity that – other than in a classic storage economy – causes the intertemporal linkages between economic agents' decisions today and tomorrow to break down. In fact, this forms the basis for electricity markets being usually characterized as very transparent with respect to their underlying economic factors, including electricity demand/consumption, available levels of generation capacity, as well as the costs for generating fuels and emissions allowances. Against this background, structural approaches taking this information explicitly into account appear especially appealing to electricity price modeling (see, e.g., Pirrong, 2012). Second, and as the above implies, the classic assumption that the evolution of all relevant pricing information, i.e., the information filtration, is fully determined by the price process of the commodity

[1] See, e.g., Johnson and Barz (1999), Burger et al. (2004) or Fanone et al. (2012).

itself, does not hold for non-storable assets such as electricity. In other words, today's electricity prices do not necessarily reflect forward-looking information that is publicly available to all market participants.[2] At the same time, legal requirements and voluntary initiatives to increase data transparency have had power transmission system operators (TSOs) publish (online) an increasing amount of data regarding the condition of their network, including, e.g., forecasts about expected electricity demand or updated schedules of planned short-term outages.[3] Pricing electricity spot and derivatives contracts based on models that make use of historical information only, may hence result in substantial errors since the model leaves aside important (forward-looking) information, although it is publicly available and very likely to play a key role for individual trading decisions.

In this paper, we hence focus on the prominent role of forward-looking information in electricity markets and investigate its impact on empirical pricing performance. As such, our study contributes to extant literature in the following ways:
First, we propose a new fundamental model for electricity pricing including fuel, demand, and capacity dynamics that successfully captures the stylized facts of this commodity and provides analytic derivatives pricing formulae.
Second, most studies that propose new fundamental electricity pricing models do not calibrate the models to market data. If so, however, they mainly focus on time-series fitting or provide pricing results for single, selected forward contracts as illustrative examples only. In this article, we test our model in an extensive empirical study, using a comprehensive data set of forward contracts traded in the British electricity market. This also allows us to investigate several interesting implementation challenges that arise during the calibration procedure.

[2] Benth and Meyer-Brandis (2009) provide several examples in support of this argument, such as the case of planned maintenance for a major generating unit, which is likely to be public information available to all market participants. Assuming a stylized setting, this outage will necessarily affect electricity spot prices expected to prevail during the time the unit is offline. Likewise, the outage will also affect today's prices of derivative contracts such as forward/futures contracts if their delivery period overlaps with the period of scheduled maintenance. However, in the absence of any means to economically store electricity bought at (cheaper) spot prices today and to sell it at higher prices during the time of the outage, there is no way to arbitrage such situation – implying, hence, that today's electricity spot prices will remain virtually unaffected by the announcement of the outage.

[3] Regulations (EC) No. 1228/2003, its follow-up No. 714/2009 and annexed "Congestion Management Guidelines" (CMG) may serve as the most prominent example, requiring, e.g., that "*the TSO shall publish the relevant information on forecast demand and on generation (...)*" (CMG, article 5.7).

Third, and to the best of our knowledge, we are the first to empirically investigate the pricing of derivative contracts in electricity markets by explicitly making use of forward-looking information. By means of an enlargement-of-filtration approach, we show how to properly integrate forecasts of electricity demand and available capacity into our setting from a technical point of view, and thus account for the apparent asymmetry between the historical filtration and the (enlarged) market filtration in electricity markets.

In a very rough distinction, existing literature on electricity *spot* price modeling can be grouped into two categories: often allowing for analytic derivatives pricing formulae, considerable attention has been devoted to reduced-form models that either directly specify dynamics for the electricity spot price process itself or, alternatively, model the term structure of forward contracts, in which case spot dynamics are derived from a forward contract with immediate delivery (see, e.g., Clewlow and Strickland, 2000, Koekebakker and Ollmar, 2001, or Benth and Koekebakker, 2008). Starting with traditional commodity modeling approaches via mean-reverting one- or two-factor models (Lucia and Schwartz, 2002), a more adequate reflection of the stylized facts of electricity spot price dynamics demands for more elaborate settings including (affine) jump diffusion processes and/or regime-switching approaches (see, e.g., Bierbrauer et al., 2007, Weron, 2009, or Janczura and Weron, 2010, for a comprehensive overview). However, this may still not be sufficient to reliably differentiate between spike- and non-spike regimes as observed in reality, or to adequately capture the (absolute) spikiness of electricity prices. As a solution, additional enhancements have been proposed, such as considering non-constant deterministic or stochastic jump intensities (see, e.g., Seifert and Uhrig-Homburg, 2007) and their impact on possibly different speeds of mean-reversion of the underlying Ornstein-Uhlenbeck (OU) process, which, in turn, negatively affects analytic tractability. The same is true when trying to mitigate other common drawbacks such as models precluding successive upward jumps or leaving jump intensities unaffected by previous jumps. Extensions like Barone-Adesi and Gigli (2002) try to address these problems but must resort to non-Markovian models, which, however, restricts the applicability for contingent claim valuation. Finally, and as a point of structural criticism, reduced-form models obviously fail to analyze the dependence structure between prices and the electricity markets' underlying drivers, which not only leaves unexplained important features such as the occurrence of price spikes, but also affects their applicability for fields such as cross-commodity option valuation (unless, e.g., a co-integration setting is employed such as in Emery and Liu, 2002, de Jong and Schneider, 2009, or Paschke and Prokopczuk, 2009). In this context, and given the above mentioned increase in publicly available (fundamental) data released by TSOs, it must be seen as a drawback of classic reduced-form models that they obviously fail to take direct benefit from this increasing transparency.[4]

On the other hand, the class of structural/fundamental electricity price models subsumes a wide spectrum of more diverse modeling approaches; starting with equilibrium-based models (Bessembinder and Lemmon, 2002, Bühler and Müller-Mehrbach, 2007) or even more richly parameterized full production cost models (Eydeland and Wolyniec, 2002) on the one end, but also including, on the other end, econometric approaches such as regression-based settings (Karakatsani and Bunn, 2008) or time-series models whose efficiency is enhanced by including exogenous fundamental variables (Weron, 2006, or Misiorek et al., 2006).

Often referred to as hybrid approach, the class of models focused on in this study may be seen in the middle of such spectrum.[5] In its most general form, fundamental settings of this kind comprise of a selection of separately modeled underlying factors, such as electricity demand, available generation capacity, and fuels. Along with a specification of the functional relationship between these factors and electricity (spot) prices, this setting

[4]We note that it is still possible to integrate information about the dynamics of fundamental state variables (such as demand or, e.g., also temperature) into reduced-form models by means of correlated processes; see Benth and Meyer-Brandis (2009) for an example. However, even though such models may bridge the gap between classic reduced-form and fundamental approaches, it is still questionable whether a single correlation parameter may be sufficient to reflect the rich dependence structures between electricity prices and a fundamental state variable – all the more if the dynamics of several underlying variables are to be taken into account at the same time.

[5]In order to avoid ambiguities, if we refer to *fundamental* electricity price models throughout the rest of this paper, we shall actually mean the hybrid class of models *within* this category.

can hence be interpreted as merit-order framework.[6] The main challenge in this context is to be seen in an adequate reflection of the characteristic slope and curvature of the merit-order curve that is usually characterized by significant convexity.[7] As a matter of simplification, many studies (see, e.g., Skantze et al., 2000, Cartea and Villaplana, 2008, or Lyle and Elliott, 2009) propose to approximate the merit-order curve with an exponential function. While there may be other functional specifications yielding a better fit, such as a piecewise defined "hockey stick" function (Kanamura and Ohashi, 2007) or power laws (Aïd et al., 2011), the exponential setting offers the key advantage of yielding log-normal electricity spot prices, allowing for analytic derivatives pricing formulae.

Requiring our model to provide timely pricing information to market participants by retaining tractability, we hence also adopt an exponential setting in order to represent the merit-order curve; as regards the inclusion of generating fuels, we follow Pirrong and Jermakyan (2008) by modeling a stylized one-fuel market, leaving aside more flexible multi-fuel approaches such as presented in Aïd et al. (2009, 2011), Coulon and Howison (2009) and Carmona et al. (2011). Whereas our one-fuel setting avoids a model-endogenous determination both of the merit-order and the marginal fuel in place, it remains to be discussed how this reduction in flexibility affects pricing results, and for which markets such a simplification may be viable at all.

Regarding the question of how to account for forward-looking information in this context, many of the above presented models could in fact be modified to accommodate short-, mid- or long-term forecasts about future levels of electricity demand or available capacity. However, extant literature mainly focuses on the benefits of using day-ahead demand/capacity forecasts in order to improve day-ahead electricity pricing performance, such as Karakatsani and Bunn (2008) or Bordignon et al. (2011). A different approach

[6] Alternatively, the functional relationship can also be seen as inverse supply curve or bid-stack, if we abstract from generators submitting bids exceeding marginal costs. Also, our setting implicitly assumes electricity demand being completely inelastic, which is a basic assumption for models of this kind. See Carmona and Coulon (2012) for further reference as well as for a general and comprehensive review of the fundamental modeling approach.

[7] This is a non-trivial issue given that the curvature is determined by both the individual composition of generating units for each marketplace as well as their (marginal) cost structure which, in turn, depends stochastically on other factors such as underlying fuel prices, weather conditions, (un-)planned outages, and daily patterns of consumption. Additional factors to be considered may include market participants exercising market power by submitting strategic bids, but also regulatory regimes awarding, e.g., preferential feed-in tariffs to renewable energy producers.

regarding the integration of forecasts into a pricing model is proposed by Cartea et al. (2009). In their study, a regime-switching setting is invoked where the ratio of expected demand to expected available capacity is used to determine an exogenous switching component that governs the changes between "spiky" and "normal" spot price regimes. In this way, the modeling of spikes present in spot prices can be improved, although the model only resorts to very few forecast points per week and available forecasts are not explicitly part of the price formation mechanism. Burger et al. (2004) also present a model that requires as input normalized electricity demand, i.e. demand scaled by available capacity. For the latter, the usage of forecasts of future capacity levels is suggested, but not focused on in more detail.

Finally, the application of the enlargement-of-filtration approach to electricity markets was initially proposed by Benth and Meyer-Brandis (2009). Focusing on risk premia rather than on forward pricing, Benth et al. (2012) use this concept in order to analyze the impact of forward-looking information on the behavior of risk premia in the German electricity market. The authors develop a statistical test for the existence of an *information premium*[8] and show that a significant part of the oftentimes supposedly irregular behavior of risk premia can be attributed to it.[9]

The remainder of this paper is structured as follows: in the next section, we develop our underlying pricing model. Section III introduces the concept of the enlargement-of-filtration approach and discusses how it can be applied in the context of fundamental electricity price modeling. The empirical part of this article starts with Section IV where the data used, the estimation of the model, and the general structure of the pricing study are described. Section V presents the empirical results, Section VI concludes.

[8]The information premium is defined as the difference between forward prices, depending on whether or not forward-looking information is entering the price formation mechanism.

[9]On a more general note, the idea to resort to forward-looking information, of course, extends to numerous other fields of academic research. Another "natural" candidate is, by way of example, the pricing of weather derivatives. For studies that resort to temperature forecasts in order to price temperature futures, see, e.g., Jewson and Caballero (2003), Dorfleitner and Wimmer (2010) or Ritter et al. (2011).

II A Fundamental Electricity Pricing Model

A. Electricity Demand

Electricity demand is modeled on a daily basis with its functional specification chosen so as to reflect typical characteristics of electricity demand such as mean-reverting behavior, distinct seasonalities as well as intra-week patterns. On a filtered probability space $\big(\Omega, \mathcal{F}^D, \mathbb{F}^D = (\mathcal{F}^D)_{t\in[0,T^\star]}, \mathbb{P}\big)$ with natural filtration $\mathbb{F}^q = (\mathcal{F}^q)_{t\in[0,T^\star]}$ (for $\mathcal{F}_t^D = \mathcal{F}_0 \vee \mathcal{F}_t^q$), demand D_t is assumed to be governed by the following dynamics:

$$\begin{aligned} D_t &= q_t + s^D(t) && (1)\\ \mathrm{d}q_t &= -\kappa^D q_t \mathrm{d}t + \sigma^D \mathrm{e}^{\varphi(t)} \mathrm{d}B_t^D && (2)\\ s^D(t) &= a^D + b^D t + \sum_{i=2}^{12} c_i^D M_i(t) + c_{WE}^D WE(t) + \sum_{j=1}^{4} c_{PH_j}^D PH_j(t) && (3)\\ \varphi(t) &= \theta \sin\big(2\pi(kt+\zeta)\big), && (4) \end{aligned}$$

where q_t is an OU-process with mean-reversion parameter κ^D and a standard Brownian motion B_t^D. Since volatility of electricity demand has often been found to exhibit seasonal levels of variation (see, e.g., Cartea and Villaplana, 2008),[10] we apply a time-varying volatility function as proposed by Geman and Nguyen (2005) or Back et al. (2012), with $\theta \geq 0$, a scaling parameter $k = \frac{1}{365}$, and $\zeta \in [-0.5; 0.5]$ to ensure uniqueness of parameters.[11] In order to also reflect absolute-level demand-side seasonality, the deterministic component $s^D(t)$ contains monthly dummy variables $M_i(t)$ as well as additional indicators for weekends $WE(t)$ and public holidays.[12] A linear trend is also

[10]As our estimation results will show, volatility of electricity demand in the British market is higher during winter months than during summer months. However, this effect may be less pronounced or even reversed for other markets where, e.g., the need for air conditioning during summer months drives electricity demand to higher (and more volatile) levels than during winter months.

[11]This volatility specification allows for continuous differentiability, which is a technical necessity in the context of the enlargement-of-filtration approach. See the technical appendix for further information.

[12]Since the extent of a demand reduction induced by a public holiday strongly depends on the respective season prevailing, three different groups of public holidays shall be distinguished: those occurring in winter (PH_2), the Easter holidays (PH_3), and the remainder (PH_4). Additionally, the days with reduced electricity demand between Christmas and New Year are treated as *quasi*-public holidays (PH_1). This may appear overly detailed, however, almost all coefficients turn out to be highly significant; see Bühler and Müller-Mehrbach (2009) for an even more detailed approach.

included in $s^D(t)$ in order to capture the effect of structural developments in the respective market that may lead to an increase or decrease of electricity demand in the long-term.

B. *Available Capacity*

Available capacity C_t is modeled in a similar manner as electricity demand. Hence, on a filtered probability space $\big(\Omega, \mathcal{F}^C, \mathbb{F}^C = (\mathcal{F}^C)_{t\in[0,T^\star]}, \mathbb{P}\big)$ with natural filtration $\mathbb{F}^m = (\mathcal{F}^m)_{t\in[0,T^\star]}$ (for $\mathcal{F}_t^C = \mathcal{F}_0 \vee \mathcal{F}_t^m$), we specify the following dynamics:

$$
\begin{aligned}
C_t &= m_t + s^C(t) && (5)\\
\mathrm{d}m_t &= -\kappa^C m_t \mathrm{d}t + \sigma^C \mathrm{d}B_t^C && (6)\\
s^C(t) &= a^C + b^C t + \sum_{i=2}^{12} c_i^C M_i(t) + c_{WE}^C WE(t) + \sum_{j=1}^{4} c_{PH_j}^C PH_j(t) + c_R^C R(t), && (7)
\end{aligned}
$$

where m_t is again an OU-process with mean-reversion parameter κ^C and constant volatility σ^C (in contrast to demand D_t, available capacity C_t is generally not found to exhibit seasonality in volatility levels). B_t^C is a standard Brownian motion and $s^C(t)$ is defined analogously to $s^D(t)$. In addition, another dummy variable $R(t)$ is included in order to reflect the fact that, other than for the electricity demand data used in this study, our capacity data includes generating units from Scotland only after April 2005.[13]

C. *Marginal Fuel*

In addition to the processes for electricity demand and available capacity, we introduce the dynamics for our third state variable, i.e., the marginal fuel used for generation. As a matter of simplification, we assume that the marginal fuel for the respective electricity market under study does not change: while this certainly is a restrictive assumption, it may still seem justified for markets that are strongly relying on one generating fuel only so that during baseload/peakload hours, spot markets are primarily cleared by plants

[13]The introduction of the British Electricity Trading and Transmission Agreements (BETTA) as per April 2005 is generally referred to as the starting point of a UK-wide electricity market. Prior to that, and although linked via interconnectors, the electricity markets of England/Wales and Scotland were operating independently.

that use the same fuel for generation. Reflecting the dominant role of natural gas as marginal generating fuel in the British market – and, more generally, in several other major electricity markets – we include it as single fuel into our overall pricing model. Although for modeling natural gas, a variety of multi-factor approaches with varying degree of sophistication have been proposed by recent literature (see, e.g., Cartea and Williams, 2008, for an overview), we seek to limit both complexity and (the already high) parametrization of the model and, therefore, apply the mean-reverting one-factor model initially proposed by Schwartz (1997). On a filtered probability space $\big(\Omega, \mathcal{F}^g, \mathbb{F}^g = (\mathcal{F}^g)_{t\in[0,T^\star]}, \mathbb{P}\big)$, the log gas price, $\ln g_t$, is assumed to be governed by the following dynamics:

$$\ln g_t = X_t + s^g(t) \tag{8}$$

$$\mathrm{d}X_t = -\kappa^g X_t \mathrm{d}t + \sigma^g \mathrm{d}B_t^g \tag{9}$$

$$s^g(t) = a^g + b^g t + \sum_{i=2}^{12} c_i^g M_i(t), \tag{10}$$

where X_t is the logarithm of the de-seasonalized price dynamics and $s^g(t)$ reflects the strong seasonality component that is inherent in natural gas prices. It should be noted that the overall structure of our power price model as well as the availability of closed-form solutions will be retained when introducing refinements such as a multi-factor log-normal model for natural gas.[14]

D. *Pricing Model*

In order to link the three state variables – marginal fuel g_t, electricity demand D_t, and capacity C_t – with electricity (spot) prices P_t, we employ an exponential setting, thus reflecting the convex relationship between prices and load/capacity as induced by the merit-order curve. At the same time, we assume power prices to be multiplicative in

[14]While applying a one-factor model for natural gas prices may be seen as simplistic (since the structure of this model implies that all natural gas forward/futures contracts are perfectly correlated across maturities), we note that in this paper, we primarily focus on pricing short-term electricity forward contracts for which only the short end of the curve may be relevant. However, when pricing longer-term electricity contracts, we suggest a two-factor natural gas price model be employed instead.

the marginal fuel; both these assumptions can be considered common practice (see, e.g., Carmona and Coulon, 2012) and yield the following structural relationship between power prices and state variables:

$$P_t = \alpha\, g_t^{\delta}\, \mathrm{e}^{\beta D_t + \gamma C_t} \tag{11}$$

Or, in log-form:

$$\ln P_t = \ln \alpha + \delta \ln g_t + \beta D_t + \gamma C_t \tag{12}$$

where δ can be interpreted as the elasticity of the electricity spot price with respect to changes in the natural gas price. Setting $\delta = 1$ would thus allow to interpret $\mathrm{e}^{\beta D_t + \gamma C_t}$ as heat rate function.[15] However, given that we primarily investigate baseload power prices in the empirical part of this paper, we acknowledge that the elasticity of baseload power prices with respect to natural gas may be varying and, hence, do not impose the restriction $\delta = 1$.

Also, and as will be seen later, there is a subtle form of dependence between the parameters α and γ. In order to give an intuition for the role of α, and providing an abstract link to structural *multi*-fuel power price models at the same time, note that Equation (11) can also be re-written as follows:

$$P_t = \underbrace{f_t^{(1-\delta)}}_{\alpha}\, g_t^{\delta}\, \mathrm{e}^{\beta D_t + \gamma C_t} \tag{13}$$

In Equation (13), α can hence be interpreted as reflecting the dynamics of another generating fuel f_t (such as coal) which, however, will be held constant for simplicity.

Following classic theory, (electricity) futures prices equal the expectation of the spot price at maturity under a suitably chosen risk-neutral measure $\mathbb{Q}$ (Cox and Ross, 1976, and Harrison and Kreps, 1979). However, the non-storability of electricity creates non-

[15] The *heat rate* indicates how many units of natural gas (or, more generally, of any other generating fuel) are required to produce one unit of electricity. In our case, the "market" heat rate would refer to the price-setting plant that generates the marginal unit of electricity.

hedgeable risks, leading to an incomplete market setting. Therefore, the risk-neutral measure $\mathbb{Q}$ cannot be determined uniquely, but will instead be inferred from market prices of traded forward contracts, as will be shown in Section IV. In order to govern the change of measure, and following Girsanov's theorem, we introduce separate market prices of risk λ^D, λ^C, and λ^g for the different sources of uncertainty in our model; these market prices of demand, capacity, and fuel price risk are assumed constant. Given that P_t is log-normal in the state variables, the log futures price, $\ln F_t(T)$, at time t and with delivery date T is given as follows:

$$\begin{aligned} \ln F_t(T) &= \mathbb{E}^{\mathbb{Q}}\big[\ln P_T \mid \mathcal{F}_t\big] + \tfrac{1}{2}\mathbb{V}^{\mathbb{Q}}\big[\ln P_T \mid \mathcal{F}_t\big] && (14)\\ &= \ln\alpha + \delta\,\mathbb{E}^{\mathbb{Q}}\big[\ln g_T \mid \mathcal{F}_t\big] + \beta\,\mathbb{E}^{\mathbb{Q}}\big[D_T \mid \mathcal{F}_t\big] + \gamma\,\mathbb{E}^{\mathbb{Q}}\big[C_T \mid \mathcal{F}_t\big] \\ &\quad + \tfrac{1}{2}\delta^2\,\mathbb{V}^{\mathbb{Q}}\big[\ln g_T \mid \mathcal{F}_t\big] + \tfrac{1}{2}\beta^2\,\mathbb{V}^{\mathbb{Q}}\big[D_T \mid \mathcal{F}_t\big] + \tfrac{1}{2}\gamma^2\,\mathbb{V}^{\mathbb{Q}}\big[C_T \mid \mathcal{F}_t\big] && (15) \end{aligned}$$

where $\mathbb{E}^{\mathbb{Q}}\big[\cdot \mid \mathcal{F}_t\big]$ and $\mathbb{V}^{\mathbb{Q}}\big[\cdot \mid \mathcal{F}_t\big]$ indicate expectation and variance[16] under $\mathbb{Q}$ and conditional on $\mathcal{F}_t$ which is defined as $\mathcal{F}_t := \mathcal{F}_t^D \vee \mathcal{F}_t^C \vee \mathcal{F}_t^g$. As is further outlined in Section III, when pricing forward contracts by making use of forecasts of electricity demand and capacity, forward prices will be computed as risk-neutral expectations of the spot during the delivery period, conditional on $\mathcal{G}_t$ rather than $\mathcal{F}_t$. Consequently, Equation (14) will need to be replaced by $\ln F_t(T) = \mathbb{E}^{\mathbb{Q}}\big[\ln P_T \mid \mathcal{G}_t\big] + \frac{1}{2}\mathbb{V}^{\mathbb{Q}}\big[\ln P_T \mid \mathcal{G}_t\big]$, where $\mathcal{G}_t := \mathcal{G}_t^D \vee \mathcal{G}_t^C \vee \mathcal{F}_t^g$ and $(\mathcal{G}_t)_{t\in[0,T^\star]}$ (or, more precisely, $(\mathcal{G}_t^D)_{t\in[0,T^\star]}$ and $(\mathcal{G}_t^C)_{t\in[0,T^\star]}$) is the enlarged market filtration containing forecasts of expected demand and capacity levels, respectively.

Also note that Equation (14) refers to a contract with delivery of electricity at some future date T, whereas standard electricity forward contracts specify the delivery of electricity throughout a delivery period $[\underline{T}, \overline{T}]$ (with $\underline{T} < \overline{T}$), e.g., one week or one month. Following Lucia and Schwartz (2002), we approximate the price of a forward contract with delivery period $[\underline{T}, \overline{T}]$, containing $n = \overline{T} - \underline{T}$ delivery days, as the arithmetic average of a portfolio of n single-day-delivery forward contracts with their maturities spanning the entire delivery

[16] Note that the second part of Equation (14) reflects our implicit assumption of all state variables being independent of each other.

period, i.e.:

$$F_t(\underline{T}, \overline{T}) \;=\; \frac{1}{\overline{T} - \underline{T}} \sum_{i=1}^{n} F_t(\tau_i) \tag{16}$$

Finally, calculating electricity forward prices based on Equation (16) also requires us to have available the corresponding fuel forward prices with single-day maturities, i.e., one also needs to compute $\mathbb{E}^{\mathbb{Q}}\left[\ln g_{\tau_i} \mid \mathcal{F}_t\right]$ (as well as the conditional variance) for every day τ_i within the delivery period $[\underline{T}, \overline{T}]$. For that purpose, we take the implied log-spot price of natural gas at time t as a starting point to compute for every day τ_i within the delivery period the price of a (hypothetical) natural gas forward contract that matures on that very day. A simplified approach could be to use only one average value for $\mathbb{E}^{\mathbb{Q}}\left[\ln g_{\tau_i} \mid \mathcal{F}_t\right]$ during the entire delivery period (e.g., based on the current value of the month-ahead natural gas forward, when pricing month-ahead electricity forwards). However, this may pose problems for non-standard delivery periods as well as would require identically defined delivery periods for gas and power.[17]

III The Enlargement-of-Filtration Approach

Non-storability of a given asset Z implies that forward-looking information can neither be inferred from, nor is reflected in the historical evolution of its price trajectory Z_t (Benth and Meyer-Brandis, 2009). Mathematically speaking, given a finite horizon $T^\star$ and letting $\left(\Omega, \mathcal{F}, \mathbb{F} = (\mathcal{F})_{t \in [0,T^\star]}, \mathbb{P}\right)$ be a filtered probability space, the natural filtration $\mathbb{F}^Z = (\mathcal{F}^Z)_{t \in [0,T^\star]}$ (with $\mathcal{F}_t = \mathcal{F}_0 \vee \mathcal{F}_t^Z$) may not reflect all forward-looking information available to market participants. Assume that agents have access to some (non-perfect) forecast of the price of Z at some future point in time $t^\star \in [0, T^\star]$. Then, there exists a sigma-algebra $\mathcal{G}_\tau$ with $\mathcal{F}_\tau \subset \mathcal{G}_\tau$ for all $\tau < t^\star$, where $\mathcal{G}_\tau$ reflects all available information including the forecast, whereas $\mathcal{F}_\tau$ does not. For $\tau \geq t^\star$, i.e. for times beyond the

[17] We note, however, that in the UK, electricity forward contracts (still) trade according to the EFA (electricity forward agreement) calendar, following which every calendar year is grouped into four quarters with three delivery months with lengths of 4/4/5 calendar weeks, respectively. Consequently, delivery months of electricity forward contracts may not exactly overlap with corresponding delivery months of traded natural gas futures contracts.

forecast horizon, we however have $\mathcal{F}_\tau = \mathcal{G}_\tau$, since no further forward-looking information is (assumed to be) available.

Next, note that whereas electricity clearly serves as most prominent example for non-storable underlyings, the above outlined incompleteness of natural filtrations with respect to forward-looking information can generally be extended to any kind of non-storable underlying. Therefore, and strictly speaking, we do not enlarge the filtration of the electricity spot price in order to incorporate forecasts, like Benth and Meyer-Brandis (2009) do in their reduced-form setting. Instead, we focus on electricity demand D_t in Equation (1) and available capacity C_t in Equation (5) which are, of course, non-storables as well, and hence do not reflect forward-looking information either. Therefore, and more precisely, it is the filtrations relating to the demand and capacity processes, respectively, that need to be enlarged in order to integrate forecasts provided by the system operator.

In the following, all formulae derived in this section relate to *available capacity* and forecasts thereof. Additional theoretical background as well as how to derive respective formulae for the more general case of deterministic, but non-constant volatility (as for *electricity demand* D_t) is provided in the technical appendix. For notational convenience, we work with de-seasonalized forecasts that relate to m_t instead of C_t; $\mathcal{F}_t^C$ and $\mathcal{G}_t^C$ are defined as further above.[18]

In this setting, the (de-seasonalized) forecast of generation capacity available at time t with forecast horizon T is interpreted as $\mathcal{G}_t$-conditional expectation and can be expressed as:

$$\mathbb{E}^{\mathbb{P}}\left[m_T \mid \mathcal{G}_t^C\right] = m_t \mathrm{e}^{-\kappa^C(T-t)} + \sigma^C \mathbb{E}^{\mathbb{P}}\left[\int_t^T \mathrm{e}^{-\kappa^C(T-u)} \mathrm{d}B_u^C \,\middle|\, \mathcal{G}_t^C\right] \tag{17}$$

This raises the question of how to treat expectations like $\mathbb{E}^{\mathbb{P}}\left[\int_t^T \mathrm{e}^{-\kappa^C(T-u)} \mathrm{d}B_u^C \mid \mathcal{G}_t^C\right]$ that

[18]One could argue that there exist, of course, numerous other forecasts about expected available capacity that market participants might also have access to. E.g., capacity forecasts released by the system operator that relate to intermittent energy sources (such as wind or solar power) might be adjusted based on a utility's proprietary model involving different meteorological assumptions, such as more windy conditions or fewer sunshine hours. Likewise, the same is, of course, true for demand forecasts if market participants expect, e.g., higher temperatures than implied by the forecast of the system operator. Therefore, if we speak of $\mathcal{G}$ as the sigma-algebra "including forecasts", we assume away the existence of other forecasts and only mean to refer to those forecasts released by the TSO.

are conditional on $\mathcal{G}_t^C$ (i.e., the sigma-algebra incl. forecasts) when B_t^C, however, is an $\mathcal{F}_t^C$-adapted Brownian motion. Consequently, B_t^C may no longer be a standard Brownian motion with respect to $(\mathcal{G}_t^C)_{t\in[0,T^\star]}$. Even more importantly, and following the "average approach" in Equation (16), the pricing of, e.g., a forward contract with delivery period of one month will require us to (ideally) have capacity forecasts at our disposal for every day within the delivery period. Yet, as is outlined in Section IV, detailed forecasts on a daily basis (as released by National Grid for the British market) always cover a window of the next 14 days only. For longer-term prognoses, such as expected available capacity in 21 days, only forecasts of weekly granularity are published. Consequently, we may at best cover a certain first part of the delivery period with daily forecasts, whereas for the rest of the period, only few weekly forecast points will be available, thus leaving several delivery days "uncovered" by forecasts. Therefore, another key question is how to consistently determine $\mathbb{E}^{\mathbb{P}}\big[m_T \mid \mathcal{G}_t^C\big]$ when forecasts for capacity on delivery day T are not available, but only for times T_1 and T_2 with $T_1 \leq T \leq T_2$. This leads to the following proposition:

Proposition III.1. *Suppose that market participants dispose of forecasts of available capacity at future points in time T_1 and T_2, i.e., $\mathbb{E}^{\mathbb{P}}\big[m_{T_1} \mid \mathcal{G}_t^C\big]$ and $\mathbb{E}^{\mathbb{P}}\big[m_{T_2} \mid \mathcal{G}_t^C\big]$. Then, for $t \leq T_1 \leq T \leq T_2$, capacity expected to be available at time T is given as:*

$$\begin{aligned}\mathbb{E}^{\mathbb{P}}\big[m_T \mid \mathcal{G}_t^C\big] &= \mathbb{E}^{\mathbb{P}}\big[m_{T_1} \mid \mathcal{G}_t^C\big]\mathrm{e}^{-\kappa^C(T-T_1)} \\ &\quad + \mathbb{E}^{\mathbb{P}}\left[\int_{T_1}^{T_2} \mathrm{e}^{\kappa^C u}\mathrm{d}B_u^C \,\middle|\, \mathcal{G}_t^C\right]\sigma^C \frac{\mathrm{e}^{\kappa^C T}\big(1-\mathrm{e}^{-2\kappa^C(T-T_1)}\big)}{\mathrm{e}^{2\kappa^C T_2}-\mathrm{e}^{2\kappa^C T_1}} \end{aligned} \tag{18}$$

The first part of the second term on the RHS of Equation (18) *can be derived as follows:*

$$\mathbb{E}^{\mathbb{P}}\left[\int_{T_1}^{T_2} \mathrm{e}^{\kappa^C u}\mathrm{d}B_u^C \,\middle|\, \mathcal{G}_t^C\right] = \frac{1}{\sigma^C}\left(\mathbb{E}^{\mathbb{P}}\big[m_{T_2} \mid \mathcal{G}_t^C\big]\mathrm{e}^{\kappa^C T_2} - \mathbb{E}^{\mathbb{P}}\big[m_{T_1} \mid \mathcal{G}_t^C\big]\mathrm{e}^{\kappa^C T_1}\right) \tag{19}$$

Proof. This directly follows from Proposition 3.5 and Proposition 3.6 in Benth and Meyer-Brandis (2009). Detailed derivations for the more general case of non-constant deterministic volatility are provided in the technical appendix. □

Note that we do not impose any specific structure on the nature of the enlarged filtration

$(\mathcal{G}_t^C)_{t\in[0,T^\star]}$ apart from *(i)* the fact that the forecasts released by the TSO are interpreted as $\mathcal{G}_t$-conditional expectations and *(ii)* that the $\mathcal{F}_t$-adapted process B_t^C (likewise B_t^D) is a semimartingale under the enlarged filtration. The latter is a common and well-studied approach in the enlargement-of-filtration theory, although more recent studies (Biagini and Oksendal, 2005, or Di Nunno et al., 2006) have shown that such assumption could in fact be relaxed. As is shown in the appendix in more detail, the general idea in this case is that B_t^C under the enlarged filtration $(\mathcal{G}_t^C)_{t\in[0,T^\star]}$ decomposes into a standard Brownian motion $\hat{B}_t^C$ and a drift term $A(t) = \int^t \vartheta(s)\mathrm{d}s$ which is usually referred to as the information drift. Hence, the additional information is essentially incorporated in the drift term $\vartheta(t)$, so that the dynamics for m_t in Equation (6) can be re-written as follows:

$$\mathrm{d}m_t \quad = \quad -\kappa^C\left(m_t - \frac{\sigma^C}{\kappa^C}\vartheta(t)\right)\mathrm{d}t + \sigma^C\mathrm{d}\hat{B}_t^C \tag{20}$$

Based on Equation (20) – or, equivalently, on Proposition III.1 – we can now compute $\mathcal{G}_t$-conditional expectations which relate to those points in time where no TSO forecasts are available, but which are still consistent with the modified stochastic dynamics as imposed by the *available* forecast points. Although a related concept, the change of the drift for the above capacity process has not been obtained through a change of the probability measure, i.e., $\hat{B}_t^C$ is a $\mathcal{G}_t$-adapted Brownian motion under the statistical measure $\mathbb{P}$. Therefore, when it comes to derivatives pricing under a risk-neutral measure $\mathbb{Q}$ in Section V, we consequently look for a $\mathcal{G}_t$-adapted standard $\mathbb{Q}$-Brownian motion $\check{B}_t^C = \hat{B}_t^C - \Lambda_{\mathcal{G}}^C(t)$, where $\Lambda_{\mathcal{G}}^C(t)$ is a finite variation process representing the market price of risk that will be inferred from prices of electricity derivative contracts.

Finally, we briefly discuss why we propose to use this specific approach of integrating demand and capacity forecasts here. In fact, one may think of two alternative ways of how the incorporation of forward-looking information could be dealt with technically.
Assuming the forecast data to be reasonably reliable, one approach would be to treat the forecasts as deterministic. In such case, demand and capacity forecasts, ultimately represented by expected values in Equation (14), would be replaced by constants, so that the corresponding variance terms vanish. Although appealing by its simplicity, this

approach raises several issues: first, when pricing, e.g., a forward contract with monthly delivery period, it is often the case that detailed forecast data on a daily basis is not available for all days of the delivery month. Especially for mid- to longer-term forecasts, granularity of forecast points tends to be rather low, i.e., only expected maximum weekly, monthly or seasonal demand (capacity) levels may be indicated. Irrespective of the question of whether longer-term forecasts are still sufficiently accurate at all in order to justifiably treat them as deterministic, choosing among different methods of how to interpolate missing forecast data may appear arbitrary and pricing results would be quite sensitive to the specific method chosen. Second, and as is analyzed further below, future capacity levels are generally known to be hard to predict, in particular for the British market (see Karakatsani and Bunn, 2008, on this issue). This results in slightly less reliable forecasts, hence invalidating the assumption of deterministic forecasts in the first place.[19]

Another approach could mitigate the above problem of interpolation in case of longer-term forecasts, although this (again) requires daily forecasts to be considered deterministic whenever available. For missing daily forecast points for periods beyond the horizon of the daily forecasts, one could proceed as follows: A demand (capacity) process is estimated based on the time series of historical data that has been extended to also include a given set of available daily forecasts, treating the latter as if they were actually observed. Missing forecasts are then replaced with expectations derived from the estimated process. This approach is proposed by Ritter et al. (2011) and Härdle et al. (2012) in the context of weather derivatives pricing. Roughly speaking, estimating parameters based on historic and forecast data at the same time may come close to the general idea of enlarging the information filtration. However, we argue that especially for electricity, this combined estimation may level out expected future (demand/capacity) fluctuations too strongly due to the (necessary) inclusion of historic data into the estimation. Assume, e.g., that

[19] Basically the same arguments apply if we instead retain the general structure of our stochastic processes for demand and capacity in Equations (2) and (6), and include the forecast data (e.g., relating to a future point in time T) into the deterministic parts $s^D(T)$ and $s^C(T)$. Additionally, and at least in the short-term, the deterministic part "within" $\mathbb{E}^{\mathbb{Q}}[\cdot]$ is clearly overlaid with the mean-reverting OU-component $q_t e^{-\kappa^D(T-t)}$ (likewise for capacity) so that the exact forecast values contained in $s^D(T)$ will be distorted.

a longer-lasting period of very high electricity demand is expected. This will, of course, be explicitly reflected in the daily forecasts. However, even if these days (for which high demand is forecast) are added to the historical time series in the "combined" estimation procedure, the *derived* demand expectations for periods thereafter will likely be lower, owing to the influence of historic long-term average demand. Note that this very issue is avoided by modifying demand and capacity dynamics as proposed in Equation (20), while retaining the basic stochastic properties of the respective processes at the same time.

IV Data and Estimation Approach

A. Fundamental Data

The data set used in this study for the fundamental variables demand and capacity comprises of ten years of historical data for the British electricity market, covering the period from 2002 up to 2011. These contain both historical realized as well as historical forecast data, and were obtained from National Grid, the British TSO,[20] and Elexon, the operator of the balancing and settlement activities in the British market.[21] Figure 1 shows the development of the realized demand and capacity data during the period from 01-Jan-2007 to 31-Dec-2011, i.e., the period covered by our pricing study (whereas the prior five years are used for estimation purposes).

With respect to electricity demand, the *realized* data used is based on the outturn average megawatt (MW) value of electricity demand in England, Wales, and Scotland during the peak half-hour of the day, as indicated by operational metering.[22] Specifically, the demand metric we use is classified as *IO14 DEM*, which includes transmission losses and station transformer load, but excludes pump storage demand and net demand from interconnector

[20]National Grid both owns and operates the systems in England and Wales. Since the start of BETTA in April 2005, it has also been operating the high-voltage networks in Scotland owned by Scottish and Southern Energy as well as Scottish Power.

[21]The following websites were accessed: http://www.nationalgrid.com, http://www.bmreports.com, and http://www.elexonportal.co.uk.

[22]Other than for most electricity markets, electricity in Great Britain is traded on a half-hourly basis, corresponding to 48 settlement periods per day.

imports/exports.[23]

As regards *forecasts* of expected electricity demand, two categories need to be distinguished: first, National Grid releases daily updated demand forecasts covering the next 2 weeks ahead with daily granularity. These are forecasts of electricity demand expected to prevail during the peak half-hour of the respective day, which is the reason why we are using peak demand instead of average (baseload) demand throughout this study. Second, longer-term forecasts of expected demand are released once a week, covering the next 2-52 weeks ahead with weekly granularity. These forecasts relate to expected demand during the peak half-hour of the respective week. Figure 2 provides a schematic overview of the different forecast horizons in the context of pricing a forward contract with monthly delivery period. Finally, note that special attention was paid to the realized and forecast data employed in our study being defined consistently.

In terms of *realized* capacity available, National Grid records maximum export limits (MEL) for each of the units that are part of the overall balancing mechanism (BM).[24] These limits quantify the maximum power export level of a certain BM unit at a certain time and are indicated by generators to the TSO prior to gate closure for each settlement period;[25] should there be an (un-)expected outage for some generation unit, generators will accordingly submit a MEL of zero during the time of the outage for this unit. Moreover, since MEL do include volatile interconnector flows as well as generation from intermittent/renewable sources, they can be seen as a good real-time proxy of available generation capacity that either is in use for production, or could additionally be dispatched into the transmission system immediately.

Capacity *forecasts* are released by National Grid, too, but primarily relate to the expected "market surplus" *SPLD*. This variable gives an indication on expected excess capacity beyond the levels required to satisfy (expected) demand and reserve requirements, but

[23]The British electricity market is connected to neighboring markets via interconnectors such as to/from France (IFA), the Netherlands (BritNed) or the Moyle Interconnector (connection to Northern Ireland).

[24]These are approx. 300 units in the UK (with one plant comprising several units).

[25]In the British market, gate closure is set at one hour before each half-hourly trading period. It refers to that point in time by when all market participants have to give notice about their intended physical positions so that the TSO can take action to balance the market.

is structurally different from the MEL-approach that we follow for the realized capacity data. This is, amongst other reasons, due to SPLD including a statistically derived reserve-allowance which is based on average loss levels and forecast errors, rather than actual reserve levels held in operational timescales (which are probably less pessimistic as well). As such, and in order to consistently define realized and forecast capacity levels, we instead use forecasts of expected total generation availability (which are also released by National Grid) and adjust these for few additional items.[26] Both timescale and updating structure of these forecasts are similar to the demand case.

When feeding the forecast data into our model, note that the weekly updated demand forecasts with a forecast horizon ranging from 2-52 weeks ahead are specified as relating to the expected peak half-hour within the respective week, i.e., it is not tied to a specific (business) day. Weekly capacity forecasts then relate to this very same half hour of expected peak demand, but do not specify an exact date either, which, however, is required in order to apply Proposition III.1. Based on historic data, the peak half-hour of demand during a given week was most often found to occur between Tuesday and Thursday. As a matter of simplification, we hence assume that weekly demand and capacity forecasts always relate to the Wednesday of the respective week.[27]

Finally, an important *caveat* applies: while using forward-looking information may presumably be beneficial for derivatives pricing purposes, using *outdated* forward-looking information may certainly lead to the opposite. In fact, depending on both maturity and length of the respective contract to be priced, it may be the case that $\mathbb{E}^{\mathbb{P}}\left[D_\tau \,\middle|\, \mathcal{G}_t^D\right]$ and

[26]Even when using generation availability instead of SPLD, and unlike for the case of demand data, capacity forecasts still slightly differ in definition from the capacity metric on which the realized data is based (i.e., MEL). There are several reasons for this: *Inter alia*, volatile interconnector flows are hard to predict and, hence, are set at float throughout all forecast horizons. Also, a small number of generating units submit a MEL which, however, is not included into the forecast of generation capacity. We roughly adjust for these items to still arrive at consistently defined metrics, e.g., by carrying over latest observed values/forecast deviations into the future. At the same time, special focus is laid on our adjustments to remain simple, easily reproducible, and hence likely to be used by market participants. Further details are available from the authors upon request.

[27]Pricing results have proven to be rather insensitive to this assumption, i.e., fixing the weekly forecasts to relate to each Tuesday or Thursday of a given week (or even alternating, based on the business day for which the weekly peak-hour during the preceding week was observed) did not visibly change pricing errors.

$\mathbb{E}^{\mathbb{P}}\left[C_\tau \;\middle|\; \mathcal{G}_t^C\right]$ for $\tau = \underline{T} \ldots \overline{T}$ are exclusively determined based on longer-term forecast points which are only updated weekly (as opposed to the *daily* updated 2-14 day-ahead forecasts). Focusing specifically on capacity forecasts, it may, however, happen that a major unplanned outage occurs just after the most recent weekly forecasts have been released. Even worse, for few periods in our data sample, forecast updates are missing altogether, leaving gaps of up to several weeks between successive forecast updates. Feeding such outdated forecasts into our (or any other) model and not updating for significant outages (whenever indicated) that move the market, hence unduly punishes the forecast-based model.

Therefore, in case of missing updates or major unplanned outages not reflected in the most recent set of capacity forecasts, we have adjusted for such events by combining the forecast data with information provided on Bloomberg's "UK VOLTOUT" page as well as in news reports from ICIS Heren. Note that this information was available to market participants at the time of trading. Prominent examples, amongst others, relate to several of the unplanned trippings of nuclear generation units during 2007/08, which (along with increased retrofitting activities of coal-based plants at that time) led to extremely tight levels of available capacity in the British market.

B. Electricity Spot and Forward Data

Following the historic development of electricity market regulation and especially since the inception of the New Electricity Trading Arrangements (NETA) regime in 2001, wholesale trading in the British market is predominantly characterized by OTC forward transactions with physical settlement. The forward market – defined as covering maturities from day-ahead up to several years ahead of delivery – hence makes up for about 90% of overall electricity volume traded in the UK (Wilson et al., 2011). Compared to other major European electricity markets such as Germany or the Nordic market, financially-settled trades are less common and mainly concentrate on limited exchange-based trading activity such as at the Intercontinental Exchange (curve) or at the APX UK exchange (prompt). More recently, the N2EX platform, operated by Nord Pool Spot and Nasdaq OMX Commodities and established in order to re-strengthen exchange-based trading, has also started to list cash-settled power futures contracts for the British market. Despite these developments, exchange-based derivatives trading activity still seems to be rather limited, with member participation in futures trading increasing at slow pace only (see OFGEM, 2011).

In view of this dispersed market structure with the vast majority of trades still being bilateral or broker-based, our electricity price data is exclusively based on OTC contracts and was obtained from two sources: first, Bloomberg provides historical forward prices which are defined as composite quotes from a panel of OTC brokers. Second, we obtained a comprehensive data set from Marex Spectron, a leading independent energy broker that operates one of Europe's largest and most established marketplaces for electricity. This second data set is entirely based on trade data (including time stamp of trade, executed through platform or voice brokers) and contains a variety of additional types of electricity contracts, out of which a second OTC sample was formed. These two samples, for which pricing errors are analyzed separately in Section V, contain the following types of contracts:

"Bloomberg Data Set":

- 1-month ahead forward contracts

"Marex Spectron Data Set":

- 1-month ahead forward contracts
- 2-months ahead forward contracts

All selected forward contracts are baseload contracts. Moreover, electricity spot (i.e., day-ahead) price data is additionally used for model calibration purposes, but is not analyzed further in the main study. We deliberately focus on pricing the above types of baseload contracts, leaving aside other instruments with quarterly, seasonal, or yearly delivery periods. This is due to the following reasons: first, we are primarily interested in the pricing impact of including demand and capacity forecasts into our model, compared to the case when disregarding such forecasts. Since these forecasts are more accurate for short-term horizons,[28] our study focuses on contracts with short maturities and delivery periods. Second, trading activity generally concentrates on front months with liquidity at the longer end of the curve rapidly decreasing (OFGEM, 2011). Finally, and again primarily for liquidity reasons, we have chosen to analyze baseload contracts instead of peakload contracts. The fact that we are pricing *baseload* contracts, although using demand and capacity during *peak* half-hours as inputs, may seem inconsistent, but is ultimately due to the forecast data being available in this format only. It might be possible to convert the peakload demand and capacity forecasts into corresponding baseload predictions, e.g., by applying scaling factors that are based on historical averages. However, this is already indirectly accounted for by the estimation procedure outlined in the following subsection. An overview of the two data samples is provided in Table 1 where descriptive statistics as well as further contractual characteristics for the day-ahead and forward contracts are summarized. As can be seen, the data exhibits well-known characteristics of electricity prices, such as substantial levels of volatility and excess kurtosis. While these effects are – as expected – more pronounced for spot than for forward contracts, we also note the obvious difference in skewness of log-returns between both types of contracts.

C. Estimation Approach and Estimation Results

The individual processes for the state variables demand (D_t) and available capacity (C_t) are estimated by discretizing Equations (2) and (6) and using maximum likelihood. Based on annually rolling windows of five years of time series data, parameters are re-estimated annually, but held constant throughout every subsequent year when used for pricing purposes. Estimation results and robust standard errors are presented in Tables 2 and 3. The reported significance levels underline the distinct seasonalities for both demand and capacity, with our chosen specifications capturing well the most prominent characteristics. Given the already very high number of parameters to be estimated, we have chosen a rather simple one-factor approach to model the dynamics of the marginal fuel used for generation, i.e., natural gas in our case. Since the spot component X_t in Equation (8)

[28] Longer-term forecasts rely on statistical averages and, thus, should convey no significant additional information as compared to the "no-forecast" case (that is characterized by filtration $(\mathcal{F}_t)_{t\in[0,T^\star]}$).

cannot be observed directly, estimation of all parameters for the natural gas model is instead performed based on futures data and using the Kalman filter and maximum likelihood. Re-casting the model into state-space representation with corresponding transition and measurement equations is a standard exercise which is outlined in more detail, e.g., in Schwartz (1997). Since our study primarily focuses on pricing short-term electricity forward contracts, we focus on the short end of the natural gas curve and hence seek to infer the log-spot natural gas dynamics from corresponding short-term futures contracts with maturities ranging from one to four months. Relevant data is sourced from Bloomberg and relates to natural gas futures contracts traded at the ICE (Intercontinental Exchange) with physical delivery at the National Balancing Point (NBP), the virtual trading hub for natural gas in Great Britain. Parameter estimates for the dynamics of natural gas are summarized in Table 4. Again, the estimates are statistically highly significant and clearly reflect the strong seasonal component that is present in natural gas prices.

Having estimated the parameters that govern the dynamics of the respective underlying variables D_t, C_t, and g_t, the parameters α, β, γ, and δ that link the three fundamental factors yet remain to be determined. Generally, two approaches appear suitable:

1. Based on Equation (12), historic log electricity spot prices $\ln P_t$ are regressed on corresponding time-series data of D_t, C_t, and $\ln g_t$. This approach is proposed by Cartea and Villaplana (2008) for a structurally similar model (that, however, does not include marginal fuel dynamics or forward-looking information)

2. Implicit (re-)estimation over time, based on a cross-section of electricity spot and forward prices

Given evidence that α, β, γ, and δ may not be constant over time, we favor the second approach: in their study on electricity spot price forecasting in the British market, Karakatsani and Bunn (2008), for example, also apply fundamentals-based models, and conclude that the models with the best pricing performance are those that allow for time-varying coefficients to link the fundamental factors. Moreover, some specific structural aspects of the model proposed by Carmona et al. (2011) may be seen in the same spirit. Therefore, and although treated as constants in our model, the time-varying nature of the parameters α, β, γ, and δ is captured by implicitly extracting and weekly re-estimating them from the cross-section of quoted power prices. Likewise, the parameters λ^D and λ^C governing the change of measure from $\mathbb{P}$ to $\mathbb{Q}$ are inferred in the same way.[29]

In order to implicitly estimate these parameters, the following objective function is minimized:

$$\begin{aligned}\Phi_W^\star &= \arg_{\Phi_W} \min RMSPE(\Phi_W) \\ &= \arg_{\Phi_W} \min \left[\sqrt{\frac{1}{N_W^P} \sum_{i=1}^{N_W^P} \left(\frac{\hat{P}_{W,i}(\Phi_W^{\mathbb{P}}) - P_{W,i}}{P_{W,i}} \right)^2} + \sqrt{\frac{1}{N_W^F} \sum_{i=1}^{N_W^F} \left(\frac{\hat{F}_{W,i}(\Phi_W^{\mathbb{Q}}) - F_{W,i}}{F_{W,i}} \right)^2} \right]\end{aligned}$$

where $\Phi_W \equiv \{\alpha, \beta, \gamma, \delta, \lambda^D, \lambda^C\}$, and with the two subsets $\Phi_W^{\mathbb{Q}}$ and $\Phi_W^{\mathbb{P}}$ defined as $\Phi_W \equiv \Phi_W^{\mathbb{Q}}$ and $\Phi_W^{\mathbb{P}} \equiv \Phi_W^{\mathbb{Q}} \setminus \{\lambda^D, \lambda^C\}$. To minimize the root mean squared percentage error (RMSPE) over the in-sample period W, we assemble all available day-ahead prices $P_{W,i}$ (totaling N_W^P quotes) as well as all available forward prices $F_{W,i}$ (N_W^F quotes) and compare against prices $\hat{P}_{W,i}$ and $\hat{F}_{W,i}$ as predicted by our model according to Equations (12) and (14). For in-sample estimation windows W, we use a length of eight weeks (e.g., $w_1 - w_8$) for the Bloomberg sample. Out-of-sample testing of the model is performed during the subsequent week (i.e., w_9), employing the parameters estimated over W – thus only using information available up to the respective pricing day. Finally, the in-sample period is shifted by one week (i.e., new window: $w_2 - w_9$) and parameters are re-estimated. For the Marex Spectron sample, we shorten the length of the in-sample estimation windows to six weeks since more price observations per week are available, thus allowing for a robust estimation with a shorter window. Furthermore, these changes in the in-sample set-up may be seen as providing additional robustness to our findings examined in Section V, so

[29]Note that for pricing power derivatives in our structural framework, risk-neutral dynamics are also required for the natural gas component of our model. The corresponding market price of risk λ^g (which is assumed constant), however, has already been determined by Kalman filter estimation (see Table 4). We hence assume that the "look-through" risk premium of natural gas indirectly inherent to power derivatives is equal to the one for (outright) traded natural gas futures contracts. While λ^g could easily be re-estimated by including it into the set of implicitly determined parameters Φ, we refrain from doing so and instead prefer to reduce the number of free parameters here.

as to ensure that pricing improvements when using forecasts do not rely on a specific mix of contracts or length of in-sample estimation windows.

Different sets of implied parameter estimates $\Phi_W^\star$ are obtained for the Bloomberg and Marex Spectron samples (which are priced separately), as well as depending on whether or not forecasts of demand and/or available capacity are used during the estimation procedure. As an example, implied estimates for the Bloomberg sample (when using both demand and capacity forecasts) are summarized in Table 5.[30] Although the table only provides an aggregate view on the estimates, their corresponding means and standard errors indicate significant weekly variation between the parameters which our model could not capture when holding constant the "fundamental" parameters α, β, γ, and δ in $\Phi_W^\star$ otherwise.

Examining more closely the development of the parameter estimates over time, we observe that β and γ, the parameters governing the sensitivity of the power price with respect to changes in demand and capacity, respectively, culminate in 2008 and gradually decline thereafter. As is further outlined in the next section, this can be well explained by the fact that in terms of (excess) capacity, the British power market was especially tight in 2008, as is clearly reflected in the behavior of day-ahead and month-ahead forward prices displayed in Figure 3. The years to follow are marked by a massive increase in installed generation capacity by more than 10 gigawatts (GW), leading to oversupply especially of thermal generation and, consequently, to tightening spreads (especially spark spreads) for generators. As a consequence of these abundant capacity levels, changes in demand and capacity are of less importance for power price dynamics at that time, as evidenced by rather small absolute values for the estimates of β and γ in the years 2009-11.[31] As will be seen, this strongly affects the relative advantage of using forecasts of demand and capacity.

Recalling that δ can be interpreted as the elasticity of the power price with respect to

[30]Estimation results for the other sets of parameters are available from the authors upon request.

[31]However, we acknowledge that it may be up for debate whether the variation of β and γ (and especially the increase in absolute values for 2008) could, at least to some extent, also be due to insufficient convexity of our functional representation of the merit order curve which is likely to be much steeper during times of low system margin than the corresponding levels implied by our exponential-form representation.

changes in the fuel price, we observe that between 2009 and 2011, the estimate for δ more than doubles. This increase in the power-gas sensitivity may come as a surprise given that at the same time, spark spreads have continued to decline. However, and as a matter of fact, it is the heavily gas-based structure of the British generation park that causes especially the short end of the power price curve to track the NBP gas curve very closely; hence, the link between gas and power markets may have become even stronger recently, owing to the fact that *(i)* the LCPD[32] has started to reduce availability levels of coal plants and that *(ii)* new generation coming online has primarily been of CCGT-type.[33] We also note that the increase in value for δ during 2009-2011 goes in line with a corresponding decrease in value for α, which appears reasonable when recalling the interpretation of $\alpha = f_t^{1-\delta}$ in Equation (13).

Finally, in view of rather large estimates for the market prices of demand and capacity risk, λ^D and λ^C, it is important to mention that since these two parameters are estimated simultaneously, they interact with each other during the estimation procedure and cannot be determined uniquely. It might hence be more convenient to think of a "combined" market price of (reserve) margin risk $\beta\lambda^D + \gamma\lambda^C$ which is also shown in Table 5.

V Pricing Results

Pricing results for 1-month ahead forward contracts from the Bloomberg data set are summarized in Table 6. In order to examine the pricing impact of using forward-looking information in more detail, we distinguish between three cases: using *(i)* no forecasts, *(ii)* demand forecasts only, and *(iii)* forecasts of both demand and available capacity. Results are reported for each of the five years covered by our study as well as on an aggregate basis (for 2007-2011). As can be seen, employing demand and capacity forecasts clearly improves pricing performance on an overall basis, reducing pricing errors by up to 50%:

[32]The UK Large Combustion Plant Directive (LCPD) limits the amount of Sulphur Dioxide, Nitrous Oxides, and dust that (coal- and oil-fired) power stations are allowed to emit. As an alternative to complying with the tighter emissions regulations, power stations that were "opted-out" either face restrictions of operational hours and/or have to close by 2015.

[33]Combined cycle gas turbine (CCGT) plants are natural gas fired generation plants which, thanks to their technology, achieve high levels of thermal efficiency and offer sufficient flexibility in generation to meet sudden fluctuations in electricity demand.

aggregate RMSPE over the entire sample period from 2007-2011 reduces to less than 6% as compared to an RMSPE of about 10% when no forecasts are used; corresponding absolute-level RMSE even halves and decreases by some £4.00/MWh, which also underlines the economic significance of the pricing improvements achieved by incorporating forecasts into our model – especially in view of average contract volumes of several thousands of megawatt hours (MWh).

In order for the analysis of pricing errors to be consistent with our estimation procedure, we mainly focus on root mean squared-based error measures, given that this objective function has also been used for estimation. However, we also note that the relative improvement in pricing performance when employing forecasts is generally smaller when looking at the absolute percentage error (MAPE) as opposed to RMSPE, which underlines that incorporating forecasts seems to pay off mainly in situations of unusually high or low demand/capacity. Hence, before analyzing the breakdown of pricing errors on a yearly basis, it is important to recall that primarily during the first 2-2.5 years covered in our study, the British power market has suffered from exceptionally poor (expected) levels of power plant availability, with reserve margins clearly falling below long-term averages (especially in 2008). Consequently, the model excluding forecasts fares clearly worse than during any other period of our study. By contrast, the model including both demand and capacity forecasts gives strong evidence of its capabilities, reducing pricing errors even in times of extreme fluctuations in day-ahead and forward price levels – i.e., during times demanding utmost flexibility from any type of model. Reconsidering Figure 3, the extreme spike in month-ahead forward prices during September/October 2008 was clearly driven by ever-increasing supply fears,[34] and it is obvious that such a trajectory can only be captured (albeit not perfectly) by a model that includes forward-looking information about the capacity levels that are expected to prevail during the respective delivery months.

The pricing performance of the models during the year 2007 provides another opportunity to further discuss what kind of forward-looking information we actually consider to be contained in the enlarged filtration $(\mathcal{G}_t^C)_{t\in[0,T^\star]}$ – and what is not contained therein. Based

[34]This is supported by our analysis of market commentary covering the respective trading days. Importantly, in these days, then prices of month-ahead natural gas were approximately flat.

on a detailed analysis of single-day pricing errors, the model including both forecast types yields very satisfactory pricing results throughout this year, except for a period of rather poor pricing performance during November and December 2007, for which forward prices are clearly underestimated. Although market commentary may generally be criticized for over-emphasizing alleged causal relationships between specific events and strong market movements, several of the reports released at that time stress, amongst other reasons, the then very high continental power prices that are said to have impacted British power prices as well. In fact, French power prices had reached record levels in November 2007, fueled by strikes in the energy sector that led to temporary production cuts by some 8,000 MW. This, in turn, raised concerns about French electricity supplies for the rest of the year, which ultimately could have resulted in Britain becoming a net exporter of power to France via its interconnector, putting an additional drain on the already tight British system.[35] However, although market commentary indicates that (British) market participants do seem to have "priced in" such a scenario, and although pricing errors for the forecast-based variant of our model would have clearly been reduced, we have decided not to incorporate this belief (i.e., interconnectors switching from imports to exports) in our capacity forecasts: $(\mathcal{G}_t^C)_{t\in[0,T^\star]}$ is only based on forecasts released by the TSO and supplemented with updates of major unplanned outages. Although likely to further improve pricing performance, starting to integrate *market beliefs* about future available import/export capacity levels would also require us to do so for the rest of our sample, i.e., during times where such market sentiment may be more difficult to infer. Moreover, it is obviously impossible to exactly observe and consistently quantify these beliefs, e.g., it is unknown how long exactly and to what extent market participants would expect the above scenario to continue.

In the years 2009-2011, the relative improvement of the forecast-based models is smaller than in previous years. As indicated by the corresponding parameter estimates for β and γ, the influence of demand and capacity as fundamental factors driving power prices has been much reduced during these years, primarily due to growing oversupply

[35] The interconnector that links British and French electricity markets has a capacity of approx. 2,000 MW; Britain has "traditionally" been an importer of French electricity – which (especially during peak hours) tends to be cheaper, also in view of the higher share of nuclear baseload generation capacity.

in generation capacities leading to permanently healthy reserve margin levels. As such, given that (short- to mid-term) power prices at that time were almost exclusively driven by natural gas dynamics under these conditions, the impact of incorporating forward-looking information vanishes accordingly. Interestingly, pricing performance of the model for the years 2009-2011 seems to be even slightly better when using demand forecasts only, and leaving capacity forecasts aside. This could be due to the fact that in the British market – as also stressed by Karakatsani and Bunn (2008) – the forecasts of available capacity levels (or, equivalently, margins) released by the TSO tend to be received with slight skepticism and, hence, are likely to be adjusted (or not used at all) by market players. This adds to other, more general problems of capacity forecasts, such as accuracy in terms of generation from renewables or their consistency in definition with realized data. This is also reflected in Figure 4 where prediction errors between forecast and realized demand and capacity levels are summarized.[36] Capturing well the regular consumption patterns that characterize the dynamics of electricity demand, related forecasts are subject to rather low forecast errors only. By contrast, predicted future levels of available capacity are significantly less accurate and this inaccuracy increases more strongly for longer forecast horizons. While this certainly impacts pricing performance during 2009-11, such generally higher inaccuracy of capacity forecasts nevertheless seems to be of minor importance during times of exceptionally low reserve margins, as shown above.

The results based on the data obtained from Marex Spectron are presented in Table 7 and Table 8. Again, we observe an improvement in pricing performance when integrating demand and capacity forecasts into our model – as evidenced by relative reductions in total RMSPE of 8% and 15% for 1-month and 2-months ahead forward contracts, respectively. Moreover, the overall pattern of pricing errors for both types of forward contracts is in general agreement with the conclusions drawn based on the Bloomberg sample: notably, integrating demand as well as capacity forecasts into our model again primarily pays off during the years 2007-2008, reducing aggregate RMSE during these years by about £1.20-2.00/MWh. Such economic significance is also confirmed statistically by applying a

[36] Note that especially for forecasts of available capacity, the input capacity data from National Grid has been subject to further adjustments by the authors.

Wilcoxon signed-rank test which shows that the reductions in errors are significant at the 1%-level. For the remaining years (during which the impact of the fundamental factors D_t and C_t has been found to be rather muted) pricing errors can still be reduced by using only demand forecasts as compared to the "no-forecast" case.

Obviously, the differences in error metrics between the models including and excluding forward-looking information are not of the same order of magnitude as those reductions in pricing errors observed for the Bloomberg sample. Importantly, however, the in-sample fitting procedure for the Marex Spectron data sample additionally includes 2-months ahead forward contracts. As such, the fact that the benefits of using forecasts still prevail when calibrating our model to a broader cross-section of forward quotes may clearly be seen as underlining the robustness of our general findings.[37]

Examining the pricing errors in more detail, the year 2008 may again serve as a key example that illustrates another (and more subtle) effect when using forecasts as compared to excluding them. For this year, and based on the *Bloomberg* data sample, pricing performance of the "no-forecast" variant of the model is especially poor, as indicated by an RMSPE of about 20%. For the *Marex Spectron* sample, by contrast, corresponding pricing errors for 1-month ahead contracts are much lower, yielding an RMSPE of less than 10%. In this context, it is important to note that amidst the height of above mentioned capacity shortage in 2008 (that led to the prominent spike in 1-month ahead forward prices in September/October shown in Figure 3), supply fears primarily concentrated on the front month. Consequently, 2-months ahead forward contracts at that time (although, of course, not completely unaffected by the shortage) were clearly less subject to such strong fluctuations in price levels. Therefore, the broader cross-section of forward quotes in the Marex Spectron sample forces the "no-forecast" variant of our model to simultaneously accommodate such contrary 1-month and 2-months ahead price dynamics, which results in a "mediocre compromise" at best: 1-month ahead contracts are now strongly underestimated (2008 MPE of -2.78% in Table 7 vs. 0.98% in Table 6), which, however, halves RMSPE to less than 10%, given that underpricing pays off after the sudden "collapse" in post-spike forward pricing levels. Yet on the other hand, the

[37] As a further robustness check, the in-sample estimation window was shortened from 6 to 4 weeks, yielding similar pricing results.

pronounced spike in 1-month ahead forwards has 2-months ahead contracts become strongly overpriced post-spike (despite an overall 2008 MPE of -0.69%), which alone contributes more than 2% to the overall RMSPE of 11.45%. By contrast, and again comparing Table 6 and Table 7, all pricing errors for the model including demand and capacity forecasts in 2008 are surprisingly similar – irrespective of whether 2-months ahead contracts are included in the cross-section or not.

Put differently, the above example provides evidence of the additional benefits that arise when including forecasts into our model. Forecasting low levels of capacity in the short-term, but healthier levels in the mid- to long-term may help govern opposed dynamics of contracts with differing maturities, such as outlined above. This flexibility is also reflected in the implicitly estimated fundamental parameters α, β, γ, and δ. In fact (and although not reported here), the implied estimates show clearly higher variation throughout 2007 and 2008 than if demand and/or capacity forecasts are accounted for during the estimation procedure. This appears reasonable given the additional flexibility for the forecast-based model variants in fitting observed prices, whereas the model variant without forecasts always has expected demand and capacity mean-revert to the same (long-term) levels. As a result, flexibility is reduced, which must be compensated for by higher variation in the set of fundamental parameters. Altogether, this again underlines that excluding forecasts from the pricing procedure not only affects pricing performance, but may also imply using a mis-specified model.

VI Conclusion

Modeling the dynamics of electricity prices has traditionally been a challenging task for market participants, such as generators/suppliers, traders, and speculators. The strong links between power prices and their fundamental drivers make structural modeling approaches especially appealing in this context, and it can be expected that both current and future developments – such as further integration of geographic markets via market coupling – will even further promote the importance of bottom-up modeling frameworks (albeit at the cost of increasing complexity). At the same time, increasing transparency as well as more reliable outturn and forecast data released by system operators help market participants face these challenges and allow for more informed trading decisions.

In this paper, we develop and implement a model for electricity pricing that takes these developments into account by integrating forward-looking information on expected levels of electricity demand and available system capacity. Special focus is laid on calibrating the model to market prices of traded electricity contracts and it is shown that the model parameters are easily interpretable in an economic way. Being one of the key advantages of the fundamental approach, this helps to provide deeper insight into the structure of the market than standard reduced-form models could ever do.

Although hard to compare with other pricing studies (that focus on different markets or periods), the pricing performance of our model appears very satisfactory. Importantly, we find that out-of-sample pricing errors can be reduced significantly by making use of forward-looking information. Especially during times of very tight reserve margins, as witnessed for the British market in 2008 (and, to some extent, also in 2007 and 2009), capacity forecasts are of crucial importance in order to track sudden outage-induced changes in forward pricing levels and, therefore, significantly reduce pricing errors. However, we have also found that if spare capacities or, equivalently, tightness of the system is not perceived as playing a "fundamental" role, the advantage of employing capacity forecasts reduces and, in some instances, may even lead to marginally lower pricing performance. This is also strongly supported by our findings that capacity forecasts are generally less accurate on average than demand forecasts. Nevertheless, in these cases, it is still beneficial to keep using *demand* forecasts (rather than using no forecasts at all), which still reduces pricing errors. This is especially true for the pricing of forwards during the years 2009-2011, where, as our parameter estimates indicate, the dynamics of natural gas prices are the main fundamental driver so that demand and capacity only play a subordinate role for pricing.

Given the above mentioned challenges and future developments, there is ample room for further research in the field of structural electricity price modeling. First, it would be interesting to conduct empirical pricing studies for other electricity markets as well.

Given that structural electricity price models may always appear somewhat “tailored” to capture the characteristics of a specific electricity market, it would be interesting to see how these types of models perform empirically in those markets where merit-order dynamics are different. Second, given that our model is cast in a log-normal setting, it is equally well-suited to option pricing like other previously proposed fundamental models (see, e.g., Carmona et al., 2011). Further *empirical* studies might not only investigate the impact of using forward-looking information on option pricing performance, but also focus on the question of how pricing performance is affected depending on whether a 1- or 2-fuel model is used. Finally, the continued shift towards renewable energy sources in the generation mix of many European power markets poses new and highly complex challenges regarding the forecasting of availability levels of intermittent generation, such as for wind or solar power. These forecasts will play an indispensable role especially when modeling geographic markets that are highly interconnected with each other, so that abundant supplies are likely to “spill over” across borders and impact price levels in neighboring markets.

A Appendix

A. *Conditional Expectations Based on Enlarged Filtrations Under the Historical Measure*

Let $\big(\Omega, \mathcal{F}^D, \mathbb{F}^D = (\mathcal{F}^D)_{t\in[0,T^\star]}, \mathbb{P}\big)$ be a filtered probability space and q_t be specified as in Equation (2). Assume that $\mathbb{E}^{\mathbb{P}}\big[q_{T_1} \bigm| \mathcal{G}_t^D\big]$ and $\mathbb{E}^{\mathbb{P}}\big[q_{T_2} \bigm| \mathcal{G}_t^D\big]$ (with $\mathcal{F}_t^D \subset \mathcal{G}_t^D$) are available from the system operator. Before computing a forecast of expected electricity demand at time T with $t \leq T_1 \leq T \leq T_2$, we first derive relevant formulae under the assumption that only one forecast point for T_1 is given by the system operator – hence neglecting for the time being the existence of $\mathbb{E}^{\mathbb{P}}\big[q_{T_2} \bigm| \mathcal{G}_t^D\big]$ – and that a forecast of electricity demand is needed for time T with $t \leq T \leq T_1$. Formally, this can be expressed as follows:

$$\mathbb{E}^{\mathbb{P}}\big[q_T \bigm| \mathcal{G}_t^D\big] \;=\; q_t \mathrm{e}^{-\kappa^D(T-t)} + \sigma^D \mathbb{E}^{\mathbb{P}}\left[\int_t^T \mathrm{e}^{\varphi(s)}\mathrm{e}^{-\kappa^D(T-s)}\mathrm{d}B_s^D \;\middle|\; \mathcal{G}_t^D\right] \tag{21}$$

In order to manipulate the conditional expectation on the RHS of (21), a standard approach (see, e.g., Protter, 2004, or Biagini and Oksendal, 2005) is to exploit the semimartingale-property of B_t^D with respect to $\mathcal{G}_t$, i.e., to decompose B_t^D as follows:

$$B_t^D \;=\; \hat{B}_t^D + A(t) \tag{22}$$

where $\hat{B}_t^D$ is a $\mathcal{G}_t^D$-martingale (standard Brownian motion) and $A(t)$ a continuous $\mathcal{G}_t^D$-adapted process of finite variation (commonly referred to as the *"information drift"*). Following Hu (2011) and Di Nunno et al. (2006), $\hat{B}_t^D$ in Equation (22) can be written more explicitly as:

$$\hat{B}_t^D \;=\; B_t^D - \underbrace{\int_0^t b_t(s) B_s^D \mathrm{d}s}_{A_1(t)} - \underbrace{\int_0^t a(s)\Big(\mathbb{E}^{\mathbb{P}}\big[Y \bigm| \mathcal{G}_s^D\big] - \rho'(s) B_s^D\Big)\mathrm{d}s}_{A_2(t)} \tag{23}$$

with $A(t) = A_1(t) + A_2(t)$. Following Theorem A.1 in Benth and Meyer-Brandis (2009) or, equivalently, Theorem 3.1 in Hu (2011) – $a(s)$ and $b_t(s)$ in above Equation (23) are given as follows:

$$a(s) = \frac{\rho'(s)}{\tau - \int_0^s \big(\rho'(u)\big)^2 \mathrm{d}u} \tag{24}$$

$$b_t(s) = \rho''(s) \int_s^t \frac{\rho'(v)}{\tau - \int_0^v \big(\rho'(u)\big)^2 \mathrm{d}u} \mathrm{d}v \tag{25}$$

where $\rho(t) = \mathbb{E}^{\mathbb{P}}[B_t^D Y]$ is twice continuously differentiable, $\tau = \mathbb{E}^{\mathbb{P}}[Y^2]$ and Y is a centered Gaussian random variable with $Y = \int_0^{T_1} \mathrm{e}^{\varphi(s)} \mathrm{e}^{\kappa^D s} \mathrm{d}B_s^D = \int_0^{T_1} \mathrm{e}^{\theta \sin(2\pi(ks+\zeta))} \mathrm{e}^{\kappa^D s} \mathrm{d}B_s^D$.

Focusing on $A_1(t)$ and since $b_s(s) = 0$, it holds that:

$$\begin{aligned} \int_0^t b_t(s) B_s^D \mathrm{d}s &= \int_0^t \int_0^s \frac{\partial b_s}{\partial s}(u) B_u^D \mathrm{d}u \mathrm{d}s \\ &= \int_0^t a(s) \left[\int_0^s \rho''(u) B_u^D \mathrm{d}u \right] \mathrm{d}s & (26) \\ &= \int_0^t a(s) \left[\rho'(s) B_s^D - \int_0^s \rho'(u) \mathrm{d}B_u^D \right] \mathrm{d}s & (27) \end{aligned}$$

where Equation (27) is derived from Equation (26) by applying Itô's Lemma to $\rho'(s)B_s^D$. Based on the above, Equation (23) can now be re-arranged to yield:

$$\hat{B}_t^D = B_t^D - \underbrace{\int_0^t a(s) \left(\mathbb{E}^{\mathbb{P}}\big[Y \mid \mathcal{G}_s^D\big] - \int_0^s \rho'(u) \mathrm{d}B_u^D \right) \mathrm{d}s}_{A(t)} \tag{28}$$

Given above definition of Y, and since it can be shown that $\rho'(t) = \mathrm{e}^{\varphi(t)} \mathrm{e}^{\kappa^D t}$, the information drift $A(t)$ can be further simplified, so that Equation (28) now reads:

$$\begin{aligned} \hat{B}_t^D &= B_t^D - \int_0^t a(s) \mathbb{E}^{\mathbb{P}} \left[\int_s^{T_1} \mathrm{e}^{\varphi(u)} \mathrm{e}^{\kappa^D u} \mathrm{d}B_u^D \mid \mathcal{G}_s^D \right] \mathrm{d}s \\ &= B_t^D - \int_0^t a(s) \mathbb{E}^{\mathbb{P}} \left[\int_s^{T_1} \rho'(u) \mathrm{d}B_u^D \mid \mathcal{G}_s^D \right] \mathrm{d}s & (29) \\ &= B_t^D - \underbrace{\mathbb{E}^{\mathbb{P}} \left[\int_t^{T_1} \rho'(u) \mathrm{d}B_u^D \mid \mathcal{G}_t^D \right] \int_0^t a(s) \exp\left(- \int_t^s \rho'(v) a(v) \mathrm{d}v \right) \mathrm{d}s}_{A(t)} & (30) \end{aligned}$$

where Equation (30) is derived from Equation (29) based on Proposition A.3 in Benth and Meyer-Brandis (2009). Hence, in our initial setting of Equation (21) where a demand forecast $\mathbb{E}^{\mathbb{P}}\big[q_T \bigm| \mathcal{G}_t^D\big]$ is to be determined that is consistent with the exogenously given forecast point relating to T_1, this can now be computed as follows:

$$\begin{aligned}
\mathbb{E}^{\mathbb{P}}\big[q_T \bigm| \mathcal{G}_t^D\big] &= q_t \mathrm{e}^{-\kappa^D(T-t)} + \underbrace{\sigma^D \mathbb{E}^{\mathbb{P}}\left[\int_t^T \mathrm{e}^{\varphi(s)} \mathrm{e}^{-\kappa^D(T-s)} \mathrm{d}B_s^D \,\middle|\, \mathcal{G}_t^D\right]}_{I_{\mathcal{G}}(t,T)} && (31)\\
&= q_t \mathrm{e}^{-\kappa^D(T-t)} + \sigma^D \mathrm{e}^{-\kappa^D T} \mathbb{E}^{\mathbb{P}}\left[\int_t^T \rho'(s) \mathrm{d}B_s^D \,\middle|\, \mathcal{G}_t^D\right] && (32)\\
&= q_t \mathrm{e}^{-\kappa^D(T-t)} + \sigma^D \mathrm{e}^{-\kappa^D T} \mathbb{E}^{\mathbb{P}}\left[\int_t^T \rho'(s) \mathrm{d}\big(\hat{B}_s^D + A(s)\big) \,\middle|\, \mathcal{G}_t^D\right] \\
&= q_t \mathrm{e}^{-\kappa^D(T-t)} + \sigma^D \mathrm{e}^{-\kappa^D T} \int_t^T \rho'(s) \mathrm{d}A(s) \\
&= q_t \mathrm{e}^{-\kappa^D(T-t)} + && (33)\\
&\quad \sigma^D \mathrm{e}^{-\kappa^D T} \underbrace{\mathbb{E}^{\mathbb{P}}\left[\int_t^{T_1} \rho'(u) \mathrm{d}B_u^D \,\middle|\, \mathcal{G}_t^D\right]}_{(\star)} \int_t^T \underbrace{\rho'(s) a(s) \exp\left(-\int_t^s \rho'(v) a(v) \mathrm{d}v\right)}_{f(s)} \mathrm{d}s
\end{aligned}$$

Note that the term $I_{\mathcal{G}}(t,T)$ is also referred to as *information premium* which is defined as $\mathbb{E}^{\mathbb{P}}\big[q_T \bigm| \mathcal{G}_t^D\big] - \mathbb{E}^{\mathbb{P}}\big[q_T \bigm| \mathcal{F}_t^D\big]$. The term $(\star)$, in turn, can be extracted from the given forecast as follows:

$$(\star) = \frac{1}{\sigma^D}\left(\mathrm{e}^{\kappa^D T_1} \mathbb{E}^{\mathbb{P}}\big[q_{T_1} \bigm| \mathcal{G}_t^D\big] - q_t \mathrm{e}^{\kappa^D t}\right) \qquad (34)$$

The integral in the second term on the RHS of Equation (33) can be further simplified significantly if volatility is constant, as is the case for the dynamics of the capacity process in Equation (6). In the case of the seasonal volatility function for the demand process (as specified in Equation (4)), however, no analytic solutions for the integral exist; still, it can be approximated computationally in an efficient way by using standard numerical integration techniques.

Having outlined the general procedure for the case $T \leq T_1$, we now turn to the more relevant case where $\mathbb{E}^{\mathbb{P}}\big[q_{T_1} \bigm| \mathcal{G}_t^D\big]$ and $\mathbb{E}^{\mathbb{P}}\big[q_{T_2} \bigm| \mathcal{G}_t^D\big]$ (with $\mathcal{F}_t^D \subset \mathcal{G}_t^D$) are released by the system operator and a forecast $\mathbb{E}^{\mathbb{P}}\big[q_T \bigm| \mathcal{G}_t^D\big]$ needs to be computed with $t \leq T_1 \leq T \leq T_2$.

We proceed as follows:

$$\mathbb{E}^{\mathbb{P}}\big[q_T \mid \mathcal{G}_t^D\big] = \mathbb{E}^{\mathbb{P}}\Bigg[q_{T_1} + \underbrace{\mathbb{E}^{\mathbb{P}}\big[q_T - q_{T_1} \mid \mathcal{G}_{T_1}^D\big]}_{(\star\star)} \Bigg| \mathcal{G}_t^D\Bigg] \tag{35}$$

Re-arranging $(\star\star)$ and taking out what is known, i.e. $\mathcal{G}_{T_1}^D$-measurable, we get:

$$\begin{aligned}\mathbb{E}^{\mathbb{P}}\big[q_T - q_{T_1} \mid \mathcal{G}_{T_1}^D\big] &= q_{T_1}\big(\mathrm{e}^{-\kappa^D(T-T_1)} - 1\big) \\ &\quad + \sigma^D \mathbb{E}^{\mathbb{P}}\Bigg[\int_{T_1}^{T} \mathrm{e}^{\varphi(s)}\mathrm{e}^{-\kappa^D(T-s)}\mathrm{d}B_s^D \Bigg| \mathcal{G}_{T_1}^D\Bigg]\end{aligned} \tag{36}$$

Combining Equations (35) and (36) and using iterated conditioning now yields:

$$\begin{aligned}\mathbb{E}^{\mathbb{P}}\big[q_T \mid \mathcal{G}_t^D\big] &= \mathbb{E}^{\mathbb{P}}\big[q_{T_1} \mid \mathcal{G}_t^D\big]\mathrm{e}^{-\kappa^D(T-T_1)} \\ &\quad + \mathbb{E}^{\mathbb{P}}\Bigg\{\sigma^D \mathbb{E}^{\mathbb{P}}\Bigg[\int_{T_1}^{T} \mathrm{e}^{\varphi(s)}\mathrm{e}^{-\kappa^D(T-s)}\mathrm{d}B_s^D \Bigg| \mathcal{G}_{T_1}^D\Bigg] \Bigg| \mathcal{G}_t^D\Bigg\} \\ &= \mathbb{E}^{\mathbb{P}}\big[q_{T_1} \mid \mathcal{G}_t^D\big]\mathrm{e}^{-\kappa^D(T-T_1)} + \mathbb{E}^{\mathbb{P}}\big[I_{\mathcal{G}}(T_1, T) \mid \mathcal{G}_t^D\big]\end{aligned} \tag{37}$$

The term $\mathbb{E}^{\mathbb{P}}\big[I_{\mathcal{G}}(T_1, T) \mid \mathcal{G}_t^D\big]$ in Equation (37), however, can be manipulated similarly to Equations (31) - (33):

$$\begin{aligned}\mathbb{E}^{\mathbb{P}}\big[I_{\mathcal{G}}(T_1, T) \mid \mathcal{G}_t\big] &= \mathbb{E}^{\mathbb{P}}\Bigg\{\sigma^D \mathrm{e}^{-\kappa^D T}\mathbb{E}^{\mathbb{P}}\Bigg[\int_{T_1}^{T_2} \rho'(u)\mathrm{d}B_u^D \Bigg| \mathcal{G}_{T_1}^D\Bigg] \int_{T_1}^{T} f(s)\mathrm{d}s \Bigg| \mathcal{G}_t^D\Bigg\} \\ &= \sigma^D \mathrm{e}^{-\kappa^D T} \underbrace{\mathbb{E}^{\mathbb{P}}\Bigg[\int_{T_1}^{T_2} \rho'(u)\mathrm{d}B_u^D \Bigg| \mathcal{G}_t^D\Bigg]}_{(\star\star\star)} \int_{T_1}^{T} f(s)\mathrm{d}s\end{aligned} \tag{38}$$

Analogous to Equation (34), the term $(\star\star\star)$ can be backed out from the given forecast points relating to T_1 and T_2:

$$(\star\star\star) = \frac{1}{\sigma^D}\Bigg(\mathrm{e}^{\kappa^D T_2}\mathbb{E}^{\mathbb{P}}\big[q_{T_2} \mid \mathcal{G}_t^D\big] - \mathrm{e}^{\kappa^D T_1}\mathbb{E}^{\mathbb{P}}\big[q_{T_1} \mid \mathcal{G}_t^D\big]\Bigg) \tag{39}$$

B. Conditional Expectations Based on Enlarged Filtrations Under an Equivalent Risk-Neutral Measure

For derivatives pricing purposes, and based on Equation (14), conditional expectations $\mathbb{E}^{\mathbb{Q}}\big[\cdot \mid \mathcal{G}_t\big]$ and variances $\mathbb{V}^{\mathbb{Q}}\big[\cdot \mid \mathcal{G}_t\big]$ under the enlarged filtration $(\mathcal{G}_t)_{t\in[0,T^\star]}$ and a risk-neutral measure $\mathbb{Q}$ need to be computed for both demand and capacity processes D_t and C_t, respectively.

Defining $A(t) = \int_0^t \vartheta(s)\mathrm{d}s$, and based on the manipulations in the previous subsection, the $\mathcal{G}^D$-adapted dynamics for D_t can now be stated as below (cf. Equation (2)):

$$\mathrm{d}q_t \;=\; -\kappa^D\left(q_t - \frac{\sigma^D \mathrm{e}^{\varphi(t)}}{\kappa^D}\vartheta(t)\right)\mathrm{d}t + \sigma^D \mathrm{e}^{\varphi(t)}\mathrm{d}\hat{B}_t^D,$$

where $\hat{B}_t^D$ is a $\mathcal{G}_t^D$-adapted standard $\mathbb{P}$-Brownian motion.[38] Applying Girsanov's theorem, and given that our market setting is inherently incomplete, we assume that under a suitably chosen risk-neutral measure $\mathbb{Q}$, $\hat{B}_t^D$ is a semi-martingale and decomposes as follows:

$$\hat{B}_t^D \;=\; \check{B}_t^D + \Lambda_{\mathcal{G}}^D(t),$$

where $\check{B}_t^D$ is a $\mathcal{G}_t^D$-adapted standard $\mathbb{Q}$-Brownian motion and $\Lambda_{\mathcal{G}}^D(t) = \int_0^t \lambda_{\mathcal{G}}^D(s)\mathrm{d}s$ is a finite variation process governing the change of measure as market price of (demand) risk. The risk-neutral dynamics for D_t under the enlarged filtration now are:

$$\mathrm{d}q_t \;=\; -\kappa^D\left(q_t - \frac{\sigma^D \mathrm{e}^{\varphi(t)}}{\kappa^D}\big(\vartheta(t) + \lambda_{\mathcal{G}}^D(t)\big)\right)\mathrm{d}t + \sigma^D \mathrm{e}^{\varphi(t)}\mathrm{d}\check{B}_t^D,$$

where conditional expectation $\mathbb{E}^{\mathbb{Q}}\big[\cdot \mid \mathcal{G}_t\big]$ and variance $\mathbb{V}^{\mathbb{Q}}\big[\cdot \mid \mathcal{G}_t\big]$ are then derived in the standard way. As outlined in Section IV, the market price of risk will be assumed constant and inferred from price quotes of traded derivative contracts. Depending on whether or not forward-looking information will be used, it will be referred to as $\lambda_{\mathcal{G}}^D$ or $\lambda_{\mathcal{F}}^D$, respectively.

[38] Recall that we assume the filtration $(\mathcal{G}_t^C)_{t\in[0,T^\star]}$ to be of such nature that $B_t^D = \hat{B}_t^D + A(t)$ is a semimartingale.

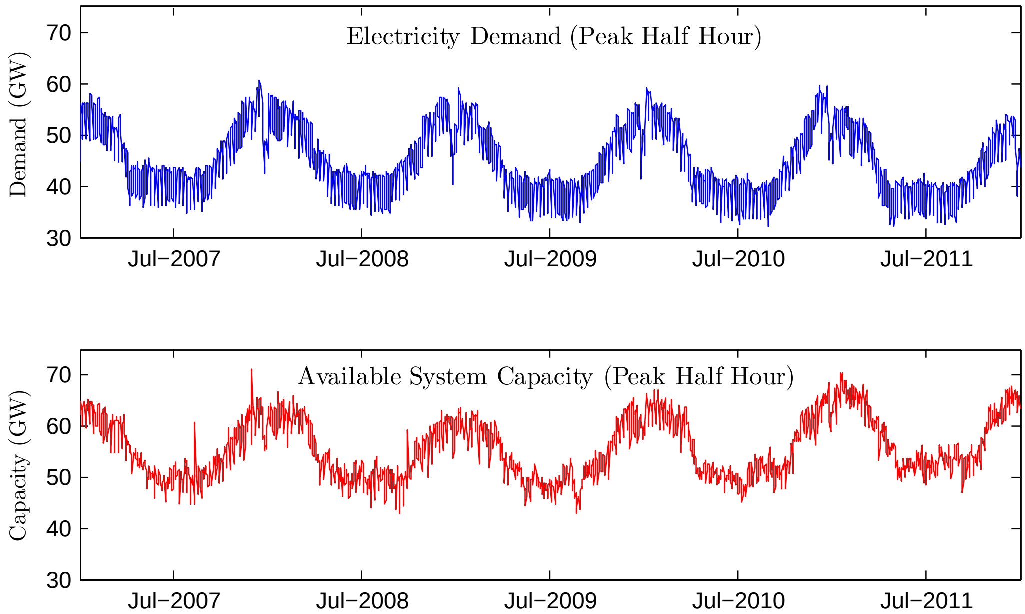

Figure 1: **Daily Electricity Demand and Available System Capacity**
This figure shows the time series of realized daily electricity demand and available system capacity in the British market during the period from 01-Jan-2007 to 31-Dec-2011. Displayed demand and capacity data both relate to the same daily peak (demand) half hour. All data shown were obtained from National Grid and Elexon.

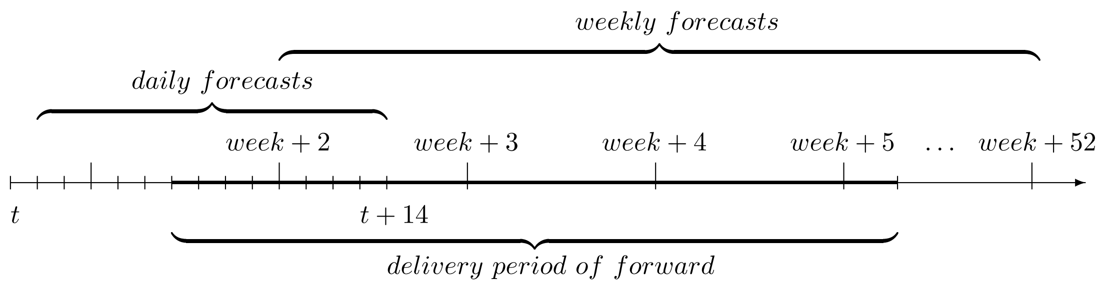

Figure 2: **Schematic Overview of Forecast Horizons for the GB Market**
Daily forecasts are available on a 2- to 14-days-ahead basis; additionally, forecasts of expected maximum demand (capacity) per week are released for weeks 2-52. In this example, the first nine delivery days of some given forward contract are covered by daily forecasts, expected demand (capacity) for each of the remaining days must be derived based on Proposition III.1.

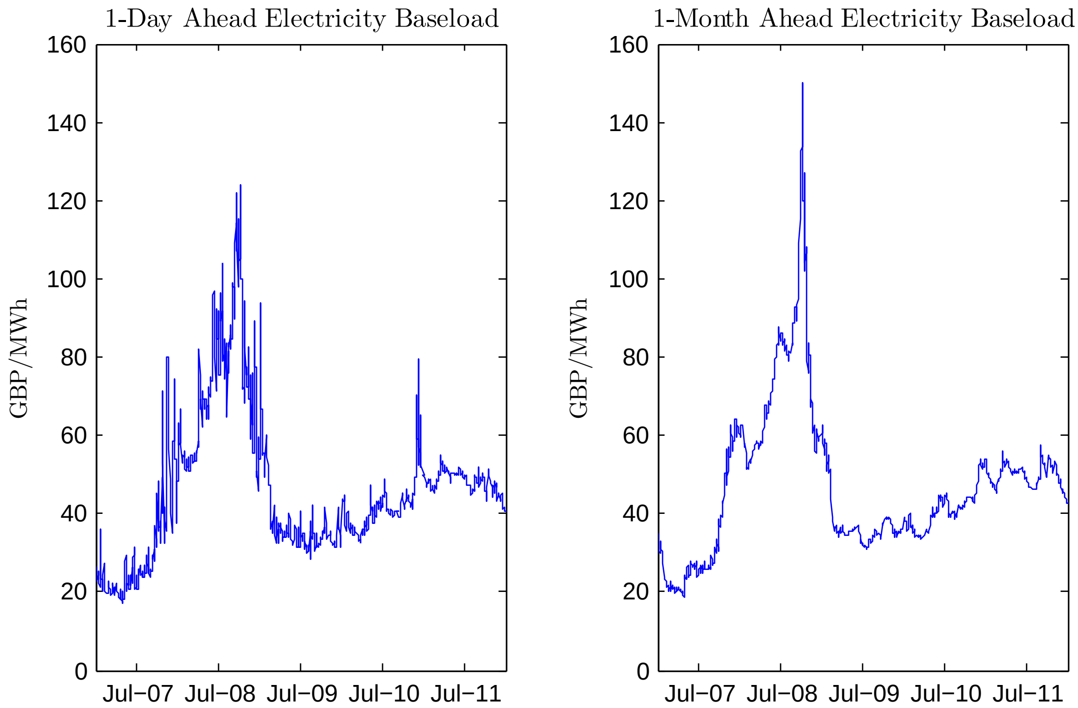

Figure 3: **1-Month Ahead Forward and 1-Day Ahead Baseload Electricity Prices**
This figure shows the time series of daily forward prices for 1-month ahead and 1-day ahead baseload electricity contracts during the period from 01-Jan-2007 to 31-Dec-2011. All data shown were obtained from Bloomberg; for dates with missing quotes/prices, the last observed historic price was carried over.

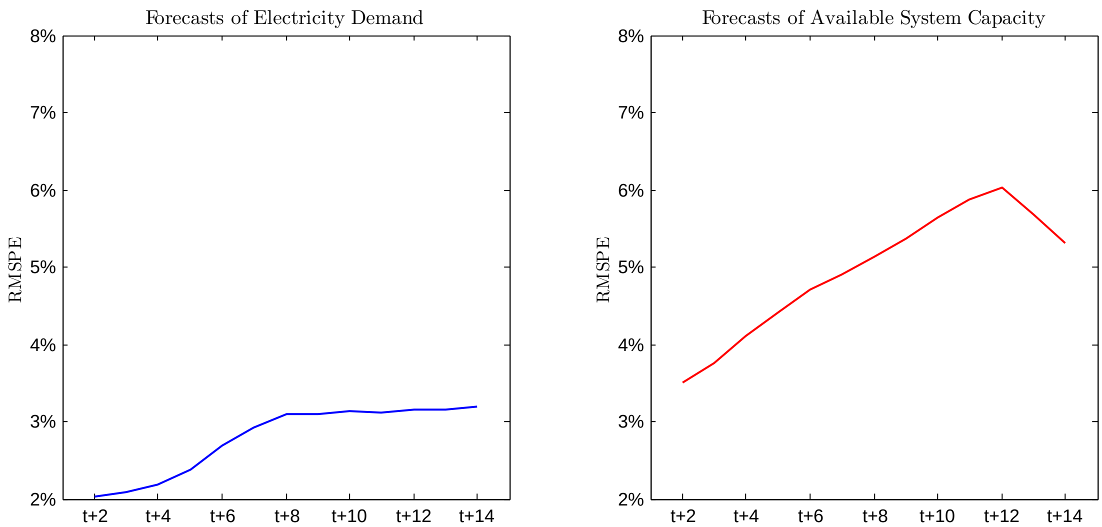

Figure 4: **Performance of Demand and Capacity Forecasts**
This figure shows the root mean squared percentage error (RMSPE) for the 2-14 days-ahead forecasts of electricity demand and available system capacity during the period from 01-Jan-2007 to 31-Dec-2011. Note that especially for capacity forecasts, inputs are based on data released by National Grid plc, yet have been further adjusted by the authors.

Table 1: **Samples of Baseload Spot and Forward Contracts**

This table reports summary statistics for the samples of electricity spot (day-ahead) and forward prices covering the period from January 2, 2007 until December 30, 2011. $[\underline{T}, \overline{T}]$ denotes the average delivery period (in days) and $\underline{T} - t$ the average maturity (in days) as measured until the start of the delivery period. All contracts from both the Bloomberg and Marex Spectron samples are baseload contracts. Displayed log-returns for 1- and 2-month(s) ahead forward contracts are adjusted to account for roll-over of contracts as well as for missing quotes.

		Mean	Median	Std. Dev.	Skewness	Kurtosis	$[\underline{T}, \overline{T}]$	$\underline{T} - t$
Bloomberg Data	**1-Day Ahead**							
	$\ln P_t$	3.7543	3.7600	0.3891	0.0818	-0.1121	1.0	1.0
	$\ln P_t - \ln P_{t-1}$	-0.0019	-0.0019	0.0717	1.2812	12.8443		
	1-Month Ahead							
	$\ln F_t$	3.7781	3.7899	0.3737	0.2196	0.2131	30.4	15.9
	$\ln F_t - \ln F_{t-1}$	-0.0009	-0.0003	0.0219	-0.2365	5.4731		
Marex Spectron Data	**1-Day Ahead**							
	$\ln P_t$	3.7500	3.7612	0.3829	0.1225	0.0516	1.0	1.0
	$\ln P_t - \ln P_{t-1}$	-0.0027	-0.0021	0.0721	1.2138	10.9920		
	1-Month Ahead							
	$\ln F_t$	3.7856	3.7956	0.3604	0.2359	0.4641	30.4	16.1
	$\ln F_t - \ln F_{t-1}$	-0.0010	-0.0014	0.0205	-0.1641	4.8328		
	2-Months Ahead							
	$\ln F_t$	3.8003	3.7975	0.3532	0.3031	0.4328	30.8	44.8
	$\ln F_t - \ln F_{t-1}$	-0.0009	-0.0005	0.0177	-0.3552	4.5943		

Table 2: **Maximum-Likelihood Parameter Estimates for Electricity Demand**

This table reports Maximum-Likelihood parameter estimates and robust t-statistics (White (1982); [in brackets]) for the electricity demand process as specified in Equations (2), (3), and (4). Parameters are estimated using five years of daily electricity demand data (in GW units), and are held constant throughout the following year for pricing purposes. Note that following the discretization of Equation (2) into an autoregressive form, we have that $B^D = 1 - \kappa^D$.

Period	**2002–2006**		**2003–2007**		**2004–2008**		**2005–2009**		**2006–2010**	
a^D	56.05	[401.72]	56.78	[395.75]	57.25	[429.66]	57.90	[394.80]	57.80	[388.69]
b^D	0.00	[13.01]	0.00	[0.47]	-0.00	[-10.38]	-0.00	[-23.81]	-0.00	[-23.28]
c^D_{WE}	-6.67	[-76.15]	-6.49	[-73.94]	-6.35	[-74.57]	-6.24	[-71.85]	-6.08	[-70.76]
$c^D_{PH_1}$	-5.70	[-10.99]	-5.56	[-11.37]	-5.17	[-10.69]	-4.90	[-11.34]	-5.51	[-12.99]
$c^D_{PH_2}$	-12.21	[-12.88]	-11.80	[-13.56]	-11.38	[-14.37]	-11.35	[-14.39]	-11.29	[-12.58]
$c^D_{PH_3}$	-7.92	[-15.47]	-7.46	[-14.63]	-7.00	[-12.63]	-6.75	[-10.32]	-5.30	[-12.73]
$c^D_{PH_4}$	-6.82	[-16.45]	-6.41	[-15.13]	-6.05	[-15.70]	-5.63	[-17.39]	-5.37	[-20.13]
c^D_2	-1.36	[-7.43]	-1.10	[-5.87]	-1.10	[-6.36]	-1.29	[-6.58]	-1.62	[-7.92]
c^D_3	-4.54	[-17.21]	-4.37	[-16.20]	-3.98	[-15.68]	-4.51	[-17.39]	-5.09	[-20.68]
c^D_4	-10.95	[-62.34]	-11.00	[-57.07]	-10.63	[-59.30]	-11.04	[-56.50]	-11.88	[-58.90]
c^D_5	-12.87	[-81.37]	-12.81	[-79.57]	-12.82	[-85.86]	-13.02	[-83.28]	-13.51	[-83.13]
c^D_6	-13.80	[-94.34]	-13.55	[-92.88]	-13.49	[-99.37]	-13.52	[-92.65]	-14.09	[-93.23]
c^D_7	-13.83	[-94.59]	-13.66	[-90.73]	-13.57	[-99.13]	-13.68	[-89.38]	-14.19	[-89.93]
c^D_8	-14.14	[-94.00]	-14.00	[-92.23]	-13.88	[-101.70]	-14.10	[-95.22]	-14.72	[-96.25]
c^D_9	-12.17	[-72.44]	-11.81	[-67.16]	-11.57	[-71.56]	-11.75	[-66.99]	-12.34	[-66.55]
c^D_{10}	-7.43	[-34.45]	-7.15	[-35.94]	-7.13	[-39.65]	-7.25	[-37.10]	-7.77	[-37.62]
c^D_{11}	-1.72	[-9.22]	-1.30	[-6.75]	-1.24	[-6.57]	-1.53	[-7.58]	-2.15	[-10.53]
c^D_{12}	-0.17	[-0.67]	0.26	[1.07]	0.27	[1.08]	0.39	[1.64]	0.50	[1.96]
B^D	0.59	[21.47]	0.62	[24.03]	0.61	[23.49]	0.64	[26.53]	0.66	[26.69]
σ^D	1.24	[40.55]	1.22	[39.64]	1.18	[40.85]	1.17	[41.89]	1.15	[41.90]
θ^D	0.41	[10.43]	0.40	[9.82]	0.41	[10.25]	0.41	[10.48]	0.40	[10.88]
ζ^D	0.21	[17.63]	0.21	[16.33]	0.21	[18.11]	0.22	[19.42]	0.23	[18.33]
LogLik	2984.32		2947.37		2890.18		2879.65		2851.13	

Table 3: **Maximum-Likelihood Parameter Estimates for Available System Capacity**

This table reports Maximum-Likelihood parameter estimates and robust t-statistics (White (1982); [in brackets]) for the capacity process as specified in Equations (6) *and* (7)*. Parameters are estimated using five years of data for daily levels of available capacity (in GW units), and are held constant throughout the following year for pricing purposes. Note that following the discretization of Equation* (6) *into an autoregressive form, we have that* $B^C = 1 - \kappa^C$.

Period	**2002–2006**		**2003–2007**		**2004–2008**		**2005–2009**		**2006–2010**	
a^C	64.30	[245.80]	63.96	[243.35]	64.78	[342.91]	64.09	[388.07]	63.32	[391.66]
b^C	-0.00	[-1.28]	0.00	[0.56]	-0.00	[-6.00]	-0.00	[-10.60]	-0.00	[-2.14]
c^C_{WE}	-4.08	[-40.96]	-3.79	[-37.23]	-3.66	[-36.24]	-3.39	[-31.79]	-3.20	[-28.14]
$c^C_{PH_1}$	-3.97	[-10.19]	-4.11	[-9.77]	-3.45	[-7.80]	-3.23	[-6.85]	-2.91	[-5.10]
$c^C_{PH_2}$	-5.69	[-10.42]	-5.47	[-9.22]	-4.49	[-7.96]	-3.88	[-6.52]	-3.83	[-7.27]
$c^C_{PH_3}$	-1.56	[-2.75]	-0.92	[-1.54]	-0.58	[-1.02]	-0.87	[-1.28]	0.21	[0.35]
$c^C_{PH_4}$	-1.93	[-2.78]	-1.22	[-1.69]	-0.82	[-1.24]	-0.86	[-1.34]	-0.31	[-0.48]
c^C_2	-1.14	[-6.70]	-1.33	[-7.91]	-1.25	[-7.66]	-1.22	[-6.46]	-1.02	[-4.89]
c^C_3	-3.51	[-16.72]	-3.70	[-18.17]	-3.09	[-15.74]	-2.74	[-12.85]	-2.54	[-11.94]
c^C_4	-8.18	[-38.50]	-8.54	[-40.54]	-8.60	[-41.70]	-8.31	[-36.69]	-8.48	[-33.64]
c^C_5	-10.89	[-48.52]	-11.40	[-52.22]	-11.58	[-55.40]	-10.97	[-49.18]	-11.32	[-47.63]
c^C_6	-12.83	[-69.10]	-13.24	[-72.03]	-13.42	[-77.23]	-12.75	[-63.98]	-12.69	[-65.04]
c^C_7	-13.86	[-77.47]	-14.02	[-80.84]	-13.68	[-78.72]	-13.11	[-66.31]	-13.21	[-66.08]
c^C_8	-13.33	[-69.21]	-13.57	[-69.54]	-12.97	[-75.28]	-12.42	[-58.00]	-12.24	[-51.63]
c^C_9	-11.86	[-61.42]	-11.86	[-61.46]	-12.32	[-61.59]	-11.73	[-51.87]	-11.73	[-52.06]
c^C_{10}	-7.68	[-36.33]	-7.98	[-38.31]	-8.28	[-40.26]	-7.81	[-35.42]	-7.81	[-31.77]
c^C_{11}	-2.42	[-12.84]	-2.63	[-13.67]	-3.09	[-15.62]	-2.59	[-11.76]	-2.69	[-11.63]
c^C_{12}	-0.47	[-2.45]	-0.68	[-3.25]	-1.00	[-4.70]	-0.53	[-2.31]	-0.25	[-0.97]
c^C_R	-1.57	[-9.65]	-1.41	[-7.89]	-1.95	[-12.64]	-1.74	[-6.85]	NA	[NA]
B^C	0.62	[31.50]	0.57	[21.41]	0.57	[20.93]	0.59	[22.62]	0.62	[24.98]
σ^C	1.43	[55.87]	1.53	[32.27]	1.53	[31.05]	1.57	[31.64]	1.62	[32.86]
LogLik	3245.46		3369.11		3370.00		3411.43		3467.47	

Table 4: **Kalman Filter Parameter Estimates for Natural Gas**

This table reports parameter estimates and robust t-statistics (White (1982); [in brackets]) for the natural gas price process as specified in Equations (9) and (10), using the Kalman filter and maximum likelihood estimation. Parameters are estimated based on five years of daily natural gas futures price data (using 1-, 2-, 3-, and 4-months ahead contracts), and are held constant throughout the following year for pricing purposes.

Period	**2002–2006**		**2003–2007**		**2004–2008**		**2005–2009**		**2006–2010**	
a^g	0.11	[7.70]	-0.09	[-2.77]	-0.11	[-3.88]	0.09	[4.57]	-0.16	[-8.88]
b^g	0.00	[9.52]	0.00	[9.07]	0.00	[-1.26]	0.00	[-2.91]	0.00	[-6.22]
c_2^g	-0.07	[-34.83]	-0.07	[-33.30]	-0.06	[-29.82]	-0.05	[-32.98]	-0.04	[-24.26]
c_3^g	-0.25	[-78.69]	-0.22	[-62.19]	-0.20	[-48.47]	-0.16	[-44.95]	-0.12	[-36.66]
c_4^g	-0.41	[-96.57]	-0.38	[-78.62]	-0.34	[-64.59]	-0.31	[-60.07]	-0.24	[-53.62]
c_5^g	-0.50	[-107.85]	-0.47	[-91.46]	-0.43	[-75.92]	-0.40	[-74.14]	-0.32	[-60.91]
c_6^g	-0.56	[-116.13]	-0.52	[-97.74]	-0.49	[-82.69]	-0.45	[-81.02]	-0.36	[-65.48]
c_7^g	-0.59	[-117.36]	-0.56	[-102.07]	-0.53	[-87.58]	-0.50	[-88.41]	-0.40	[-70.95]
c_8^g	-0.58	[-108.87]	-0.54	[-92.92]	-0.52	[-81.27]	-0.49	[-84.04]	-0.40	[-66.70]
c_9^g	-0.64	[-123.86]	-0.61	[-103.24]	-0.58	[-90.65]	-0.54	[-94.63]	-0.44	[-70.82]
c_{10}^g	-0.41	[-80.88]	-0.40	[-67.10]	-0.39	[-62.47]	-0.35	[-64.60]	-0.28	[-55.10]
c_{11}^g	-0.20	[-42.95]	-0.18	[-32.71]	-0.17	[-28.83]	-0.13	[-26.62]	-0.09	[-25.51]
c_{12}^g	-0.08	[-20.91]	-0.07	[-16.29]	-0.06	[-14.46]	-0.04	[-12.61]	-0.02	[-10.82]
κ^g	0.00	[5.13]	0.00	[5.87]	0.00	[6.45]	0.00	[10.68]	0.00	[10.80]
λ^g	3.72	[19.75]	7.54	[7.70]	7.74	[12.46]	5.63	[28.02]	6.38	[24.20]
σ^g	0.02	[13.28]	0.03	[15.67]	0.03	[15.55]	0.03	[15.34]	0.03	[16.51]
LogLik	11067.23		10535.82		10357.80		10749.18		10972.14	

Table 5: **Implied Parameter Estimates for $\Phi^\star_W$ (Bloomberg Sample / Forecasts for D_t and C_t)**

This table reports yearly average values and corresponding standard errors [in brackets] of the implied estimates for the "fundamental" and risk-neutral parameters $\Phi^\star_W = \{\alpha, \beta, \gamma, \delta, \lambda^D_{\mathcal{G}}, \lambda^C_{\mathcal{G}}\}$. Parameters are obtained by minimizing the root mean squared percentage errors (RMSPE) between observed market prices and theoretical model prices for both 1-day ahead and 1-month ahead forward contracts from the Bloomberg (BBG) data sample. Throughout the estimation procedure, forecasts of both electricity demand as well as available capacity are used. In-sample estimation is performed based on a time window W of eight weeks with weekly shifting. Then, for every parameter in $\Phi^\star_W$, the below displayed average values are computed based on the set of estimates implied from all in-sample windows in the respective year.

	2007	**2008**	**2009**	**2010**	**2011**	**07-11**
α	1.5203	3.0155	2.7241	2.0338	1.8217	2.2220
	[0.1386]	[0.1956]	[0.0981]	[0.1025]	[0.1276]	[0.0697]
β	0.0279	0.0286	0.0164	0.0135	0.0106	0.0193
	[0.0020]	[0.0027]	[0.0009]	[0.0008]	[0.0003]	[0.0008]
γ	-0.0175	-0.0344	-0.0150	-0.0116	-0.0117	-0.0180
	[0.0015]	[0.0035]	[0.0006]	[0.0004]	[0.0004]	[0.0009]
δ	0.4317	0.4369	0.2713	0.4674	0.5578	0.4324
	[0.0334]	[0.0388]	[0.0269]	[0.0253]	[0.0317]	[0.0151]
$\lambda^D_{\mathcal{G}}$	-16.3475	3.9719	-45.1915	-30.9245	-58.2689	-29.5412
	[10.2246]	[14.4244]	[12.5963]	[7.5293]	[8.8900]	[5.0647]
$\lambda^C_{\mathcal{G}}$	20.2954	11.3382	52.4444	36.0655	64.8668	37.1602
	[15.0584]	[18.2663]	[14.5850]	[7.6381]	[9.5236]	[6.1353]
$\beta\lambda^D_{\mathcal{G}}+$	-0.0817	-0.8503	-1.5816	-0.6957	-1.2935	-0.9034
$\gamma\lambda^C_{\mathcal{G}}$	[0.4304]	[0.8125]	[0.4360]	[0.1589]	[0.1913]	[0.2087]

Table 6: **Out-of-Sample Pricing Results: 1-Month Ahead Forward Contracts (BBG)**

This table reports yearly (and aggregate) pricing errors for 1-month ahead electricity forward contracts from the Bloomberg (BBG) data sample; implemented models either do not rely on forecasts ("No FC") or incorporate demand and/or capacity forecasts ("FC for D_t" and "FC for D_t & C_t", respectively). We use weekly subperiods for out-of-sample pricing, with in-sample fitting of the model being performed based on the cross-section of 1-day ahead and 1-month ahead forward quotes collected during the preceding eight weeks. Error measures shown are mean percentage error (MPE), mean absolute percentage error (MAPE), root mean squared error (RMSE) and root mean squared percentage error (RMSPE).

	MPE	MAPE	RMSE	RMSPE	MPE	MAPE	RMSE	RMSPE
	2007				**2008**			
No FC	-3.33%	5.19%	£2.90	7.17%	0.98%	9.83%	£17.22	20.57%
FC for D_t	-4.54%	5.86%	£3.78	7.76%	0.71%	7.73%	£10.29	12.04%
FC for D_t & C_t	-4.30%	5.53%	£3.47	7.49%	-1.19%	6.65%	£7.73	8.99%
	2009				**2010**			
No FC	-0.79%	3.18%	£2.06	5.99%	-1.02%	2.00%	£1.31	2.96%
FC for D_t	-0.11%	2.68%	£1.36	3.90%	-0.36%	1.94%	£1.16	2.69%
FC for D_t & C_t	-0.28%	2.83%	£1.46	4.16%	-0.31%	2.17%	£1.32	3.03%
	2011				**2007-2011**			
No FC	1.15%	2.59%	£1.73	3.63%	**-0.62%**	**4.56%**	**£7.90**	**10.31%**
FC for D_t	1.06%	2.18%	£1.37	2.81%	**-0.67%**	**4.09%**	**£5.00**	**6.87%**
FC for D_t & C_t	1.47%	2.41%	£1.57	3.23%	**-0.94%**	**3.93%**	**£3.95**	**5.91%**

Table 7: **Out-of-Sample Pricing Results: 1-Month Ahead Forward Contracts (OTC)**

This table reports yearly (and aggregate) pricing errors for 1-month ahead electricity forward contracts from the Marex Spectron (OTC) data sample; implemented models either do not rely on forecasts ("No FC") or incorporate demand and/or capacity forecasts ("FC for D_t" and "FC for D_t & C_t", respectively). We use weekly subperiods for out-of-sample pricing, with in-sample fitting of the model being performed based on the cross-section of 1-day ahead, 1-month ahead, and 2-months ahead forward quotes collected during the preceding six weeks. Error measures shown are mean percentage error (MPE), mean absolute percentage error (MAPE), root mean squared error (RMSE) and root mean squared percentage error (RMSPE).

	MPE	MAPE	RMSE	RMSPE	MPE	MAPE	RMSE	RMSPE
	2007				**2008**			
No FC	-2.73%	6.11%	£4.50	8.97%	-2.78%	7.18%	£9.04	9.51%
FC for D_t	-3.60%	6.14%	£4.26	8.57%	-1.88%	8.11%	£9.25	10.23%
FC for D_t & C_t	-3.16%	5.77%	£3.90	7.74%	-1.17%	6.71%	£7.49	8.60%
	2009				**2010**			
No FC	2.44%	3.34%	£1.78	4.65%	0.13%	2.32%	£1.38	3.03%
FC for D_t	2.07%	3.15%	£1.63	4.31%	0.12%	2.25%	£1.37	2.98%
FC for D_t & C_t	2.52%	3.38%	£1.76	4.63%	0.33%	2.40%	£1.44	3.21%
	2011				**2007-2011**			
No FC	2.25%	3.18%	£1.87	3.82%	**-0.02%**	**4.30%**	**£4.52**	**6.40%**
FC for D_t	1.81%	3.03%	£1.74	3.61%	**-0.19%**	**4.39%**	**£4.53**	**6.42%**
FC for D_t & C_t	2.36%	3.54%	£2.05	4.19%	**0.28%**	**4.26%**	**£3.87**	**5.91%**

Table 8: **Out-of-Sample Pricing Results: 2-Months Ahead Forward Contracts (OTC)**

This table reports yearly (and aggregate) pricing errors for 2-months ahead electricity forward contracts from the Marex Spectron (OTC) data sample; implemented models either do not rely on forecasts ("No FC") or incorporate demand and/or capacity forecasts ("FC for D_t" and "FC for D_t & C_t", respectively). We use weekly subperiods for out-of-sample pricing, with in-sample fitting of the model being performed based on the cross-section of 1-day ahead, 1-month ahead, and 2-months ahead forward quotes collected during the preceding six weeks. Error measures shown are mean percentage error (MPE), mean absolute percentage error (MAPE), root mean squared error (RMSE) and root mean squared percentage error (RMSPE).

	MPE	MAPE	RMSE	RMSPE	MPE	MAPE	RMSE	RMSPE
	2007				**2008**			
No FC	-5.04%	6.86%	£3.75	8.72%	-0.69%	8.61%	£9.76	11.45%
FC for D_t	-4.23%	7.26%	£3.95	9.31%	0.66%	7.08%	£8.11	9.56%
FC for D_t & C_t	-4.59%	6.97%	£3.64	8.72%	-0.02%	6.21%	£6.81	7.85%
	2009				**2010**			
No FC	0.49%	2.88%	£1.65	4.46%	-0.84%	2.15%	£1.25	2.74%
FC for D_t	0.34%	2.55%	£1.33	3.55%	-0.84%	2.19%	£1.34	2.90%
FC for D_t & C_t	0.10%	2.99%	£1.55	4.12%	-0.65%	2.63%	£1.51	3.51%
	2011				**2007-2011**			
No FC	0.81%	2.19%	£1.48	2.91%	**-0.95%**	**4.34%**	**£4.57**	**6.72%**
FC for D_t	-0.07%	1.76%	£1.20	2.29%	**-0.79%**	**3.99%**	**£3.96**	**6.17%**
FC for D_t & C_t	0.25%	1.78%	£1.22	2.38%	**-0.92%**	**3.96%**	**£3.47**	**5.70%**

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