Project work

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Day-ahead electricity market bidding under uncertainty for a complex river system

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Preface

This report is written as a project thesis within the field of Applied Economics and Operations Research at the Department of Industrial Economics and Technology Management, University of Science and Technology (NTNU). We would like to thank our teaching supervisor Professor Stein-Erik Fleten for his helpful assistance. From Agder Energi we thank Jarand Røynstrand for initiative and valuable input. Discussions with Gro Klæbø have also been of great help. In addition, we would like to thank Turid Follestad at SINTEF for help with scenario generation and Henrik Andersson for help with model implementation.

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Abstract

This report considers the bidding problem facing a Nordic hydropower producer, specifically Agder Energi. A stochastic mixed-integer model based on Fleten and Kristoffersen (2007) is developed and tested. Uncertainty in market prices and inflow is the reason why a stochastic model is appropriate. The model concerns bidding in the spot market for power and short-term production scheduling based on the realized market price. Uncertainty is modeled in several stages. The modeling of reservoirs and watercourses are general and includes time delay and different watercourses for discharge, bypass and spill. Results from the stochastic model are compared with results from a deterministic case, and we find that it produces a more realistic unit commitment schedule.

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1. Introduction

Selling power into the day-ahead market constitutes a substantial part of the revenues for Nordic power producers. Finding a good solution for the bidding problem is hence one of the most important tasks the power producers are faced with. In the Nordic countries the day-ahead market for physical contracts is called the Elspot market, which is run by Nord Pool Spot AS (from now called Nord Pool). Buyers and sellers submit their bids for the day-ahead operations to Nord Pool. The participants deliver their bids in the form of a bidding matrix before12:00 the day before operation, and Nord Pool then calculates the prices by aggregating the sales- and purchase-curves for every hour for the following day. The spot price is found at the intersection between demand and supply, trough a mixed-integer program algorithm. A producer's bid for a given hour is accepted if the bid price is equal to or lower than the system spot price.

The transfer of power is subject to capacity constraints and if power flow in or out of one area exceeds the available transmission capacity, the prices are lowered in surplus areas and raised in deficit areas to facilitate the flow. This results in different area prices. The area prices are calculated by Nord Pool and are published together with the spot price every day before 13:30.

Once a producer has participated in the Elspot marked he is obligated to deliver the volume bidded for the realized area price. This volume is calculated by interpolation between the two nearest price points in the bidding matrix for a given hour. The market is cleared up to 36 hours before actual delivery of power and due to uncertainties in the production situation the produced volume may not be in exact balance with the committed volume. This imbalance can be handled in the Elbas market, where continuous trading of physical power is available up to one hour prior to delivery. The Elbas market enables the producers to make trades much closer to the operating hour.

In the hour of operation, total production has to be instantly balanced by consumption, and this is the responsibility of the TSOs (in Norway Statnett). Producers may participate in the market for regulating power by committing to maintain available capacity to be called upon in case of unbalances. These obligations are committed after the spot market clears, but prior to the operating hour.

Both the Elbas market and the regulating power market are important properties of the Nordic power system. Still, the objective of this project is to optimize the bidding strategy for the Elspot market, and therefore neither Elbas nor the regulating market will be modelled in detail. The power system should be planned such that production is balanced by demand, and hence it is wanted by the TSO that producers trade their expected production in the Elspot market (Statnett, 2012).

Our motivation for this project is to find an alternative for the short term scheduling of hydropower production used by Agder Energi. Bidding in the Elspot market is the most important part of the short term production scheduling. Agder Energi and most other Nordic power producers are using a deterministic model to solve the bidding problem, although future prices and inflow are stochastic variables in the short-term perspective. The deterministic model generates a bidding matrix based on a set of forecasted price profile scenarios using fixed price points and operating hours. This matrix is then submitted to NordPool and one hopes that the realized price profile matches the underlying forecasted profile used for the bidding matrix.

The problem in Agder Energi's bidding strategy arises when the realized price profile does not match the profile of the price scenarios. It may happen that the realized price profile crosses the price scenarios, as in figure 1-1. This can cause unfortunate outcomes for the production plan, making costly adjustments for production scheduling necessary.

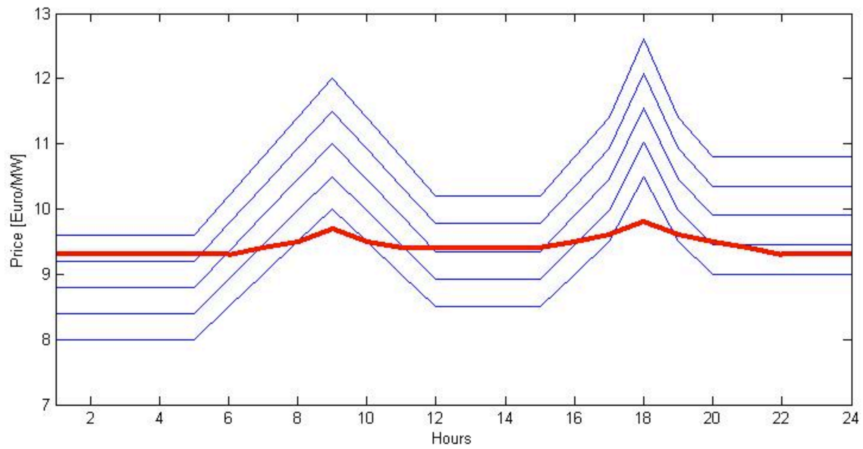

Figure 1-1: The realized price (red) profile and scenario profiles

In the situation displayed in figure 1-1 the realized price is intersecting a high price scenario for the first hours of the day and will therefore result in planned production in large parts of the system. But when the morning peak emerges the realized price increases less than the predicted profile of the price scenario, resulting in crossing of a lower price scenario. The price scenario that the realized price now intersects has over all such a low price that the production is never started in this scenario. Hence you can risk that the production stops even though the price actually increases in the morning hours. As shown in the figure, the same problem can occur for the afternoon peak.

Situations like this is today sorted out by skillful production planners at Agder Energi, but a model which can avoid such problems is desirable, and hence the motivation for this project.

Our approach to solving the problem described above is by the use of stochastic programming. We will develop a model for optimal bidding and short-term production planning with the objective to maximize the total profit from a given system. Our model is based on Kristoffersen and Fleten (2007), which develop a stochastic MIP model for short-term production planning given uncertain prices. The system presented in the paper is a cascaded two-reservoir system without the option of bypass or spill. We, however, implement the stochastic model on a more complicated real life system, Mandalsvassdraget. This is a large Norwegian hydropower cascade with several reservoirs, river courses and power stations linked together. A more complex system makes the formulation and implementation part more challenging.

The goal of this report is to show Agder Energi the possibilities that stochastic programming gives and that they potentially can benefit from using such a model.

Stochastic programming takes into account the uncertainty of different parameters within a problem. As input to the model we have to generate different price scenarios that capture the uncertainty in the possible realized prices and inflow. The stochastic model explicitly takes the uncertainty into account and solves the problem by summing the different scenarios multiplied by their corresponding probability. The short term planning problem can be seen as a problem in two stages. The first stage is done before the spot price is revealed and determines the optimal bidding strategy. The next stage of the problem is to determine the unit commitment, that is, from which reservoirs the committed volume is to be produced.

This report presents related work on the subject in section 2 and discusses stochastic programming and scenario generation in section 3. In section 4, the mathematical formulation of the model is given, before the specific case study of Mandalsvassdraget is presented in section 5. Results from the case study can be found in section 6. Finally, general conclusions and suggestions for further work can be found in section 7.

2. Literature

In this section related work on the short-term production planning problem is presented. Our modeling approach is to a far extent based on the paper by Fleten and Kristoffersen (2007). There, a stochastic mixed-integer model for creating bids and scheduling the short-term production is developed and compared to the deterministic optimization tools in common use among Nordic hydroelectric producers today, as presented in Fosso, Gjelsvik, Haugstad, Mo and Wangensteen (1999) or Fosso and Belsnes (2004).

The model developed by Fleten and Kristoffersen (2007) is applicable to Nordic price-taking hydropower producers that participate in in a pool-based day-ahead market, and try to optimize the bid curves for both hourly and block bids under uncertainty in prices. The modeling of the hydropower production and reservoir topology is fairly simple, but we have used the same principles in our model, adapting them to our case study. We disregard block bids in our model, although Fleten and Kristoffersen (2007) holds that block bids protect against unexpected price changes when day-ahead prices are uncertain.

Belsnes, Fleten, Fleischmann, Haugstvedt and Steinsbø (2011) develop an extension of Fleten and Kristoffersen (2007) that looks at a time horizon from the day-ahead to the end of the drawdown season. This makes the model less dependent on input in the form of water values from other long- or intermediate-term models, but also creates a multistage problem where in every stage one must balance the value of producing water now against the value of storing it for later use.

Löhndorf, Wozabal and Minner (2011) take yet another programming approach, where the bidding problem is solved using stochastic dual dynamic programming. This formulation integrates short-term intra-day decisions such as bidding and scheduling with longer-term inter-day decisions of managing the reservoirs over time. Pritchard, Philpott and Neame (2005) also use a dynamic programming approach with stages. Here, a stage can have variable length and the first few stages represent a single trading period, whereas the later stages represent gradually longer time periods up to several days or even a week. The intra-stage sub problem computes the bids for every trading period in the current stage to give maximum expected revenue for a specified mean and variance of the water release over the current stage. The inter-stage problem uses the values from the first sub-problem to choose the mean and variance of the water release over the stage to maximize revenue from the current stage plus the expected revenue from future stages.

Both Fleten and Kristoffersen (2007) and our model assume that generation is proportional to discharge, which means that the relationship between produced volume and discharged volume, often called the production function, is linear. This is a simplification, as the generation efficiency is dependent on discharge level and water head, thus making the problem non-linear. The generation efficiency, or the relationship between produced volume and dispatched volume, could be modeled by concave functions or piecewise linear approximations of concave functions, such as in Fleten and Kristoffersen (2008), Faria and Fleten (2009) or Conejo, Arrayo, Contreras and Villamor (2002). Catalao, Mariano, Mendes and Ferreira (2005) consider the short-term scheduling problem with head-dependency in a deterministic setting. The head-dependency makes the problem non-linear, but the article holds that for cascaded hydro systems formed by several small reservoirs modeling of head-dependency gives more realistic and feasible results. Péréz-Diaz, Wilhelmi and Sánchez-Fernández (2010) also propose a non-linear method to model head-dependency, and states that more accurate modeling of these non-linear effects are most important when the reservoirs are small and their volumes can be significantly changed on a daily or hourly basis depending on the generation schedule.

Faria and Fleten (2009) consider the possibility of adjusting the dispatched power in the Elbas market. After the spot market is cleared, some adjustments may be necessary due to uncertain prices, inflow and load, and the possibility of doing this can influence the bidding strategy for the day-ahead market. Bidding strategies with intentional imbalances is not wanted by the TSOs, as the spot market should be regarded as the 'real' market. We, and Fleten and Kristoffersen (2007) model the Elbas market by having penalties for producing in imbalance with the committed volumes. In this way, the producer still has the possibility to participate in the balancing market, but does not let this influence the bidding strategies to any great extent.

3. Stochastic programming and scenario generation

In this section we present our modeling approach and show the mathematical formulation of the short-term planning problem.

3.1. Converting a real-life system into a model

When converting a complex system in to a model it is challenging to know which parts of the system needs to be described in detail and which parts can be approximated by easier expressions. How detailed and how close to the real world the model is, does not necessarily measure the quality of the model. Instead one has to look at the purpose of the model and find out which parts are essential to getting a satisfying result. In production planning it is important to model the bidding process as accurate as possible. It is also important to get a god description of the physical system and what restrictions that has to be followed. Approximations that can be made without reducing the quality of the solution is for instance to assume the turbine efficiency to be constant or to assume that every station has only one turbine. We will discuss different assumptions when the relevant constraints are presented.

3.2. Why stochastic programming?

The system that we want to model is a relatively complex system. There are several parameters that are uncertain. In this section we will discuss how uncertainty affects how the system is modelled and why we choose to solve the problem stochastically.

There are two main ways to solve an optimization problem; deterministic and stochastic modeling. A deterministic model builds on the assumption that every parameter in the model is predictable and known with absolute certainty. On the other hand, a stochastic model takes uncertain parameters into account when solving the problem, under the assumption that the probability distribution of the uncertain parameters is known. Most real life decision problems have to deal with uncertainty in one or more of the parameters. This is also the case for the short-term production planning problem. Here the uncertain parameters are the future prices and future inflow. Today's practice is to solve the problem for many deterministic scenarios, and then somehow combine the solutions.

Although a deterministic model will solve the short-term production planning problem sufficiently in most cases, there are cases where a deterministic model does not give good enough answers. This is the case for the problem we received from Agder Energi, which is described in the introduction. The problem arises when the realized price profile does not match the price scenarios used by the deterministic model. Our theory is that by solving the problem stochastically Agder Energi will get reasonable results in the situations where today's model fails to give an adequate answer.

As mentioned before, at the time of bidding the prices for tomorrow as well as all future prices are unknown. The same holds for inflow to the reservoirs. There is, however, a correlation between tomorrow's prices and the prices to come, as well as between future rainfall and future prices. As opposed to a deterministic model where you model as if you know the outcomes of future events, a stochastic model explicitly accounts for the uncertainty through the use of scenarios with corresponding probabilities. How these scenarios are made is discussed later in this section.

3.3. Deterministic equivalent (Recourse functions)

When solving the problem stochastically we can describe the vector of uncertain parameters as ξ, where the probability distribution of ξ is assumed known. The decision variables, x, is independent of the distribution of ξ. Hence one can present the general stochastic formulation as (Kall and Wallace, 1994, Wets, 1974):

$$\min g_0(x, \tilde{\xi}) \tag{3.1}$$

$$\begin{aligned} s.t. \quad & g_i(x, \tilde{\xi}) \leq 0, i = 1, \dots, m, \\ & x \in X \in \mathbb{R}^n \end{aligned} \tag{3.2}$$

But the model needs to have a way to describe how to take good decisions on x, before knowing the realization of ξ. It is therefore necessary to do a revision of the modeling process. The model needs some kind of recourse action to compensate for the deviation between the decision made under uncertainty and the optimal solution when ξ is revealed. The difference between this recourse or second stage activity and the choices made under uncertainty has an extra cost or penalty, q, which will be minimized in the objective function (Kall and Wallace, 1994, Wets 1974).

$$Q(x, \xi) = \min_y \left\{ \sum_{i=1}^{m} q_i y_i \right\} \tag{3.3}$$

$$s.t. \quad y_i(\xi) \geq g_i^+(x, \xi), i = 1, \dots, m \tag{3.4}$$

This gives the total cost to be minimized in the objective function:

$$f_0(x, \xi) = g_0(x, \xi) + Q(x, \xi) \tag{3.5}$$

We can relate this to the short-term production planning problem because there the production company will experience a cost when the produced volume differs from the committed volume. If the production company have a committed volume that is lower than the produced volume there is a surplus of power that can be sold on the balancing market. The cost associated with this is due to the fact that the price the production company gets in the balancing market is lower then the spot price. Contrary, if the production company have more committed power than the volume produced, power has to be bought in the balancing market to a higher price than the spot price. So either way, if there is an unbalance between committed volume and produced volume the power company will loose profit when the system is unbalanced, and this can be seen as a part of the recourse function.

3.4. Scenario generation

The quality of the stochastic modeling approach heavily depends on the capability to adequately model the uncertainty inherent in the real-world problem. Integrating too little uncertainty will give too optimistic results, while on the other hand, incorporating too much uncertainty will lead to prohibitively large models, that are either unsolvable, too complex to be informative, or both.

Uncertainty is represented trough scenario trees, which can be generated by many different methods. A review of methods most commonly used for scenario generation is given in Mitra (2006) or Kaut and Wallace (2003). In addition, Kaut and Wallace (2003) also give quality and suitability measures for different scenario-generation methods. The method explained later in this section is only a suggestion for how the scenario generation could be done, and we empathize that the model itself and the input scenarios are independent from each other, as the model can be used with any scenario generation method available. Scenario generation is related to forecasting of price and inflow which both are important aspects of short term scheduling. We regard model formulation and implementation as the main contribution of this report, and hence use a simple way of generating scenarios.

A scenario tree is used to analyze how the future prices and inflow influence what we bid in the market for tomorrow. What we bid is dependent on the balance between expected profits for tomorrow versus expected profits after the short-term planning period, and the scenarios represent the uncertainty in prices and inflow for tomorrow and the days after. The expected profits after the short-term horizon are represented by the water value. The bid volumes and produced volumes are dependent on the uncertain prices, which again are dependent on uncertain inflow, and hence the profits are themselves uncertain. Our objective is to find the bidding volumes that give the optimal balance between quantity of power produced tomorrow and quantity of water stored for later use.

Short-term scheduling of hydropower production can have a time horizon of up to seven days. From longer-term models the water values are known for a time step of one week, so the short-term problem deals with production scheduling within the week. If the water values are calculated each Sunday, then the scenario tree needs to cover all possible realizations of prices and inflow for the days until next Sunday. The resulting scenario tree consists of nodes representing different realizations of the future prices and inflow, and one path trough this tree is equivalent to one specific realization of prices and inflow for a whole week. If scheduling is done for Monday, then all days up to Sunday have to be represented in the tree, and hence the tree will have 7 stages.

The tree structure is needed to represent the information flow inherent in the problem; that is, it represents the timing of when new information becomes available to the decision maker. On any given day, the bidding decisions for the day after are dependent on the price realizations of all following days in the week, not just the day one is actually bidding for. This is exactly what the tree represents; it takes into account all future possible realizations of prices for the remaining days of the week, and also the timing of when these prices become known.


Figure 3-1: One individual scenario (deterministic)

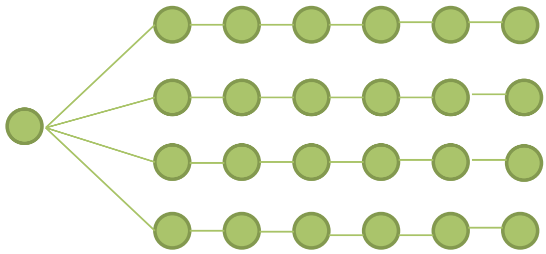

Figure 3-2: One-stage tree

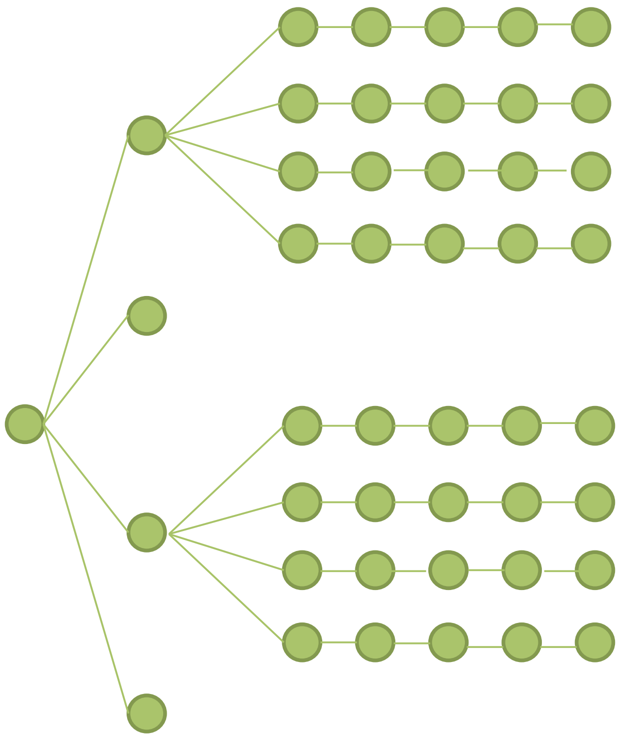

Figure 3-3: Two-stage tree

Looking at figure 3-1, 3-2 and 3-3, the scenario tree approach can be explained more thoroughly. Figure 3-1 shows one individual scenario for prices and inflow in the 7 days to come. If the uncertainty of the prices and inflow is represented in this way, we actually have a deterministic problem since this single scenario is the only one that can occur and hence it has a probability of 1. Figure 3-2 shows a scenario tree with 4 different possible realizations for the price and inflow tomorrow. After the price tomorrow becomes known, the prices for the remaining days also become known, and there is no more uncertainty. Hence this is a one-stage tree; we only face uncertainty once while moving in the tree. Finally, figure 3-3 shows a tree with 2 stages and 16 scenarios in total, although not all of them are fully drawn. There are 4 different possible realizations for the price tomorrow, and then there are 4 different realizations for the day after tomorrow. At this point the prices for the remaining days are assumed certain.

The tree in figure 3-3 can be expanded to include more stages and more realizations per stage. In our case we need up to 7 stages in the tree. If we only have 4 realizations in each stage as in the tree above, we get 4^7 = 16384 scenarios. This illustrates that the tree structure grows rather quickly, and considering that we need much more than 4 realizations per stage to adequately represent the uncertainty in the real world problem, it is easy to imagine that the problem can become too large to solve. We also have to incorporate the uncertainty in inflow into the tree, making the number of possible realizations in each stage even larger.

In our approach, we first develop individual scenarios for price and inflow, and then we combine these two together and use a scenario tree generation algorithm to construct a scenario tree. This tree is then used as input in the model, but as noted above, any sound method for generating the scenario input could be used.

The price scenarios used in our model are constructed using historical data for price forecasts and realized prices for the same day. From this we get a distribution of the error term between the forecast and the realized prices, and from this distribution we then draw random errors to add to the latest forecast to get different scenarios for the price. We use a normal distribution for the error term, and even though plots of the errors seem to follow a normal distribution, this may not always be a correct assumption. Other distributions could be used, and we could also have drawn price forecasts directly from the data set and used this as individual scenarios, and thus not needing to state any distribution for the error terms. For our purposes, we consider the method with normal distribution to be valid. Hence, as a first approach, each price scenario is constructed by the equation

$$\rho_h^s = Price\ Forecast_h + e_h \tag{3.6}$$

where

$$e_h = N^{-1}(\mu_\varepsilon, {\sigma_\varepsilon}^2) \tag{3.7}$$

Stating that the price in hour h for scenario s, ρ_h^s, is given by the latest price forecast for hour h, $Price\ Forecast_h$, plus a random number, e_h , drawn from the normal distribution with average equal to the average of the historical error terms, μ_ε, and standard deviation equal to the standard deviation of the historical error terms, ${\sigma_\varepsilon}^2$.

The above method has its limitations, however. If error terms are drawn randomly from the distribution, we will not capture correlations between the hours of the day. It is a crude assumption to assume that the deviations in each hour are totally uncorrelated, and it gives a too simplified representation of the uncertainty. It is very likely that the deviations for neighboring hours of the day are highly correlated, since some hours may be harder to predict than others and a high price in one hour is often followed by a high price in the next hour. This suggests that an autoregressive (AR) method should be used. AR processes are useful in describing situations in which the present value of a time series depends on its preceding values plus a random variation (Wei, 2006). Here, we let the price in one hour be the forecasted value for that hour plus a weighted share of the error in the preceding hour and a random error term, as in the equation below.

$$\rho_h^s = Price\ Forecast_h + \alpha * e_{h-1} + e_h \tag{3.8}$$

The weight given to the error from the last hour is found trough regression analysis of the error terms in following hours. As the above equation states, the price is dependent on the forecasted price and not on the preceding value of the price as is required for a time series. Hence, we use the AR-model only to make up different values of the price and not to develop the price as a time series.

The above method is the simplest form of AR model, AR(1). This could be extended by letting the price be dependent on the error terms in several hours before the current hour, as in

$$\rho_h^s = Price\ Forecast_h + \alpha_1 * e_{h-1} + \alpha_2 * e_{h-2} + \cdots + \alpha_n * e_{h-n} + e_h \tag{3.9}$$

Here the errors from the different hours are given different weights, and some may be weighted with zero. This method lets us incorporate even more correlation between hours.

We know that the prices for electricity follow a typical 24-hour profile with low prices during the night, a morning peak, a small dip in the early afternoon and then an evening peak, an example of this can be seen in figure 3-4. The day hours are harder to predict than the night hours, and there is also more uncertainty around the peak hours. This could be captured by letting the price in one hour be dependent on the error made in the corresponding hour the day before. For instance, the error made at hour 00-01 one night may also be correlated to the error made between 00-01 the night before, in addition to the correlation to the directly preceding hour. Which hours are more or less correlated to each other could be found trough regression. To keep our model simple, we only use AR(1).

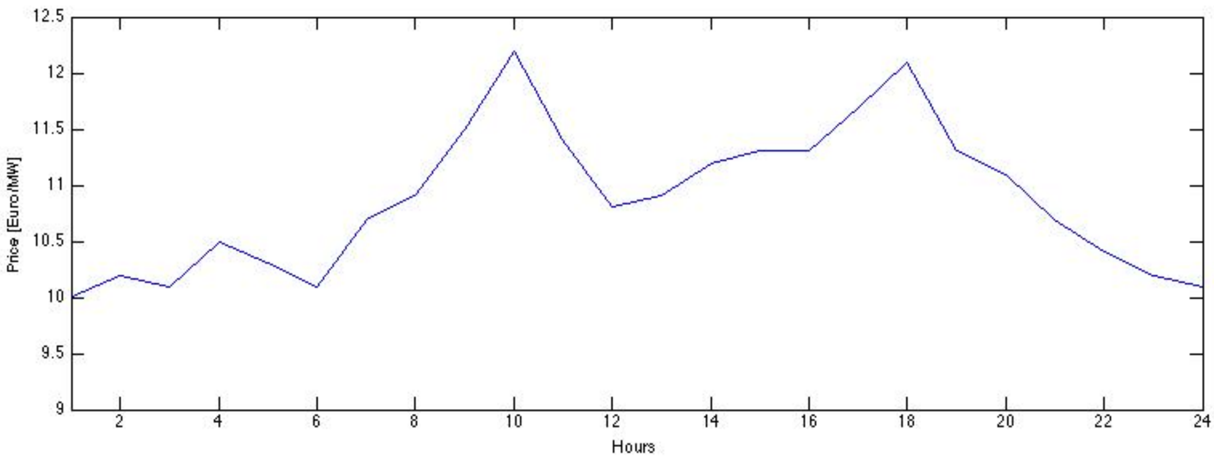

Figure 3-4: Exemple of a 24-hour profile for the spot price

There are some additional problems with the above method. By using the same price forecast as base for the all the scenarios, we are assuming that the uncertainty in prices can be represented by just the random error terms, and hence we assume that the forecasted values are always correct when it comes to the 24-hour profile; that is, we assume that the forecasting technique always matches the profile down to a normally distributed error term. For a given location and a given season, the 24-hour profile is known with some certainty, but we still want our model to handle extreme scenarios where the profile may look completely different. This can be accomplished by manually selecting some extreme scenarios.

For our purposes, we use the AR(1) method described above for price scenarios. For the inflow scenarios, historical data of scenarios used by Agder Energi has been obtained. These data are from the same period as we have price data and price forecasts for. We do not reflect too much about the effects of uncertainty in inflow in this report, due to the fact that we have data for the fall season when reservoir levels and inflow are quite high. There are other times of the year where uncertainties in inflow are far more influential on the results.

We develop N price scenarios and combine these with the M inflow scenarios, making N*M individual scenarios. This gives a discrete realization of the prices and inflow for the remaining days in the planning horizon, but it is still not represented as a tree. An individual scenario is equivalent to a particular path through the scenario tree. To construct a tree that is representative for the scenarios, we use the software SCENRED developed by Heitsch and Römisch (2006). See also GAMS Software (2002) for more information about the algorithm. The algorithm reduces the tree to the minimum number of nodes still needed for the tree to be representative. Follestad, Wolfgang and Belsnes (2007) discuss the effects on the objective function by reducing the tree for a case with uncertainty in inflow.

The tree structure suitable for our model will look different from figure 3-3, and an example of our tree structure with 4 realizations per stage is given in figure 3-5 below. The number of realizations in each stage will vary according to the individual input scenarios and the reduction parameter settings in the SCENRED algorithm. Another feature is that our tree will have several time steps in each stage, representing the 24 hours of a day. The next stage occurs when the market is cleared and the prices for all hours of the next day are revealed.

We choose to have a tree covering 72 hours, that is, we assume that the model is run three days before the water values are updated. The model given by Fleten and Kristoffersen (2007) has a time horizon of 24 hours. This means that they only have uncertainty in one stage, and the value of water after the operating day is described by water values. Their trees structure is often referred to as a fan (Follestad, Wolfgang, and Belsnes, 2007). With a longer time horizon, we have to have a representation of the possible profits in hours 25 to 72, which can be either deterministic or stochastic. A deterministic representation, as shown in figure 3-2, will give a smaller problem to solve, but the bid matrix for hour 1 to 24 will be based on a less realistic description of hour 25 to 72. To find the best possible bids we have chosen to describe hour 25 to 72 stochastically. We therefore need to calculate the bid matrix for hour 25 to 72 to validate the value of water and the profits we make in these hours, although the bids for hours 25-72 are never implemented since we use the model on a rolling time horizon.

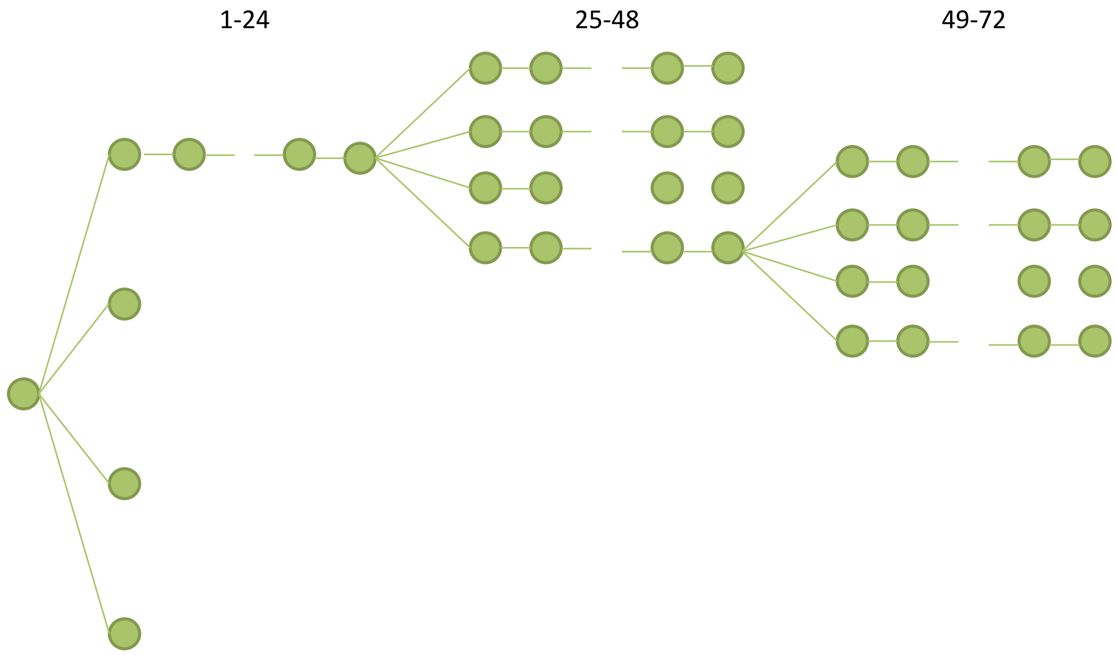

Figure 3-5: 72-hour tree with uncertainty in three stages

4. Mathematical formulation

In this section the mathematical model formulation is presented. For reasons of clarity, the model is first introduced without uncertainty. This makes the representation and explanation of constraints easier. Uncertainty is introduced in the model after all relevant explanations are given, and then the model with uncertainty is presented as a whole.

4.1. Objective function

The objective of the model is to maximize the profit from bidding in the day-ahead market. The profit is the income from the spot market less the start-up costs and the penalty for producing in unbalance with the committed volume. The value of the water for future use must also be taken into consideration. The short-term production planning problem can be described as finding the optimal balance between water used today and water stored for future production.

The income from the day-ahead market is noted $\rho_h y_h$, where the index h is defined for every hour of the operating day. In hydropower production the variable costs of production are negligible, and the real cost of producing is the opportunity cost of water. The marginal opportunity cost of water in a reservoir is called the water value. Since production means using water, this water value can be seen as the resource cost of water. The water value can be calculated as a function of water available in the reservoirs. How this calculation is done will be discussed further when the water value constraint is introduced.

As mentioned in section 3.3, the objective function needs a recourse function to compensate for the difference between volume bidded under uncertainty and the optimal volume when the actual inflow and prices becomes known. If the producer has a surplus of power, power must be sold in the balancing marked at a price lower than the spot price. If the producer has a power deficit the balancing volume is the amount to be purchased in the balancing market, to a price higher than the spot price. Hence there is a penalty related to producing in imbalance with committed volume. To model this we need to introduce a new variable representing the balancing volume.

The last part that needs to be included in the objective function is the minimization of start–up costs. This is the cost related to starting a turbine that has been out of production. This is an important part of the problem since it is undesirable to have frequently starts and stops of turbines, due to tear on the turbines. The start-up cost should reflect the fact that whenever there is a start or stop in the production, water is lost. In addition frequently stops causes unnecessary exhaustion of the turbines, increases the risks of component failure and requires more work from the operator. The real cost of start-up are hard to measure, and therefore assigning a value to the start-up price is a difficult task. The important part is that the start-up cost punishes start and stop in such a way that one reduces the occurrence of starts and stops of the same turbine within a few hours. To model start up costs we introduce a binary variable that describes which state a turbine is in (On/Off).

Hence, the objective function is:

$$\max \sum_{h\in\mathcal{H}} \rho_h y_h - \sum_{r\in\mathcal{R}} (\Lambda_{1,r} L_r - \Lambda_{72,r} l_{72,r}) + \sum_{h\in\mathcal{H}} (\mu_h^+ z_h^+ - \mu_h^- z_h^-) - \sum_{h\in\mathcal{H}} \sum_{r\in\mathcal{R}} S_r \delta_{h,r} \tag{4.1}$$

Where:

ρ_h: The price in the spot market in hour h

y_h: The volume bidded in the spot market in hour h

$\Lambda_{1,r}$: Water value at the start of the period

L_r: Initial storage level in reservoir r

$\Lambda_{72,r}$: Water value in hour 72, in reservoir r

$l_{72,r}$: Storage level in hour 72, in reservoir r

μ_h^+: The price in the balancing market for up-regulation

z_h^+: The positive unbalance volume, sold in the balancing market

μ_h^-: The price in the balancing market for down-regulation

z_h^-: The positive unbalance volume, sold in the balancing market

S_r: Start-up cost for reservoir r

$\delta_{h,r}$: Binary variable, 1 in reservoir r is turned on in hour h, 0 otherwise

4.2. Modeling bids to Nord Pool

There are several ways to bid in the Nord Pool spot market; the most common are day-ahead bids, both hourly and block bids. The concept of block bids is to bid the same volume over several hours to the same price. The spot price that gets valid for the block is the average spot price over the hours of the block. In a stochastic approach to production planning block bids are important elements. A block bid secures a steady production over several consecutive hours, and this can be used to build a more robust solution that considers the average price over a few hours and not just the price in one single hour. The average price will be more certain than the single-hour price. In fact, we regard block bids as only relevant to stochastic models, since if you know the prices with certainty, as in a deterministic model, there is no reason for bidding anything else than the optimal volume for all hours. Regardless, our model does not take block bids in to account when finding the optimal bidding volume. The main reason for this is the original problem from received from Agder Energi, where the main problem is abrupt price changes over a short period of time, even though the average may not change as much. We therefore want to study how stochastic programming affects the hourly bids, where rapid changes in price have the most impact. Another reason for neglecting block bids is Agder Energi´s request for a model as simple as possible.

The bids are modelled as piecewise linear functions. This is because the problem of finding the optimal volume as a function of price is a non-linear problem. We avoid the non-linearity by using fixed price points and optimizing the volume corresponding to each of the fixed prices (Fleten and Pettersen, 2005). The committed volume is found by interpolation between neighbouring price points. The bidding curve can be expressed as:

$$y_h = \frac{\rho_h - p_{i-1}}{p_i - p_{i-1}} x_{i,h} + \frac{p_i - \rho_h}{p_i - p_{i-1}} x_{i-1,h}, \qquad p_{i-1} \le \rho_h \le p_i, \qquad i \in \mathcal{I}, h \in \mathcal{H} \tag{4.2}$$

Where y_h is the committed volume from the bids in hour h that is decided by the market clearing spot price, ρ_h. As mentioned above, y_h is found by a linear interpolation between the to nearest price-volume points $(p_i, x_{i,h})$ and $(p_{i-1}, x_{i-1,h})$

Due to the rules of how to submit bids in the spot market, the bids have to be strictly increasing. Because of this we get the following constraint for the bidding volumes:

$$x_{i,h} \leq x_{i+1,h}, i \in \mathcal{I} \setminus \{I\}, h \in \mathcal{H} \tag{4.3}$$

4.3. Modeling stations and reservoirs

The system consists of several power stations, each station with one or more overlying reservoirs. To model this we need several constraints; e.g. how the stations and reservoirs are linked together, how the discharge from one reservoir leads to production in an underlying station and a reservoir balance for each reservoir. A general system can be modelled as seen in figure 4-1.

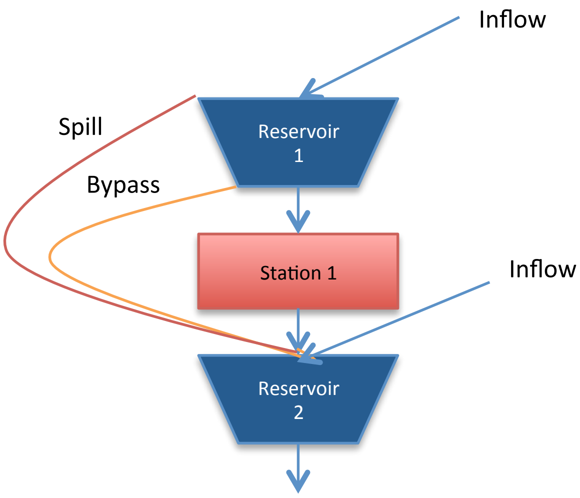

Figure 4-1: General system containing two reservoirs and one station

In our model, the reservoirs and the underlying stations are modelled together as one entity, and are referred to as both reservoir and station depending on the context.

4.3.1. Production

First we model the production capacity of a station with a constraint stating that if there is production in a station in a given hour, then this has to be above the minimum production level, w_{min}. Similarly, the production has to be less then w_{max}.

$$u_{h,r} w_{min,r} \leq w_{h,r} \leq u_{h,r} w_{max,r} \tag{4.4}$$

$w_{h,r}$: The production from reservoir r in hour h

$u_{h,r}$: Binary variable, 1 if there is production from reservoir r in hour h. 0 otherwise.

How much power the station generates, w, from one unit of water discharged, v, is dependent on the efficiency of the turbines and generators. Generally, this is a non-linear relationship, due to dependency of water head and discharge level. A higher plant-head increases the energy equivalent as seen in equation 4.5 below. To keep our model linear, we make piecewise linear approximations of the production function for each station. The production function relates produced effect to discharged volume. The efficiency of the turbine is given as percentage efficiency for different levels of discharge volume, in m^3/s. The efficiency of the generator is given as percentage efficiency for levels of produced volume, in MW. We have to convert m^3/s to MW, hence we need the energy equivalent.

$$e = \frac{\gamma\, g\, H\, \eta}{3{,}6 * 1000^6} \quad [MWh/m^3] \tag{4.5}$$

Where:

γ: Water density, 1000 $[kg/m^3]$
g: Gravity acceleration $[m/s^2]$
H: Plant head $[m]$
η: Plant efficiency

The produced volume, w, is given by

$$w = v * e \ * 3600/1000 \quad [MW] \tag{4.6}$$

where v is discharge in m^3/s and e is the energy equivalent.

To utilize the water as best as possible the producers want to run the turbines at the best efficiency point. But as the spot price increases, the producers get an incentive to produce above the best point. Normally an optimal unit commitment will give production volumes at either best point or maximum point. But start-up costs and bounds for discharge may give optimal production both below best-point and in-between best-point and maximum production.

Power plants may have several turbines, and each of these has a corresponding production function. We aggregate the functions for each turbine in a plant to a total curve for each plant. This is done in order to simplify the model, making it unnecessary to have variables for each turbine and thus reducing the model size. To do this simplification, it is assumed that the start-up cost for each turbine is known and that the cheapest turbine in a plant is driven to its maximum capacity before the next turbine is started.

The aggregated curves are in general non-linear, and thus we have to approximate them with piecewise linear cuts. In figure 4-3 it is indicated how the linearization is done. We define a set of breakpoints of the form $(v_{h,i}^k, w_{h,i}^k)$ on the curve and constrain the production, w_h, to always be below the minimum of these cuts. Hence, we have

$$w_h \leq \frac{v_h - v_{h,i-1}^k}{v_{h,i}^k - v_{h,i-1}^k}\left(w_{h,i}^k - w_{h,i-1}^k\right) + w_{h,i-1}^k, \quad v_{h,i-1}^k \leq v_h \leq v_{h,i}^k, \ i \in \mathcal{I}, h \in \mathcal{H} \tag{4.7}$$

Where $v_{h,i}^{k}$ and $w_{h,i}^{k}$ are known volumes on the curve and w_h is the produced volume corresponding to the discharged volume v_h. The breakpoints are selected so that they cover the entire range of possible values for discharge and production, that is, one breakpoint each for maximum and minimum. To best capture the effects of the efficiency curves on the bidding strategy, one of the breakpoints should be the best-point volume. More breakpoints give a better approximation of the non-linear production functions, but also increase the solution time.

Figure 4-2 below show a plot of the production function for a one-unit station (left), which is a nearly straight line, and a two-unit station (right), which has a kinked point where one unit ends and the next one starts. Figure 4-3 shows the linear cuts that approximate the production function.

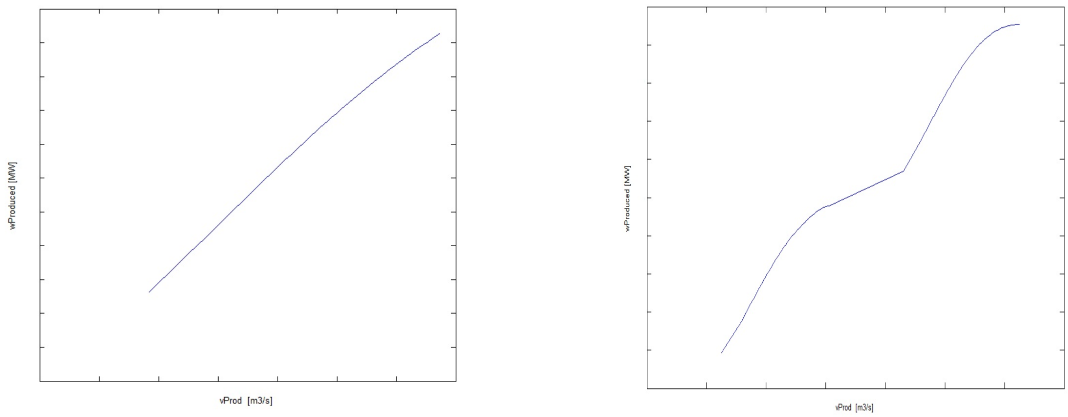

Figure 4-2: Plot of the production function of a one-unit station (left) and a two-unit station (right)

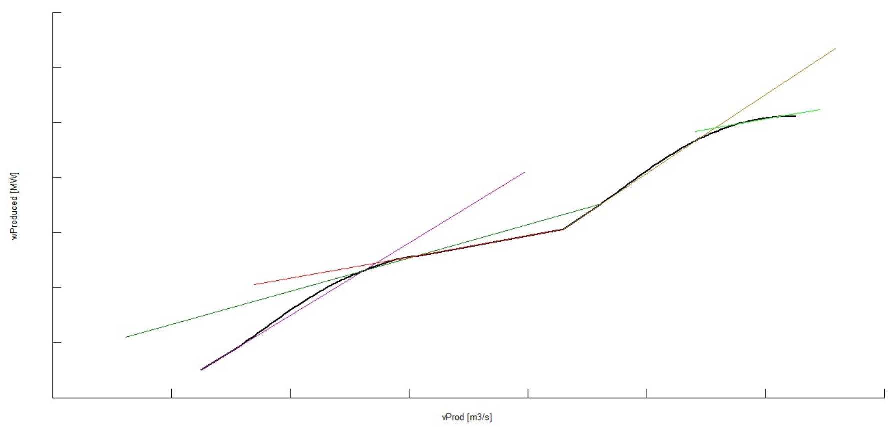

Figure 4-3: Cuts approximating the non-linear production function

4.3.2. Discharge

As for production there are bounds for the discharge from a reservoir. The main reason for these restrictions is that its neither desirable to let a river course run dry, nor to flood it. There are three different waterways out of a reservoir; discharge for production, bypass and spill. Hence there are different bounds for the different kinds of discharge. Spill is unbounded, due to the fact that a flood cannot be regulated.

$$v_{prod_{min,r}} u_{h,r} \leq v_{prod_{h,r}} \leq v_{prod_{max,r}} u_{h,r} \tag{4.8}$$

$$v_{Bypass_{min,r}} \leq v_{Bypass_{h,r}} \leq v_{Bypass_{max,r}} \tag{4.9}$$

The constraint for production discharge, $v_{prod_{h,r}}$, is multiplied with the binary on/off variables, $u_{h,r}$, to make sure that we do not let any production discharge run trough the station when it is not running.

4.3.3. Reservoir balance

To model a reservoir we need to include a balance equation that accounts for discharge in to the reservoir, inflow, discharge out of the reservoir and change in storage level. The storage level gives the volume of water in a given reservoir. The change in storage level is given by the volume at the end minus the volume in the beginning of the time period. For the first hour of the model, that is, when time step h equals 1, then $h-1$ in the equation below refers to the initial value of the storage levels, which is given as input to the model.

The reservoir receives discharge form overlying reservoirs after a time delay in the waterway between the reservoirs, τ. If the time step, h, minus the time delay, τ, is less then 1, the discharge on the way from the overlying reservoirs has to be given as input parameters to the model; this is water released from overlying reservoirs in the hours before the model has started, but that has not yet reached its destination. We need to keep track of how much water is on the way into the reservoir at h=1, and when this discharge will arrive. This must be done for discharge for production, bypass and spill, and each of these may have different watercourses and different time delays.

The reservoir topology may be more complex than just one reservoir directly above the next as in figure 4-1. A system can have parallel river courses, or some completely different layout, and it is therefore not given that the discharge from reservoir r ends up in reservoir r+1, see figure 4-4. Hence, we need a matrix that gives which reservoirs are connected to each other, $C_{r,k}$. This matrix consists of binary parameters, equal to 1 if there is a direct connection between reservoir r and k, and zero otherwise. Since there can be different waterways for discharge from production, bypass and spill, we need different connection matrixes. The watercourses for discharge for production and bypass will be the same, and this is shown in the equation below.

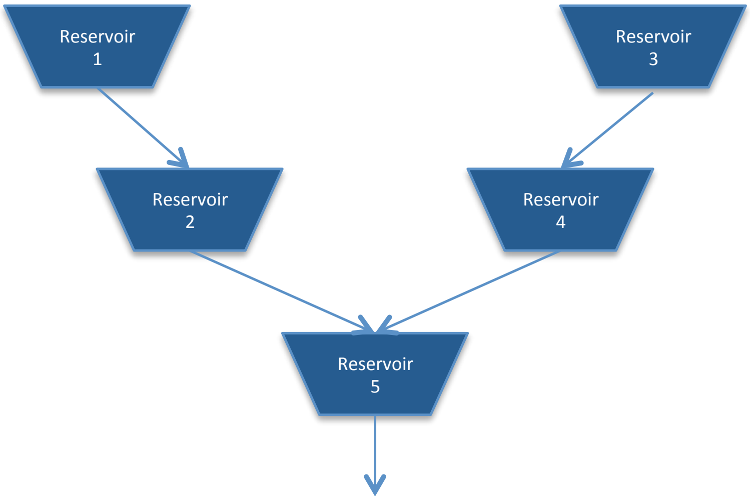

Figure 4-4: System with parallel river courses

Hence, the reservoir balance equation is:

$$l_{h-1,r} - l_{h,r} + \sum_{k \in \mathcal{R}} \{(v_{prod,h,k} + v_{bypass,h,k})C_{r,k} + v_{spill,h,k}C_{spill_{r,k}}\} + R_{inflow_{h,r}} - v_{prod,h,r} - v_{bypass,h,r} - v_{spill,h,r} = 0 \quad (4.10)$$

The storage level in the reservoirs can only be regulated within an upper, $l_{max,r}$, and a lower boundary, $l_{min,r}$. These boundaries are either set by the physical constraints of the reservoir or by boundaries set by the government through the Norwegian Water Resources and Energy Directorate, NVE. Hence, we have

$$l_{min,r} \leq l_{h,r} \leq l_{max,r} \quad (4.11)$$

4.3.4. Modeling the water value

The water value is the marginal opportunity cost of water in the reservoirs. The water value is a function of future development depending on demand, market prices and inflow. The water value is therefore both non-linear and uncertain. For the short term planning period, the expected future water value is known from the longer-term models down to time step of one week. That means that the water value at beginning and the end of our planning horizon is known. Some power companies also update these values within the week, so the water value will be known by for instance three days apart. This means that the water values can be taken as parameters in the model, and has to be multiplied with the reservoir levels at the beginning and at the end of the short-term time horizon to give the total value of the water in each reservoir.

There are more theoretical methods to represent the water value in the model, and one such approach is explained in Mo and Gjelsvik (1997) or Gjelsvik, Belsnes and Håland (1997). This method is more complicated, and requires coupling to longer-term models. We do not use this in our model due to the fact that Agder Energi do not use this method in their present model and that we want to keep the model as simple as possible.

In addition, we need to give value to the water that is discharged from an overlying reservoir but has not reached the underlying reservoir before the end of the short-term planning horizon, due to time delay between reservoirs. This water needs to be valued to avoid end of horizon effects, which for instance may be that stations are turned off in the last hours of the modeled time horizon to 'save' water in the overlying reservoir, where is has value. If the water in the watercourses at the end has value zero, the optimization will avoid releasing water in the last hours. We let all water in the watercourses at the end, that is, production discharge, bypass and spill, have the same value as the water in the reservoir where it is heading.

4.3.5. Modeling start-up costs

As discussed in section 4.1, the start-up cost is an important part of the short-term production-planning problem. If we had excluded the start-up cost from the problem, the unit commitment would always produce from the reservoir with the lowest water value. But as the committed volume shifts from hour to hour, the capacity of the cheapest reservoir may not be enough to cover the committed volume a couple of hours ahead. The unit commitment optimization may then want to start up production from another reservoir. As the unit commitment always produces from the cheapest reservoir, it doesn't take into account that production in a station may be switched on and off several times due to rapid changes in price. As discussed above, this is undesirable and has to be accounted for by the model.

To model this cost we have to create a variable for each station saying if there is production or not. This variable, $u_{h,r}$, is binary, with the value 1 if there is production and zero otherwise. To find out if a station has changed state from one time step to another, we need a new binary variable, $\delta_{h,r}$. This variable is 1 if a station has started up in time step t, and zero otherwise. This can be modelled as:

$$\delta_{h,r} \geq u_{h,r} - u_{h-1,r} \tag{4.12}$$

4.4. Modeling Balancing power

As discussed in section 4.1 the model needs to have a way to cope with differences between volume dispatched and volume produced. If there is imbalance between the committed and the produced volume the producer has the flexibility to bid this capacity into the balancing market close to real time (Fleten and Kristoffersen, 2007). z_h^+ and z_h^- represent the imbalance in the system and is calculated as the difference between the committed volume, y_h, and the sum of production from the different power stations. As shown in the objective function, the balance variables are multiplied with a penalty for purchasing or selling in the balancing market. We set the penalty value quite high to make sure that the balancing market is only used as a last resort.

$$\sum_{r \in \mathcal{R}} w_{h,r} - y_h + z_h^+ - z_h^- = 0 \tag{4.13}$$

4.5. The stochastic model

We have now presented a general model for the short-term production-planning problem without uncertainty. In this section the stochastic model is presented. A scenario tree is given as input to the model. Each scenario has a known probability, π_s. The index s is defined as a set of all scenarios. Then each parameter and variable in the model that may have different realizations in the different scenarios has to be defined over all scenarios in addition to whatever indices they already have. For instance, the committed volume will vary with scenario and is now denoted by $y_{s,h}$ instead of just y_h as before. The input parameters that differ in the different scenarios are the price forecast, ρ_h, and inflow, $R_{inflow_{h,r}}$. The water value in hour 1 and 72 is equal in every scenario, as they are given as input to the model without uncertainty. As are the initial values for storage level, discharge, spill and initial state of the stations. Every variable calculated in the model, y_h , $l_{h,r}, z_h^+$, z_h^- , $\delta_{h,r}$, $u_{h,r}$, $w_{h,r}$, $v_{prod_{h,r}}$, $v_{bypass_{h,r}}$ and $v_{spill_{h,r}}$ are all dependent on inflow and price, which vary in the different scenarios. Therefore all the variables calculated in the model have to be denoted with s. The stochastic model is thus given as:

$$\max \sum_{s \in \mathcal{S}} \pi_s \left\{ \sum_{h \in \mathcal{H}} \rho_{s,h} y_{s,h} - \sum_{r \in \mathcal{R}} (\Lambda_{1,r} L_r - \Lambda_{72,r} l_{s,72,r}) + \sum_{h \in \mathcal{H}} (\mu_h^+ z_{s,h}^+ - \mu_h^- z_{s,h}^-) - \sum_{h \in \mathcal{H}} \sum_{r \in \mathcal{R}} S_r \delta_{s,h,r} \right\} \quad (4.14)$$

$s.t.$

$$y_{s,h} = \frac{\rho_{s,h} - p_{i-1}}{p_i - p_{i-1}} x_{i,h} + \frac{p_i - \rho_{s,h}}{p_i - p_{i-1}} x_{i-1,h} \qquad p_{i-1} \leq \rho_{s,h} \leq p_i, \; s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \quad (4.15)$$

$$x_{i,h} \leq x_{i+1,h} \qquad s \in \mathcal{S} i \in \mathcal{I} \setminus \{I\}, h \in \mathcal{H} \quad (4.16)$$

$$u_{s,h,r} w_{min,r} \leq w_{s,h,r} \leq u_{s,h,r} w_{max,r} \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \quad (4.17)$$

$$w_{s,h} \leq \frac{v_{s,h} - v_{h,i-1}^k}{v_{h,i}^k - v_{h,i-1}^k} \left(w_{h,i}^k - w_{h,i-1}^k\right) + w_{h,i-1}^k \qquad v_{h,i-1}^k \leq v_{s,h} \leq v_{h,i}^k, \; s \in \mathcal{S}, h \in \mathcal{H}, i \in \mathcal{I} \quad (4.18)$$

$$v_{prod_{min,r}} \leq v_{prod_{s,h,r}} \leq v_{prod_{max,r}} \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \quad (4.19)$$

$$v_{Bypass_{min,r}} \leq v_{Bypass_{s,h,r}} \leq v_{Bypass_{max,r}} \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \quad (4.20)$$

$$l_{s,h-1,r} - l_{s,h,r} + \sum_{k \in \mathcal{R}} \{(v_{prod_{s,h,k}} + v_{bypass_{s,h,k}}) C_{r,k} + v_{spill_{s,h,k}} C_{spill_{r,k}}\} + R_{inflow_{s,h,r}} - v_{prod_{s,h,r}} - v_{bypass_{s,h,r}} - v_{spill_{s,h,r}} = 0 \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \quad (4.21)$$

$$l_{min,r} \leq l_{s,h,r} \leq l_{max,r} \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \quad (4.22)$$

$$\delta_{s,h,r} \geq u_{s,h,r} - u_{s,h-1,r} \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \quad (4.23)$$

$$\sum_{r \in \mathcal{R}} w_{s,h,r} - y_{s,h} + z_{s,h}^{+} - z_{s,h}^{-} = 0 \qquad s \in \mathcal{S}, h \in \mathcal{H} \qquad (4.24)$$

$$y_{s,h} \geq 0 \qquad s \in \mathcal{S}, h \in \mathcal{H} \qquad (4.25)$$

$$x_{i,h} \geq 0 \qquad h \in \mathcal{H}, i \in \mathcal{I} \qquad (4.26)$$

$$u_{s,h,r} \in \{0,1\} \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \qquad (4.27)$$

$$\delta_{s,h,r} \in \{0,1\} \qquad s \in \mathcal{S}, h \in \mathcal{H}, r \in \mathcal{R} \qquad (4.28)$$

$$z_{s,h}^{+} \geq 0 \qquad s \in \mathcal{S}, h \in \mathcal{H} \qquad (4.29)$$

$$z_{s,h}^{-} \geq 0 \qquad s \in \mathcal{S}, h \in \mathcal{H} \qquad (4.30)$$

5. Case study: Mandalsvassdraget

We will now implement the general model given in section 4 to a large Norwegian hydro system, Mandalsvassdraget. Mandalsvassdraget is a large river course in Aust-Agder and Vest-Agder, owned by Agder Energi. It contains twelve reservoirs and six power plants. When the system is modeled we accumulate some of the reservoirs, Storevatn, Kværnevatn, Langevatn, Stegil and Nåvatn in to one big reservoir called Nåvatn. The reason for doing this is that the reservoirs above Nåvatn have a low degree of regulation, which means that the water contained in these reservoirs will end up in Nåvatn within a short period of time. Also, Agder Energi has plans to build a bigger dam at Nåvatn, and demolish the dams of the overlaying reservoirs (NVE, 2011). A simple representation of Mandalsvassdraget is found in figure 5-1 below. Here the red lines represent waterways for spill, the yellow lines waterways for bypass and the blue lines the regular waterways between reservoirs and stations. The system also includes other smaller river courses that have their outlets in Mandalsvassdraget. These are included as inflow in the model.

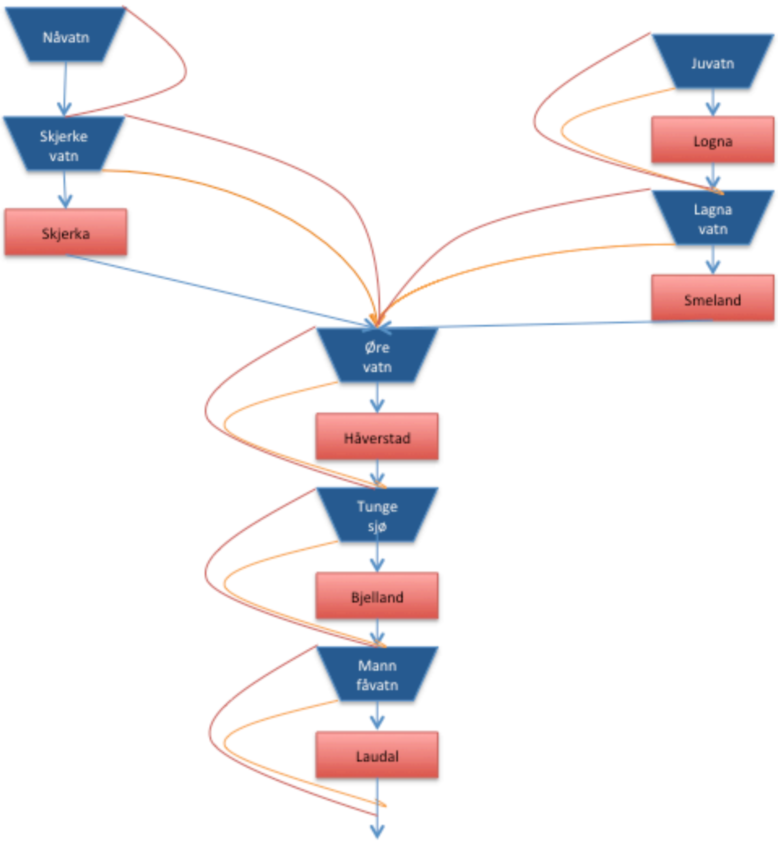

Figure 5-1: Mandalsvassdraget

5.1. Assumptions

The general assumptions for the formulation of the mathematical model are reasonable also for Mandalsvassdraget. Agder Energi is a large hydropower producer, but we assume that what they bid in the market does not influence the overall price level and hence Agder Energi is a price taker in the sense that they must take the spot price as an exogenous random variable. We also assume that this random variable can be well represented by the scenario generation method used, and that the probability distribution of the scenarios is known, although our scenario generation method may have its limitations.

In our formulation it is not necessary to model both reservoirs and stations, since each reservoir only has maximum one station directly below it. The reservoirs and corresponding stations are thus modelled together as one entity, and hence we have 7 reservoirs called Nåvatn, Skjerkevatn, Juvatn, Lognavatn, Ørevatn, Tungesjø and Mannflåvatn. Nåvatn does not have its own station, but this is handled by stating that its production curve is flat so that we can release water from it without power generation.

The waterways in Mandalsvassdraget are shown in figure 5-1, and here it is evident that the waterways for spill, bypass and production discharge are the same. The model handles cases where spill has different waterways than other discharge, and this may be used if the complete river system with the reservoirs currently combined into Nåvatn should be modelled as entities on their own.

As discussed in section 4.3.1 the power generation depends on plant head. In Mandalsvassdraget there is only one plant where the effects of changes in head are significant enough that Agder Energi has included them in the SHOP model in current use. This plant has 3 different efficiency curves given for a plant-head at 130, 140 and 150 meter, as shown in figure 5-2.

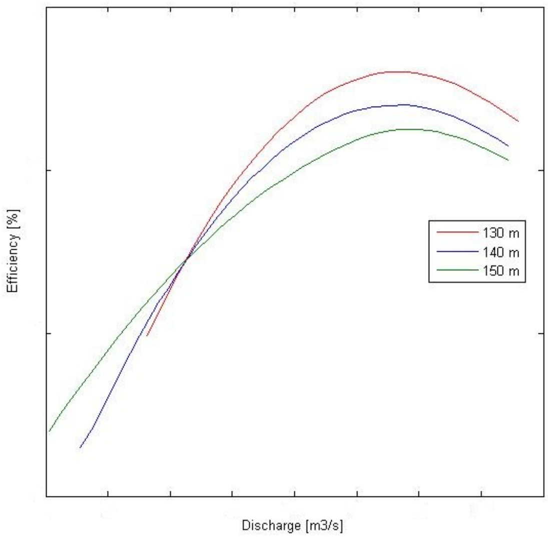

Figure 5-2: Efficiency curves for three different heads

Due to non-linearity and the small amount of plants with head effects we have chosen to neglect the head effects in Mandalsvassdraget. Hence we make an approximation at this plant, using the efficiency curve for 140-meter head at all levels of the reservoir.

5.2. Bounds

We have obtained all relevant data on bounds for the reservoirs such as minimum and maximum production capacity, minimum and maximum discharge capacity for spill, bypass and discharge and minimum and maximum storage level. Where no explicit boundary exists, we have set the values to 0 or a very high value to keep the formulation general.

Some of the reservoirs have a stated lower bound on storage level, and this makes the coupling of the reservoir more strongly linked. It may happen that the initial storage level and the inflow to one reservoir is not enough to keep this reservoir over its minimum bound while at the same time satisfying lower bounds on bypass or production discharge. Hence water can be 'drawn' down from overlying reservoirs just to keep the water flow within its bounds. We observe this effect when we deliberately set the water values at the end so high that we in all scenarios should hold back as much water as possible, and we still get results showing that water is discharged from some of the overlying reservoirs. In Mandalsvassdraget, it is the last reservoir, Mannflåvatn, which has a lower bound on storage and also the largest minimum bound on bypass that creates this difficulty. In some scenarios, this means that we produce from some stations even though the price we get for the power is less than we would get by saving the water.

5.3. Time horizon

The time horizon chosen in the model is 72 hours, starting at hour 00:00 the following day from the time the model is run. Agder Energi updates the water values of the reservoirs once a week, with new values starting every Wednesday. The water values are then adjusted if necessary during the week before they are updated again next Wednesday. Agder Energi has water value meetings every day to discuss the situation in the market and expected inflow to decide how the adjustment of the water values should be done. Because the water values are updated once a week the period before the next time the water values are updated is at its most 7 days. We have data for water values for 72 hours apart, and use these as constant parameters in the model.

We only care about what to bid in the market for tomorrow, but to make sound bids, the uncertainty in prices and inflow for all hours of the next three days are taken into account in the stochastic optimization. The model should be used on a rolling time horizon, which is common practice for short-term scheduling.

5.4. Scenario trees

As explained in section 3.4, the scenario trees is generated by combining scenarios for inflow with scenarios for prices and constructing a tree with the SCENRED algorithm. Scenarios for inflow are obtained from Agder Energi and are basically the same scenarios that are currently used in the deterministic model. We have 51 scenarios for inflow to each of the seven reservoirs.

To test the model and validate the results, we develop several sets of scenarios for price. These are summarized in table 5-1.

Table 5-1: Summary of scenarios

Name	Number of scenarios	Generation method	Reduced number of scenarios
Deterministic	1*1	Average values	1
Historical	50*51	AR(1)	17,47,158
Extreme	10*51	Manual	12

5.2.1. Deterministic average value scenario

To be able to compare the results of the stochastic solution to a deterministic model, we use a single scenario as input to the model. This means that there is no uncertainty and hence the model is deterministic. The average values of the scenarios developed using an AR(1) model and historical estimates of μ and σ^2 are used as the single scenario case. These are combined with the average values of the inflow scenarios, creating 1*1 = 1 scenarios in total.

5.2.2. Scenarios using historical mean and standard deviation

We use forecasted values obtained from Agder Energi as the starting point for scenario generation. This forecast, with some added noise, can be seen in figure 5-3.

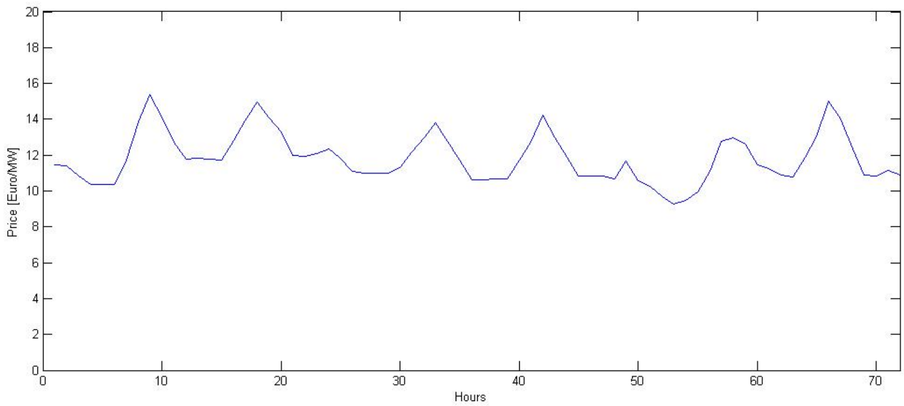

Figure 5-3: Plot of the given forecasted price profile for all 72 hours

To make price scenarios, we use historical data to estimate the mean and standard deviation of the distribution of the error between the forecasted prices and the realized prices. We have tested the historical data for a three-month period and observed that they resemble a normal distribution. Therefore the assumption of a normal distribution is validated. We then obtain the distribution:

$$Forecasting\ error: e_h \sim N(0.148\ \ , 1.816) \tag{5.1}$$

Where 0,148 is the average forecasting error and 1,816 is the standard deviation of the error of the historical data obtained from Agder Energi. An observation is that the average error is not zero, but has a positive value. We do not know if this is a general result or if it is specific for our selection of data. In addition, we have not tested if this result is statistically significant, and hence we are not able to determine the importance of this observation. We then use the AR(1) model

$$\rho_h^s = Price\ Forecast_h + \alpha * e_{h-1} + e_h \tag{5.2}$$

where e_h is drawn from the historical normal distribution. We use this to make 50 scenarios for price that we combine with the inflow scenarios to a total of 50*51 = 2550 scenarios. We assume that the probability distribution of the scenarios is uniform and hence each individual scenario has a probability of 1/2550. Using these scenarios as input, SCENRED constructs a tree with 17 scenarios, as shown in figure 5-4.

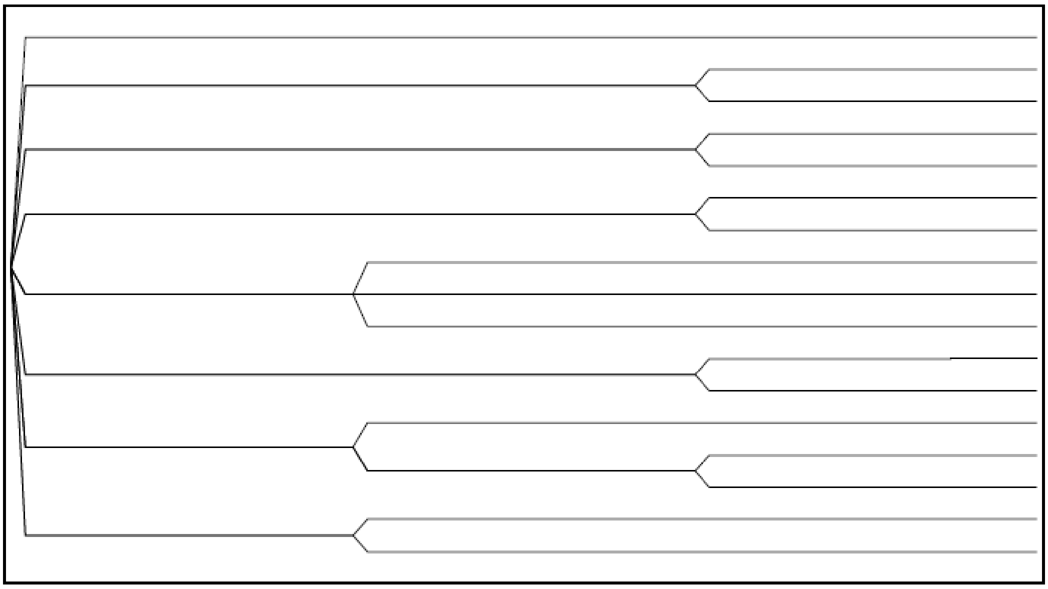

Figure 5-4: Plot of the base case with 17 scenarios

This tree has small variance, and the minimum and maximum price arising in the scenarios are 8.95 and 16.42€/MWh, respectively. This is quite a narrow spread, and may stem from the fact that the forecasted error is small or that we use a relatively flat price profile. The AR(1) model used is a simplified way to model uncertainty and the reduction algorithm in SCENRED detects that the underlying structure of the individual scenarios are similar, and it is hence able to reduce the tree to a great extent. We want to test our model for more complicated inputs. Still, we regard this as our main scenario input since it is based on historical data.

In the scenario tree given in figure 5-4 each node consists of prices and inflow for the 7 reservoirs. The price scenarios and the inflow scenarios at one of the reservoirs are given in the figures 5-5 and 5-6.

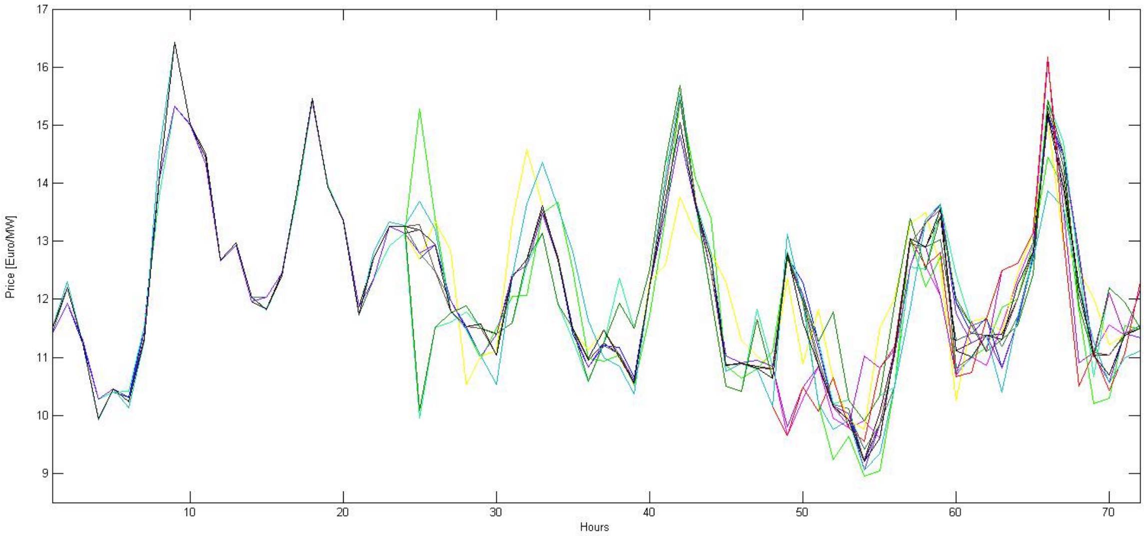

Figure 5-5: Plot of the 17 price scenarios

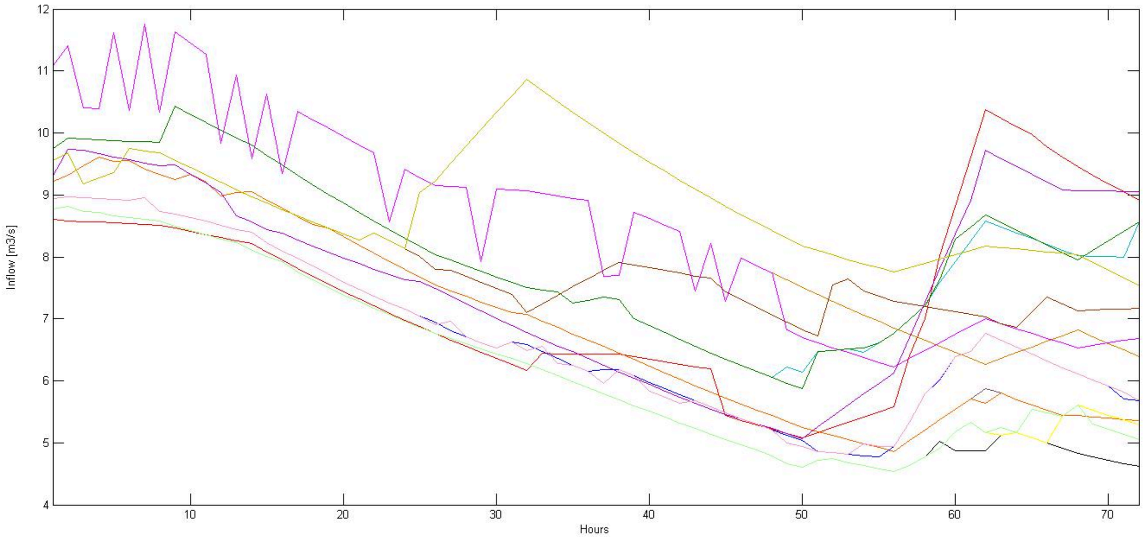

Figure 5-6: Plot of the 17 inflow scenarios

We also use the same underlying 2550 individual scenarios to run the SCENRED algorithm several times with different reduction parameters. This gives us the same underlying structure of the uncertainty, but increases the number of scenarios. In this way we get 17, 47 and 158 scenarios. We use this to investigate the stability of the stochastic solution, see section 7.2.

Figures 5-7 and 5-8 show plots of the scenario trees with 47 and 158 scenarios, respectively.

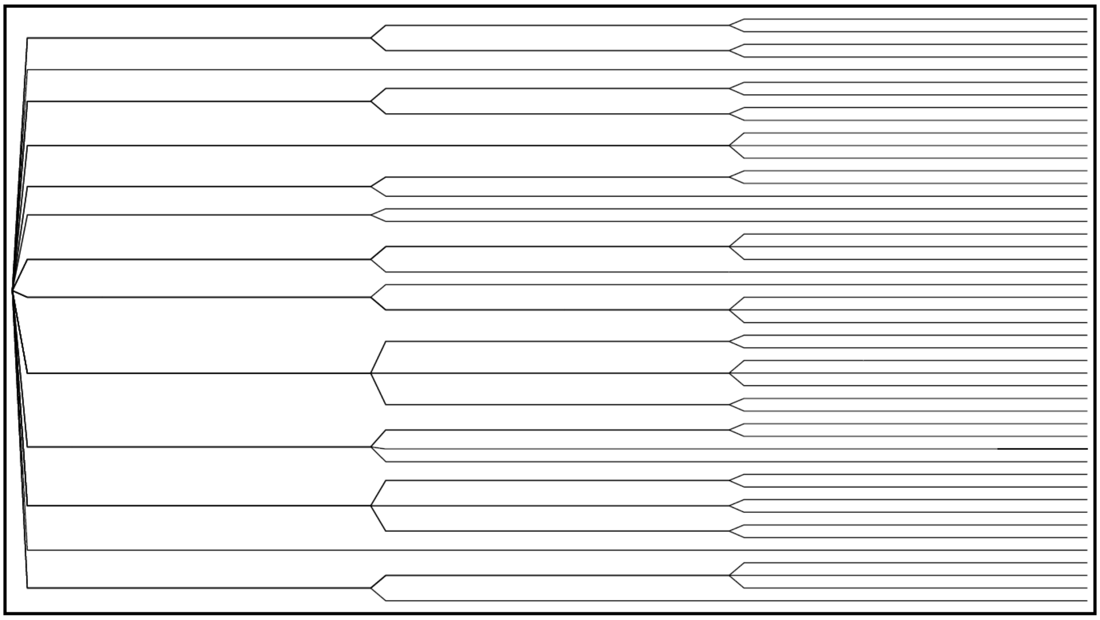

Figure 5-7: Plot of the base case with 47 scenarios

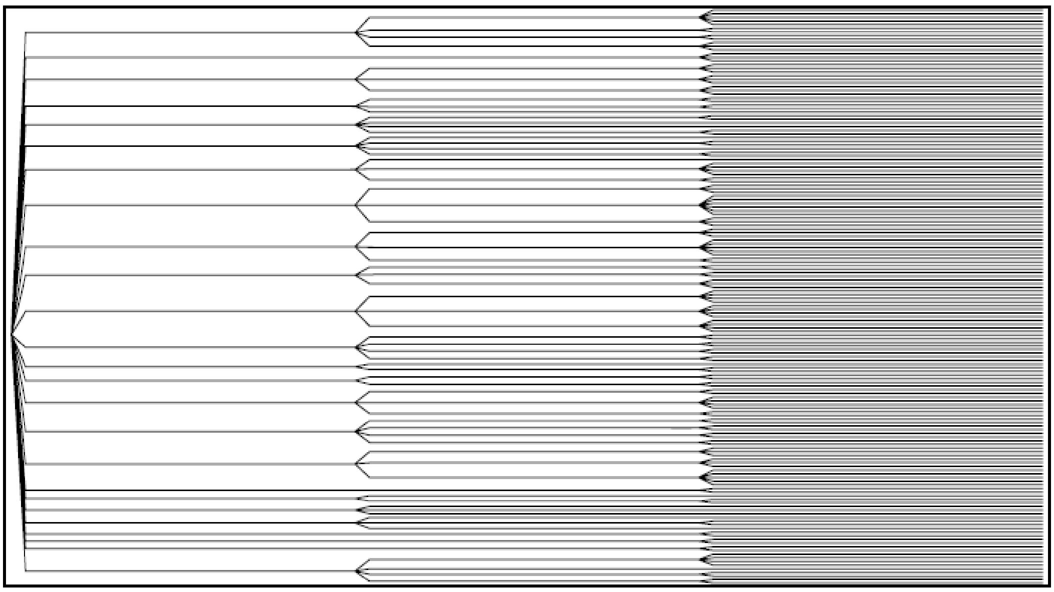

Figure 5-8: Plot of the base case with 158 scenarios

5.2.3. Extreme scenarios

As an alternative method we manually make up 10 scenarios that have odd or extreme profiles. We want to see how this influences the bidding strategy. The 10 scenarios are composed by doing some scaling of the original price forecast or simply by making up some profiles. We include scenarios where the prices are always low, always high, start low and have a very high peaks, start high and have small peaks or have a peak at the 'wrong' time of day, that is in the hours from 12-15 and not in the morning or evening. The individual scenarios are plotted in figure 5-9 below.

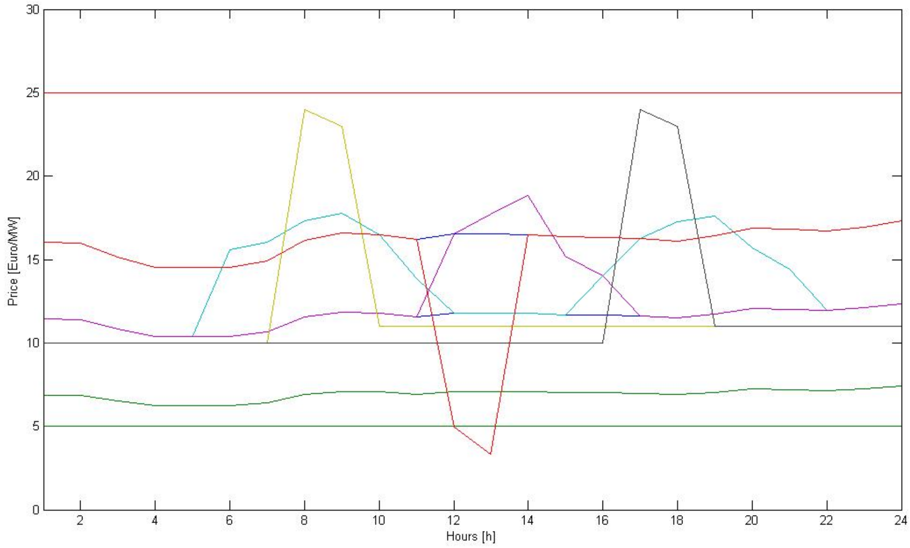

Figure 5-9: Plot of individual scenarios used for generating the extreme scenarios

These are as usual combined with inflow scenarios making 10*51 = 510 scenarios and then reduced to a tree with 12 scenarios trough SCENRED. A plot of the scenario tree is shown in figure 5-10.

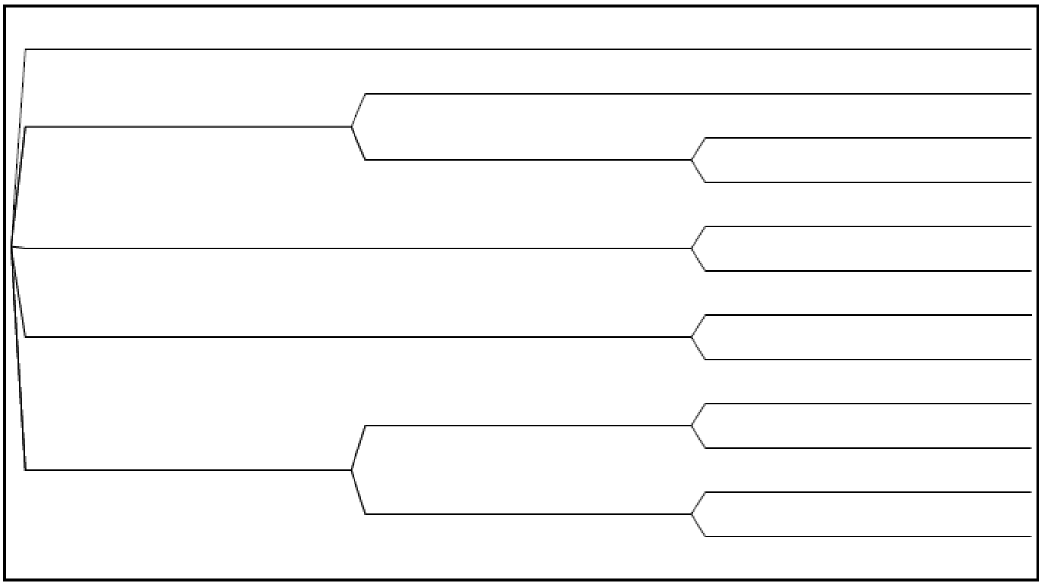

Figure 5-10: Plot of the 12 reduced scenarios for the extreme case.

6. Results from case study

First we present a short table of the obtained results from the case study, and then the results from the test cases are discussed in more detail individually.

Table 6-1: Table of results from the different test cases

Name	Number of Scenarios	Objective function value	Solution time
Deterministic	1	$3.1260*10^6$	0.3s
Historical A	17	$3.0345*10^6$	36.6s
Historical B	47	$3.1251*10^6$	21083.9s
Historical C	158	$3.0345*10^6$	265937.4 s
Extreme	12	$2.8281*10^6$	30.8s

We regard the objective function value and the bidding matrix as the major output from the model, but also look at other relevant results such as the number of unfortunate starts and stops of stations.

6.1. Deterministic average value results

In the case using average values we get a bidding matrix where we mostly bid our maximum capacity as soon as the price in a given hour exceeds the fixed price point. Figure 6-1 below shows a plot of the bidding function for 4 randomly selected hours (hour 1, 4, 9 and 19). For instance, looking at the purple line in the figure, we start bidding at a price point corresponding to a price of 11.5 €/MWh. This is reasonable as the price in this hour – which is deterministically given - turned out to be 11.45 €/MWh. The linear interpolation between price points give us a volume of 204 MW in this hour, since we bid 0 MW at 11.4 €/MWh and our maximum capacity of 319.5 MW at 11.5 €/MWh.

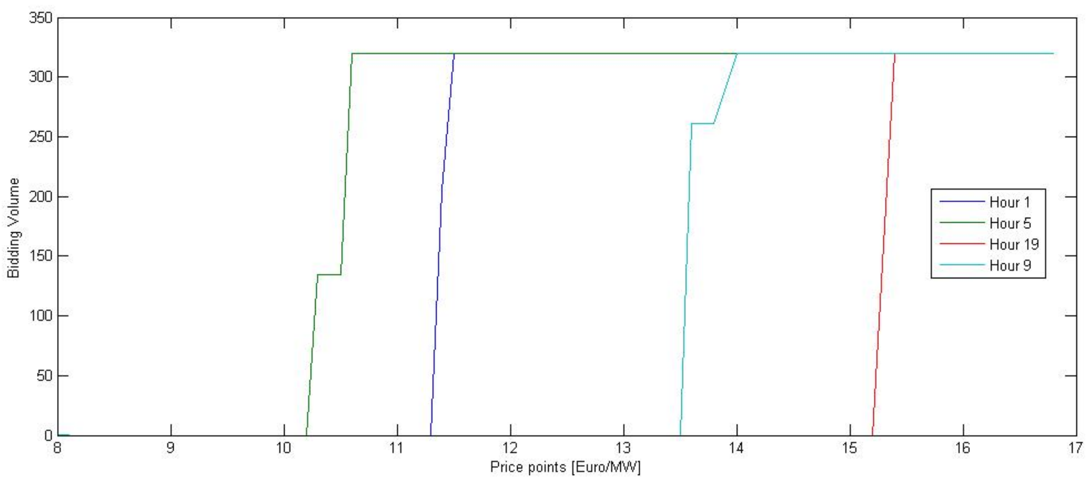

Figure 6-1: Plot of the bidding function for some selected hours.

For some hours, like hour 5 and 9 above, we bid at an intermediate level and not just 0 or maximum capacity. This can be seen from the small step on the lines going from 0 to 319.5 MW. This means that it is not optimal to bid our total capacity for these prices in this given hour, and that we should hold some water back awaiting higher prices. We would expect the bids to be a combination of units run at best point or maximum capacity. Due to the fact that other constraints may be binding of the solution, the stations are run at other volumes than best point or maximum point. For instance the constraints on Mannflåvatn causes water to be 'drawn' from the overlying reservoirs, and hence may force the production to be at other volumes than best point.

In the deterministic model, we do not use the balancing market, as anticipated. There is no need for recourse actions when all information is known in advance. Hence we do not get any penalty for being in unbalance with committed volume, and the overall objective function value is higher than for the stochastic test cases. This shows the fact that we know everything – both price and inflow - in the first stage and do not have to make a solution that is weighted for all possible future events, and hence we get an overly optimistic objection function value.

6.2. Historical mean and standard deviation results

At first glance, the solution in the stochastic case looks a lot like the deterministic solution, as can be seen from the plot of the bidding functions in figure 6-2. In most hours, and at the highest price points, we bid our maximum capacity. But in the stochastic case, there are far more hours where we bid somewhere between 0 and 319.5 MW, and we also make use of the balancing market. In fact, we bid intermediate volumes in 65 of 72 hours, compared to 43 of 72 hours in the deterministic case.

When price and inflow are uncertain, there is a value of holding water back and wait for higher prices. We keep the possibility of producing the water at a later point in time, and this has added value when decisions are made under uncertainty. In the deterministic case, we bid as soon as the price exceeds the bid point price – and this is the only criterion to decide if we should bid or not. In the stochastic case, the criterion for bidding is more complicated. We now have to take into consideration the value of saving the water and the loss of value by being in unbalance with committed volume. At each price point, we want to bid a volume that lets us benefit if the realized price is high; while at the same time not generate losses if the price turns out to be low. This is why uncertainty makes the problem conceptually more difficult, and also the reason why we need a recourse action.

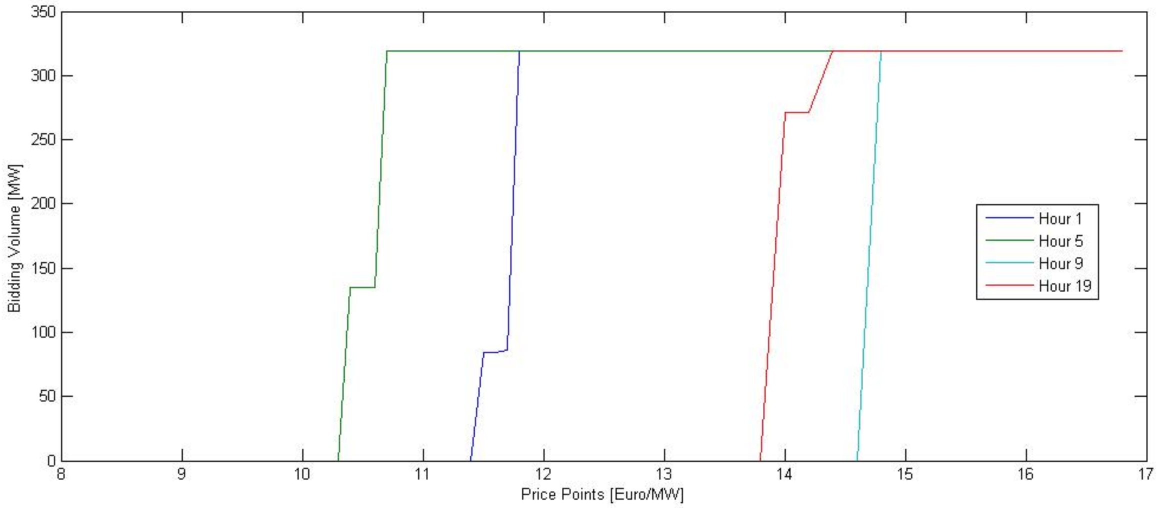

Figure 6-2: Plot of the bidding function in some selected hours

As noted, the effect of uncertainty is that we more often bid some intermediate volume. These bids are weighted decisions between the possibility for profit in the spot market and the loss of water and penalty for using the balancing market. The effect of start-up costs is also weighted in the bidding functions, making the decision even more complex. For higher price points, we incrementally turn on more stations and bid a new volume. In the stochastic case we also get bid curves with several intermediate steps before we reach the total capacity.

In the deterministic case, the balancing market was not used, but for the stochastic solution this is used occasionally. This gives a penalty in the objective function, and the total objective function value for the stochastic solution is lower than for the deterministic case. This is due to the effects of uncertainty.

6.2.1. Value of stochastic solution

To emphasize the effect of using stochastic models, we compare the objective function values for using the deterministic bidding matrix in the stochastic case and using stochastic bids. We do this by solving the average value case and fixing the bidding matrix in the stochastic case to these values. Hence, our first stage decisions are deterministically determined and then used to solve for the second stage variables. This means that we can calculate the value of the stochastic solution (Birge, 1982) as

$$VSS = RP - EEV \tag{6.1}$$

That is, the value of the stochastic solution (VSS) is the difference between the stochastic or recourse value solution (RP) and the expected result of using the deterministic solution (EEV). For the 17 scenario case, we get

$$VSS = 3.03454 * 10^6 - 2.86328 * 10^6 = 0.17126 * 10^6 \tag{6.2}$$

The effect of using deterministic models when prices and inflow are uncertain is that the bids are perfectly adapted to the average value case and hence we get large penalties when other realizations of prices and inflow occur. This means that the balancing market is used to a great extent, since an accurate balance between committed and produced volume only happens by coincidence. When fixing the bid matrix to the deterministic values, we use the balancing market in all hours, and this is the reason why the objective function of the expected value solution is much lower than when using the stochastic bid matrix. VSS is a measure of how well the stochastic program is approximated by the deterministic average value solution, and shows potential gains by using a stochastic model. Our VSS is about 6% of the deterministic objection function value, which shows that Agder Energi could potentially increase their profits by using a stochastic model.

6.2.2. Odd starts

A result that directly relates to the original problem received from Agder Energi is the number of 'odd' starts and stops of the stations. To turn the stations on or off for just a few hours is an unwanted production schedule due to the fact that frequent start and stops cause tear on turbines and other equipment and loss of water. We define an 'odd' start as when a station is turned on or off for 1 or 2 hours, and compare the results for the deterministic and stochastic case. It is, however, not straightforward to compare the unit commitment in the deterministic and stochastic case, and hence we make a probability weighted sum of the 'odd' start and stops for the stochastic solutions and compare this to the deterministic solution

Table 6-2: Summary of number of odd starts and stops in two of the test cases

Name	Number of 'odd' starts and stops
Deterministic	3
Historical A	5.3
Deterministic bids + stochastic model	47.8

From the numbers in the table above, we get less 'odd' starts and stops in the deterministic case than in the stochastic case, although the difference is not as large. Still, several of the scenarios have no starts, while others have some, so the number of starts and stops are dependent on what actually happens within the scenario, which is deterministic. The model finds the scenario optimal unit commitment based on what actually happens on an individual scenario level. But by making the bidding decision based on an aggregation of what happens in the scenarios the unit handling will be more robust against fluctuations in the price and hence more stable. When fixing the first-stage bid matrix decisions to the solutions obtained from the deterministic model and then solving the stochastic program, very many 'odd' starts and stops occur. This shows that a deterministic model generates bids that are not suited as a base for production planning. Using a stochastic model will aid Adger Energi in making sound production schedules.

6.2.3. Solution stability

For the test cases with 47 and 158 scenarios we also get a solution of the same nature as described above, where the bid is a weighted decision of spot market profits and costs of start-ups, penalties and loss of water. To test the stability of the solution, we can compare the objective value for the different number of scenarios. If the model is run several times with different numbers of scenarios as input, one can eventually determine about how many scenarios are needed to get a stable solution. With only three tests it is impossible for us to state anything about stability, but we note that the function is not steadily increasing, see table 6-1. The percentage change in objective function value between our three instances with 17, 47 and 158 scenarios is about $\pm 3\%$.

6.3. Extreme results

When using the extreme scenarios as input, we get bidding functions that have the same shape as for the historical mean and standard deviation test case, that is; we bid several steps of intermediate volumes before we reach maximum capacity. It happens far more often now that we bid in several stages, so that a certain combination of units is only optimal for a few price points. This is taken to mean that larger uncertainty makes the decision problem more complex, and it is harder to find an optimal combination of units for the entire range of possible prices. We bid intermediate volumes in a total of 67 of 72 hours.

An observation made from this case is that it is very varying how much the balancing market is used in the different scenarios. In some scenarios we need to buy large volumes in the balancing market and incur a large penalty. In other scenarios the balancing market is used to a lesser extent, meaning that the bids are well suited to what actually happens within that particular scenario. The amount of balancing volume used is thus related to the input scenario profile. If a particular profile is always low, the bid - which is a weighted decision for all scenarios – is too high and large volumes need to be bought, and vice versa if the profile is always high. This can be seen as an example of what happens when making decisions under uncertainty; penalties occur depending on whether the realized outcome of the random variables is good or bad, and since there is no way of knowing what will happen, the bids have to produce acceptable results for all possible events.

The results from the extreme scenarios illustrate how important good and updated forecasts are for the unit commitment scheduling.

7. General results and suggestions for further work

As a conclusion to this report, we here generalize some of the results found in the case study and make suggestions for future work.

7.1. General results

From the results of the preceding section, some general comments can be made. For instance, we see that the deterministic solution exceeds the stochastic solution. This was anticipated, as prices and inflow are modelled as certain and hence there is no need for using the balancing market and no penalty is incurred. In the stochastic case these penalties reduce the objective function. This holds generally; if production and demand always completely balance each other, there would be no need for regulation or balancing market solutions. Also, the stochastic solution makes bids that are more suited for use under uncertainty, and hence the bid volumes are more modest in anticipation of random events. Using the deterministically calculated bids in the stochastic case causes very large penalties. In this sense we say that the bids from the stochastic model are more robust. In general terms; when faced with uncertainty, decisions made tend to be less extreme to safeguard against bad outcomes.

Another result is that we get a more acceptable production schedule, which was wanted from Agder Energi. When using the deterministically calculated bids in the stochastic case, we get far more 'odd' starts and stops, showing that using the deterministic bid matrix as base for unit commitment causes an unwanted production schedule. Hence, we say that the stochastic solution give a more realistic production schedule.

In addition, from our test case using historical mean and standard deviation, the objective value is not yet stable with respect to the number of scenarios. An increased number of scenarios give an increasingly better approximation of the real continuous distribution and we would expect that the objective would be increasing for more scenarios. The computational burden increases with increasing number of scenarios, and this is an unavoidable disadvantage when the number of scenarios and hence the number of variables increases. Hence there is a trade-off between correct modeling and solution time.

7.2. Suggestions for further work

We have implemented the model by Fleten and Kristoffersen (2007) on a complicated real-life system and hence added constraints related to watercourses and reservoir connections. This makes the model more applicable for use by real producers and can be seen as a first step towards introducing stochastic models to the industry. In addition, we have uncertainty in both price and inflow, which is new compared to Fleten and Kristoffersen (2007), which only have uncertainty in price. Thus far we have not to any great extent explored how the effects of uncertainty in inflow influence the solution due to the fact that we have data from a period of the year when reservoir levels and inflow are high and stable. For further work, data from other parts of the year may be used in the model and effects of uncertainty in inflow may be investigated more thoroughly.

7.2.1. Improving the scenario generation method

The quality of a stochastic model is strongly linked to the quality of the scenarios used as input. This is due to the fact that the scenarios provide information to the model of how the stochastic variables vary. If the scenarios are of low quality, then the quality of the stochastic solution is consequently also low. Therefore, part of the future work should concern improving the generation of scenarios, investigate different methods and their applicability to this particular problem. The scenario generation method and model development are independent, and could be developed apart from each other. Our results show that the objective value varies with the number of scenarios. Hence it is interesting to generate larger sets of scenarios. With an increasing number of scenarios it is also interesting to see if the objective value converges toward a certain value.

7.2.2. Simulation

To compare the results from the model with a deterministic model, a simulation procedure where the suggested production schedule and bidding matrices from the stochastic model are implemented over a longer period of time should be performed. The revenues, unit commitment and reservoir handling from this simulated production could then be compared with the results from the deterministic model in the same time period. If the model produces reasonable results, this would be a validation of the modeling approach.

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