# Investment analyses of CCGT power plants

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# Preface

This paper is prepared as our Master Thesis at the NTNU at the Department of Industrial Economics and Technology Management.

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# Executive summary

This thesis analyses the investment decision of building CCGT power plants in Norway using real options theory. The investment timing for a firm owning a license to build a natural gas fired power plant, and hence the option value to delay investment is investigated. The base case scenario considers an 800 MW base load plant located at Tjeldbergodden, Norway.

The valuation models are based on simulated price movements on electricity and natural gas. The commodity prices are modelled to follow different stochastic processes according to two model frameworks; a spot price model (SPM) and a forward curve model (FCM). The spot price model is considered the main model. Historic price data on electricity from Nord Pool and on natural gas from the IPE are used to construct expected forward prices for the full length of the project life time. For both models, the valuation is done for a spark spread process and for natural gas and electricity as separate processes.

The FCM is more tractable when it comes to estimate parameters since it has fewer than the SPM, whereas the SPM is assumed to be easier to communicate and is a well proven method in real options theory. In both model frameworks the spark spread models seem to give sufficient information compared to the accuracy obtained by increasing variables. The authors believes the valuation methods for FCM need more development before it is ready for real option analysis. The SPM, is considered to be the best model for the valuation.

The cost of $CO_2$ emission is set to 20 Euro/ton in the base case valuation. Reducing the cost of $CO_2$ does not alter the investment decision. The SPM is extended to allow $CO_2$ emission cost to follow a stochastic process and to incorporate tax effects. Including tax effects lead to a negative project value. Further the impact of competition to the option value is studied. The value of holding the license to invest is considerably reduced if there is a risk of loosing the project to competitors.

Risk management of the option and project value is considered. Hedging the option value using long term energy contracts traded at Nord Pool and IPE is possible, but is considered too costly to carry out.

All models consent on the decision to delay investment, and extensive sensitivity analyses do not alter the conclusion. The main reason for delaying investment is to wait for better information on the prices of electricity and natural gas. The spark spread spot price model is considered to be the best model. The model gave a value of investing today of 13 MNOK and the option value was found to be 2800 MNOK. The authors advice the license holders to delay investment.

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# 1 Introduction

Natural gas fired power plant is on the public agenda in Norway at present. Insufficient generation capacity in parts of the country calls for new production facilities. To this day no natural gas fired power plant supplying the main grid is built. The discussion continues of whether high natural gas prices, low electricity prices or uncertainty about $CO_2$ handling are the reason for why investors do not invest.

This master thesis applies real option theory to value the license to invest in a base load Combined Cycle Gas Turbine plant (CCGT) in Norway under uncertainty. The license is modeled as an option to invest. Prices of electricity and natural gas effecting future cash flows are considered uncertain and are modeled as stochastic processes. Both spot price and forward curve models are presented and used. These stochastic processes are fitted to publicly available price data from Nord Pool and the International Petroleum Exchange using Kalman filter, root mean square error methods and Principal Components Analysis. As closed form solutions for the option value in most cases do not exist, Monte Carlo simulation is used to value the license.

The first part of the thesis introduces real option theory, discusses the need for more generation capacity in Norway, presents the financial markets for electricity and natural gas and outlines the assumptions which the models are based on. In the second part several commodity models from the literature are presented, and deductions of the models used in the thesis and description of parameter estimations are outlined. The final part presents the results from calculations and a discussion of results and assumptions is provided before the conclusion. The appendix provides a thorough discussion of the costs and technical aspects of a CCGT.

It is assumed that the reader have a basic understanding of financial derivatives theory and some knowledge of stochastic differential equations. Ton refers always to metric ton.

# 2 Theory on real options

A firm owning real assets has several choices to make about how to manage it's portfolio of assets. It can choose to add more assets to the portfolio or remove others. The firm can also decide whether to use an asset today or not. Different options can represent a value and have to be considered when making decisions. Especially in a world of uncertainty such options are important. More generally on can divide real options into:

- The option to wait (for instance wait to invest)
- The option to abandon (for instance to sell an asset)
- The option to be flexible
- The option to expand
- Etc.

The real option theory is introduced by among others Brennan and Schwartz (1985), Pindyck (1988), Dixit and Pindyck (1994) and Trigeorgis (1996).

## 2.1 The option to postpone an investment

This paper investigates the option to wait and by that also the timing of the investment. A firm owning a license to build a natural gas fired power plant has the right but not an obligation to invest in the power plant. The firm can decide to invest whenever it wants in the whole period for which the license is valid. Thus the firm has an option to invest at any time during the life of the option (the period when the license valid). In financial terms such an option is of American style.

Consider a firm having the right to invest in a power plant. When the investment decision is made the investment cost is paid and the power plant is built. The investment cost cannot be retrieved once the investment is made and is said to be "sunk". According to general investment theory the investment should be made once the net present value (NPV) of the expected cash flows exceeds zero as shown in equation (2.1). Following the real option approach this may not be true because the option to wait (or other options) is not considered. When there is a possibility to delay investment the option to wait for better prices can have a greater value than investing today. Thus only considering the NPV of the future expected cash flows one assumes that either the investment cost is fully retrievable or the opportunity to invest is now or never so the option to wait does not exist. This is one of the major criticisms of the general NPV approach.[1]

Real option theory implies that an investment should not be made before the NPV of future expected cash flows exceeds the value of the option to wait. The option to wait should not be "killed" (making an investment kills the option) before it is more profitable to have the power plant as illustrated in equation (2.2). In the setting of this paper this means that at the last day of the license a regular NPV analysis is valid. Before that the option to postpone the investment must be considered.[2]

$$\text{NPV} = \text{PV(future cash flows)} - \text{Investment} \geq 0 \qquad (2.1)$$

$$\text{PV(future cash flows)} - \text{Investment} \geq \text{Option to wait} \qquad (2.2)$$

## 2.2 Real Options in the Energy Business

Real option analysis is very suitable for projects in the energy business since such projects incorporate many of the features that set the basis for real option theory. Such features are among others

- Large sunk investment cost. Investment in a hydro power plant or transmission equipment cannot be retrieved.
- Production flexibility. The power plant should only be operated when prices make it profitable.
- Option to expand capacity. Already existing power plants can have possibilities to install new capacity.

Existence of futures and other derivatives that serve as spanning assets that reveal important info about the value of the project (spanning assets are explained in chapter 8.2). In recent years much work has been done using real options to value different kinds of projects in the energy business. Deng et al (2001) has used real options analysis to value peak load power

[1] Dixit and Pindyck (1994)

[2] Dixit and Pindyck (1994)

plants and transmission capacity. Thomson (1995) uses real option theory to value take-or-pay contracts. In Schwartz' work from 1997 and 98 methods for investment decisions are outlined. Dobbe, Fleten and Sigmo (2003) uses real option theory to value a peak load natural gas fired power plant in Norway and to value an infinite license to build such a plant. Fleten and Näsäkkälä (2003) uses real option theory to decide the optimal investment plan for a base load CCGT plant.

# 3 Plans for gas fired power plants in Norway

The construction of gas fired power plants in Norway is in 2004 a relevant topic with respect to the electrical power balance. Several plans have been proposed, all accompanied by controversy related to $CO_2$ emissions. By the first quarter of 2004, no commitments to build a gas fired power plant for delivery of electricity to the main grid have been announced.

## 3.1 Electrical power balance

Norway is integrated in the Nord Pool electricity system, comprising the larger part of the Nordic countries[3]. About 50% of the generation capacity in the Nord Pool area is hydropower and is mainly located in the North and Western part of the Nordpool area consisting of Norway and the Northern part of Sweden. The Thermal and Nuclear facilities are mainly located to the south and east. With as much as 50% of the electricity production generated by hydropower, the price in the entire market area and in Norway in particular, is highly affected by precipitation.

The Norwegian electricity supply is based on hydro power (99.5% of domestic production) and imports. During a year with normal precipitation hydro power supplies the electricity market with 118 TWh per year. The actual production can vary between 89 TWh for a dry year and 150 TWh for a wet year. The Norwegian electricity demand is about 125 TWh per year, resulting in an import demand of 7 TWh on a regular basis. NVE states that the import under normal conditions will increase to 12 TWh in 2010[4]. In dry years limited import capacity can become a restriction.

### 3.1.1 Transmission constraints[5]

The power transmission grid does not by 2004 have the sufficient capacity to handle extreme deviations from the normal power balance. The transmission grid within the Nord Pool area is operated by Transmission System Operators (TSO) being Statnet in Norway. New investments and upgrading of the exiting transmission grid capacity are considered in a long term perspective and are subject to a socioeconomic validation. Compared to other countries,

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[3] A thorough description of the Nord Pool area is given at the Nord Pool homepage; www.nordpol.com

[4] Kristensen et. al (2003)

[5] The chapter is based on Hoelsæther (2004)

Norway has small power transmission capacity over its national borders (18% of maximum consumption). In addition to the current level of about 4300 MW, additional transmission capacity of 2400 MW is needed to reach the same level as Sweden (28%).

The main transmission constraints of today are limited transmission capacity over the Nord Pool area border and some bottlenecks within the Nord Pool area. In case of an extreme dry year Norway will have to import as much as 30 TWh. The import capacity is limited to 15-20 TWh, resulting in a need for additional 10-15 TWh. If no new generation capacity is installed in Norway, and the transmission grid not upgraded, the anticipated consumption growth will lead to aggravated electrical power balance between Norway and Sweden.

Future bottlenecks with respect to Norwegian electricity import have been identified as the line over Skagrak to Denmark and the connections to Sweden that will not make due if the power balance worsens. New transmission capacity to Denmark and out of the Nordic area to UK have been considered but put aside for the time being. Currently a new line between Norway and the Netherlands is evaluated. Within Norway, the axis North-South has low capacity as of today.

### 3.1.2 New capacity

The prevailing power balance in the southern half of Norway calls for installation of new electricity generation capacity if not the import capacity is greatly increased. Gas fired power plants are only one of several solutions. New hydro power capacity on large and small scale and wind power are the other most likely alternatives, however, having some constraints on its potential.

Enova estimates a wind power production of 3 TWh in 2010[6]. The sum of existing, coming and planned wind power totals to 5.1 MW with a production potential of 16.4 TWh. 2.7 MW out of the total 4.8 MW planned wind power production is located in Finmark and the northern parts of Troms.[7] The North-South transmission capacity is already limited, and in order to transport the generated power to southern Norway the transmission grid will have to be upgraded at a cost of several billion NOK.

[6] Arnstad (2002)

[7] Hoelsæther (2004)

The remaining hydro power potential in Norway is about 25 TWh, including the larger part of an upgrading and expanding potential on existing hydro power installations of 10 TWh[8]. The remaining hydro power projects are to a large extent made up of controversial and increasingly costly projects. New surveys suggest that the potential from micro scale hydro power plants is highly underestimated and the total potential to be 30-40 TWh. The applicable part is still not determined.[9] A coming revision on a law on waterway conservation is expected to further reduce the total potential.

The energy-intensive industry in Norway uses 30-35 TWh a year. At least half of the consumption is bought on old long term contracts with very favorable conditions measured by the price level of today. Large contract volumes with a price of 50 NOK/MWh expire in 2007-2008. It remains to be seen how large part of the industry that will remain when buying electricity on market prices. The result could be affecting the electrical power balance greatly.[10]

## 3.2 Existing plans for Gas fired power plants in Norway

Table 1 shows the formal plans for gas fired power plants in Norway. The projects at Kollsnes and Kårstø were granted permission in 1996 and these two permissions are liable till October 2007[11]. The power plant at Skogn was granted permission in 2000[12]. The permissions usually have a life time of 10 years. 10 years is considered the normal life of a licence in this setting.

[8] Paaske (2002)

[9] Kvål (2004)

[10] Kvål (2004)

[11] Rønning (2003)

[12] Industrikraft Midt-Norge (2003)

**Table 1**

**Existing plans**

| **Project** | **Company** | **Status** | **MW** | **TWh** | **Investment cost MNOK** | **Investment cost (NOK/TWh)** |
|---|---|---|---|---|---|---|
| Kollsnes | Naturkraft | License granted | 390 | 3.1 | 1800 [1)] | 580.6 |
| Kårstø | Naturkraft | License granted | 380 | 3.0 | 1800 [2)] | 600 |
| Skogn | Industrikraft Midt-Norge | License granted | 800 | 6.4 | 3600 [3)] | 562.5 |
| Karmøy | Norsk Hydro | Reported | 1300 | 10.5 | - | - |
| Melkøya | Statoil | Under construction | 230 | 1.7 | - | - |
| Tjeldbergodden | Nordenfjelske Energi | Reported | 400 | 3.2 | - | - |
| Tjeldbergodden | Naturkraft | Reported | 800 | 6.4 | 3000 [4)] | 562.5 |
| Hammerfest | Hammerfest Elektrisitetsverk | Reported | 100 | 0.7-0.8 | 1300 [5)] | 1625 |
| Grenland | Skagerak Kraft AS | Reported | 400-1000 | 3.2-8.0 | - | - |

Formal projects permitted by or reported to NVE. 1,2) According to SFT. 3) According to Nord Trøndelag fylkeskommune (1999). 4)According to Hegerberg(2003), Adressavisen. 5) According to Hammerfest E-verk.

Statoil, one of the larges investors in Naturkraft, has signalled the preference of locating a new natural gas fired power plant in connection with already existing petrochemical industry[13]. It implies that the original projects at Kollsnes and Kårstø are less likely to be undertaken. A CCGT plant at Tjeldbergodden or Kårstø and a CHP plant at Mongstad have been proposed as potential project. Elkem which owns 50% of Industrikraft Midt-Norge claims the reason the plant at Skogn is not built is due to too high prices on natural gas. There is also a discussion of who is to cover the large cost of gas infrastructure investment connected to this project.[14] According to Managing Director of Naturkraft, Ole Rønning, a stable electricity price of 300 NOK/MWh will make a gas fired power plant highly profitable.[15]

[13] Petromagasinet (2003)

[14] Sundsbø (2003)

[15] Steensen (2003)

A Gas fired power plant is already under construction on Melkøya, Finmark, (See Figure 1) as part of the Snøhvit project, a large scale LNG production facility. The plant is for industrial purposes only, and has not been subject to the controversy at the same scale as the other suggested projects. Other gas fired electrical power production facilities are to commence production within the Nord Pool area the coming years, e.g. a CHP plant near Gothenburg, Sweden with 260MW electricity production to be operational in 2006[16]. Hammerfest El. verk has recently announced plans to build a 100 MW CCGT plant with $CO_2$ sequestration equipment in Hammerfest as a pilot plant[17].

**Potential locations**


**Figure 1**: Potential locations of a CCGT plant in Norway.

[16] Goteborg Energi 2003

[17] Hammerfest El.verk (2004)

The current Norwegian government has stated that no new licenses to invest in gas fired power plants with $CO_2$ emissions will be given. Several option holders have suggested switching the licenses to other projects. It is not certain political acceptance will be gained to support this view. It has also been suggested that political pressure on the license holders has prevented exercise of the options[18]. Political risk is discussed further in chapter 13. Statoil has declared that gas fired power plants will be constructed if they will be given equal conditions as investors in the EU[19]. With Norway struggling to uphold the country's commitment to its Kyoto obligations, as mentioned in chapter 7, it is still undetermined whether there will be political acceptance to allow large $CO_2$ emissions.

## 3.3 Price effects of new generation capacity and the need for base load

According to the efficient market hypothesis, the market price is already reflecting the general belief in the market with respect to new generation capacity. This is discussed further in chapter 4.5. Electricity market analysts can provide estimates of the impact a new 800 MW power plant will have on the electricity prices in the Nordpool area. The location of new capacity of this order is of large importance due to constraints on transmission capacity between the grid areas, and can thus highly influencing the local area price. A SINTEF study of the old regulated market without considering transmission constraints concluded that a new plant of 800 MW at Skogn would lead to a significant decrease in electricity prices in the order of 10-20 NOK/MWh for the whole year. The price effect in the deregulated market is considered to be smaller and the exact impact on the price is hard to estimate. The expectations of new generation capacity will already be priced in the market. Regional effects can, however, be large making local energy producers more reluctant to invest in new projects.

The Nordic electricity grid has abundance of hydro power generation capacity with good regulatory characteristics, excellent for peak load production. A gas fired power plant located in Norway will as a consequence be used for base load production. Nuclear power and coal fired power plants in other parts of the Nordpool area will operate with a lower marginal cost than a CCGT plant, but the recent power balance and transmission constraints in the Nordic

[18] Werner (2004)

[19] Hagen (2004)

market do not exclude the possibility of running a base load plant at full load continuously.[20] It should be possible to sell the production from the base load plant at long-term fixed contracts to e.g. Hydro expanded aluminum plant at Sunndalsøra.

[20] Botterud (2003)

# 4 Market for electricity, natural gas and spark spread

Norway has a well developed electricity market as being part of the Nordic commodity Exchange for electrical power, Nord Pool. There is no similar market for natural gas in Norway, and it is assumed that natural gas would be sold on long term contracts with a take-or-pay clause (TOP). Price data from the International Petroleum Exchange (IPE), subtracted for the transportation cost from the North Sea to UK, is used to approximate "Norwegian" prices on natural gas prior to investment.

## 4.1 The IPE and natural gas financial contracts

The larger part of Norwegian natural gas production is exported to the UK and Continental Europe through an extensive net of transport pipelines. The price of natural gas in Southern half of Norway will be determined by the alternative price achieved by export. The International Petroleum Exchange in London is one of Europe's leading energy futures and options exchange and lists natural gas financial contracts.

### 4.1.1 The market for natural gas

Natural gas is expensive to transport, and thus the price obtained for natural gas becomes very dependent on the location of the natural gas field. As long as the natural gas fired power plant is supplied from a field connected to the main European natural gas grid, the price will be given by the market price in UK or continental Europe. This will be the case for a natural gas fired power plant located in the Southern part of Norway. As long as there are no transportation constraints, the market price in Norway will be set by the price achieved by export.

IPE data is chosen to represent market prices as the UK gas market is the only fully deregulated gas marked in Europe, and since export to the UK is a likely alternative to domestic consumption. IPE Natural Gas contracts are the pricing benchmark for the UK Gas Industry. A Nordic gas market centered in Denmark is currently at the planning stage. For locations like Hammerfest, IPE data will not be representative since the regional gas fields are not linked up to the European gas grid.

### 4.1.2 Natural gas financial contracts at IPE[21]

Natural gas futures have been traded at the IPE since 1997. Depending on the date of the trading day, the contracts listed below are subject to trade. Figure 2 displays the term structure graphically.

- Monthly futures for delivery in the 9-11 next months
- Quarterly futures for delivery in the 6-7 quarters after the monthly contracts. (Denoted Q(number, year))
- A six-month futures contract for the nearest season after the quarterly contracts, and a six-month futures contract for the consecutive season. (Denoted Summer/Winter(year))

**Futures at the IPE**

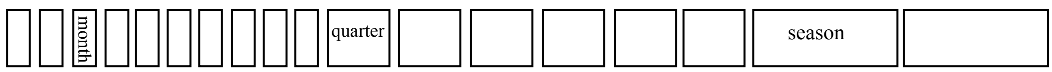


**Figure 2**: Illustration of futures contracts available at the IPE.

The Season and Quarter contracts are split up as time closes in on the delivery period. As an example, the Q103 is split into monthly contracts after the last trading day in March 2002. The Q203 is split into monthly contracts and the WI04 is split into two quarterly contracts of Q404 and Q105 after the last trading day in June 2002. Examining data from the IPE-historical databank reveals that contracts with more than a year to delivery are not very liquid.

[21] The rest of this chapter is based on public information from the web pages of IPE (2004)

**Natural gas term structure**

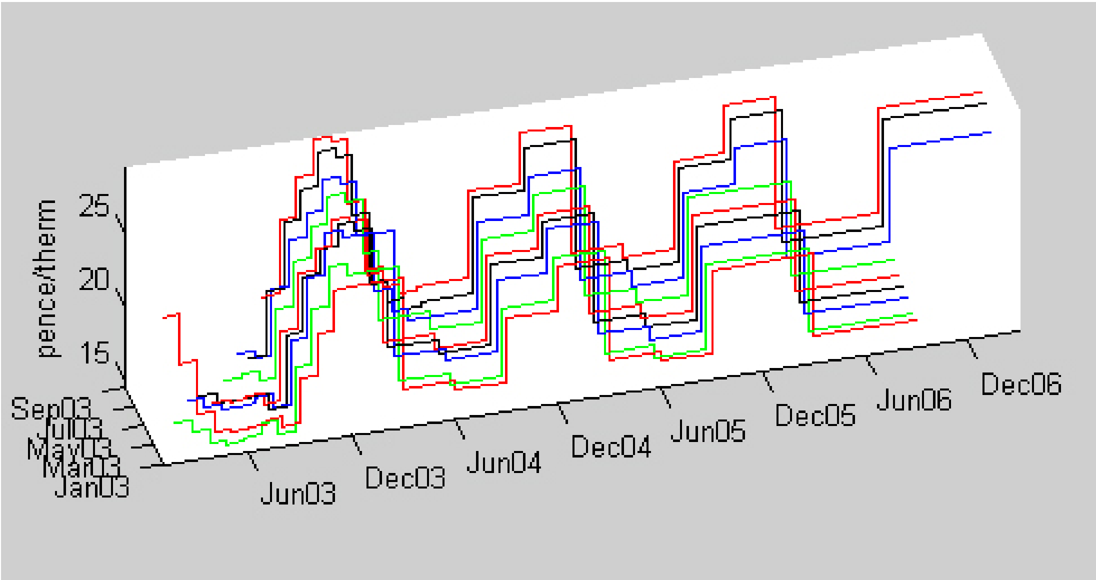


**Figure 3**: The term structure of natural gas futures at the IPE. Each path represents the term structure on the first trading day in each month from January to September 2003. The prices are noted pence/therm (1 therm equals 29.3071 kWh, conversion to NOK/MWh is outlined in chapter 5)

Figure 3 shows the term structure of natural gas at IPE for the first trading day in each month from January to September 2003. From this figure one can clearly see how the futures prices have seasonal variations as the prices vary from 0.15£/therm during the summer months to about 0.30£/therm during the winter months. In January 2003 contracts out September 2006 are available.

Natural gas is a collective term for the smallest hydrocarbons. The energy per volumetric entity, $Sm^3$, is determined by the exact composition of hydrocarbons and the energy quality may vary considerable between gas fields. The CCGT measures the gas input in terms of energy. The natural gas futures contracts are traded in £/therm. Since the financial contracts are listed in terms of energy and not volume the quality of the natural gas does not have to be taken into account when using the IPE gas data. The IPE natural gas futures are traded in quantities of 5 lots of 1000 therm per day.

## 4.2 Financial electricity contracts at Nord Pool[22]

The Nord Pool area covers the mainland part of the Nordic countries. The Nordic power exchange consists of a physical and a financial electricity market as well as a clearing service for the OTC market. Founded in 1996, Nord Pool was the first electricity exchange in the world and comprised at that time Norway and Sweden. The other countries joined the following years. 32% of the financial marked was traded directly at Nord Pool in 2003. The Nord Pool price is used as a reference in the OTC market and acts as the main clearing service. The traded volume in the financial market is nearly halved since the wake of the Enron scandal. As a result, the market liquidity has decreased and only few contracts are traded in high volumes on a regular basis.

Forward and futures financial energy contracts and European options are the contract types listed at the Nord Pool financial market. The products offered are changed according to market demand. Nord Pool is currently restructuring the financial contracts to structure more similar to that of IPE. The transition will be completed in the beginning of 2006. The new structure will consist of forward contracts for month, quarter and year futures contracts for days and weeks. All contracts within the new structure will be traded in EUR, replacing NOK. The financial contracts are not split up as delivery date approaches.

Figure 4 displays the contract configuration under the old and the new structure. Depending on the date, under the old structure the following contracts were available:

- Weekly futures for 4 – 7 next weeks.
- 8 – 12 block futures each block consisted of 4 weekly futures and was split into weekly contracts in accordance with rules determined in the product specifications.
- 6 – 8 seasonal forward contracts, starting the next season.
- Three yearly contracts the first starting next calendar year.

The new structure was initiated April 7th 2003. The contracts left from the old structure will be traded till they mature. When the new structure is fully introduced it will be as shown in Figure 4 and the following contracts are then available depending on the date:[23]

- 8 weekly futures contracts

[22] This chapter is mainly based on public information from the web pages of Nord Pool (2004)

[23] Nordpool (2003)

- 6 monthly forward contracts the first starting next month.
- 8–11 quarterly forward contracts the first maturing the next quarter.
- Three yearly contracts the first starting next calendar year.

Since historical data is used, the old structure makes up a larger part of the empirical data set. All Nord Pool financial contracts are listed in MW for delivery of 1 MW per hour for the full length of the contract period.

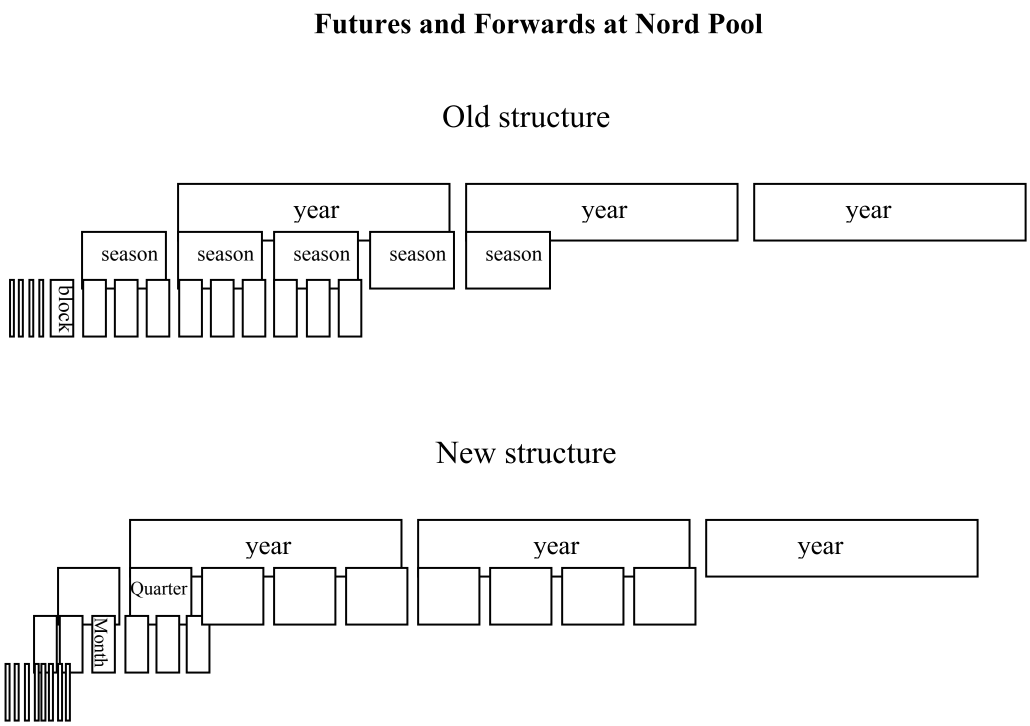


**Figure 4**: Illustration of the futures and forward contracts available at Nord Pool under the old and the new structure.

## 4.3 Long term contracts

The project will have an expected lifetime of 30 years, and knowing prices on contracts for the entire period of production would be preferable for project valuation. The exchanges Nord Pool and IPE only offer financial contracts on a relatively short horizon. A comparable market for long term contracts does not exist. Long term price projections for a 30 year horizon are often based on analyzing various scenarios.[24] This is not done in this thesis.

[24] Bergli (2003)

### 4.3.1 Long term electricity contracts

Although not traded on exchange, long term power-contracts of e.g. 10 years delivery period are traded in the OTC market. The ten-year contract is a forward similar to those traded at Nord Pool. Elkem Energy has provided the price of a ten year contract to be used in this thesis[25]. According to various brokers, the prices on 10-year contracts today are not very liquid but the price information is believed to be representative [26]. Most energy brokers list a bid ask spread on 10 year contracts. The contracts are rarely traded and the spread is large. Actors would be interested in long term contracts if the price is right and by that compensated for the risk involved. Market analyst states that long term electricity contracts are not in demand from end users.[27]

### 4.3.2 Long term natural gas contracts – the Take-or-pay clauses

The long term contracts in the former regulated European gas market were usually made up of a set of provisions. Among the most important pricing provisions were to correlate the price of natural gas to the price development of other energy sources and the Take-or-pay (TOP) clause.[28] The price of natural gas in long term contracts was in the past independent of natural gas spot price as no such existed.

Buying natural gas on a long term contract with a TOP-clause is considered the most realistic scenario for a gas fired power plant located in Norway.[29] The contract price today will be affected by the price of natural gas in the European market, and the price offered in Norway will be determined by the alternative value of selling the natural gas to UK or other European countries.

In every long term contract the provisions are subject to separate negotiations and the details are not publicly available. The general contract structure is described by Brautaset et al (1998). The "take or pay" concept can be defined as a provision, written into a contract, whereby one party has the obligation of either taking delivery of goods or paying a specified amount. TOP includes a package of provisions in which flexibility in delivered volume and

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[25] Dobbe (2003)

[26] Florholmen (2003)

[27] Kvål (2004)

[28] Austvik (2003) p42

[29] Sigmo (2003)

price settling are the most important. Price information on long-term contracts is scarce. However, Dobbe (2003) has provided a natural gas 10-year forward price for UK.

The concept of the long term contracts with a TOP clause leads to the buyer taking the volume risk, whereas the seller takes the price risk. The logic behind is that the seller needs volume to support large investments in infrastructure, and the buyer needs to get insurance of a competitive price to be willing to make a long time commitment.

### Volume

The TOP-clause takes into consideration that the buyer can experience variations in demand of natural gas. Delivery flexibility is obtained by allowing departure within a given interval from a set of reference volumes, a Daily Contract Quantity (DCQ) and Annual Contract Quantity (ACF). The allowed deviation will typical be in the range of 40-110% for DCQ and 90-110% for ACQ, further departure initiating payment obligations.[30]

The flexibility in the contract will be dependent on the location of the gas plant. Terms as described above can only be obtained in a larger infrastructure where production and consumption of natural gas can be smoothed. By locating the power plant in connection with the existing natural gas infrastructure, similar contract terms should apply as is the case for most of the projects listed in Table 1.

### Price

The price of natural gas is determined by a combination of a fixed base price and a set of variables, the most important being the price development of relevant substitutes[31] and a few other variables with impact on energy consumption such as temperature. In the European gas market, the natural gas contracts have largely been formulated in a way that prices react to changes in prices on alternative energy sources, with a certain time lag. The most important has traditionally been the oil price, but the price of electricity is gaining importance.[32]

[30] Osmundsen (2002)

[31] Osmundsen (2002)

[32] Tomasgard (2003)

The spot price of natural gas itself has gained importance as the long term contracts coexist with the spot market. The effect of deregulation has not yet fully influenced the European natural gas market. The price of natural gas has gradually become more determined by the supply and demand of natural gas, but not considered to be fully decoupled from the oil price before 2010.[33]

The contract details will specify which energy carriers and other factors that are to be included in the pricing formula, the weighting of the carriers and the initial relationship. The escalation mechanisms between changes in prices for the alternative energy carriers and the price of natural gas are also considered. Applying the concept of price risk taken by the seller, a TOP contract for a gas fired power plant in Norway would be highly affected by the price of electricity.

#### Renegotiation

Long term natural gas contracts normally include a clause on renegotiation. Changes in market conditions will be reflected by renegotiating the terms of the long term contract. As an example, Troll contracts contain clauses to ensure that either buyer or seller can demand renegotiations every 3rd year if market conditions have changed so much that the pricing formulas no longer reflect the competitive position of natural gas in the market. The renegotiation term has been included to prevent the break up of long term contracts as cheaper natural gas has become available in the deregulated marked. The renegotiation clause has to this date mainly benefited the buyer. However, including a renegotiation clause will add risk to the project of building a gas fired power plant since it can result in an increased cost of natural gas.

### 4.3.3 Fitting a TOP into our model

Prior to the investment decision natural gas is assumed to follow a stochastic price process. IPE price movements are assumed to be the best approximation of this price process, and will be used to estimate parameters. Once the investment decision is made it is assumed that the power plant will buy natural gas on a TOP-contract.

[33] UBS Warburg (2001)

It is assumed that IPE quotations are a good representation for natural gas prices in Norway. No rational participant would commit to a contract on other terms than those reflected in the market price. As consequence, it is assumed that using the forward curve derived from the IPE quotations will be a good approximation for a TOP contract. Since only relatively short term financial contracts are listed at IPE, it would have been favorable to include long term OTC contracts in the estimation of the forward curve since TOP contracts are long term commitments. Unfortunately only one such contract was available to the authors.

## 4.4 Spark spread

Spark spread is defined as the price of 1 MWh electricity less the cost of fuel needed to make 1 MWh of electricity as given in equation (4.1).

$$\text{Spark Spread} = P_{\text{el}} - P_{\text{gas, el equivalent}} = P_{\text{el}} - HP_{\text{gas}} \tag{4.1}$$

$P_{\text{el}}$ - price of electricity [NOK/MWh$_{\text{el}}$]

$P_{gas}$ - price of natural gas [NOK/MWh$_{\text{gas}}$]

$H$ - heat rate [MWh$_{\text{gas}}$/MWh$_{\text{el}}$] as defined in chapter 5.

Spark spread contracts representative for the CCGT valuation are not traded at energy exchanges. Spark spread financial contracts are traded OTC in liberalized energy markets with high thermal electricity production. Spark spread contracts between coal, oil or natural gas and various financial electricity contracts are available. The spark spread contracts are offered by e.g. RWE Trading which states that the spark spread contracts are mainly of financial art. For the valuation in this thesis, spark spread contracts are constructed from the gas and electricity data set according to equation (4.1). A detailed description is given in chapter 9.7.

## 4.5 The efficient market

Throughout the main part of the thesis it is assumed that the Nord Pool electricity market and the market for natural gas at the IPE are efficient, and that all information is reflected in the market price. This includes that the market's belief of new generation capacity and the future cost of $CO_2$ emission is embedded in the forward prices. In general this means that the effect of competition to invest in new generation capacity as described in detail in chapter 12 does not have to be taken into consideration as the market price reflects the probability of new investments. The forward prices will at any point in time reflect the prevailing market expectations. If e.g. the market's expectations with regard to new generation capacity change, the forward prices will change accordingly. Further the models do not need to incorporate increased electricity prices due to increased $CO_2$ emission costs.

# 5 Heat rate and conversion factors

This paragraph describes how the cost of producing 1 MWh of electricity can be calculated from the prices at the IPE using these as proxy for future gas prices in Norway. This requires some conversion factors and subtraction of the transport cost from the North Sea to the UK. As stated in chapter 4, electricity is measured in NOK/MWh and natural gas at IPE in £/therm. Since this thesis considers a power plant built in Norway it is most convenient to convert the price of natural gas at IPE into NOK/MWh produced electricity. The actual exchange rate at the day of observation was used. Below these matters are outlined. Table 2 shows the relevant conversion factors together with other convenient conversion factors.

### Transportation costs

The cost of transporting natural gas from the North Sea to the UK will not be relevant in when landing natural gas in Norway. This cost must be subtracted from the price quoted at IPE. The transportation cost, c, is usually in NOK/$Sm^3$, so a conversion factor, $\vartheta$, between $Sm^3$ and thermal units is needed.

### Heat rate

A CCGT requires a certain amount of natural gas to produce one unit of electricity. This is governed by the efficiency, $\eta$, of the plant. In general heat rate, $H$, is defined as the inverse of the efficiency. However, if the units in which electricity and natural gas are measured are different, then the heat rate should be adjusted to incorporate the conversion of units. The efficiency is discussed in chapter 6 and Appendix I. In this thesis natural gas measured in thermal units must be converted to MWh in which electricity in Norway is measured using the conversion factor $\phi$.

### Conversion from £/therm to NOK/$MWh_{el}$

By using the conversion factors and transportation cost as mentioned above and in addition the exchange rate, $x$ for NOK/£ the equation to find the fuel cost of producing electricity from natural gas, $P_{\text{gas, el eqv}}$, in Norway can be written as

$$P_{gas,eleqv} = (P_{IPE}x - c\vartheta)H$$

where

$$H = \frac{1}{\eta}\frac{1000}{\theta}$$

$P_{\text{gas, el eqv}}$ – [NOK/$MWh_{el}$]

$P_{IPE}$ – price quoted at IPE [£/therm]

H – [therm/$MWh_{el}$]

In the preceding chapter the price of natural gas is always given as the cost of producing 1 MWh electricity, $P_{\text{gas, el eqv}}$.

**Table 2**

**Conversion factors and transportation cost**

| Conversion | symbol | value | units |
|---|---|---|---|
| Transportation cost | c | 0.15 | NOK/ $Sm^3$ |
| $Sm^3$ to therm | $\vartheta$ | 2.64 | $Sm^3$/therm |
| efficiency | $\eta$ | 0.58 | $MWh_{el}/MWh_{gas}$ |
| therm to MWh | $\phi$ | 29.3071 | kWh/therm |
| | | | |
| $Sm^3$ to $MWh_{el}$ | | 155 | $Sm^3/MWh_{el}$ |
| $Sm^3$ to MJ | | 40 | MJ / $Sm^3$ |
| therm to MJ | | 105,5 | MJ /therm |

The table contains conversion factors for converting natural gas prices at the IPE to NOK/$MWh_{el}$. Some additional conversion factors are also added.

# 6 The power plant

This chapter provides a brief description of the gas-fired power plant and presents the technical input values to be used in the Real Options analysis. A more detailed description together with a discussion of some of the most important issues of plant and process design is given in Appendix I.

## 6.1 Assumptions

The following assumptions are made based on what is considered to be a likely scenario with respect to operation strategy and location of a gas fired power plant in Norway. The scenario is "based on" Statoil's project of building an 800MW gas-fired power plant at Tjeldbergodden.[34]

*Assumption 1: Operating strategy*

The plant will be producing electricity only and operate as a base load plant. Using large Combined Cycle Gas Turbines (CCGT) will be the best choice to match the operating strategy.

*Assumption 2: Location*

The plant will be located close to existing gas processing facilities in the costal region of western Norway.

The type and size of the gas turbines installed will determine the plant's part load efficiency and ramping time. The gas turbines are the most expensive components of the plant components, and it is important that the technology used suits the intended production in a cost efficient way.

The location is important for two main reasons. Ambient conditions are affecting the thermal efficiency of the process. Relatively cold climate with small temperature variations as well as access to a large and stable source of cooling are desirable[35]. Minimizing of the infrastructure

[34] Hegerberg (2003)

[35] Appendix I

investments with respect to both gas transport and electricity transmission is also a key objective.

The costal climate at Tjeldbergodden provides favorable ambient conditions and the direct access to stable cold costal water provides an excellent source of cooling. Tjeldbergodden is already connected to the main gas pipeline system through the pipeline Haltenpipe and is located relativity close to new large customers of electricity, minimizing the need for investment into new infrastructure.

## 6.2 Input parameters used for Real Option analysis

The input values of investment, operating- and maintenance and emission costs are considered. A more detailed cost allocation is given in Appendix I.

### 6.2.1 Investment costs

The total investment cost is set to 3000 MNOK.[36] This figure does not include large investment in gas or electricity infrastructure. The existing gas pipeline to Tjeldbergodden (Haltenpipe) has enough excess capacity to supply the power plant.[37] Investment of around 300 MNOK is needed in a new power transmission. This is omitted and considered to be a separate project financed by e.g. transmission fees. Standard assumptions of 30 year lifetime and a three year construction period are assumed.

### 6.2.2 Operating costs and cost of emissions

Operating and maintenance cost can be set to 2%, and insurance to 0.5 % of planned investment cost per year.[38] The plant will be subject to restrictions on $NO_X$- emission. This requires the use of e.g. a dry-low $NO_X$ turbine. $NO_X$ reducing measures will add an operating cost of 14 NOK per MWh produced.[39]

Uncertainty concerning taxes and restrictions on $CO_2$-emissions makes it difficult to provide accurate forecasts for the price on $CO_2$ emissions. The technology on $CO_2$-seqrustration is

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[36] Hegerberg (2003)

[37] Arnstad 2003

[38] Bolland (2003)

[39] Bolland (2003)

associated with higher cost pr MWh produced than the expected quota-price on $CO_2$.[40] $CO_2$-seqrustration will not be considered further. Cost of $CO_2$ emission is included by adding the expected quota price as an operating cost. The cost of $CO_2$ emissions is further discussed in chapter 7.

### 6.2.3 Efficiency and availability

The base load plant is assumed to produce full load when operational, setting operating hours to the technical maximum. The plant is assumed to have an availability factor of 90%,[41] equivalent to 7900 operating hours. As full load operation is assumed at all time, the equivalent operating hours will be equal to operating hours.

Degradation and fouling will lead to reversible and irreversible decrease in output and efficiency and are incorporated by reducing equivalent operating hours. The equivalent operation hours are reduced by additional 2% for fouling and 2% for degradation, resulting in a total of 7600 (rounded) equivalent operating hours to be used in the valuation[42].

A standard CCGT plant with F-series technology can reach up to 58 % thermodynamic efficiency by Norwegian conditions. The efficiency, μ, is as defined as the electrical output divided by the energy used for its production (6.1).

$$\mu = \frac{electricity, produced}{energy, gas} \tag{6.1}$$

### 6.2.4 Input values – base case

Table 3 summarizes the values that will be the input parameters to the models concerning the power plant. The plant will produce a yearly average of 6.06 TWh during the production period under the assumptions made.

[40] Lont (2003)

[41] Appendix I

[42] Appendix I

**Table 3**

**Input parameters**

| Cost | Value | Unit |
|---|---|---|
| Investment cost | 3000 | MNOK |
| Project life time | 30 | Year |
| Construction period | 3 | Year |
| O & M | 60 | MNOK/year |
| Insurance | 15 | MNOK/year |
| $NO_X$ | 14 | NOK/MWh |
| $CO_2$ | 20 | NOK/MWh |
| Eqv operating hours | 7600 | hours/year |
| Plant efficiency | 58 | % |
| Risk free interest rate | 6 | % annually |

Input parameters – Base case

The yearly production cost less fuel cost for the CCGT plant is given in Table 4. The cost per MWh is given for a yearly production of 6.06 TWh. The production costs include annualized investment cost, operation and maintenance cost and insurance. The variable costs include the cost of $NO_x$ and the base case cost of $CO_2$. The fuel cost of natural gas is not included as it is considered to be a stochastic variable. The production cost less fuel cost provides a good measure of the spark spread level needed for profitable operations. Table 4 shows that an average spark spread of 118 NOK/MWh is need for profitable operations with emission costs of 20 EUR/ton $CO_2$ and 62 NOK/MWh with zero emission costs. The implication of income taxes is discussed in chapter 11.1.

The risk free interest rate is set to the interest rate of government bond with 30 years to maturity, equal to the lifetime of the plant. 30 year government bond are not offered in Norway. To find the interest rate, the yield curve from the date of evaluation is constructed, and a 30 year interest rate is extrapolated. The interest rate is found to be about 6% annually or 5.8% continuously compounded.

**Table 4**

**Production cost less natural gas cost**

| Cost | NOK per year | NOK per MWh |
|---|---|---|
| Investment (annuity) | 213 925 081 | 35 |
| O & M | 60 000 000 | 10 |
| Insurance | 15 000 000 | 2 |
| $NO_X$ | 85 120 000 | 14 |
| $CO_2$ | 340 348 000 | 56 |
| | | |
| Sum fixed costs | 288 925 081 | 48 |
| Sum variable costs | 425 600 000 | 70 |
| Total costs | 714 525 081 | 118 |
| Total costs without $CO_2$ | 374 045 081 | 62 |

The table shows the annual costs less fuel cost for owning a gas fired base load CCGT producing 800 MW 7600 hours per year. All figures are in NOK

# 7 $CO_2$ handling and emission costs

The uncertainty about future handling of $CO_2$ regarding taxes and restrictions on emissions, $CO_2$-quotas and green certificates have made investors reluctant to initiate plan for gas fired power plants. The development in $CO_2$ prices will affect the price level of both electricity and natural gas as well as the electricity production structure of entire Europe.

## 7.1 The Kyoto protocol and trading mechanism

The Kyoto Protocol of 1997[43] is an international agreement intended to reduce the greenhouse gas emissions of developed economies, during the years from 2008 to 2012 by at least 5 percent from their aggregate 1990 levels. Norway is set to reduce its emissions by 1% and the EU by 8%. Both the EU and Norway have stated they will meet their Kyoto obligations regardless of the protocol's legal status. Norway is up for a hard struggle to fulfil the country's obligations as the $CO_2$ emissions by 2003 had increased by 23 percent since 1990[44].

The Norwegian government has been vague on signalling how the restriction on $CO_2$ emissions as imposed by the Kyoto agreement is going to be implemented, but it is likely to follow the system of the EU. The EU- commission has proposed a so called cap-and-trade-scheme. The scheme sets annual limits on the aggregate amount of greenhouse gases and will be mandatory for large emitters including power plants above 20 MW. Large parts of the national quotas are initially given free of charge to the industries and are subject to trade. A regulatory authority establishes the cap which is considerably lower than the historic level of emissions. If the quotas are exceeded additional ones will have to be bought on the market or a penalty has to be paid.[45]

A pilot market will be introduced from 2005-07 and regular market will be in place for the Kyoto period of 2008-2012. During the pilot period the industrial quotas are only allowed to be traded domestically and national quotas traded between the EU member countries. The quotas are free of charge and the cost for exceeding quotas is 40 EUR/ton during the pilot period increasing to 100 EUR/ton when the regular market comes into effect. The regular

[43] United Nations (1998)

[44] SFT (2004)

[45] Platts (2004) (1)

market allows for all trading mechanisms described in the Kyoto protocol. 90% the quotas are free of charge and can be traded at an international level.

## 7.2 $CO_2$ trade and price forecasts

As the trading scheme is not yet in effect, $CO_2$ is currently only traded OTC in Europe.
$CO_2$ quotas are already traded at the Chicago Climate Exchange but the market had by first quarter of 2004 low volume liquidity as shown by Figure 5. There has been some trading in European OTC market for 2005, 2006 and 2007, but volumes remain relatively small and the market is not liquid. According to market markers, 2005 quotas are priced at around 7 EUR/ton, 2006 at 7-8 EUR/ton and 2007 slightly above 2006 (May 2004)[46]. As market data remains scarce, most $CO_2$ predictions are based on scenario forecasts.

**$CO_2$ trade at Chicago Climate Exchange**

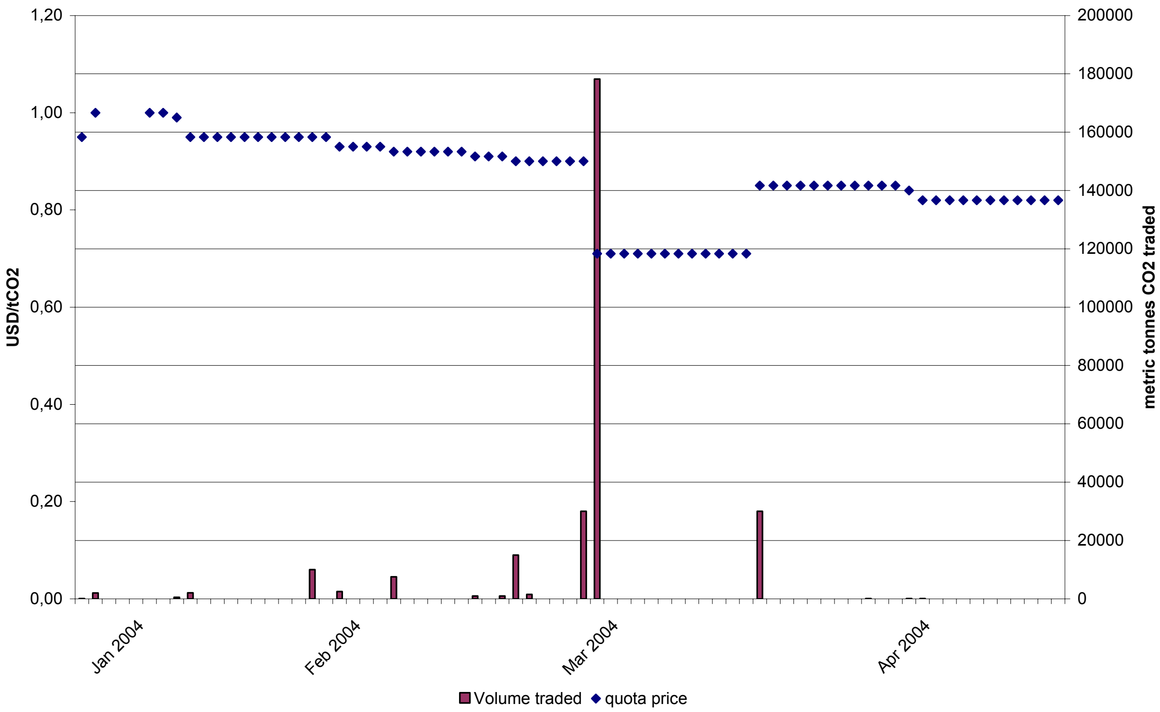



**Figure 5**: The figure shows prices and volumes for traded $CO_2$ quotas for the 2005 vintage at Chicago Climate Exchange. The period on the graph is from 12.12.2003 to 15.04.2004.

The forecasts on $CO_2$-quoutas vary greatly. Eckert (2003) estimates price in 2010 to be around 10 EUR/ ton, increasing towards 20 EUR in 2020[47]. The estimates are coherent with

[46] Platts (2004) (2)

[47] Eckert (2003)

the RWE Powers assumption of a price somewhere in the range of 5-20 EUR/MT.[48] The long term price will be determined by development in supply and demand and accurate estimates can hardly be predicted. Point Carbon analysis suggests that carbon prices may fall in the range 2-20 EUR per ton $CO_2$ in the period 2005-2007, depending on the overall reduction target, with a 'best estimate' of about 7 EUR/ton in 2005.[49]

Leyva and Lekander (2004) states that most studies forecast a price between 15 and 30 EUR/ton and expects the prise to settle at around 25 EUR/ton in 2008 after some initial volatility. 25 EUR/ton is referred to as a switching point as they argue higher prices makes it profitable to reduce emission by switching from coal to gas fired electricity production rather than buying quotas. Renewable energy sources and carbon sequestration will not have a large contributionon in reaching the emission targets during Kyoto period ending in 2012.

The $CO_2$ quota price level within the EU will be greatly affected by the size of $CO_2$ quotas available from the new east European members. A determining factor is how the 1990 emission levels are calculated and when the new members are included into the EU scheme. The actual size of the 1990 emission level is yet to be determined. A high quantity can pull the bottom out of the quota market which might also be the case if Russia joins the market at a later stage.

## 7.3 Implications on the electricity price

The added cost of $CO_2$ emissions is expected to have a large impact on electricity prices in Europe as it increases the marginal cost of electricity production from fossil fuelled power plants. The increase will be most dramatic for coal plants having the largest $CO_2$emission per MWh produced.

According to market analysts a quota price of 12 EUR/ton will lead to an increase in the Nord pool prices of approximately 75 NOK/MWh. Market analysts argue that the full cost of $CO_2$ is not reflected in prices of the Nord Pool forward contracts. At a European level, market analysts suggest that a quota price on 25 EUR/ton would lead to a 40% increase in European electricity prices. A 30% increase is caused by the direct effect of $CO_2$ cost an additional 10%

[48] Bergli (2003)

[49] Point Carbon (2004)

increase is believed come as an indirect effect as extensive fuel switching will increase the price of gas by 15%.[50] It can be questioned to which extent forward prices on electricity and natural gas reflect the future cost of $CO_2$ emissions. According to the assumption made in chapter 4.5 future expectations of $CO_2$ are reflected in the future prices.

## 7.4 Emission costs for the CCGT

The cost of $CO_2$ emission is set to 20 EUR/ton in the calculations. As discussed in the previous chapter, the estimates on emission cost are within a wide range. 20 EUR/ton is close to the high end estimates, and is used to give a conservative project valuation.

Quotas on 20 EUR/ton $CO_2$ result in an additional production cost of 56 NOK/MWh. Green certificates that will promote renewable energy are estimated to induce a cost equivalent to that of $CO_2$-qoutas[51]. A gas fired power plant will need a permit on $CO_2$ emissions from the Norwegian Pollution Control Authority (SFT) to commence operation. Permits are currently given to all projects with a granted licence Table 1. The models are designed to include the cost of $CO_2$ emission by adding cost of $CO_2$ per MWh produced. In a CCGT with 58% efficiency about 0.35 ton $CO_2$ is produced per MWh resulting in the conversion factor 0.35 ton $CO_2$/MWh.

[50] Levy et al. (2004)

[51] Lont (2003)

# 8 A general introduction to futures and forward curves

Modelling the futures and forward market is important for many applications including production hedging and valuation of financial and real assets. As most modellers recognize stochastic behaviour in commodity prices recent models incorporate this feature. The two common but different approaches to model the forward curve under uncertainty are presented in the chapters below and are used in the models built for valuing the investment option.

## 8.1 Theoretic introduction

In brief one method assumes that a set of state variables describes the current state of the market and that the forward curve is given by the processes these state variables are assumed to follow. This is referred to as a *spot price model* and abbreviated *SPM*. A second approach is to model the evolution of the entire forward curve given the current initial forward curve by assuming that various market factors affect the forward curve in a specific manner. Models built under this assumption are denoted *forward curve models* and abbreviated *FCM*.

Some authors have taken these two model approaches further and shown the connection between the two or used them together. Such papers are written by Miltersen and Schwartz (1998) and Manoliu and Thompaidis (2000). This thesis does not go that far but keeps the models separated through the discussion. The use of forward curve models in valuing real options is rarely seen. In the preceding chapters the SPM must be considered as the main model and is most thoroughly discussed.

## 8.2 Tradable and spanning assets

Electricity and natural gas are either very expensive or often impossible to store, making the spot price highly dependent on instantaneous supply and demand. Thus one unit of electricity or natural gas at present is not the same as one unit at a later point in time. It can be said to be two different commodities[52]. As a consequence, the commodity electricity or natural gas cannot be traded in a bulk market and arbitrage arguments involving storage, such as the cash-and-carry argument for the relationship between futures and spot prices, are largely invalid. Futures and forward contracts on these commodities are, however, tradable and their prices

[52] Lucia & Schwartz 2002

viewed as stochastic processes are martingales under the risk neutral probability measure[53]. Futures close to maturity which show much the same structure as the spot prices are used as a proxy spot price.

As will be show in chapter 9 the proxy spot price is made up of a short and long term state variable which are not traded or even observable in the market. It is, however, sufficient that other assets whose risk track or span the uncertainty in the state variables are traded. A "spanning asset" is used to replicate the non tradable asset by tracking its risk[54]. This follows the assumption of complete markets which says that a complete market is one in which the complete set of possible gambles on future states-of-the-world can be constructed with existing assets[55]. This implies that every contingent claim can be replicated by a trading strategy, and therefore it can be priced under risk neutral probability measure.

Under this framework it can be assumed that all the traded futures and forwards at Nord Pool and IPE spans the risk in the short term and long term variables of electricity and natural gas. Further it can be anticipated that the futures close to maturity spans the risk in the short term factor, where as the futures and forward with long delivery periods spans the risk in the long term factor. Under such assumptions contingent claims written on the commodities can be replicated. Whether these assumptions are valid can be discussed. When modelling the entire forward curve as in the forward curve models the same assumptions can be made; all the traded futures and forwards spans the risk in the 30 year long forward curve.

Futures prices close to maturity are ruled by uncertainty factors as weather and demand in the near future, whereas futures and forward prices for longer periods are more influenced by expectation of future demand, production and transmission capacity and long run production costs. For the problem of valuing an investment it would from this perspective be reasonable to believe that the uncertainty appearing in contracts for longer periods and thus also the long term state variables tracks much of the risk in the investment. Political uncertainty about future $CO_2$ handling or exchanges rate which will influence the investment cost can be reflected in the expectation of future production capacity. For instance many investors in

[53] Manoliu and Tompaidis 2000

[54] Dixit et al 1994

[55] About.com

Norway does not invest in a CCGT because future cost of handling $CO_2$ are very uncertain something which leads to lack of future production capacity. Similar arguments can be made for the transmission capacity to continental Europe. An increase in this capacity could make it less profitable to own production capacity. In chapter 13 the issue of whether there actually are contracts to be used for hedging is discussed further in debt.

# 9 Spot price models

This chapter discusses models of spot commodity prices based on stochastic differential equations. First there is a general presentation of important models from literature where advantages and suitability are discussed. The second part of this chapter presents the models chosen for the calculations, procedures for estimating parameters, estimated parameters and the valuation methods.

## 9.1 Literature

There are several contributors of commodity models. Those mentioned here are models presented by Gibson, Schwartz, Brennan, Smith, and Deng. Many of these models are similar, but the small differences are important. In this chapter the equations are presented as they were in the original papers. The purpose of this chapter is to show the evolution of models and how various models fit different commodities and situations.

### 9.1.1 Gibson-Schwartz

Brennan and Schwartz (1985) presents a model where natural resource spot prices follow a geometric Brownian motion. It is on the form:

$$\frac{dS}{S} = (\mu - \delta)dt + \sigma dZ$$

where $\delta$ is the convenience yield (see Brennan et al (1985) for definition) assumed to be constant. In this paper the model is used to evaluate investments in natural resources.

Research on commodity prices reveals that they are most likely mean reverting, and do not follow a GBM. This is pointed out by Gibson-Schwartz (1990) when they extend the Brennan Schwartz model and add a stochastic mean reverting convenience yield to better describe the spot price dynamics of commodities. The model of the commodity spot price is then given by:

$$\frac{dS}{S} = (\mu - \delta)dt + \sigma_1 dZ_2$$
$$d\delta = \kappa(\alpha - \delta)dt + \sigma_2 dZ_2$$
$$dZ_1 dZ_2 = \rho dt$$

The aim of this paper is to find a model that prices contingent claims on oil. Gibson and Schwartz find this model to fit crude oil futures traded at the NYMEX.

### 9.1.2 Schwartz (1997/98)

Schwartz (1997) introduces three models for the commodity spot prices. The three models are:

- a one factor mean revering model:

$$\frac{dS}{S} = \kappa(\mu - \ln S)dt + \sigma dZ$$

- a two factor model is the Gibson Schwartz model stated above and
- a three factor model which is a combination of the Gibson Schwartz model and Vasicek's interest rate model:

$$\frac{dS}{S} = (r - \delta)dt + \sigma_1 dZ_1^*$$
$$d\delta = \kappa(\hat{\alpha} - \delta)dt + \sigma_2 dZ_2^*$$
$$dr = \alpha\left(m^* - r\right)dt + \sigma_3 dZ_3^*$$
$$dZ_1^* dZ_2^* = \rho_{12} dt, dZ_1^* dZ_3^* = \rho_{13} dt, dZ_2^* dZ_3^* = \rho_{23} dt$$

These three models are tested on the commodities crude oil, copper and gold. The general conclusion is that there is not much gain in using a three factor model. But the two factor model performs far better than the one factor model. In this paper Schwartz introduce the Kalman filter to estimate the parameters and the unobservable state variable $\delta$.

In Schwartz (1998) the two factor model stated above is converted into a one factor long term model. The long term model and the two factor model give the same forward price when time to maturity is more than three years. The parameters in this model are estimated using the two factor model. The main advantage with this model is that it simplifies calculations of long term complex investments with multiple options values, and keeping most of the properties from the two factor model.

### 9.1.3 Schwartz and Smith (2000)/Lucia and Schwartz (2002)/Manoliu and Thompaidis (2000)

Schwartz and Smith (2000) presents a two factor model where one factor is a short term mean reverting factor which aims to model the short term fluctuations in the spot price. The other factor is a long term equilibrium factor which will capture the long term dynamics of the prices. The model is

$$\ln S = \chi + \xi$$
$$d\xi = \mu dt + \sigma_\xi dZ_\xi$$
$$d\chi = -\kappa \chi dt + \sigma_\chi dZ_\chi$$
$$dZ_\xi dZ_\chi = \rho dt$$

Schwartz et al (2000) show that this model is a reparameterisation of the Gibson Schwartz model above.

In this *long- term-short- term-model* the long term variable is the important one when time to maturity increases. Thus when valuing long term investments only the long term variable needs to be considered. This and the fact that the two state variables are a linear combination gives this model an advantage over the Gibson Schwartz model. Schwartz et al (2000) test this model on crude oil prices. In this paper an extensive explanation of the procedure to estimate parameters by using the Kalman filter is given.

Lucia et al (2002) investigates how a one factor and a two factor model, both with deterministic seasonal terms describe the forward curve dynamics at Nord Pool. In the one factor model the spot price process is assumed to follow seasonal function with a mean reverting component:

$$\ln S = f(t) + \chi$$
$$d\chi = -\kappa \chi dt + \sigma_\chi dZ_\chi$$

The two factor model is the long-term-short-term-model stated above added a the function $f(t)$. The function $f(t)$ is either a discrete one or cosine function:

$$f(t) = \alpha + \beta D_t + \gamma \cos\left( (t-\tau)\frac{2\pi}{365} \right)$$
$$D_t = \begin{cases} 1 \text{ if date t is weekend or holiday} \\ 0 \text{ otherwise} \end{cases}$$

The motivation for adding the seasonal component is that inspection of spot and forward prices at Nord Pool show significant seasonal pattern. Manoliu and Thompaidis (2003) use almost the same models as Lucia et al (2002) to model natural gas prices at the Henry Hub. The seasonal term is added since the natural gas futures exhibit seasonal patterns.

### 9.1.4 Deng

Deng has written several papers on valuation of generation assets. In the papers the models investigated are among geometric Brownian motion and mean reversion similar to those mentioned above. But Deng also adds another interesting feature, a jump process. Electricity prices in the United States and especially California occasionally show huge jumps. Deng shows that this should be considered when valuing peak load generation assets in particular.

### 9.1.5 Summary of stochastic processes

Summarizing the discoveries above one can see that there exist several models for modelling commodity prices. Common for the models that perform the best are those which incorporate mean reversion. Models have been tested on crude oil, copper, gold, natural gas and electricity. Common for the three first commodities are that they are not seasonal dependant most probably due to their storability. Investigating spot and future prices for these commodities does not reveal any specific seasonal pattern. In line with this none of the models described above which are tested on these commodities have any extra term like a seasonal or jump term. Models for electricity and natural gas which exhibit seasonal or jumpy patterns have specific terms to take care of such dynamics. Based on this it seems crucial to use a model that fit the type of data to be modelled.

## 9.2 Choice of models for valuation

To perform the investment valuation of a gas fired power plant one approach is to use models where prices are modelled as stochastic processes as those mentioned above. By doing so the uncertainty in the prices is considered. An investment in such a plant depends on either the price of electricity and natural gas or the spark spread. This means that one can model the spark spread directly or can one model electricity and natural gas explicitly.

The investment under consideration is in a long term perspective. Based on this either the long term model from Schwartz (1998) or the long term short term model from Schwartz et al

(2000) seems appropriate. As stated above the long term short term model has some advantages over the long term model. Since this thesis is dealing with electricity and natural gas prices it seems reasonable to consider seasonal components since both prices show such characteristics.

Based on this it seems reasonable to assume that the price dynamics for natural gas and electricity can be modelled with the general model:

$$
\begin{aligned}
&\ln S = f(t) + \chi + \xi \\
&d\xi = \mu dt + \sigma_\xi dZ_\xi \\
&d\chi = -\kappa\chi dt + \sigma_\chi dZ_\chi \\
&dZ_\xi dZ_\chi = \rho dt \\
&f(t) = \overline{S} + \gamma \cos\big((t-\tau)2\pi\big)
\end{aligned}
$$

and spark spread with the model:

$$
\begin{aligned}
&S = f(t) + \chi + \xi \\
&d\xi = \mu dt + \sigma_\xi dZ_\xi \\
&d\chi = -\kappa\chi dt + \sigma_\chi dZ_\chi \\
&dZ_\xi dZ_\chi = \rho dt \\
&f(t) = \overline{S} + \gamma \cos\big((t-\tau)2\pi\big)
\end{aligned}
$$

The two models are basically the same. The only difference is that electricity and natural gas prices can never become negative and thus the log price must follow the given processes. Spark spread however can easily become negative and the model must allow this. In both models the mean reverting component is meant to capture short term deviations from the long term equilibrium component following Brownian motion. The seasonal term aims to absorb seasonal variations in the spot and future prices.

## 9.3 Modelling frame work – Spot price model

This chapter deduces the mathematical models used for calculations in this thesis under the SPM paradigm. The models used are those selected in chapter 9.2.

## Nomenclature – Spot price model

The nomenclature presented below is only for use in models deducted in chapter 9.

$\Theta$ – vector of state variables

$X$ – short term state variable

$\varepsilon$ – long term state variable

$V_t(\Theta)$ – value of the project

$F_{t,T} = F_{t,T}(\Theta_t, T)$ – Forward price in per MWh produced

$S$ – sparks spread

$P_{el}$ – Electricity price

$P_{gas}$ – Natural gas price

$t$ – time of calculation

$r$ – risk free interest rate

$T$ – maturity of a forward contract

$t_u$ – expected time of building the plant

$L$ – expected life time of project in years

$K$ – capacity of the plant in MW

$E$ – equivalent operating hours per year

$R$ – cost of removing $NO_x$, NOK/MWh

$Q$ – cost of $CO_2$, NOK/MWh

$D$ – sum of operation and maintenance cost per year, NOK/year

$G$ – sum of insurances cost per year, NOK/year

$I$ – sunk investment cost

$\tau$ – dummy variable used in integration

$\mu$ – risk neutral drift rate of long term state variable

$\kappa$ – mean reversion rate of short term state variable

$\sigma$ – volatility. Subscript denotes of which variable

$\rho$ – correlation. Subscripts denotes between which variables

$\gamma$ – amplitude of seasonal term

$\eta$ – periodic dislocation

$\upsilon$ – half life

### 9.3.1 Project value

The assumption of the base load plant always producing maximum load made in chapter 6 exclude any operational option. The value of the project once an investment is determined is just the present value of the cash flow generated by the plant discounted at the risk free interest rate since all cash flows are considered to be certain due to the forward curve in (9.1). Thus the general formula for the value of the power plant is given by (9.1).

$$V(\Theta) = \int_{t+t_u}^{t+t_u+L} e^{-r(\tau-t)} \left[KE\left(F_{t,\tau}(\Theta_t,\tau) - R - Q(\tau)\right) - D - G\right] d\tau \tag{9.1}$$

When the decision to invest is made it is considered that a sunken investment cost is paid. Thus the traditional net present value (NPV) of the power plant is given by (9.2). According to real option theory, an investment is not made before the traditional NPV is greater than the value of the option as mentioned in chapter 2.

$$NPV = V(\Theta) - I \tag{9.2}$$

### 9.3.2 Spark spread spot price model – SPM1

The spark spread is modelled by a two factor model given by equation (9.3). In this model it is assumed that the risk free proxy price of the real spot prices, i.e. the futures with shortest time to maturity is governed by the five equations (9.3a)-(9.3e). The price (9.3a) is made up of a time-varying deterministic component (9.3e), a short-term mean reverting component (9.3b) and a long-term equilibrium price level following an arithmetic Brownian motion (9.3c). The Wiener processes are correlated through equation (9.3d).

$$S = f(t) + X + \varepsilon \tag{9.3a}$$

$$d\varepsilon = \mu dt + \sigma_\varepsilon dZ_\varepsilon \tag{9.3b}$$

$$dX = -\kappa X dt + \sigma_X dZ_X \tag{9.3c}$$

where

$$dZ_\varepsilon dZ_X = \rho dt \tag{9.3d}$$

$$f(t) = \overline{S} + \gamma \cos\left((t-\eta)2\pi\right) \tag{9.3e}$$

Schwartz and Smith show that given $X_0$ and $\varepsilon_0$, $X$ and $\varepsilon$ are jointly normal distributed with mean

$$\mathrm{E}^*[(X_t, \varepsilon_t)] = [X_0 e^{-\kappa t}, \varepsilon_0 + \mu t] \tag{9.4a}$$

and covariance matrix

$$\mathrm{Cov}^*[(X_t, \varepsilon_t)] = \begin{bmatrix} \left(1 - e^{-2\kappa t}\right)\frac{\sigma_X^2}{2\kappa} & \left(1 - e^{-\kappa t}\right)\frac{\rho \sigma_\varepsilon \sigma_X}{\kappa} \\ \left(1 - e^{-\kappa t}\right)\frac{\rho \sigma_\varepsilon \sigma_X}{\kappa} & \sigma_\varepsilon^2 t \end{bmatrix} \tag{9.4b}$$

The asterisk, * denotes the risk neutral probability measure which is further discussed in 10.2.

Since the spark spread price is just a linear combination of the state variable the expected spot price under the risk neutral probability measure is just the sum of two means and the deterministic seasonal term.

$$\mathrm{E}^*[S_t] = \left[f(t) + X_0 e^{-\kappa t} + \varepsilon_0 + \mu t\right] \tag{9.6a}$$

and the variance

$$\mathrm{Var}^*[S_t] = \left(1 - e^{-2\kappa t}\right)\frac{\sigma_X^2}{2\kappa} + \sigma_\varepsilon^2 t + 2\left(1 - e^{-\kappa t}\right)\frac{\rho \sigma_\varepsilon \sigma_X}{\kappa} \tag{9.6b}$$

Since the forward price equals the expected spot price under the risk neutral probability measure the forward price for model 1 is given by (9.7).

$$F_{t,T}(S_t, T) = f(t) + e^{-\kappa(T-t)} X_t + \varepsilon_t + A(T-t)$$

where (9.7)

$$A(T-t) = \mu(T-t)$$

The value of the power plant is found by inserting equation (9.7) into equation (9.1) and integrating. The result is not stated here but can easily be found by using algebraic software like Maple.

Half life, denoted $\upsilon$, is the time it takes for the short term variable to return halfway back to the mean reverting level after a deviation. $\upsilon$ is defined as $\upsilon = \frac{\ln 2}{\kappa}$. This can be deducted as follows:

$E(dX) = -\kappa X_t dt$ where $E$ denotes the expectation. This can be integrated:

$$\int_{S_0}^{S_1} \frac{dX}{(-X_t)} = \int_{t_0}^{t_1} \kappa dt \rightarrow \ln\left(\frac{X_0}{X_1}\right) = \kappa(t_1 - t_0)$$ Using that $(t_1 - t_0) = \upsilon$, and that

$\frac{X_0}{X_1} = 2$ by definition gives $\upsilon = \frac{\ln 2}{\kappa}$.

### Valuing the option

Under the assumption that the there exist assets in the energy market that spans the risk in the underlying state variable as described in chapter 8.2, then a risk neutral portfolio, $\pi$ ,can be constructed of the option, W, and $\Delta_i$ units of the two underlying $(\mathrm{i} = X \text{ and } \varepsilon)$ as in equation (9.8a). By using Ito's lemma, setting the $\Delta_\varepsilon = -\frac{\partial W}{\partial \varepsilon}$ and $\Delta_X = -\frac{\partial W}{\partial X}$ and demanding that a risk neutral portfolio must earn the risk free return the differential equation for the option value is given by (9.8b). For a general deduction of this see Dixit et al (1994). The option to invest can be valued using least squares Monte Carlo simulation (LSM). Monte Carlo simulation is described in chapter 9.10.

If $\kappa$ is relatively large compared to the construction time (assuming cash flow commence after the construction time) the short term component is irrelevant as stated in chapter 9.1. Under such conditions the model becomes mathematically very tractable since one only need to consider the long term factor and analytical solutions exist for the value of infinite options. With only one variable in the model the NPV of the plant can be written as $\mathrm{NPV} = C_1\varepsilon + C_2 - I$ where $C_1$ and $C_2$ are constants. The differential equation for the infinite option is then given in (9.8c) with boundary conditions (9.8d) and solution (9.8e). $\varepsilon^*$ denotes the trigger lever, that is, the level of $\varepsilon$ where investment is feasible.

$$\pi = W + \Delta_X X + \Delta_\varepsilon \varepsilon \tag{9.8a}$$

$$\frac{\partial W}{\partial t} + \frac{\partial^2 W}{\partial \varepsilon^2}\sigma_\varepsilon^2 + \frac{\partial^2 W}{\partial X^2}\sigma_X^2 + \frac{\partial^2 W}{\partial X \partial \varepsilon}\rho\sigma_\varepsilon\sigma_X + \frac{\partial W}{\partial \varepsilon}\mu + \frac{\partial W}{\partial X}(-\kappa X) - rW = 0 \tag{9.8b}$$

$$\frac{\partial^2 W}{\partial \varepsilon^2}\sigma_\varepsilon^2 + \frac{\partial W}{\partial \varepsilon}\mu - rW = 0 \tag{9.8c}$$

$$\begin{aligned} &W(\varepsilon^*) = \mathrm{NPV}(\varepsilon^*) \\ &W'(\varepsilon^*) = \mathrm{NPV}'(\varepsilon^*) \\ &W(-\infty) = 0 \end{aligned} \tag{9.8d}$$

$$\begin{aligned} &W(\varepsilon) = \frac{C_1}{\beta_1} e^{\beta_1(\varepsilon - \varepsilon^*)} \\ &\beta_{1,2} = \frac{-\mu \pm \sqrt{(\mu)^2 + 2\sigma^2 r}}{\sigma^2}, \beta_1 > 0, \beta_2 < 0, \\ &\varepsilon^* = \frac{C_1 - C_2\beta_1 + I\beta_1}{C_1\beta_1} \end{aligned} \tag{9.8e}$$

### 9.3.3 Electricity and natural gas spot price model – SPM2

Both electricity and natural gas are modelled as two factor models very similar to SPM1. The only difference is that now the log price instead of the price itself is modelled. The same assumption as made for the preceding model that the risk free proxy log price of the log spot can be described by the following model:

$$\ln P_i = f_i(t) + X_i + \varepsilon_i \tag{9.9a}$$

$$d\varepsilon_i = \mu_i dt + \sigma_{\varepsilon_i} dZ_{\varepsilon_i} \tag{9.9b}$$

$$dX_i = -\kappa_i X_i dt + \sigma_{X_i} dZ_{X_i} \tag{9.9c}$$

where

$$dZ_{\varepsilon_i} dZ_{X_i} = \rho_i dt \tag{9.9d}$$

$$f_i(t) = \overline{P}_i + \gamma_i \cos((t - \eta_i)2\pi) \tag{9.9e}$$

$i = \mathrm{el}, \mathrm{gas}$

The expectation and covariance of the state variables is still given by equation (9.4), and the expected log spot price and its variance are given by

$$\mathrm{E}^*\left[\ln P_{i,t}\right] = \left[f_i(t) + X_{i,0}e^{-\kappa_i t} + \varepsilon_{i,0} + \mu_i t\right] \tag{9.10a}$$

and

$$\mathrm{Var}^*[\ln P_{it}] = \left(1-e^{-2\kappa t}\right)\frac{\sigma_X^2}{2\kappa} + \sigma_\varepsilon^2 t + 2\left(1-e^{-\kappa t}\right)\frac{\rho\sigma_\varepsilon\sigma_X}{\kappa} \qquad (9.10b)$$

Since the proxy spot price is log-normally distributed the expected spot price is now given by $\mathrm{E}[P_{it}] = \exp\left(\mathrm{E}[\ln P_{it}] + \frac{1}{2}\mathrm{Var}[\ln P_{it}]\right)$ and the expected log proxy spot price or the log futures price is given by

$$\ln F_{t,T}(P_{it},T) = f(t) + e^{-\kappa(T-t)}X_t + \varepsilon_t + A(T-t)$$
where (9.11)
$$A(T-t) = \mu(T-t) + \frac{1}{2}\left(\left(1-e^{-2\kappa(T-t)}\right)\frac{\sigma_X^2}{2\kappa} + \sigma_\varepsilon^2(T-t) + 2\left(1-e^{-\kappa(T-t)}\right)\frac{\rho_{\varepsilon X}\sigma_X\sigma_\varepsilon}{\kappa}\right)$$

### Valuing the option

If one where to include all the variables, both long term and short term for both electricity and natural gas when valuing the investment and the option this would become a model dependant on four state variables. As stated in the deduction of SPM1 the short term variable is not import when valuing a long term investment and the option to invest in such a project. To simplify the calculations the model can then be reduced to only depend on the long term state variables of electricity and natural gas. Thus, for the valuation the model forward prices are given by

$$\ln F_{t,T}(P_{it},T) = \varepsilon_t + A(T-t)$$
where (9.12)
$$A(T-t) = \mu(T-t) + \frac{1}{2}\left(\frac{\sigma_X^2}{2\kappa} + \sigma_\varepsilon^2(T-t) + 2\frac{\rho_{\varepsilon X}\sigma_X\sigma_\varepsilon}{\kappa}\right)$$

When $\kappa$ is fairly large the exponential terms in equation (9.11) approaches zero before the cash flow starts. Thus the error made by doing this is very small. Also note that the seasonal term is not used during the valuation since it is assumed that such variations over the year will cancel out.

When the model is dependant on the two long term state variables only, the value of any contingent claim written on the two state variables is found by solving equation (9.13) with the right boundary conditions. (9.13) is derived in the same manner as (9.8). In this thesis the equation is solved using LSM simulation as explained in chapter 9.10.

$$\frac{\partial W}{\partial t}+\frac{\partial^2 W}{\partial \varepsilon_{el}^2}\sigma_{el}^2+\frac{\partial^2 W}{\partial \varepsilon_{gas}^2}\sigma_{gas}^2+\frac{\partial W}{\partial \varepsilon_{el}}\mu_{el}+\frac{\partial W}{\partial \varepsilon_{gas}}\mu_{gas}-rW=0 \qquad (9.13)$$

## 9.4 Estimating the parameters

The models outlined in chapters 9.3.2 and 9.3.3 need accurate parameters in order give the correct option and project value. Such parameters can be estimated by fitting the models to historical and current futures and forward data. In this chapter two general procedures for estimating parameters to the models are described. A test of the two procedures and a discussion of criteria for accepting the parameters are included. Finally the actual method used in the thesis for estimating parameters is described.

### 9.4.1 Futures and forward prices vs $F_t(\Theta,T)$

Quoted prices on futures and forwards are prices that are constant over a certain range in time as described in chapter 4. The forward price obtained by equations (9.7) and (9.11) gives the forward price at a certain point in time, not over a period. It is a continues function of the forward price. The relationship between the quoted price, $F_{t,T1,T2}$, and the function for the forward price is

$$F_{t,T1,T2}=\frac{\int_{t+T1}^{T2} e^{-r\tau}F_t(\Theta,\tau)d\tau}{\int_{t+T1}^{T2} e^{-r\tau}d\tau} \qquad (9.14)$$

For futures and forwards quoted for a short period the approximation

$$F_{t,T1,T2}=F_t(\Theta,T1) \text{ or } F_{t,T1,T2}=F_t(\Theta,\frac{T1+T2}{2}) \qquad (9.15a,b)$$

will not give a big error. Forwards for a year or longer one should use equation (9.14). If a close form solution of (9.14) exists it is easily found using algebraic software like Maple. The advantage of using the close form solution is that it speeds up calculation when big data sets are used. Alternatively eq (9.14) can be calculated numerically.

### 9.4.2 Method 1 – minimizing the RMSE

This method is based on the idea of finding the model parameters that minimize the root mean square error (RMSE) of all the futures and forward contracts in the data set by using an iterative procedure. The RMSE is defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\hat{F}_n - F_n\right)^2}$$

where $\hat{F}_n$ and $F_n$ are the nth calculated and observed forward price respectively.

The iteration follows a three step procedure. For a given set of parameters, the state variables for each date are set to minimize the sum of squared errors of the futures and the forward contracts that given day. Step two uses these state variables in the regression procedure explained in next paragraph to find estimates for $\rho, \sigma_X$ and $\sigma_\varepsilon$. These parameters given, step three involves estimating the remaining parameters by minimizing RMSE of the complete set of futures and forward data. The iteration procedure is repeated until RMSE converges to a minimum level. In this thesis 0.00001 was used as convergence level. This procedure is described in Lucia et al (2002). When estimating parameters with this procedure equation (9.14) was used instead of equation (9.15).

The regression mention in the second step above is done as follows. The processes of the two state variables given in (9.3) and (9.9) presented at discrete form are

$$X_t - X_{t-1} = -\kappa \Delta t X_{t-1} + \xi_t^X \qquad (9.16a)$$

$$\varepsilon_t - \varepsilon_{t-1} = \mu \Delta t + \xi_t^\varepsilon \qquad (9.16b)$$

where $\xi_t^X$ and $\xi_t^\varepsilon$ are normally distributed variables with zero mean and standard deviation $\sigma_X$ and $\sigma_\varepsilon$ respectively and covariance $\rho\sigma_X\,\sigma_\varepsilon$. Given the state variables in period *t*-1 the expected value in period *t* is

$$\left[\hat{X}_t, \hat{\varepsilon}_t\right] = E\left[\left(X_t, \varepsilon_t\right)\right] = \left[\left(1-\kappa\Delta t\right)X_{t-1}, \mu\Delta t + \varepsilon_t\right] \approx \left[e^{-\kappa\Delta t}X_{t-1}, \mu\Delta t + \varepsilon_t\right], \Delta t \to 0 \qquad (9.17)$$

Now $\hat{\xi}_t^X = X_t - \hat{X}_t$ and $\hat{\xi}_t^\varepsilon = \varepsilon_t - \hat{\varepsilon}_t$. Estimates for the standard deviations can now be found by calculating the standard deviation of all the $\hat{\xi}_t^X$ and $\hat{\xi}_t^\varepsilon$ respectively and correlation is found by calculating the correlation between all the $\hat{\xi}_t^X$ and $\hat{\xi}_t^\varepsilon$. This is easily done in a spread sheet.

### 9.4.3 Method 2 – Kalman filter

"*The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modelled system is unknown.*"[56]

The posterior state variables estimates are based on a prior knowledge of the initial state variables and on observations that depend on the state variables given a set of parameters for the processes. The concept is described in Figure 6. Given this tool and a function to calculate the likelihood of the observations based on the parameters the Kalman filter can be used to estimate the parameters.

**Principles of the Kalman filter**

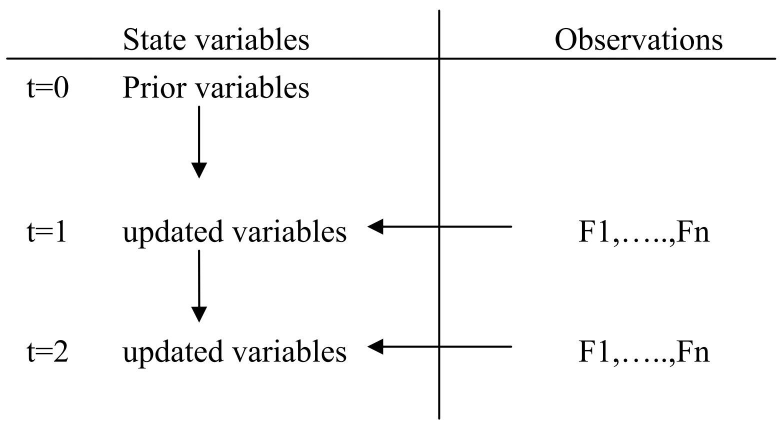



**Figure 6**: The figure illustrates the principles of the Kalman filter. By prior knowledge of the state variables in time step 0 and the observations in time step 1 the Kalman filter calculates the state variables in time step 1 that minimizes the RMSE for a given set of parameters. As time passes and new observations arrive the state variables can be updated.

[56] Bishop and Welch (2004)

### The equations needed to use the Kalman filter

First the model which parameters are to be estimated must be cast in a state space form consisting of a *measurement equation* and a *transition equation*. The measurement equation describes the relationship between the observations and the state variables, while the transition equation generates the unobservable state variables. For the models in this thesis the observations are prices or log prices of quoted futures and forward data.

The *measurement equation* can be obtained from (9.7) or (9.11) and when having N observations per time step it can be written as[57]

$$\mathbf{y}_t = \mathbf{Z}_t \mathbf{x}_t + \mathbf{d}_t + \mathbf{v}_t \tag{9.18a}$$

where

$$\mathbf{y}_t = \left[F_t(T_i)\right]' \text{ or } \left[\ln(F_t(T_i))\right]' \quad i = 1..N \qquad \text{N x 1 vector} \tag{9.18b}$$

$$\mathbf{Z}_t = \left[e^{-\kappa T_i}, 1\right] \qquad \text{N x 2 matrix} \tag{9.18c}$$

$$\mathbf{x}_t = \left[X_t, \varepsilon_t\right] \qquad \text{1 x 2 vector} \tag{9.18d}$$

$$\mathbf{d}_t = \left[A(T_i)\right] \qquad \text{N x 1 vector} \tag{9.18e}$$

$\mathbf{v}$ N x 1 vector of serially uncorrelated disturbances with (9.18f)

$\mathrm{E}(\mathbf{v}_t) = 0$ and $\mathrm{Cov}(\mathbf{v}_t) = \mathbf{H}$

Equation (9.18b,c) assumes that quoted prices are prices for delivery at a specific point in time instead of over a period. As mentioned in chapter 9.4.1 this works quite well when $\mathbf{y}_t$ is a vector of short futures like week or months. Then either equation (9.15a) or (9.15b) can be used. However when $\mathbf{y}_t$ contains long contracts like season futures or yearly forwards it would be more appropriate to use the exact equation (9.14). Then

$$\mathbf{y}_t = \left[F_t(T1_i, T2_i)\right]' \text{ or } \left[\ln(F_t(T1_i, T2_i))\right]' \quad i = 1..N \qquad \text{N x 1 vector} \tag{9.18g}$$

$$\mathbf{Z}_t = \left[ \frac{1}{\int_{T1_i}^{T2_i} e^{-r\tau} d\tau} \int_{T1_i}^{T2_i} e^{-r\tau} e^{-\kappa\tau} d\tau, 1 \right] \qquad \text{N x 2 matrix} \tag{9.18h}$$

and

[57] Schwartz et al (2000), Schwartz (1997)

$$\mathbf{d}_t = \left[ \frac{1}{\int_{T1_i}^{T2_i} e^{-r\tau} d\tau} \int_{T1_i}^{T2_i} e^{-r\tau} A(\tau) d\tau \right] \qquad \text{N x 1 vector} \qquad (9.18i)$$

The transition equation is obtained from (9.3) or (9.9) and is about the same as the equation (9.16), only on a matrix form[58]

$$\mathbf{x}_t = \mathbf{G}\mathbf{x}_{t-1} + \mathbf{c} + \boldsymbol{\omega}_t \qquad (9.19a)$$

where

$$\mathbf{G} = \begin{bmatrix} e^{-k\Delta t} & 0 \\ 0 & 1 \end{bmatrix} \qquad (9.19b)$$

$$\mathbf{c} = [0, \mu\Delta t] \qquad (9.19c)$$

$\boldsymbol{\omega}_t$ 2 x 1 vector of serially uncorrelated disturbances with

$$\mathrm{E}(\boldsymbol{\omega}_t) = 0 \text{ and } \mathrm{Cov}(\boldsymbol{\omega}_t) \equiv \mathrm{Cov}[(X_t, \varepsilon_t)] \qquad (9.19d)$$

$\Delta t$ is the length of the time step from $t$-1 to $t$.

For an excellent description on how the Kalman filter can be used to update the state variables based on observations please see Schwartz and Smith (2000).

The likelihood function used in the maximization procedure is found in Harvey (1989) and can be written in the notation used in Schwartz and Smith (2000) as

$$\log L = -\sum_{t=1}^{T} \log|\mathbf{Q}_t| - \sum_{t=1}^{T} \mathbf{v}_t' \mathbf{Q}_t^{-1} \mathbf{v}_t$$

where (9.20)

$$\mathbf{v}_t = \mathbf{y}_t - \mathbf{f}_t$$

The constant terms are left out since one is only interested in maximizing this function.

### Parameter estimation

The previous paragraph explained how to setup the model for the Kalman filter. This paragraph gives a short description on how to use the Kalman filter to estimate the parameters of the models.

---

[58] Schwartz et al (2000), Schwartz (1997)

The likelihood function given in (9.20) can be expressed as a function of the unknown parameters since the Kalman filter calculates $\mathbf{Q}_t$ and $\mathbf{f}_t$ for a given set of parameters. Thus the likelihood function is $L(\Xi)$ where $\Xi$ is a vector of the parameters to be estimated. $\Xi = [\mu, \kappa, \sigma_X, \sigma_\varepsilon, \rho, \gamma, \eta, \varepsilon_0, \bar{S}]$. The initial or prior long term factor, $\varepsilon_0$, is not really a parameter but since it cannot be observed it is also unknown.

The procedure for estimating the parameters is then recursive. The first step is to select an initial $\Xi$ and run the Kalman filter. Then select new parameters. These new parameters can be found by running a Newton Raphson procedure. Then run the Kalman filter again. Repeat these steps until the likelihood function converges to some level.[59]

## 9.5 Testing the methods

In order to verify that the methods outlined above really could estimate parameters correlated state variables were generated. The generated processes had parameters: $\mu$=0.5, $\kappa$=2, $\rho$=0.1, $\sigma_X$=50, $\sigma_\varepsilon$=10 and 300 periods were generated with time step 0.01. Three forward prices were calculated with different maturities.

With both methods $\mu$ and $\kappa$ are estimated correctly, but with method 1 the time of estimating depends greatly of the initial guess. This is not the case with the Kalman filter. Method 2 finds a $\rho$ 0.0892 and the volatilities are $\sigma_X$ = 50.6767 and $\sigma_\varepsilon$ = 10.3169. Method 1 is slightly more inaccurate. Both methods give equally good estimates of the parameters, but the Kalman filter is much faster. Method 1 was implemented in Excel and method 2 was implemented in Matlab. For a true test of which method is faster both should be implemented in Matlab. This was not done.

In most papers using models like these presented here the true process is described, and the risk neutral process is deducted from the true process using prices of risk. The methods were also tested when the models includes the true drift, mean reversion rate and prices of risk. Both methods worked poorly under this scenario. Method 1 was able to estimate the risk adjusting parameter in the short term process when the initial guess was close to the correct

[59] Shumway & Stoffer (2000)

value. Both methods failed completely to estimate the true drift of the process, especially when $\mu << \sigma_{\varepsilon}$ which might be the case for the data to be estimated. The reason for this is that $\mathrm{Var}(\hat{\mu}) = \sigma_{\varepsilon}^{2} / T$ where T is the length of the sample period[60]. When $\mu << \sigma_{\varepsilon}$ the relative size of $\mathrm{Var}(\mu)$ compared to $\mu$ is large. To estimate $\mu$ correctly when this is the case the sample period must be very long. This can easily be verified in a spread sheet.

In many applications the true drift is of no interest. However if one uses the latter situation with risk adjustments and wishes to use Monte Carlo simulation the results can become really wrong since the true drift will not be correctly estimated. Fortunately in this thesis the true process is of no interest and there is no need to distinguish between the true and the risk adjusted drift rate.

## 9.6 The actual method

Based on the knowledge gained from testing and other criteria a procedure using elements from both methods was used to estimate the parameters. Schwartz and Smith 2000 suggest several such methods. The best way to do this depends on the applications of the model.

The procedure used is:

1. The Kalman filter was used on the entire data set to estimate $\kappa, (\lambda, \tau), \rho, \sigma_X$ and $\sigma_{\varepsilon}$. $\sigma_X$ and $\sigma_{\varepsilon}$ and thus $\rho$ is best estimated using as many observations as possible. The nature of the seasonal variations are not expected to change over the data set.
2. Base on the parameters in step one the $\mu$ and $\kappa$ that minimizes the RMSE of the most recent long maturities forward/futures is selected. This is done in order to use only the most recent market information in the calculations. $\mu$ are very sensitive to the length of the sample. Thus recent data is believed to be the most correct data set for estimating the long term drift.
3. Given all the parameters the $\mu$ is again selected to minimize the RMSE of all of the contracts in the data set the given day of valuation including the 10 year forward that day.

[60] Söderlind, P (2003)

Using this procedure the important parameter $\mu$ is estimated based on recent market information. Some modifications are made to this procedure on the various commodities (electricity, natural gas and spark spread). For instance the state variable must be somewhat adjusted to fit the current forward curve.

## 9.7 The data sets

This paragraph briefly describes the three data sets used for estimation in this thesis. When constructing the spark spread data set one must be sure to compare similar contracts. A problem then arises since the term structures of electricity and natural gas are different. In order to overcome this problem the contracts found at Nord Pool were set as basis and then natural gas contracts from several periods were combined to make equivalent long period contracts. Since the interest rate is assumed to be constant, there is no difference between futures and forward contracts. Thus for a given date it is possible to construct a yearly contract maturing January 1st the three next years. When combining forwards equation (9.14) was used. Below is an example of the procedure. For some contracts the method does not work out as nicely as in the example.

Considering a day in January 2004 and a forward contract for delivery over one year starting 01.01.2005 is desired. Then according to equation (9.14) the price of this one year forward will be

$$F(01.01.2005,31.12.2005) = \frac{\dfrac{F_{Q0105}}{e^{-rt_1} - e^{-rt_2}} + \dfrac{F_{Q0205}}{e^{-rt_2} - e^{-rt_3}} + \dfrac{F_{Q0305}}{e^{-rt_3} - e^{-rt_4}} + \dfrac{F_{Q0405}}{e^{-rt_4} - e^{-rt_5}}}{e^{-rt_1} - e^{-rt_5}}$$

where $t_i$ and $t_{i+1}$ is the start and the end of the $i$th quarter respectively.

More generally

$$F(T1,T2) = \frac{\sum_{i=1}^{n} \dfrac{F_i}{e^{-rt_i} - e^{-rt_{i+1}}}}{e^{-rt_1} - e^{-rt_{n+1}}} \tag{9.21}$$

where $i$ refers to the $i$th contract in the period.

#### Electricity data set

When estimating the parameters for the electricity process the following contracts were used: The block or equivalent block contract closest to maturity (the equivalent consist of four weekly contracts constructed in the manner described above) and the contracts for delivery in the next, second and third nearest calendar year.

#### Natural gas

The data set for natural gas consisted of the first nine monthly contracts, the quarterly contracts for delivery after 11 and 25 months and the two seasonal contracts available. All the prices were then converted to NOK/MWh$_{el}$ as described in chapter 5.

#### Spark spread

Natural gas contracts equivalent to those explained for electricity were constructed by using the method above. Then the prices were adjusted to NOK/MWh$_{el}$ and subtracted from the electricity contracts to make out the spark spread contract.

## 9.8 Results of parameter estimation

This chapter presents the parameters estimated for spark spread, electricity and natural gas. The parameters given here are considered to be the base case parameters from which sensitivity analyses are performed.

### 9.8.1 Spark spread

Table 5 presents the parameters estimated for the spark spread process. When estimating these parameters equation (9.14) was used both in the Kalman filter and later when the $\mu$ and $\kappa$ was fitted to the recent data. The seasonal component was not considered when estimating the parameters for the spark spread since the constructed data set mostly contained yearly data and thus seasonal fluctuations are assumed to disappear.

The half life defined in chapter 9.3.2 is 0.42 years or about 5 months for the short term process. As expected the short term process has much higher volatility than the long term process. The correlation between the two is almost zero. This means that the long term movements are not affected by short term variations, something which seems quite reasonable having the discussion in 8.2 in mind. In Figure 7 the long term state variable is graphed together with the proxy spot price. There are no large jumps in the long term variable when the spot price rose during the end of 2002.

As is seen from Figure 8 the forward curve fits the current constructed term structure quite well. The one year contract starting in the third next calendar year is not priced by this curve. This is a clear weakness.

The estimated parameters imply a forward price for delivery in 2033 at about 190 NOK/MWh.

**Table 5**

**Spark spread parameters**

| **Parameter** | **Estimate** | **Denominator** | **Standard error** | **95 % confidence interval** | **Comments** |
|---|---|---|---|---|---|
| $\sigma_X$ | 191.42 | NOK/MWh | 9.7 | ± 19 | |
| $\sigma_\varepsilon$ | 34.71 | NOK/MWh | 0.07 | ± 0.14 | |
| $\rho$ | 0.08 | | | | |
| $\kappa$ | 1.66 | year$^{-1}$ | | | |
| $\mu$ | 3.42 | NOK/MWh | | | |
| $\gamma$ | 0 | NOK/MWh | | | Held constant |
| $\eta$ | 0 | year | | | Held constant |
| $S$ | 0 | NOK/MWh | | | Held constant |
| $\varepsilon_0$ | 76.62 | NOK/MWh | | | |
| $X_0$ | 119.98 | NOK/MWh | | | |

This table shows the estimated parameters for the spark spread process. The data used for estimation is electricity and natural gas futures and forwards from Nord Pool and IPE respectively from every Friday from 07.01.2000 to 27.09.2003. A set of 10 year contracts was also available for 12.09.2003. Subscript zero refers to 12.09.2003.

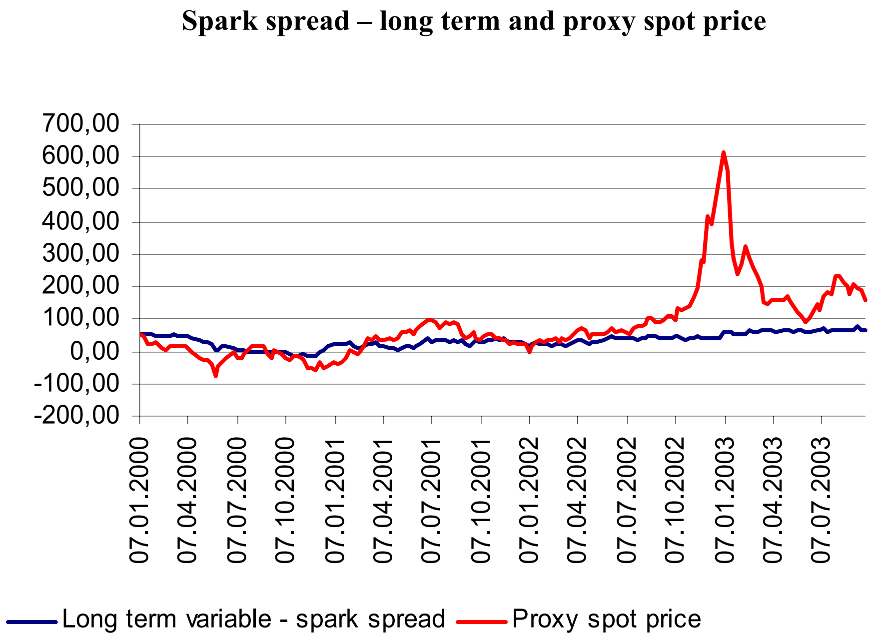



**Figure 7**: The figure shows long term variable and the proxy spot price of spark spread.

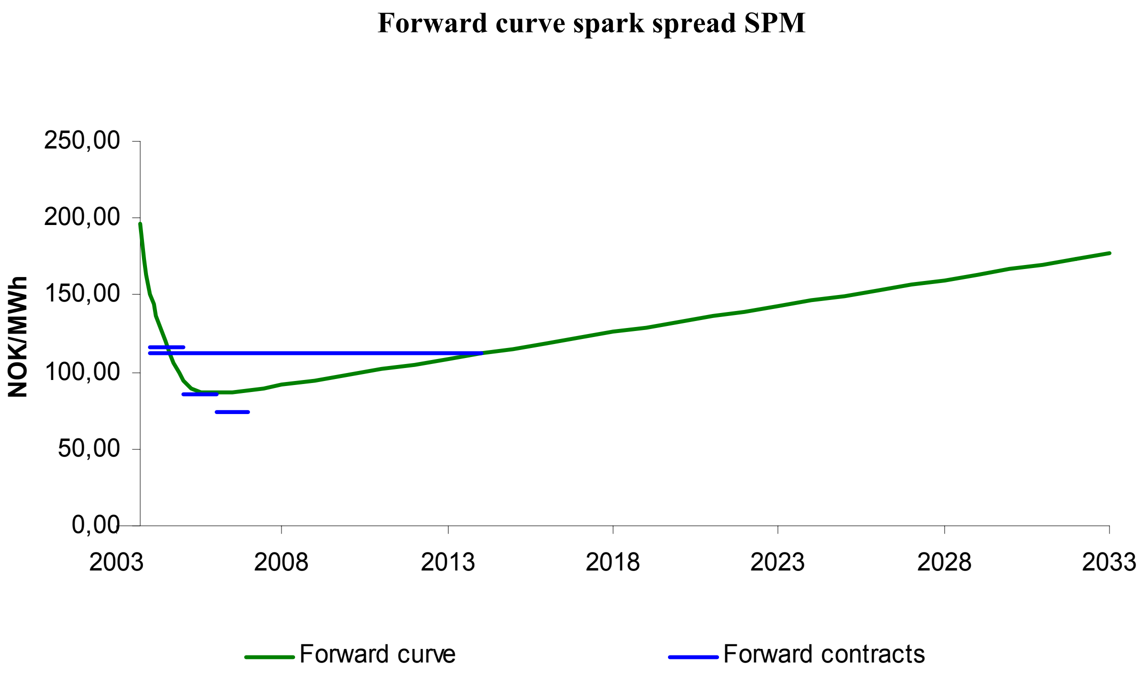



**Figure 8**: The figure shows the spark spread forward curve 12.09.03 and the constructed forward contracts. The forward curve prices the first one year contract correctly but with some errors on the other contracts. This is the forward curve that minimizes the RMSE.

### 9.8.2 Electricity

Table 6 displays the estimated parameters for the electricity price. In the Kalman filter equation (9.15b) was used setting the time of maturity to the middle of the contract period. In step 2 in the procedure outlined in paragraph 9.4.3 equation (9.14) was used. It was integrated using the trapezoidal method. Also here since only the yearly contracts quoted at Nord Pool were used, seasonal variations were assumed not to be relevant and these parameters were held constant.

The half life of the short term process of the electricity price is 0.64 years or 7.9 months. Also for electricity the two processes are almost uncorrelated and the short term variable is more volatile than the long term. Figure 9 graphs the long term variable together with the proxy spot price. No big shifts or jumps are discovered during the end of 2002 and beginning of 2003. Figure 10 shows the how well this forward curve fits the contracts considered for that day.

**Table 6**

**Electricity parameters**

| **Parameter** | **Estimate** | **Denominator** | **Standard error** | **95% Confidence interval** | **Comment** |
|---|---|---|---|---|---|
| $\sigma_X$ | 70.56 | % | 0.027 | ±0.054 | |
| $\sigma_\varepsilon$ | 9.38 | % | 0.002 | ±0.004 | |
| $\rho$ | -0.047 | | | | |
| $\kappa$ | 1.0823 | $year^{-1}$ | | | |
| $\mu$ | 3.06 | % | | | |
| $\gamma$ | 0 | % | | | Held constant |
| $\eta$ | 0 | Year | | | Held constant |
| $S$ | 0 | % | | | Held constant |
| $\varepsilon_0$ | 166.65 | NOK/MWh | | | |

This table shows the estimated parameters for the electricity process. The data used for estimation is electricity futures and forwards from Nord Pool every Friday from 07.01.2000 to 31.10.2003. A 10 year contract was also available for 12.09.2003. Subscript zero refers to 12.09.2003.

**Electricity – long term variable and proxy spot price**

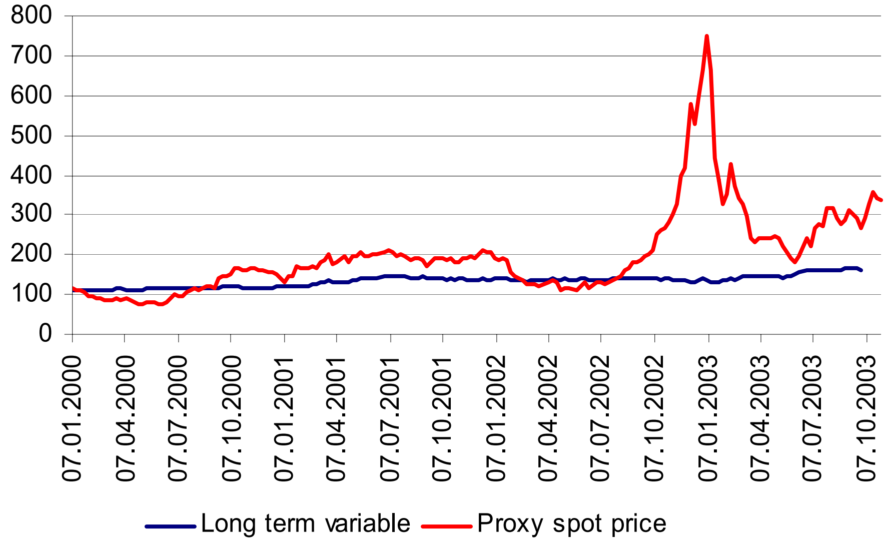



**Figure 9**: The figure shows long term variable and the proxy spot price of electricity.

**Electricity forward curve**

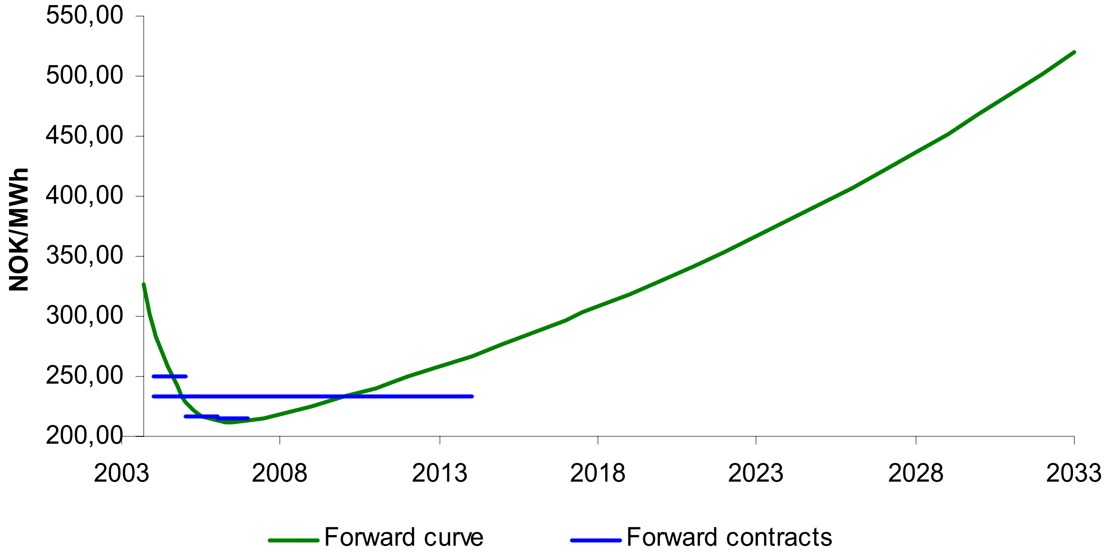



**Figure 10**: The figure shows the electricity forward curve on 12.09.2003. The forward curve prices all the contracts almost correctly.

### 9.8.3 Natural Gas

When estimating the parameters for natural gas, the seasonal component was used. The reason for this is that all the futures quoted at IPE show extensive seasonal behaviour as can be seen from Figure 3. There are no yearly contracts available at IPE that would level out such variations. The reason for not using the constructed yearly contracts as described in chapter 9.7 is that the whole reason for analyzing electricity and gas separately is to use only real contracts.

Though using the seasonal component the long term volatility estimated by the method described above were still high (45%), and the correlation is higher than the two other models mentioned above but negative (-0.19) as displayed in Table 7. This suggests that the to processes move with opposite movements to movements in the term structure. This could indicate a weakness of the parameters compared to the two other models. The high long term volatility and the corresponding negative drift (-8.7 %) do not seem very realistic. Due to this an ad hoc estimation was performed by assuming that the long term volatility is somewhat higher than the equivalent for electricity, 10 %, and the drift was adjusted to fit the forward curve the given day. The reason why the long term natural gas volatility are assumed somewhat higher than electricity is just that estimation procedure gave very high volatility.

The half life of the short term process is 0.26 years or 3 months. Since $\gamma$ is negative the $\eta$ shows that the seasonal peak comes in December each year. Figure 12 shows the how the forward curve implied by these parameters fits the term structure 12.09.03.

In Table 8 the correlations of the state variables in SPM2 are listed. Generally the correlation is just around zero. This shows that the state variables and especially the long term variables of electricity and natural gas are indeed uncorrelated. This supports the assumption that the electricity and natural gas prices are uncorrelated.

**Table 7**

**Natural gas parameters**

| **Parameter** | **Estimate** | **Denominator** | **Standard error** | **95% Confidence interval** | **Comment** |
|---|---|---|---|---|---|
| $\sigma_X$ | 66.53 | % | 0.002 | ±0.004 | |
| $\sigma_\varepsilon$ | 10 (45.77) | % | 0.007 | ±0.014 | |
| $\rho$ | -0.195 | | | | |
| $\kappa$ | 2.6952 | year$^{-1}$ | | | |
| $\mu$ | 1.2 (-8.7) | % | | | |
| $\gamma$ | -32.68 | % | | | |
| $\eta$ | 0.46 | Year | | | |
| $S$ | 0 | % | | | Held constant |
| $\varepsilon_0$ | 111.05 | NOK/MWh | | | |

This table shows the estimated parameters for the natural gas process. The data used for estimation is natural gas futures from IPE every Friday from 08.10.1999 to 27.09.2003. A 10 year contract was also available for "the summer of 2003". Subscript zero refers to 12.09.2003. The numbers in parenthesis are the numbers given by the estimation procedure.

**Table 8**

**Correlation matrix**

| | $\varepsilon_{El}$ | $\varepsilon_{Gas}$ | $X_{El}$ | $X_{gas}$ |
|---|---|---|---|---|
| $\varepsilon_{El}$ | 1 | **-0.002** | -0.05 | 0.064 |
| $\varepsilon_{Gas}$ | -0.002 | 1 | -0.011 | -0.195 |
| $X_{El}$ | -0.05 | -0.011 | 1 | 0.078 |
| $X_{gas}$ | 0.064 | -0.195 | 0.078 | 1 |

The correlation matrix between the state variables that make up the electricity and natural gas prices. Almost all state variables are uncorrelated expect the short and long term variable for natural gas. It is interesting to see that the long term variables of electricity and natural gas are uncorrelated. This suits the situation well since prices used for estimation purposes are taken from two separated markets.

**Natural gas – long term variable and proxy spot price**

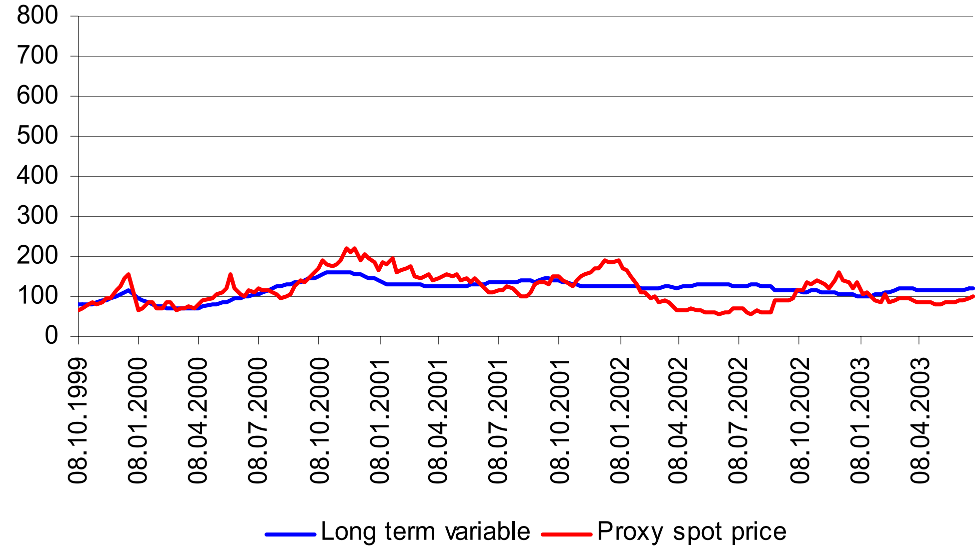



**Figure 11**: The long term state variable and the proxy spot price for natural gas. The same axis as in Figure 7 and Figure 9 is used on this graph so that the reader will get the right picture of the smoothness of the curves.

**Natural forward curve**

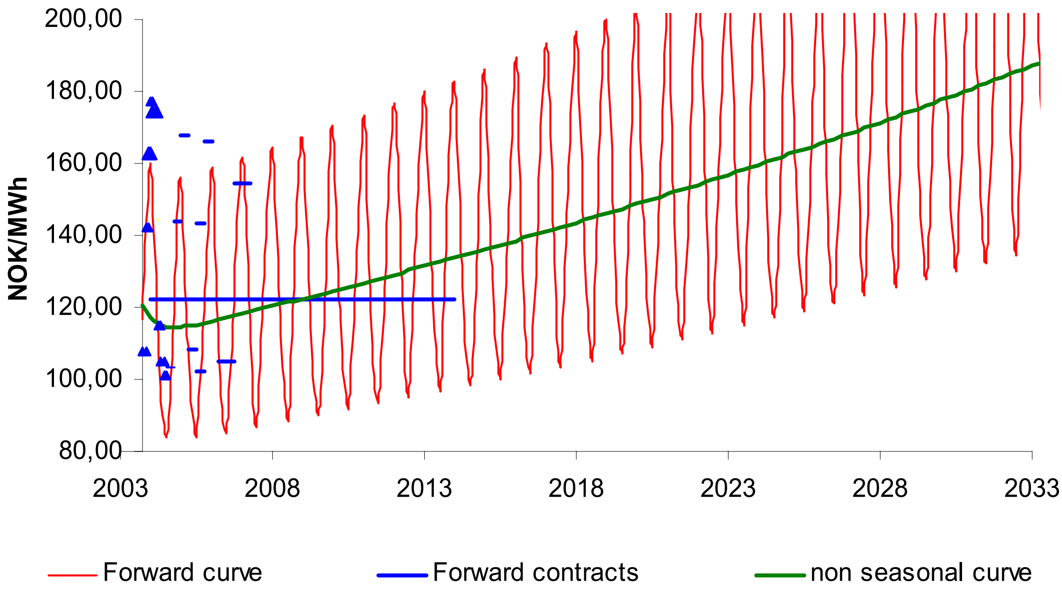



**Figure 12**: The figure displays the forward curve with and without the seasonal term. As is evident from the figure the seasonal term is crucial to price the contracts correctly. The forward curve without the seasonal term is used for calculation of the option value.

## 9.9 Statistical analysis of the estimated parameters

In order to tell how good the estimated parameters are it would be convenient to perform statistical analyses on the parameters. The estimation procedure outlined above does not provide such analysis directly. However it is possible to find the parameter for each date in the data set by finding the parameters $\mu$ and $\kappa$ that minimize the RMSE on the prices of the futures and forwards a given day (only changing one parameter at the time). But since $\mu$ and sometimes $\kappa$ are estimated based on the long 10 year contract which is only available one day, this method will not give the desired result when constructing confidence intervals. On the contrary the volatilities are estimated over the whole data set, so given the $\mu$ and $\kappa$ one can use GARCH(1,1) on the residual terms given in (9.16a,b) to find the volatility each date in the data set. Having such a time series of volatilities one can find confidence intervals and one can get a picture of whether volatilities are dependant on the state variable and whether it is correct to assume constant volatility or not.

Panel 1 and panel 2 in Figure 13 show the long term and the short term volatility of the stochastic processes for spark spread together with the spark spread proxy spot price respectively. As one can see the long term volatility does not seem to be affected by movements in the proxy spot price. However the short term volatility seems very dependant on variations in the proxy spot. As pointed out earlier and verified by calculations in chapter 14 the volatility of the short term variable in SPM1 is irrelevant when it comes to valuing the option so even thought it seems to be highly dependant on the proxy spot no large error is made setting this parameter constant.

A Wilcoxon rank sum test was performed on several parts of the long term volatility time series by finding the mean volatilities on these parts and test whether it is likely that the means are equal. The test concluded that on can reject the null hypothesis that the means are equal. However, inspecting the graph of the long term volatility the level seems relatively constant so the conclusion is that using constant long term volatility is a good approximation.

Similar tests were performed on the long term volatility of electricity and natural gas. For both electricity and natural gas the null hypothesis that the volatility is indeed constant is rejected. In panel A in Figure 14 the long term volatility of electricity is graphed versus the proxy spot price of electricity. The figure shows that volatility has been steadily increasing in recent years and made a shift to a higher level during the high electricity prices in December 2002. Such increasing volatility is in accordance with the Nordic electricity market. Investigating option prices at Nord Pool for the recent years reveals that implied volatility has increased. The long term volatility for electricity seems to be somewhat dependant on the proxy spot price since it increases as the electricity price increases. Thus modelling with constant volatility can lead to errors. Panel B in figure shows the same graph for natural gas. The long term volatility seems to make small jumps as prices increase, but falls back to about the same level as before.

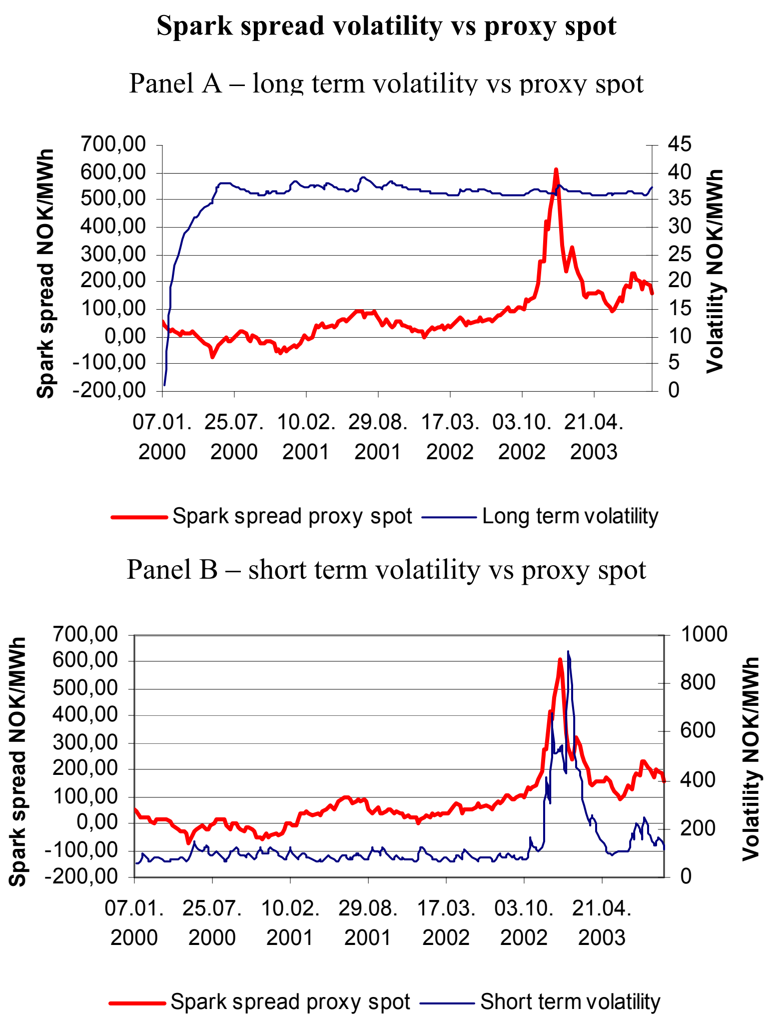
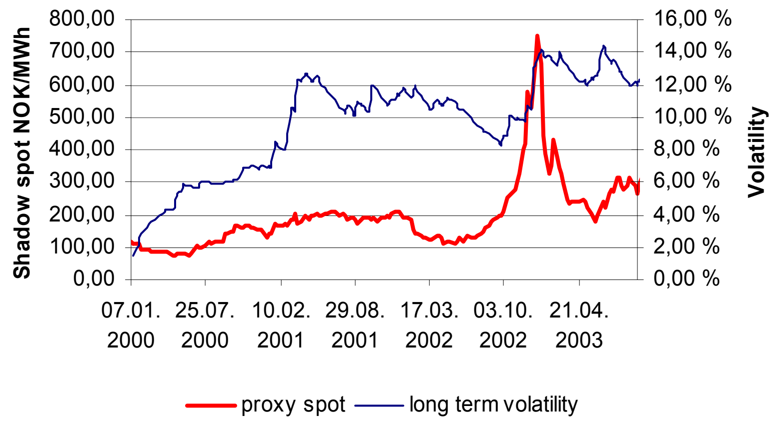



**Figure 13**: Panel A shows the long term spark spread volatility vs. the spark spread proxy spot. Panel B shows the short term spark spread volatility vs. the spark spread proxy spot

**Electricity and Natural gas long term volatility**

Panel A – Electricity long term volatility vs. electricity proxy spot




Panel B – Natural gas long term volatility vs. natural gas proxy spot

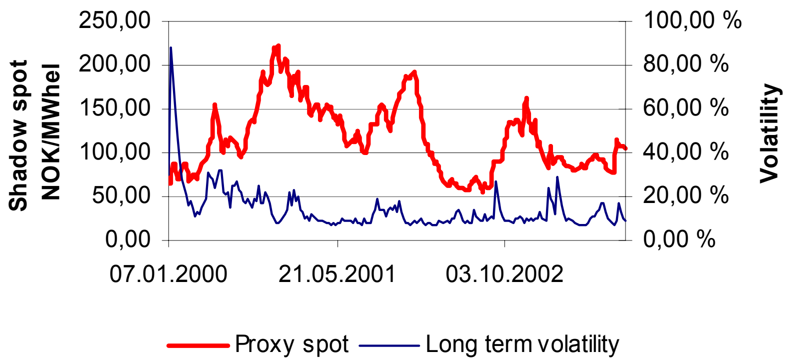



**Figure 14**: Panel A displays the long term volatility for electricity vs. the proxy spot for electricity. Panel B shows the same for natural gas.

## 9.10 Calculation method – spot price models

This chapter describes the general procedure which is used to value the power plant and the option to invest. Since many of the integrals and differential equations given above do not have any analytical solution numeric methods must be used. For the value of the power plant numerical integration is used in SPM2, and for solving the differential equations Monte Carlo simulation is used.

When log price models are used the integral of equation (9.1) does not have an analytical solution. Thus numerical integration must be used. In this thesis the trapezoidal or Simpson's method is used. For a general description, see Kreizig (1999)

The differential equations are solved numerically by using Monte Carlo simulation to approximating the value of the American options, using the approach given by Longstaff and Schwartz (2001), and known as the least squares Monte Carlo (LSM).

The holder of an American option has a continues right to exercise her option at any point in time. This can not be modelled numerically so the American option must be modelled as a Bermudan option. A Bermudan option is an option contract of American style, but exercise is only allowed at given points in time. As the exercise points go to infinity the value of the Bermudan option approaches the American option. The idea is then to select sufficient exercise points when valuing the American option.

A holder of an American option will at each discrete point in time optimally compare the payoff from immediate exercise with the expected payoff from the continuation, and then exercises if the immediate payoff is higher. Longstaff et al (2001) argues that the conditional expectations on holding the option can be estimated from the cross-sectional information in a simulation by using least squares. Least square analysis is used to determine the best-fit relationship between the value of continuing and the values of state variables at each time an early exercise decision has to be made. At each exercise point, the *b* paths that are in the money at that point are singled out to reduce computation efforts.

Accordingly, the expected value of holding the option is found by (9.21) where $\Psi_r$ is some basis function of one or more state variables, $\beta_{ir}$ is the weight given to function $r$ in time step *i* and $C_i$ is the value of continuing in period *i*.

$$C_i = \sum_{r=1}^{M} \beta_{ir} \psi_r(x) \qquad (9.21a)$$

To find the coefficients, $\beta_{ir}$, general least square regression is used at each exercise point. Briefly

$$\hat{\beta}_i = \hat{B}_{\psi}^{-1} \hat{B}_{\psi V}$$

where

$\hat{B}_{\psi}$ is an M x M matrix with entries $qr$

$$\frac{1}{b} \sum_{j=1}^{b} \psi_q(X_{ij}) \psi_r(X_{ij})$$

and

$\hat{B}_{\psi V}$ is an M vector with entries $r$

$$\frac{1}{b}\sum_{k=1}^{b}\psi_r(X_{ik})V_{i+1}(X_{i+1,k})$$

where $V_{i+1}$ is the value of the option when continuing to the next step and $X_{ij}$ is the state variable(s) in the *i*th period and path *j*. The value to wait is then given on matrix form

$C_i = \hat{\beta}_i^T \psi(x)$ [61]

Finally the value of the option is determined by discounting each cash flow back to time zero at the risk-free rate and calculating the mean of the result.

Given the state variable S1 and S2 some typical basis functions are shown in Table 9.

**Table 9**

**Typical basis functions**

| **Basis function** | **Combination of state variables** |
|---|---|
| $\psi_1$ | $S_1$ |
| $\psi_2$ | $S_2$ |
| $\psi_3$ | $S_1^2$ |
| $\psi_4$ | $S_2^2$ |
| $\psi_5$ | $S_1 S_2$ |
| $\psi_6$ | $S_1^2 S_2$ |
| $\psi_7$ | $S_1 S_2^2$ |

The table shows some typical basis functions that can be used. Longstaff and Schwartz (2001) suggests more complicated basis functions as Laguerre polynomials and Bessel functions.

[61] Glasserman (2004)

# 10 The Forward Curve Model

This chapter gives a general introduction to the framework of forward curve modeling emphasizing on commodities. Further the models used for valuation and the procedure for estimation of parameters are presented.

## 10.1 The HJM Model

The Forward curve model was introduced by Heath, Jarrow and Morton (1992) making it known as the HJM-model. They developed a no-arbitrage model of the stochastic movements of the term structure of interest rates. The model takes as given the initial forward curve process consistent with no arbitrage. Although originally an interest rate model, the framework can also be applied to commodities. The salient point is modeling the movements along the whole forward curve and not just the spot rate. The benefit of modeling the forward curve directly is that there is no problem to fit the model to current forward curve.

The forward curve model describes evolution in the forward curve given an initial forward curve and a mechanism which describes how it fluctuates. The stochastic differential equation for the risk –neutral forward curve is given in equation (10.1) $T$ denotes time-to-maturity and by that the location on the forward curve. A thorough deduction of the equation is given by e.g. Hull (2003).

$f(t,T)$ :instantaneous forward rate at time $t$ for a contract maturing at time $T$

$m(t,T)$ :forward curve drift

$\sigma_i(t,T)$ :The $i$ sources of forward curve volatility

$$df(t;T) = m(t,T)dt + \sum_{i=1}^{K} \sigma_i(t,T)dW_i \qquad (10.1a)$$

$$m(t,T) = \sum_{i=1}^{K} \sigma_i(t,T)\int_t^T \sigma_i(t,s)ds \qquad (10.1b)$$

(10.1) is a multifactor HJM model where the motions follow a $K$-dimensional Wiener process.

## 10.2 HJM models applied to commodities

Empirical investigations of forward curve models in commodity markets have been conducted by among other Cortazar and Schwartz (1994) and Clewlow and Strickland (2000). Similar to the general HJM models, these models take the initial term structure of commodity futures prices and derive its stochastic movement consistent with no arbitrage. The models can be used to value all types of commodity derivates. Bjerksund et al. (2000) and Koekebakker and Ollmar (2001) study Nordic electricity prices in the HJM modeling framework.

The forward price models are used to describe the stochastic evolution under equivalent martingale measure, the Q measure. Harrison and Kreps (1979) and Harrison and Pliska (1981) show that under very general conditions, the absence of arbitrage opportunities in the economy implies the existence of a probability measure under which asset prices follow a martingale which is a zero drift stochastic process. This probability measure has been called risk-neutral probability measure and is also known as equivalent-martingale measure. Constant risk free interest rate is assumed, and is also assumed in this thesis. Given constant interest rates the futures and forward prices are by construction martingales under the measure Q. Under Q measure the drift term from equation (10.1) vanishes.

In the following, a few models using the HJM framework on commodities are presented. Cortazar and Schwartz (1994) uses equation (10.2) to price copper contingent claims.

$$\frac{df(t,T)}{f(t,T)} = \sum_{k=1}^{K} \sigma_k(t,T) \cdot dW_k \tag{10.2}$$

The evolution of the forward curve for a given time to maturity $T$ evolves according to $K$ independent Brownian motions $W_1, W_{2,} \ldots, W_K$ under the equivalent martingale measure as is the case for the following models presented. The number of independent processes needed to explain the evolution of the forward curve is found using principal components analysis.

Bjerksund et al. (2000) uses a one factor model (10.3a) to value options in the Norwegian electricity market. They also propose a model with a richer volatility structure for risk management purposes (10.3b).

$$\frac{df(t,T)}{f(t,T)} = \left( \frac{a}{T-t+b} + c \right) \cdot dW(t) \tag{10.3a}$$

$$\frac{df(t,T)}{f(t,T)} = \frac{a}{T-t+b} \cdot dW_1(t) + \left( \frac{2ac}{T-t+b} \right)^{\frac{1}{2}} \cdot dW_2(t) + c \cdot dW_3(t) \tag{10.3b}$$

Kokebakker et al. (2001) uses special cases of the general multifactor term structure models developed for commodity markets in Miltersen and Schwartz (1998) to look at the volatility structure of Nord Pool electricity data. They use (10.4a) to look at price changes and (10.4b) to look at price returns.

$$df(t,T) = \sum_{i=1}^{K} \sigma_i^A(t,T) dW_i(t) \tag{10.4a}$$

$$\frac{df(t,T)}{f(t,T)} = \sum_{i=1}^{K} \sigma_i^B(t,T) dW_i(t) \tag{10.4b}$$

Audet et al. (2002) models the electricity forward curve with parameterized volatility and correlation structure (10.5). The correlation structure of the Brownian motions for the different time to maturities is given in equation (10.5b).

$$\frac{df(t,T)}{f(t,T)} = e^{-\alpha(T-t)} \sigma(T) \cdot dW_T(t) \tag{10.5a}$$

$$, \; dW_T(t) \cdot dW_{T^*}(t) = e^{-\rho\left|T-T^*\right|} \tag{10.5b}$$

Reviewing the models, it is clear that the volatility structure is the determining factor when modelling commodity futures in a HJM framework.

## 10.3 The forward curve models used

The models used to describe the evolution of the forward curves are similar to those of Koekebakker et al. (2001). A forward curve model for spark spread and forward curve models for the gas and electricity as separate processes are presented. The volatility function will be further specified in chapter 10.4

### 10.3.1 Forward curve model 1: Spark spread (FCM1)

The forward price at time $t$ for a commodity with delivery at time $T$ is denoted $f(t,T)$. The spark spread forward curve is set to evolve according to equation (10.6) which allows for the spark spread taking on negative values. This is a zero drift model according to the earlier discussion.

$$df(t,T) = \sum_{i=1}^{K} \sigma_i^s(t,T)dW_i(t) \tag{10.6}$$

The evolution of the forward curve for a given time to maturity $T$ evolves according to $K$ independent Brownian motions $W_1, W_2, \dots, W_K$ under the equivalent martingale measure. The spark spread volatility, $\sigma_i^s$, are time dependent volatility functions independent of the forward price level. The spark spread volatility is further discussed in chapter 10.3.3. The forward curve expressed on stochastic integral form is given in (10.7).

$$f(t,T) = f(0,T) + \sum_{i=1}^{K} \int_0^t \sigma_i^s(s,T)dW_i(s) \tag{10.7}$$

The forward prices are then normally distributed $N(g,v)$ as given in equation (10.8) with mean $g$ and variance $v$.

$$f(t,T) \sim \mathrm{N}\left( f(0,T), \sum_{i=1}^{K} \int_0^t \sigma_i^1(s,T)dW_i(s) \right) \tag{10.8}$$

### 10.3.2 Forward curve model 2: Electricity and natural gas (FCM2)

The forward price at time $t$ for a commodity with delivery at time $T$ is denoted $f(t,T)$ and the natural gas and electricity forward curves are set to evolve according to equation (10.9). This is like (10.6) a zero drift model according to the earlier discussion.

$$\frac{df(t,T)}{f(t,T)} = \sum_{i=1}^{K} \sigma_i^u(t,T)dW_i(t) \tag{10.9}$$

The evolution of the forward curve for a given time to maturity $T$ evolves according to $K$ independent Brownian motions $W_1, W_{2,} \ldots, W_K$ under the equivalent martingale measure. The superscript $u$ denotes the commodity and can be either gas or electricity. The volatility of natural gas or electricity, $\sigma_i^u$, is time dependent volatility functions dependent on the forward price level and are further discussed in chapter 10.3.3. The forward curve expressed on stochastic integral form is given in (10.10).

$$f(t,T) = f(0,T) + \exp\left( -\frac{1}{2} \sum_{i=1}^{K} \int_0^t \sigma_i^u (s,T)^2 \, ds + \sum_{i=1}^{K} \int_0^t \sigma_i^u (s,T) dW_i(s) \right) \qquad (10.10)$$

The logarithm of the forward prices are then normally distributed $N(g,v)$ as given in equation (10.11) with mean $g$ and variance $v$.

$$\ln f(t,T) \sim N\left( \ln f(0,T) - \frac{1}{2} \sum_{i=1}^{K} \int_0^t \sigma_i^u (s,T)^2 \, ds, \sum_{i=1}^{K} \int_0^t \sigma_i^u (s,T)^2 \, ds \right) \qquad (10.11)$$

### 10.3.3 Volatility

In the following it is assumed that the volatility of the forward curve depends only on the time to maturity, according to Wilmot (2001) a usually made assumption. The assumption is necessary to perform the principal component analysis in chapter 10.4. Denoting time to maturity by $\tau$, the general volatility function is reduced to a function where the volatility is only dependent on time to maturity (10.12).

$$\sigma_i = \sigma_i (T - t) = \sigma_i (\tau) \qquad (10.12)$$

The models described in chapter 10.2 propose several suggestions for volatility function for commodities in general and for the Nordic electricity market in particular. Volatility functions suggested for natural gas and spark spread are scarcer in the literature. The volatility function (10.12) is fitted to the empirical volatility in chapter 10.4.

### 10.3.4 Modeling the forward curve

For modeling purposes, the discrete approximations given in this chapter are used in the project valuation.

The spark spread forward curve is discrete approximated as given equation (10.13).

$$f(t,\tau) = f(t-1,\tau) + \sigma(\tau) \cdot \sqrt{\Delta t} \cdot \varepsilon \qquad (10.13)$$

where $\varepsilon$ is a normally distributed random variable with a mean of zero and a standard deviation of 1.

Similar, the forward curves of electricity and natural gas are discretely approximated as given in equation (10.14).

$$f(t,\tau) = f(t-1,\tau) + \sigma \cdot f(t-1,\tau) \cdot \sqrt{\Delta t} \cdot \varepsilon \qquad (10.14)$$

where $\varepsilon$ is a normally distributed random variable with a mean of zero and a standard deviation of 1.

## 10.4 Estimation of parameters

Making the assumption of equation (10.12), the volatility functions for electricity, gas and spark spread can be determined empirically from time series data thorough the use of Principle Component Analysis (PCA). Following the work of Cortazar et al. (1994), Clewlow et al. (2000) and Koekebakker et al. (2001), PCA is used to analyze the volatility factor structure of the forward curve on gas, electricity and spark spread.

Whereas the price model is derived under the risk neutral Q measure, the observations used for volatility estimation are made under objective probability measure, known as P measure or real world measure. Cortazar et.al (1994) show that the diffusion terms are equal under both measures and further states that this allows real world data to be used in the estimation of the forward curve volatility when observations are sampled continuously. The observations used serves as a proxy for continuously sampled data.

### 10.4.1 Construction the forward curve

When working with the whole forward curve as in the HJM framework using the forward and future market data directly would not work. The financial energy contracts differ from e.g. forward/future contracts on interest rates as they are only traded for a set of fixed delivery dates. As the contracts are fixed to the calendar the time to maturity quoted each day would be different. As an example the one year contract for next year will be traded from one year to one day before start of delivery period and shows a dramatic increase in volatility as the delivery period draws closer.

What is needed is data for specific times to maturity. Due to this one must assume that there exists forwards, $f(t,T)$, for delivery in week $T$ for the whole range of interest $T = 0$ to $T$ = *construction time* + *plant life time* denoted by $R$. In order to get a data set of contracts with equal time to maturity a continuous forward curve is constructed for each observation day.

Koekebakker et al. (2001) uses commercial software to compute a smooth forward curve function that prices all the traded forwards and futures within the bid/ask spread. Fleten and Lemming (2003) uses a similar approach combined with bottom-up models. The smoothed forward curves in this thesis are based on the method of Fleten et al. (2003).

### 10.4.2 Smoothed forward curves - Method

It is assumed that the forward prices $f(t,T)$ can be found for all $T$ in the interval $R$. A smoothed curve can be found by minimizing the integral of the squared second derivative of the function to be found over the whole interval.

$$\text{Min} \int_1^T \left(f''(t,x)\right)^2 dx \tag{10.15}$$

subject to constraints

The method presented here uses discrete points instead of a continuous function. The discrete representation of the second derivative is $\frac{f_{t-1} - 2f_t + f_{t+1}}{dt^2}$, and the minimization problem can be written as

$$\underset{f_t}{\text{Min}} \sum_{t=2}^{T-1} \left( f_{t-1} - 2f_t + f_{t+1} \right)^2 \qquad (10.16)$$

subject to constraints

This is the same as minimizing the price difference between adjacent points in time in order to create a smooth curve. This makes sense since one will expect that to contracts which are close do not differ too much in price.

The price of a forward contract for delivery between $T_1$ and $T_2$, $F(T_1, T_2)$, is given in (9.14) and is given on discrete form

$$F(T_1, T_2) = \frac{1}{\sum_{t=T_1}^{T_2} e^{-rt}} \sum_{t=T_1}^{T_2} e^{-rt} f_t \qquad (10.17)$$

This equality could be used as a constraint in equation (10.16). For the natural gas futures this would work since only the closing prices are quoted. At Nord Pool only the bid/ask spread is quoted for those contracts that are not traded and the appropriate constraint would be

$$F(T_1, T_2)_{BB} \leq \frac{1}{\sum_{t=T_1}^{T_2} e^{-rt}} \sum_{t=T_1}^{T_2} e^{-rt} f_t \leq F(T_1, T_2)_{BS} \qquad (10.18)$$

where BB and BS are “best buyer” and “best seller” respectively. Since equality constraints are much tighter than inequality constraints, inequality constraints are also used for natural gas. The bid/ask spread for natural gas was constructed by subtracting/adding 1 % from/to the quoted closing price. One can argue that the contracts close to maturity which are more traded should have a smaller bid/ask spread than the illiquid contracts. For simplicity we used 1 % for all contracts.

The minimization problem which must be solved to find the smoothed curve is then given by

$$\underset{\mathrm{f_t}}{\mathrm{Min}} \sum_{\mathrm{t=2}}^{\mathrm{T\text{-}1}} \left(f_{t-1} - 2f_t + f_{t+1}\right)^2$$

subject to

$$\frac{1}{\sum_{t=T_{1,j}}^{T_{2,j}} e^{-rt}} \sum_{t=T_{1,j}}^{T_{2,j}} e^{-rt} f_t \geq F\left(T_{1,j}, T_{2,j}\right)_{BB} \quad \forall\ \mathrm{j} \in \mathrm{P}$$

$$\frac{1}{\sum_{t=T_{1,j}}^{T_{2,j}} e^{-rt}} \sum_{t=T_{1,j}}^{T_{2,j}} e^{-rt} f_t \leq F\left(T_{1,j}, T_{2,j}\right)_{BS} \quad \forall\ \mathrm{j} \in \mathrm{P} \tag{10.19}$$

where

$f_t$ are variables.

$F\left(T_{1,j}, T_{2,j}\right)_{BB}$ is the best buyer price for contract j.

$F\left(T_{1,j}, T_{2,j}\right)_{BS}$ is the best seller price for contract j.

$T_{1,j}$ start of delivery for contract j.

$T_{2,j}$ end of delivery for contract j.

*T* the last week in the set.

*P* the set of forward products quoted at Nord Pool a given day.

In the electricity forward market there are overlapping contracts. According to Fleten and Lemming (2003) the minimization problem above does not have a solution if there are arbitrage opportunities present. The natural gas futures market does not have any overlapping contracts. The time needed to solve the problem increases when overlapping contracts are present. One can remove some of the contracts that overlap from the product set *P* to decrease the computational time. That is not done in this thesis.

The minimization problem is nonlinear and is solved using Matlab. The code is shown in Appendix III Some examples of the smoothed curves are presented below.

**10 year smoothed forward curve – electricity**

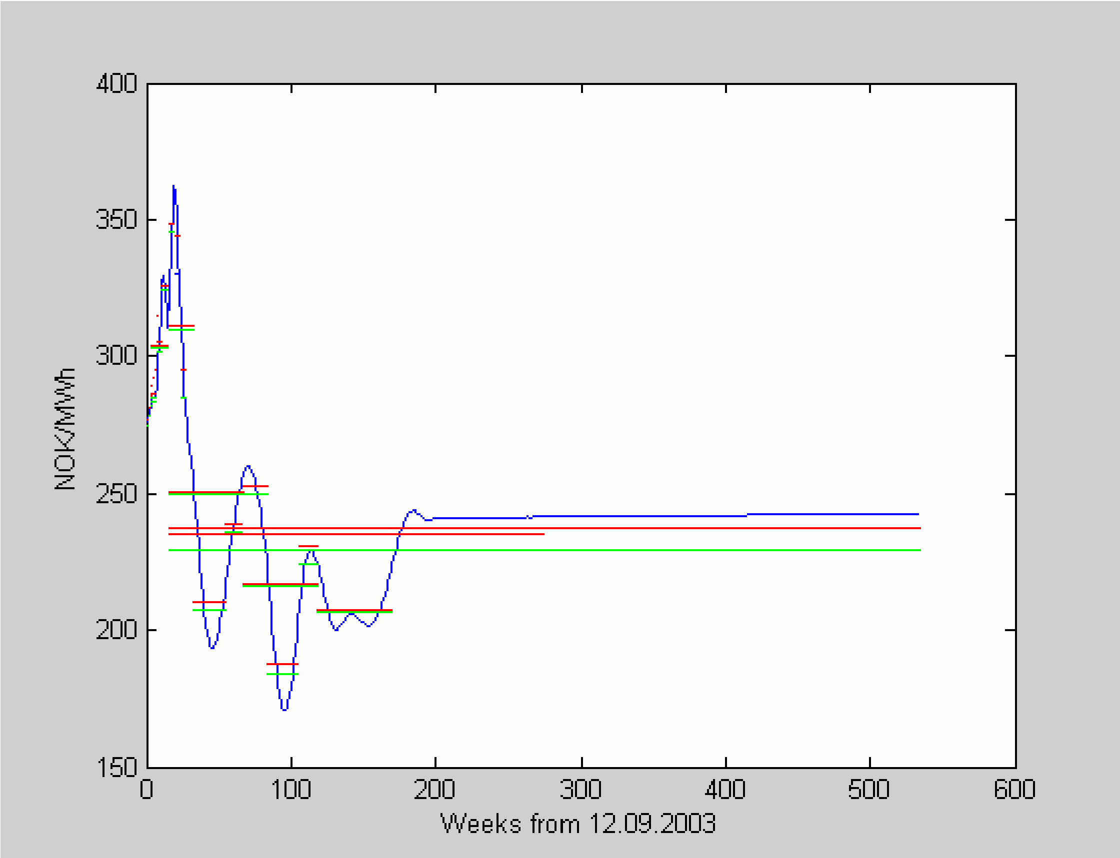



**Figure 15**: The red and green lines are prices for best seller and best buyer respectively. The length of the lines indicates the length of the contract. The blue curve is the smoothed forward curve that prices all the contracts this day.

**Smoothed forward curve – natural gas**

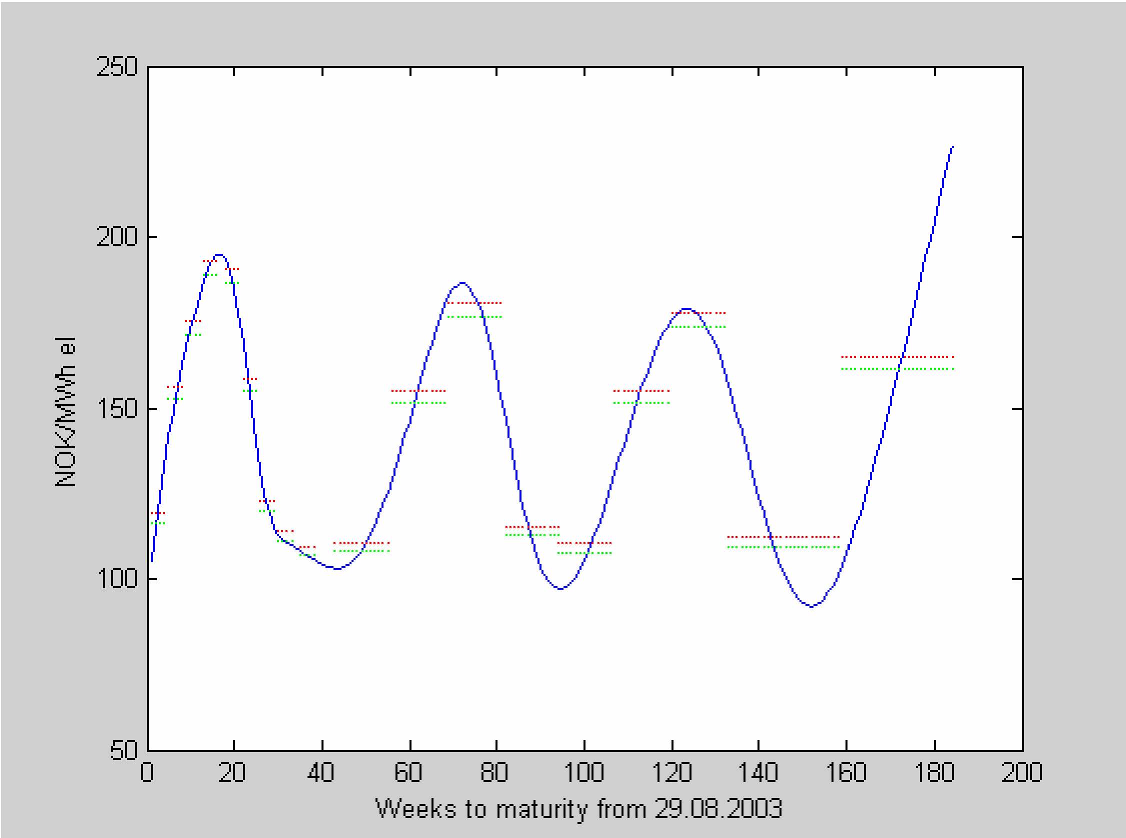



**Figure 16**: The red and green lines are prices for best seller and best buyer respectively. The length of the lines indicates the length of the contract. The blue curve is the smoothed forward curve.

### 10.4.3 Principle Component Analysis (PCA)

A Principle Component Analysis is concerned with explaining the variance-covariance structure of a set of variables through a few linear combinations of these variables. Its general objectives are data reduction and interpretation. The Principal Components are a set of variables that define a projection that encapsulates the maximum amount of variation in a dataset and is orthogonal and by that uncorrelated to the previous principle component of the same dataset. The objective is to explain as much of the total variance with as few components as possible. PCA is a commonly used technique and a thorough description can be found in e.g. Johnson and Wichern (1998).

The volatility explained by the principal components is considered systematic volatility whereas the excess volatility is considered to be specific to the variables, in this case specific to the time to maturity. The PCA is carried out so that the constructed volatility function for the whole length of the project life $R$ is based on systematic volatility only and to determine the number of independent Brownian Motions needed to explain the evolution of the forward curve.

### The equations for the PCA

A data set consists of N observation of M different variables. Each data set is gathered in an N x M matrix referred to as the Data-matrix and denoted $\mathbf{X}$.

$$\mathbf{X} = \begin{bmatrix} \mathbf{x} & \mathbf{x} & \dots & \mathbf{x} \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1M} \\ x_{21} & x_{22} & \cdots & x_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{NM} \end{bmatrix}$$

Each Data matrix has a corresponding sample covariance matrix denoted $\mathbf{\Sigma}$. Eigenvalue-eigenvector pairs for $\mathbf{\Sigma}$ are needed for the PCA. The eigenvalue matrix $\mathbf{\Lambda}$ and eigenvector matrix $\mathbf{P}$ are given below. The orthogonal decomposition of the covariance matrix is $\mathbf{\Sigma} = \mathbf{P\Lambda P^T}$. The eigenvalues are of convention ordered such as $\lambda_{11} \geq \lambda_{22} \geq .. \geq \lambda_{MM}$

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_{11} & 0 & \cdots & 0 \\ 0 & \lambda_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots \\ 0 & 0 & \cdots & \lambda_{MM} \end{bmatrix} \quad , \qquad \mathbf{P} = \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & .. & \mathbf{e}_M \end{bmatrix} = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1M} \\ e_{21} & e_{22} & \cdots & e_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ e_{M1} & e_{M2} & \cdots & e_{MM} \end{bmatrix}$$

The principal components are uncorrelated and have variances equal to the eigenvalues of $\mathbf{\Sigma}$. The matrix $\mathbf{Y} = \mathbf{PX}$ is called the matrix of the principal components and the $i$th principal component is given by equation (10.20).

$$\mathbf{Y}_i = \mathbf{e}_i^{'}\mathbf{X} = e_{i1}\mathbf{X}_1 + e_{i2}\mathbf{X}_2 + ... + e_{iM}\mathbf{X}_M \qquad i = 1,2,..., p \qquad (10.20)$$

Expressed mathematically, the data matrix $\mathbf{X}` = \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_M \end{bmatrix}$ has covariance matrix $\mathbf{\Sigma}$ with eigenvalue-eigenvector pairs $(\lambda_1, \mathbf{e}_1), (\lambda_2, \mathbf{e}_2), \dots, (\lambda_M, \mathbf{e}_M)$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$. Then $\mathbf{Y}_1 = \mathbf{e}_1^{'}\mathbf{X}$, $\mathbf{Y}_2 = \mathbf{e}_2^{'}\mathbf{X}$, ..., $\mathbf{Y}_M = \mathbf{e}_M^{'}\mathbf{X}$ are the principle components and $\sigma_{11} + \sigma_{22} + \cdots + \sigma_{MM} = \sum_{i=1}^{M} Var(\mathbf{X}_i) = \lambda_1 + \lambda_1 + \cdots + \lambda_M = \sum_{i=1}^{M} Var(\mathbf{Y}_i)$

The following proof is given by Johnson et al. (1998). $\mathbf{\Sigma} = \mathbf{P\Lambda P`}$ , where $\Lambda$ is the diagonal matrix of eigenvalues and $\mathbf{P} = \begin{bmatrix}\mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_M\end{bmatrix}$ so that $\mathbf{PP`} = \mathbf{P`P} = \mathbf{I}$. The trace of a $kxk$ matrix being the sum of the diagonal elements and is denoted $tr("kxk - matrix")$, that is $tr(\mathbf{A}) = \sum_{i=1}^{k} a_{ii}$ . Using the property $tr(\mathbf{AB}) = tr(\mathbf{BA})$ where $\mathbf{A}$ and $\mathbf{B}$ are $kxk$ m $kxk$ matrixes, then $tr(\mathbf{\Sigma}) = tr(\mathbf{P\Lambda P`}) = tr(\mathbf{\Lambda P`P}) = tr(\mathbf{\Lambda}) = \lambda_1 + \lambda_2 + \cdots + \lambda_M$

Thus

$$\sum_{i=1}^{M} Var(\mathbf{X}_i) = tr(\mathbf{\Sigma}) = tr(\mathbf{\Lambda}) = \sum_{i=1}^{M} Var(\mathbf{Y}_i)$$

The result states that the total variance can be explained as the sum of the eigenvalues and consequently, the proportion of total variance explained by a principle component is the eigenvalue divided by the sum of all eigenvalues. If large values can be assigned to the first components, then these components can “replace” the original $M$ variables without much loss of information. The volatility function given in equation (10.21) follows directly from the eigenvalue decomposition.

$$\sigma_i(\tau_j) = \sqrt{\lambda_i}(e_i)_j \qquad (10.21)$$

Equation (10.23) gives the volatility explained by the *i*th principal component for a time-to-maturity $\tau_j$.

### Choosing the number of components

All $M$ components are needed to explain all variance in the sample. There is no definite answer to how many components to retain. Determining factors in choosing the appropriate number are the amount of total sample variance explained, the relative sizes of the eigenvalues and the subject-matter interpretations of the components. One graphical method is called the scree plot[62]. With the eigenvalues ordered from largest to smallest, a scree plot is a plot of the magnitude of the eigenvalue versus its number. The number of components is taken to be the point at which the remaining eigenvalues are relatively small and all about the same size.

### 10.4.4 The data sets

The data from witch the parameters are estimated is weekly price return for electricity and gas and weekly price differences for spark spread. The observations are the Friday closing price of Nordpool and IPE from January 1999 to October 2003. The constructed forward curves yields data for each week to maturity close up to 170 weeks to maturity for every observation of both natural gas and electricity.

The spark spread price difference is discrete approximated as given in equation (10.22) and the price return of electricity and natural gas is discrete approximated as given in equation (10.23).

$$df(t_n, T_m) \approx f(t_n, T_m) - f(t_{n-1}, T_m) \tag{10.22}$$

$$\frac{df(t_n, T_m)}{f(t_n, T_m)} \approx \ln\left(\frac{f(t_n, T_m)}{f(t_{n-1}, T_m)}\right) \tag{10.23}$$

Separate data sets for spark spread, electricity and natural gas are constructed in Data matrixes as mentioned in the previous chapter. The $M$ different variables consist of $M$ series of selected weeks to maturity, and are sampled for a total of $N$ observations. The chosen weeks to maturity are considered to reflect the actual traded contracts described in chapter 4. Hence the observed series occur more frequent within the first year on the forward curve. Recall from chapter 4 the differences in term structure between IPE and Nordpool. Since the spark spread contracts are constructed from data on electricity and natural gas, the spark spread maturities are chosen to reflect both traded electricity and gas contracts in a representative way.

The week-to-maturities chosen for electricity contracts are:

[62] Johnson et al. (1998)

[1, 2, 4, 6, 8, 10, 12, 16, 20, 24, 28, 32, 40, 50, 60, 80, 110, 140]

The week-to-maturities chosen for gas contracts are:
[4, 6, 8, 10, 12, 16, 20, 24, 28, 32, 44, 48, 61, 74, 87, 100, 131, 162]

The week-to-maturities chosen for spark spread contracts are:
[2, 4, 6, 8, 10, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 61, 74, 80, 100, 140]

PCA is sensitive to outliers and a filtering procedure was conducted to detect deviating observations. The filtering was carried out since for a small number of observation dates the generated forward curves failed to give a reasonable curve structure. The missing observations lead to great leaps in price which resulted in much noise in the factor analysis. The spark spread data set was filtered for observations containing weekly differences price differences on more than 100 NOK. The data set on price returns for gas and electricity was filtered for observations containing weekly changes of more than 20%.

The data sets were tested for the assumption of normality using a Q-Q plot and an $r_Q$- test.[63] All data series failed when tested for the assumption of normality. The statistical analysis showed nonzero skewness and kurtosis. The data sets were also tested for drift but since the confidence region included zero, a nonzero drift can not be claimed under $P$ measure.

### 10.4.5 Estimation Procedure and PCA results

The PCA is carried out using Microsoft Excel with VBA code and an R-software package is used to compute the eigenvectors and eigenvalues from the covariance matrix.

[63] The methods are described by e.g. Johnson (1998) chapter 4

**Table 10**

| Factor relevance - Cumulative distribution | | | |
|---|---|---|---|
| | **Spark spread** | **Natural gas** | **Electricity** |
| PC1 | 73 % | 51 % | 53 % |
| PC2 | 80 % | 67 % | 68 % |
| PC3 | 85 % | 75 % | 74 % |
| PC4 | 87 % | 80 % | 78 % |
| PC5 | 90 % | 84 % | 82 % |
| PC6 | 92 % | 87 % | 85 % |
| PC7 | 94 % | 90 % | 88 % |
| PC8 | 95 % | 92 % | 90 % |
| PC9 | 96 % | 94 % | 92 % |
| PC10 | 97 % | 95 % | 93 % |

The table shows the cumulative distribution of the part of the total variance explained when retaining *i* components for spark spread, natural gas and electricity respectively

The cumulative distribution of factor explanatory value is given in Table 10. Scree plot is used to determine the number of components to be retained and two components are retained for all sets. Figure 17, Figure 18 and Figure 19 show scree plot for electricity, natural gas and spark spread respectively. The choice was clear in the case of electricity whereas the number of principal components could also have been one for spark spread and three for natural gas. PC2 is retained for spark spread as it gives a clear pattern in factor movement. Based on the same criteria, PC3 from the natural gas data set is omitted as it fails to show a clear pattern for the longer time-to-maturity series.

**Scree plot – Electricity**

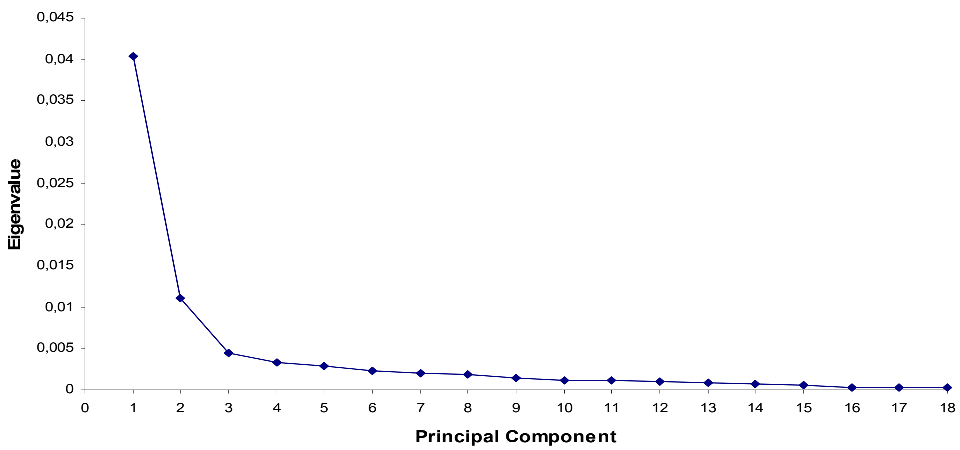



**Figure 17**: The scree plot shows that two principal components should be used for estimation of the electricity volatility

**Scree plot – Natural gas**

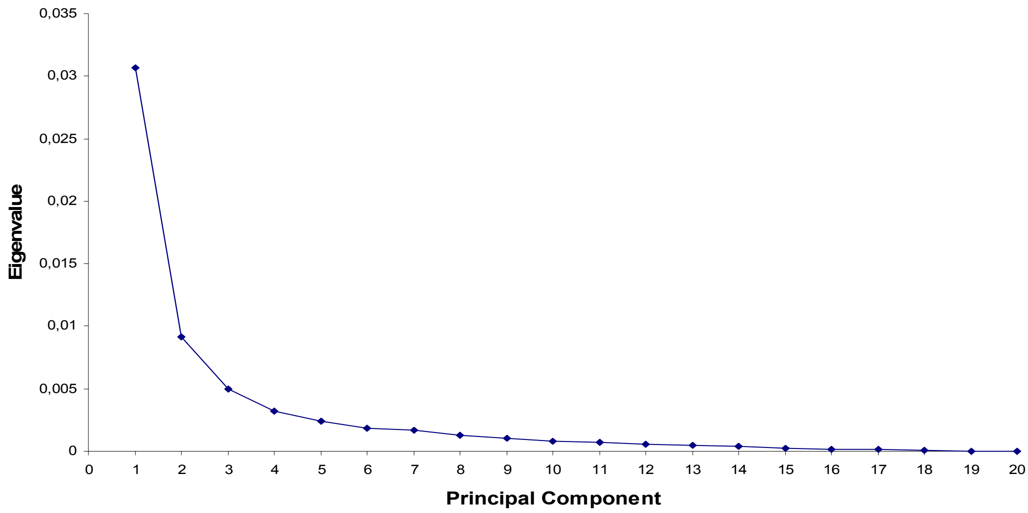



**Figure 18**: The scree plot shows that two or three principal components should be used for estimation of the natural gas volatility

**Scree plot – Spark spread**

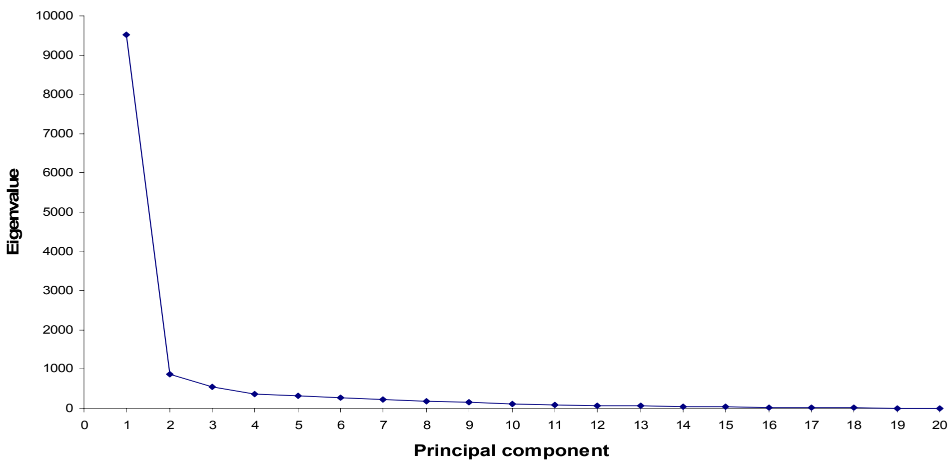



**Figure 19**: The scree plot shows that one or two principal components should be used for estimation of the spark spread volatility

Table 10 shows that the first Principal component (PC1) explains 51% of total variation in the natural gas data set, 53% in case of electricity and 73% of spark spread total variance. PC1 show correlated movements for all data series and can be interpreted as representing a shift factor. Movements will result in the values of all contracts to move in the same direction. The higher explanatory value of the spark spread model is believed a result of seasonal noise from the two original data sets being even out.

**Volatility patterns –Spark spread**

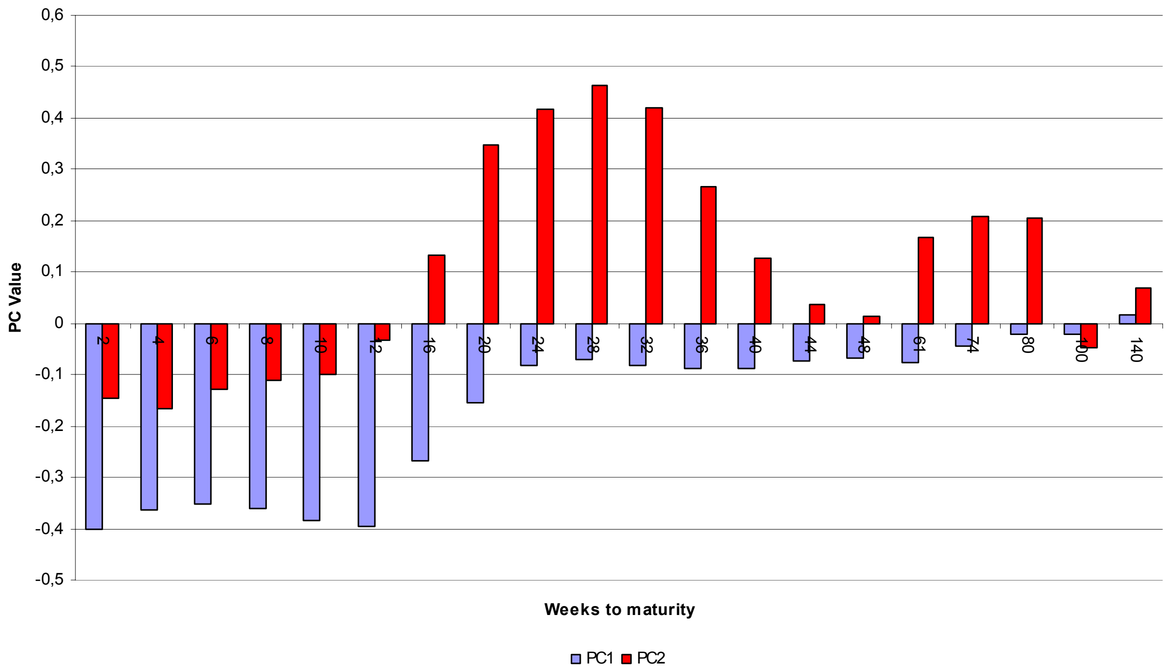



**Figure 20**: The figure shows that PC1 represents shocks that cause prices for all maturities to move in the same direction and that movements in PC2 represent shocks that cause the prices for short and long time to-maturity-contracts to move in opposite directions. A similar pattern was also found for electricity and natural gas.

The second Principal Component (PC2) represents 16%, 15% and 7% of total variability for gas, electricity and spark spread respectively. As shown in figure, PC2 causes causing short and long term contracts to move in opposite directions, and can be interpreted as a tilt factor.

PC3 showed resemblance to a smile factor, but too much noise on the longest time-to-maturity observations lead to dismissal of this component. The structures were similar for all data sets and the three Principal Components had the commonly found pattern of “shift”, “tilt” and “smile”.

The retained components explain the term structure movement common to all maturities and represent 67 % of total variance for natural gas and 68% for electricity compared to 80% for spark spread. (See Table 10) The rest of the variance is considered specific to each maturity. The spark spread data set is constructed from the data sets used for gas and electricity in the PCA. As comparison, Koekebakker et.al (2001) performed PCA on price returns and price differences from Nordpool financial data and got 68% and 70% variation explained from the first component. Further, they get equal explanatory value for price returns as for price changes, and it is therefore assumed that the differences in explanatory value between the data sets of this thesis are not a result in differences in patterns between price returns and price changes.

### 10.4.6 The volatility function

The PCA analysis was partly carried out to determine the number of independent Brownian Motions needed to explain the forward curve movements. However, as the components individual volatility reaches a constant level before the end of the construction period, there is no need to model as separate processes. If more data for the long time to maturity region was available, the volatility functions from the PCA could be modeled as independent processes. The simplification of the volatility structure is mainly affecting volatility for short time-to-maturities contracts. Assuming a construction period of 3 years[64] the short time fluctuations will not affect the project valuation and the simplification can be made without much loss of information.

The forward curve volatility was found by first adding the factor volatility of the two retained Principal Components. Recall the volatility formula for a given time to maturity $\tau$ resulting in the total volatility given in equation (10.24).

$$\sigma(\tau_j) = \sum_{i=1}^{2} \sqrt{\lambda_i}\,(e_i)_j \tag{10.24}$$

A continuous volatility function is needed for the whole valuation period. Recalling from chapter 10.2, Bjerksund et al. (2000) proposes the volatility (10.25) for the one factor model (10.3a) in the case of electricity.

$$\sigma(t,T) = \frac{a}{T-t+b} + c \tag{10.25}$$

The volatility function in (10.6) and (10.9) are assumed to follow (10.26) which have elements from the volatility functions of the previously discussed models, being most similar to that of Bjerksund et al. given in (10.25).

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[64] See chapter 6

$$\sigma(\tau) = \alpha e^{-\beta\tau} + c \qquad (10.26)$$

The continuous forward curve volatility is set to follow equation (10.26) to obtain a volatility function for the full length of the forward curve. (10.26) proposes decreasing volatility with time to maturity to a level given by the constant parameter $c$.

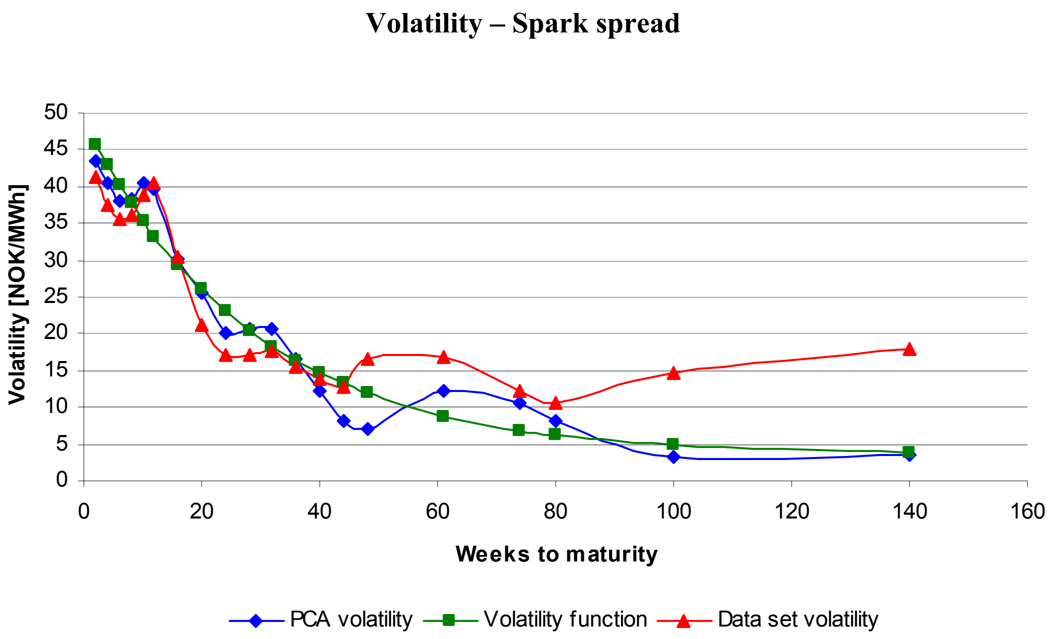



**Figure 21**: The figure shows the volatility function for spark spread graphed against the PCA volatility and the simple volatility derived from the data series.

The parameters to the volatility function (10.26) were estimated by minimizing the RMSE[65] of the difference between the volatility function and the resulting volatility from the PCA. The RMSE routine was conducted using Microsoft Excel/VBA solver. The parameter values are given in Table 11.

[65] See chapter 9.4.2 for a description of the RMSE procedure

**Table 11**

| | | | | |
|---|---|---|---|---|
| | **Parameters – Weekly volatility functions** | | | |
| | α | β | c | **RMSE** |
| Electricity | 0.0881 | 0.0284 | 0.0139 | 0.0132 |
| Natural gas | 0.0599 | 0.0437 | 0.0339 | 0.0087 |
| Spark spread | 45.4 | 0.0349 | 3.44 | |

The table displays the parameter values to be used in equation (10.26) for electricity, natural gas and spark spread.

### 10.4.7 Long term volatility

Testing the factor relevance for each week, the result states that the two principle components explain a smaller part of the total variance for the far end of the forward curve. This can also be seen from Figure 21, Figure 22 and Figure 23 where the volatility from the PCA procedure is graphed against the historic volatility from the sample set. The data set volatility is a simple volatility estimate for different weeks to maturities extracted from the constructed forward curves.

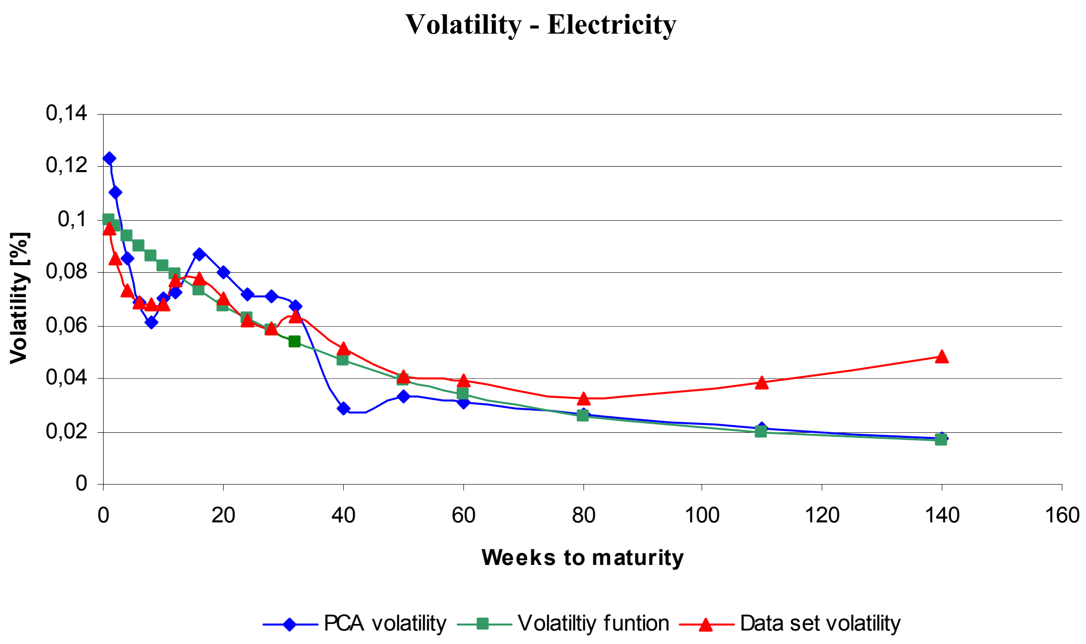



**Figure 22:** The figure shows the volatility function for electricity graphed against the PCA volatility and the simple volatility derived from the data series.

Figure 22 shows that the PCA results gives a good fit in the region on the forward curve were comprehensive data is available. The constructed forward curves might be a source of error for the long time-to-maturity contracts. For the contract with the longest time to maturity, there will only be a point of reference in the short end and by that hard to judge whether or not the constructed curve gives a good approximation for this part of the forward curve. The increase in volatility from 80 to 140 weeks to maturity may be a result of the noise generated by the constructed forward curves. The volatility derived from the PCA analysis is therefore considered to be a good measure of long term volatility despite the relatively low explanatory value.

The PCA is conducted for various other weekly combinations. Some deviation in the results occurs, especially when changing weeks with longer maturities which add to the belief of much noise in the long end of the data set. The maturities used for the PCA results presented in this thesis are considered to be representative.

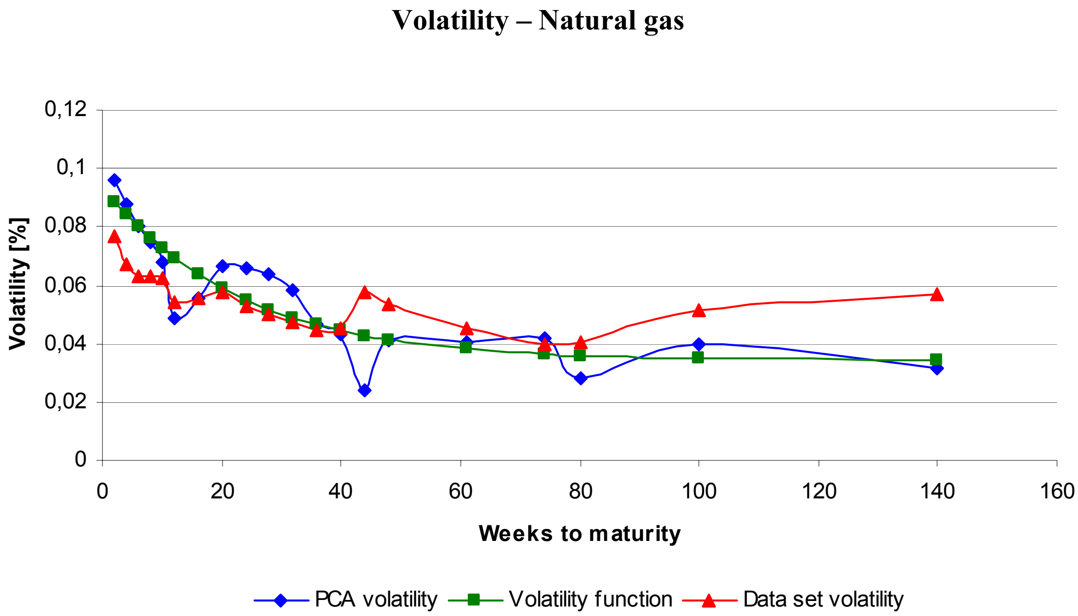



**Figure 23** The figure shows the volatility function for natural gas graphed against the PCA volatility and the simple volatility derived from the data series.

It has been assumed that the volatility factor is constant with respect to time. The volatility was tested using a non-parametric test as the data series failed the test of normal distribution.

The Wilcoxon rank sum test[66] was used to test the data set for constant volatility for various times to maturities. Several observations series with varying length were tested. The result gave a clear rejection of the presence of constant volatility.

## 10.5 Computational method – forward curve model

This paragraph describes how the calculation of the project and option value is done for the forward curve model. To remind the reader that in order to value the option under the risk neutral probability measure one must be able to set up a risk neutral portfolio. The construction of such a portfolio is repeated below. Consider a portfolio consisting of the option $W$ and $\Delta$ of the underlying variable in this case the forward curve, $f$, given by equation (10.6) or (10.9) under which is assumed to be spanned by existing assets under the special case where $i = 1$. By using Ito's lemma and setting $\Delta$ equal to $-\frac{\partial W}{\partial f}$ it can easily be shown the value of any contingent claim written on the forward curve follows the differential equations (10.27) or (10.28) depending on whether $f$ is assumed given by (10.6) or (10.9) with the appropriate boundary conditions. The dividend is $\delta$.

$$\frac{\partial W}{\partial t} + \frac{\partial^2 W}{\partial f^2}\sigma^2 + (r-\delta)W\frac{\partial W}{\partial f} - rW = 0 \qquad (10.27)$$

$$\frac{\partial W}{\partial t} + \frac{\partial^2 W}{\partial f^2}W^2\sigma^2 + (r-\delta)W\frac{\partial W}{\partial f} - rW = 0 \qquad (10.28)$$

The same result must be achieved by using the Bellman equation under risk neutral probability measure: $rdtW = \mathrm{E}^*\left[dW\right]$ where by Ito's lemma $\frac{\partial W}{\partial t} + \frac{\partial^2 W}{\partial f^2}\sigma^2 - rW = 0$. This suggests that the dividend in the forward curve equals the risk free rate, and this is consistent with the zero drift mentioned in 10.2. In the paragraphs below a procedure to solve (10.27 and 10.28) numerically is outlined.

### 10.5.1 The initial forward curve

The initial forward curve on 12.09.03 is set to be the smoothed 10 year curves over all the available contracts this day as shown in Figure 15. It is not satisfactory to use the curve with

[66] Walpole, Myers & Myers (1998)

seasonal variations as it is found in the smoothing procedure because as time evolves these seasonal fluctuations will move to the left. This is not accounted for in the model. Due to this the smoothed curves are made linear. Linear regression is used on the long ends of these curves to find the slope of the curves. Then the curves are linearly extended out to 30 years. The seasonal terms can be omitted as seasonal fluctuations will even out for when valuation the whole project period.

### 10.5.2 The computational algorithm

The forward curves mentioned in chapter 10.5.1 are not described by a specific function, but describe by discrete points in time. In order to find the value of the power plant numerical integration is used making trapezes between each point discounted back to the time of valuation. A distance between each point of one to four weeks gives a close enough approximation to the actual integral. Given the initial forward curve the procedure to find the option value is outlined below.

1. Partition the lifetime of the option into *n* exercise points.
2. Simulate the forward curve *n* periods ahead in time.
3. Find the project value at each period. Points 2 and 3 are one simulation.
4. Repeat points 2 and 3 for *m* simulations.
5. A *m* x *n* matrix with exercise values has now been generated. Use the Longstaff and Schwartz method to work backwards through the matrix. This is; for each period single out the points of where the exercise value is positive and use least squares regression to approximate a function for the value to wait. Use this function to make a decision of whether to exercise or wait at each of the paths in the simulation. The procedure was described in chapter 9.10.

### 10.5.3 The concept of a trigger value

In the spot price models the concept of a trigger value is clearly defined as the value of the state variables where the immediate exercise value is greater than the value of continuing one more period. When working with the entire forward curve the forward curve itself is the state variable and the shape of this curve can under general assumptions take any shape. However, under the model frame work outlined in paragraph 10.3 the parts of the curve used in the valuation (from 3 years) are all shifted with the same value. Thus one can assume that any future forward curve can be approximated by such a shift. It is then possible for each period in the simulation find the shift where immediate exercise is more profitable than waiting, and use this as proxy for the trigger value.

# 11 Extended models

This chapter describes some minor extensions that can be made to the models. These extensions are applied to the spark spread spot price model and the results are given in chapter 14.4.

## 11.1 Investment and taxes

In real investments taxes should be considered since most companies pay taxes. It is assumed that the plant will not pay property tax. Considering perfect capital markets, financing have a zero net present value so tax deduction of interests will not be considered in the following. When including income taxes the power plant will be deprecated at 5 % per year according to Norwegian tax legislation[67]. To include income taxes equation (9.1) and (9.2) must be changed to:

$$V(\Theta) = \sum_{i=t+t_u}^{t+t_u+L-1} Min\left((1-\upsilon)\Omega_i, \Omega_i\right)$$

where

$$\Omega_i = \int_i^{i+1} e^{-r_\upsilon(\tau - t)}\left[KE\left(F_{t,\tau}(\Theta_t, \tau) - R - Q(\tau)\right) - D - G\right]d\tau \tag{9.1b}$$

$$r_\upsilon = (1-\upsilon)r$$

$$NPV = V(\Theta) - I + \frac{I\upsilon d}{r_\upsilon + d} \tag{9.2b}$$

The last term in equation (9.2b) is the infinite sum of tax shields due to depreciations.

## 11.2 Uncertainty in future $CO_2$ price

There is a great uncertainty about what future $CO_2$ emissions will cost. As described in chapter 7 an extensive quota market is planned in the EU, and at the Chicago Climate Exchange such quotas are already traded. Assuming that such markets exist and that the

[67] Skatteloven §14-43 (1) g elektronisk utrustning i kraftverk

prices on the traded vintages span all the risk in the future $CO_2$ price, uncertainty in $CO_2$ prices can be modelled as geometric Brownian motion.

The model for the price, C, on the closest vintage is then assumed to follow the model $dC = \alpha C dt + s C dZ$ where $\alpha$ and $s$ are the risk free drift rate and volatility respectively. This model can be fitted to market data if such data exists. As mentioned in chapter 7 such data is still scarce, and a drift rate of zero and volatility at 20% are used in the calculation example. This must only be considered as an example of incorporating cost of $CO_2$ emission in the model. The $dZ$ term is considered to be uncorrelated with other state variables.

# 12 Competition

It has been assumed in the previous chapters 8 to11 that the option holder has monopoly on the option to invest and/or that the forward prices reflect the markets expectations of new generation capacity completely. Del Sol and Ghemawat (1999) argues that typical option valuation models fail to recognize the competition for a limited number of investment opportunities and tend to recommend waiting too long before investing. A general approach to real options and competition is given by Dixit et al (1994). The literature published on real option valuation where competition is taken into account follows two main approaches. The first uses an approach with market equilibrium under the assumption of perfect competition and free entry as done by e.g. Leahy (1993). The second line uses a game theoretic approach as done by e.g. Grendadier (1996) and Murto and Keppo (2002). Game theory and Real Options is a relatively new field of research and several papers on the topic have been published in the recent years. Using a game theoretic approach, the models will be expanded to include the presence of multiple option holders and used to look into the impact of competition.

## 12.1 A Game theoretic approach[68]

The game theoretic concept models the option holders as players with a set of actions and strategies. The actions a player can undertake in the given setting will be "invest" or "not invest". A strategy tells which action to execute under given conditions. If more than one strategy are available, the one resulting in the most favorable outcome is called the dominant strategy, and will be the strategy chosen.

The Nash equilibrium is a basic solution concept in game theory and is used to find the dominant strategies. Tirole (1998) defines it as "if a set of actions is in Nash equilibrium if, given the actions of its rivals, a firm cannot increase its own profit by choosing an action other than its equilibrium action".

Recall from chapter 9.3 that the players invest when the project value reaches a trigger level. The strategy when holding the monopoly to invest can be formulated as "invest if the trigger level is reached". It is in the following discussion assumed that there is only one, or at least a

[68] A thorough description about game theory is given by Tirole (1998)

limited number of investment opportunities available and that all option holders are rational investors.

A scenario of limited investment opportunities is based on the assumption that new generation capacity exceeding the market's expectation will lead to a decrease in electricity prices, in turn decreasing the profit from building additional base-load plants. In the Norwegian market, construction of one power plant is believed to reduce the willingness to invest amongst other actors. There will be a first mover advantage affecting the option value. An assumption of only one investment opportunity results in zero option value for all competitors when an investor exercises the option. The assumption implies that if there is a positive project value, the investor will exercise the option early to prevent a losing the project to a competitor.

The impact of competition on the investment decision is dependent on the given situation at hand. An important factor is the information on the competitor. Including competition lowers the option value and in some cases reduces the investment decision to a simple NPV decision.

### 12.1.1 Full information

In the case of full information on competitors' project value, the strategy can be formulated as invest just before the trigger value of the competitor is reached as long as there is a positive project value. If the option holders have identical cost structure the investment decision is reduced to a simple NPV decision.

In the case of a natural gas fired power plant, differences in cost structure would most likely be caused by the different need to invest in infrastructure on the various locations, e.g. the extra cost of building a pipeline to Skogn or Grenland compared to Tjeldbergodden where the pipeline is already in place.

### 12.1.2 No information

In the case of no information the trigger value of the competitor is unknown. An approach done by Murto et al. (2002) is to model the chance of a competitor exercising the option by a "hazard rate" denoted $\lambda$. The hazard rate is modeled by including a Poisson term in the option value formula where the value of the Poisson parameter reflects the probability of a competitor making the investment. The investments trigger decreases with increasing λ. As the likelihood of competitors making investments increases, other investment holders will not dare to wait in fear of loosing the investment opportunity.

The resulting Nash equilibrium has a trigger level somewhere between the monopoly trigger and the level giving zero NPV depending on the value of λ. Murto et al. (2002) model λ as a decreasing function of the competitors trigger level, representing an increased risk of exercise from competitors when their trigger level is believed to be low.

The hazard rate can be modeled by making λ dependent on the price level. If implemented to the simulation procedure described in chapter 9.10 and 10.5, λ will be large when the simulation path reaches high levels and by that reducing the option value further. The main problem in using this approach is the difficulty to provide an estimate for λ as the hazard rate is no more than the firm's beliefs of competitors' investment likelihood. If the option holders use the same governing processes for valuation, their project values will be highly correlated. A high project value will lead to high probability for a competitor making the investment.

## 12.2 Modelling Competition

A Poisson term is added to the model to incorporate competition and the possibility of losing the investment opportunity. Any contingent claim written on the equilibrium variable now evolves according to equation (12.1) given below.

$$dW = (W_t + \mu W_\varepsilon + \frac{1}{2}F_{\varepsilon\varepsilon}\sigma^2)dt + W_\varepsilon \sigma dz - Wdq \qquad (12.1)$$

where $dq = 1$ with probability $\lambda dt$ and $dq = 0$ with probability $1 - \lambda dt$. A risk free portfolio is constructed in the same way as in chapter 9.3.2 and the differential equation is now given by (12.2). Solved with boundary conditions (9.8d) the solution to (12.2) is given in (12.3) when setting $W_t = 0$ since the option is assumed to have an infinite life time.

$$\frac{1}{2}W_{\varepsilon\varepsilon}\sigma^2 + \mu W_\varepsilon - (\lambda + r)W = 0 \qquad (12.2)$$

$$W(\varepsilon) = \frac{C_1}{\beta_1} e^{\beta_1(\varepsilon - \varepsilon^*)}$$

$$\beta_{1,2} = \frac{-\mu \pm \sqrt{(\mu)^2 + 2\sigma^2(r+\lambda)}}{\sigma^2}, \beta_1 > 0, \beta_2 < 0 \tag{12.3}$$

$$\varepsilon^* = \frac{C_1 - C_2\beta_1 + I\beta_1}{C_1\beta_1}$$

If more investors can be viewed upon as competitors, the the spark spread level which triggers investment is for the i'th firm is given as

$$\varepsilon_i^* = \frac{C_1 - C_2\beta_i + I\beta_i}{C_1\beta_i}, \qquad \beta_i = \frac{-\mu + \sqrt{(\mu)^2 + 2\sigma^2(r+\lambda)}}{\sigma^2} \tag{12.4}$$

As $\beta_i$ increases with λ, the spark spread trigger level decreases towards that of the NPV decision. To the firm holding an option, the only matter is whether some competing firm exercises its option, not which firm. The total hazard rate can then be written as $\sum_{j \neq i} \lambda^j (\varepsilon_j^*) = \lambda_C$ where $\lambda^j$ is the hazard rate of the *jth* firm, leading to the Nash equilibrium in equation (12.5)

$$\varepsilon_A^N = \frac{C_1 - C_2\beta_A^N + I\beta_A^N}{C_1\beta_A^N}, \quad \beta_A^N = \frac{-\mu + \sqrt{(\mu)^2 + 2\sigma^2(r+\lambda_C)}}{\sigma^2} \tag{12.5}$$

*A* denotes "our" firm,
*C* denotes competition

A lower trigger level results in earlier investment. As firms are allowed to optimize their trigger level with respect to their beliefs, which in turn affects others' trigger levels leads to Nash equilibrium for the trigger level of all firms. The model was implemented in Microsoft Excel and the result is presented in chapter 14.5. Only the infinite option to invest was considered.

# 13 Risk management

Risk management involves identifying the sources of risk affecting the option and project values. Risk can be avoided by hedging the various sources of risk. In the case of holding an option to invest, risk management will concern hedging the option to invest until its exercise, and hedging of the project value once the decision to invest is made. Readers are advised to see e.g. Hull (2003) for an in-dept discussion about hedging and risk management.

The main sources of risk identified are the price development in gas and electricity prices. Other sources of risk are the cost of $CO_2$ emission, currency exposure and political risk. A hedging strategy using financial energy contracts are presented for the risk associated with the development in the prices electricity and natural gas. The other sources of risk will be limited to a discussion only. This thesis only considers hedging within the framework of the spot price models.

## 13.1 The risk of $CO_2$ emission cost and political and currency risk

The cost of $CO_2$ emission can be hedged by buying emission quotas, options on quotas or using other Kyoto mechanisms. Hedging beyond the end of the Kyoto period of 2012 will be rather speculative as the framework for trade is yet to be determined.

If the marginal electricity producer is subject to $CO_2$ emission costs, the variations in emission costs will be reflected in the electricity price. Hence the risk associated with $CO_2$ emission costs will largely be embedded in the electricity and gas prices as these market prices will reflect the increase in marginal production cost and demand respectively.

Political risk is defined as the uncertainty about government intervention and policy change that results from having to deal with various governments. Political risk is considered to be minimal in Norway, however, ranked slightly behind countries such as USA and UK.[69] Although political risk in general is considered to be small, it will increase in controversial issues such as the building of a gas fired power plant.

[69] Click and Coval (2002)

Norway's struggle to meet the countries Kyoto obligations on $CO_2$ emissions as mentioned in chapter 7 have made several political parties refuse the building of gas fired power plants without $CO_2$ sequestration facilities. Despite the harsh debate regarding emission growth, the political risk should be considered minimal once the option is given. There has not been serious attempt to withdraw the options already given. Political pressure and lobbyism against the power plant poses a sort of indirect political risk. Member of the Norwegian parliament Klungland has expressed there has been considerable political pressure on the firms holding the option to invest.[70]

Although political risk cannot be directly hedged, it should be dealt with and assessed to minimize it. Such a strategy will involve employing lobby firms, (calling in favors and supporting political campaigns) to counter the opponents' efforts to persuade the decision makers. Handling political risk can easily become a grey area if the company does not have a set of strict rules to follow and should be conducted with outmost care.

The electricity price, natural gas price and investment cost contribute to the largest cash flows during the project lifetime. If noted in different currencies, currency fluctuations could lead to large variations in net results. Disregarding sales to end users, the electricity will be sold in EUR. With the full liberalization of the European market for natural gas, representative natural gas contracts will be listed in EUR as well. The investment costs are largely made up of the turnkey costs which are sold at an international market with USD as the main currency of reference. However, producers are located worldwide, some with EUR as main currency. It should be possible make the investment in EUR or make a currency hedge at the time of investment. All things considered it should be possible to make currency exposure minimal and currency hedging superfluous.

## 13.2 Hedging against price movements in electricity and natural gas

The hedging of electricity contracts and other flow commodities are different from hedging stocks. Koekebakker et al. (2001) warns against hedging long term contracts by short term contracts as the long and short time-to-maturities contracts are influenced differently by movements in various factors. In order to hedge the option value, there has to be an asset or

[70] Werner (2004)

financial contracts that are governed by the same underling process as the option value. The option value is determined by development in the long term variable for the spot model, and by shifts in the forward curve in case of the forward curve model. Under the assumption of spanning assets as presented in chapter 8, hedging is possible.

Recalling from chapter 4, only forward contracts with up to 3 years to maturity are traded at Nord Pool and the IPE. The long time-to-maturity contracts traded can be used as a hedging instrument in the forward curve model if they follow the same process as the determining variable in the valuation models. As pointed in the following the strong correlation between the movements in the long term variable and the forward contract with the longest time-to-maturity make them suitable for hedging purposes. As long as the listed forward contracts provide a sufficient hedge the 10 year OTC contracts are avoided as they are even more illiquid. The time-to-maturity is kept high by rolling the hedge over to contracts with longer time-to-maturity as time passes and new contracts come available.

### 13.2.1 Hedging of the option to invest

The delta of an option is the sensitivity of the option to the underlying. Delta is given as (13.1) and can be discrete approximated using (13.2). The delta hedge is made by constructing a new portfolio consisting of the option to invest and a short position in "long term" financial contract. Such a contract is not traded at exchange and the year three forward contract is used as a proxy. A delta-neutral portfolio is hedged against the random movements of the underlying.

$$\Delta = \frac{\partial V}{\partial S} \tag{13.1}$$

$$\Delta = \lim_{h \to 0} \frac{V(S+h,t) - V(S-h,t)}{2h} \tag{13.2}$$

The option value is estimated twice using Monte Carlo simulation, finding the values of the option for small step change, $h$, in the underlying, $S$, to be $V(S+h)$ and $V(S-h)$ at time *t*.[71] The error when using Monte Carlo simulation is quite large and is given by the

[71] Wilmott (2001) pp 465

function $O(1/hN^{\frac{1}{2}})$. To reduce the error term the two values could be estimated using the same set of random numbers, and the errors in the MC simulation will cancel each other out.

The option delta on stock when the stock price itself is the underlying is just a ratio telling how many stocks to go long/short to hedge the position. In this setting however, examining the denomination of the delta reveals that delta is denominated in MWh. The delta then says how many MWh of the underlying needed hedge the position.

### 13.2.2 The size of delta

The delta for the spark spread model is calculated to be 40 TWh using step of 1 NOK/MWh making it necessary to enter into a short position covering that amount of energy. To put it into perspective one need 4560 one year contracts or 456 ten year contracts of 1 MW. Spark spread is not a directly tradable in Norway. Spark spread contracts can be entered OTC by companies such as RWE trading in Germany. It should be possible to enter into similar contracts in Norway, but possibly at a higher liquidity premium. Figure 24 display how correlated the movements of the long term variable are to the price of the year contract maturing in the third calendar year. The actual correlation is 0.8 over the whole period. This suggests that contracts with long time to maturity can theoretically be used in hedging the real option.

In the models where natural gas and electricity are modeled separately, delta can be related to contracts traded at the IPE and Nordpool. The electricity delta the current date was 102 TWh, and the natural gas delta was -114 TWh electricity equivalents, corresponding to -196 TWh or -6.7 Gtherms natural gas. This means that in order to hedge the real option the license holder must short 102 TWh electricity corresponding to 11644 one year contracts or 1164 ten year contracts, and enter into a long position of 6.7 Gtherms corresponding to 7444 seasonal contracts.

For electricity the contract maturing in the third calendar year is closely correlated to the long term variable of electricity, having a correlation coefficient of 0.93 over the whole period from 2000 – 2003. The two are graphed in Figure 25. Liquidity on the contracts is very low with a daily traded volume at Nord Pool of only a few to zero MW. Thus the contracts will have to be bought OTC. Entering into such a hedge using Nord Pool as the clearing house this hedge would consist of 3 % of the total contract volume traded and cleared at Nord Pool.

Considering natural gas holding an equal amount of the two seasonal contracts at IPE could work as a proxy for the long term natural gas variable. The correlation between the long term variable and a portfolio consisting of both the seasonal contracts are 0.77. Figure 26 graphs the two. Also for natural gas there is liquidity problem. Using futures the hedge will be about 6 times the total yearly volume traded of natural gas at the IPE.

Holding a delta-neutral position could mean constant rebalancing and rising transaction costs, especially if the products are illiquid. Gamma, the second derivative of the position with respect to the underlying, is used as a measure to how often and how much a position must be rehedged to remain a delta-neutral position. Gamma can be calculated similar to delta using Monte Carlo simulation.[72] A gamma close to zero implies that there is no need for rebalancing of the portfolio and managing the hedging strategy reduces to roll the portfolio over to new contracts as they come available. If gamma is different from zero the portfolio can be gamma hedge by adding other options to the portfolio. This is not discussed further in this thesis. Active rebalancing of the hedge will lead to increased transaction costs.

So far it has only been considered to enter into existing contracts. A better hedge, however, assuming that the price would span the risk in the long term variable would be to enter into a contract for delivery/purchase of 800 MW 7600 hours per year in a period long enough to cover the needed energy amount. I.e. when delta is 102 TWh this would result in entering into a contract for 16.8 years. An issue would be when the contract should mature. One suggestion would be to start the *building-time-years* after the expected long term variable reaches the trigger level. As the equilibrium price raises the option delta will approach the total production of the planned power plant so when investment is commenced the owner has sold the whole production forward from the day production starts. It is not likely that any counterparty would engage in such a contract.

[72] Wilmott (2001)

### Example

Considering a firm holding the license to invest in a gas fired power plant. At the day of calculation the long term electricity price was 166 NOK/MWh and the option value was 5017 MNOK. A move of the long term state variable of 1 NOK/MWh would increase the option value to 5139 MNOK. The option delta is 102 TWh and hedging would result in selling 11644 one year forward contracts maturing in the third calendar year at 214 NOK/MWh totally worth 21828 MNOK. When the underlying long term variable increases 1 NOK/MWh also the forward contract is assumed to rise likewise and the short position would be worth 21930 MNOK (remember that correlation is 0.93.) This would lead to a gain on the real option of 122 MNOK and a loss on the short position of 102 MNOK almost preserving the market value of the firm. The inaccuracy is due to the numerical computed delta.

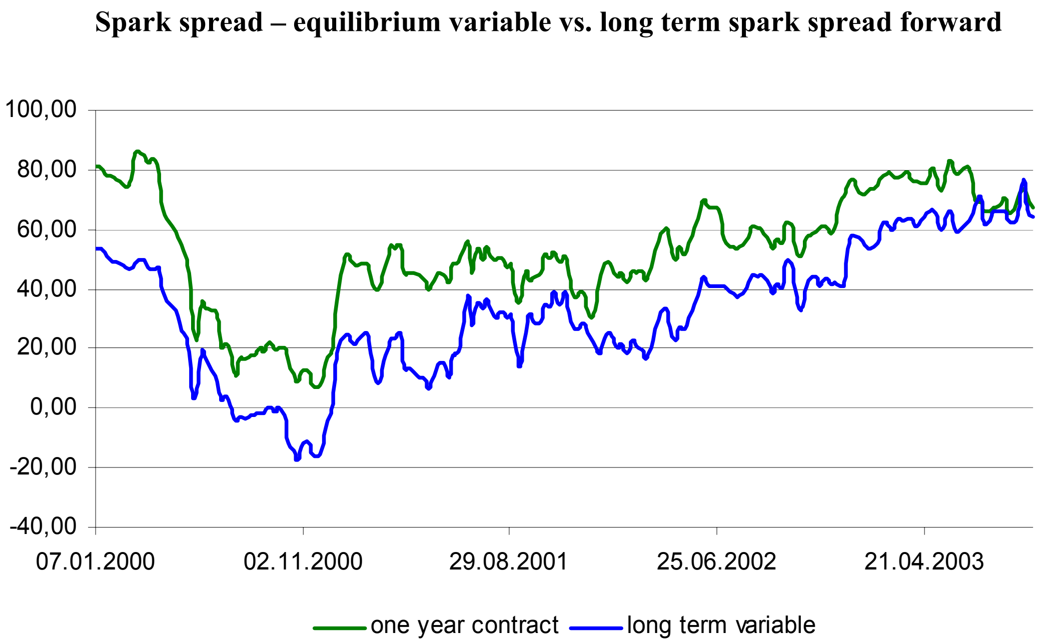



**Figure 24**: The figure displays the long term state variable of spark spread versus the price of the one year spark spread contract maturing in the third calendar year from today.

**Electricity – equilibrium variable vs. long term spark spread forward**

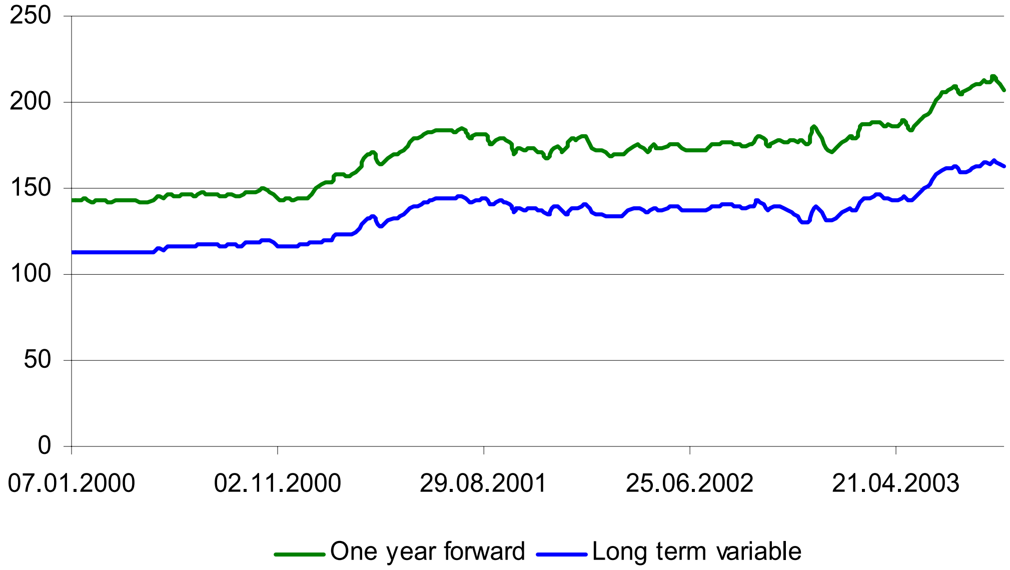



**Figure 25**: The figure displays the long term state variable of electricity versus the price of the one year electricity contract maturing in the third calendar year from today.

**Natural gas – equilibrium variable vs. long term spark spread forward**

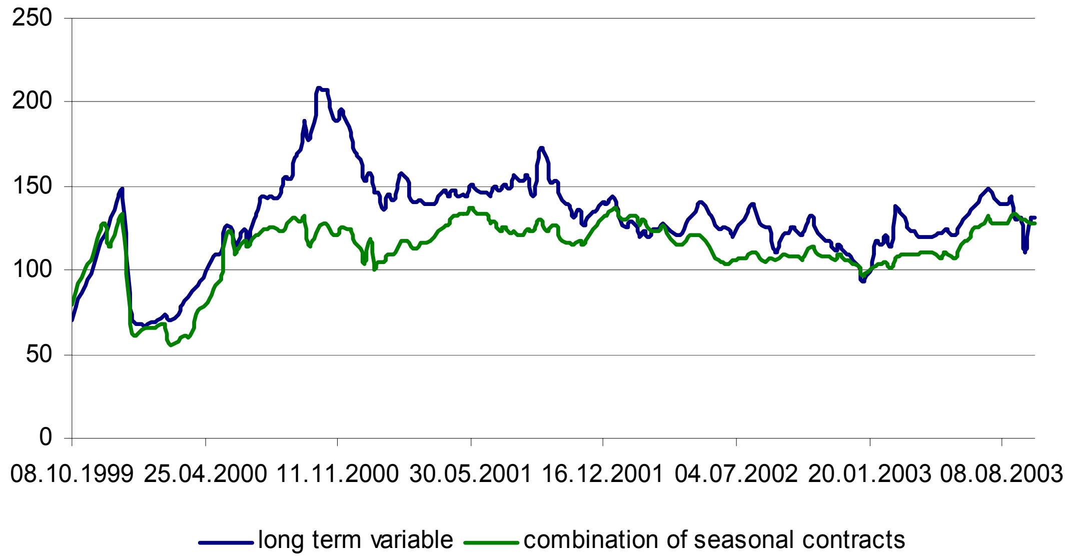



**Figure 26**: The figure displays the long term state variable of natural gas versus the price of the average price of the two seasonal contracts available.

### 13.2.3 The cost of hedging and accounting issues

Movements in the option value will not result in cash flows or change the book value. The forward contracts do not require cash settlement until the price is fixed for the delivery period. If a rolling strategy is followed, a cash settlement must be made as the hedge is rolled over to the next year forward contract. Cash settlement will also be made when rebalancing the hedge. How the outlays are balanced against the option value will be an accounting issue. The risk of cash outlays will make project managers more reluctant to hedge the option to invest. In addition large security deposit and a bank guarantee will be needed. The natural gas futures contracts will require a daily cash settlement.

The yearly cost of holding a rolling delta hedge of 102 TWh of electricity will cost almost 4.5 MNOK a year in clearing fees. Holding a rolling delta hedge of 6.7 Gtherm of natural gas will lead to clearing fees of around 4.5 MNOK. Assuming broker fees in the same range, the yearly cost of maintaining delta will come at a cost between 15 and 20 MNOK. The numbers do not include rebalancing of the portfolio in-between the yearly rollover. Holding a delta hedge for the full length of the option period is associated with considerable costs even when the cost of bank guarantees is not considered.

Using the hedging strategy outlined above the market value of the company owning the license will be maintained as described in the example above. However, the real option will never show in the financial statements of the firm, whereas the gain and losses on financial contract will be recorded. This can result in large influences on the income statement. If the 102 TWh is sold forward at price of 214 NOK/MWh and the price increases 20 NOK/MWh over the year assuming the delta is fairly constant the position has lost more than 2000 MNOK. This would reduce the net income by 1440 MNOK assuming 28 % corporate tax and profit being made. This would mean a decrease in profits of 9 % for Statoil and 13.5 % for Norsk Hydro.  Such a scenario is not unlikely as prices on ENOYR-06 rose by more than 20 NOK/MWh in the first half of 2003. No board of directors would probably ever put such numbers at risk since this would shake the share price and investors would leave.

## 13.3 Hedging the project value

In theory, the project value can be completely hedged by entering long term contracts for the entire project lifetime. In the case of electricity there would be substantial risk premium involved due to the illiquidity in the market for long term contracts. Whether or not the given exposure should be hedged depends on the option holders risk aversion. A strategy can be to hedge larger part of the production as time to maturity approaches. There is no best way of how to hedge project risk. Using illiquid contracts may cause a considerable premium to be paid and would therefore be avoided by most investors.

The Take-or-pay contract will be a long term contract. If it contains provision linking it to the development of the electricity price, it will pose a natural hedge. The take-or-pay contract can be indirectly hedged by hedging for the price provisions involved.

## 13.4 The investment opportunity as an hedging strategy

The strategies suggested are valid only if the project is a separate project or not linked to the other business activities of the option holders company or its owners. The option to invest in power production facility can e.g. be a hedging strategy for large energy consumers. The real option can be a better way to hedge against a future rise in electricity prices than entering illiquid long term contracts. If the option is used as a hedge or is a small part of a bigger strategy, the hedging strategy must be based on the net exposure of the entity as a whole.

# 14 Calculations and Results

This chapter provides the results from the empirical analysis. Below the general results are discussed briefly. Then sensitivity analysis and important parameters are presented. When performing sensitivity analysis only one parameter is changed at a time unless otherwise stated. At the end of this chapter the effects of adding income tax and uncertainty regarding $CO_2$ to the model are also shown.

The main results from the calculations are shown in Table 12. As is seen all models consent on the same investment decision, to delay investment. The value of investing today is positive in the spot price models, but negative in the forward curve models. The values of waiting vary from 2500 MNOK to 5000 MNOK.

**Table 12**

**Main results**

| | **Spot price models** | | **Forward curve models** | |
|---|---|---|---|---|
| | Spark spread | Electricity/ natural gas | Spark spread | Electricity/ natural gas |
| Value of investing today | 13.00 | 238.5 | -504.9 | -504.9 |
| Value of waiting | 2849.20 | 5017.7 | 3027.9 | 2511.5 |
| Decision | Wait | Wait | Wait | Wait |

All numbers are in million NOK. The table shows investment decision under the base case scenario. All models agree on delaying the investment.

## 14.1 Spark spread – spot price model

The investment decision of this model stated in Table 12 does not change when the underlying parameters are changed one at a time. $\kappa$ must be close to zero in order to increase the NPV significantly. $\sigma_X$ can take on any value without changing the option value. Changing $\mu$ does not alter the decision, probably due the level of $\sigma_\varepsilon$. An interesting observation is that setting $\sigma_\varepsilon$ close to zero still gives positive option value. This means that even in a world of certainty one should not invest before the day of license expiry. $\mu$ and $\sigma_\varepsilon$ seems to be important parameters. Figure 27 shows a 3 dimensional plot of the option value and exercise value when both parameters are changed. The blue surface is the NPV of exercising the option today, the black surface is the zero plane and the rainbow coloured surface represents the option value. As the figure shows, even if the volatility is zero the investment decision is not changed.

**Sensitivity to long term drift and volatility**

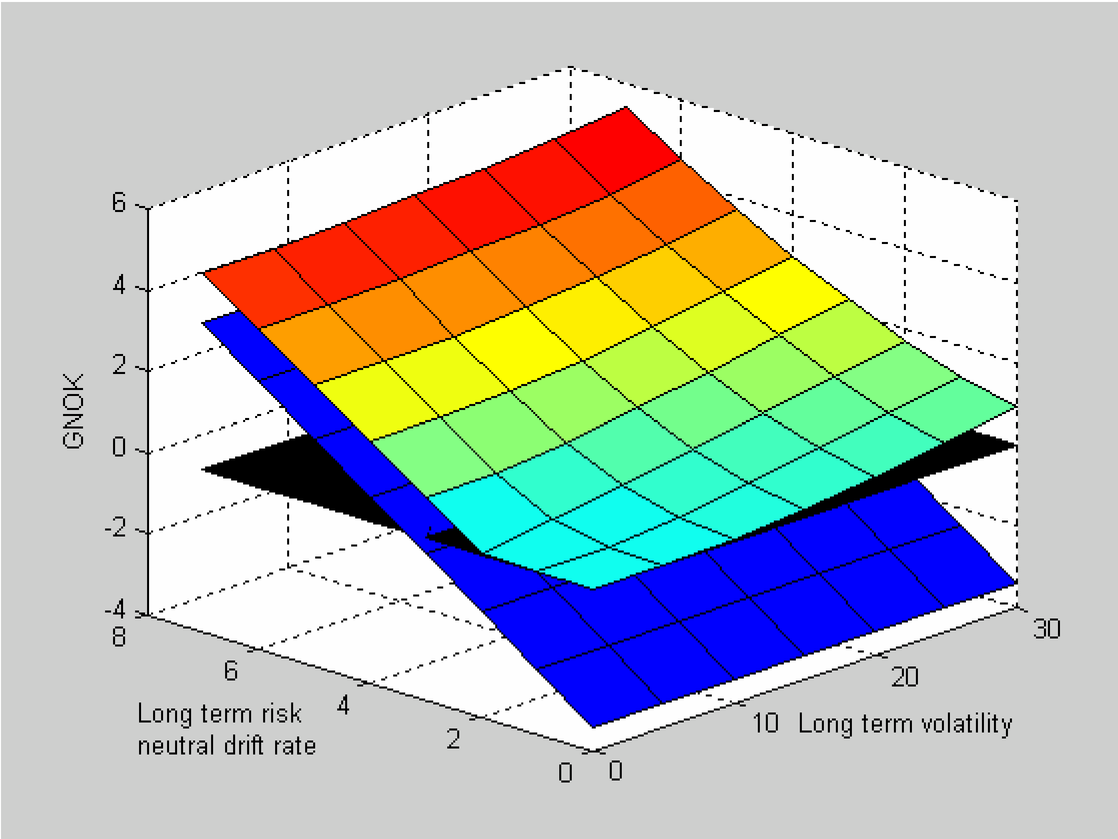
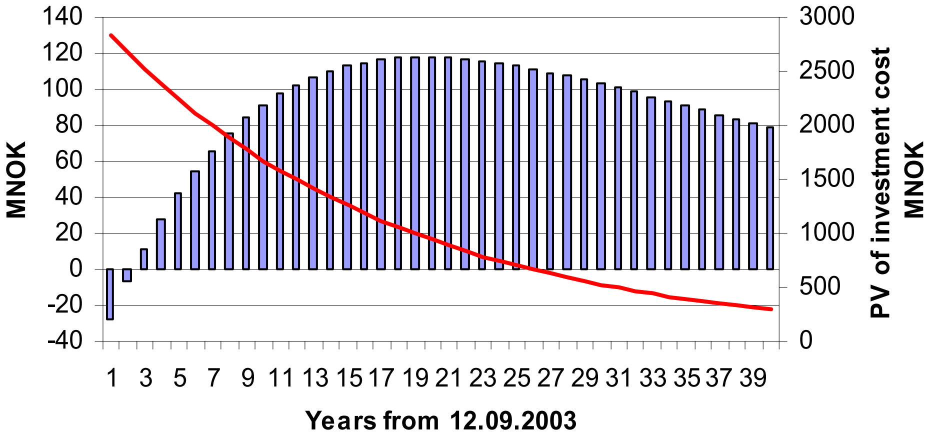



**Figure 27**: The figure shows the option value and the NPV as a function of $\mu$ and $\sigma_\varepsilon$. The rainbow coloured surface represents the option value, the blue plane is the NPV. The zero level is also shown by the black plane.

The fact that the option has value even in a certain world needs more investigation. Panel A and panel B in Figure 28 show the PV of the expected yearly cash flows from the gas fired power plant considered here and the NPV of investing *t* years from now respectively. The PV of the cash flows increases by a decreasing ratio out to about 20 years. After that the PV of cash flows decreases slowly. If the investment opportunity is a now or never opportunity it should be commenced today since the NPV is greater than zero. The investor would then be entitled to the 30 next cash flows starting with the 4$^{th}$ cash flow since the construction time is 3 years. However, if the world is certain and investor has the option to wait, she can choose to wait another year and for sure be entitled to the 30 next cash flows starting with the 5$^{th}$ cash flow. From the graph in panel B of Figure 28 one can see that the NPV of investing next year is greater than the value of investing today. Actually it is better to wait 17 years if the option lasts that long.

**PV of cash flows and investment under certainty**

Panel A




Panel B

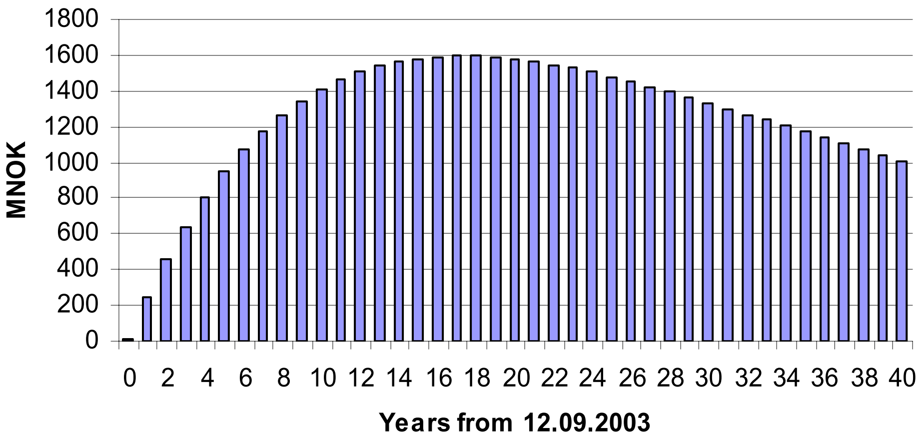



**Figure 28**: Panel A show the PV 12.09.2003 of the future expected cash flows on the left axis and the PV of the investment cost on the right axis. The line curve is the PV of the investment cost. Panel B shows the NPV of investing *t* years from now.

The investment trigger is here defined as the value of the underlying state variables where investing today is at least as profitable as waiting. In Figure 29 there is a surface plot of option value and NPV as functions of the state variables. As can be seen from the figure the investment decision is almost insensitive to the short term variable. In the figure below the importance is exaggerated due to low resolutions on the chosen state variable used to make the plot. This suggests that one could find a trigger value only for the long term variable.

Doing this one find the trigger value on the long term variable to be about 203 NOK/MWh compared to today's value of 76 NOK/MWh. This is represented by the red line in the figure. The blue line is the current level. This suggests that there must be a significant change in the long term variable in order to make early exercise feasible.

**NPV and option value as function of state variables**

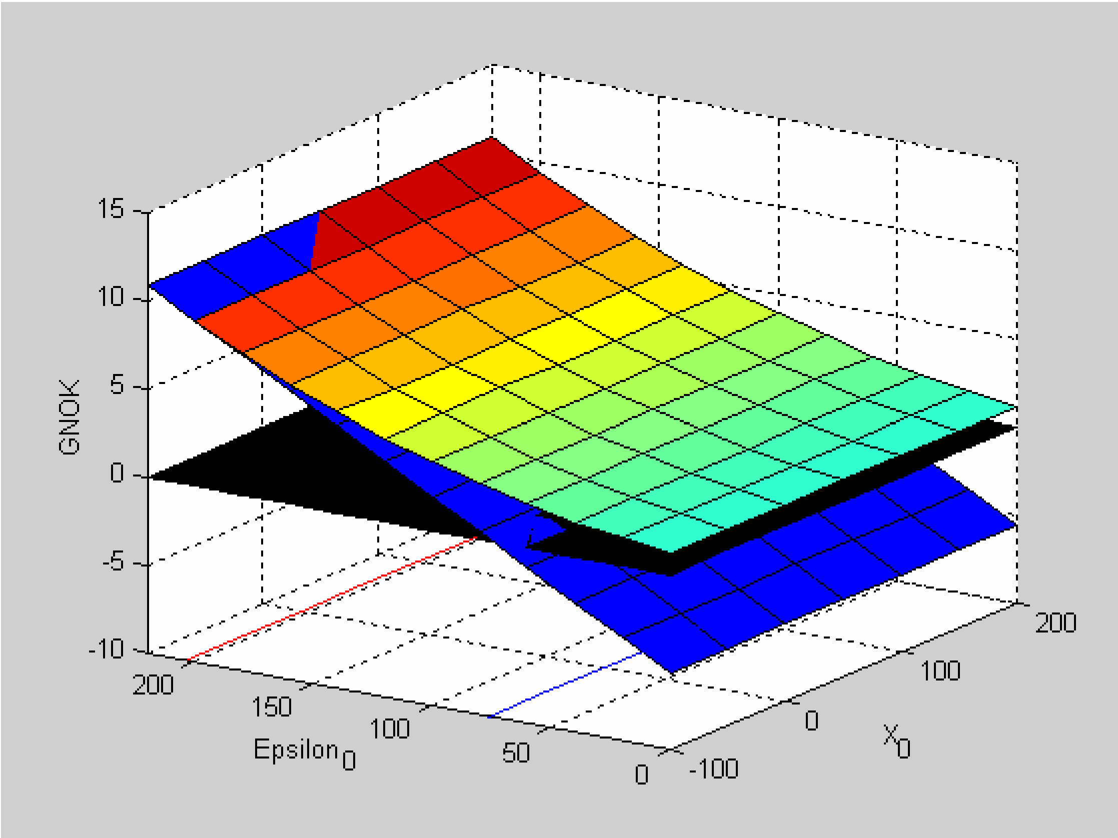



**Figure 29**: The figure shows the option value and the NPV as a function of the two state variables. The rainbow coloured surface represents the option value, the blue plane is the NPV. The zero level is also shown by the black plane. The blue and red lines are the current value and the trigger lever of ε, the long term variable, respectively.

When time to maturity decreases the investment trigger also decreases. Figure 30 shows how the investment trigger changes as time to maturity decreases. From the figure one clearly sees that the trigger falls to the value where NPV equals zero only the day when the option matures. The trigger value is always more than twice the break even point as long as the option is alive. The break even point is just below 76 NOK/MWh.

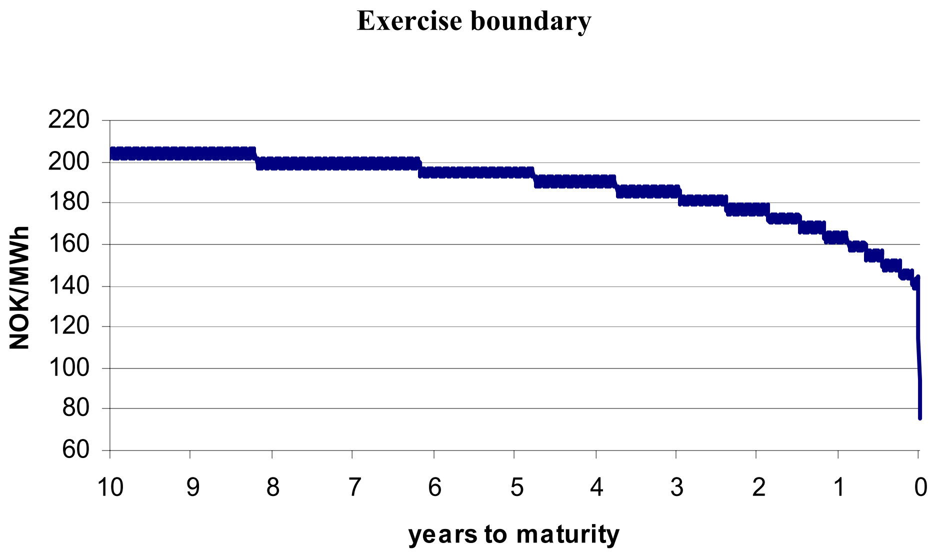



**Figure 30**: The figure shows decreasing trigger value as time to maturity goes to zero. The graph is discontinuous at the expiry date.

Figure 31 shows the importance of plant life time. As the figure shows the last years are very significant to make the plant profitable today since any shorter life time will result in a negative NPV. The importance of the years far out is due to the slope of the forward curve. From this one can conclude that the life time is important for a base load plant.

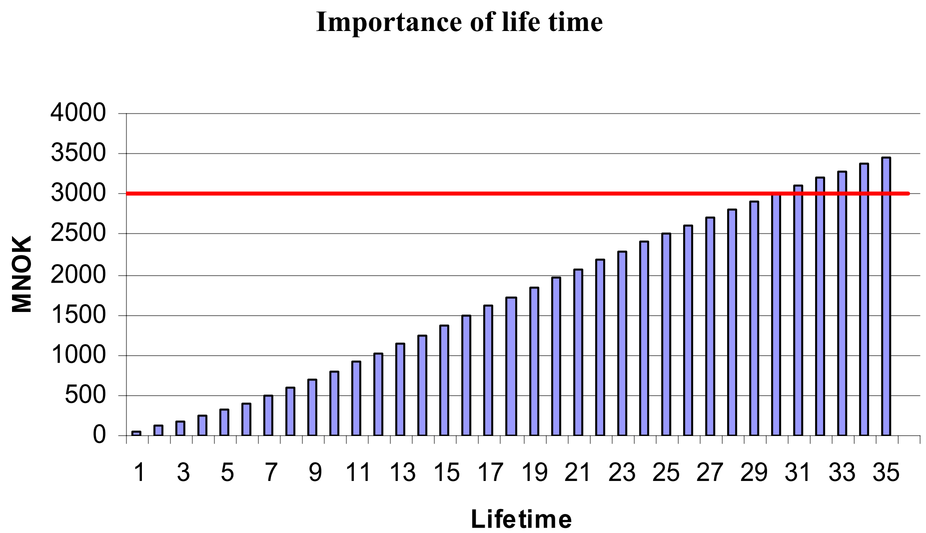



**Figure 31**: The figure shows the PV of investing in a base load CCGT today with a life time of *t* years. The line is the investment cost today. As the figure displays a plant with a life of 30 years is just profitable whereas a plant with a life of only 29 years is unprofitable if investment is made today. (not considering the real option)

## 14.2 Electricity/natural gas – spot price model

As explained in chapter 9 the short term variables are insignificant. When doing calculations on this model the short term variables were excluded and the original four factor model was converted to a simpler two factor model using only the long term variables for both electricity and natural gas.

This model has many parameters and before performing sensitivity analysis they will be grouped in two groups. The parameters $\sigma_{X_{el}}, \sigma_{X_{gas}}, \kappa_{el}, \kappa_{gas}, \rho_{X_{el}\varepsilon_{el}}$ and $\rho_{X_{ga}\varepsilon_{gas}}$ do only contribute to the level of the forward curve and then indirectly the option price, whereas the parameters $\mu_{el}, \mu_{gas}, \sigma_{\varepsilon_{el}}$ and $\sigma_{\varepsilon_{gas}}$ influence directly both the option value and the forward curve. From equation (9.12) this is evident since the short term variable is considered irrelevant.

For the first group of parameters it is interesting to test whether changes to them that lead to a positive shift in the implied spark spread forward curve alter the investment decision. $\sigma_{X_{gas}}$ and $\kappa_{el}$ should be decreased in order to make this effect, but a lower short term volatility of natural gas or longer half life of electricity is not considered likely. An increase in $\rho_{X_{gas}\varepsilon_{gas}}$ would be realistic since this value is below zero, but an increase in this parameter will not make the desired effect. A decrease is considered less likely. Remaining is to check whether an increase in $\sigma_{X_{el}}$ and $\kappa_{gas}$ alter the investment decision. Testing these parameters does not change the investment decision.

When making changes to the volatility parameters in the second group one should have in mind that for instance increasing the $\sigma_{\varepsilon_{el}}$ will lead to increased drift in the implied spark spread forward curve, but also to an increase in the option value due to higher volatility. Likewise increasing $\sigma_{\varepsilon_{gas}}$ will on one hand increase the option value but also decrease the slope of the implied spark spread forward curve. Testing the volatilities with different value does not alter the investment decision, and setting both volatilities to zero still gives a positive option value. Making changes to the risk free drifts will not induce investment.

Figure 32 shows NPV and option value as function of the two state variables. Under the current conditions the long term natural gas price must fall about 50% and the long term electricity price must more than double. Such a scenario is not very likely.

Also in this model the trigger value will be greater than the break even value as long as the option is alive and drop to the break even level the day the option matures. Like the curve in Figure 30, the plane illustrating the trigger level now, will show discontinuity the day the option matures. Such a plane is not showed here since very many simulations must be done in the LSM. The authors did not give this task a high priority. The value of the American option is always very close to the European option value.

From the sensitivity analysis above one can conclude that the best decision today under this model is to delay investment. The electricity price must increase dramatically and the gas price must be halved before investment is feasible when having a ten year license.

**NPV and option value as function of electricity and natural gas prices**

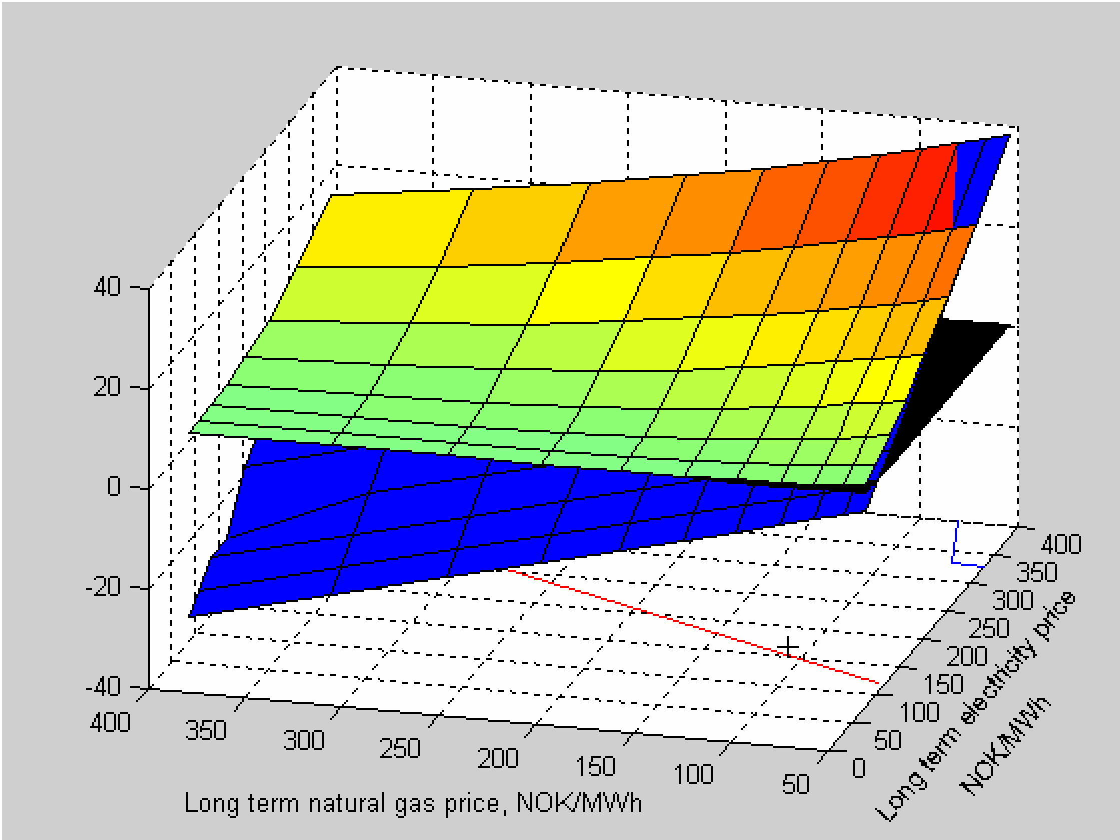



**Figure 32**: The figure displays the option and NPV as function of the long term electricity and natural gas prices. The rainbow coloured surface is the option value, the blue surface is the NPV and the black plane is the zero level. The "+" is the current prices, to the right of the red line the NPV is positive and the area bounded by the blue lines indicates where the decision to invest today is feasible. (The seemingly "dip" in the NPV plane is due to an error in Matlab)

## 14.3 Forward curve models

In these types of models there are not as many parameters to test. The relevant parameter to test is the constant $c$ in the volatility function given in (10.14). Changing the parameter $\alpha$ within reasonable values would not make too much difference since the valuation of the plant uses the part of the forward curve where the effects of $\alpha$ are cancelled out given the current value of $\beta$. Sensitivity analysis could be performed on the constant $\beta$, and choosing a lower value would give higher option values. However higher volatilities on contracts maturing in more than 200 weeks, or about 4 years are not very reasonable. Thus for the volatility function only the constant $c$ have been tested for sensitivity. Since the NPV of this model is negative and as long as the constant $c$ is greater than zero the option has value, the investment decision to wait is not questioned. The slope of the initial forward curve was also tested without changing the investment decision.

As discussed in chapter 10.5 the concept of trigger values can apply if one considers such a shift as the only possible way the curve can change. Under this assumption the spark spread forward curve have to shift upwards by about 170 NOK/MWh to make early exercise feasible. This trigger shift will of course decrease as the time to maturity declines and end up at the break even shift when the option matures. The forward curve must be shifted up about 5 NOK/MWh to make the NPV greater than zero.

For FCM2 one can make the same observations for the constants $\alpha_i$, $\beta_i$ and $c_i$ of the volatility functions, and the option will always take on a value greater than zero even though either $c_{el}$ or $c_{gas}$ are set to zero.

**Simulated forward curves**

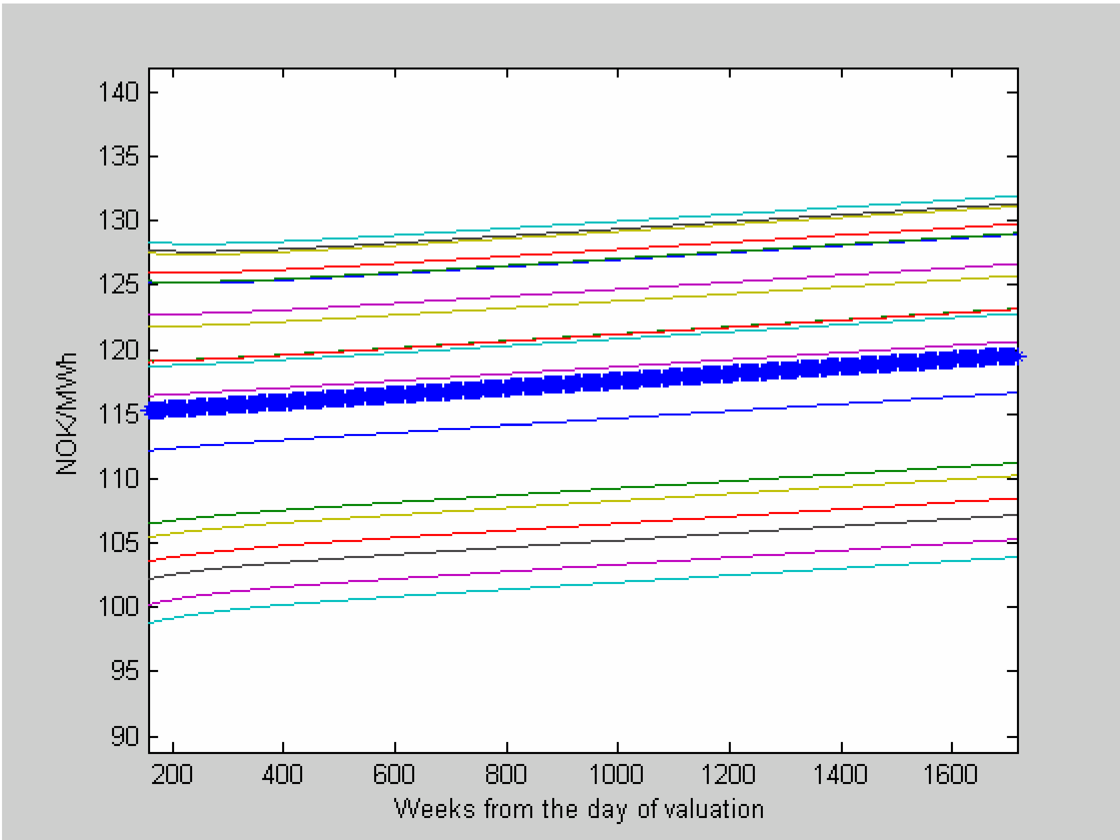



**Figure 33**: The figure displays the forward curve simulated 20 weeks ahead in time. The initial forward curve is the thick blue line.

## 14.4 Extended spark spread spot price model

When extending the model to incorporate taxes the NPV falls with 23.5 MNOK to -10.5 MNOK, and the option value decreases 114.7 MNOK to 2734.5 MNOK. Thus the tax shield from deducting the depreciations does not cover the taxes paid. However the investment decision is the same.

This model has also been extended to take into consideration uncertainty in the future price of emitting $CO_2$. As stated in previous chapters parameter for such a stochastic process cannot be obtained from the market since market data so fare is scarce.  Due to this the risk adjusted drift rate is set to zero and volatility is set to 20%. Since the short term variable is insignificant, the model is made a two factor model where the long term variable and the $CO_2$ price are the state variables. Figure 34 shows the NPV and option value depending on the values of the state variables. In this model investment today is only for low costs of $CO_2$ emission and high long term spark spread prices.

**Uncertainty in $CO_2$**

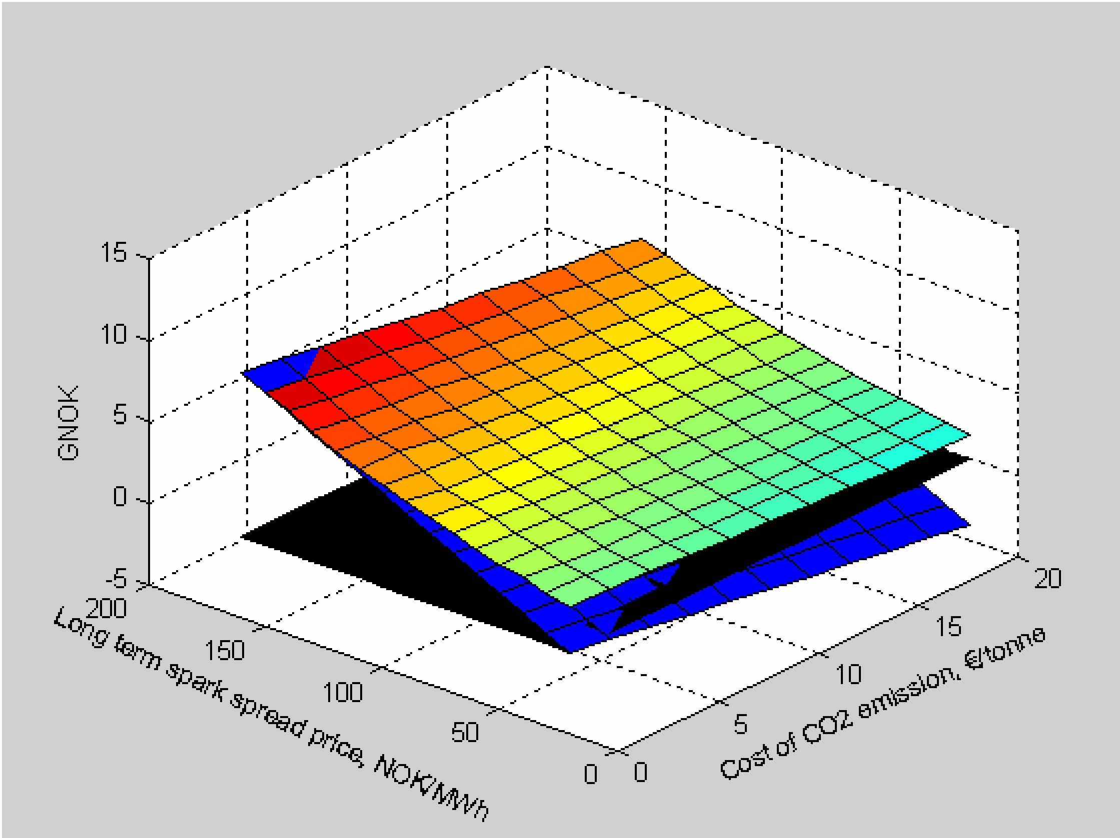



**Figure 34**: The figure displays the option value and NPV as function of $CO_2$ emission costs and long term spark spread price. Investment today is feasible when the current cost of $CO_2$ emissions are low and spark spread approaches the trigger level given in Figure 30. Where the blue surface shows through the rainbow surface investment today is feasible.

## 14.5 The impact of competition on the investment decision

This paragraph presents calculations under the model outlined in chapter 12. The investment decision for the CCGT under competition is modeled using the framework of no information. The option to invest is held by more than one investor and an investment terminates the option of the other firms. The game is modeled as a game between an option holder and a competitor in which all sources of competition are embedded. Large parts of the investment decision, such as the technology to be used and the total investment costs can be considered public available information. Not known is the competitors' prediction on future price level of electricity and gas, and further the competitors' belief in development concerning $CO_2$ taxation etc. Despite using the model for no information, all information on the competitor will contribute to a better estimation of $\lambda$.

## Hazard rate

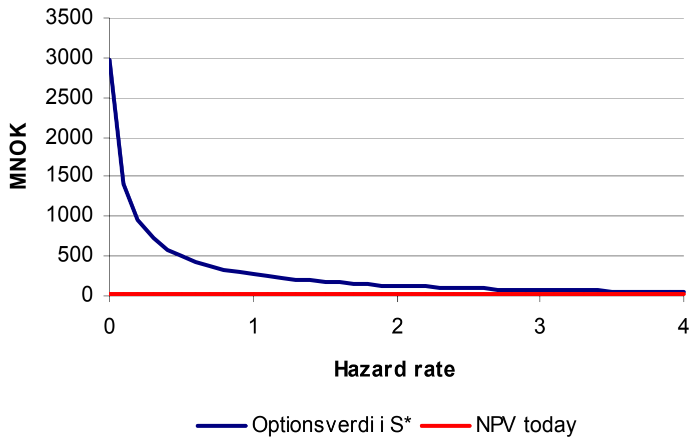



**Figure 35**: The figure displays the option value as a function of the hazard rate. If the investor finds it likely that someone will invest within the next quarter she should pronounce her investment now.

Figure 35 displays the option value as a function of the hazard rate $\lambda$. The option value decreases rapidly with increasing hazard rate for $\lambda$ less than 0.5. For larger values of lambda, the option value is less sensitive to the hazard rate. The likelihood of investments from competitors can be interpreted by the mean time to investment, given by $1/\lambda$. A hazard rate of 0.5 gives a mean time to investment of 2 years. Hence, if the option holder considers it likely that competitors will invest within the few next years, the option value decreases considerably.

# 15 Discussion and critics

In this chapter the models, assumption and results are commented and discussed. Some relevant questions about the models and assumptions are introduced. Further a short discussion of whether investors will find the results presented valid is given.

## 15.1 The parameters

In estimating the parameters for the SPM both a Kalman filter and an implied method were used. A uniform procedure combining the two methods that was robust was sought. However, this did not succeed and minor adjustments had to be made to the estimation procedure to get reasonable parameters. Especially the parameters of the long term natural gas process estimated by the Kalman filter seemed somewhat unreasonable and adjustments in addition to the outlined procedure had to be made. The forward curve of the spark spread model and the natural gas model do not price the current term structure as well as they ought to. It would be preferable that either the Kalman filter or the implied method always worked and gave sensible parameters. If this worked than a tool to continuously update parameters and state variables as new information arrived could be made. Since the SPM1 implied far less modifications than the natural gas parameters in SPM2 this calls in favour of SPM1.

In order to find parameters to the FCM by using PCA smoothed forward curve had to be constructed as explained in chapter 10.4.1.The far end of the constructed curves can contain errors due to the lack of more contracts to govern the price movements far ahead in time. This can lead to a higher estimated volatility on the long term basis than what is actually true. The authors have tried to adjust for this by filtering the data.

PCA is a data reduction technique and is used to find systematic volatility in forward curve data. The systematic volatility found by PCA comprised only 70 to 80 % of the total volatility in the data sets. The remaining volatility is unsystematic; that is specific to time to maturity. The authors consider such unsystematic volatility not to relevant when making a volatility function for the full length of the plants life time.

In general the FCM1 is better fitted than FCM2. This can be due to the fact that the FCM models do not incorporate seasonal variations as such fluctuations can lead to higher estimated volatility than what is actually true. For spark spread the seasonal variations is less in value than for electricity and natural gas separately. This can explain why FCM1 is better than FCM2.

The models in this thesis assume constant parameters. If this was not the case one could continue to make the parameter stochastic and obtain very detailed and difficult models. As mentioned in chapter 9.9 the SPM fail the tests for constant parameters. For spark spread the long term volatility seems independent of prices whereas the long term volatility for natural gas and electricity seem to some extent dependant on the price. More advanced volatility functions or structure has not been tried, but maybe stochastic volatility would perform better. The draw back of this is that it complicates the model. Also other important parameters as the risk neutral drift show large fluctuations over time and are not constant. For the FCM the tests of constant parameters in chapter 10.4.7 confirm that one can rule out constant volatility. In general SPM have far more parameters to be estimated than the FCM. More parameters to be estimated can lead to additional inaccuracy so at this point the FCM are superior. Statistical data as standard error would be preferable to have on all the parameters. The authors did not find a satisfactory method to find such statistics. The authors believe that the weakness of the parameters in general is a source of error in the models.

## 15.2 The models and results

All the four models give the same investment decision as presented in chapter 9; delay investment. For the SPM the NPV are positive whereas for the FCM the NPV are negative. However, the option has a positive value in all cases. SPM1 and FCM1 show that there is need of a significant increase in spark spread in order to make investment feasible when an investor have a ten year licence. Likewise the SPM2 and the FCM2 demand an increase in the electricity price and/or decrease in the natural gas price. The investment decision is not altered in any of the models as parameters are changed one at a time. Thus the investment decision is believed to be robust under the general assumptions.

As pointed out only the long term variables are significant. This makes it easy to obtain an investment rule under the SPM; invest as the equilibrium price moves into the exercise region. An investor using this kind of model would need a tool to continuously update the state variables in order to decide when to invest. This makes the SPM very tractable and easy to communicate to investors. In the FCM the forward curve itself is the state variable and theoretically any forward curve can give a desired value. Thus the forward curve itself cannot be used in the investment rule. In the FCM the value of the plant must be used in the investment rule; Invest when the value of the plant equals the option value. However, this is somewhat more diffuse since a lot more than the price of the commodities can be assumed to make up the value. It must then be communicated that the plant value depends only on prices, but that it is difficult to tell exactly what price will induce investment. An investor using this model will need a tool to find the plant value based on the given forward prices in the markets. When it comes to making an investment rule the SPM are superior. Work on real option theory is usually based on SPM something which explains parts of why it is more tractable. This thesis gives some views of how to use FCM in a real option frame work, but more research is needed.

If one assumes that expected production capacity investments are not reflected in the forward market a game theoretic approach as outlined in chapter 12 can be used by introducing a hazard rate. The problem doing this it that it is impossible to observe and estimate the hazard rate, and the whole analysis boils down to a subjective guess on the hazard rate. The inability to determine the hazard rate from market data makes the model less satisfactory.

The simplest model, the SPM1 is extended to incorporate taxes. As calculations proved, the tax shield of depreciation deductions are smaller than the actual taxed paid, and the NPV of the plants become slightly negative. However, the investment decision is not changed. SPM1 is also extended to take into consideration uncertainty in the future $CO_2$ emission costs. Regardless of which assumptions one make about future $CO_2$ emission costs, investing today is infeasible. That is, whether the cost of future $CO_2$ emissions is zero, 20 €/ton or uncertain the best investment rule is to wait. It can be questioned to which extent forward prices on electricity and natural gas reflect the future cost of $CO_2$ emissions.

All over the authors believes that SPM1 is the best model compared to the complexity, parameters and tractability of the other models. It is fairly easy to extend the model, it has few parameters to be estimated and calculations using this model can be done in a binomial three instead of LSM, the latter being much slower.

## 15.3 Investors and market assumptions

A real option approach to the investment decision is used in this thesis. However, it is uncertain whether this method is actually used by investors. Do the investors take the option value as real, or is it more an academic approach with no use outside the four walls of universities? The lack of investments today can be due to negative NPV or that investors recognize the option value and thereby postpones the investment. The rush to get licenses by many firms indicates that the options have value.

Another question remaining is to which extent the building of new generation capacity is reflected in the forward prices of today. The market players do not necessarily have the same way of interpreting the data and modelling prices, making it harder to judge the impact new information will have on the underlying variables. Further, as questioned above, the investment rules of investors can differ. Many investors may still invest by the old NPV rule and take the investment before it is profitable. This leading to that new generation capacity arrives earlier than predicted by the models in this thesis.

The 800 MW CCGT plant will have a yearly production of 6 TWh, a considerable amount compared to the yearly consumption in Norway of about 120 TWh and the yearly consumption within the Nord Pool of about 380 TWh. Many market players will argue that additional 6 TWh will have a considerable price effect on the local area price and also affect the Nord Pool system price to some extent. If this is the general belief then the assumption made in chapter 4.5 are questionable and the results in this thesis can be wrong.

In chapter 8.2 the assumption of complete markets and the assumptions of spanning assets are presented. The assumption that a set of futures and forwards traded today can span the risk of gas fired power plant over 30 years might be rejected from investors. Likewise a forward curve for 33 years ahead in time constructed on the basis of only a few listed contracts together with a 10 year OTC contract may possibly be regarded as quite optimistic.

It can be argued that such forward curves are wild guesses of future prices and that price estimates based on scenario analysis are equally good. Such issues and many more can be debated. The counter argument will be that as long as there are many market players each using its own models and beliefs to trade in the deregulated market the general expectations could be said to be reflected in the market as a weighted sum of all the different beliefs. Then the use of current futures and forward data will be the best information available to construct forward prices.

This thesis has used some quite advanced models to describe the dynamics of energy commodity prices. However, taking into consideration all the aspects mentioned in chapter 15 regarding parameters, models and market assumptions the authors feel that it would be preferable that the models captured some evident properties in a better manner. For instance the prices seem to have a stochastic volatility and stochastic risk neutral drift rate, and models incorporating one or both features would be an interesting study. Further a forward curve model that incorporates the fact that the seasonality always “moves to the left” as time passes can make the forward curve models more realistic. Energy term structure is quite different from interest rates, but the authors think that this distinction is not recognized in the literature. The models and results in this thesis can be used as investment support, but for final investment conclusion authors encourage investors to support the decision with additional models. Even thought investors do not trust this blindly, the large option value compared to the NPV can signal that waiting is profitable. If more long term data was available to the authors investors may find the results more credible.

Finally, the authors wonder if there are special conditions in Norway preventing investments in gas fired power plants. Appendix 2 provides some thoughts of why gas fired power plants are built in other European countries, but does not seem profitable in Norway.

# 16 Concluding remarks

Briefly summarized the FCM is more tractable when it comes to estimate parameters since it has fewer than the SPM whereas the SPM is assumed to be easier to communicate. For both model frame works the spark spread models seems to give sufficient information compared to the accuracy obtained by increasing variables. As the authors believe the valuation methods for FCM need more development before it is ready for real option analysis the SPM1, spark spread spot price model, is considered to be the best model to use.

Based on the calculations, the assumptions made and the discussion above an investor having a license to build a base load CCGT in Norway is recommended to delay her investment. All models consent on this decision and extensive sensitivity analysis does not alter the conclusion. The main reason for delaying investment is to wait for better information on the future prices of electricity and natural gas. SPM1 is chosen as the best model. The value of investing today is 13 MNOK, whereas the value of postponing the investment is 2800 MNOK.

# 17 Further work

This chapter provides some suggestions for further work that can be done to improve the analyses and conclusions.

Standard errors are provided by among Schwartz (1997) for all parameters and future researchers in this area are encouraged to develop statistical methods and share them to the world. In general the authors feel a lack of statistical knowledge, and students doing similar work are encouraged to take courses in stochastic statistics and statistical time series.

Estimating parameters by the Kalman filter a Matlab solver was used to maximize the likelihood function. In order to see if one could better estimate the parameters of natural gas using two mean reverting processes and one arithmetic Brownian motion the Kalman filter was set up for this. However, Matlab ran for 24 hours without getting a solution. If one intends to use the Kalman filter one should use a Newton method to find the change in parameters needed to improve the likelihood function at each iteration. It is recommended to use algebraic software to find the equation. Several books on calculus and optimization provide the basis for the Newton method, and Shumway et al. (2000) provides a procedure to find such equations.

The method of using least squares regression to estimate the value of waiting some times fails as the American options a few times have proven to be less worth than the European, a clear proof that something is wrong. The scaling of the input variables to the regression is crucial for it to work.

When regarding the model they could be extended to incorporate stochastic volatility and risk neutral drift rate, or one could add more stochastic processes to capture more of the short term movements. Further the forward curve models should be extended with seasonal terms and the construction of smoothed forward curves needs an "anchor point" in the far future such as long run marginal costs.

The models can also be extended to consider the option to reinvest.

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# Appendix 1 - The power plant

The different categories of gas fired power plants produce either electricity in simple cycle or combined cycle gas turbines (CCGT), or are used for cogeneration of heat and power (CHP). Simple cycle consists of a gas turbine only, whereas the CCGT includes both gas and steam turbine(s). Simple cycle is a cheaper investment, smaller in size and more flexible but has lower total efficiency than the CCGT. A CCGT plant will be the preferable alternative for base load electricity production. A CHP plant uses CCGT with the ability to extract steam from the process. Building a CHP plant is relevant if there is industrial need for high temperature steam.

The CCGT process utilises a combination of fossil fuelled and gas turbine plant and equipment. Generally one or a number of combustion gas turbines feed their exhausts to a unitised or common gas duct which then passes the GT exhaust to a steam boiler where the heat is used to change water to steam. This is in turn passed to a conventional steam turbine, which is complete with its own generator. Overall cycle efficiencies can be as high as 60%. The CCGT has flexibility in purchase and low capital installation costs compared to competing sources of electricity production such as coal fired or nuclear electricity plants.

It should be noted that in the event of a loss incident at one of the gas turbines, the other units will be able to operate independently. If the steam turbine suffers an incident the GT (s) will still be able to continue in operation but obviously at reduced efficiency. There is therefore a considerable amount of flexibility in operation.

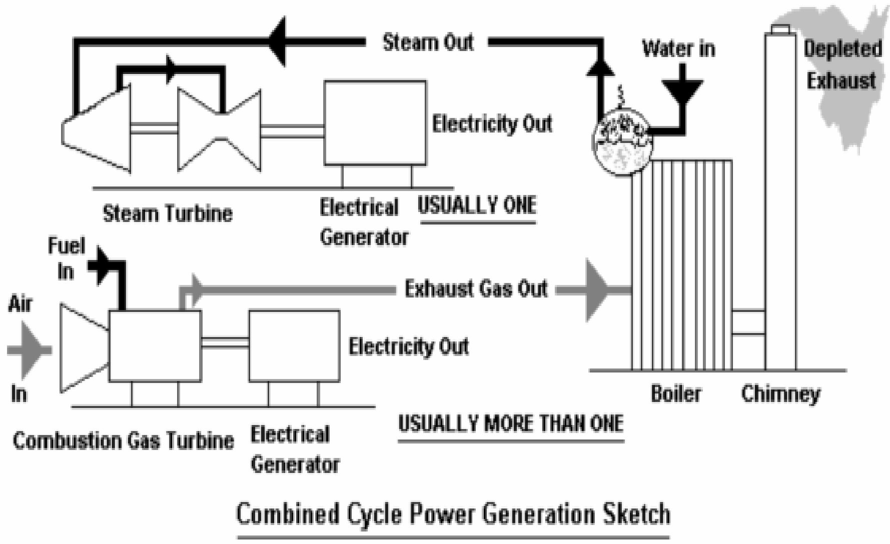


**Figure A 1: A simple display of a CCGT process**

## Description of the different components

The following chapter contains a brief discussion of the main considerations in plant-design with respect to the main components. For a more in depth discussion, interested readers are advised Moran et al. for more details on the thermodynamic processes.

### Gas and Steam turbines

The power production in a CCGT plant is generally made up of 2/3 from gas turbine(s) and 1/3 from the steam turbine. The efficiency of gas turbines are highly load dependent, whereas the steam turbine is more flexible. To avoid running on low efficiency, a power plant will be designed with several smaller turbines if good part-load characteristics are an important design criteria. The operating strategy of a gas power plant is that the gas turbines are either running at optimal load or is not running at all to cope with the major load variations. An important part in designing the power plant is determining the optimal number and size of gas turbines. There will be little need for such flexibility assuming a constant delivery of base load.

The cost per MW installed is reduced with increasing unit capacity, the units should be chosen as large as possible. The largest gas turbines commercially available today are of 300-

400 MW.  A CCGT plant intended for delivery of 800 MW relatively stable base load will most likely consist of two large gas turbines and a large steam turbine.

The gas turbines are normally standard equipment due to very high development costs. Steam turbines come in a wide range, and it is not difficult to find a suitable one to match the gas turbines.[73] The plant will have one steam turbine with two or three pressure levels. The steam turbine is a cheap component compared to the gas turbines, and can be used to regulate output without much decrease in efficiency.

### The other main components

The evaporator fixates the load point. With variable load, a larger number of evaporators are needed to secure good part load characteristic. In the case of a base load plant one or two evaporators will be sufficient. The heat from combustion not delivered to water in the tubes of the boiler and superheater are extracted in the economizer. The electrical generator and power generating components are important parts of designing the power plant, but will not be discussed in this paper.

In terms of components, better thermodynamic efficiency results in higher investment costs. A plant specific analysis must be conducted to find the optimal trade off. Heat-recovery equipment, added flue-work and building area increase the cost. The fixed costs due to increased investment and the saving in operation costs due to reduction of flue –gas temperature should balance and show the justification for the use of heat-recovery apparatus.

### Choosing design

Choosing the optimal design of components is made based on the type of production. (e.g. Base- or peak load). The main rule is that operational flexibility is expensive. The main consideration in the design process is choosing the optimal set of gas turbines.

For each combination of fuel cost, load factor and capacity factor, there is an economical pressure and temperature of steam which will result in the lowest cost of power. There is a trade off between increasing the efficiency due to increase in steam pressure and temperature and the additional investment in equipment required for the purpose. After determining the

[73] Bolland (2003)

main specifications, the exact design with respect to setting temperatures and mass flows for the process in details can be solved with computer tools such as GTPRO.[74]

### Efficiency

A CCGT plant equipped with the most modern generation of gas turbines, H-series, can obtain a net plant efficiency of 60%. The older generation of F-series turbines can obtain 56-58 % depending on the exact model, e.g. the General Electric S109 FA and the Toshiba 109 FA with net plant efficiency of 56,7% based on standard assumptions.[75] The newest turbine technology generally includes dry low $NO_X$ combustion technology. A net plant efficiency of around 58% by Norwegian conditions can be assumed for F-series technology conditions for F-series technology.[76]

### Calculating costs

The turnkey costs for a CCGT plant as found in Gas Turbine World involves all components concerning the power generation process. The turnkey costs for a 400 MW CCGT plants are 200 MUSD with H-series and around 140 MUSD with F-series turbines. An 800 MW CCGT with F-series turbines will have a turn key cost of around 240 MUSD.[77] (year 2000 USD)

The total cost of the plant, is according to Bolland (2003) as a rule between 1.3 and 1.5 times the turnkey cost. It can be higher if substantial infrastructure investments have to be made. Competition and market conditions will also heavily affect the prices listed and could also affect terms of insurance and maintenance conditions.

The investment is highly exposed to currency fluctuation. As an example, the total investment cost of an 800 MW CCGT will 360 MUSD by adding 1.5 to the turnkey cost. By using the average of daily exchange rates from 2003, the resulting investment cost is 2560 MNOK. Comparably, the same plant will have an investment cost of 3168 MNOK if the average of daily exchange rate from 2000 is used. The difference of 600 MNOK is a considerable amount and states that currency fluctuations should be taken into account.

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[74] Bolland (2003)

[75] Gas Turbine World (2000)

[76] Bolland (2003)

[77] Gas Turbine World (2000)

The range of the investment cost is fairly wide with a total investment cost of 2218 MNOK using the most and 3168 MNOK using the least favorable conditions. Statoil has estimated the investment costs at Tjeldbergodden to be in the order of 3000 MNOK. The estimate does not include large investments into gas or electricity infrastructure and is still in the high end. Without knowing a more detailed cost allocation or the exchange rates used for the calculations it difficult to interpret the number further than label it as a conservative cost estimate.

**Table A 1**

**Investment cost for a CCGT power plant**

| | | |
|---|---|---|
| **Turnkey costs (based on GT PRO)** | | **[kUSD]** |
| Total installed capital equipment | 71% | 113 000 |
| Contractor's engineering | 8% | 12 528 |
| Contractor's construction | 10% | 15 660 |
| Miscellaneous & star-up costs | 3% | 4 176 |
| Contingency (10%) | 9% | 14 536 |
| **Total turnkey cots** | | **159 900** |
| **Additional costs (based on experience- highly)** | **In % of turnkey cost** | |
| Turnkey contr. | 5.0% | 7 995 |
| Connection to cooling water | 4.0% | 6 396 |
| Connection to HB overhead lines/upgrade and HV lines | 5.0% | 7 995 |
| Connection to gas terminal | 6.0% | 9 594 |
| Conceptual eng., etc | 5.0% | 7 995 |
| Power and fuel for commissioning | 0.5% | 800 |
| Spare parts (not capitalized) | 1.0% | 1 599 |
| Verifications, insurance, risk analysis, environm. studies | 2.5% | 3 998 |
| Contingency, additional costs | 15.0% | 23 985 |
| **Total additional costs** | | **70 356** |
| **Total project investment cost** | | **230 256** |

Allocation of investment cost for a 390 MW F-series CCGT plant, Norwegian conditions (Bolland 2003)

Table A 1 presents a detailed allocation of investment costs for a 390 MW CCGT in a Norwegian setting. Building two identical blocks or a single 800 MW block should result in a lower investment cost per MW installed.

If the option to upgrade the plant later shall be taken into consideration, another turbine set may be favorable as shown by Fleten and Näsäkkälä (2003) but is not considered in this paper.

## Location issues and the impact of ambient conditions

The efficiency of the CCGT - process is affected by both the temperature of cooling water and the ambient air temperature. The cooling water has the largest influence.

To run the CCGT process with high efficiency requires a good and stabile source of cooling. Locating the plant in according to assumption 2, chapter 6, allows for heat exchanging with low-temperature sea water, which provides an excellent and stabile source of cooling the whole year. The sea-water will be taken from a depth of 50 m where the temperature shows little seasonal variation and is in the range of 6-10 ˚C.

The performance of the gas turbine is heavily affected by ambient conditions. Any parameter affecting the mass flow of the air entering the gas turbine will have an impact on the performance. Increasing ambient temperature will have a negative effect on power output.

The two major categories of plant output enhancement are gas turbine inlet air cooling and power augmentation[78] Evaporating cooling is the most cost efficient method when ambient temperature is high and the relative humidity is low, but not ideal for Norwegian costal climate. To enhance performance decreasing the inlet temperature to the compressor, inlet cooling, will improve power output and heat rate. As the temperature in the costal areas of western Norway rarely is above 20˚C, the ambient conditions will only to a small extent reduce power output. It will not be an economical incentive to install inlet cooling equipment.

## Wear, failure and Availability

Wear and failure of components and the issue of availability can have a huge impact of the profitability of the plant and should be taken into consideration both in terms of design- and operational issues. The base load plant in question should optimally be running constantly according to the assumptions made. It will, however, experience downtime due to both planned and unplanned outages.

[78] Kiameh (2003)

**Table A 2**

| Estimated Operations and Maintenance Life Cycle Costs | | |
|---|---|---|
| **Operations Cost** | | |
| Direct Salary & Labor | 2 249 520 | USD/year |
| Direct Materials & Subc | 745 913 | USD/year |
| **Total** | **2 995 433** | **USD/year** |
| **Maintenance – Planned** | | |
| Gas Turbines | 2 929 950 | USD/year |
| HRSGs | 144 283 | USD/year |
| SCR Replacement | - | |
| ST-Gs | 145 986 | USD/year |
| Instruments | 37 162 | USD/year |
| System BOP | 154 514 | USD/year |
| **Total** | **3 411 895** | **USD/year** |
| **Maintenance – unplanned** | 686 379 | USD/year |
| **Total O & M** | **7 093 707** | **USD/year** |

Allocation of Operation and Maintenance cost (in 1998 USD) estimated for CCGT plant using a GE STAG 109FA (approx 350 MW). (Bolland 2003)

The gas turbine, and particularly the turbine blades, is the most expensive component with respect to maintenance as shown in Table A 2. Using new and unconventional technology increases the risk of failure.

Availability and reliability are important characteristics for a base load plant and the definitions are as follows. A plant is considered reliable if no unplanned service breakdown occurs as a result of disturbance. A plant is available when it is ready for operation, non-availability also including planned outages. The term RAM (Reliability, Availability, and Maintainability) is often used as a general description of analysis and estimates/assessments where availability and reliability are connected to maintenance. The work and experience concerning these issues are normally confidential, but some databases exist. The two most important databases with RAM information about gas- and steam-turbines are the NERC-GADS and SPS-ORAP. The databases containing Empirical RAM- data are usually subject to payment as in the case of the SPS-ORAP database which provides good RAM-data for base-load plants.

The plant is considered to have 7900 operating hours a year (90% availability). From publicly available sources, average availability for large CCGT plants is found to be 85% in the WEC annual report of 2001[79] and to 90% for CCGT plants in general from NERC-GADS.[80] The data does not take into account whether the size nor the age and operating strategy of the actual turbines. Bolland (2003) uses 7900 hours.

### Degradation and fouling

Degradation and fouling with time will lead to reduction in performance and efficiency, mainly of the gas turbines. A CCGT plant will suffer reduction in power of 3-10% and efficiency of 2-5%. Using natural gas as fuel will lead to reductions in the lower end and up to 50-80%, mainly fouling, can be avoided by performing regularly maintenance such as washing the turbines.[81] Veer (2003) shows that the irreversible degradation effect occurs within the first 10000 operating hours and output reduction stabilizes at around 4 %. During the plants lifetime, degradation will be reduced by maintenance and repair programs. Degradation and fouling can be incorporated into the model by reducing equivalent operating hours or lifetime of the plant. In the valuation model, a 2% reduction in equivalent operating time is made for both the fouling and degradation effects. The simplification of using a flat instead of decreasing adjustment factor is made to accommodate the valuation model.

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[79] World Energy Council (2001)

[80] Generating Availability Data System (2003)

[81] Bolland (2203)

# Appendix II

Is the investment in large CCGT power plants unprofitable in general, or is it specific factors that make investment in Norway more unprofitable?

Although abundant with natural gas, the Norwegian domestic prices will be determined by the prices of continental Europe. The price of natural gas in Western Europe is high but slightly lower in Norway due to less cost of transportation.

The price level of electricity in Norway is considerable lower than in European countries like e.g. Germany where the expected price level for the coming years is 3 to5 Euros above that of Nord Pool. The price difference is for base load whereas peak load prices are much higher. In thermal dominated electrical systems, the building of a gas fired power plant for electricity production only would be designed to produce peak load at a flexible operational pattern. Running a peak load plant will be far more lucrative. However, as discussed in chapter 3, using gas power plant for peak load production will not be lucrative in the hydro dominated Norwegian power system.

New generation capacity will to some extent lead to a decrease in electricity prices, and by that reduce income from existing power generating facilities. In theory, this should not prevent investment since there will always be investors without existing generating facility willing to commit to a profitable project. However, in small market with limited players this could have an effect.

The building of gas fired power plants is presented as an environmental friendly project in all countries other than Norway. Usually natural gas replaces coal and reduces $CO_2$ emissions, whereas Norwegians are used to zero emissions from hydro power. The origin of import electricity does not seem to be of interest to the general public.

The environmental issues concerning $CO_2$ emissions are closely linked to politics. The political acceptance for building gas fired power plants is larger in neighboring countries. Due to the existing electricity generation structure, gas fired power plants will lead to emission reductions in these countries whereas they will lead to increased $CO_2$ discharge in Norway. The lack of political acceptance for investments into $CO_2$ emitting activities makes it less likely that any of the large state owned companies will undertake such investments. Investing in a power plant is expensive, and since Norway suffers from a small private sector there are relatively few companies with the financial strength and the interest to invest in power generating facilities with a bad public image.

CHP plants might be more lucrative if the excess heat generates high income as often the case in central Europe. The existing gas infrastructure in Norway is mainly located far away from the population and industrial centers. Building a CHP in proximity will only be possible as a part of a larger investment in new gas infrastructure.

Conditions in Norway are not favorable. The cost of natural gas is relatively high, whereas the price of electricity is relatively low. In addition, lack of investors and public skepticism are prevailing. The investment in gas fired power plants is more likely to take place outside Norway.

# Appendix III

In this master thesis a tremendous amount of computer code is produced. In this appendix the code of the most important programs are printed.

## The Kalman filter

Below is code created to use the Kalman filter to estimate parameters. Language: Matlab.

```
%---------------------------------------------------------------------
%| Authors:  Rolf M. Alstad & Jørgen T. Foss                          |
%| Date:    Spring 2004                                               |
%| Version: Final                                                     |
%| Description:  This program estimates uses the Kalman filter to     |
%| estimate parameters for two factor model based on observations in the|
%| forward market. The code is based on the paper of Schwartz & Smith   |
%| 2000. Any users of this code is advised to read this paper carefully.|
%| The authors like to thank Erkka Näsäkkälä for the basis of this code.|
%----------------------------------------------------------------------
close all;
clear all;
%starts the timer
tic
%loads the data file. The datafile is organized as follows. The first
%column holds the relative time of observations. The M next columns hold
%the observed forward prices. The next M-1 columns hold the time for start
%of delevery and the M-1 remaining columns holds the times for end of
%delivery. It is assumed that the first contract has zero time to maturity
%and has point delivery; the proxy spot.
load spread.txt -ascii

%initial guess on parameters
init=[0 1 1.5 0.5 50 7 5 5 5 5 0 0 0 0 50];
%lower and upper boundaries for the parameters. Setting to wide boundaries
%Matlab uses more time than necessary.
lower=[0 -10  0.1  -1  1  0.1   0.01 0.01 0.01 0.01  0  0 0  0 -100];
upper=[0 500 10   1  500 50    3000 3000 3000  3000  0  0 0  0 100];

% parameter 1  mu
% parameter 2  mu*
% parameter 3  kappa
% parameter 4  rho
% parameter 5  sigma_X
% parameter 6  sigma_e
% parameter 7  V1
% parameter 8  V2
% parameter 9  V3
% parameter 10 V4
% parameter 11 SS
% parameter 12 lambdaX
% parameter 13 gamma
% parameter 14 tau
% parameter 15 prior long term variable

%continously compunded interest rate
r=0.058;

%puts the data in the data file into convenient vectors
time=spread(:,1);
price=[spread(:,2) spread(:,3) spread(:,4) spread(:,5)];
matTime=[zeros(length(spread(:,6)),1),spread(:,6) spread(:,7) spread(:,8)];
endTime=[zeros(length(spread(:,6)),1),spread(:,9),spread(:,10),spread(:,11)];

% This line pass the objective function, the initial guess, the boundaries and the other
%function
% inputs to the Matlab optimization function "fmincon". This function
% returns the optimal parameters, the value of the objective function and
% some runtime information.
```

```
[param,min_like,exitflag,output]=fmincon(@likelihood2dim,init,[],[],[],[],lower,upper,[],optim
set('TolFun',1e-4,'MaxIter',1e+5,'MaxFunEvals',1e+5),r,time,price,matTime,endTime);

% running the Kalman filter a last time with the optimal parameters to get the state
%variables.
[expect,stdev,t_out,price_out,Q_out,statevar]=kalmanfilter(param,r,time,price,matTime,endTime)
;
% stopping the timer
toc




%---------------------------------------------------------------------
%| Authors:  Rolf M. Alstad & Jørgen T. Foss                          |
%| Date:    Spring 2004                                               |
%| Version: Final                                                     |
%| Description:  This is the objective function used in the Kalman    |
%| filter. The log likelihood function is given in Harvey (1989).     |
%---------------------------------------------------------------------

function out=likelihood2dim(param,r,t,price,tmat,etime)

% runs the Kalman filter with the given parameters
[exp,stdev,t_out,price_out,Q_out]=kalmanfilter(param,r,t,price,tmat,etime);

% declaring dummy variables
first=0;
second=0;

% calculating the log likelihood function
for i=1:1:length(Q_out)
first = first +log(abs(det(Q_out{i})));
second= second + (price_out{i}-exp{i})'*inv(Q_out{i})*(price_out{i}-exp{i});
end

% returns the value of the log likelihood function for the given set of
% parameters.
%min because we maximize
out=-(-first-second);




%---------------------------------------------------------------------
%| Authors:  Rolf M. Alstad & Jørgen T. Foss                          |
%| Date:    Spring 2004                                               |
%| Version: Final                                                     |
%| Description:  This is the the Kalman filter made on the basis of   |
%| Schwartz and Smith (2000)                                          |
%---------------------------------------------------------------------

function [expected,stdev,t_out,price_out,Q_out,m]=kalmanfilter(param,r,t,price,T1,T2)
% extracting the parameters from the param vector for more easy reference.
% The filter is set up to be able to compute both the true and the risk
% neutral drift.
myy=param(1);
myys=param(2);
kappa=param(3);
roo=param(4);
sigma_x=param(5);
sigma_e=param(6);
V0=param(7);
V1=param(8);
V2=param(9);
V3=param(10);
SS=param(11);
lambdaX=param(12);
gamma=param(13);
tau=param(14);
E=param(15);

%initial state varaibles
m(1,:)=[0, E];

%init covar for prior
C_s(1,:)=[0 0 0 0];
C=[C_s(1,1) C_s(1,2);C_s(1,3) C_s(1,4)];
```

```
% for each obeservation in the data set
for i=2:1:length(t)
    % the length between observations
    dt=t(i)-t(i-1);
    % covariance matrix
    W(1,:)=[(1-exp(-2*kappa*dt))*sigma_x^2/(2*kappa) (1-exp(-
kappa*dt))*roo*sigma_x*sigma_e/kappa];
    W(2,:)=[(1-exp(-kappa*dt))*roo*sigma_x*sigma_e/kappa sigma_e^2*dt];

%------- measurement equation.
    %the F matrix and d vector for the forward prices. Takes into account that this is a flow
    %commodity
    for j=2:4

       F(j,:)=[r*(exp(-T2(i,j)*(r+kappa))-exp(-T1(i,j)*(r+kappa)))/((-exp(-r*T1(i,j))+exp(-
r*T2(i,j)))*(r+kappa)), 1];

       d(j,:)=myys*(-exp(-r*T1(i,j))*r*T1(i,j)-exp(-r*T1(i,j))+exp(-r*T2(i,j))*r*T2(i,j)+exp(-
r*T2(i,j)))/((-exp(-r*T1(i,j))+exp(-r*T2(i,j)))*r)+...
            (-lambdaX/kappa)*(-r*exp(r*T2(i,j))-exp(r*T2(i,j))*kappa+exp(r*T2(i,j)-
kappa*T1(i,j))*r+r*exp(r*T1(i,j))+exp(r*T1(i,j))*kappa-exp(r*T1(i,j)-kappa*T2(i,j))*r)*exp(-
r*(T1(i,j)+T2(i,j)))/((-exp(-r*T1(i,j))+exp(-r*T2(i,j)))*(r+kappa))+...
            SS+...
            r*gamma*(-r*exp(-r*T1(i,j))*cos(-2*pi*T1(i,j)+2*pi*tau)-2*pi*exp(-r*T1(i,j))*sin(-
2*pi*T1(i,j)+2*pi*tau)+r*exp(-r*T2(i,j))*cos(-2*pi*T2(i,j)+2*pi*tau)+2*pi*exp(-
r*T2(i,j))*sin(-2*pi*T2(i,j)+2*pi*tau))/((-exp(-r*T1(i,j))+exp(-r*T2(i,j)))*(r^2+4*pi^2));
   end
    %the F matrix and d vector for the proxy spot
   F(1,:)=[1 1];
   d(1,:)=SS+(1-exp(-kappa*0))*(-lambdaX/kappa) + myys*0+gamma*cos(2*(0-tau)*pi);

    % Observation errors
    V=diag([V0 V1 V2 V3]);
%------ transition equation.
    %c vector in the transition equation.
    c=[0,myys*dt];
    % G matrix in transition equation.
    G=[exp(-kappa*dt) 0 ; 0 1]; %del av state eq
%----- the Kalman filter
    a=(c'+G*m(i-1,:)')'; %best estimator of state var [x e]
    R=G*C*G'+W; % Covar to state var contingent on the previous time step. 2X2
    a_out(i-1,:)=a; % not needed
    t_out(i-1)=t(i); %not needed
    %stores the obeservations in a price vector
    price_out{i-1}=[price(i,1) price(i,2) price(i,3) price(i,4)]';
    Q=F*R*F'+V; %covar of predicted observations contingent on the previous time step. 4X4
    Q_out{i-1}=Q;%stores Q in a vector
    A=R*F'*inv(Q); %correction to a
    C=R-A*Q*A'; % up dates the covar for the state variables
    C_s(i,:)=[C(1,1) C(1,2) C(2,1) C(2,2)];
    f=d+F*a';%mean of predicted observations contingent on the previous time step.
    f_out{i-1}=f; %stores f in a matrix
    m(i,:)=a+(A*(price(i,:)'-f))'; %calculates the new state variable
end

%function returns:
price_out=price_out;
t_out=t_out';
Q_out=Q_out;
expected=f_out;
stdev=0; %not needed
statevar=a_out;
```

## LSM SPM1

Below is the code used to value the option  under SPM1. Language: Matlab.

```
%----------------------------------------------------------------------
%| Author:  Rolf M. Alstad & Jørgen T. Foss                            |
%| Date:    Spring 2004                                                |
%| Version: Final                                                      |
%| Description:  This program calculates the value of an American option|
```

```
%|              using Monte Carlo simulation.                            |
%-----------------------------------------------------------------------
clear all;
tic %starting the timer
%input for the stochastic processes.
X0=119.98%   ;%(NOK/MWh)
epsilon0=76.62;%(NOK/MWh)
SS=0;%(NOK/MWh)
rente=log(1+0.01*6); % per year
alfas=0;% %(NOK/MWh)
mu_per_ar=3.42;%%(NOK/MWh)per year
kappa=1.66; %per year
sigmaShort=191.42;% %(NOK/MWh) per year
sigmaLong=34.71;%(NOK/MWh) per year
correl=0.08;% %correlation between X and epsilon

% input for the simulation
totaltid=10;%year, lenght of the option
timesteps=100; %periods in simulation
simulations=7000;% number of simulations
periodelengde=totaltid/(timesteps)

% input for the project
invest=3;
kapasitet=800;%MW
driftstid=7600;%t
driftskost=invest*0.02*1e+9;%NOK/year
forsikring=invest*0.005*1e9;%NOK/year
rensing=14; %(NOK/MWh)
co2certificate=0.35*20*8; %(NOK/MWh)
t1=3; % years
t2=33; % years

%Parameters to the mean reverting prosess
sigma_mean_rev=sqrt(sigmaShort^2*periodelengde);
kappa1=kappa*periodelengde;
meanrevert=alfas;
%parameters to the ABM process
sigmaABM=sqrt(sigmaLong^2*periodelengde);
mu=mu_per_ar*periodelengde
%generates X
Rx=random(simulations,timesteps+1);
X=mcMeanRev(X0,meanrevert,kappa1,sigma_mean_rev,simulations,timesteps+1,Rx);
%generates epsilon
Re=random(simulations,timesteps+1);
epsilon=mcAMB(epsilon0,mu,sigmaABM,simulations,timesteps+1,Rx,Re,correl);
% deleting the random variables
Re=0;
Rx=0;
% calls the function that values the option.
svar=TwoFacAmOption(X,epsilon,rente*periodelengde,driftstid,kapasitet,rente,kappa,alfas,mu_per
_ar,SS,driftskost,forsikring,rensing,co2certificate,t1,t2,invest);
F=svar{1,1};
% Prints the option value
'option value'
verdi=svar{1,2}
exbound=svar{1,3};
% Prints the value of exercise today
'exercise today'
exToday=exercise_meanrev_twofactor(X0,epsilon0,driftstid,kapasitet,rente,kappa,alfas,mu_per_ar
,SS,driftskost,forsikring,rensing,co2certificate,t1,t2,invest)
toc %stops the timer

function
value=TwoFacAmOption(X,epsilon,diskont,E,K,r,kappa,alfa,mu,SS,opcost,insure,clean,certif,t1,t2
,invest)
%----------------------------------------------------------------------
%| Author:  Rolf M. Alstad & Jørgen T. Foss                              |
%| Date:    Spring 2004                                                  |
%| Version: Final                                                        |
%| Description: Calculates an American option using Monte Carlo.        |
%|              This function is constructed on basis of Longstaff &    |
%|              Schwartz 2001.                                           |
%-----------------------------------------------------------------------


% X is a matrix containing all the simulated X paths
```

```
% epsilon is a matrix containing all the simulated epsilon paths
% diskont is the discount rate per periode: exp(-diskont*t) gives the
%         discount factor
% E is the equivelant operation time in hours per year
% D is the capacity in MW
% r is yearly interest rate in %
% kappa is yearly mean reversion for short term process
% alfa is price of risk in NOK/MWh
% SS is the long term expected price in NOK/MWh
% D is cost of operations in NOK/MWh
% mu is drift in long term process in NOK/MWh per year
% t1 start of operations in years
% t2 end of operations in years
% invest investment i GNOK


%finding the number of simulations
[simulations periodes]=size(X);
%Defines the matrix F for holding all the option values for each simulation
F=zeros(simulations,periodes);

%When the option matures the option is exercised if V(S)-I >=0
for i=1:simulations

F(i,periodes)=max(exercise_meanrev_twofactor(X(i,periodes),epsilon(i,periodes),E,K,r,kappa,alf
a,mu,SS,opcost,insure,clean,certif,t1,t2,invest),0);
end
%Prints the value of the European option.
temp_eur=sum(F(:,periodes))/simulations*exp(-diskont*(periodes-1))

%for the rest of the periodes we check whether it is optimal to wait or to
%exercise. Each price path can only have one exercise point.
for j=1:periodes-2
    exboundp=0;
    n=periodes-j;
    % Since computational time is long a count down for every 100th period
    % is printed to the screen.
    if mod(n,20)==0
        n
    end


    k=0; %k is a counter that counts how many exercises points in a periode that has to be
checked
    % A is vector holding some dummy values. It is set to zero each
    % iteration.
    A=[0 0 0 0];
    for i=1:simulations
        if
exercise_meanrev_twofactor(X(i,n),epsilon(i,n),E,K,r,kappa,alfa,mu,SS,opcost,insure,clean,cert
if,t1,t2,invest)>0
            k=k+1;
            A(k,1)=i;  %this holds on to the simulation number
            A(k,2)=X(i,n); %+q*epsilon(i,n); %the dummy price with weighted X and epsilon
            A(k,3)=epsilon(i,n);
            %discounting the exercise value of the given path back to the
            %current time step.
            [big time]=max(F(i,:)); %finding the point of exercise and its value
            A(k,4)=exp(-diskont*(time-n))*big;
        end
    end

    % if any price path was picked out above we need to check whether early exercise is
optimal.
    if A(1,1)>0
        p=multireg(A(:,4), [A(:,2), A(:,3), A(:,2).^2, A(:,3).^2, A(:,2).*A(:,3)]); %Finding
the function for the value of waiting;
        %Wait(S)=a+bX+cE+dX^2+eE^2+fX*E
        a=p(1);
        b=p(2);
        c=p(3);
        d=p(4);
        e=p(5);
        f=p(6);

        [rows column]=size(A);%finding the number of rows in A
        for k=1:rows
```

```
        % Value of early exercise

exercise=exercise_meanrev_twofactor(X(A(k,1),n),epsilon(A(k,1),n),E,K,r,kappa,alfa,mu,SS,opcos
t,insure,clean,certif,t1,t2,invest);
        % Value of waiting
        wait_val=wait(A(k,2),A(k,3),a,b,c,d,e,f);
             if wait_val < exercise %if the value of waiting is less than the vaule of
exercising
                F(A(k,1),n)=exercise; %sets the value of the option equal exercise
                %since the option will be exercised prematuerly the remaining periodes is set
to zero
                sets the exercise boundary
                exboundp(k)=A(k,3);
                for m=n+1:periodes
                    F(A(k,1),m)=0;
                end
            else
                exboundp(k)=inf;
            end
        end
        %finds the exercise boundary for each period.
        exbound(n)=min(exboundp);
    end
end
% Each row in the F matrix should now have no more than one non zero number (or all zero).
% At each timestep the option values are summed, discounted back to the
% first periode and averaged.

temp_value=0; %temporary variable to hold the option value
for j=2:periodes
    temp_value=temp_value+exp(-diskont*(j-1))*sum(F(:,j))/simulations;
end
%either you exercise today or you wait.
temp_value=max(temp_value,exercise_meanrev_twofactor(X(1,1),epsilon(1,1),E,K,r,kappa,alfa,mu,S
S,opcost,insure,clean,certif,t1,t2,invest));
value{1,1}= F;
value{1,2}=temp_value; %returning the option value
value{1,3}=exbound
%-------------------------------------internal functions---------------------
%defining a function for the value of waiting
function sub_1=wait(X,E,a,b,c,d,e,f)
sub_1=(a+b*X+c*E+d*X^2+e*E^2+f*X*E);

%------------------------------------------------------------------------
%| Author:  Rolf M. Alstad & Jørgen T. Foss                               |
%| Date:    Spring 2004                                                    |
%| Version: Final                                                          |
%| Description: Uses general least squares regression to estimate         |
%|              coefficients in a polynom.                                 |
%-------------------------------------------------------------------------


function [w, regM]=multireg(Y,X)

% Y column vector of n obersvations
% X n by D matrix for D variables

n=size(X,1);
for i=1:n
ones(i)=1;
end
%adding a column of ones since the first term in the polynomial is a
%constant
X=[ones' X];

D_1=size(X,2);
for k=1:D_1
    for j=1:D_1
    R(k,j)=1/n*sum(X(:,k).*X(:,j));
    end
end

for j=1:D_1
    p(j)=1/n*sum(X(:,j).*Y(:));
end
%finds the coefficents
w=inv(R)*p';
```

```
function value=random(simulations,periodes)
%---------------------------------------------------------------------
%| Author:  Rolf M. Alstad & Jørgen T. Foss                           |
%| Date:    Spring 2004                                               |
%| Version: Final                                                     |
%| Description:  this function returns a (simulations)X(periodes-1)   |
%|               matrix with random normal distributed numbers with   |
%|               mean=0 and stdev=1                                   |
%----------------------------------------------------------------------
value=randn(simulations,periodes-1);


function
value=exercise_meanrev_twofactor(X,epsilon,E,K,r,kappa,alfa,mu,SS,opcost,insure,clean,certif,t
1,t2,invest)
%---------------------------------------------------------------------
%| Author:  Rolf M. Alstad & Jørgen T. Foss                           |
%| Date:    Spring 2004                                               |
%| Version: Final                                                     |
%| Description:  this function returns the exercise value at the given X|
%|               and epsilon.                                         |
%----------------------------------------------------------------------

% X
% epsilon
% E is the equivelant operation time in hours per year
% D is the capacity in MW
% r is yearly interest rate in %
% kappa is yearly mean reversion for short term process
% alfa is price of risk in NOK/MWh
% SS is the long term expected price in NOK/MWh
% D is cost of operations in NOK/MWh
% mu is drift in long term process in NOK/MWh per year
% t1 start of operations in years
% t2 end of operations in years
% invest investment i GNOK

% uses some dummy variables
E1=(exp(-t1*(r+kappa))-exp(-t2*(r+kappa)));
E2=(exp(-t1*r)-exp(-t2*r));
E3=((r*t1+1)*exp(-t1*r)-(r*t2+1)*exp(-t2*r));

c1=K*E/(r+kappa)*E1;
c2=K*E/r*E2;
c3=-K*E*alfa/(r+kappa)*E1;
c4=K*E*(SS+alfa)/r*E2;
c5=K*E*mu/r^2*E3;
c6=-K*E/r*(clean+certif)*E2;
c7=-(opcost+insure)/r*E2;

temp=(c1*X+c2*epsilon+c3+c4+c5+c6+c7)*1e-9; %returns in GNOK
value=temp-invest;%returns in GNOK

function value=mcAMB(start,alfa,sigma,simulations,periodes,random1,random2,correl)
%---------------------------------------------------------------------
%| Author:  Rolf M. Alstad & Jørgen T. Foss                           |
%| Date:    Spring 2004                                               |
%| Version: Final                                                     |
%| Description:  a function that simulates ABM for n=periodes 'periodes'|
%|               and m=simulations 'simulations'                      |
%----------------------------------------------------------------------

% start is the initial value of the process
% alfa is the per periode value of the drift
% sigma is the per periode value of standard deviation
% simulations is the number of simulations
% periodes is the number of periods
% random1 is a matrix(simulations, periodes-1) containing random normal
% distributed numbers with mean=0 and stdev=1 for the ABM process (this
% process)
% random2 is a matrix(simulations, periodes-1) containing random normal
% distributed numbers with mean=0 and stdev=1 for the meanreverting
% process(the other process)
% correl is the correlation between the meanreverting and ABM process

% defining the matrix to hold the function value
```

```
temp=zeros(simulations,periodes);
%doing the simulations
for i=1:simulations
    for j=1:periodes
        if j==1
            temp(i,1)=start;
        else
        temp(i,j)=temp(i,j-1) + alfa + sigma*(correl*random1(i,j-1)+random2(i,j-1)*sqrt(1-
correl^2)); %the actual simulation
        end
    end
end
value=temp;


function value=mcMeanRev(start,alfas,kappa,sigma,simulations,periodes,random)
%------------------------------------------------------------------------
%| Author:  Rolf M. Alstad & Jørgen T. Foss                              |
%| Date:    Spring 2004                                                  |
%| Version: Final                                                        |
%| Description:  a function that simulates a mean reverting process for |
%|               n=periodes 'periodes' and m=simulations 'simulations'  |
%-------------------------------------------------------------------------

% start is the initial value of the process
% alfas is the value to which the process mean revert
% kappa is the per periode value of mean reversion
% sigma is the per periode value of standard deviation
% simulations is the number of simulations
% periodes is the number of periods
% random is a matrix(simulations, periodes-1) containing random normal
% distributed numbers with mean=0 and stdev=1

% defining the matrix to hold the function value
temp=zeros(simulations,periodes);

%doing the simulations
for i=1:simulations
    for j=1:periodes
        if j==1
            temp(i,1)=start;
        else
        temp(i,j)=temp(i,j-1) + alfas*(1-exp(-kappa)) + (exp(-kappa)-1)*temp(i,j-1) +
random(i,j-1)*sigma*sqrt(1/(2*kappa)*(1-exp(-2*kappa)));      %sqrt(1/(2*kappa)*(1-exp(-
2*kappa)));   %sqrt(1-exp(-2*kappa*(periodes-j))); %the actual simulation
        end
    end
end
value=temp;
```

## LSM SPM2

Below is the code used to value the option in SPM2. Functions that are the same as in LSM SPM1 is not shown again. Language: Matlab.

```
clear all;
close all;
%------------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                            |
%| Date:    Spring 2004                                                  |
%| Version: Final                                                        |
%| Description:  This program calculates the value of an american opiton|
%|               using Monte Carlo simulation for a two factor ABM model|
%-------------------------------------------------------------------------
tic %starting the timer

%--input for the stochastic processes---
%-----Common----
rente=log(1+0.01*6); % per year
correl_gas_long_el_long=-0.002; %correlation between epslion_gas and epslion_el
correl_gas_long_el_short=0;%-0.011; %correlation between epslion_gas and X_el
```

```
correl_gas_short_el_long=0;%0.064; %correlation between X_gas and epslion_el
correl_gas_short_el_short=0;%0.078; %correlation between X_gas and X_el

%-------------Electricity-----------------
%X0_el=0.6749;%(NOK/MWh)
epsilon0_el=5.1159;%(NOK/MWh)
SS_el=0;%(NOK/MWh)
alfas_el=0; %(NOK/MWh)
mu_per_ar_el=0.0306;%(NOK/MWh)per year
kappa_el=1.0823; %per year
sigmaShort_el=0.7056; %(NOK/MWh) per year
sigmaLong_el=0.0001%0.0938;%(NOK/MWh) per year
correl_el=-0.05; %correlation between X and epsilon

%------Natural gas ------------------
%X0_gas=0.08;%(NOK/MWh)
epsilon0_gas=4.701;%(NOK/MWh)
SS_gas=0;%(NOK/MWh)
alfas_gas=0; %(NOK/MWh)
mu_per_ar_gas=0.012;%(NOK/MWh)per year
kappa_gas=2.6952; %per year
sigmaShort_gas=0.6653; %(NOK/MWh) per year
sigmaLong_gas=0.00001%0.1; %0.4577;%(NOK/MWh) per year
correl_gas=-0.195;% %correlation between X and epsilon


%----input for the simulation-----
totaltid=10;%year, lenght of the option
timesteps=20; %periods in simulation
simulations=7000;% number of simulations
periodelengde=totaltid/(timesteps);

%------input for the project------
invest=3;
kapasitet=800;%MW
driftstid=7600;%t
driftskost=invest*0.02*1e+9;%NOK/year
forsikring=invest*0.005*1e9;%NOK/year
rensing=14; %(NOK/MWh)
co2certificate=0.35*160; %(NOK/MWh)
t1=3; % years
t2=33; % years
total_opcost=driftskost/(rente)*(exp(-rente*t1)-exp(-rente*t2));
total_forsikring=forsikring/(rente)*(exp(-rente*t1)-exp(-rente*t2));
total_clean=kapasitet*driftstid*rensing/(rente)*(exp(-rente*t1)-exp(-rente*t2));
total_co2certif=kapasitet*driftstid*co2certificate/(rente)*(exp(-rente*t1)-exp(-rente*t2));
total_cost=total_opcost+total_forsikring+total_clean+total_co2certif+invest*1e+9;%total costs
except fuel

 rho=correl_gas_long_el_long;
 Z_long_el=randn(simulations,timesteps+1);
 tmp1=randn(simulations,timesteps+1);
 Z_long_gas=rho.*Z_long_el+tmp1.*sqrt(1-rho^2);
 tmp1=0;


%---parameters to the el ABM process----
sigmaABM_el=sqrt(sigmaLong_el^2*periodelengde);
mu_el=mu_per_ar_el*periodelengde;
%---parameters to the gas ABM process---
sigmaABM_gas=sqrt(sigmaLong_gas^2*periodelengde);
mu_gas=mu_per_ar_gas*periodelengde;



%----- generate the state variables -----

%------ long term el variable, epsilon_el
epsilon_el=mcAMB(epsilon0_el,mu_el,sigmaABM_el,simulations,timesteps+1,Z_long_el);
Z_long_el=0;
%------ long term el variable, epsilon_el
epsilon_gas=mcAMB(epsilon0_gas,mu_gas,sigmaABM_gas,simulations,timesteps+1,Z_long_gas);
Z_long_gas=0;
%----deleting the random variables to save memory

%---- calls the function that values the option.
```

```
svar=TwoFacAmOption(epsilon_el,kappa_el,alfas_el,mu_per_ar_el,SS_el,sigmaShort_el,sigmaLong_el
,correl_el,...

epsilon_gas,kappa_gas,alfas_gas,mu_per_ar_gas,SS_gas,sigmaShort_gas,sigmaLong_gas,correl_gas,.
..
    rente*periodelengde,driftstid,kapasitet,rente,total_cost,t1,t2,invest);

%F=svar{1,1};
%---- Prints the option value
disp('option value')
verdi=svar{1,2}
%----Prints the value of exercise today
disp('exercise today')
extoday=svar{1,3}
ex_el=svar{1,4};
ex_gas=svar{1,5};
toc %stops the timer


function value=TwoFacAmOption(eps_el,kappa_el,alfa_el,mu_el,SS_el,sigX_el,sigEps_el,rho_el,...
    eps_g,kappa_g,alfa_g,mu_g,SS_g,sigX_g,sigEps_g,rho_g,...
    diskont,E,K,r,total_cost,t1,t2,invest)
%-----------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                          |
%| Date:    Spring 2004                                                |
%| Version: Final                                                      |
%| Description: Calculates an American option using Monte Carlo.       |
%|              This function is constructed on basis of Longstaff &   |
%|              Schwartz 2001.                                         |
%-----------------------------------------------------------------------

%finding the number of simulations
[simulations periodes]=size(eps_el);
%Defines the matrix F for holding all the option values for each simulation
F=zeros(simulations,periodes);
invest=invest*1e+9;
%When the option matures the option is exercised if V(S)-I >=0
for i=1:simulations
    ex_value=quad(@value_fourfactor,3,33,'1+e-
3',[],eps_el(i,periodes),kappa_el,alfa_el,mu_el,SS_el,sigX_el,sigEps_el,rho_el,...
    eps_g(i,periodes),kappa_g,alfa_g,mu_g,SS_g,sigX_g,sigEps_g,rho_g,...
    E,K,r)-total_cost-invest;
    F(i,periodes)=max(ex_value,0);
end
%Prints the value of the European option.
temp_eur=sum(F(:,periodes))/simulations*exp(-diskont*(periodes-1))

siste=F(:,periodes);
%for the rest of the periodes we check whether it is optimal to wait or to
%exercise. Each price path can only have one exercise point.
for j=1:periodes-2
    n=periodes-j;
    % Since computational time is long a count down for every 100th period
    % is printed to the screen.
    if mod(n,5)==0
        n
    end


    k=0; %k is a counter that counts how many exercises points in a periode that has to be
checked
    % A is vector holding some dummy values. It is set to zero each
    % iteration.
    A=[0 0 0 0 0];
    for i=1:simulations
        per_val=quad(@value_fourfactor,3,33,'1+e-
3',[],eps_el(i,n),kappa_el,alfa_el,mu_el,SS_el,sigX_el,sigEps_el,rho_el,...
        eps_g(i,n),kappa_g,alfa_g,mu_g,SS_g,sigX_g,sigEps_g,rho_g,...
    E,K,r)-total_cost-invest;
        if per_val>0
            k=k+1;
            A(k,1)=i;  %this holds on to the simulation number
            %A(k,2)=X_el(i,n);  %
            A(k,2)=eps_el(i,n);
            %A(k,4)=X_g(i,n); %
            A(k,3)=eps_g(i,n);
            %discounting the exercise value of the given path back to the
```

```
            %current time step.
            [big time]=max(F(i,:)); %finding the point of exercise and its value
            A(k,4)=exp(-diskont*(time-n))*big/1e+9;
            A(k,5)=per_val/1e+9;

        end
    end


    if A(1,1)>0
        p=multireg(A(:,4), [A(:,2), A(:,3), A(:,2).^2, A(:,3).^2, A(:,2).*A(:,3) , A(:,5)-
invest/1e+9 ] );
        %Wait(S)=a+bE_el+cE_g+dE_el^2+eE_g^2+fE_el*E_g+g*E_e^3+hE_g^3
        %, A(:,2).^4 , A(:,3).^4

        a=p(1);
        b=p(2);
        c=p(3);
        d=p(4);
        e=p(5);
        f=p(6);
        g=p(7);
        [rows column]=size(A);%finding the number of rows in A
        for k=1:rows
        % Value of waiting
        wait_val(k)=wait(A(k,2),A(k,3),A(k,5)-invest/1e+9,a,b,c,d,e,f,g);%,a3,a4,a1,a2,g,h
        pro(k)=A(k,5);
             if wait_val(k) < A(k,5) %if the value of waiting is less than the vaule of
exercising
                F(A(k,1),n)=A(k,5)*1e+9; %sets the value of the option equal exercise
                %since the option will be exercised prematuerly the remaining periodes is set
to zero
                for m=n+1:periodes
                    F(A(k,1),m)=0;
                end
                exerc_el(n,k)=A(k,2);
                exerc_gas(n,k)=A(k,3);
             end
         end
    end
end
% Each row in the F matrix should now have no more than one non zero number (or all zero).
% At each timestep the option values are summed, discounted back to the
% first periode and averaged.

temp_value=0; %temporary variable to hold the option value
for j=2:periodes
    temp_value=temp_value+exp(-diskont*(j-1))*sum(F(:,j))/simulations;
end
%either you exercise today or you wait.
tmp2=quad(@value_fourfactor,3,33,'1e-
3',[],eps_el(1,1),kappa_el,alfa_el,mu_el,SS_el,sigX_el,sigEps_el,rho_el,...
    eps_g(1,1),kappa_g,alfa_g,mu_g,SS_g,sigX_g,sigEps_g,rho_g,...
    E,K,r)-total_cost-invest;
temp_value=max(temp_value,tmp2);
value{1,1}= F;
value{1,2}=temp_value; %returning the option value
value{1,3}=tmp2; %returning the exercise value today
value{1,4}=exerc_el; %exercise values el
value{1,5}=exerc_gas; %exercise values gas
%-----------------------------------internfunksjoner---------------------
%defining a function for the value of waiting
function sub_1=wait(E_el,E_g, exval,a,b,c,d,e,f,g)
%Wait(S)=a + bX_el + cE_el + dX_g + eE_g + fX_el^2 + gE_el^2 + hX_g^2 + a1E_g^2 +
        %b1E_el*E_g + c1E_el*X_el + d1E_g*X_g
sub_1=(a + b*E_el + c*E_g + d*E_el^2 + e*E_g^2 + f*E_el*E_g + g*exval);


function
value=value_fourfactor(t,eps_el,kappa_el,alfa_el,mu_el,SS_el,sigX_el,sigEps_el,rho_el,...
    eps_g,kappa_g,alfa_g,mu_g,SS_g,sigX_g,sigEps_g,rho_g,...
    E,K,r)
%-----------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                          |
%| Date:    Spring 2004                                                |
%| Version: Final                                                      |
%| Description:  integrating this function returns the value at        |
```

```
%|              the given epsilon.                                     |
%-----------------------------------------------------------------------


F_el=exp(lnForw(eps_el,t,SS_el,alfa_el,kappa_el,mu_el,sigX_el,sigEps_el,rho_el));
F_g=exp(lnForw(eps_g,t,SS_g,alfa_g,kappa_g,mu_g,sigX_g,sigEps_g,rho_g));
F=F_el-F_g;
value=exp(-r*t).*K*E.*(F);

function sub1=lnForw(eps,t,SS,alfa,kappa,mu,sigX,sigEps,rho)
%----------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                          |
%| Date:    Spring 2004                                                |
%| Version: Final                                                      |
%| Description:  This is function for the log of the forward price.    |
%-----------------------------------------------------------------------

sub1=SS  + eps + mu*t + (1)*alfa ...
    +0.5*((1)*sigX^2/(2*kappa)+sigEps^2*t+2*(1)*rho*...
    sigX*sigEps/kappa);
```

## Option value FCM1

Below is the code for finding the option value in FCM1. The code for finding the option value is very similar to this code.

```
%----------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                          |
%| Date:    Spring 2004                                                |
%| Version: Final                                                      |
%| Description:  This program finds the option value when using a     |
%|              spark spread forward curve model.                      |
%-----------------------------------------------------------------------

clear all;
close all;
tic
%loading the initial forward curves
load init_el_forward.txt -ascii
load init_gas_forward.txt -ascii
init_spark=init_el_forward-init_gas_forward;
%---- parametervalues
%------input for the project------
invest=3;
kapasitet=800;%MW
driftstid=7600;%t
driftskost=invest*0.02*1e+9;%NOK/year
forsikring=invest*0.005*1e9;%NOK/year
rensing=14; %(NOK/MWh)
co2certificate=0.35*160; %(NOK/MWh)
t1=3*52; % weeks
t2=33*52; % weeks
rente=log(1+0.06);
total_opcost=driftskost/(rente)*(exp(-rente/52*t1)-exp(-rente/52*t2));
total_forsikring=forsikring/(rente)*(exp(-rente/52*t1)-exp(-rente/52*t2));
total_clean=kapasitet*driftstid*rensing/(rente)*(exp(-rente/52*t1)-exp(-rente/52*t2));
total_co2certif=kapasitet*driftstid*co2certificate/(rente)*(exp(-rente/52*t1)-exp(-
rente/52*t2));
total_cost=total_opcost+total_forsikring+total_clean+total_co2certif+invest*1e+9;%total costs
except fuel

%--- option input
totaltime=520; %time to option matures in weeks
steps=100; %steps in simulation (exercise points)
timestep=totaltime/steps;

% ---- forward input
timebetween=4; % time(in weeks) between weeks to simulate
dt=timebetween;%tidssteg som brukes i integreringen
```

```
a=45.35; %per week
b=0.035; %per week
c= 3.44; %per week

simulations=1000;
parts=round((t2-t1)/timebetween);
% creating a vector containing discounted init prices to simulate
for i=0:parts
    points(i+1)=t1+timebetween*(i);
    F_sp(1,i+1)=(init_spark(points(i+1)))*exp(-rente/52*points(i+1));
end
%finding the value of investing today
init_value=kapasitet*driftstid/52*sparkValue(F_sp(1,:),dt)-total_cost
for i=1:simulations
    %The first column holds the initial value
    Value(i,1)=init_value;
end

%simulate the forward curves
for m=1:simulations
    if mod(m,50)==0
        m
        toc
    end
    for j=2:steps+1
        random=randn(1,1);
        for n=1:length(F_sp)
            t=t1-1+n;
            %el:
            el_vol=el_volf1(t,a,b,c);
            F_sp(j,n)=F_sp(j-1,n)+(el_vol*sqrt(timestep)*random);

             end
       %find the value at each exercise point at each path
       Value(m,j)=kapasitet*driftstid/(52)*sparkValue(F_sp(j,:),dt)-total_cost;
    end
end
%finding the option value
svar=AmOption(Value,rente/52*timestep);
am_opt=svar{2}
eur_opt=svar{3}
val=init_value
toc

%-------internal function
function sub1=el_volf1(t,a,b,c)
sub1=a*exp(-b*t)+c;



function value=AmOption(V,diskont)
%-----------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                          |
%| Date:    Spring 2004                                                |
%| Version: Final                                                      |
%| Description: Calculates an American option using Monte Carlo.       |
%|              This function is constructed on basis of Longstaff &   |
%|              Schwartz 2001.                                         |
%-----------------------------------------------------------------------




%finding the number of simulations
[simulations periodes]=size(V);
%Defines the matrix F for holding all the option values for each simulation
F=zeros(simulations,periodes);
%When the option matures the option is exercised if V(S)-I >=0
for i=1:simulations
    F(i,periodes)=max(V(i,periodes),0);
end
%Prints the value of the European option.
tmp2=V(1,1);
for i=1:simulations
    temp_eur(i)=max(F(i,periodes),0);
end
```

```
europt=max(sum(temp_eur)/simulations*exp(-diskont*(periodes-1)),tmp2);

siste=F(:,periodes);
%for the rest of the periodes we check whether it is optimal to wait or to
%exercise. Each price path can only have one exercise point.


for j=1:periodes-2
    n=periodes-j;
    % Since computational time is long a count down for every 100th period
    % is printed to the screen.
    if mod(n,20)==0
        n
    end


    k=0; %k is a counter that counts how many exercises points in a periode that has to be
checked
    % A is vector holding some dummy values. It is set to zero each
    % iteration.
    A=[0 0 0];
    for i=1:simulations
        if V(i,n)>0
            k=k+1;
            A(k,1)=i;  %this holds on to the simulation number
            A(k,2)=V(i,n)/1e+9;  %
            %discounting the exercise value of the given path back to the
            %current time step.
            [big time]=max(F(i,:)); %finding the point of exercise and its value
            A(k,3)=exp(-diskont*(time-n))*big/1e+9;
        end
    end

   %A
    % if any price path was picked out above we need to check whether early exercise is
optimal.
    if A(1,1)>0
        p=multireg(A(:,3), [A(:,2),A(:,2).^2,A(:,2).^3,A(:,2).^4]); %Finding the function for
the value of waiting;
        %Wait(S)=a+bV+cV^2+dV^3+eV^4
        a=p(1);
        b=p(2);
        c=p(3);
        d=p(4);
        e=p(5);


        [rows column]=size(A);%finding the number of rows in A
        for k=1:rows
        % Value of early exercise
        exercise=V(A(k,1),n);
        % Value of waiting
        wait_val=wait(A(k,2),a,b,c,d,e);
             if wait_val < exercise %if the value of waiting is less than the vaule of
exercising
                F(A(k,1),n)=exercise; %sets the value of the option equal exercise
                %since the option will be exercised prematuerly the remaining periodes is set
to zero
                for m=n+1:periodes
                    F(A(k,1),m)=0;
                end
            end
        end
    end
end
% Each row in the F matrix should now have no more than one non zero number (or all zero).
% At each timestep the option values are summed, discounted back to the
% first periode and averaged.

temp_value=0; %temporary variable to hold the option value
for j=2:periodes
    temp_value=temp_value+exp(-diskont*(j-1))*sum(F(:,j))/simulations;
end
%either you exercise today or you wait.

optval=max(tmp2,temp_value);
```

```
%V=exercise_meanrev_twofactor(X,epsilon,E,K,r,kappa,alfa,mu,SS,opcost,insure,clean,certif,t1,t
2,invest);
value{1,1}= F;
value{1,2}=optval; %returning the option value
value{1,3}=europt; %europt value
%value{1,4}=tmp2; %returning the exercise value today

%------------------------------------internal function---------------------
%defining a function for the value of waiting
function sub_1=wait(V,a,b,c,d,e)
        %Wait(S)=a+bV+cV^2+dV^3+eV^4
sub_1=(a + b*V + c*V^2 + d*V^3 + e*V^4)*1e+9;
```

## Smoothed forward curves

Below is the code for constructing smoothed forward curves for electricity, natural gas is very similar.

```
%---------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                         |
%| Date:    Spring 2004                                               |
%| Version: Final                                                     |
%| Description: This program calculates the smoothed forward curve for  |
%|              electricity.                                          |
%----------------------------------------------------------------------
clear all;
close all;
rente=0.05;
%loading the forward data
data=xlsread('startdata.xls');
antall=size(data,1)
set_lengde=size(data,2)
maxuker=zeros(antall,1);
%finding the greatest number of variables that will occur
maxmax=0;
for i=1:antall
    v=1;
    while data(i,21+4*(v-1))>0
        maxuker(i)= max(maxuker(i),data(i,18+4*(v-1))+data(i,19+4*(v-1))-1);
        lengde(i)=21+4*(v-1);
        maxmax=max(maxmax,maxuker(i));
        v=v+1;
    end
end

priser=zeros(antall,maxmax);
tid=zeros(antall,1);
iter=zeros(antall,1);
dato=zeros(antall,1);

%for each date in the observations
for n= 1:antall
%starting the timer
tic
%finding all closing prices
for i=1:(lengde(n)-1)/4
        temp(i,1)=(data(n,5+4*(i-1))+data(n,5+4*(i-1)))/2;
end
%finding the mean price of the forward curve
gjsnitt=mean(temp);
%establishing the variables
kont_priser=zeros(maxuker(n),1);
for i=1:length(kont_priser)
    kont_priser(i,1)=gjsnitt;
end
gjsnitt=0;
%no prices are less than zero
lb=zeros(maxuker(n),1);
%establishing the discounting variables
for i=1:maxuker(n)
    diskont(i,1)=exp(-i/360*rente);
```

```
end
% This line pass the objective function, the initial variables, the boundary and the other
function
% inputs to the Matlab optimization function "fmincon". This function
% returns the optimal variables

[f, fval,exitflag,output1,lambda,grad] =
fmincon(@minsec,kont_priser,[],[],[],[],lb,[],@confun,optimset('TolFun',1e-
4,'MaxIter',5e+2,'MaxFunEvals',1e+5),data(n,:),diskont,lengde(n));
f;
fval;

%adjusting data before writing to file

dato(n,1)=data(n,1);
iter(n,1)=output1.iterations;
for i=1:length(f)
priser(n,i)=f(i,1);
end
toc
t=toc;
tid(n,1)=t;
output=[iter tid dato priser ];
dlmwrite('output.dat',output,';');

%plotting the smoothed curve. Should be commented out if you "antall">1
bid=zeros(maxuker(n),(lengde(n)-1)/4);
for i=1:(lengde(n)-1)/4
    start=data(1,2+4*(i-1));
    varighet=data(1,3+4*(i-1));
    for j=start:start+varighet-1
        bid(j,i)=data(1,5+4*(i-1));
    end
end

ask=zeros(maxuker(n),(lengde(n)-1)/4);
for i=1:(lengde(n)-1)/4
    start=data(1,2+4*(i-1));
    varighet=data(1,3+4*(i-1));
    for j=start:start+varighet-1
        ask(j,i)=data(1,4+4*(i-1));
    end
end
t=1:length(f);
plot(t,f)
hold on
for i=1:size(ask,2)
plot(t,bid(:,i),'.r')
plot(t,ask(:,i),'.b')
end
n
end
'OBS  OBS  OBS  OBS  OBS'
'Husk å ta output.dat inn i excel og lagre datene før du kjører en ny runde med nye data'


%----------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                          |
%| Date:    Spring 2004                                                |
%| Version: Final                                                      |
%| Description: This is the constraint function used in the smoothed   |
%|              curves. The price must be less than Best Seller or      |
%|              greater than Best Buyer                                 |
%-----------------------------------------------------------------------

function [c, ceq]=confun(kont_priser,data,diskont,lengde)
m=0;
for i=1:(lengde-1)/4
    if data(1,5+4*(i-1))~=0
    m=m+1;
    tmp1=0;
    tmp2=0;
    start=max(1,data(1,2+4*(i-1)));
    varighet=data(1,3+4*(i-1));
    for j=start:1:start+varighet-1
        tmp1=tmp1+diskont(j)*kont_priser(j);
        tmp2=tmp2+diskont(j);
```

```
    end
    c(m,1)=tmp1-data(1,5+4*(i-1))*tmp2;%less than constraint
    c(m+(lengde-1)/4,1)=data(1,4+4*(i-1))*tmp2-tmp1;%greater than constraint
    end
end

ceq=[];

%-----------------------------------------------------------------------
%| Author:  Jørgen T. Foss and Rolf M. Alstad                           |
%| Date:    Spring 2004                                                 |
%| Version: Final                                                       |
%| Description: This is the objective function used in the smoothed     |
%|              curves. It finds the sum of squared difference between  |
%|              adjacent prices                                         |
%-----------------------------------------------------------------------

function svar=minsec(kont_priser,data,diskont,lengde)
tmp=0;
for i=2:length(kont_priser)-1
    tmp=tmp+(kont_priser(i-1,1)+kont_priser(i+1,1)-2*kont_priser(i,1))^2;
end
svar=tmp;
```