

Real Option Analysis of Gas Fired Power Plants

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Preface

This paper is prepared as our Project Thesis at the NTNU by the Department of Industrial Economics and Technology Management. Our work falls within the Group of Investment and Finance Management.

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Executive summary

The prevailing power situation in Norway opens for investments into new power generation capacity. Several plans for building gas fired power plants have been announced, but no commitment has yet been made. This paper investigates the profitability of investing in a CCGT power plant by using a real option approach for the investment decision.

This paper has investigated the investment timing for a firm owning a license to build a gas fired power plant, and hence the option value to delay investment. Spark spread, the difference between the price of electricity and cost of natural gas, forms the basis of the valuation. Spark spread is modeled to follow different stochastic processes according to three models; a one factor arithmetic Brownian motion, a single factor mean reverting and a two factor model with short time mean reversion and long time drift.

A base case scenario is calculated using representative values for a 800 MW CCGT base load power plant located at Tjeldbergodden. The assumptions made with respect to the power plant represents a sort of “best case” conditions, but are still considered to be realistic. The investment can not be made more profitable by changing plant location unless synergy effects can be obtained.

The uncertainty regarding cost of CO₂ emissions as much discussed in the public media is captured by model 1 and 2. Since this is not reflected in model 3, cost of CO₂ emission cannot be claimed to be the most important aspect for delaying investing. All models point out the future spark spread level, e.g. the future price of natural gas and electricity as important for the investment decision. Based on this long-term future price levels of natural gas and electricity are considered to be the most significant decision factor.

We conclude that the two factor model is the most accurate. Applied to the base case scenario it gives a value to invest of about 2900 MNOK and an option value of 3500 MNOK. The best investment decision is to wait.

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1 Introduction

Gas fired power plant is on the public agenda in Norway at present. Insufficient generation capacity in parts of the country calls for new production capacity. To this day no gas fired power plant is build, and the discussion of whether high natural gas prices, low electricity prices or uncertainty about CO₂ handling is the reason for why investors do not invest goes on. Through real option theory these factors are analyzed.

This paper applies real option theory to value the license to invest in a base load Combined Cycle Gas Turbine plant (CCGT) under uncertainty. The license is modeled as an option to invest.

In order to do such valuation several input parameters are need such as cost and operational parameters. Through a thorough discussion of properties as size, availability, number of turbines etc. in a CCGT, and its location these parameters are determined. Costs concerning NO_x reduction and CO₂ quotas are also considered.

To model uncertainty in the price difference of electricity and natural gas, called spark spread, stochastic processes are used. These stochastic processes and their equivalent functions for expected future price are fitted to real price data from Nord Pool and the International Petroleum Exchange through regression and least squares methods. In this paper three different stochastic processes for the prices are investigated.

To perform the valuation of the options numerical procedures are used. Closed form solutions are only possible for infinite option for the simplest stochastic process. It is the options with finite life time which are most realistic. The numerical procedures used in this paper are binomial trees, trinomial trees and Monte Carlo simulation.

2 Theory on Real Options

In the real world choices are made every day. Many of these are considered to be obvious and other requires more thought. All these choices represent options. When buying clothes there is an option to wait for the “back to school sale”. Then money could be saved if so is chosen and the option to wait has a value. This is an example of a real option.

A firm owning real assets has several choices to make about how to manage it’s portfolio of assets. It can choose to add more assets to the portfolio or remove others. The firm can also decide whether to use an asset today or not. Different options can represent a value and has to be considered when making decisions. Especially in a world of uncertainty such options are important. More generally one can divide real options into:

- The option to wait (for instance wait to invest)
- The option to abandon (for instance to sell an asset)
- The option to be flexible
- The option to expand
- Etc.

The real option theory is introduced by among others Brennan and Schwartz (1985), Pindyck (1988), Dixit and Pindyck (1994) and Trigeorgis (1996).

2.1 The option to postpone an investment

This paper investigates the option to wait and by that also the timing of the investment. A firm owning a license to build a gas fired power plant has the right but not an obligation to invest in the power plant. The firm can decide to invest whenever it wants in the whole period for which the license is valid. Thus the firm has an option to invest at any time during the life of the option (the period when the license valid). In financial terms such an option is of American style.

Consider a firm having the right to invest in a power plant. When the investment decision is made the investment cost is paid and the power plant is built. The investment cost cannot be fully retrieved once the investment is made. The investment cost is said to be “sunk”. According to general investment theory the investment should be made once the net present

value (NPV) of the expected cash flows exceed zero as shown in (2.1). Following the real option approach this may not be true because the option to wait (or other options) is not considered. The next period the NPV can easily become negative and the “sunk” investment cost is wasted. Thus only considering the NPV of the future expected cash flows assume that either the investment cost is fully retrievable or the opportunity to invest is now or never so the option to wait does not exist. This is one of the major criticisms of the general NPV approach.¹

$$PV(\text{future cash flows}) - \text{Investment} \geq 0 \quad (2.1)$$

Real option theory implies that an investment should not be made before the net present value of future expected cash flows exceed the value of the option to wait. The option to wait is not “killed” (making an investment kills the option) before it is more profitable to have the power plant as shown in (2.2). In our setting this means that at the last day of the license a regular NPV analysis is valid. Before that the option to postpone the investment must be considered.²

$$PV(\text{future cash flows}) - \text{Investment} \geq \text{Option to wait} \quad (2.2)$$

2.2 Real Options in the Energy Business

Real option analysis is very suitable for projects in the energy business since such projects incorporate many of the features that set the basis for real option theory. Such features are among others

- Large sunk investment cost. Investment in a hydro power plant or transmission equipment cannot be retrieved.
- Production flexibility. The power plant could only be operated when prices make it profitable.
- Option to expand capacity. Already existing power plants can have possibilities to install new capacity.
- Existence of futures and other derivatives that serve as spanning assets that reveal important info about the value of the project (spanning assets are explained in chapter 5.1)

¹ Dixit and Pindyck (1994)

² Dixit and Pindyck (1994)

In recent years much work has been done using real options to value different kinds of projects in the energy business. Deng et al (2001) has used real options analysis to value peak load power plants and transmission capacity. Thomson (1995) uses real option theory to value take-or-pay contracts. In Schwartz' work from 1997 and 98 methods for investment decisions are outlined. Dobbe et al (2003) uses real option theory to value a peak load gas fired power plant in Norway and to value the an infinite license to build such a plant.

3 Plans for gas fired power plants in Norway

The construction of gas fired power plants in Norway is in 2003 a relevant topic with respect to the power situation. Several plans have been proposed, all accompanied by controversy related to CO₂ emissions. By the end of 2003, no commitments to build a gas fired power plant for delivery of electricity to the main grid has been announced.

3.1 Power situation

The Norwegian electricity supply is based on hydro power (99.5% of domestic production) and imports. In a year with normal precipitation hydro power supply the electricity market with 118 TWh per year. The actual production can vary between 89 TWh for a dry year and 150 TWh for a wet year. The Norwegian electricity demand is about 125 TWh per year, resulting in an import demand of 7 TWh on a regular basis. NVE states that the import will increase to 12 TWh in 2010³. In dry years limited import capacity can become a restriction.

The current situation in Norway calls for installation of new electricity generation capacity. Gas fired power plants is only one of several possibilities. New hydro power capacity on large and small scale and wind power are other most likely possibilities, however, having some constraints on its potential. Enova estimates a wind power production of 3 TWh in 2010⁴. Remaining hydro power potential is about 25 TWh, including the larger part of an upgrading and expanding potential on existing hydro power installations of 10 TWh⁵. The remaining hydro power project is to a large extent made up of controversial and increasingly costly projects. New transmission capacity to UK and Denmark has been considered but put aside for the time being. Though there are alternatives to a gas fired power plant none of these exclude the possibility to build a gas fired power plant.

3.2 Existing plans for Gas fired power plants in Norway

Table 1 shows the formal plans for gas fired power plants in Norway. The projects at Kollsnes and Kårstø were granted permission in 1996 and these two permissions are liable till October

³ Kristensen et. al (2003)

⁴ Arnstad (2002)

⁵ Paaske (2002)

2007⁶. The power plant at Skogn was granted permission in 2000⁷. The permissions usually have a life time of 10 years. 10 years is considered the normal life of a licence in this setting.

| Project | Company | Status | MW | TWh | Investment cost GNOK |
|----------------|-----------------------------|-----------------------|------------------|------------------|-----------------------------|
| Kollsnes | Naturkraft | License granted | 390 | 3.1 | 1.8 ¹ |
| Kårstø | Naturkraft | License granted | 380 | 3.0 | 1.8 ² |
| Skogn | Industrikraft Midt-Norge | License granted | 800 | 6.4 | 3.6 ³ |
| Karmøy | Norsk Hydro | Reported | 1300 | 10.5 | |
| Snøhvit | Statiol | Under construction | 230 ⁴ | 1.7 ⁴ | |
| Tjeldbergodden | Nordenfjeldske Energi | Reported | 400 | 3.2 | |
| Tjeldbergodden | Naturkraft | Reported | 800 | 6.4 | 3.0 ⁵ |

Table 1: Formal projects permitted by or reported to NVE.⁸ 1,2) According to SFT. 3) According to Nord Trøndelag fylkeskommune (1999). 4) According to Statnett. 5)According to Hegerberg(2003), Adressavisen.

Statoil, one of the largest investors in Naturkraft, has signalled the preference of locating a new gas power plant in connection with already existing petrochemical industry⁹. It implies that the original projects at Kollsnes and Kårstø are less likely to be undertaken. A CCGT plant at Tjeldbergodden or Kårstø and a CHP plant at Mongstad have been proposed as potential project. Elkem which owns 50% of Industrikraft Midt-Norge claims the reason the plant at Skogn is not built is due to too high prices on natural gas. There is also a discussion of who is to cover the large cost of gas infrastructure investment connected to this project.¹⁰ According to Managing Director of Naturkraft, Ole Rønning, a stable electricity price of 300 NOK/MWh is will make a gas fired power plant highly profitable.¹¹

⁶ Rønning (2003)

⁷ Industrikraft Midt-Norge (2003)

⁸ NVE (2003)

⁹ Petromagasinet (2003)

¹⁰ Sundsbø (2003)

¹¹ Steensen (2003)



Figure 1. Potential locations of a CCGT plant.

3.3 Price effects of new generation capacity

According to the efficient market hypothesis, the market price is already reflecting the general belief in the market with respect to new generation capacity. It is hard to estimate exactly how large impact a new 800 MW power plant will have on electricity prices in the Nordpool area. The location of new capacity of this order is of large importance due to constraints on transmission capacity between the grid areas, and can thus highly influencing the local area price. A SINTEF study of the old regulated market without considering transmission constraints concluded that a new plant of 800MW at Skogn would lead to a significant decrease in electricity prices in the order of 10-20 NOK/MWh for the whole year. A similar effect can be assumed to occur in the deregulated market.¹²

¹² Wangensten (1999)

For a real option valuation approach which includes the price effects, see Keppo and Lu (2003) who argues that large investors must also take the negative price effects on existing plants into account before making the investment decision.

Assuming that a new power plant does have an impact on the market price of electricity, the profit from building additional base-load plants will decrease. In the Norwegian market, construction of one power plant will reduce the willingness to invest amongst other actors. There will be a first mover advantage affecting the option value. Interested readers are advised to see Dixit and Pindyck (1994) a general real option approach.

3.4 The need for base load

The Nordic electricity grid has abundance of hydro power generation capacity with good regulatory characteristics, excellent for peak load production. A gas fired power plant located in Norway will as a consequence be used for base load production.

Nuclear power plants in Sweden will operate with a lower marginal cost than a CCGT plant, but the recent power balance in the Nordic market does not exclude the possibility of running a base load plant at full load most of the time.¹³ It should be possible to sell the power from the base load plant at long-term fixed contracts to e.g. Hydro expanded aluminum plant at Sunndalsøra. The producer is facing the risk of having to pay a risk premium in order to secure the production several years into the future.

¹³ Botterud (2003)

4 Market for electricity and natural gas

Norway has a well developed electricity market as a part of the Nordic commodity Exchange for electrical power, Nord Pool. There is no similar market for natural gas in Norway, and it is assumed that natural gas would be sold on long term contracts with a take-or-pay clause (TOP). Price data from the International Petroleum Exchange (IPE), subtracted for the transportation cost from the North Sea to UK, are used to approximate “Norwegian” prices on natural gas. The markets for electricity and natural gas are assumed to be perfect.

4.1 Natural gas financial contracts at IPE¹⁴

Natural gas futures are among the energy derivatives traded at IPE in London. Depending on the date of the trading day, the contracts listed below are subject to trade:

- Monthly futures for delivery in the 9-11 next months
- Quarterly futures for delivery in the 6-7 quarters after the monthly contracts. (Denoted Q(number, year))
- A six-month futures contract for the nearest season after the quarterly contracts, and a six-month futures contract for the consecutive season. (Denoted Summer/Winter(year))

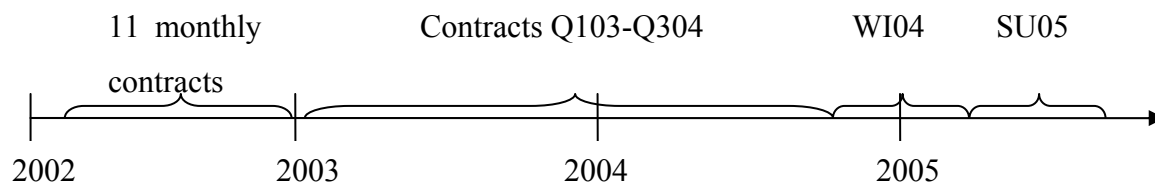


Figure 2: Future contracts available at IPE in January 2002.

The Season and Quarter contracts are split up as time closes in on the delivery period. As an example, the Q103 is split into monthly contracts after the last trading day in March 2002. The Q203 is split into monthly contracts and the WI04 is split into two quarterly contracts of Q404 and Q105 after the last trading day in June 2002. Examining data from the IPE-historical databank reveals that contracts with more than a year to delivery are not very liquid.

¹⁴ The chapter is based on public information from the web pages of IPE (2003)

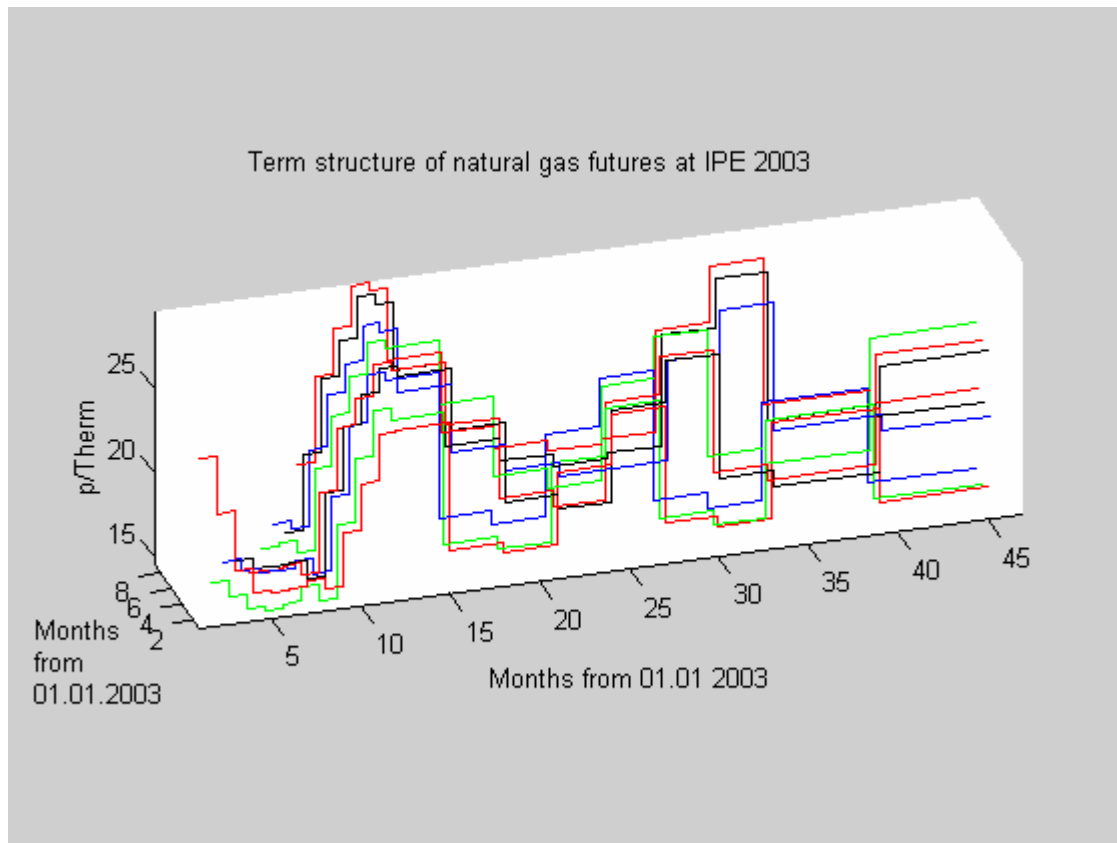


Figure 3: The term structure of Natural gas futures at IPE. Each path represents the term structure on the first trading day in each month from January to September 2003. The prices are noted pence/therm. The axis ranging from 0 to 45 represents the months starting January 2003. The axis ranging from 1 to 9 represents the starting point month counted from January 2003.

Figure 3 shows the term structure of natural gas at IPE for the first trading day in each month from January to September 2003. From this figure one can clearly see how the short-term futures prices have seasonal variations. The top of around months 10 to 15 from January 2003 are prices for the winter in 2004.

4.2 Financial electricity contracts at Nord Pool

A variety of financial electricity contracts are traded at Nord Pool, among them monthly and yearly forwards. For any given trading day, monthly forward contracts for the next six months and yearly forward contracts for the next three years are available. Up until April 7th 2003 Nord Pool offered block futures contracts for the 8-12 next months. One block was a 4 week contract which was split into weekly contracts some weeks prior to expiry in accordance with rules determined in the product specifications. After April 7th 2003 the monthly forward contracts replaced the 4 week blocks.¹⁵

¹⁵ Nord Pool 2003

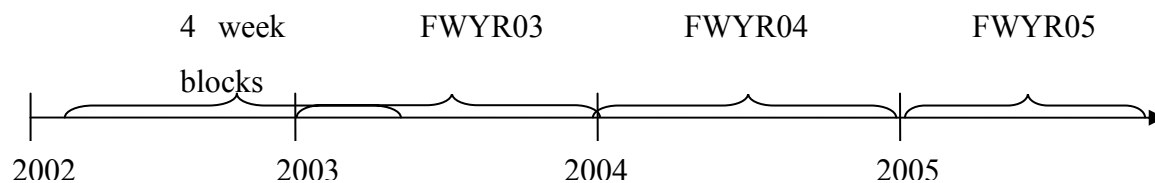


Figure 4: Future and forward contracts available at Nordpool in January 2002.

Figure 4 shows the block futures contracts and the three yearly forward contracts of interest to this paper available for January 2002. Since historical data are used, block contracts will make up a larger part than the new monthly forwards in the empirical data set.

4.3 Long term contracts

The project will have an expected lifetime of 30 years, and knowing prices on contracts for the entire period of production would be preferable for project valuation. Exchanges such as Nord Pool and IPE only offer financial contracts on a relatively short horizon. A comparable market for long term contracts does not exist. Long term price projections for a 30 year horizon are often based on analyzing various scenarios.¹⁶

4.3.1 Long term electricity contracts

Although not traded on exchange, long term power-contracts of up to 10 years delivery period are traded in the OTC market. The ten-year contract is a forward similar to those traded at Nord Pool. Elkem Energy has provided the price of a ten year contract to be used in this paper¹⁷. According to various brokers, the prices on 10-year contracts are today not very liquid but the price information is believed to be representative¹⁸.

4.3.2 Long term natural gas contracts – the Take-or-pay clauses

The long term contracts in the former regulated European gas market were usually made up of a set of provisions. Among the most important pricing provisions were to positive correlate the price of natural gas to the price development of other energy sources and the Take-or-pay

¹⁶ Bergli (2003)

¹⁷ Dobbe (2003)

¹⁸ Florholmen (2003)

(TOP) clause.¹⁹ The price of natural gas in long term contracts was in the past independent of natural gas spot price as no such existed.

Buying natural gas on a long term contract with a TOP-clause is considered the most realistic scenario for a gas fired power plant located in Norway.²⁰ The contract price today will be affected by the spot price of natural gas in the European market, and the price offered in Norway will be determined by the alternative value of selling the natural gas to UK or other European countries.

The provisions in every long term contracts are subject to separate negotiations and the details not publicly available. The general contract structure is described by Brautaset et al (1998). The “take or pay” concept can be defined as a provision, written into a contract, whereby one party has the obligation of either taking delivery of goods or paying a specified amount. TOP includes a package of provisions in which flexibility in delivered volume and price settling are the most important. Price information on long-term contracts is scarce. However, Dobbe (2003) has provided a 10-year price estimate for UK.

The concept of the long term contracts with a TOP clause leads to the buyer taking the volume risk, whereas the seller takes the price risk. The logic behind is that the seller needs volume to support large investments in infrastructure, and the buyer needs to get insurance of a competitive price to be willing to make a long time commitment.

Volume

The TOP-clause takes into consideration that the buyer can experience variations in demand of natural gas. Delivery flexibility is obtained by allowing departure within a given interval from a set of reference volumes, a Daily Contract Quantity (DCQ) and Annual Contract Quantity (ACQ). The allowed deviation will typical be in the range of 40-110% for DCQ and 90-110% for ACQ, further departure initiating payment obligations.²¹

¹⁹ Austvik (2003) p42

²⁰ Sigmo (2003)

²¹ Osmundsen (2002)

The flexibility in the contract will be dependent on the location of the gas plant. Terms as described above can only be obtained in a larger infrastructure where production and consumption of natural gas can be smoothed. By locating the power plant in connection with the existing gas infrastructure, similar contract terms should apply as is the case for most of the projects listed in Table 1.

Price

The price of natural gas is determined by a combination of a fixed base price and a set of variables, the most important being the price development of relevant substitutes²² and a few other variables with impact on energy consumption such as temperature. In the European gas market, the natural gas contracts have largely been formulated in a way that prices react to changes in prices on alternative energy sources, with a certain time lag. The most important has traditionally been the oil price, but the price of electricity is gaining importance.²³

The spot price of natural gas itself has gained importance as the long term contracts coexists with the spot market. The effect of deregulation has not yet fully influenced the European gas market. The price of natural gas has gradually become more determined by the supply and demand of natural gas, but not considered to be fully decoupled from the oil price before 2010.²⁴

The contract details will specify which energy carriers and other factors that are to be included in the pricing formula, the weighting of the carriers and the initial relationship. The escalation mechanisms between changes in prices for the alternative energy carriers and the price of natural gas are also considered. Applying the concept of price risk taken by the seller, a TOP contract in Norway would be highly affected by the price of electricity.

Renegotiation

Long term natural gas contracts normally include a clause on renegotiation. Changes in market conditions will be reflected by renegotiating the terms of the long term contract. As an example, Troll contracts contain clauses to ensure that either buyer or seller can demand

²² Osmundsen (2002)

²³ Tomasgard (2003)

²⁴ UBS Warburg (2001)

renegotiations every 3rd year if market conditions have changed so much that the pricing formulas no longer reflect the competitive position of natural gas in the market. The renegotiation term has been included to prevent the break up of long term contracts as cheaper natural gas has become available in the deregulated market. The renegotiation clause has to this date mainly benefited the buyer. However, including a renegotiation clause will add risk to the project of building a gas fired power plant since it can result in an increased cost of natural gas.

5 Spark spread

This paper uses spark spread as the underlying asset to value investment options. Spark spread is defined as the price of 1 MWh electricity less the cost of natural gas needed to make 1 MWh of electricity as given in equation (5.1).

$$\text{Spark Spread} = P_{\text{el}} - P_{\text{gas,el equivalent}} = P_{\text{el}} - HP_{\text{gas}} \quad (5.1)$$

P_{el} - price of electricity [NOK/MWh_{el}]

P_{gas} - price of natural gas [NOK/MWh_{gas}]

H - heat rate [MWh_{gas}/MWh_{el}] as defined in chapter 6.2.3.

Electricity and natural gas are both very expensive to store, making the spot price highly dependant on instantaneous supply and demand. Thus one unit of electricity or natural gas at present is not the same as one unit at a later point in time. It can be said to be two different commodities.²⁵ As a consequence, the commodity electricity or natural gas cannot be traded in a market. Futures and forward contracts on these commodities are, however, tradable.

5.1 “Spark spread spot”

Since the commodities itself are not traded in the market, the difference between spot prices of electricity and natural gas can not be used as the underlying spark spread. It is, however, sufficient that other assets whose risk tracks or spans the uncertainty are traded. A “spanning asset” is used to replicate the non tradable asset by tracking its risk.²⁶ Spark spread forwards are used as spanning asset and called spark spread “spot”. This is the price referred to as “spot” in the text. “Dummy spot” is used when long-term market information gives the variable as explained in chapter 9.2.

The “spark spread spot” must be constructed from some financial contracts close to the given date considered. Such contracts could be a monthly forward contract on electricity and monthly futures on natural gas. Since the closest available futures contract on natural gas is maturing in the beginning of the second month from a given date this contract and the equivalent electricity contract is used to construct the spark spread “spot” price. Data from

²⁵ Lucia and Schwart (2002)

²⁶ Dixit and Pindyck (1994)

2000 till October 2003 is used. The constructed spark spread “spot” is assumed to be a spanning asset of the commodities.

5.2 “Spark spread” forwards

Since the models use spark spread, and not the gas or electricity prices explicitly, the construction of a set of spark spread forwards must be conducted in order to estimate parameters for the models. The construction is done by first setting the electricity forwards from Nord Pool as basis. Then yearly natural gas contracts are constructed from the available contracts at IPE following the theory on swaps in McDonald (2003):

$$F_{\text{year}} \sum_i d_i = \sum_i d_i F_i \text{ or } F_{\text{year}} = \frac{\sum_i d_i F_i}{\sum_i d_i} \quad (5.2)$$

F_{year} - price of equivalent yearly contract

F_i - price of available contract in period i .

d_i - discount factor used in period i . In the discount factor the risk free rate is used and the contract is discounted from the middle of the contract period.

i – a period where a future contract exists. I.e. $i=Q0103$, see Figure 2.

The financial contracts on gas and electricity have different term structures, and some adjustments had to be made to achieve coherence. Since the interest rate is assumed to be constant, there is no difference between futures and forward contracts. Thus for a given date it is possible to construct a yearly contract maturing January 1st the two next years and a 9 month contract that matures January 1st in 3 years. These are used in the estimation of parameters in chapter 9.

5.3 Modeling Spark Spread

Prior to the investment decision spark spread is assumed to follow a stochastic price process. Price data from Nord Pool and IPE are assumed to be the best approximation of this price process, and will be used to estimate parameters. Once the investment decision is made it is assumed that the power plant will buy natural gas on a TOP-contract.

Further the TOP-contract will be assumed to have the prices of electricity and natural gas as its only variables. It allows for using the spark spread forward curve to reflect the market value of the power plant at the time of investment. If other variables were to be included in the TOP- contract, they would also have to be included in the valuation. Omitting other variables can be considered a relatively fair assumption as the price of natural gas moves towards a gas-to-gas price and electricity being the main relevant substitute for natural gas in Norway. An alternative approach would be to model the price of natural gas and electricity as separate processes.

6 The power plant

This chapter provides a brief description of the gas-fired power plant and presents the technical input values to be used in the Real Options analysis. A more detailed description together with a discussion of some of the most important issues of plant and process design is given in Appendix I.

6.1 Assumptions

The following assumptions are made based on what is considered to be a likely scenario with respect to operation strategy and location of a gas fired power plant in Norway. The scenario is “based on” Statoil’s project of building an 800MW gas-fired power plant at Tjeldbergodden.²⁷ A more detailed general discussion of cost structure is given in appendix I.

Assumption 1: Operating strategy

The plant will be producing electricity only and operate as a base load plant. Using large Combined Cycle Gas Turbines (CCGT) will be the best choice to match the operating strategy

Assumption 2: Location

The plant will be located close to existing gas processing facilities in the costal region of western Norway.

The type and size of the gas turbines installed will determine the plant’s part load efficiency and ramping time. The gas turbines are the most expensive components of the plant components, and it is important that the technology used suits the intended production in a cost efficient way.

The location is important for two main reasons. Ambient conditions are affecting the process as described later in appendix I. The location should also be chosen to minimize the investments into infrastructure with respect to both gas transport and electricity transmission.

Tjeldbergodden provides favorable conditions with respect to both assumptions. The costal climate and proximity to the coast, the fact that Tjeldbergodden already is connected to the

²⁷ Hegerberg (2003)

main gas grid through the pipeline Haltenpipe and is relatively close to new large customers of electricity, minimizing the need for investment into new infrastructure.

6.2 Input parameters used for Real Option analysis

The input values of investment, operating- and maintenance and emission costs are considered.

6.2.1 Investment costs

The total investment cost is set to 3000 MNOK.²⁸ This figure does not include large investment in gas or electricity infrastructure. The existing gas pipeline to Tjeldbergodden (Haltenpipe) has enough excess capacity to supply the power plant.²⁹ Investment of around 300 MNOK is needed in a new power transmission. This is omitted and considered to be a separate project financed by e.g. transmission fees. 30 year lifetime and a three year construction period are assumed.

6.2.2 Operating costs and cost of emissions

Operating and maintenance cost can be set to 2%, and insurance to 0,5 % of planned investment cost per year.³⁰

The plant will be subject to restrictions on NO_x- emission. This requires the use of e.g. a dry-low NO_x turbine. NO_x reducing measures will add an operating cost of 14 NOK per MWh produced.³¹

Uncertainty concerning taxes and restrictions on CO₂-emissions makes it difficult to make accurate price forecasts. CO₂-separation equipment is more expensive than the expected quota-price on CO₂.³² Cost of CO₂ emission is included by using the expected quota price added as an operating cost. CO₂ – quotas are further discussed in chapter 7.

²⁸ Hegerberg (2003)

²⁹ Arnstad 2003

³⁰ Bolland (2003)

³¹ Bolland (2003)

³² Lont (2003)

6.2.3 Efficiency and availability

The base load plant is assumed to produce as much as possible. Operating hours is set to the technical maximum. The plant is assumed to have an availability factor of 90%,³³ equivalent to 7900 operating hours. As full load operation is assumed at all time, the equivalent operating hours will be equal to operating hours.

A standard CCGT plant with F-series technology can reach up to 58 % thermodynamic efficiency by Norwegian conditions. The efficiency, μ , is as defined as the electrical output divided by the energy used for its production (6.1). Heat rate (HR) is defined as the inverse of μ (6.2).

$$\mu = \frac{\text{electricity, produced}}{\text{energy, gas}} \quad (6.1)$$

$$HR = \frac{1}{\mu} \quad (6.2)$$

Degradation of the CCGT plant will lead to irreversible decrease in output and efficiency. This is not considered in the valuation, but can be included by reducing the input parameter of equivalent operating hours or the expected lifetime.

6.2.4 Input values – base case

Table 2 summarizes the values that will be the input parameters to the models concerning the power plant.

| Cost | Value | Unit |
|----------------------|-------|------------|
| Investment cost | 3000 | MNOK |
| Project lifetime | 30 | year |
| Construction period | 3 | year |
| O & M | 60 | MNOK/year |
| Insurance | 15 | MNOK/year |
| NOX | 14 | NOK/MWh |
| CO2 | 160 | NOK/MWh |
| Eqv. Operating hours | 7900 | hours/year |
| Plant efficiency | 58 | % |

Table 2. Input parameters – Base case.

³³ Appendix I

7 CO₂ quotas

The Norwegian government has been vague on signalling how the restriction on CO₂ emissions as imposed by the Kyoto agreement is going to be implemented. The uncertainty about future handling of CO₂ regarding taxes and restrictions on emissions, CO₂-quotas and green certificates have made investors reluctant to initiate plan for gas fired power plants.

The forecast on CO₂-quotas varies. Eckert (2003) estimates price in 2010 to be around 10 EUR/metric ton (MT), increasing towards 20 EUR in 2020³⁴. The estimates are coherent with the RWE Powers assumption of a price somewhere in the range of 5-20 EUR/MT.³⁵ The long term price will be determined by development in supply and demand and accurate estimates can hardly be predicted. Quotas has recently been traded for 11-12 EUR/MT, however, all details remain disclosed.³⁶

A cost of 20 EUR/MT is used in the calculations. The high end estimate is used to give a conservative project valuation. Quotas on 20 EUR/MT CO₂ results in an additional cost of 56 NOK/MWh produced. Green certificates that will promote renewable energy are estimated to have a cost equivalent to that of CO₂-quotas³⁷. A gas fired power plant will need a permit on CO₂ emissions from the Norwegian Pollution Control Authority (SFT) to commence operation. Permits are currently given to all projects with a granted licence (Table 1). The models are designed to include the cost of CO₂ emission by adding cost of CO₂ per MWh produced.

³⁴ Eckert (2003)

³⁵ Bergli (2003)

³⁶ PointCarbon (2003)

³⁷ Lont (2003)

8 Mathematical models

Chapter 8 describes and deduce the mathematical models which are used for calculations in this paper. Three models are presented assuming the spark spread “spot” following a one factor arithmetic Brownian motion, a one factor mean reverting process and a two factor mean reverting process.

Recall from chapter 5.1 that the spark spread “spot” is defined as the difference between the nearest monthly contracts on 1 MWh of electricity and the heat rate adjusted price on natural gas. This difference can become negative. Thus all three models must support this property.

The assumption of the base load plant always producing maximum load exclude any operational option. The value of the project once an investment is determined is just the present value of the plant discounted at the risk free interest rate since all cash flows are considered to be certain due to the forward curve in (8.1). Thus the general formula for the value of the power plant is given by (8.1).

$$V(S) = \int_{t+t_u}^{t+t_u+L} e^{-r(\tau-t)} [KE(F_{t,\tau}(S_t, \tau) - R - Q) - D - G] d\tau \quad (8.1)$$

$V(S)$ – value of the project

t – time of calculation

r – risk free interest rate

T – maturity of a forward contract

t_u – expected time of building the plant

L – expected life time of project in years

K – capacity of the plant in MW

E – equivalent operating hours per year

$F_{t,T} = F_{t,T}(S_t, T)$ – price on the future spark spread contract per MWh produced

R – cost of removing NO_x , NOK/MWh

Q – cost of CO_2 , NOK/MWh

D – sum of operation and maintenance cost per year, NOK/year

G – sum of insurances cost per year, NOK/year

When the decision to invest is made it is considered that a sunk investment cost is paid. Thus the traditional net present value (NPV) of the power plant is given by (8.2). According to real option theory, an investment is not made before the traditional NPV is greater than the value of the option.

$$NPV = V(S) - I \quad (8.2)$$

I – sunk investment cost

The variables and parameters do not necessarily mean the same for all the models.

8.1 Model 1 - Arithmetic Brownian motion

Model 1 is based on the real option theory presented by Dixit and Pindyck (1994). The value of an infinite option must satisfy an Ordinary Differential Equation (ODE) which can be solved analytically, and the value of the finite option must satisfy a Partial Differential Equation (PDE) which cannot be solved analytically. The finite option is calculated using a binomial tree.

8.1.1 Spark spread

In the first model denoted model 1, the spark spread “spot” is assumed to follow an arithmetic Brownian motion (ABM). This process allows the spark spread “spot” to become negative.

$$dS = \alpha^* dt + \sigma dz^* \quad (8.3)$$

α^* – the risk free drift rate

σ – the volatility of spark spread

dz^* - the risk free equivalent to dz in a regular process

The forward price of an asset following the process in (8.3) is according to Dixit et al (1994) given by (8.4).

$$F_{t,T} = S_t + a^*(T - t) \quad (8.4)$$

8.1.2 The value of an asset, $F(S,t)$

The value of an asset is derived based on contingent claims analysis following Dixit et al (1994). A portfolio consisting of a risk free asset and n of the underlying asset is constructed to replicate the asset F . The risk free asset earns the return $r dt$ over the period dt . Given that the underlying S follows the non risk free process given in (8.3) the underlying asset pays the dividend $nS\delta dt$ and the random capital $ndS = n\alpha dt + n\sigma dz$

The total return on the portfolio is then given by (8.5)

$$\frac{1r + (S\delta + \alpha)n}{1 + nS} dt + \frac{n\sigma}{1 + nS} dz \quad (8.5)$$

The capital gain, dF , of the asset F is according to Ito's Lemma as given in (8.6).

$$dF = (F_t + \alpha F_s + \frac{1}{2} F_{ss} \sigma^2) dt + F_s \sigma dz \quad (8.6)$$

If the asset F pays a dividend denoted π , then the return on the asset F is given in (8.7).

$$\frac{(\pi + F_t + \alpha F_s + \frac{1}{2} F_{ss} \sigma^2)}{F} dt + \frac{F_s \sigma}{F} dz \quad (8.7)$$

If the portfolio is to replicate the asset both the dt and dz terms in (8.5) and (8.7) has to be equal. This results in (8.8) and (8.9).

$$\frac{n}{1 + nS} = \frac{F_s}{F} \quad (8.8)$$

$$\frac{(\pi + F_t + \alpha F_s + \frac{1}{2} F_{ss} \sigma^2)}{F} = \frac{1r + (S\delta + \alpha)n}{1 + nS} \quad (8.9)$$

Substituting for n , the expression in (8.9) becomes (8.10).

$$\frac{(\pi + F_t + \alpha F_S + \frac{1}{2} F_{SS} \sigma^2)}{F} = \frac{r(F - F_S) + (S\delta + \alpha)F_S}{F} \quad (8.10)$$

Simplifying (8.10) when $a^* = S(r-\delta)$ gives (8.11).

$$\frac{1}{2} F_{SS} \sigma^2 + \alpha^* F_S + F_t - rF + \pi = 0 \quad (8.11)$$

The same equation will apply when S is a spanning asset.

8.1.3 Value of the project

The value of the project is given by (8.1). Inserting (8.4) into (8.1) for the forward price and integrating setting $t_l = t + t_u$ and $t_2 = t + t_u + L$ gives (8.12).

$$V(S_t) = C_1 S_t + C_2 \quad (8.12)$$

$$C_1 = \frac{KE}{r} (e^{-r(t_1-t)} - e^{-r(t_2-t)})$$

$$C_2 = \frac{\alpha^* KE}{r^2} [(r(t_1-t) + 1)e^{-r(t_1-t)} - (r(t_2-t) + 1)e^{-r(t_2-t)}] - \frac{(e^{-r(t_1-t)} - e^{-r(t_2-t)})}{r} [KE(R+Q) + (D+G)]$$

8.1.4 Value of an infinite option to invest

Let $F(S)$ denote the option to invest. F is independent of time since it is infinite. Also, since an option pays no dividends the value of the option must, according to (8.11), satisfy the following ODE (8.13)

$$\frac{1}{2} F_{SS} \sigma^2 + \alpha^* F_S - rF = 0 \quad (8.13)$$

(8.13) has to satisfy the following boundary conditions

$$F(S^*) = V(S^*) - I \quad (8.14)$$

$$F'(S^*) = V'(S^*) \quad (8.15)$$

$$\lim_{S \rightarrow -\infty} F(S) = 0 \quad (8.16)$$

(8.13) has the general solution

$$F(S) = Ae^{\beta_1 S} + Be^{\beta_2 S} \quad (8.17)$$

$$\beta_{1,2} = \frac{-\alpha^* \pm \sqrt{(\alpha^*)^2 + 2\sigma^2 r}}{\sigma^2}, \beta_1 > 0, \beta_2 < 0$$

Solving (8.17) according to (8.14), (8.15) and (8.16) gives

$$S^* = \frac{C_1 - C_2\beta_1 + I\beta_1}{C_1\beta_1} \quad (8.18)$$

and

$$F(S) = \frac{C_1}{\beta_1} e^{\beta_1(S-S^*)} \quad (8.19)$$

8.1.5 Value of an option to invest with a finite lifetime

The option value, $F(S,t)$, is now dependant of time but does still not pay dividend. According to (8.11) the value of the option then has to satisfy the PDE (8.20).

$$\frac{1}{2}F_{SS}\sigma^2 + \alpha^*F_S + F_t - rF = 0 \quad (8.20)$$

8.1.6 The binominal tree

To value the finite option, a binomial tree as first introduced by Cox, Ross & Rubinstein (1979) are used. The value in the next period will increase by u with probability p , or decrease by d with probability $(1-p)$ as illustrated in Figure 5.

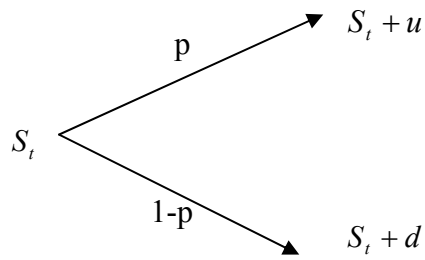


Figure 5: Binomial tree

Using the binomial tree requires finding the values of p , u and d . The three unknown are found by solving equations (8.21-23)

The forward price equals the expect value as show in (8.21), and rearranged in (8.22).

$$S_t + \alpha^* T = p(S_t + u) + (1 - p)(S_t + d) \quad (8.21)$$

$$p = \frac{\alpha^* T - d}{u - d} \quad (8.22)$$

By definition, the variance for a small time interval δt is given as $\sigma^2 \delta t = E(Q^2) - \{E(Q)\}^2$

Inserting for the expected values, the equation becomes (8.23).

$$pu^2 + (1 - p)d^2 - (pu + (1 - p)d)^2 = \sigma^2 dt \quad (8.23)$$

Recombination is required in the binomial tree, meaning that a step up and a step down results in the original value, leading to (8.24)

$$s + u + d = s$$

$$u = -d \quad (8.24)$$

Solving (8.22), (8.23) and (8.24) gives the following expressions for u, d and p.

$$u = \sqrt{(\alpha^* T)^2 + \sigma^2 T} \quad (8.25)$$

$$d = -\sqrt{(\alpha^* T)^2 + \sigma^2 T} \quad (8.26)$$

$$p = \frac{\alpha^* T + \sqrt{(\alpha^* T)^2 + \sigma^2 T}}{2\sqrt{(\alpha^* T)^2 + \sigma^2 T}} \quad (8.27)$$

8.1.7 Testing

An Excel spread sheet has been developed to calculate the finite option. In order to verify that the model outlined above works, a test is performed setting $\alpha^* = 3$ NOK/MWh $\sigma = 8$ NOK/MWh and doing 3 periods. Figure 6 shows that the tree is recombining. The code is given in appendix II.

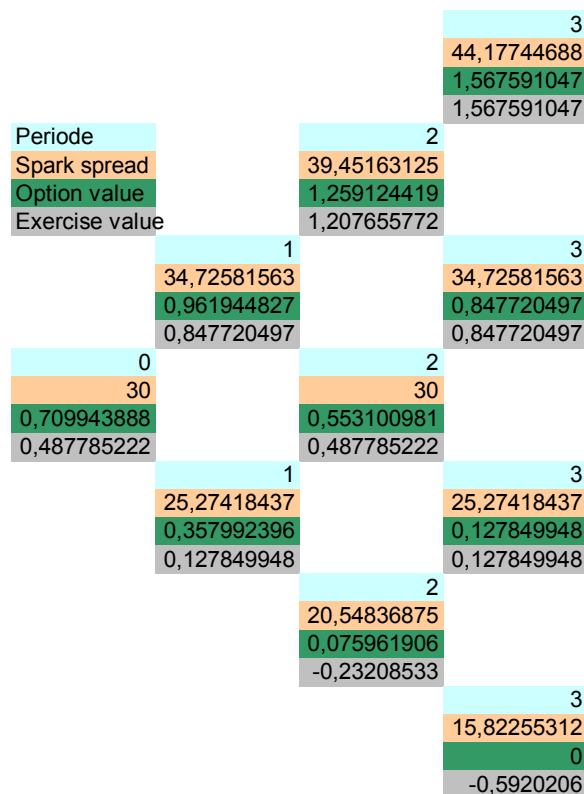


Figure 6: Test of the binomial tree. Screenshot from Excel.

8.2 Model 2 - Mean reversion one factor model

Model 2 is derived based on theory proposed by Lucia and Schwartz (2002). The value of the finite option is found by construction a trinomial tree.

8.2.1 Spark spread

Lucia and Schwartz (2002) propose that the spot price, P , of a commodity is given by (8.28). $f(t)$ is a known deterministic function. X_t follows the risk neutral process given in (8.29) where $\alpha^* \equiv -\lambda\sigma/k$ and dZ^* is the risk neutral equivalent to dZ in general Brownian motion. λ is the price of one unit risk.

$$P_t = f(t) + X_t \quad (8.28)$$

$$dX_t = \kappa(\alpha^* - X_t)dt + \sigma dZ^* \quad (8.29)$$

Equation (8.28) can be rewritten as (8.30), allowing (8.29) to be written as (8.31).

$$X_t = P_t - f(t) \quad (8.30)$$

$$d(P_t - f(t)) = \kappa(\alpha^* - (P_t - f(t)))dt + \sigma dZ^* \quad (8.31)$$

The equations are adapted to fit the scenario in this paper by substituting P with S , where S is the spark spread. Seasonal variations as shown in Figure 3 are disregarded since the project last for several years and seasonal variations will cancel out. Hence $f(t) = \bar{S}$, where \bar{S} denotes some long-term historic level of the spark spread “spot”. (8.31) can then be written as (8.32).

$$d(S) = \kappa(\alpha^* - (S_t - \bar{S}))dt + \sigma dZ^* \quad (8.32)$$

Based on Lucia and Schwartz (2002) a commodity following (8.32) has the following expression for the forward price at time t maturing at T . (8.33)

$$F_t(S_t, T) = \bar{S} + (S_t - \bar{S})e^{-\kappa(T-t)} + \alpha^*(1 - e^{-\kappa(T-t)}) \quad (8.33)$$

κ and α^* are the mean revering coefficient and the market price of risk respectively.

Half life, denoted η , is the time it takes for the spark spread “spot” to return halfway back to the mean reverting level $(\alpha^* + \bar{S})$ after a deviation. η is defined as $\eta = \frac{\ln 2}{\kappa}$. This can be deducted as follows:

$E(dS) = \kappa((\alpha^* + \bar{S}) - S)dt$ where E denotes the expectation. This can be integrated:

$$\int_{S_0}^{S_1} \frac{dS}{((\alpha^* + \bar{S}) - S)} = \int_{t_0}^{t_1} \kappa dt \rightarrow \ln\left(\frac{(\alpha^* + \bar{S}) - S_0}{(\alpha^* + \bar{S}) - S_1}\right) = \kappa(t_1 - t_0)$$
 Using that $(t_1 - t_0) = \eta$, and that

$$\frac{(\alpha^* + \bar{S}) - S_0}{(\alpha^* + \bar{S}) - S_1} = 2 \text{ by definition gives } \eta = \frac{\ln 2}{\kappa}.$$

8.2.2 The value of an asset F(S,t)

By following the same procedure as for the ABM in model 1, and setting $S(r - \delta) = \kappa(\alpha^* - (S_t - \bar{S}))$ will give the PDE (8.34).

$$\frac{1}{2}F_{SS}\sigma^2 + \kappa(\alpha^* - (S_t - \bar{S}))F_S + F_t - rF + \pi = 0 \quad (8.34)$$

8.2.3 Value of the project

The value of the project is given by (8.1). Inserting (8.33) into (8.1) for the forward price and integrating setting $t_l = t + t_u$ and $t_2 = t + t_u + L$ gives the project value by (8.35).

$$V(S_t) = C_1 S_t + C_2 \quad (8.35)$$

$$C_1 = \frac{KE}{r + \kappa} \left(e^{-r(t_1-t) - \kappa(t_1-t)} - e^{-r(t_2-t) - \kappa(t_2-t)} \right)$$

$$C_2 = \frac{KE}{r} (\bar{S} + \alpha^*) \left(e^{-r(t_1-t)} - e^{-r(t_2-t)} \right)$$

$$- \frac{KE}{r + \kappa} (\bar{S} + \alpha^*) \left(e^{-r(t_1-t) - \kappa(t_1-t)} - e^{-r(t_2-t) - \kappa(t_2-t)} \right)$$

$$- \frac{\left(e^{-r(t_1-t)} - e^{-r(t_2-t)} \right)}{r} [KE(R + Q) + (D + G)]$$

8.2.4 Value of an option to invest with a finite lifetime

The option value, $F(S,t)$, is dependant of time and does still not pay dividend. Then according to (8.34) the value of the option has to satisfy the PDE (8.36).

$$\frac{1}{2}F_{SS}\sigma^2 + \kappa(\alpha^* - (S_t - \bar{S}))F_S + F_t - rF = 0 \quad (8.36)$$

To value the finite option, the two stage procedure for constructing trinomial trees as proposed by Hull and White is used.³⁸ (8.33) is similar to a simplification of the original Hull-White model.³⁹ A branch of the trinomial tree is shown in Figure 7. From the node (i,j) , there is a possibility of p_u of an increase in S of δS , p_m of no movement and p_d of a decrease in S of δS

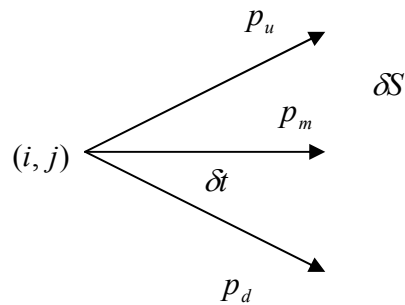


Figure 7: Trinomial tree

The first stage of the procedure is to construct a tree for a variable S^* that is defined to be initially zero and follows the process given in (8.37). The variable $S^*(t + \delta t) - S^*(t)$ is normal distributed with mean $-\kappa S^* \delta t$ and variance $\sigma^2 \delta t$.

$$dS^* = -\kappa S^* dt + \sigma dz^* \quad (8.37)$$

The jump size is set to $\delta S = \sigma\sqrt{3\delta t}$ to minimize error.⁴⁰ To prevent the probability of reaching any of the three branches ever being negative, alternative branching methods are introduced. For a maximum value, j_{\max} , an alternative branch with the possibilities no movement, one

³⁸ Hull and White (1994)

³⁹ Hull and White (1990)

⁴⁰ Hull (2000)

down and two down is introduced. The opposite is done for a minimum value. This is useful for incorporating mean reversion when the spark spread is very high or low. Hull and White (1994) show that probabilities are always positive if j_{\max} is set equal to the smallest integer greater than $0,184/\kappa\delta t$, and $j_{\min}=-j_{\max}$. The probabilities at each node are chosen to match the mean and standard deviation of the change in S^* for the process in equation (8.37).

The only unknowns are the probabilities of the three movements. The unknowns are found by solving the equations for the expected value, the variance and the law of total probability. If the branches has the form as shown in Figure 7, the following equations must be satisfied at node (i,j) . At node (i,j) , $S^* = j\delta S$.

$$p_u \delta S - p_d \delta S = -Kj \delta S \delta t \quad (\text{the expected value})$$

$$p_u \delta S^2 - p_d \delta S^2 = \sigma^2 \delta t + K^2 j^2 \delta S^2 \delta t^2 \quad (\text{the variance})$$

$$p_u + p_m + p_d = 1$$

The three probabilities are then easily solved, and the similar is done for the two other branching methods.

The second stage is to convert the tree for S^* into a tree for S by displacing the nodes on the S^* tree to that the initial term structure of S is exactly matched. $S(t)$ and $S^*(t)$ differs only by some function of time here defined as $\psi(t)$ as given in (8.38)

$$\psi(t) = S(t) - S^*(t) \quad (8.38)$$

By using (8.32) and (8.37) which gives

$$d\psi = \{\kappa(\alpha^* + \bar{S}) - \kappa\psi(t)\}dt \quad (8.39)$$

(8.39) is a separable first order ODE and comparing the solution (8.40) to (8.33), the solution of ψ is equal to the Forward curve in $t = 0$.

$$\psi(t) = \bar{S} + (S_0 - \bar{S})e^{-\kappa t} + \alpha^*(1 - e^{-\kappa t}) \quad (8.40)$$

$$\psi(t) = F(0, t)$$

The values of S at time $i\delta t$ is found by setting S equal to S^* plus the value of ψ at time t and keeping the probabilities the same.

8.2.5 Testing

The model is implemented in Matlab. The code is given in appendix II. For ten periods and $\kappa = 0.2$ which gives a tree with $j_{\max} = 1$ is shown in Figure 8. The tree is recombining and has only three vertical nodes. The tree structure at j_{\max} and j_{\min} are different from the middle node.

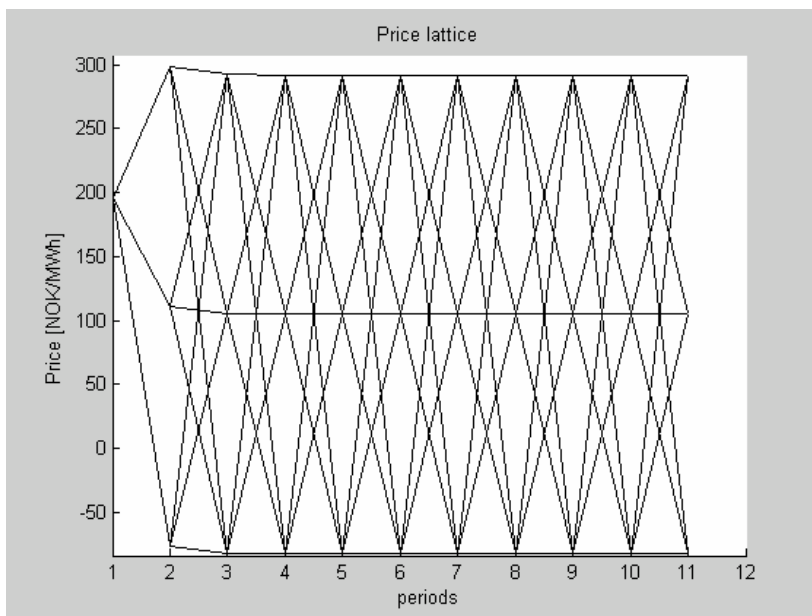


Figure 8: Trinomial tree, Matlab screenshot

Figure 9 show option values for different times to maturity. The shape of the graphs and increasing option values as time to maturity increases are consistent with option theory. This shows that the Matlab script values the options correctly.

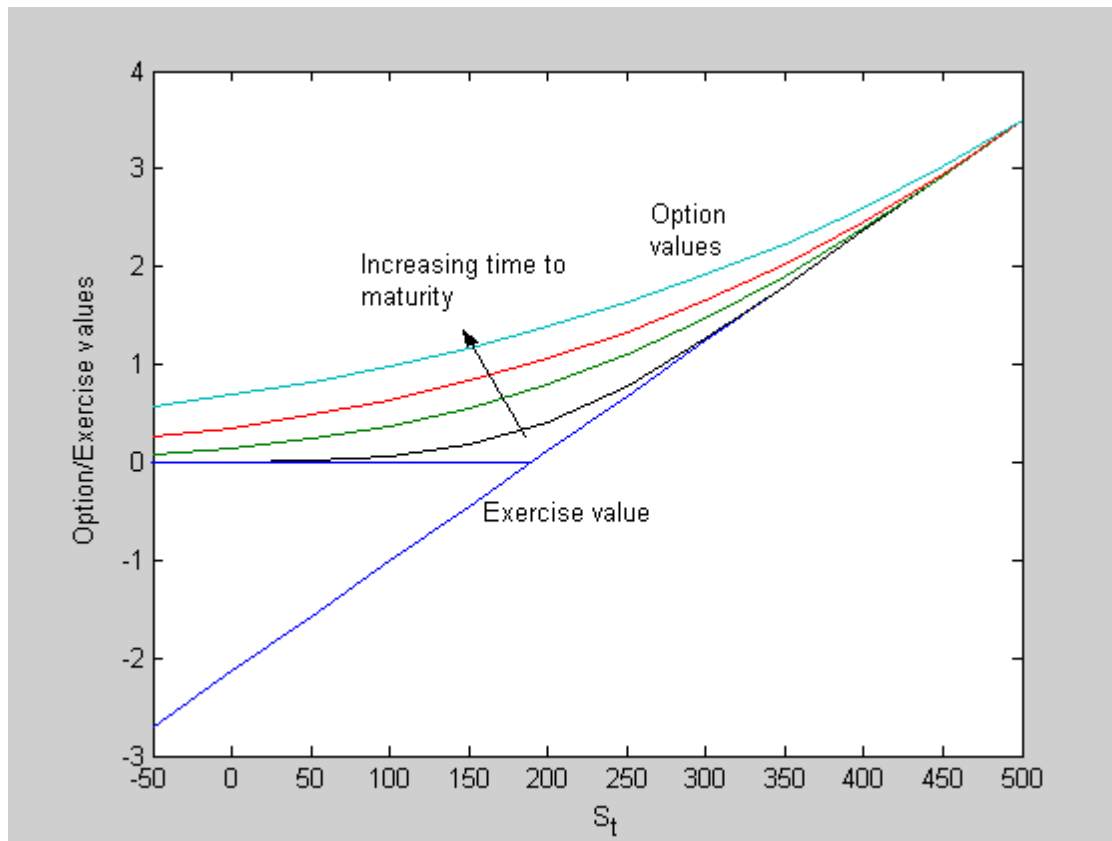


Figure 9: Option values for different maturities. Matlab Screenshot

8.3 Model 3 - Mean reversion two factor model

Model 3 is based on theory derived by Lucia and Schwartz (2002). The value of a finite option to invest is computed using Monte Carlo simulation for American options as suggested by Longstaff and Schwartz (2001).

8.3.1 Spark spread

Preliminary analysis conducted by Lucia and Schwartz (2002) suggests that modelling electricity prices can be improved by using a two-factor model (8.41). The price (8.41) is made up of a time-varying deterministic component, a short-term mean reverting component (8.42) and a long-term equilibrium price level following an arithmetic Brownian motion (8.43). The Wiener processes are correlated through equation (8.44).

$$P_t = f(t) + X_t + \varepsilon_t \quad (8.41)$$

$$dX = \kappa(\alpha^* - X)dt + \sigma_X dZ_X^* \quad (8.42)$$

$$d\varepsilon = \mu^* dt + \sigma_\varepsilon dZ_\varepsilon^* \quad (8.43)$$

$$dZ_X dZ_\varepsilon = \rho dt \quad (8.44)$$

In this case the spark spread, S , follows a similar motion where $f(t)$ is assumed to be \bar{S} . The Spark Spread process reduces to (8.45) and is still subject to (8.42), (8.43) and (8.44).

$$S = \bar{S} + X_t + \varepsilon_t \quad (8.45)$$

Lucia and Schwartz (2002) find the forward curve to be (8.46)

$$F_{t,T}(S_t, T) = \bar{S} + e^{-\kappa(T-t)} X_t + \varepsilon_t + (1 - e^{-\kappa(T-t)}) \alpha^* + \mu^*(T-t) \quad (8.46)$$

κ, α^* and μ^* are the mean revering coefficient, the market price of risk and the risk free drift of the long term factor respectively. Further $\alpha^* \equiv -\lambda_X \sigma / k$ and $\mu^* = \mu - \lambda_\varepsilon$.

8.3.2 The value of an asset $F(X, \varepsilon, t)$

By using Ito's Lemma for functions dependant on more than one stochastic process⁴¹ together with (8.42), (8.43) and (8.44) will give the PDE in (8.47) for the value of an asset.

$$\frac{1}{2} F_{XX} \sigma_X^2 + F_{X\varepsilon} \sigma_X \sigma_\varepsilon \rho + \frac{1}{2} F_{\varepsilon\varepsilon} \sigma_\varepsilon^2 + \kappa(\alpha^* - X_t) F_X + F_\varepsilon \mu^* + F_t - rF + \pi = 0 \quad (8.47)$$

where π is the dividend paid by the asset F .

8.3.3 Value of the project

The value of the project is given by (8.1). Inserting (8.46) into (8.1) for the forward price and integrating setting $t_1 = t + t_u$ and $t_2 = t + t_u + L$ gives (8.48).

$$V(X_t, \varepsilon_t) = C_1 X_t + C_2 \varepsilon_t + C_3 \quad (8.48)$$

$$C_1 = \frac{KE}{r + \kappa} \left(e^{-r(t_1-t) - \kappa(t_1-t)} - e^{-r(t_2-t) - \kappa(t_2-t)} \right)$$

$$C_2 = \frac{KE}{r} \left(e^{-r(t_1-t)} - e^{-r(t_2-t)} \right)$$

⁴¹ Dixit and Pindyck pp 81

$$\begin{aligned}
C_3 &= \frac{KE}{r} (\bar{S} + \alpha^*) (e^{-r(t_1-t)} - e^{-r(t_2-t)}) \\
&+ \frac{\mu^* KE}{r^2} [(r(t_1-t) + 1)e^{-r(t_1-t)} - (r(t_2-t) + 1)e^{-r(t_2-t)}] \\
&- \frac{\alpha^* KE}{r + \kappa} (e^{-r(t_1-t) - \kappa(t_1-t)} - e^{-r(t_2-t) - \kappa(t_2-t)}) \\
&- \frac{(e^{-r(t_1-t)} - e^{-r(t_2-t)})}{r} [KE(R + Q) + (D + G)]
\end{aligned}$$

8.3.4 Value of an option to invest with a finite lifetime

The value of the finite investment option must satisfy the PDE (8.49).

$$\frac{1}{2} F_{XX} \sigma_X^2 + F_{X\varepsilon} \sigma_X \sigma_\varepsilon \rho + \frac{1}{2} F_{\varepsilon\varepsilon} \sigma_\varepsilon^2 + \kappa(\alpha^* - X_t) F_X + F_\varepsilon \mu^* + F_t - rF = 0 \quad (8.49)$$

Equation (8.49) is solved numerically by using Monte Carlo simulation for approximating the value of American options, using the approach given by Longstaff and Schwartz (2001), and known as the least squares Monte Carlo (LSM).⁴²

The holder of an American option, at each discrete point in time, optimally compares the payoff from immediate exercise with the expected payoff from the continuation, and then exercises if the immediate payoff is higher.

Longstaff & Schwartz (2001) argues that the conditional expectations on holding the option can be estimated from the cross-sectional information in a simulation by using least squares. Least square analysis is used to determine the best-fit relationship between the value of continuing and the values of relevant variables at each time an early exercise decision has to be made. At each node, the paths that are in the money at that point are singled out to reduce computation efforts.

Accordingly, the expected value of the holding the option is found by (8.50) where S is a variable reflecting the state at the point and V is the value of continuing discounted back to the point. Least squares regression is then used at all the paths where early exercise has to be

⁴² Longstaff and Schwartz (2001)

considered (8.51). Longstaff & Schwartz uses a second degree polynomial. Testing showed that using a third degree polynomial gave faster convergence and is used in the valuation.

$$V = a + bS + cS^2 + dS^3 \tag{8.50}$$

$$\sum_{i=1}^N (V_i - a - bS_i - cS_i^2 - dS_i^3)^2 \tag{8.51}$$

The value of the option is determined by discounting each cash flow back to time zero at the risk-free rate and calculating the mean of the result. The procedure was implemented in Matlab. Matlab has function called “polyfit” which uses least square methods to approximate an n’th degree polynomial. This function is used in the code given in appendix II.

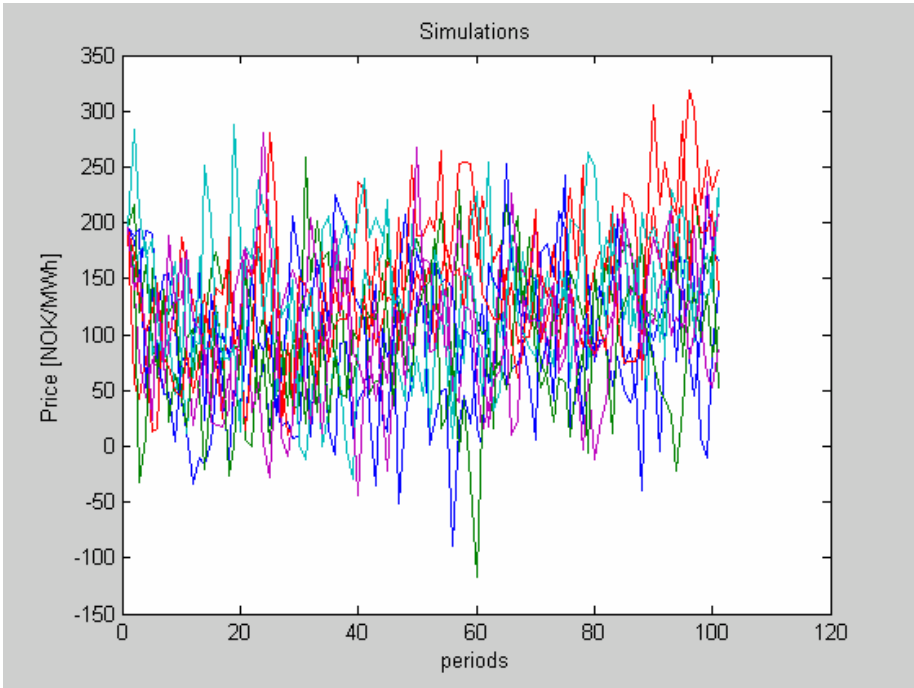


Figure 10: Some sample paths from a Monte Carlo simulation

In order to verify whether this procedure gave satisfactory results it was tested for model 1 and 2 against the values from the binomial and trinomial respectively. The results from the tests are shown in Table 3 below. Panel A shows the test for model 1 with 1000 simulations, and panel B shows the test for model 2 with 2000 simulations for some arbitrary parameters.

The procedure gives poor results when the option is far out of the money. When this is the case only a few price paths are in the money and thus singled out for calculation of (8.50) at each time step. Thus the approximation of (8.50) fails to give good results. More simulations in this case would help. However, as the option approaches the “in the money” state the results are very satisfactory. Also note that the values obtained using 2000 simulations are fare better than those of 1000 simulations.

Panel A

| Initial spark spread | MC Am option | Bin tree Am option | % error of Bin tree | Initial project value |
|----------------------|--------------|--------------------|---------------------|-----------------------|
| 0 | 0,0038 | 0,0018 | 111 % | -4,96 |
| 20 | 0,0232 | 0,0172 | 35 % | -3,44 |
| 40 | 0,1159 | 0,1 | 16 % | -1,9 |
| 60 | 0,4061 | 0,4066 | 0 % | -0,39 |
| 80 | 1,1899 | 1,1972 | 1 % | 1,13 |

Panel B

| Initial spark spread | MC Am option | Tri tree Am option | % error of Bin tree | Initial project value |
|----------------------|--------------|--------------------|---------------------|-----------------------|
| 0 | 0,745 | 0,7173 | 4 % | -2,78 |
| 20 | 0,7737 | 0,7759 | 0 % | -2,28 |
| 40 | 0,8339 | 0,8421 | 1 % | -1,7898 |
| 60 | 0,9444 | 0,9182 | 3 % | -1,2944 |
| 80 | 1,0411 | 1,0071 | 3 % | -0,799 |

Table 3: Comparison of values from MC simulation and bi- and trinomial tree method

One calculation of the American option value with 5000 simulations takes about 15 minutes. Performing simulations with 2500 and 10000 simulations suggests that computation time increase by a factor of three each time the number of simulations is doubled. Working backwards takes about 20 times the time of valuing the European option. When performing the calculations in chapter 10 and 11 we use 5000 simulations. This is assumed to give satisfactory results for the option value. Table 4 shows how the option value differs in six simulations. The deviations from the average value of these six are negligible and will not affect the investment decision since the computations do not require any higher level of accuracy.

| Simulated option value | Deviation from mean |
|------------------------|---------------------|
| 3,6686 | 0,3 % |
| 3,6416 | -0,4 % |
| 3,6679 | 0,3 % |
| 3,6686 | 0,3 % |
| 3,6535 | -0,1 % |
| 3,6459 | -0,3 % |

Table 4: Deviation from mean by 5000 simulations

Figure 10 and Figure 11 show some screenshot from Matlab. Figure 10 show a few sample paths of a simulation of the “spot” price. Since the option value is dependant on two variables the option value can graphically be represented as surface in space. This can be seen in Figure 11.

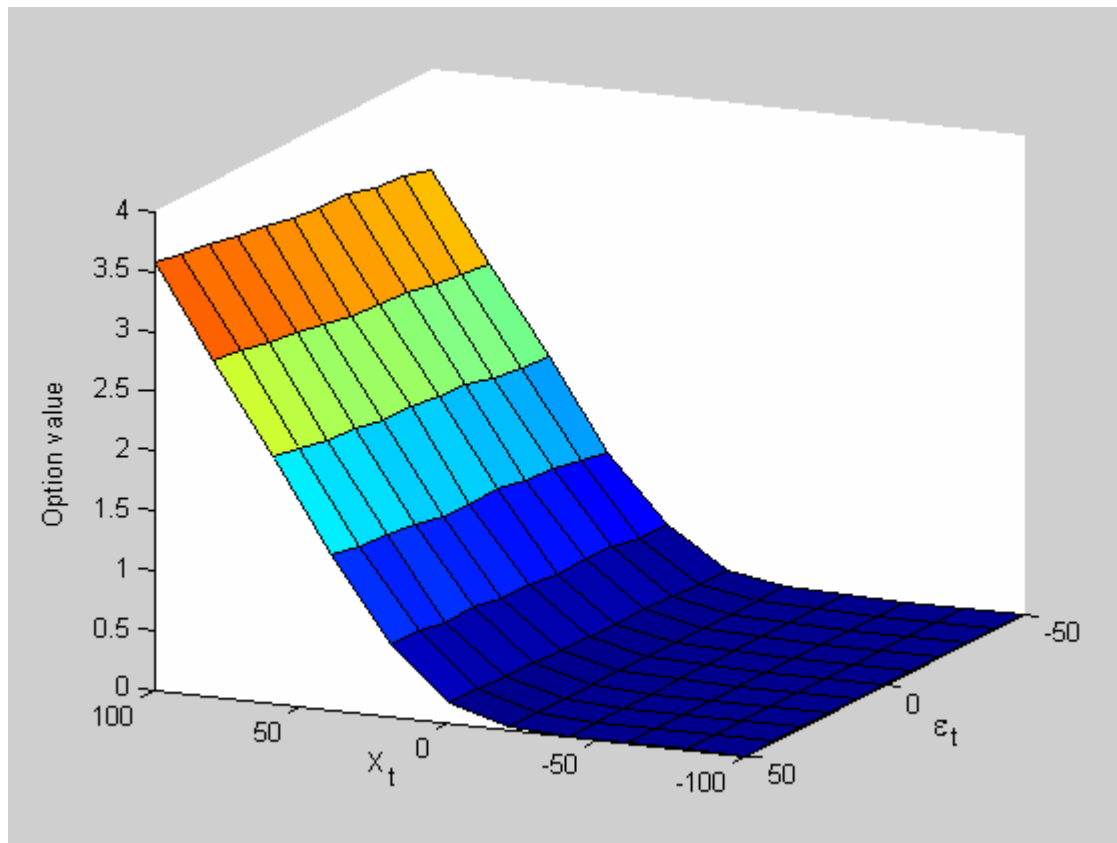


Figure 11: The surface represent the option value for some arbitrary parameter values. Matlab screenshot.

9 Estimating the parameters

The estimation of parameters was carried out by using regression and minimizing the Root Mean Square Error on constructed Spark Spread forwards. The Spark Spread forwards were constructed by combining historical data on natural gas futures from IPE and electricity futures and forward contracts from Nord Pool as explained in chapter 5. Data from 2000 till October 2003 are used in the estimation as explained in chapter 5.1 and 5.2.

9.1 Long term contracts – constructing a forward curve

In lack of financial contracts with more than three years to maturity it was necessary to construct proxy contracts on which RMSE could be computed. These contracts were found by constructing a forward curve as done by Dobbe and Sigmo (2002).

Non-arbitrage conditions imply that the discounted value of a ten-year forward contract equals a series of consecutive one-year forward contracts over the same period. This is consistent with the swap theory presented in 5.2. By using the price of a known ten-year contract starting 2004 and the prices of one-year contracts for the years 2004, 2005 and 2006 an approximation for the forward curve was obtained by finding the implied increase rate in forward price which satisfy equation (5.2).

A ten-year Spark Spread forward was constructed by using long term price information given by Elkem.⁴³ The price of a ten-year contract, delivered in UK and starting 1/1-2004 was priced to 0.21 £/Therm. The price in Norway was found by deducting transportation costs of 0.15 NOK/Sm³.⁴⁴ On a corresponding ten-year contract on electricity, delivery starting 1/1-2004, 234 NOK/Sm³ was considered a fair price.

The volatility on forward contracts are assumed to increase as time to delivery decreases. Due to this the price on the contract for delivery in 2004 are quite volatile while the prices on contracts for delivery in 2005 and 2006 are more stable. Since this is only done for one date the fluctuations around September 12th 2003 are leveled out by using average values from August to September 2003 on closest one year contract. The same is done with the risk free

⁴³ Dobbe (2003)

⁴⁴ Sigmo (2003)

interest rate⁴⁵. The average value for the forward contract was 120.95 NOK/MWh compared to the actual value of 146.17 NOK/MWh. Table 5 show the results from the construction.

| Year | One-year contract | One-year discounted | Accumulated | Days discounted | 10-year contract | 10-year discounted | Accumulated |
|------|-------------------|---------------------|-------------|-----------------|------------------|--------------------|-------------|
| 2004 | 120,95 | 116,25 | 116,25 | 288 | 111,53 | 107,20 | 107,20 |
| 2005 | 85,94 | 78,62 | 194,87 | 648 | 111,53 | 102,02 | 209,22 |
| 2006 | 90,49 | 78,78 | 273,65 | 1008 | 111,53 | 97,10 | 306,32 |
| 2007 | 96,81 | 80,21 | 353,86 | 1368 | 111,53 | 92,41 | 398,72 |
| 2008 | 103,57 | 81,66 | 435,52 | 1728 | 111,53 | 87,94 | 486,67 |
| 2009 | 110,80 | 83,15 | 518,67 | 2088 | 111,53 | 83,70 | 570,36 |
| 2010 | 118,53 | 84,66 | 603,33 | 2448 | 111,53 | 79,65 | 650,02 |
| 2011 | 126,81 | 86,19 | 689,52 | 2808 | 111,53 | 75,81 | 725,82 |
| 2012 | 135,66 | 87,76 | 777,28 | 3168 | 111,53 | 72,15 | 797,97 |
| 2013 | 145,14 | 89,35 | 866,63 | 3528 | 111,53 | 68,66 | 866,63 |

Table 5: Construction of the spark spread forward curve

The forward curve can be approximated to be on the form $f(T) = aT + b$ by using linear regression. The result gave $a=4.94$ $b=86.15$ with an explanatory power of $R^2 = 0.59$. The low R^2 value of the regression is mainly a result of the first data point being an outlier compared to the rest of the series. The values of a and b are quite high compared to Dobbe et al. (2002), who claims the equivalent a to be zero and b to be 19. The regression line (fitted curve) is shown in Figure 12 together with the actual 10 year forward contract, the observed forward prices and the constructed forwards on September 12th 2003.

⁴⁵ Norges Bank (2003)

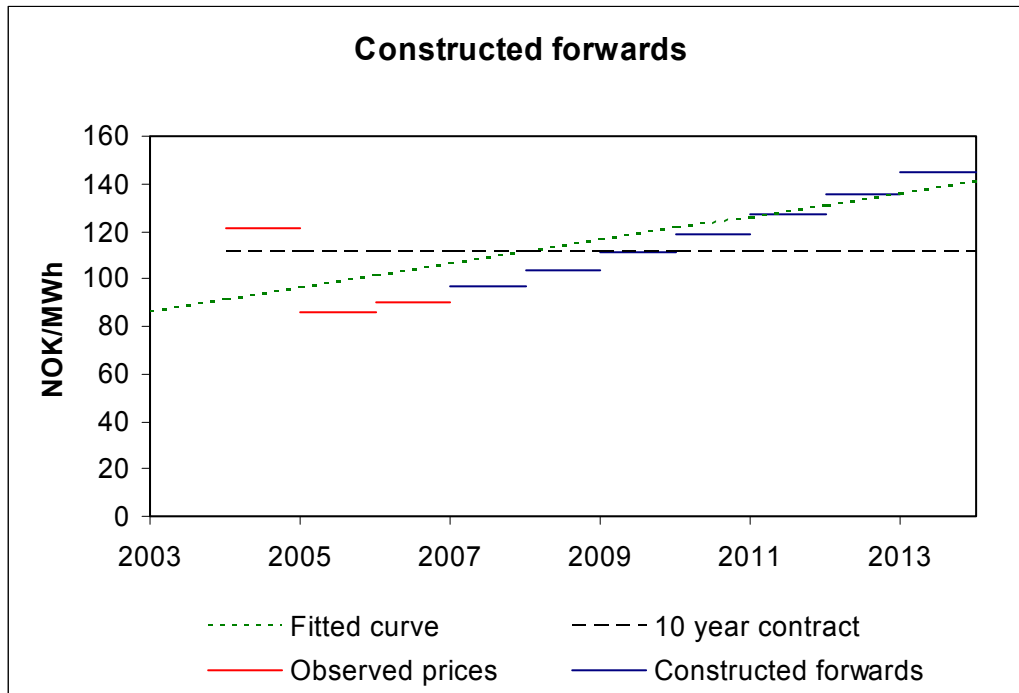


Figure 12: Constructed forward curve, regression and known forwards at September 12th 2003.

9.2 The Basic model – Parameters

Equation (8.4) for the forward curve of this model says that the future prices are given by the “spot” price and some drift rate. This is very dependant on the “spot” price and thus almost impossible to fit to reflect the information in the market. I.e. chapter 9.1 suggests a drift of 4.94. If the “spot” today is 200 NOK/MWh, which is not unlikely for the time considered, then this model says that the forward price is $200 + 4.94T$ where T is time to maturity. There is no resemblance between this and the constructed forward curve in Figure 12. Thus, for this model to make sense S_t in equation (8.4) must be some dummy price given by the forward market. An estimate for such a dummy price would be the regression parameter b in chapter 9.1. Then an estimate for the drift rate is the regression parameter a in chapter 9.1. Since these parameters come from long term market information and not “spot” data the volatility should also be estimated from such long term data. This is not possible with the data set used in this paper without modifications of the data. Another approach could be to use implied volatility from options. However, this is not done and a volatility given by Fleten (2003) of 8 NOK/MWh is used. The parameters used in this model are summarized in Table 6.

| Parameter | Estimate | Denom |
|------------|----------|----------|
| α^* | 4.94 | per year |
| σ | 8.0 | NOK/MWh |
| S_t | 86.15 | NOK/MWh |

Table 6: Estimated parameters to model 1. The t in S_t is equal September 12th 2003.

9.3 The one factor mean reversion model – Parameters

The estimation of parameters to this model mainly follows the procedure done by Lucia and Schwartz (2002). Estimation of κ, σ and \bar{S} is performed by doing a regression on the full range of “spot” data, and then fitting the forward curve to the forward data by estimating λ . Since the “spot” price data are discrete values the models must be expressed in discrete form in order to perform a regression. Lucia and Schwartz (2002) express the non risk adjusted version of equation (8.29) on discrete form as given in (9.1)

$$X_t = (1 - \kappa)X_{t-1} + \xi_t \quad (9.1)$$

ξ_t is identical independent distributed normal variables with mean zero and variance σ^2 . The spark spread on discrete form can be expressed as (9.2) where u_t is the residual.

$$\begin{aligned} S_t &= \bar{S} + X_t \\ X_t &= \phi X_{t-1} + u_t \end{aligned} \quad (9.2)$$

Following Lucia and Schwartz (2002) this can be expressed in a general form as (9.3).

$$\begin{aligned} S_t &= f(\bar{S}, X_t) + \xi_t \\ \xi_t &= \phi \xi_{t-1} + u_t \end{aligned} \quad (9.3)$$

The second equation in (9.3) is the first order autoregressive structure of the disturbance term ξ_t in the first equation. Rearranging the first equation and substituting for ξ_t in the second equation gives (9.4) which can be expressed as in (9.5)

$$S_t = \phi S_{t-1} + f(\bar{S}, X_t) - \phi f(\bar{S}, X_{t-1}) + u_t \quad (9.4)$$

$$S_t = \phi S_{t-1} + \bar{S} + X_t - \phi(\bar{S} + X_{t-1}) + u_t \quad (9.5)$$

Equation (9.1) of the explanatory variable and equation (9.5) of the dependant variable together with the “spot” price data are used to estimate \bar{S} , σ and ϕ by minimizing the squared error between the real “spot” price and the “spot” price given by equation (9.5). $\hat{\kappa} = 1 - \hat{\phi}$ is an estimate of κ and the standard error of regression is an estimate of σ . Further, κ and σ are estimated by using the full “spot” data set.

The parameter α^* remains to be estimated. Recall that $\alpha^* \equiv -\lambda\sigma/k$ and the estimate for α^* as given by $\hat{\alpha}^* = -\hat{\lambda}\hat{\sigma}/\hat{k}$. α^* is estimated by fitting the forward curve given in (8.33) to recent forward data for 2003 and the constructed forward curve in paragraph 9.1 by minimizing the RMSE. Figure 13 shows the regressed spot price together with the real spot price. As seen from the figure the regressed curve fits the actual quite well, something which is reflected in the R^2 of the regression which was 0.90. The standard error of regression was 34.89 NOK/MWh on a weekly basis. Lucia and Schwartz (2002) had an R^2 of 0.98 and a standard error of regression of 9 NOK/MWh on a daily basis. Although the R^2 in this estimation is lower, the parameters concerning the “spot” data must be said to be properly estimated.

$$F_t(S_t, T) = \bar{S} + (S_t - \bar{S})e^{-\kappa(T-t)} + \alpha^*(1 - e^{-\kappa(T-t)}) \quad (8.33)$$

| Parameter | Estimate | Denom. |
|------------|----------|----------|
| σ | 251.59 | NOK/MWh |
| κ | 2.69 | per year |
| α^* | 6.85 | NOK/MWh |
| λ | 0.00147 | |
| \bar{S} | 97.95 | NOK/MWh |
| RMSE | 23.23 | NOK/MWh |

Table 7: Estimated parameters to model 2

The resulting parameter values are given in Table 7. Volatility seems to be high compared to the one of model 1, but since this is a different model where long term volatility is not considered there is no contradiction. κ of 2.69 per year implies a half-life of 0.26 year or 94 days. From Lucia and Schwartz’ results a half life of 69 days can be calculated. Since the natural gas market at IPE seems to be less volatile than the electricity prices at Nord Pool a longer half life seems reasonable for spark spread. The parameter \bar{S} reflects the historic level of the prices while α^* reflects the information in the forward market. Together they make up

the level where prices will stabilize in the long run according to this model. The sum of these two parameters is 104.8 NOK/MWh which is close to the 10 year forward contract mentioned in 9.1. At first glance this seems reasonable, but as Figure 14 show this fits the forward curve at September 12th 2003 poorly. Implications of this will be discussed in chapter 10 and 11.

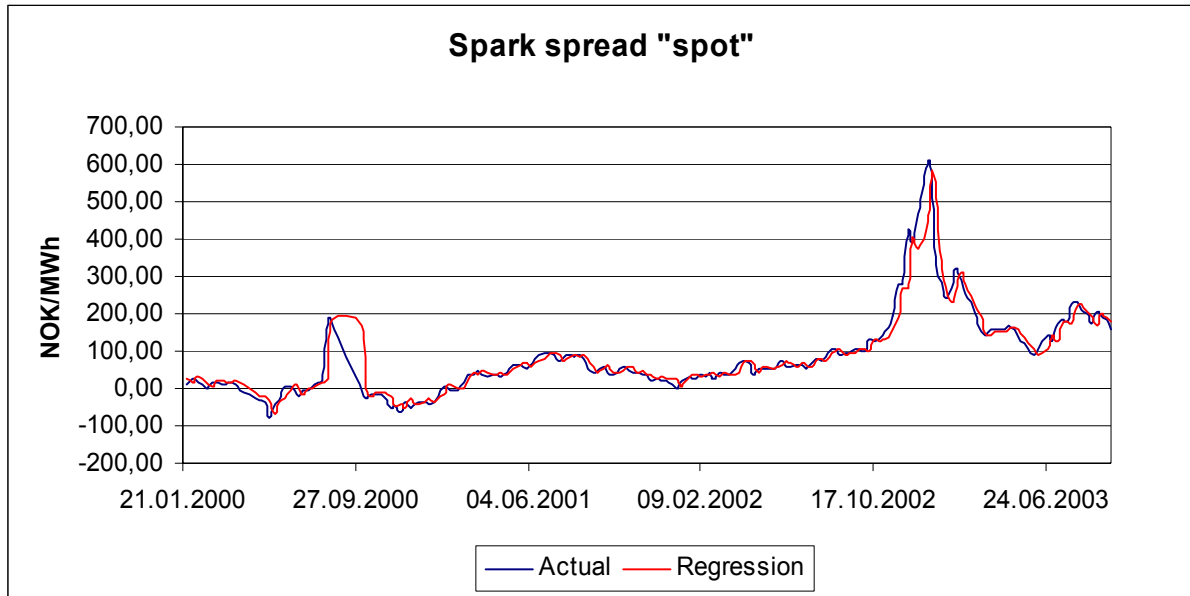


Figure 13: Regression of spark spread “spot” – Model 2

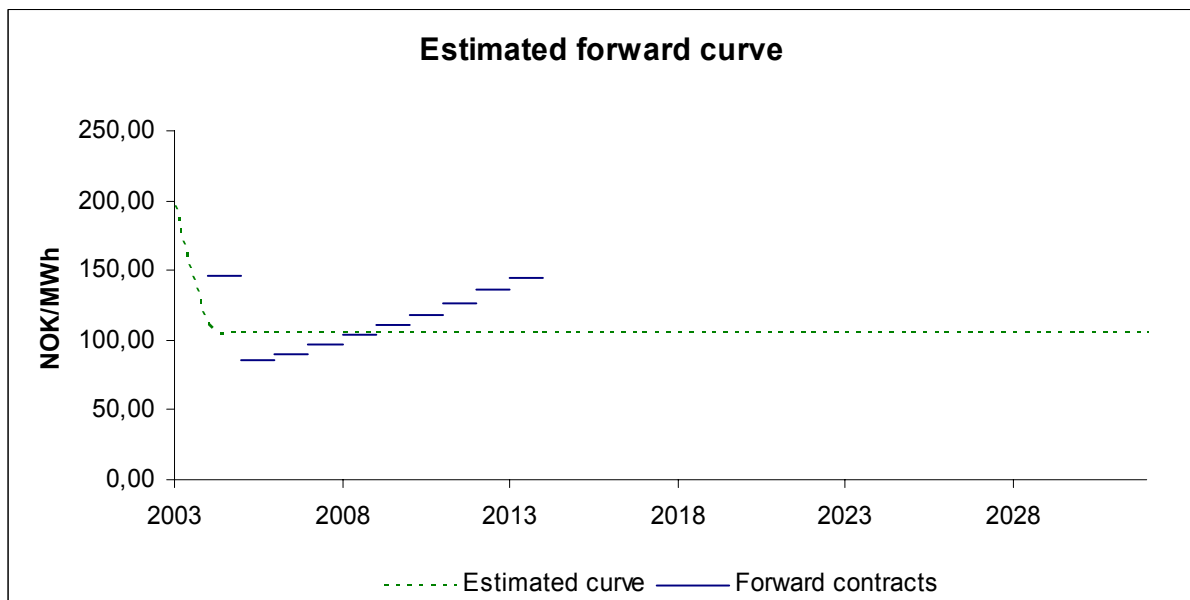


Figure 14: Estimated forward curve – Model 2

9.4 The two factor mean reverting model – Parameters

The estimation of parameters to this model also follows Lucia and Schwartz (2002) but with some modifications according to the following procedure. “Spot” data from the whole data set is used to estimate \bar{S} . Then the “spot” and forward data together with the constructed forward curve in paragraph 9.1 is used to estimate the remaining parameters and the state variables for each date in the data set using an iterative procedure to minimize the RMSE.

The iteration follows a three step procedure. For a given set of parameters, the state variables for each date is set to minimize the sum of squared errors of the spot and the forward contracts. Step two uses these state variables in the regression procedure explained in next paragraph to find estimates for $\rho, \mu, \kappa, \sigma_X$ and σ_ε . The other parameters given, step three involves estimating λ_X and λ_ε by minimizing RMSE of recent forward data from 2003. Recall that $\alpha^* \equiv -\lambda_X \sigma / k$ and $\mu^* = \mu - \lambda_\varepsilon \sigma_\varepsilon$. The iteration procedure is repeated until RMSE converges.

The regression mention in the second step above is done according to Dixit et al. (1994). A simple mean reverting process as given by equation (8.42) is expressed on discrete form as

$X_t - X_{t-1} = a + bX_{t-1} + \xi_t$, where the estimate for κ is given by $\hat{\kappa} = -\ln(1 + \hat{b})$ and an estimate σ_X is given by $\hat{\sigma}_X = \sigma_{X,\xi} \sqrt{\frac{\ln(1 + \hat{b})}{(1 + \hat{b})^2 - 1}}$ where $\sigma_{X,\xi}$ is the standard error of regression. Similarly the regression equation for the arithmetic Brownian motion is $\varepsilon_t - \varepsilon_{t-1} = \mu + \xi_t$.

In order to achieve convergence in the iterative procedure, μ^* was set equal to the regression parameter “a” of the constructed forward curve in paragraph 9.1, and λ_ε was calculated instead of found by trial. \bar{S} was found by simply computing the average of all the “spot” data. The result from the parameter estimation is given in Table 8.

| Parameter | Estimate | Denom |
|-----------------------|----------|---------|
| σ_x | 225.99 | NOK/MWh |
| σ_ε | 97.52 | NOK/MWh |
| ρ | 0.55 | |
| κ | 8.61 | |
| μ^* | 4.94 | NOK/MWh |
| α^* | 12.32 | NOK/MWh |
| λ_x | -0.0013 | |
| λ_ε | -0.0024 | |
| \bar{S} | 83.39 | NOK/MWh |
| RMSE | 13.06 | NOK/MWh |

Table 8: Estimated parameters to model 3

In this model parameters of two unobservable state variables are estimated. The volatility of the short term variable seems reasonable compared with model 2. However, the volatility of the long term variable given by the parameter estimation implies very high option values. Investment would “never” be feasible. σ_ε is therefore set to 8 NOK/MWh as for a general ABM. As for model 1 this is done since computing the long term volatility is not conducted.

The half life implied by the mean reversion coefficient of 8.61 per year is 30 days. When the short term movements are absorbed by a specific variable it seems reasonable that this variable has a shorter half life than the variable in model 2 where both the short and long term movements are reflected in one variable. \bar{S} reflects the historic level of the prices as in model 2, whereas α^* reflects the expected level in the forward market. This together with the expected long term drift seems to be at better estimate of the forward prices.

Figure 15 show the forward curve September 12th 2003 according to the two factor model. The forward curve fit the forward contracts much better than what is the case for the one factor model, which is also reflected by a 44% decrease in the RMSE. As comparison, Lucia and Schwartz (2002) had an improvement of 37% from the one factor to the two factor model. The standard errors of regression were 13.5 NOK/MWh for the long term variable and 53.35 NOK/MWh for the short term variable on a weekly basis.

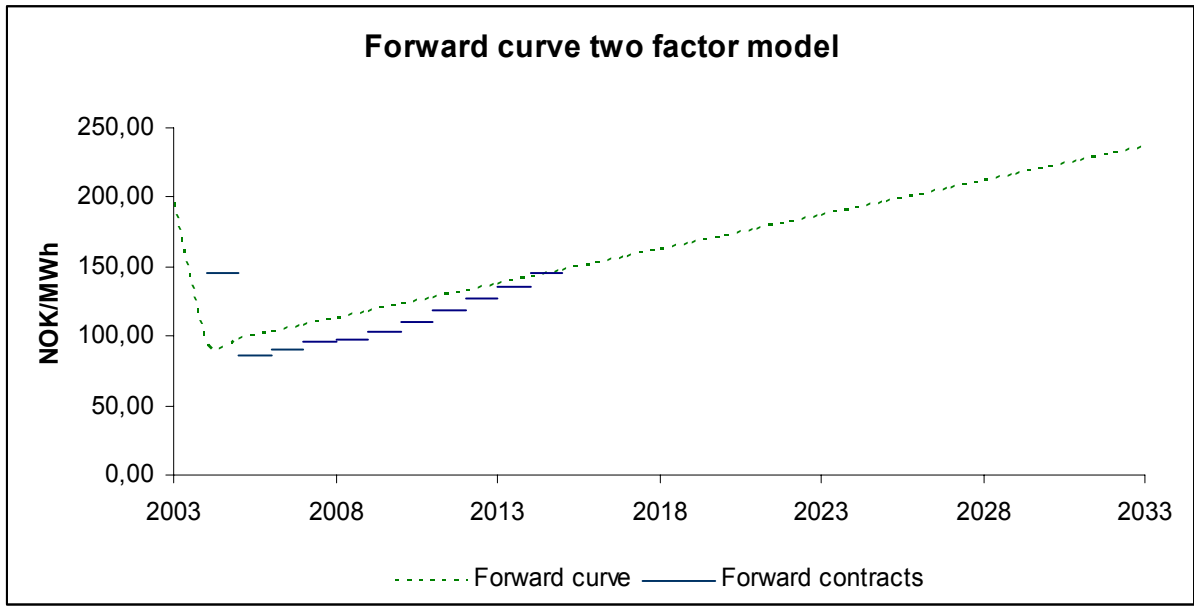


Figure 15: Estimated Forward curve – Model 3

10 The investment decision - Numerical analysis

The option to invest in a CCGT power plant is investigated for the finite and infinite option to invest for the ABM model, and for the finite option only for the mean reverting and two factor model. September 12th 2003 is chosen to be the day considered, and is referred to as “today”. On this date the spark spread “spot” was 196 NOK/MWh. The other input values are as given in chapter 6.

This chapter investigates the investment decision under the three models outlined in the preceding chapters using analysis conducted with parameters estimated in chapter 9 will be referred to as base case. During these analyses all cost data except the future cost of CO₂ emission are assumed to be known. The cost of CO₂ emission is considered to be a cost per metric tonne (MT) CO₂ as explained in chapter 7. Sensitivity analysis with respect to several variables is conducted, long-term expected prices and the price of CO₂ being the most important.

10.1 Model 1 – ABM

The ABM model is the only model where both the finite and infinite option to invest is investigated. The finite option is computed using binomial tree. A weakness with the model is that the spark spread “spot” most likely does not follow an ABM. The implications of this is discussed in chapter 9.2, and “spot” price used in this model the “dummy” price given by the information in the forward market. The “dummy spot” for September 12th 2003 was 86.15 NOK/MWh.

In this model both the infinite and finite licence to invest can be valued, and it is interesting to observe some similarities between these two options. Under the base case an infinite option to invest gives an investment trigger of 144.22 NOK/MWh for the “dummy spot”. A finite option with a life of 10 years will have the boundary for early exercise as shown in Figure 16. The boundary decreases, though not much, as time to maturity decrease. This is consistent with general option theory.

An interesting observation is that when the licence is granted, the investment trigger is about the same for the infinite and the 10 year option which has an early exercise boundary of about 144 NOK/MWh. Figure 16 also shows that the general net present value is positive for spark

spreads above about 50 NOK/MWh which is where the NPV is zero. When the option matures the investment decision is reduced to a regular NPV-decision, due to the take it or leave it situation as described in chapter 2 about real options theory. Figure 16 also suggests that the option holder have about the same investment threshold through the whole license period.

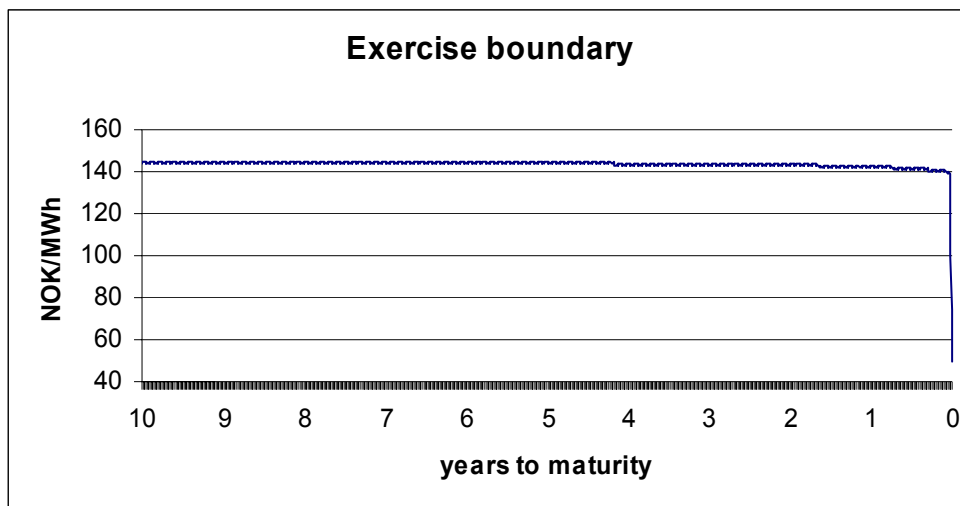


Figure 16: Exercise boundary – Model 1

Figure 17 shows how the investment trigger of an infinite option varies with price per MT CO₂. Investment is only profitable for a cost of CO₂ emissions close to zero with the level of future prices today. Such low costs for CO₂ emission are not likely to occur at least not in the last part of the project lifetime as mentioned in chapter 7, and thus this model can explain why there is yet no investment in a plant in Norway.

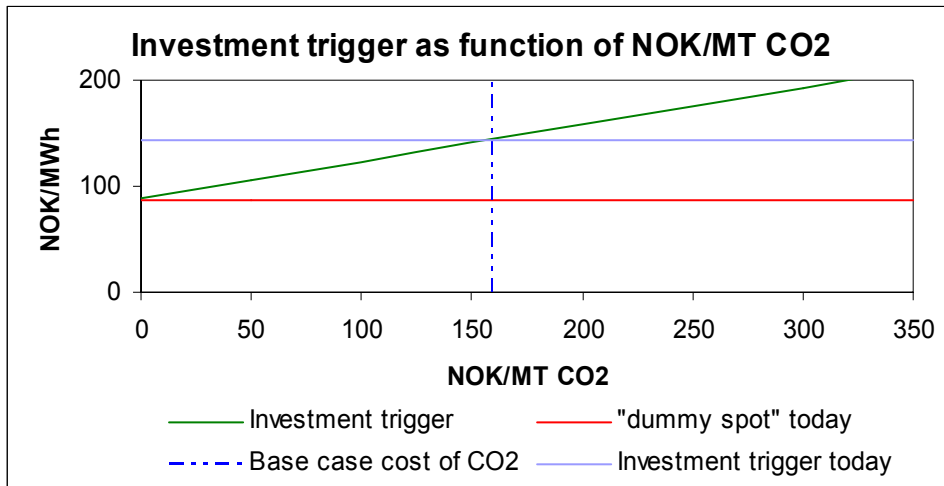


Figure 17: Investment trigger as a function of NOK/MT CO₂ emitted – Model 1, 10 year license.

Figure 18 shows how the value of the 10 year option varies with the “dummy spot” spark spread. The investment threshold and today’s value are drawn in the graph. The investment value, and thus the option value must be quite large before investment is commenced. The value of an option with ten years to maturity is very close to the infinite option. This model gives an option value of 3498.9 MNOK and a NPV of 2463.3 MNOK September 12th 2003.

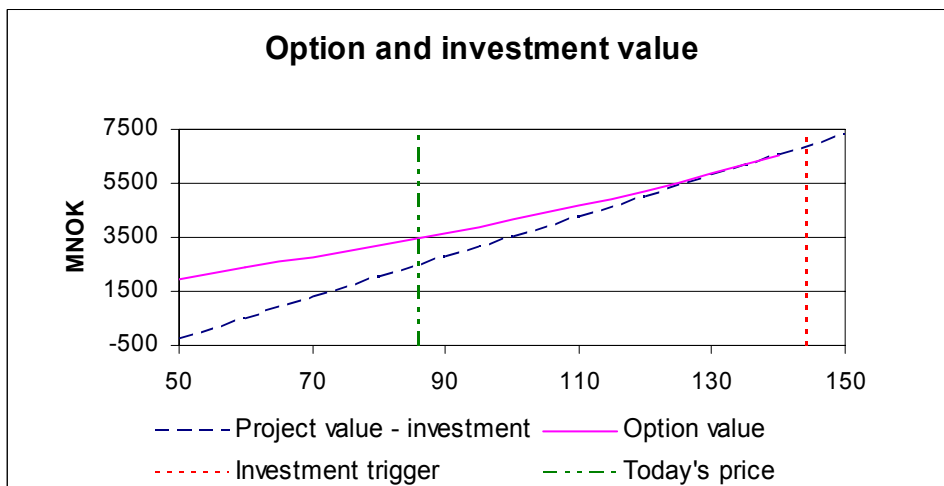


Figure 18: Project and Option value – Model 1, 10 year license using the binomial tree.

10.2 Model 2 - One factor mean reversion

Under this model the finite option to invest is investigated using trinomial trees. The estimation resulted in a strong mean reversion coefficient for this model and hence the

forward curve soon stabilizes on the level determined by the information in the forward market as can be seen of Figure 14.

As a consequence, the value of investment will never have positive NPV in the base case scenario for reasonable values of spark spread “spot”. Neither will the value of the option for any reasonable initial spark spread “spot” or volatility. Regardless of the spark spread “spot”, the forward price given by the forward curve will drop to the long run price level (in this model) almost immediately. The recent level of expected future prices is too low resulting in the investment not being feasible.

The mean reverting coefficient, κ , has to be as low as 0.6 per year for the option to take a value under the base case. Such low values of κ are not likely since the regression where κ is estimated is quite accurate having a R^2 as high as 0.9. Thus the investment decision is reduced to a regular NPV analysis of whether to invest today or not.

A sensitivity analysis conducted with respect to α^* revealed that a change in this parameter will never lead to a positive option value. An increase in α^* of 17 NOK/MWh will give a positive NPV as shown in Figure 19. This is interesting since α^* reflects the expected future prices. In comparison α^* today is about 6.85 NOK/MWh.

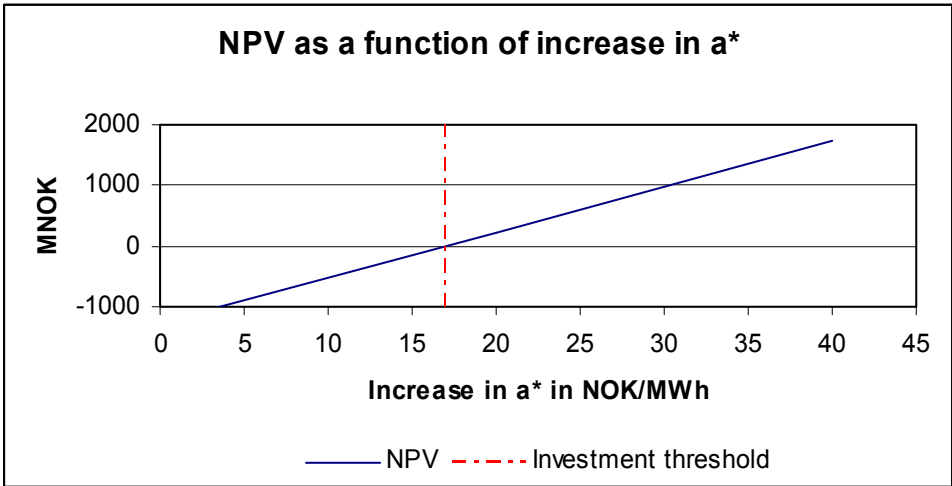


Figure 19: NPV of model 2 as function of an increase in α^* from level of today.

Figure 20 shows the NPV of investment as function of the cost of CO₂ emission. The cost of CO₂ emission has to be below 112 NOK/MT CO₂ to obtain a positive project value. Recalling from chapter 7, the predictions on cost of CO₂ suggest a lower price on emission in the near

future. However, the break even cost is exceeded when using the conservative long run estimate of CO₂ emission cost used in the base case scenario. This could explain why uncertainty in cost of CO₂ delays investment.

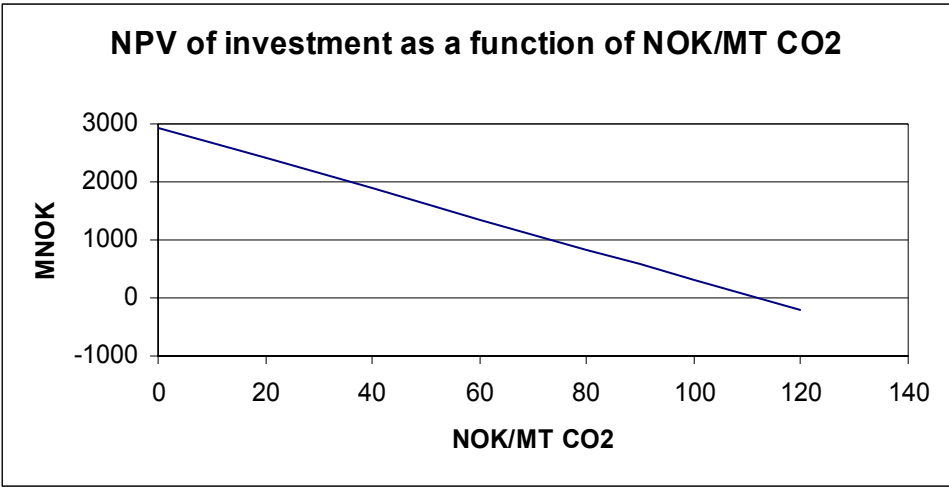


Figure 20: NPV as function of cost of CO₂ emissions – Model 2

10.3 Model 3 – The two factor model

The finite option to invest is investigated using Monte Carlo simulation for American options. Due to long computational time only 5000 simulations are used. The resulting values from 5000 simulations do not show large variations, and is considered to provide the accuracy needed for the investment decision as shown in Table 4.

Model 3 is governed by two unobservable stochastic processes. The short term process shows strong mean reversion, a property which makes the long term variable dominant when it comes to value the investment and the option. Short-term fluctuations in the “spot” price do only to a certain degree have a positive effect on the value. Figure 21 displays the spark spread “spot” price and the two state variables. The long term variable is less volatile than the short term.

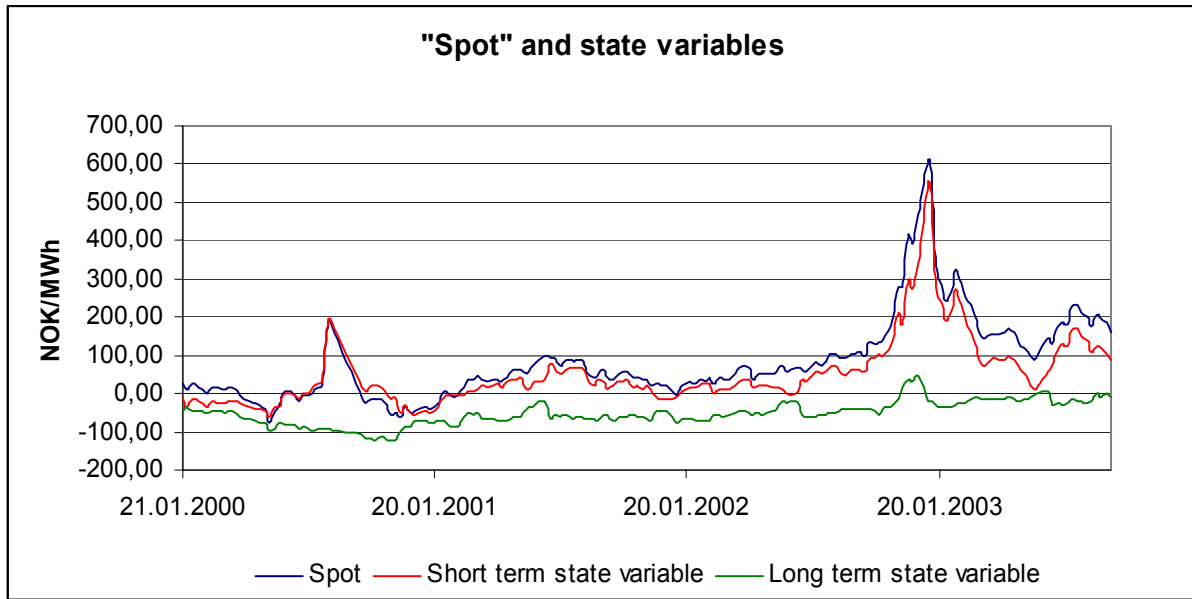


Figure 21 “Spot” and state variables – Model 3

Under the base case scenario model 3 gives an option value of 3537.4 MNOK and a NPV of 2862.3 MNOK. This calls for delaying the investment.

Performing sensitivity analysis on the parameters related to the short-term stochastic process shows that changes in these parameters have very little effect on the valuation. κ , is assumed to be greater than the mean reverting coefficient of model 2. The short-term volatility can take on almost any value without making any changes the values of option and investment.

Sensitivity analysis performed on the risk free drift parameter μ^* in the long-term stochastic process states that this variable is very important to the option and investment value. Figure 22 shows how the option- and investment value vary with the yearly risk free drift. Changes in the drift will not alter the investment decision since the exercise value never exceeds the option value. The lines in Figure 22 and Figure 23 are not straight due to the inaccuracy in Monte Carlo simulation technique.

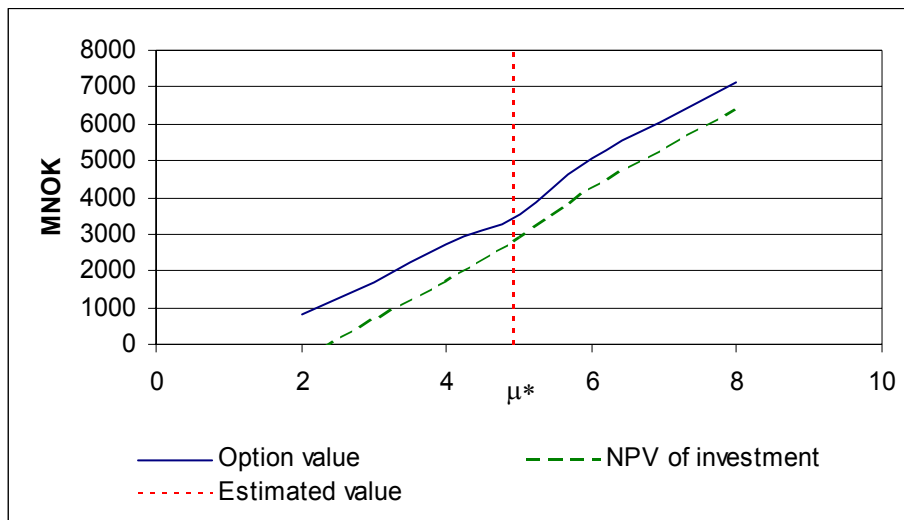


Figure 22: Varying risk free drift – Model 3

The value of \bar{S} , the long-term average of the historic market data, has a major impact on the investment decision. Increasing this parameter in the range 0 NOK/MWh to 60 NOK/MWh, Figure 23 shows how the option value closes in on the investment value. If \bar{S} is increased by more than about 56 NOK/MWh, an investment should commence. Since the mean reversion coefficient is very strong the difference in increasing α^* instead of \bar{S} is negligible. Thus increasing α^* , which reflects the long-term expected price level, with about 56 NOK/MWh shows the same result. The uncertainty lays in α^* where the estimation had a RMSE of 13 NOK/MWh. In comparison α^* and \bar{S} are 12.3 NOK/MWh and 83.4 NOK/MWh respectively as given in Table 8.

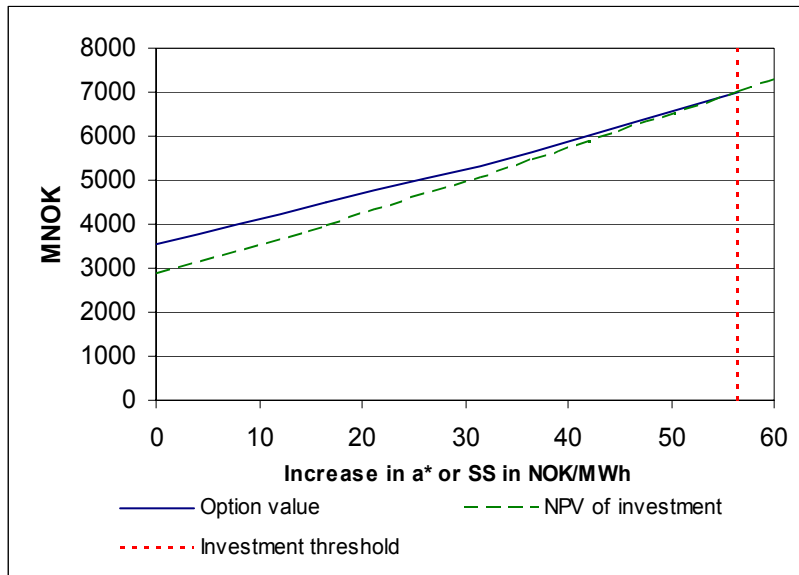


Figure 23: Option value and NPV as a function of an increase in α^* or \bar{S} – Model 3

Varying the risk free interest rate in the range from 2%-10% per year have a large impact on the values of both investment and the option, but the investment decision is not altered. Similar results appear when varying the price of CO₂ quotas. The price per quota does not seem to revise the investment decision except when the expected price of CO₂ is zero. In general the conclusion that investment should be delayed is robust.

To say something about how the option value varies with the “spot” price is not straight forward since there is no one-to-one relation between the state variables and the spark spread “spot”. One procedure could be to find spot prices in the historical data and then use the equivalent state variables found in the parameter estimation for the given date to do the calculation. This however, is not a consistent procedure. Valuing the option when the “spot” price increased in December 2002 gives higher values than valuing the option at the same price when the “spot” price decreased during the winter of 2003 because the long-term variable had a higher value in the December 2002 than during the following winter for equivalent “spot” prices.

The investment decision is hardly affected by the short term fluctuations. An alternative model could be to remove the terms in (8.46) that originate from the short term process except α^* , since it contains future market information, and let the stochastic process be given by (8.43). This is will be a model very similar to model 1 but with $S_t = \bar{S} + \varepsilon_t + \alpha^*$ in the

equation (8.4) of the forward price. Analysing the investment decision using the outlined model also calls for delaying the investment decision. The investment trigger of an infinite option, which is very similar to an option with 10 years to maturity, is 144 NOK/MWh. Subtracting $\varepsilon_0 = -7.7$ NOK/MWh and $\alpha^* = 12.3$ NOK/MWh from this threshold requires $\bar{S} = 140$ NOK/MWh which is the same result as in the sensitivity analysis of \bar{S} . This will be referred to as model 3b.

11 Discussion of the analyses

The main results from chapter 10 are summarized in Table 9. The models do not consent on a clear investment decision. In this chapter the underlying causes for these differences will be explained and analyzed, in order to make a choice of the best model. The parameters that are most likely to alter the decisions are also summarized.

| Model | Option value MNOK | Investment threshold NOK/MWh | Equivalent spot price* NOK/MWh | NPV of investment MNOK | Investment decision |
|-------|-------------------|------------------------------|--------------------------------|------------------------|---------------------|
| 1 | 3499 | 144 | 86 | 2463 | "Wait" |
| 2 | 0 | N/A** | 195 | -1280 | "Never invest" |
| 3a | 3540 | N/A*** | 195 | 2860 | "Wait" |
| 3b | 3580 | 144 | 88 | 2600 | "Wait" |

Table 9: *For model 2 and 3a this is the spark spread “spot” reflected by the constructed “spot” price as described in chapter 5.2. In model 1 and 3b the “spot” price is some price determined by the long term information in the market as described in 9.2 for model 1 and 10.3 for model 3b. ** Since the option never will take on any value no exercise boundary can be found. *** Model 3a has two parameters and the exercise boundary will be a line in the $[X, \varepsilon]$ space.

11.1 Comparison of the results

The value of investing today varies between the models. Identical cost being used in all models implies the difference to be caused by the expected future price. The differences can best be explained by a closer look at the forward curves incorporated in the three models. Figure 24 is a simplified picture of the three forward curves. Thinking of the NPV of producing 1 MWh every year in 30 years as the area under the forward curve one can easily see that $NPV(\text{Model 1 or Model 3}) > NPV(\text{Model 2})$. Model 1 and 3 give very similar results.

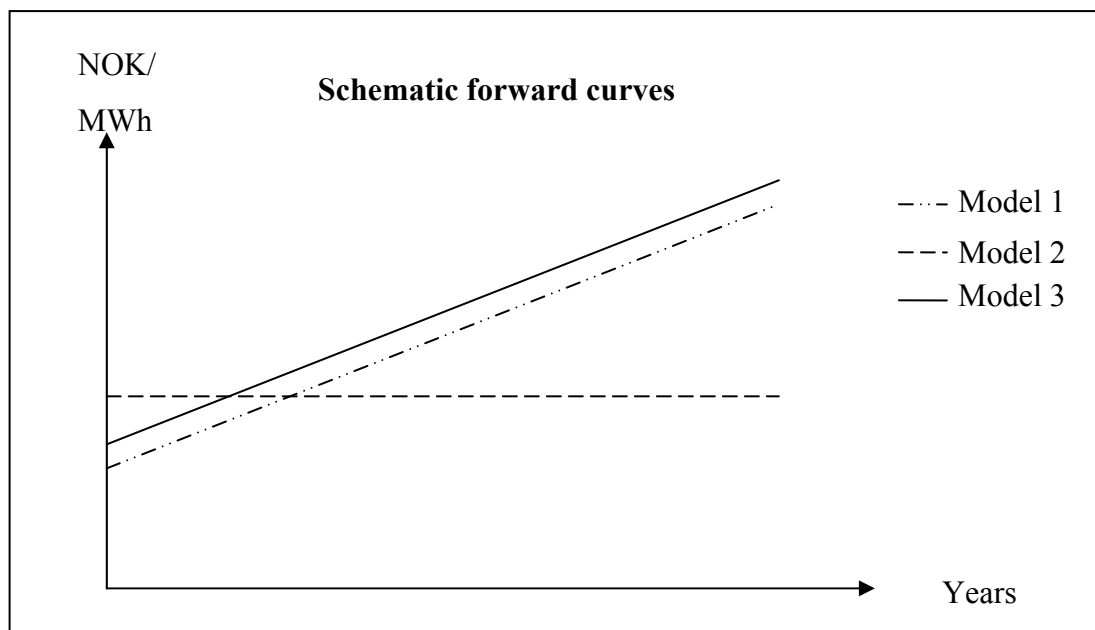


Figure 24: Schematic forward curves.

Table 9 shows that model 1 and 3 have positive option values. In these two models waiting for better information is the best choice. Model 2 has a zero option value because the investment value will always be negative under the base case scenario.

11.2 Analysing Parameters

Through the sensitivity analysis some weaknesses and strengths about the models are revealed. Some parameters can vary a lot without affecting the investment decision. This paragraph will shortly discuss the parameters affecting the investment decision the most. The cost related parameters are derived from various sources whereas the price parameters are estimated from historical price data. The technology related to gas power plants is mature, and it is unlikely that large variations in the technology related parameters will occur.

The cost of CO₂ emission is an uncertain and much discussed parameter. Model 1 and 2 captures some of the uncertainty about future CO₂ quotas. As mentioned in chapter 10 these models will recommend to invest today if the expected future cost of CO₂ is low enough. Model 3 however the investment decision does not seem to be affected by CO₂ costs.

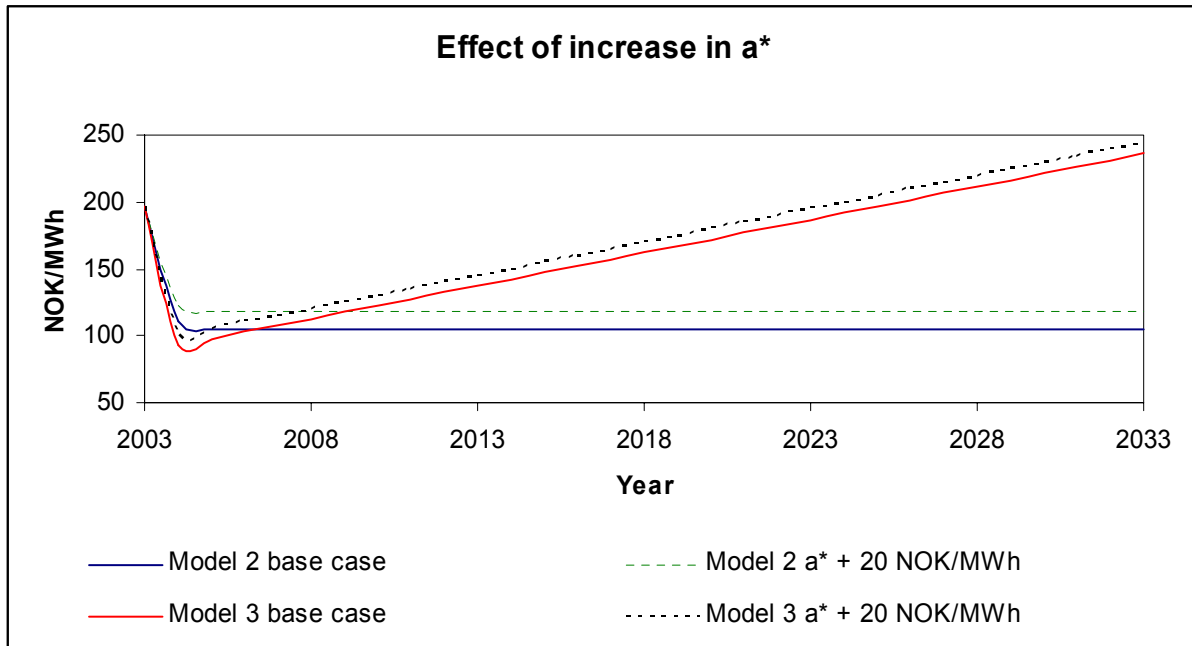


Figure 25: Effect on spark spread forward curve of increased α^* for model 2 and 3

In model 2 and 3 \bar{S} is determined by the historical data and α^* reflects the information in the forward market. If the spark spread prices in the future were expected to be higher then this would result in an increase in α^* . In chapter 10.2 and 10.3 the effect of increase in α^* on option values are discussed. Figure 25 show how an increase in α^* affects the estimated forward curves for model 2 and 3. In both models an upward shift occurs. This shows the importance of α^* on the level of the term structure, and is by that crucial to the investment decision.

The results states that the investment decision is more affected by the choice of model than variation in input parameters. In order to get proper valuation and investment decision support, care must be taken when choosing the model to be used. In this paper both the simplest and the most advanced model seems to give the most reasonable answer since both models show resemblance to the long-term market information.

11.3 Comparing the models

The advantage of model 1 is that a value of a closed form solution is available and gives the opportunity to compare the finite with the infinite option. This is not possible for the two other models and numerical procedures must be used. Considering the computational tools available, this advantage is no more important.

Model 2 is the most applicable as it only needs the current spark spread “spot” to compute a forward curve and obtain an investment decision. Model 1 and Model 3 are more cumbersome. The whole procedure outlined in chapter 9.1 must be carried out for each different date of valuation when using model 1. In model 3, the state variables must be computed for the given date in question. The additional effort needed should not be a constraint in using Model 1 and 3 when facing large investments.

Model 3 is best fitted to the actual term structure. This will always be the case regardless of the drift in the forward curve. Model 2 is not able to capture any drift in the term structure, but will a good approximation to a flat forward curve. Model 1 can be adjusted to both cases when looking at a long horizon. It is less useful for short term valuation.

The ability to capture the long-term term structure has proven to be the important feature in this valuation. Short term movement can be ignored as it only to a limited degree affects the investment decision. For the valuation presented in this paper, model 1 and 3 are equally suitable. In general, model 3 is recommended.

12 Concluding remarks

The construction of gas fired power plants in Norway is in 2003 a relevant topic based on the prevailing power situation. This paper has investigated the investment timing for a firm owning a license to build a gas fired power plant, and hence the option value to delay investment.

The models used are based on pricing models suggested by Lucia and Schwartz, modified to fit the situation at hand. Realistic operational and cost data have been determined through a thorough study of CCGT power plants. The parameters used are estimated by fitting the models to empirical data such as historical spot prices and current forward data. To perform the computations, software tailored for these models have been made. These elements are vital building blocks on the way to value the license to invest in a CCGT power plant in Norway.

The assumptions made with respect to the power plant represents a sort of “best case” conditions, but are still considered to be realistic. The investment can not be made more profitable by changing plant location unless synergy effects can be obtained.

The uncertainty regarding cost of CO₂ emissions as much discussed in the public media is captured by model 1 and 2. Since this is not reflected in model 3, cost of CO₂ emission cannot be claimed to be the most important aspect for delaying investing. All models point out the future spark spread level, e.g. the future price of natural gas and electricity as important for the investment decision. This view is shared by the possible investors. Based on this long-term future price levels of natural gas and electricity are considered to be the most significant decision factor.

Model 1 and 3 produces approximately the same result and identical investment decision. As pointed out in chapter 10.3 short term fluctuations do not alter the investment decision. Thus for making an investment of long lived projects Model 1 can be used as long as initial start price are derived from long-term forward data and not current “spot” data. When using model 1 on short lived projects, care should be taken since the short end of the forward curve will be more important. We conclude by ranking model 3, the two factor model, as the most realistic. Accordingly, the value to invest is about 2900 MNOK and the option value is 3500 MNOK. The best investment decision is to wait.

13 Critics and further work

In this chapter some critics of the analyses in this paper are mentioned. Suggestions for further work are outlined in the final paragraphs.

The implications of estimating volatility were not fully understood before the time to submitting the paper was growing short. Thus a proper method to find an estimate for long term volatility was not conducted. Further, measures of confidence should have been calculated to be able to say something about uncertainty in the estimated parameters.

The historical data on natural gas obtained from IPE is noted in GBP whereas the historical data from Nordpool is noted in NOK. The data set is constructed using actual currency exchange rates instead of some forward rate for the period. As such, movement in spark spread can be a result from currency fluctuations and not from movement in the commodity prices alone as assumed in the models. This is a weakness of the analyses and should preferably be avoided.

When using Monte Carlo simulation to perform valuation under model 3 very few simulations are conducted. The 5000 used are assumed to be sufficient to get a proper estimate. However, to get an accurate value up to 100 000 simulations should be used. To speed up calculations antithetic variables could be used. Methods such as control variate methods could be used to remove the inaccuracy implied in Monte Carlo simulations.

In future work the critics above should be considered. Further it is possible to extend the analysis to incorporate competition and first mover advantages and further investigate effects on the market price when adding capacity. Electricity and natural gas can be modelled separately allowing them to move according to different price processes. Another possibility is to look into how availability and optimal life time affects the investment decision.

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Appendix 1 - The power plant

The different categories of gas fired power plants produce either electricity in simple cycle or combined cycle gas turbines (CCGT), or are used for cogeneration of heat and power (CHP). Simple cycle consist of a gas turbine only, whereas the CCGT includes both gas and steam turbine(s). Simple cycle is a cheaper investment, smaller in size and more flexible but has lower total efficiency than the CCGT. A CCGT plant will be the preferable alternative for base load electricity production. A CHP plant uses CCGT with the ability to extract steam from the process. Building a CHP plant is relevant if there is industrial need for high temperature steam.

The CCGT process utilises a combination of fossil fuelled and gas turbine plant and equipment. Generally one or a number of combustion gas turbines feed their exhausts to a unitised or common gas duct which then passes the GT exhaust to a steam boiler where the heat is used to change water to steam. This is in turn passed to a conventional steam turbine, which is complete with its own generator. Overall cycle efficiencies can be as high as 60%. The CCGT has flexibility in purchase and low capital installation costs compared to competing sources of electricity production such as coal fired or nuclear electricity plants.

It should be noted that in the event of a loss incident at one of the gas turbines, the other units will be able to operate independently. If the steam turbine suffers an incident the GT (s) will still be able to continue in operation but obviously at reduced efficiency. There is therefore a considerable amount of flexibility in operation.

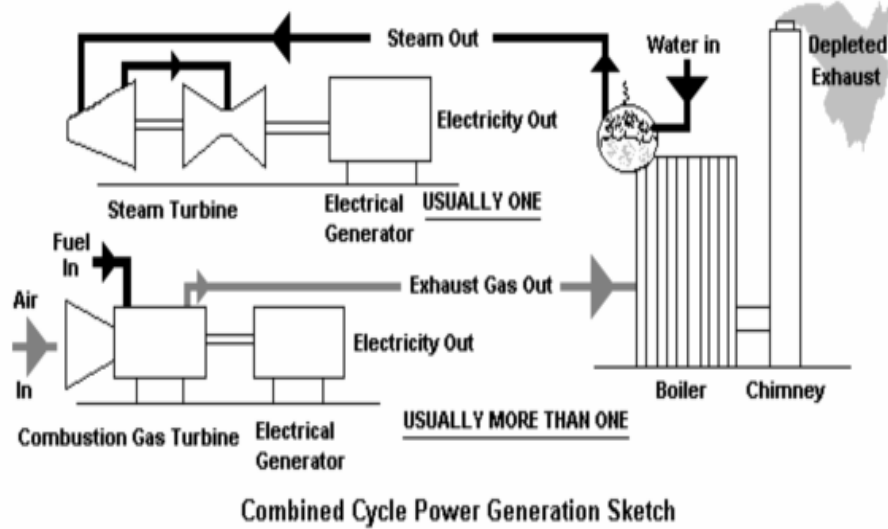


Figure A 1: A simple display of a CCGT process

Description of the different components

The following chapter contains a brief discussion of the main considerations in plant-design with respect to the main components. For a more in depth discussion, interested readers are advised Moran et al. for more details on the thermodynamic processes.

Gas and Steam turbines

The power production in a CCGT plant is generally made up of 2/3 from gas turbine(s) and 1/3 from the steam turbine. The efficiency of gas turbines are highly load dependent, whereas the steam turbine is more flexible. To avoid running on low efficiency, a power plant will be designed with several smaller turbines if good part-load characteristics is an important design criteria. The operating strategy of a gas power plant is that the gas turbines are either running at optimal load or is not running at all to cope with the major load variations. An important part in designing the power plant is determining the optimal number and size of gas turbines. There will be little need for such flexibility assuming a constant delivery of base load.

The cost per MW installed is reduced with increasing unit capacity, the units should be chosen as large as possible. The largest gas turbines commercially available today are of 300-400 MW. A CCGT plant intended for delivery of 800 MW relatively stable base load will most likely consist of two large gas turbines and a large steam turbine.

The gas turbines are normally standard equipment due to very high development costs. Steam turbines come in a wide range, and it is not difficult to find a suitable one to match the gas turbines (Bolland 2003). The plant will have one steam turbine with two or three pressure levels. The steam turbine is a cheap component compared to the gas turbines, and can be used to regulate output without much decrease in efficiency.

The other main components

The evaporator fixates the load point. With variable load, a larger number of evaporators are needed to secure good part load characteristic. In the case of a base load plant one or two evaporators will be sufficient. The heat from combustion not delivered to water in the tubes of the boiler and superheater are extracted in the economizer. The electrical generator and power generating components are important parts of designing the power plant, but will not be discussed in this paper.

In terms of components, better thermodynamic efficiency results in higher investment costs. A plant specific analysis must be conducted to find the optimal trade off. Heat-recovery equipment, added flue-work and building area increase the cost. The fixed costs due to increased investment and the saving in operation costs due to reduction of flue –gas temperature should balance and show the justification for the use of heat-recovery apparatus.

Choosing design

Choosing the optimal design of components is made based on the type of production. (e.g. Base- or peak load). The main rule is that operational flexibility is expensive. The main consideration in the design process is choosing the optimal set of gas turbines.

For each combination of fuel cost, load factor and capacity factor, there is an economical pressure and temperature of steam which will result in the lowest cost of power. There is a trade off between increasing the efficiency due to increase in steam pressure and temperature and the additional investment in equipment required for the purpose. After determining the main specifications, the exact design with respect to setting temperatures and mass flows for the process in details can be solved with computer tools such as GTPRO.⁴⁶

⁴⁶ Bolland (2003)

Efficiency

A CCGT plant equipped with the most modern generation of gas turbines, H-series, can obtain a net plant efficiency of 60%. The older generation of F-series turbines can obtain 56-58 % depending on the exact model, e.g. the General Electric S109 FA and the Toshiba 109 FA with net plant efficiency of 56,7% based on standard assumptions.⁴⁷ The newest turbine technology generally includes dry low NO_x combustion technology. A net plant efficiency of around 58% by Norwegian conditions can be assumed for F-series technology conditions for F-series technology.⁴⁸

Calculating costs

The turnkey for a CCGT plant as found in Gas Turbine World involves all components concerning the power generation process. The turnkey costs for a 400 MW CCGT plants is 200 MUSD with H-series and around 140 MUSD with F-series turbines. An 800 MW CCGT with F-series turbines will have a turn key cost of around 240 MUSD.⁴⁹ (year 2000 USD)

The total cost of the plant, is according to Bolland (2003) as a rule between 1.3 and 1.5 times the turnkey cost. It can be higher if substantial infrastructure investments have to be made. Competition and marked conditions will also heavily affect the prices listed and could also affect terms of insurance and maintenance conditions.

The investment is highly exposed to currency fluctuation. As an example, the total investment cost of an 800 MW CCGT will 360 MUSD by adding 1.5 to the turnkey cost. By using the average of daily exchange rates from 2003, the resulting investment cost is 2560 MNOK. Comparably, the same plant will have an investment cost of 3168 MNOK if the average of daily exchange rate from 2000 is used. The difference of 600 MNOK is a considerable amount and states that currency fluctuations should be taken into account.

The range of the investment cost is fairly wide with a total investment cost of 2218 MNOK using the most and 3168 MNOK using the least favorable conditions. Statoil has estimated the investment costs at Tjeldbergodden to be in the order of 3000 MNOK. The estimate does not include large investments into gas or electricity infrastructure and is still in the high end.

⁴⁷ Gas Turbine World (2000)

⁴⁸ Bolland (2003)

⁴⁹ Gas Turbine World (2000)

Without knowing a more detailed cost allocation or the exchange rates used for the calculations it difficult to interpret the number further than label it as a conservative cost estimate.

| Investments CCGT power plant of 390 MW | | |
|--|---------------------|----------------|
| Turnkey costs (based on GT PRO) | | [kUSD] |
| Total installed capital equipment | 71 % | 113 000 |
| Contractor's engineering | 8 % | 12 528 |
| Contractor's construction | 10 % | 15 660 |
| Miscellaneous & start-up costs | 3 % | 4 176 |
| Contingency (10%) | 9 % | 14 536 |
| Total turnkey cost | | 159 900 |
| Additional costs (based on experience - highly) | % of turnkey | |
| turnkey contr. | 5,0 % | 7 995 |
| Connection to cooling water | 4,0 % | 6 396 |
| Connection to HB overhead lines/upgrade og HV lines | 5,0 % | 7 995 |
| Connection to gas terminal | 6,0 % | 9 594 |
| Conceptual eng., etc. | 5,0 % | 7 995 |
| Power and fuel for commissioning | 0,5 % | 800 |
| Spare parts (not capitalized) | 1,0 % | 1 599 |
| Verifications, insurance, risk analysis, environm. studies | 2,5 % | 3 998 |
| Contingency, additional costs | 15,0 % | 23 985 |
| Total additional costs | | 70 356 |
| Total project investment cost | | 230 256 |

Table A 1: Allocation of investment cost for a 390 MW F-series CCGT plant, Norwegian conditions (Bolland 2003)

Table A 1 presents a detailed allocation of investment costs for a 390 MW CCGT in a Norwegian setting. Building two identical blocks or a single 800 MW block should result in a lower investment cost per MW installed.

If the option to upgrade the plant later shall be taken into consideration, another turbine set may be favorable as shown by Fleten and Näsäkkälä (2003) but is not considered in this paper.

Location issues and the impact of ambient conditions

The efficiency of the CCGT - process is affected by both the temperature of cooling water and the ambient air temperature. The cooling water has the largest influence.

To run the CCGT process with high efficiency requires a good and stabile source of cooling. Locating the plant in according to assumption 2, chapter 6, allows for heat exchanging with

low-temperature sea water, which provides an excellent and stable source of cooling the whole year. The sea-water will be taken from a depth of 50 m where the temperature shows little seasonal variation and is in the range of 6-10 °C.

The performance of the gas turbine is heavily affected by ambient conditions. Any parameter affecting the mass flow of the air entering the gas turbine will have an impact on the performance. Increasing ambient temperature will have a negative effect on power output.

The two major categories of plant output enhancement are gas turbine inlet air cooling and power augmentation⁵⁰ Evaporating cooling is the most cost efficient method when ambient temperature is high and the relative humidity is low, but not ideal for Norwegian coastal climate.

To enhance performance decreasing the inlet temperature to the compressor, inlet cooling, will improve power output and heat rate. As the temperature in the coastal areas of western Norway rarely is above 20°C, the ambient conditions will only to a small extent reduce power output. It will not be an economical incentive to install inlet cooling equipment.

Wear, failure and Availability

Wear and failure of components and the issue of availability can have a huge impact of the profitability of the plant and should be taken into consideration both in terms of design- and operational issues. The base load plant in question should optimally be running constantly according to the assumptions made. It will, however, experience downtime due to both planned and unplanned outages.

⁵⁰ Kiameh (2003)

| Estimated O & M Life Cycle Costs | |
|---|------------------|
| Operations Cost | [kUSD] |
| Direct Salary & Labor | 2 249 520 |
| Direct Materials & Subc | 745 913 |
| Total | 2 995 433 |
| Maintenance - Planned | |
| Gas Turbines | 2 929 950 |
| HRSGs | 144 283 |
| SCR Replacement | - |
| ST-Gs | 145 986 |
| Instruments | 37 162 |
| Systems BOP | 154 514 |
| Total | 3 411 895 |
| Maintenance - unplanned | 686 379 |
| Total O & M [\$ /year] | 7 093 707 |

Table A 2: Allocation of Operation and Maintenance cost (in 1998 USD) estimated for CCGT plant using a GE STAG 109FA (approx 350 MW). (Bolland 2003)

The gas turbine, and particularly the turbine blades, is the most expensive component with respect to maintenance as shown in table A 2. Using new and unconventional technology increases the risk of failure.

Availability and reliability are important characteristics for a base load plant and the definitions are as follows. A plant is considered reliable if no unplanned service breakdown occurs as a result of disturbance. A plant is available when it is ready for operation, non-availability also including planned outages. The term RAM (Reliability, Availability, and Maintainability) is often used as a general description of analysis and estimates/assessments where availability and reliability is connected to maintenance. The work and experience concerning these issues are normally confidential, but some databases exist. The two most important databases with RAM information about gas- and steam-turbines are the NERC-GADS and SPS-ORAP. The databases containing Empirical RAM- data are usually subject to payment as in the case of the SPS-ORAP database which provides good RAM-data for base-load plants.

The plant is considered to have 7900 operating hours a year (90% availability). From publicly available sources, average availability for large CCGT plants is found to be 85% in the WEC annual report of 2002⁵¹ and to 90% for CCGT plants in general from NERC-GADS.⁵² The

⁵¹

⁵²

data does not take into account whether the size nor the age and operating strategy of the actual turbines. Bolland (2003) uses 7900 hours.

Degradation

Degradation with time will lead to reduction in performance and efficiency, mainly of the gas turbines. A CCGT plant will suffer reduction in power of 3-10% and efficiency of 2-5%. Using natural gas as fuel will lead to reductions in the lower end and up to 50-80% can be avoided by performing regularly maintenance such as washing the turbines (Bolland 2003). Veer (2003) shows that the irreversible degradation effect occurs within the first 10000 operating hours and output reduction stabilizes at around 4 %. Degradation can be incorporated into the model by reducing equivalent operating hours or lifetime of the plant.

Appendix 2 – programming code

VBA script for binomial tree

```
Public Sub AMBoption_tree(ByVal drawBinTree As String, ByVal exboundary As String)
'-----
'|Author: Alstad, R.M. & Foss, J.T. |
'|Version: Final |
'|Date: Fall 2003 |
'|Description: This sub evalutates an americian option using a binomial |
'| tree. It contains code print the tree to the spread |
'| sheet and to print the array of early |
'| exercise boundary to the spread sheet. |
'-----
Dim Spark, invest, interest, alfas, sigma, opcost, capacity, eqvoptime, costnox, costco2,
insurance As Double
Dim t1, t2, T, n As Integer
Dim shtTreeSheet, shtOutput As String
shtTreeSheet = "Tree"
shtOutput = "Exercise boundary"
'Input parameters
Spark = Input_sheet.Range("spark").Value
invest = Input_sheet.Range("investment").Value
interest = Input_sheet.Range("interest").Value
alfas = Input_sheet.Range("alfas").Value
sigma = Input_sheet.Range("sigma").Value
opcost = Input_sheet.Range("opcost").Value
capacity = Input_sheet.Range("capacity").Value
eqvoptime = Input_sheet.Range("eqvoptime").Value
costnox = Input_sheet.Range("costnox").Value
costco2 = Input_sheet.Range("costco2").Value
insurance = Input_sheet.Range("insurance").Value
t1 = Input_sheet.Range("tone").Value
t2 = Input_sheet.Range("ttwo").Value
T = Input_sheet.Range("tee").Value
n = Input_sheet.Range("periods").Value

If (drawBinTree = "Yes") And (n > 200) Then
MsgBox ("The tree cannot have more than 200 time steps")
Exit Sub
End If

If (drawBinTree = "Yes") And (n > 10) Then
MsgBox ("Use 10 or less periodes to make add colors to a tree")
End If

Dim U, D, p, dummy As Double
Dim a, b, x, y, i, offset_down, offset_up As Integer
ReDim timing(2, n) As Variant
Dim startprint As Range

'sets up, down and the up probabillity
U = ((alfas * T / n) ^ 2 + sigma ^ 2 * T / n) ^ 0.5
D = -U
p = (alfas * (T) / n + U) / (2 * U)
offset_up = -4
offset_down = -offset_up

'defines an array for the value of spark spread
ReDim periodeTabell(0 To n, 0 To n)
'defines an array for the value of the offset_up and offset_down moves used to print the
option tree
ReDim offsetTable(0 To n, 0 To n)
'upper left cell = initial spark spread
periodeTabell(0, 0) = Spark
'upper left cell = 0 offset
offsetTable(0, 0) = 0
'the tree of spark spread are calculated left to right
For x = 0 To n - 1 'column
For y = 0 To x 'row
If y = 0 Then
```

```

'an up move is set in the cell right of the given spark spread
periodeTabell(y, x + 1) = periodeTabell(y, x) + U
offsetTable(y, x + 1) = offsetTable(y, x) + offset_up
End If
'an down move is set in the cell down and right of the given spark spread
periodeTabell(y + 1, x + 1) = periodeTabell(y, x) + D
offsetTable(y + 1, x + 1) = offsetTable(y, x) + offset_down
Next y
Next x

'defines an array for the value of the option
ReDim OptValue(0 To n, 0 To n)

'the option value is calculated left to right
For i = 0 To n
    dummy = periodeTabell(i, n)
    If ProjectValue(dummy, interest, alfas, opcost, capacity, eqvoptime, costnox, costco2,
insurance, t1, t2) _
        - invest > 0 Then
        OptValue(i, 0) = ProjectValue(dummy, interest, alfas, opcost, capacity, eqvoptime,
costnox, costco2, insurance, t1, t2) _
            - invest
    Else
        OptValue(i, 0) = 0
    End If
Next i
i = 0
For a = 1 To n 'column
    For b = 0 To n - a 'row

        dummy = periodeTabell(b, n - a)
        OptValue(b, a) = Application.Max(ProjectValue(dummy, interest, _
alfas, opcost, capacity, eqvoptime, costnox, costco2, insurance, t1, t2) - invest,
(OptValue(b, a - 1) * p + (1 - p) * OptValue(b + 1, a - 1)) * Exp(-interest * (T) / n))

        'returns the optimal investment point
        If ProjectValue(dummy, interest, _
alfas, opcost, capacity, eqvoptime, costnox, costco2, insurance, t1, t2) - invest > _
(OptValue(b, a - 1) * p + (1 - p) * OptValue(b + 1, a - 1)) * Exp(-interest * (T) / n)
Then
            'makes an array of optimal investment points
            investpoint = (n - a) * T / n
            timing(1, a) = investpoint
            timing(2, a) = dummy
        Else
            investpoint = (n - a) * T / n
            timing(1, a) = investpoint
        End If
    Next b
Next a

If drawBinTree = "Yes" Then
'----- this section writes the binomial tree
'activates worksheet
Worksheets(shtTreeSheet).Activate
'clears the active sheet
ActiveSheet.Cells.ClearContents
ActiveSheet.Cells.ClearFormats
'activates cell to write in
Worksheets(shtTreeSheet).Range("A" & 4 * n - 4).Activate
explain = Array("Periode", "Spark spread", "Option value", "Exercise value")
colorTab = Array(20, 40, 50, 15)
For i = 0 To 3
    ActiveCell.Offset(i, 0).Value = explain(i)
    If n <= 10 Then
        ActiveCell.Offset(i, 0).Select
        With Selection.Interior
            .ColorIndex = colorTab(i)
            .Pattern = xlSolid
            .PatternColorIndex = xlAutomatic
        End With
        Worksheets(shtTreeSheet).Range("A" & 4 * n - 4).Activate
    End If
Next i
Worksheets(shtTreeSheet).Range("A" & 4 * n + 4).Activate
'writes binomial tree to the worksheet

```

```

Application.ScreenUpdating = False
For j = 0 To n 'column
  For i = 0 To j 'row
    'activates the first node in the tree
    Worksheets(shtTreeSheet).Range("A" & 4 * n + 4).Activate
    'writes the time step
    ActiveCell.Offset(offsetTable(i, j), j).Value = j

    If n <= 10 Then
      ActiveCell.Offset(offsetTable(i, j), j).Select
      With Selection.Interior
        .ColorIndex = 20
        .Pattern = xlSolid
        .PatternColorIndex = xlAutomatic
      End With
      'activates the first node in the tree
      Worksheets(shtTreeSheet).Range("A" & 4 * n + 4).Activate
    End If

    ActiveCell.Offset(offsetTable(i, j) + 1, j).Value = periodeTabell(i, j)
    If n <= 10 Then
      ActiveCell.Offset(offsetTable(i, j) + 1, j).Select
      With Selection.Interior
        .ColorIndex = 40
        .Pattern = xlSolid
        .PatternColorIndex = xlAutomatic
      End With
      'activates the first node in the tree
      Worksheets(shtTreeSheet).Range("A" & 4 * n + 4).Activate
    End If
    ActiveCell.Offset(offsetTable(i, j) + 2, j).Value = OptValue(i, n - j)
    If n <= 10 Then
      ActiveCell.Offset(offsetTable(i, j) + 2, j).Select
      With Selection.Interior
        .ColorIndex = 50
        .Pattern = xlSolid
        .PatternColorIndex = xlAutomatic
      End With
      'activates the first node in the tree
      Worksheets(shtTreeSheet).Range("A" & 4 * n + 4).Activate
    End If

    ActiveCell.Offset(offsetTable(i, j) + 3, j).Value = ProjectValue(periodeTabell(i, j),
interest, _
alfas, opcost, capacity, eqvoptime, costnox, costco2, insurance, t1, t2) - invest

    If n <= 10 Then
      ActiveCell.Offset(offsetTable(i, j) + 3, j).Select
      With Selection.Interior
        .ColorIndex = 15
        .Pattern = xlSolid
        .PatternColorIndex = xlAutomatic
      End With
    End If

  Next i
Next j
Worksheets(shtTreeSheet).Range("A" & 4 * n + 4).Activate
Application.ScreenUpdating = True
End If
'----- end of tree writing

If exboundary = "Yes" Then
'----- this section graphs the optimal exercise boundary
'activates worksheet
Worksheets(shtOutput).Activate
'clears the active sheet
ActiveSheet.Cells.ClearContents
ActiveSheet.Cells.ClearFormats
'activates cell to write in
Worksheets(shtOutput).Range("A1").Activate
Application.ScreenUpdating = False
For i = 0 To n
  ActiveCell.Offset(i, 0).Value = timing(1, n - i)
  ActiveCell.Offset(i, 1).Value = timing(2, n - i)
Next i
MsgBox ("You must adjust the ranges on the graph manually")
Application.ScreenUpdating = True

```

```

End If
'MsgBox (OptValue(0, n))

End Sub

Public Function AMBOption(Spark As Double, invest As Double, interest As Double, _
alfas As Double, sigma As Double, opcost As Double, capacity As Double, _
eqvoptime As Double, costnox As Double, costco2 As Double, insurance As Double, _
t1 As Integer, t2 As Integer, T As Integer, n As Integer) As Double
'-----
'|Author: Alstad, R.M. & Foss, J.T.
'|Version: Final
'|Date: Fall 2003
'|Description: This function evalutates an americian option using a
'|              binomial tree.
'-----
Dim U, D, p, dummy As Double
Dim a, b, x, y, i As Integer
ReDim timing(2, 1) As Variant
Dim startprint As Range

'sets up, down and the up probabillity
U = ((alfas * T / n) ^ 2 + sigma ^ 2 * T / n) ^ 0.5
D = -U
p = (alfas * (T) / n + U) / (2 * U)

'defines an array for the value of spark spread
ReDim periodeTabell(0 To n, 0 To n)
'upper left cell = initial spark spread
periodeTabell(0, 0) = Spark

'the tree of spark spread are calculated left to right
For x = 0 To n - 1 'column
  For y = 0 To x 'row
    If y = 0 Then
      'an up move is set in the cell right of the given spark spread
      periodeTabell(y, x + 1) = periodeTabell(y, x) + U
    End If
    'an down move is set in the cell down and right of the given spark spread
    periodeTabell(y + 1, x + 1) = periodeTabell(y, x) + D
  Next y
Next x

'defines an array for the value of the option
ReDim OptValue(0 To n, 0 To n)

'the option value is calculated left to right
For i = 0 To n
  dummy = periodeTabell(i, n)
  If ProjectValue(dummy, interest, alfas, opcost, capacity, eqvoptime, costnox, costco2,
insurance, t1, t2) _
- invest > 0 Then
    OptValue(i, 0) = ProjectValue(dummy, interest, alfas, opcost, capacity, eqvoptime,
costnox, costco2, insurance, t1, t2) _
- invest
  Else
    OptValue(i, 0) = 0
  End If
Next i

Next i
i = 0
For a = 1 To n 'colum
  For b = 0 To n - a 'row

    dummy = periodeTabell(b, n - a)
    OptValue(b, a) = Application.Max(ProjectValue(dummy, interest, _
alfas, opcost, capacity, eqvoptime, costnox, costco2, insurance, t1, t2) - invest, _
(OptValue(b, a - 1) * p + (1 - p) * OptValue(b + 1, a - 1)) * Exp(-interest * (T) / n))

  Next b
Next a

dummy = OptValue(0, n)
AMBOption = dummy
End Function

Function ProjectValue(ByVal Spark As Double, ByVal interest As Double, _

```

```

ByVal alfas As Double, ByVal opcost As Double, ByVal capacity As Double, ByVal _
eqvoptime As Double, ByVal costnox As Double, ByVal costco2 As Double, ByVal insurance As
Double, _
ByVal t1 As Integer, ByVal t2 As Integer) As Double
'-----
'|Author: Alstad, R.M. & Foss, J.T.
'|Version: Final
'|Date: Fall 2003
'|Description: This function evalutates the value of the project.
'-----
Dim c1, c2 As Double

c1 = capacity * eqvoptime * (Exp(-interest * t1) - Exp(-interest * t2)) / interest

c2 = capacity * eqvoptime * alfas * ((interest * t1 + 1) * Exp(-interest * t1) - (interest *
t2 + 1) * Exp(-interest * t2)) / interest ^ 2 _
- (capacity * eqvoptime * (costnox + costco2) + (opcost + insurance)) * (Exp(-interest * t1) -
Exp(-interest * t2)) / interest

ProjectValue = (c1 * Spark + c2) * 0.000000001 ' i mrd NOK
End Function

Function S_star(ByVal Spark As Double, ByVal interest As Double, _
ByVal alfas As Double, ByVal opcost As Double, ByVal capacity As Double, ByVal _
eqvoptime As Double, ByVal costnox As Double, ByVal costco2 As Double, ByVal insurance As
Double, _
ByVal t1 As Integer, ByVal t2 As Integer, beta As Double, invest As Double) As Double
'-----
'|Author: Alstad, R.M. & Foss, J.T.
'|Version: Final
'|Date: Fall 2003
'|Description: This function finds the exercise threshold for an
'| infinite option.
'-----
Dim c1, c2 As Double

c1 = capacity * eqvoptime * (Exp(-interest * t1) - Exp(-interest * t2)) / interest

c2 = capacity * eqvoptime * alfas * ((interest * t1 + 1) * Exp(-interest * t1) - (interest *
t2 + 1) * Exp(-interest * t2)) / interest ^ 2 _
- (capacity * eqvoptime * (costnox + costco2) + (opcost + insurance)) * (Exp(-interest * t1) -
Exp(-interest * t2)) / interest

S_star = (c1 - c2 * beta + invest * 1000000000 * beta) / (c1 * beta)

End Function

Public Function EurAMBOption(Spark As Double, invest As Double, interest As Double, _
alfas As Double, sigma As Double, opcost As Double, capacity As Double, _
eqvoptime As Double, costnox As Double, costco2 As Double, insurance As Double, _
t1 As Integer, t2 As Integer, T As Integer, n As Integer) As Double
'-----
'|Author: Alstad, R.M. & Foss, J.T.
'|Version: Final
'|Date: Fall 2003
'|Description: This function evalutates an european option using a
'| binomial tree.
'-----
Dim U, D, p, dummy As Double
Dim a, b, x, y, i As Integer
ReDim timing(2, 1) As Variant
Dim startprint As Range

'sets up, down and the up probabillity
U = ((alfas * T / n) ^ 2 + sigma ^ 2 * T / n) ^ 0.5
D = -U
p = (alfas * (T) / n + U) / (2 * U)

'defines an array for the value of spark spread
ReDim periodeTabell(0 To n, 0 To n)
'upper left cell = initial spark spread
periodeTabell(0, 0) = Spark

'the tree of spark spread are calculated left to right
For x = 0 To n - 1 'column
    For y = 0 To x 'row
        If y = 0 Then

```

```

'an up move is set in the cell right of the given spark spread
periodeTabell(y, x + 1) = periodeTabell(y, x) + U
End If
'an down move is set in the cell down and right of the given spark spread
periodeTabell(y + 1, x + 1) = periodeTabell(y, x) + D
Next y
Next x

'defines an array for the value of the option
ReDim OptValue(0 To n, 0 To n)

'the option value is calculated left to right
For i = 0 To n
    dummy = periodeTabell(i, n)
    If ProjectValue(dummy, interest, alfas, opcost, capacity, eqvoptime, costnox, costco2,
insurance, t1, t2) _
        - invest > 0 Then
        OptValue(i, 0) = ProjectValue(dummy, interest, alfas, opcost, capacity, eqvoptime,
costnox, costco2, insurance, t1, t2) _
            - invest

    Else
        OptValue(i, 0) = 0
    End If

Next i
i = 0
For a = 1 To n 'column
    For b = 0 To n - a 'row

        dummy = periodeTabell(b, n - a)
        OptValue(b, a) = (OptValue(b, a - 1) * p + (1 - p) * OptValue(b + 1, a - 1)) * Exp(-
interest * (T) / n)

    Next b
Next a

dummy = Application.Max(ProjectValue(periodeTabell(0, 0), interest, _
    alfas, opcost, capacity, eqvoptime, costnox, costco2, insurance, t1, t2) - invest, _
    OptValue(0, n))
EurAMBOption = dummy
End Function

```

Matlab code trinomial tree.

```

clear all
%-----
%| Author:  Jørgen T. Foss          |
%| Date:    Fall 2003              |
%| Version: Final                  |
%| Description: Program to find the value of an American option using   |
%|                the trinomial tree.                                   |
%-----
%input values for the underlying asset.
S0=195%(NOK/MWh)
SS=97.95;%(NOK/MWh)
alfas=6.85; %(NOK/MWh)
rente_ar=6; % in percent per year
kappa_ar=2.69; %per year
sigma_ar=251 %per year
totaltime=10;%year
periodes=520;## of periodes in the tree

% input value for the project
invest=3;
kapasitet=800;%MW
driftstid=7900;%hours
driftskost=invest*0.02*1e9; %NOK/year
forsikring=invest*0.005*1e9;%NOK/year
rensing=14; %(NOK/MWh)
co2certificate=0.35*160; %(NOK/MWh)
t1=3;
t2=33;
%-----

% Call the function that makes the tree and calculates the option value.

```



```

Resultat=trinomisk_tre(S0,SS,alfas,rente_ar,kappa_ar,sigma_ar,totaltime,perioodes,kapasitet,dri
ftstid,driftskost,forsikring,rensing,co2certificate,t1,t2,invest);
%trinomisk_tre(S0,SS,alfas,rente_ar,kappa_ar,sigma_ar,totaltime,perioodes,kapasitet,driftstid,d
riftskost,insure,clean,certif,t1,t2,Invest)

% Store the return values in seperate variables
Ex=Resultat{1,5};
Opsjonen=Resultat{1,4};
Treet=Resultat{1,3};
DTre=Resultat{1,2};
Sanns=Resultat{1,1};

% Prints the option value to the screen.
'option value'
max(Opsjonen(:,1))
% Prints the value of exercise today to the screen
'exercise today'
exToday=prosjekt(S0,driftstid,kapasitet,log(1+0.01*rente_ar),kappa_ar,alfas,SS,driftskost,fors
ikring,rensing,co2certificate,t1,t2)-invest

% Call the function that graphs the tree.
plott_trinomisk_tre(Treet)

% plot the early exercise boundary.
t=1:1:length(Ex);
plot(t,Ex)
grid on

%S=-350:5:500;
%V=zeros(1,length(S));
%for i=1:1:length(S)
%
V(1,i)=prosjekt(S(i),driftstid,kapasitet,log(1+0.01*rente_ar),kappa_ar,alfas,SS,driftskost,fors
ikring,rensing,co2certificate,t1,t2)-invest;

%prosjekt(S(f_j(j),i),driftstid,kapasitet,log(1+0.01*rente_ar),kappa_ar,alfas,SS,driftskost,in
sure,clean,certif,t1,t2)
    %prosjekt(Spark,eqv,cap,interest,kappa,alfas,SS,opcost,insure,clean,certif,t1,t2)
    %end

%plot(S,V)

function
d=trinomisk_tre(S0,SS,alfas,rente_ar,kappa_ar,sigma_ar,totaltime,perioodes,kapasitet,driftstid,
driftskost,insure,clean,certif,t1,t2,Invest)
%-----
%| Author:   Jørgen T. Foss                               |
%| Date:     Fall 2003                                   |
%| Version:  Final                                       |
%| Thanks to Gunnar Aronsen for large parts of the code. |
%| Description: Calculates an American option using a trinomial tree |
%|             The underlying asset follows a simple mean reverting |
%|             process. The tree is constructed from the Hull & White |
%|             model 1990.                                  |
%-----
% S0 - today's price in NOK/MWh
% SS - Long term price level
% Risiko_ar - risk free interest rate in percent per year.
% Kappa_ar - mean-reversion rate per year.
% Sigma_ar - volatility.
% Totaltime - time to maturity of the option.
% Perioodes - number of step in the tree. perioodes + 1 = Imax
% Other input parameters are specific for this problem
%-----
% Scaling volatility, discount faktor and the mean reversion coeff
% to the step size.
global delta_X kappa_periode % Makes variables global in order to use
them in sub functions.
time_per_periode=totaltime/perioodes;
kappa_periode=kappa_ar*time_per_periode
sigma_periode=sigma_ar*sqrt(time_per_periode);
rente_periode=rente_ar*time_per_periode;
diskont = exp(-log(1+0.01*rente_periode)); % Discount rate per timestep.
%-----
% makes a matrix of sigma. allows for time dependant sigmas
sigma_t = zeros(1,perioodes +1);
for i =1:(perioodes +1)
    s_temp= ((sigma_periode^2)/(2*kappa_periode)*(1-exp(-2*kappa_periode)))^0.5;

```

```

        sigma_t(1,i) =s_temp;
end

% makes a matrix of the step size.
delta_X =zeros(1,perioedes +1);
for i =1:(perioedes +1)
delta_X(1,i) =sqrt(3)*sigma_t(1,i); % Regner ut den stokastiske stegstørrelsen pr tidsteg.
end

%-----
% Finds the high of the tree.
% Defines the variables Imax and Jmax which determine the size of the tree.

global Jmax Imax      % Makes variables global in order to use them in sub functions.

Imax = perioedes + 1;

jmax_temp = ceil(0.184/kappa_periode)

Jmax =min(jmax_temp,Imax-1);

Jmin = -Jmax;
%-----
% Declares the matrices needed in the function

P = zeros(f_j(Jmin) + 1, (Imax-1),3);      % Matrix for transition probabillities.
X = zeros(f_j(Jmin) + 1,Imax);            % Matrix for stochastic price element.
S = zeros(f_j(Jmin) + 1,Imax);            % Matrix fro the spot price S(t)=S*(t)+F(0,t)
F = zeros(f_j(Jmin) + 1,Imax);            % Matrix for the value of the option.

%ProVal = zeros(f_j(Jmin) + 1,Imax);      % Matrise for verdien av prosjektet
%-----
% Calculates the P matrix.
for i=1:(Imax-1)
    for j= max((-i+1),Jmin):min((i-1),Jmax)

        if (j == Jmax)&(Jmax ~= Imax-1)
            temp_u = 1;                    % if you are in the top node but not the last node use
the c tree
        else
            temp_u = 0;
            end

        if (j == Jmin)&(Jmax ~= Imax-1)
            temp_d = 1;                    % if you are in the top node but not the last node use
the b tree
        else
            temp_d = 0;
            end

        P(f_j(j),i,1:3)=temp_u*p_1(j) + (1-temp_u -temp_d)*p_2(j) +temp_d*p_3(j); % stores the
right probability in a 3D matrix
    end;
end;
%-----
% Calculates the values in the X matrix
% This matrix contains the values of the "dummy" process.
for i=1:Imax

    for j=max((-i+1),Jmin):min((i-1),Jmax)
        if(i==1)
            X(f_j(j),i) = j*delta_X(1,1); %delta_X(1,1)=0
        end
        X(f_j(j),i) = j*delta_X(1,i);

    end

end
end
%-----
% Calculates the values in the S matrix
% This matrix contains the values of the actual process.
for i=1:Imax

    for j=max((-i+1),Jmin):min((i-1),Jmax)
        S(f_j(j),i) = X(f_j(j),i) + forward(SS,S0,kappa_periode,alfas,i);

```

```

end
end
%midlertidig matrise for prosjektverdi
%for i=1:Imax
%   for j=max((-i+1),Jmin):min((i-1),Jmax)
%
%
%-----
% Calculates the value of the option by starting at the last period. In the
% last periode max(V(S)-I,0) is calculated. Then one works backward like in
% a binomial tree.
%-----
% Declares a vector to hold the lowest exercise value.
ex=zeros(1,Imax);
% The last periode.
for j= Jmin:Jmax
    % value of project in node j.

    Prosjektverdi=prosjekt(S(f_j(j),Imax),driftstid,kapasitet,log(1+0.01*rente_ar),kappa_ar,alfas,SS,driftskost,insure,
clean,certif,t1,t2);
    F(f_j(j),Imax)=max(Prosjektverdi-Invest,0);
    if (F(f_j(j),Imax)>0)&(ex(1,Imax)==0)
        ex(1,Imax)=S(f_j(j),Imax);
    end
end

end

% Working backwards from the last periode
for i=1:Imax-1
    n=Imax-i;
    for j=max((-n+1),Jmin):min((n-1),Jmax)
        % discounting back one time step.
        % Case 1: nodes not being top or bottom nodes.
        if (j>Jmin)&(j<Jmax)
            F_1 = zeros(1,3);
            for k=-1:1
                F_1(1,k+2)=P(f_j(j),n,2+k)*F(f_j(j+(-k)),n+1);%Pu*F(j+1,n+1),Pm*F(j,n+1),Pd*F(j-
1,n+1)
            end
        end
        % Case 2: nodes being bottom nodes.
        if (j==Jmin)
            F_1 = zeros(1,3);
            for k=-1:1
                F_1(1,k+2)=P(f_j(j),n,2+k)*F(f_j(j+(-
k+1)),n+1);%Pu*F(j+2,n+1),Pm*F(j+1,n+1),Pd*F(j,n+1)
            end
        end
        % Case 3: nodes being top nodes.
        if (j==Jmax)
            F_1 = zeros(1,3);
            for k=-1:1
                F_1(1,k+2)=P(f_j(j),n,2+k)*F(f_j(j+(-k-1)),n+1);%Pu*F(j,n+1),Pm*F(j-1,n+1),Pd*F(j-
2,n+1)
            end
        end
        % value of project in node j.

        Prosjektverdi=prosjekt(S(f_j(j),n),driftstid,kapasitet,log(1+0.01*rente_ar),kappa_ar,alfas,SS,
driftskost,insure,clean,certif,t1,t2);
        % F(j,n) is sett equal to max(exercise today, keep the option)
        F(f_j(j),n)=max(Prosjektverdi-Invest,sum(F_1)*diskont);
        % finds the lowest exercise value.
        if (Prosjektverdi-Invest > sum(F_1))&(ex(1,n)==0)
            ex(1,n)=S(f_j(j),n);
        end
    end
end

end
end

%-----

```

```

% fixes ex so that the first periods were early exercise is not done equal the highest
exercise price
for i=1:length(ex)
    if ex(1,i)==0
        ex(1,i)=max(ex);
    end
end
end
%-----d{1,1}=kutt(P);
d{1,2}=kutt(X);
d{1,3}=kutt(S);
d{1,4}=kutt(F);
d{1,5}=ex;
%-----% Sub function to
handle indexes. Matlab cannot use negative indexes in the
% matrices.
function sub_1 =f_j(j)

global Jmax;

temp = Jmax + 2 - j;

sub_1= temp;
%-----
% Sub function to calculate transition probabilities when the tree is
% according to figure 21.7-(c) Hull (1993).
% It is used at the upper egde off the tree.
function sub_2 =p_1(j);

global kappa_periode

% Probability to move straight forward from node (i,j) to (i+1,j)
pu = 7/6 +((kappa_periode*j)^2 -3*kappa_periode*j)/2;

% Probability to move on step down from from node (i,j) to (i+1,j-1)
pm = -1/3 - (kappa_periode*j)^2 + 2*kappa_periode*j;

% Probablitiy to move two step down from node (i,j) to (i+1,j-2)
pd = 1- pu -pm;

sub_2 =[ pu pm pd];
%-----
% Sub function to calculate transition probabilities when the tree is
% according to figure 21.7-(a) Hull (1993).
% It is used when you are not at an egde of the tree.

function sub_3 =p_2(j);

global kappa_periode

% Probability to move on step up from from node (i,j) til (i+1,j+1)
pu = 1/6 +((kappa_periode*j)^2 -kappa_periode*j)/2;

% Probability to move straight forward from node (i,j) to (i+1,j)
pm = 2/3 - (kappa_periode*j)^2 ;

% Probability to move on step down from from node (i,j) to (i+1,j-1)
pd = 1- pu -pm;;

sub_3 =[ pu pm pd];
%-----
% Sub function to calculate transition probabilities when the tree is
% according to figure 21.7-(b) Hull (1993).
% It is used at the lower egde off the tree.

function sub_4 =p_3(j);

global kappa_periode

% Probability to move two step up from fra node (i,j) to (i+1,j+2)
pu = 1/6 +((kappa_periode*j)^2 + kappa_periode*j)/2;

% Probability to move on step up from fra node (i,j) to (i+1,j+1)
pm = -1/3 - (kappa_periode*j)^2 -2*kappa_periode*j ;

% Probability to move straight forward from node (i,j) to (i+1,j)
pd = 1- pu -pm;

sub_4 =[ pu pm pd];

```

```

%-----
% sub function for the forward price
function sub_5 = forward(SS,S0,kappa_periode,alfas,i)
sub_5 = SS+(S0-SS)*exp(-kappa_periode*(i-1))+alfas*(1-exp(-kappa_periode*(i-1)));
%-----
% The matrices produces has a zeros in the upper and lower row. This
% function deletes those rows.
function sub_6 = kutt(matrise)
[rader kolonner]=size(matrise);
temp_mat=zeros(rader-2,kolonner);
for i=1:kolonner
    for j=1:rader-2
        temp_mat(j,i)=matrise(j+1,i);
    end
end
sub_6=temp_mat;

function d=plott_trinomisk_tre(F)
%-----
%| Author:   Jørgen T. Foss
%| Date:     Fall 2003
%| Version:  Final
%| Thanks to Gunnar Aronsen for large parts of the code.
%| Description: Plots the trinomial tree produced in 'trinomisk tre'.
%-----
[jrange periods] = size(F);
hold off
axis([1 periods+1 ceil(1.03*min(min(F))) ceil(1.03*max(max(F))])
hold on

global jmax;
jmax = (jrange-1)/2;
jmin = -jmax;

for i = 1:(periods-1)
    for j=max((-i+1),jmin):min((i-1),jmax)
        jadj = 0;
        if (j == jmax)
            jadj = -1;
        end
        if (j == jmin)
            jadj = 1;
        end

        for k = -1:1

            plot([i i+1],[F(f_j(j),i) F(f_j(j+k+jadj),i+1)],'k')
        end;
    end;
end;
xlabel('periods')
ylabel('Price [NOK/MWh]')
title('Price lattice')
d = 0;
%-----% Intern funksjon
for å håndtere indekser. Denne må lages fordi Matlab ikke kan bruke
% negative verdier i matrise indekseringen indeksering.

function sub_1 =f_j(j)

global jmax;

temp = jmax + 1 - j;

sub_1= temp;

function svar=prosjekt(Spark,eqv,cap,interest,kappa,alfas,SS,opcost,insure,clean,certif,t1,t2)
%-----
%| Author:   Jørgen T. Foss
%| Date:     Fall 2003
%| Version:  Final
%| Description: Calculates the value of the project.
%-----
%dummy variables
E1=(exp(-t1*(interest+kappa))-exp(-t2*(interest+kappa)));
E2=(exp(-t1*interest)-exp(-t2*interest));

C1=cap*eqv/(interest+kappa)*E1;

```

```

C3=cap*eqv*(SS+alfas)*E2/interest;
C4=-cap*eqv*(clean+certif)*E2/interest;
C5=-cap*eqv*(SS+alfas)*E1/(interest+kappa);
C6=-(opcost+insure)*E2/interest;
C2=C3+C4+C5+C6;
svar=(C1*Spark+C2)*1e-9;

```

Matlab script for Monte Carlo simulation two factor model

```

clear all;
%-----
%| Author:  Jørgen T. Foss          |
%| Date:    Fall 2003              |
%| Version: Final                  |
%| Description: This program calculates the value of an American option |
%|              using Monte Carlo simulation.                          |
%-----
%input for the stochastic processes.
X0=90;% (NOK/MWh)
epsilon0=23;% (NOK/MWh)
SS=89.39;% (NOK/MWh)
rente=log(1+0.01*6); % per year
alfas=12.319; % (NOK/MWh)
mu_per_ar=4.94;% (NOK/MWh)per year
kappa=8.6; %per year
sigmaShort=251; % (NOK/MWh) per year
sigmaLong=8;% (NOK/MWh) per year
correl=0.55; %correlation between X and epsilon

% input for the simulation
totaltid=10;%year, lenght of the option
timesteps=520; %periods in simulation
simulations=5000;% number of simulations
periodelengde=totaltid/(timesteps);

% input for the project
invest=3;
kapasitet=800;%MW
driftstid=8000;%t
driftskost=invest*0.02*1e+9;%NOK/year
forsikring=invest*0.005*1e9;%NOK/year
rensing=14; % (NOK/MWh)
co2certificate=0.35*160; % (NOK/MWh)
t1=3; % years
t2=33; % years

%Parameters to the mean reverting process
sigma_mean_rev=sqrt(sigmaShort^2*periodelengde);
kappal=kappa*periodelengde;
meanrevert=alfas;
%parameters to the ABM process
sigmaABM=sqrt(sigmaLong^2*periodelengde);
mu=mu_per_ar*periodelengde;
%generates X
Rx=random(simulations,timesteps+1);
X=mcMeanRev(X0,meanrevert,kappal,sigma_mean_rev,simulations,timesteps+1,Rx);
%generates epsilon
Re=random(simulations,timesteps+1);
epsilon=mcAMB(epsilon0,mu,sigmaABM,simulations,timesteps+1,Re,Rx,correl);
% deleting the random variables
Re=0;
Rx=0;
% calls the function that values the option.
svar=TwoFacAmOption(X,epsilon,rente*periodelengde,driftstid,kapasitet,rente,kappa,alfas,mu_per_ar,SS,driftskost,forsikring,rensing,co2certificate,t1,t2,invest);
F=svar{1,1};
% Prints the option value
'option value'
verdi=svar{1,2}
% Prints the value of exercise today
'exercise today'
exToday=exercise_meanrev_twofactor(X0,epsilon0,driftstid,kapasitet,rente,kappa,alfas,mu_per_ar,SS,driftskost,forsikring,rensing,co2certificate,t1,t2,invest)

```

```

% prints the simulated price paths.
S=SS+X+epsilon;
t=1:timesteps+1;
hold off
xlabel('periods')
hold on
ylabel('Price [NOK/MWh]')
title('Simulations')
plot(t,S)
%-----
%X=-100:5:300;
%eps=-200:5:200;
%V=zeros(length(eps),length(X));
%for i=1:1:length(X)
%   for j=1:1:length(eps)
%
V(i,j)=exercise_meanrev_twofactor(X(i),eps(j),driftstid,kapasitet,rente,kappa,alfas,mu,SS,drif
tskost,forsikring,rensing,co2certificate,t1,t2,invest);

%exercise_meanrev_twofactor(X,eps,driftstid,kapasitet,rente,kappa,alfas,mu,SS,opcst,insure,cl
ean,certif,t1,t2,invest)
%end
%end
%[Y,B]=meshgrid(X,eps);
%plot3(Y,B,V)
%grid on

function
value=TwoFacAmOption(X,epsilon,diskont,E,K,r,kappa,alfa,mu,SS,opcst,insure,clea,certif,t1,t2
,invest)
%-----
%| Author:   Jørgen T. Foss
%| Date:     Fall 2003
%| Version:  Final
%| Description: Calculates an American option using Monte Carlo.
%|           This function is constructed on basis of Longstaff &
%|           Schwartz 2001.
%-----
% X is a matrix containing all the simulated X paths
% epsilon is a matrix containing all the simulated epsilon paths
% diskont is the discount rate per periode: exp(-diskont*t) gives the
%   discount factor
% E is the equivalent operation time in hours per year
% D is the capacity in MW
% r is yearly interest rate in %
% kappa is yearly mean reversion for short term process
% alfa is price of risk in NOK/MWh
% SS is the long term expected price in NOK/MWh
% D is cost of operations in NOK/MWh
% mu is drift in long term process in NOK/MWh per year
% t1 start of operations in years
% t2 end of operations in years
% invest investment i GNOK

% The value of exercise is dependant on both X and epsilon. X and epsilon
% have different 'weights' in the value function. When X and epsilon are
% added to make a 'dummy price' in order to find the function for the value of
% waiting they have to be weighted in this dummy price in accordance with
% the weights in the value function. In the value function X is weighted c1
% and epsilon is weighted c2. Thus the dummy price will be: X+c2/c1*epsilon
c1=K*E/(r+kappa)*(exp(-t1*(r+kappa))-exp(-t2*(r+kappa)));
c2=K*E/r*(exp(-r*t1)-exp(-r*t2));
global q
q=c2/c1;

%finding the number of simulations
[simulations periodes]=size(X);
%Defines the matrix F for holding all the option values for each simulation
F=zeros(simulations,periodes);

%When the option matures the option is exercised if V(S)-I >=0
for i=1:simulations

F(i,periodes)=max(exercise_meanrev_twofactor(X(i,periodes),epsilon(i,periodes),E,K,r,kappa,alf
a,mu,SS,opcst,insure,clea,certif,t1,t2,invest),0);
end
%Prints the value of the European option.
temp_eur=sum(F(:,periodes))/simulations*exp(-diskont*(periodes-1))

```

```

siste=F(:,perioodes);
%for the rest of the perioodes we check whether it is optimal to wait or to
%exercise. Each price path can only have one exercise point.

for j=1:perioodes-2
    n=perioodes-j;
    % Since computational time is long a count down for every 100th period
    % is printed to the screen.
    if mod(n,100)==0
        n
    end

    k=0; %k is a counter that counts how many exercises points in a periode that has to be
checked
    % A is vector holding some dummy values. It is set to zero each
    % iteration.
    A=[0 0 0];
    for i=1:simulations
        if
exercise_meanrev_twofactor(X(i,n),epsilon(i,n),E,K,r,kappa,alfa,mu,SS,opcost,insure,clea,cert
if,t1,t2,invest)>0
            k=k+1;
            A(k,1)=i; %this holds on to the simulation number
            A(k,2)=X(i,n)+q*epsilon(i,n); %the dummy price with weighted X and epsilon
            %discounting the exercise value of the given path back to the
            %current time step.
            [big time]=max(F(i,:)); %finding the point of exercise and its value
            A(k,3)=exp(-diskont*(time-n))*big;
        end
    end

    % if any price path was picked out above we need to check whether early exercise is
optimal.
    if A(1,1)>0
        p=polyfit(A(:,2)/q,A(:,3),3); %Finding the function for the value of waiting;
Wait(S)=a+bS+cS^2+dS^3
        d=p(1);
        c=p(2);
        b=p(3);
        a=p(4);

        [rows column]=size(A);%finding the number of rows in A
        for k=1:rows
            % Value of early exercise

exercise=exercise_meanrev_twofactor(X(A(k,1),n),epsilon(A(k,1),n),E,K,r,kappa,alfa,mu,SS,opcs
t,insure,clea,certif,t1,t2,invest);
            % Value of waiting
            wait_val=wait(A(k,2),a,b,c,d);
            if wait_val < exercise %if the value of waiting is less than the vaule of
exercising
                F(A(k,1),n)=exercise; %sets the value of the option equal exercise
                %since the option will be exercised prematurely the remaining perioodes is set
to zero
                for m=n+1:perioodes
                    F(A(k,1),m)=0;
                end
            end
        end
    end

end
% Each row in the F matrix should now have no more than one non zero number (or all zero).
% At each timestep the option values are summed, discounted back to the
% first periode and averaged.

temp_value=0; %temporary variable to hold the option value
for j=2:perioodes
    temp_value=temp_value+exp(-diskont*(j-1))*sum(F(:,j))/simulations;
end
%either you exercise today or you wait.
temp_value=max(temp_value,exercise_meanrev_twofactor(X(1,1),epsilon(1,1),E,K,r,kappa,alfa,mu,S
S,opcost,insure,clea,certif,t1,t2,invest));
%V=exercise_meanrev_twofactor(X,epsilon,E,K,r,kappa,alfa,mu,SS,opcost,insure,clea,certif,t1,t
2,invest);
value{1,1}= F;
value{1,2}=temp_value; %returning the option value

```

```

%-----internfunksjoner-----
%defining a function for the value of waiting
function sub_l=wait(S,a,b,c,d)
global q
sub_l=(a+b*S/q+c*(S/q)^2+d*(S/q)^3);

function
value=exercise_meanrev_twofactor(X,epsilon,E,K,r,kappa,alfa,mu,SS,opcost,insure,clean,certif,t
1,t2,invest)
%-----
%| Author:   Jørgen T. Foss
%| Date:     Fall 2003
%| Version:  Final
%| Description:  this function returns the exercise value at the given X|
%|              and epsilon.
%-----
% X
% epsilon
% E is the equivalent operation time in hours per year
% D is the capacity in MW
% r is yearly interest rate in %
% kappa is yearly mean reversion for short term process
% alfa is price of risk in NOK/MWh
% SS is the long term expected price in NOK/MWh
% D is cost of operations in NOK/MWh
% mu is drift in long term process in NOK/MWh per year
% t1 start of operations in years
% t2 end of operations in years
% invest investment i GNOK

% uses some dummy variables
E1=(exp(-t1*(r+kappa))-exp(-t2*(r+kappa)));
E2=(exp(-t1*r)-exp(-t2*r));
E3=((r*t1+1)*exp(-t1*r)-(r*t2+1)*exp(-t2*r));

c1=K*E/(r+kappa)*E1;
c2=K*E/r*E2;
c3=-K*E*alfa/(r+kappa)*E1;
c4=K*E*(SS+alfa)/r*E2;
c5=K*E*mu/r^2*E3;
c6=-K*E/r*(clean+certif)*E2;
c7=- (opcost+insure)/r*E2;

temp=(c1*X+c2*epsilon+c3+c4+c5+c6+c7)*1e-9; %returns in GNOK
value=temp-invest;%returns in GNOK

function value=mcAMB(start,alfa,sigma,simulations,perodes,random1,random2,correl)
%-----
%| Author:   Jørgen T. Foss
%| Date:     Fall 2003
%| Version:  Final
%| Description:  a function that simulates ABM for n=perodes 'perodes'|
%|              and m=simulations 'simulations'
%-----
% start is the initial value of the process
% alfa is the per periode value of the drift
% sigma is the per periode value of standard deviation
% simulations is the number of simulations
% perodes is the number of periods
% random1 is a matrix(simulations, perodes-1) containing random normal
% distributed numbers with mean=0 and stdev=1 for the ABM process (this
% process)
% random2 is a matrix(simulations, perodes-1) containing random normal
% distributed numbers with mean=0 and stdev=1 for the meanreverting
% process(the other process)
% correl is the correlation between the meanreverting and ABM process

% defining the matrix to hold the function value
temp=zeros(simulations,perodes);

%doing the simulations
for i=1:simulations
    for j=1:perodes
        if j==1
            temp(i,1)=start;
        else

```

```

        temp(i,j)=temp(i,j-1) + alfa + sigma*(correl*random1(i,j-1)+random2(i,j-1)*sqrt(1-
correl^2)); %the actual simulation
    end
end
end
value=temp;

function value=mcMeanRev(start,alfas,kappa,sigma,simulations,perioes,random)
%-----
%| Author:  Jørgen T. Foss          |
%| Date:    Fall 2003              |
%| Version: Final                  |
%| Description:  a function that simulates a mean reverting process for |
%|               n=perioes 'perioes' and m=simulations 'simulations' |
%-----
% start is the initial value of the process
% alfas is the value to which the process mean revert
% kappa is the per perioe value of mean reversion
% sigma is the per perioe value of standard deviation
% simulations is the number of simulations
% perioes is the number of periods
% random is a matrix(simulations, perioes-1) containing random normal
% distributed numbers with mean=0 and stdev=1

% defining the matrix to hold the function value
temp=zeros(simulations,perioes);

%doing the simulations
for i=1:simulations
    for j=1:perioes
        if j==1
            temp(i,1)=start;
        else
            temp(i,j)=temp(i,j-1) + alfas*(1-exp(-kappa)) + (exp(-kappa)-1)*temp(i,j-1) +
random(i,j-1)*sigma*sqrt(1/(2*kappa)*(1-exp(-2*kappa))); %sqrt(1/(2*kappa)*(1-exp(-
2*kappa))); %sqrt(1-exp(-2*kappa*(perioes-j))); %the actual simulation
        end
    end
end
value=temp;

function value=random(simulations,perioes)
%-----
%| Author:  Jørgen T. Foss          |
%| Date:    Fall 2003              |
%| Version: Final                  |
%| Description:  this function returns a (simulations)X(perioes-1) |
%|               matrix with random normal distributed numbers with |
%|               mean=0 and stdev=1 |
%-----
value=randn(simulations,perioes-1);

```