

Robustness Analysis of an Aluminium Smelter with Operational Flexibility using Least Squares Monte Carlo



NTNU – Trondheim
Norwegian University of
Science and Technology

Eivind Fossan Aas and Sven Henrik Andresen

Department of Industrial Economics and Technology Management
Norwegian University of Science and Technology

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Stein-Erik Fleten

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Abstract

Aluminium smelters are important cornerstone businesses that are exposed to a wide range of risk factors, and have been under strong pressure the last few years due to oversupply and low aluminium prices in the market. Under unfavourable market conditions temporary or permanent shutdowns of a smelter may help limit losses, and a valuation of a smelter should incorporate this optionality.

In this thesis we model such an aluminium smelter with operational flexibility and analyse to what extent, and under which conditions this operational flexibility is most value adding. We apply a modification of the least squares Monte Carlo method to solve the combined optimal control and valuation problem numerically. The aluminium price, electricity prices and relevant exchange rates are treated as correlated mean reverting stochastic variables. Since the model is based on cash flows from a generic smelter, it is straightforward to evaluate the profitability and draw general conclusions based on the results. Especially the choice of electricity sourcing is an important aspect as it is a major input cost, and choosing the right type of electricity contracts can strongly impact the profitability as well as the probability of having to shut down the smelter over its lifetime.

We found that adding temporary shutdown options when already having permanent shutdown options is value adding, and of highest value for expected aluminium mean prices where permanent shutdown options go from being in the money to out of the money. The level of this region was found to be dependent on the length of pre-purchased electricity contracts. For long-term contracts, the region is centered around higher expected mean levels of the aluminium price than for short-term contracts. However, the value added was found to be higher for short-term contracts.

Finally, we found that having full operational flexibility compared to no flexibility is most value adding for long-term electricity contracts. The slope for the decrease in probabilities of permanent shutdowns with respect to an increasing mean of the aluminium price was found to be steeper for short-term contracts. Correspondingly, we found that the range of mean aluminium prices for which permanent shutdowns are likely is wider for the long-term contracts.

Preface

This thesis was written as a part of our master's degree at the Department of Industrial Economics and Technology Management (IOT) at the Norwegian University of Science and Technology (NTNU). We also have experience from the Department of Electric Power Engineering and the Department of Computer and Information Science at NTNU. The combined knowledge from all three departments mentioned above was utilized to a large extent in order to study the complexity of option valuation and risk management in the aluminium industry. The project thesis will be the basis for a master's thesis to be written in the spring.

We would like to express a special appreciation and thanks to our supervisor, professor Stein-Erik Fleten at IOT, for the assistance and guidance offered. His participation, constructive criticism and in-depth knowledge in among other topics, option pricing, price forecasting and commodity markets was indeed of high importance in order to complete the thesis. We also want to acknowledge the participation from professor Sjur Westgaard at IOT, who assisted us in issues regarding empirical finance.

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Sven Henrik Andresen

Eivind Fossan Aas

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Chapter 1

Introduction

Aluminium production is a classic industrial process, in which a smelter transforms alumina and carbon into aluminium through a very power-intensive electrolysis process. A convenient way to look at the profitability of a smelter is thus to regard it as a spread option between the volatile aluminium and power prices. Power is a dominating production cost, and reliable access to power is critical (Øye and Sørli, 2011). Access to energy is therefore an important aspect in deciding where to locate an aluminium smelter. Smelters are thus typically constructed close to reliable and cheap power sources, and often take the role of being cornerstone businesses in their respective districts due to labour demands. This introduces a whole set of political aspects when the aluminium producer considers and decides on future strategies as e.g. secure workplaces. A proper valuation of an aluminium smelter to be used as basis for decision-making is therefore of high importance both in business and social terms. Especially an evaluation of the risk of permanent or temporary shutdowns is of high importance, as the consequences of such outcomes are undesirable from a socioeconomic point of view. However, including all considerations, such as political issues, in the valuation of a smelter would make the model highly subjective, and in many cases it would be very hard to derive realistic estimations of the monetary impacts from such considerations. The socioeconomic aspect of running an aluminium smelter is thus considered to be a motivation for precise valuations rather than part of the valuations. A static approach to valuating a smelter does not encapsulate the value of operational flexibilities that may be available to management, such as temporary shutdown options. Using the static approach thus produces less precise results of the value of a smelter. An alternative approach is to use real options valuation which encapsulates the full optionality, produces a more precise value estimate and derives an optimal operating policy. Being able to properly evaluate the option to temporarily shut down operations for a few years may in fact reduce the risk of

a permanent shutdown, which is the worst case outcome in local communities. Since the value of the smelter may appear to be higher when using a real options approach, rather than a static NPV approach, the risk of all shutdown types may be reduced by providing management with a more accurate decision basis.

The literature on real options is extensive. Dixit and Pindyck (1994) were the first authors to publish a textbook devoted to real options theory. They characterized the investment decision as partially or completely irreversible, subjective to uncertainty over future awards and time dependent. As opposed to the static net present value approach, Dixit and Pindyck argued that the real options approach to value an investment decision can capture the value of being able to delay investments, which comes from gaining more information about the market conditions before investing. The investment opportunity is thus not regarded as a now-or-never opportunity, but as a call option with strike price equal to the investment cost and maturity equal to how long you may postpone the investment. Another important characteristic of the investment decision described by Dixit and Pindyck (1994), is that the option to close down a non-profitable plant could be value adding, which lowers the investment threshold, and should therefore be included in the capital budgeting process. This option has the characteristics of a put option with strike price equal to the saved costs and a potential salvage value.

An extension to the single option approach is discussed by Kulatilaka and Trigeorgis (1994), which is relevant to the smelter problem in terms of including the optionality to switch between multiple operating modes. The authors propose a general method to value a project with options to switch between alternative technologies or operating modes. The optionality can thus be considered as a strip of call options on switching operating modes, with a strike prices equal to the switching cost. The valuation problem studied has one stochastic continuous state variable, and discrete-time approximations (binomial lattices or Markov chains) are used in order to define points in time where switching decisions are undertaken. This method will however be computationally inefficient when modelling an aluminium smelter, where we have a multivariate problem to be solved, due to an exponential increase in possible states.

In fact, most capital budgeting decisions are dependent on multiple state variables, studied by Boyle (1977). Boyle develops a Monte Carlo simulation method to obtain numerical solutions to option valuation problems where the expected payoff of the underlying asset depends on multiple state variables. Cortazar and Schwartz (1998) later use the Monte Carlo approach in a real option study to evaluate an oil field investment opportunity. They define a two-factor stochastic model and use the Monte Carlo method to determine the optimal time

of investment, that is the time of investment that maximises today's value of the payoff from the investment opportunity. Even though the Monte Carlo approach allows us to describe the payoff of an underlying asset dependent on multiple stochastic variables, the approach becomes biased in a real option case where the underlying asset has the switching optionality discussed by Kulatilaka and Trigeorgis (1994). This is because the original Monte Carlo approach introduced by Boyle (1977) is a forward-looking technique.

Brekke and Øksendal (1994) redefine the optimal switching problem as a generalized impulse control problem in order to give a mathematical proof that optimal entry and exit strategies exist. Under the assumption that the state of the system is a stochastic process, they optimize the timing of when to open and close a multi-activity project, called an impulse control, given the cost of opening, operating and closing the activities. The optimal impulse control is then used to derive the value of the option to switch operating modes.

The least squares Monte Carlo (LSM) approach is a modification of the Monte Carlo method, first introduced by Carriere (1996), and later used by Tsitsiklis and Roy (2001) and Longstaff and Schwartz (2001). This method applies the concept of Monte Carlo, but uses least squares regression to determine the continuation value at each time step. The regression is used to approximate the continuation value of keeping the option alive for one more time step, thus penalising the unrealistic assumption that one knows what scenario has materialised. Values of the state variables at the current time step are used as explanatory variables in the regression, and the continuation values of the next time step are regressed on these. Gamba (2003) later extends this model by applying the LSM method in real options problems. Hence, he defines an extension to Longstaff and Schwartz (2001) by studying investment projects with embedded switching options using the optimal impulse control introduced by Brekke and Øksendal (1994). The valuation technique used in this thesis is based on the method introduced in Gamba (2003), since it allows us to use backwards dynamic programming to solve the switching option problem. The method also allows us to describe the payoff of the underlying asset using multiple correlated stochastic variables, which makes it suitable for the aluminium smelter case.

Bastian-Pinto et al. (2013) is another paper of high relevance. They study the effects of operational flexibility for the specific case of an aluminium smelter. Their approach is to use a Monte Carlo method to estimate the value of a smelter with flexible choices regarding power sourcing, and with embedded temporary shutdown and restart options. The paper includes a thorough description on how to model the spot electricity and aluminium price using a mean reverting process with jumps.

The problem studied in this thesis is similar to the one in Bastian-Pinto et al. (2013),

but with some important differences. Rather than a bundle of European options, we use embedded American options to value the aluminium smelter. The options, referred to as switching options, are valued by applying the LSM approach, using backwards dynamic programming and multivariate regressions to estimate the continuation values at each time step. The latter is an important extension to Bastian-Pinto et al. (2013) where they assume perfect foresight.

The dynamics of the options regarding operational flexibility are defined by consulting with industry sources, thus switching costs and time constraints are carefully chosen in order to realistically estimate the value of a generic smelter. As opposed to Bastian-Pinto et al. (2013) we also include a third unique state in which the smelter can shut down permanently, thus differentiating between temporarily and permanently shut down. We will evaluate the market conditions for which the temporary shutdown options are value adding. For power sourcing, we allow flexibility in contract lengths and currencies, and discuss how the choice of power sourcing affects the probability for shutdowns as well as risk profile of the smelter under different market conditions. Hence, we believe that the insight offered in this paper will not only be helpful in terms of smelter valuation, but can also give valuable input regarding risk management in the aluminium industry. Finally, the paper offers an up-to-date overview of the aluminium value chain, and important market characteristics for readers not familiar with the aluminium market.

The paper is organized as follows. Chapter 2 will give an introduction to the value chain in aluminium production and the aluminium market, and introduce some valuation techniques known to be applied by industry players. In chapter 3 we develop a model to value the smelter with switching options by using the LSM method, while in chapter 4 we evaluate the results of solving the latter and analyse the probability of smelter shutdowns in varying market conditions. We conclude in chapter 5 by discussing our approach and highlighting possible extensions and improvements for further work.

Chapter 2

Market and Institutional Context

This chapter starts by giving a brief introduction to the value chain of aluminium producers. Section 2.2 describes a general mathematical model for the cash flows of a generic smelter, while a more specific model for the case of own power sourcing is formulated in 2.3. Further on, in sections 2.4, 2.5 and 2.6 different closure triggers are evaluated based on NPV analysis of cash flows that are adjusted by applying the respective closure triggers in different scenarios. The analysed closure triggers are known to be applied by current industry players. An average of the NPV calculated in each scenario is used as proxy for the true value of the smelter when comparing different closure triggers.

2.1 Aluminium producer value chain

Following is up-to-date business insight for readers not familiar with the aluminium market. Three different inputs are required for aluminium production; alumina, electricity and carbon. Alumina is the direct base for aluminium and is refined from bauxite, a mineral that contains about 15-25% aluminium. It is mostly found several meters underground in a belt around the equator (Norsk Hydro, 2012). After recovery of bauxite the mineral is transported to crushing or washing plants before it is processed into aluminium oxide, commonly known as alumina. Bauxite extraction requires a large logistics network as well as high investment and operating costs. Mining of bauxite has become multinational and large aluminium producers tend towards complete vertical integration. Aluminium producers often have their own mining facilities or engage in joint ownerships with mining companies (Garen et al., 2009). Bauxite is heterogeneous in terms of chemical characteristics based on its origin. In addition bauxite is bulky in nature. Therefore, alumina refineries are often constructed close to and dedicated to specific areas of bauxite mining. The coun-

tries with the largest production of bauxite are Australia (30%), China (18%), Brazil (13%) and Indonesia (12%) (U.S. Geological Survey, 2013). However, in January 2014 Indonesia banned bauxite exports in order to motivate investments in domestic aluminium smelters. This could potentially have some impact on the global market for bauxite, as China is a net importer of bauxite, mainly from Indonesia. Thus global prices of bauxite could strengthen somewhat due to lower supply (Norsk Hydro, 2014).

Carbon accounts for about 13% of the total production cost of primary aluminium (Garen et al., 2009), and is used for the cathodes and anodes in the electrolysis step of aluminium production described later in this section. It is common for aluminium producers to own carbon electrode plants close to the smelter. The usage of carbon electrodes does lead to carbon emissions and certain countries have introduced a tax on carbon emissions, giving local producers a competitive disadvantage.

Electrical power stands on average for one third of the production cost of primary aluminium, but may be a source of competitive advantage for some producers. The average cost per mt produced aluminium can vary from \$400-1,000 between industry players (Garen et al., 2009). Aluminium producers have two options for sourcing of electricity; they can enter into long-term commitments with electricity producers or invest in power plants. Often, industry players with long-term commitments to electricity producers pay a lower price than players with short-term commitments. Despite investing in power plants being considered a capital-intensive strategy, several European aluminium producers own power generating assets since electricity is a dominating cost. In addition, to soften the effect of electricity price spikes aluminium producers also commonly trade in energy derivatives.

The production process of aluminium goes as follows. In a metal plant alumina is processed into aluminium using the Hall-Héroult process. In this process alumina is dissolved into molten cryolite and undergoes an electrolytic reduction to obtain aluminium. The Hall-Héroult process is extremely energy intensive, as a direct current of 150 to 250 kA is necessary to obtain the electrolytic reduction (Harton, 2003). The process takes place in a bath of hot cryolite (around 960°C), hence access to reliable power sources is a necessity to ensure a high temperature in the bath at all times (Dubal Aluminium, 2014).

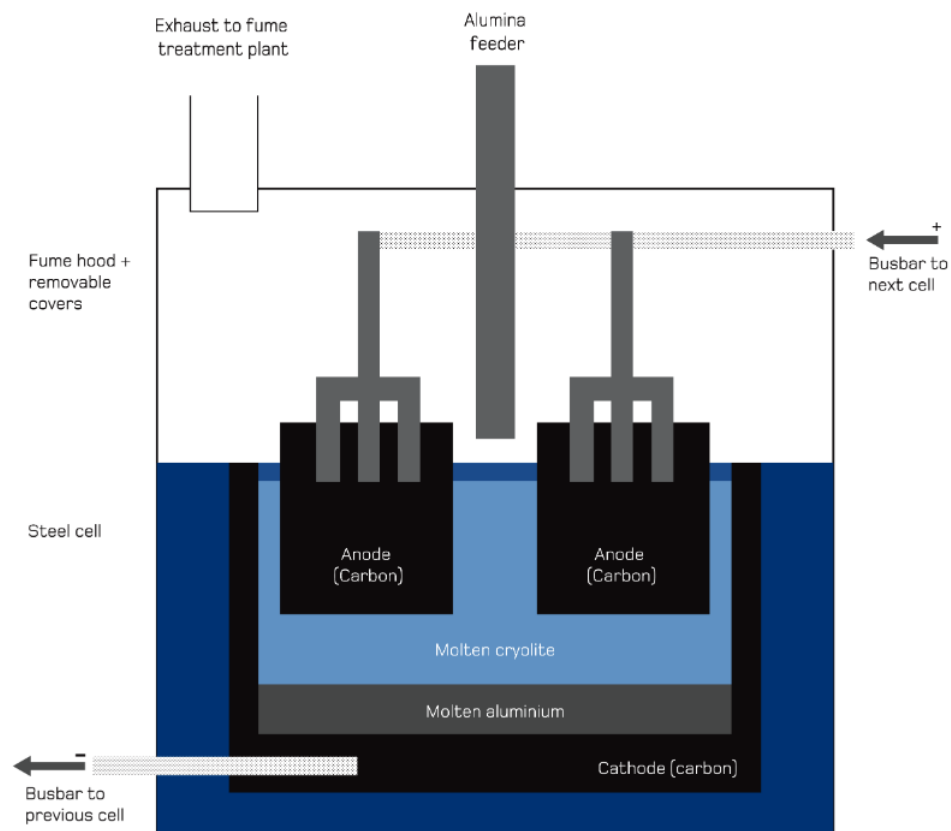


Fig. 2.1 Illustration of the Hall-Héroult process (Dubal Aluminium, 2014)

After the molten aluminium is extracted from the smelter, the aluminium is placed in large furnaces before being casted into other products. In the furnaces, the pure aluminium holds a temperature higher than 700°C while it is alloyed by combining it with other elements to further strengthen the material. The metal is then casted into different products specified by the end user. This final step is done in a casthouse.

Producing aluminium is considered continuous, meaning that once the smelter plant is operating, it must continue to operate at all times in order to maintain a high temperature in the electrolytic baths. Short interruptions in the production process could potentially damage and reduce the lifetime of the pots due to cooling cracks in the cathode (Øye and Sørli, 2011). Aluminium producers do, however, have the optionality to shut down the smelter for longer time periods in cases of unfavourable market conditions, but restarting the smelter entails high costs.

In recent years, the aluminium market has been experiencing low prices due to oversupply. As a consequence, high-cost producers have been forced to shut down production. As optimism returns to the market the producers speculate at which price level it is economi-

cally viable to restart production again. Thus, developing a composite model for valuating a smelter with operational flexibility is of high importance to evaluate the risk of shutting down or starting up aluminium smelters.

A final remark is that we in this thesis always will refer to the unit USD per mt produced aluminium, and value the smelter on this basis rather than based on an aggregate amount of USD derived from total amount of installed capacity. We thus disregard economies of scale. To see why this is a reasonable approach it is important to understand the rationale behind the geographic location of an aluminium smelter, which is mainly motivated by variations in energy prices and not the prices of other inputs as alumina or bauxite (Nappi, 2013). This is despite the fact that alumina and electricity account for approximately equal fractions of the total production costs. The reason for this is that the cost of electricity varies much more between different geographical locations than the cost of other inputs, as e.g. alumina for which the cost is approximately at the same level regardless of geographic location (Center for European Policy Studies and Economisti Associati, 2013). In fact, according to Nappi (2013) and the latter, energy cost is the major determinant of international differences in aluminium production costs. In addition, electricity together with other costs as alumina and carbon are approximately linearly related to the amount of aluminium produced. Thus, having optimised the location of the smelter to minimise energy costs, the remaining economies of scale are mainly related to the initial capital investment since indirect variable costs only constitute a small fraction of total costs (Center for European Policy Studies and Economisti Associati (2013)). Having already invested, the depreciated capital cost is of less interest when comparing relative cost positions. One would rather look at the direct costs, which by the arguments above do not exhibit strong economies of scale. Modelling the smelter on a basis of USD per mt produced aluminium is thus relevant.

2.2 General cash flows of a smelter

In order to study the problem of an aluminium smelter with permanent and temporary shut-down options, a description of the smelter cash flows under different scenarios must be derived. The following section introduces such a description based on the cash flows of a generic aluminium smelter located in Norway, with cash flows stated in terms of USD/mt aluminium produced. Note that the parameters and variables presented in this section are limited to the most important ones, while additional parameters and variables can be found in appendix A.

In reality smelters are assumed to have a very long lifetime, but for simplification of

the calculations we model the smelter within a time frame of 20 years. After 20 years we assume that the smelter continues operating forever, but estimate the value of this as a perpetuity of the cash flow received in the 20th year. Further on we assume that the investment has already been made and that the operating mode for the following year is decided at the beginning of each year. Scenarios for the stochastic correlated input variables are generated in order to create an outcome space to be used for the numerical solution. Further on, as electricity is an important input factor and a dominant operating cost, we assume management has several choices when deciding on the type of contracts for electricity sourcing. Long-term contracts may offer hedging opportunities against spikes in the spot electricity price, thus reducing the risk of closing the aluminium smelter when electricity prices are high. Locking in electricity prices may, on the other hand, increase the risk of shutting down if aluminium prices drop. To evaluate how this choice may affect the value of a smelter as well as shutdown probabilities, we assume management can choose between pre-purchased electricity contracts of different lengths and in different currencies. More specifically the contracts in this problem can have lengths of 1 year, 5 years, 10 years or 20 years, and may be in the currencies NOK, EUR or USD. The dynamics of such contracts are that for the 5-, 10- and 20-year contracts the aluminium producer agrees to buy a certain amount of power for a fixed price over the defined time period. This fixed price is the long-term power forward price. Base currency for power is assumed to be EUR/MWh so for contracts in NOK or USD the producer must pay the EUR/MWh base price multiplied with the relevant exchange rate that materialises. When considering contracts with length of one year the producer buys power in the spot market and only the EUR currency is considered, as this is an approximate of the spot price that is stated in EUR/MWh. In addition we consider the case in which the smelter owns a power asset from which it sources electricity. This choice of electricity sourcing will be referred to as contract length *Own sourcing* and is in currency NOK.

Thus we have the following sets:

$$\begin{aligned}
 \mathbf{N} &= \mathbf{time} [1 : T] \\
 \mathbf{S} &= \mathbf{scenarios} [1 : M] \\
 \mathbf{L} &= \mathbf{contract length} [1, 5, 10, 20, \textit{Own sourcing}] \\
 \mathbf{C} &= \mathbf{contract currency} [\textit{NOK}, \textit{EUR}, \textit{USD}]
 \end{aligned}$$

The letters t , s , l , c will be used to index time, scenario, contract length and contract currency respectively.

We assume here that the smelter is exposed to two different electricity prices, three different exchange rates and finally the aluminium price. All of the above prices are assumed

to follow some time series model calibrated to historical data (refer to section 3.5). We define the following variables to be used in the model:

- $l_{s,t}$ = spot aluminium price, London Metal Exchange, at time t in scenario s
- $y_{s,t}^1$ = electricity price 1-year (EUR/MWh) at time t in scenario s [Price for a 1-year electricity forward contract, used as proxy for the spot electricity price]
- $y_{s,t}^3$ = electricity price 3-year (EUR/MWh) at time t in scenario s [Price for a 3-year electricity forward contract, used as proxy for the long-term electricity price]
- $x_{s,t}^N$ = exchange rate USD/NOK at time t in scenario s
- $x_{s,t}^E$ = exchange rate USD/EUR at time t in scenario s

The aluminium price is based on the official price from the London Metal Exchange (LME) and we will later use LME as an abbreviation for aluminium price. The aluminium variable will be explicitly included in the simplified cash flow description, while the other variables will only be implicitly included through help variables in order to avoid making the description too complex to interpret. These help variables are:

- $o_{s,t}$ = net operational expenses from electrolysis and LME casthouse at time t
- $p_{s,t,l,c}$ = power contract price (USD/mt produced aluminium) at time t in scenario s with contract length l and in currency c
- $tax_{s,t,l,c}$ = tax paid (USD/mt produced aluminium) at time t in scenario s with contract length l and in currency c
- $q_{s,t,l,c}$ = remaining exposure (USD/mt production) from power contracts at time t , in scenario s , with contract length l and in currency c

Finally, the following parameters are used:

- Raw = raw materials (USD/mt prod. aluminium)
- ρ = weighted average cost of capital (%)
- O^{temp} = yearly operating cost for a temporarily shut down smelter (USD/mt prod. aluminium)
- $K^{direct perm}$ = cost of switching mode from operating to permanently shut down (USD/mt prod. aluminium)
- K^{temp} = cost of switching mode from operating to temporarily shut down (USD/mt prod. aluminium)
- K^{perm} = cost of switching mode from temporarily shut down to permanently shut down (USD/mt prod. aluminium)
- $K^{operate}$ = cost of switching mode from temporarily shut down to operating (USD/mt prod. aluminium)

Having introduced the above notation, we can now state the expression for the unadjusted cash flow (USD/mt) from a smelter at time t in scenario s , under power contract length l and with power contract in currency c , $FCF_{s,t,l,c}^{unadjusted}$, as:

$$\begin{aligned}
 & l_{t,s} \\
 & - Raw \\
 & - P_{s,t,l,c} \\
 & - o_{s,t} \\
 & \hline
 & = PretaxCF_{s,t,l,c} \\
 & - tax_{s,t,l,c} \\
 & \hline
 & = FCF_{s,t,l,c}^{unadjusted} \\
 & \hline
 \end{aligned}$$

This FCF will be used in upcoming subsections in order to value the smelter under different closure triggers assumptions.

2.3 Cash flows with power sourcing

Electricity for the electrolysis process is, as earlier described, one of the main sources of production costs for an aluminium producer. It is therefore of interest to study the case where the aluminium producer owns a power asset from which electricity is sourced. The cash flows related to such power assets are highly industry specific and will therefore not be discussed in detail in this thesis. However, we will elaborate on a regulatory requirement for hydropower assets located in Norway.

By regulation, a hydropower asset located in Norway is required to sell a fixed share of its production to the municipality for a fixed price that is lower than the spot price (Norwegian Water Resources and Energy Directorate, 2001). Due to this, for each MWh produced by the power asset to supply the smelter, a certain percentage of the production must be sold externally to a regulated price. If the power asset only produces the amount of MWh that is required for the smelter, a way to fulfil its regulatory requirements is to buy the remaining required amount of electricity in the spot market and sell it for the regulated price. The owner of the power asset thus incurs a cost to meet the regulation, and this cost must be included in the contract price for the smelter. Refer to appendix A for the extensive model

with all details on how to derive the smelter cash flows in the case of own power sourcing.

2.4 Value of smelter without optionality

When there is no option to permanently shut down the smelter it is straightforward to calculate the NPV of the smelter in each scenario, as the cash flows are just the unadjusted free cash flows calculated in section 2.2 .

$$NPV_{s,l,c}^{noclosure} = \sum_{t=1}^{20} \frac{FCF_{s,t,l,c}^{unadjusted}}{(1+\rho)^t} \quad \forall s, \forall l, \forall c$$

The estimated NPV of the smelter is then just the average of the NPV across all scenarios.

2.5 Value of smelter with cumulative margins trigger

Cumulative margins trigger is an approach that investigates whether unadjusted free cash flows have been negative the two previous years and will be negative the next two years in the current scenario. In other words, if the smelter experiences negative cash flows in four consecutive years during a given scenario, the smelter is permanently shut down and the cash flow of the upcoming year is not received. In other words, if the following condition is true, the smelter shuts down in year τ :

$$\prod_{\tau=t-2}^{t+1} 1_{\{FCF_{\tau} < 0\}} = 1 \quad (2.1)$$

Upon permanent shutdown the company incurs a shutdown cost, and is further on assumed to sell remaining exposure from power contracts to the spot price at the time of the shutdown.¹ The cash flow in the closure year is then adjusted to include the sale of remaining power exposure and the shutdown cost that is incurred. Having calculated the adjusted free cash flows, $FCF_{s,t,l,c}^{adjusted}$, it is straightforward to calculate the NPV of the smelter in each scenario:

¹In the case of power sourcing the remaining exposure is the NPV at time t of all future after-tax stand-alone cash flows from the power asset. Refer to appendix A for calculations of remaining exposure in the case of own power sourcing

$$NPV_{s,l,c}^{c.trigger} = \sum_{t=1}^{20} \frac{FCF_{s,t,l,c}^{adj.c.trigger}}{(1+\rho)^t} \quad \forall s, \forall l, \forall c$$

The estimated NPV of the smelter is then just the average of the NPV across all scenarios.

2.6 Perfect foresight - permanent shutdown

Perfect foresight is an unrealistic closure trigger, since it assumes that one knows exactly how the current scenario will materialise and hence at any point in time can calculate the NPV of continuing operations. Comparing the NPV with the costs of shutting down (direct shutdown cost and remaining exposure) at any point in time, one makes the decision whether to continue operations or shut down. In the case of own power sourcing the NPV is only compared with the direct shutdown costs. In other words, the remaining exposure can be considered to be zero. For the perfect foresight method the smelter shuts down permanently if the following condition holds:

$$NPV_{s,t-1,l,c}^{table} < K^{direct perm} + q_{s,t,l,c} \quad \forall t, \forall s, \forall l, \forall c \quad (2.2)$$

Where we calculate the rolling NPV at each time step:

$$NPV_{s,t,l,c}^{table} = \sum_{i=t}^{20} \frac{FCF_{s,i,l,c}}{(1+\rho)^i} \quad \forall t, \forall s, \forall l, \forall c \quad (2.3)$$

Once again it is straightforward to calculate the NPV of the smelter in each scenario knowing the adjusted free cash flows under the *perfect foresight* approach:

$$NPV_{s,l,c}^{perf.foresight} = \sum_{t=1}^{20} \frac{FCF_{s,t,l,c}^{adj.perf.foresight}}{(1+\rho)^t} \quad \forall s, \forall l, \forall c$$

The estimated NPV of the smelter is then again the average of the NPV across all scenarios. Note that with the procedure above the value of the smelter is calculated in a forward-looking manner, hence it does not represent the true upper bound to the smelter value. Calculating this true upper bound would require a backwards dynamic-programming approach. In chapter 4 all references to perfect foresight refer to the true upper bound derived through backwards dynamic programming.

Chapter 3

Least Squares Monte Carlo Option Valuation

3.1 Least squares Monte Carlo option valuation

In the previous chapter, different closure triggers were applied and the free cash flows were adjusted according to what state of operation the respective closure trigger implied. There are several drawbacks with these methods, whereas the most significant drawback is that one assumes that one knows exactly which scenario has materialised. This is a bias independent of whether looking two years ahead or until the end of the period. In reality one has no way of knowing which scenario has materialised. Therefore one has to stick with expectations based on the current values of fluctuating variables as e.g. exchange rates. A more sophisticated way to evaluate a smelter with shutdown options is through a real options approach. As the complexity of the problem prevents us from deriving an analytical solution, a numerical method must be applied. One such method is the least squares Monte Carlo (LSM) method which we will apply in this thesis. In short terms, the procedure in the LSM approach is to use dynamic programming working backwards from $t=T$ to $t=0$, and for each time step compare the continuation value from operating the smelter with the value of exercising the option to shut down. The key idea of the LSM approach is to estimate the continuation value by regressing the discounted continuation value of the next time step on the current state of the stochastic variables. This means that for each time step the approximation of the continuation value at time t is the expected continuation value given the values of the stochastic variables at time t . A clear advantage of using this technique is that one avoids perfect foresight, and given a sufficiently high number of scenarios, the approximated value is a fair expectation as it is based on an exhaustive sample space. In

addition, one is able to capture the full value of the available operational flexibility in the valuation of the smelter.

3.2 Operational flexibility in terms of permanent and temporary shutdown options

In chapter 2 the only operational flexibility available to management was the option to close down operations permanently. When applying the LSM method, a more realistic case may be considered in which management has full flexibility regarding operating modes. A way to look at this combined optionality is to note that the smelter has three different operating modes; fully operating, temporarily shut down or permanently shut down. This implies that in order to find the NPV of the smelter with operational flexibility one must also determine the optimal operating mode at any given time. A technique for solving this combined problem, commonly referred to as a problem of switching options, is presented in Brekke and Øksendal (1994), where two interdependent problems are solved; the *Valuation Problem* and the *Impulse control problem*. An extension of the former paper is to apply the findings in a least squares Monte Carlo setting. This is done by Gamba (2003), whose method and notation will be used as framework for the solution of the problem studied in this thesis. Problems with a similar structure have also been discussed by Mazières and Boogert (2013), who study the use of LSM valuation for gas storage. In this section, a general model is presented before a model specific to our problem is formulated.

Let there be three operating modes, where z denotes the general operating mode and $Z = \{\text{operating, temporarily shut down, permanently shut down}\}$. The different operating modes are described in table 3.1 and an example operating plan is illustrated in figure 3.1.

Operating mode	Description
Fully operating	Smelter is operating and owners receive the cash flows from sale of aluminium.
Temporarily shut down	Smelter is currently not operating. Owners receive the proceeds from sale of pre-purchased electricity and may have to pay some operating expenses. The work force has been laid-off and production may restart if favourable market conditions occur.
Permanently shut down	Owners have closed the smelter permanently and there is no optionality to restart operations. After paying the closure cost the smelter will not generate any future cash flows, but upon closure the owners receive today's value of the remaining amount of pre-purchased electricity.

Table 3.1 Explanation of operating modes

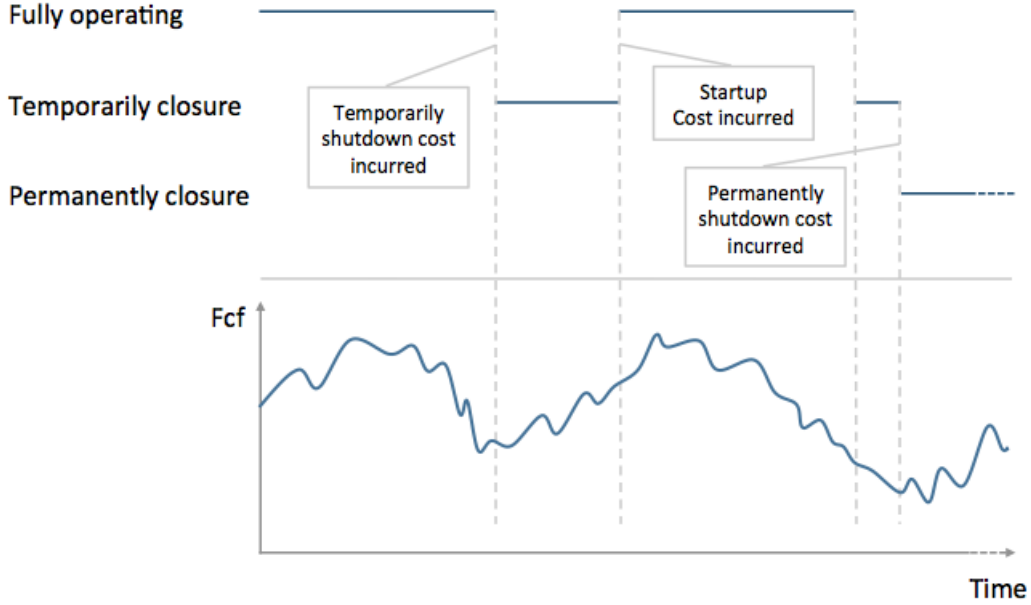


Fig. 3.1 Example of optimal operating policy in a given scenario

Figure 3.1 illustrates how shutdown and restart decisions are undertaken given the level of free cash flow from the aluminium smelter. Since the problem is multivariate with complex interdependencies, we cannot define one single cash flow threshold level for each operational decision, as in a single factor model.

Let $Y_t(t, X_t, Z_t)$ denote the state of the system at time t . An impulse control is defined by $w = \{\tau_1, \tau_2, \dots, \tau_k; \zeta_1, \zeta_2, \dots, \zeta_k\}$ where $\tau_n \leq T$ denotes an optimal switching time and ζ_n is the associated operating mode that is switched to at $t = \tau_n$, $n + 1 \leq k$, both based on the current system state $Y_t(t, X_t, Z_t)$. Note that k is the number of resulting switching times where $k \leq T$ and e.g. $k = 5$ means that there are five points in time where the operating mode is switched. We can now let W denote the set of optimal controls for the problem. If an impulse

control $w \in W$ is applied at Y_t then we get that $Y_t = Y_t^{(w)} = \begin{pmatrix} t \\ X_t \\ \zeta_n \end{pmatrix} \forall \tau_n \leq t \leq \tau_{n+1}$. The cost

of switching from z to ζ at time t and given state x is denoted by $K(t, x, z, \zeta)$ and is assumed to be strictly positive for every $z \neq \zeta$. In addition, the cheapest way to switch from z to ζ is to do it directly and not through another mode ζ' . That is, the cheapest way to go from operating to permanently shut down at time t is to do a permanent shutdown directly, and not stepwise through a temporary shutdown. This implies that $K_{z, \zeta}(t_n, X_{t_n}) \leq K_{z, \zeta'}(t_n, X_{t_n}) + K_{\zeta', \zeta}(t_n, X_{t_n})$. To take into consideration irreversible states, such as permanently shut down, we just set the switching cost from this state arbitrarily large. Let $\Pi(y)$ be the cash flow paid out at $y(t, x, z)$. The optimal switching problem given state $y = (t, x, z)$ is now the one finding

w such that the expected discounted total cash flow net switching costs is maximised. This is formulated as:

$$F(y) = F(t, x, z) = \max_{w \in W} \left\{ \mathbb{E}_y^* \left[\int_{i=t}^T e^{-\rho(i-t)} \Pi(Y_i^{(w)}) di - \sum_{n=1}^{\infty} e^{-\rho(\tau_n - t)} K(Y_{\tau_n}^w, \zeta_n) \right] \right\} \quad (3.1)$$

We have that $F(Y)$ is the value function at y with the corresponding Bellman equation formulated as:

$$F(y) = F(t, x, z) = \max_{\tau, \zeta_\tau} \left\{ \mathbb{E}_y^* \left[\int_{i=t}^{\tau} e^{-\rho(i-t)} \Pi(Y_i) di - e^{-\rho(\tau-t)} (K(\tau^-, X_\tau, z, \zeta_\tau) - F(Y_\tau)) \right] \right\} \quad (3.2)$$

We can now tackle the optimal switching problem. Given that we are in operating mode z , the net payoff of a transition to mode h can be written as $\Pi'_{z,h}(t_n, X_{t_n}) = \frac{\Delta t \Pi_h(t_n, X_{t_n})}{1+\rho} - K_{z,h}(t_n, X_{t_n})$ where $\Pi_h(t_n, X_{t_n})$ is the payoff from mode h and $K_{z,h}(t_n, X_{t_n})$ the cost of switching from mode z to h , e.g. cost of switching from operating to temporarily shut down (note that discrete discounting is used in our problem as opposed to continuous discounting in Gamba (2003)). Using the latter and expressing the optimal stopping time problem in discrete time, equation (3.2) can be rewritten to:

$$F_z(t_n, X_{t_n}) = \max_h \left\{ \Pi'_{h,z}(t_n, X_{t_n}) + \frac{1}{(1+\rho)^{\Delta t}} \times \mathbb{E}_{t_n}^* [F_h(t_{n+1}, X_{t_{n+1}})] \right\} \quad (3.3)$$

The intuition of equation (3.3) is that it is optimal to switch from mode z to mode h , $z \neq h$, at (t_n, X_{t_n}) if the expected continuation value of mode h plus immediate payoff from mode h exceeds the expected continuation value from staying in mode z plus the immediate payoff from mode z . By defining $\Phi_h(t_n, X_{t_n}) = \frac{1}{(1+\rho)^{\Delta t}} \times \mathbb{E}_{t_n}^* [F_h(t_{n+1}, X_{t_{n+1}})]$ to be the continuation value of mode h , an explicit decision rule for switching mode along a path ω is formulated

to:

$$\text{If } \left(\Pi'_{z,z}(t_n, X_{t_n}(\omega)) + \Phi_z(t_n, X_{t_n}(\omega)) < \max_{h \neq z} \left\{ \Pi'_{h,z}(t_n, X_{t_n}(\omega)) + \Phi_h(t_n, X_{t_n}(\omega)) \right\} \right)$$

Then

$$\tau = t_n \text{ and } \zeta(\tau, \omega) = \arg \max_h \left\{ \Pi'_{h,z}(t_n, X_{t_n}(\omega)) + \Phi_h(t_n, X_{t_n}(\omega)) \right\}$$

Otherwise

Switching time not updated

It is at this point that the least squares Monte Carlo method is applied. Since we want to avoid perfect foresight in the valuation, the continuation value, $\Phi_z(t_n, X_{t_n}(\omega))$, of the smelter along each path is estimated by using the values of a set of explanatory variables known at each point in time. There are five stochastic variables in the smelter problem described in this thesis, all which are implicitly included in the help variable $FCF_{s,t}$. Further on the remaining time of a power contract $\delta_{t,l}$ is an important input variable when determining whether to switch operating modes, as it explicitly defines the magnitude of the smelter's obligations from pre-purchased electricity contracts that must be fulfilled even in the case of a permanent shutdown. The latter together with the electricity spot price $y_{s,t}^1$ are used to determine the value of these obligations, later referred to as remaining exposure.¹ Finally, as pre-purchased electricity contracts are based on the 3-year electricity price $y_{s,t}^3$, quoted in terms of euros, we also include this and the USD/EUR exchange rate $x_{s,t}^E$ as explanatory variables.

Based on the above discussion we therefore choose $FCF_{s,t}$, $\delta_{t,l}$, $y_{s,t}^1$, $y_{s,t}^3$ and $x_{s,t}^E$ as explanatory variables in a multivariate linear regression performed at each time step to approximate the continuation value of a given operating mode at time t . Choosing the functional form of the regression is also a challenge as the function should resemble the shape of the value function. Rodrigues and Armada (2006), Carmona and Ludkovski (2010), Areal et al. (2008), Longstaff and Schwartz (2001) and Moreno and Navas (2003) argue that regressing on simple powers of the explanatory variables and cross products provide fairly accurate numerical results compared to other forms of the explanatory variables. This is, among other, because the least squares Monte Carlo algorithm only depends on the fitted value of the regression, and not on the correlation between the independent variables. An additional

¹Note that if the electricity spot price is high the remaining exposure may have a positive sign meaning that the smelter can sell pre-purchased electricity with a net gain. Evidently, such considerations are of relevance in the decision-making process.

argument is that that the method is fairly robust to the choice of basis functions, as using different polynomials of any degree yield highly similar results due to the fact that these polynomials are linear combinations of each other. Hence, we perform the regression on powers of the chosen explanatory variables, up to and including degree 2, with cross products. This yields the following regression equation for each type of contract currency, c , and contract length, l over a number of scenarios:

$$\begin{aligned}
 \Phi_{s,t} = & \beta_1^t \times FCF_{s,t} + \beta_2^t \times \delta_{t,l} + \beta_3^t \times y_{s,t}^1 + \beta_4^t \times y_{s,t}^3 + \beta_5^t \times x_{s,t}^E + \beta_6^t \times FCF_{s,t}^2 + \beta_7^t \times \delta_{t,l}^2 \\
 & + \beta_8^t \times (y_{s,t}^1)^2 + \beta_9^t (y_{s,t}^3)^2 + \beta_{10}^t (x_{s,t}^E)^2 + \beta_{11}^t \times FCF_{s,t}^3 + \beta_{12}^t \times \delta_{t,l}^3 + \beta_{13}^t \times (y_{s,t}^1)^3 \\
 & + \beta_{14}^t (y_{s,t}^3)^3 + \beta_{15}^t (x_{s,t}^E)^3 + \beta_{16}^t \times FCF_{s,t} \times \delta_{t,l} + \beta_{17}^t \times y_{s,t}^1 \times \delta_{t,l} + \beta_{18}^t \times FCF_{s,t} \times y_{s,t}^1 \\
 & + \beta_{19}^t \times FCF_{s,t} \times y_{s,t}^3 + \beta_{20}^t \times FCF_{s,t} \times x_{s,t}^E + \beta_{21}^t \times \delta_{t,l} \times y_{s,t}^3 + \beta_{22}^t \times \delta_{t,l} \times x_{s,t}^E \\
 & + \beta_{23}^t \times y_{s,t}^1 \times y_{s,t}^3 + \beta_{24}^t \times y_{s,t}^1 \times x_{s,t}^E + \beta_{25}^t \times y_{s,t}^3 \times x_{s,t}^E
 \end{aligned} \tag{3.4}$$

Since this is a multivariate regression with cross products, we need to calculate a large number of coefficients for each approximation. A more sophisticated way to do this could be to perform the regression using radial basis functions, a procedure described by Mazières and Boogert (2013), but this is left as a proposed further extension.

To initiate the LSM method, we define realistic continuation values for the last time step when $t = T$. In this thesis we use a relationship between the weighted average cost of capital and the cash flow in year T as an approximation of the continuation value when all options regarding operational flexibilities expire:

$$\Phi_z(T, X_T) = \frac{\Pi'_{z,z}(T, X_T(\omega))}{\rho}, \quad \forall z \tag{3.5}$$

Having defined the continuation values at the last time step, we can now use backwards dynamic programming and multivariate regression to estimate the value of the aluminium smelter at each time step from $t = T$ to $t = 0$. The discounted expected continuation values of keeping the smelter operating or temporarily shut down at time t in scenario s are easily calculated by $\beta^t \mathbf{R}^{t,s}$, where $\mathbf{R}^{t,s}$ is the matrix (25×1) with the values of the explanatory variables at time t in scenario s and β^t is the vector of the regression coefficients at time t .² To ensure that we do not get a biased approximation of the continuation values, we

²We actually do two regressions at each time step. The first regression is conducted in order to approximate the continuation value of an operating smelter while the second regression is conducted in order to approximate the continuation value of a temporarily shut down smelter.

generate the coefficients matrix, β^t , by regressing on a different set of simulated values for the explanatory variables. This means that we generate two sets of scenarios; an in-sample set of M' scenarios to generate the coefficient matrix via conducting multivariate regressions and an out-of-sample set with M scenarios for the explanatory variables used to estimate the continuation values. Hence, the $\mathbf{R}^{t,s}$ matrix will be the explanatory variables at time t in scenario s from the out-of-sample set of scenarios. The expected continuation values of keeping the smelter operating or temporarily shut down are then calculated by multiplying the coefficients generated from the in-sample set with the values of the explanatory variables from the out-of-sample set.

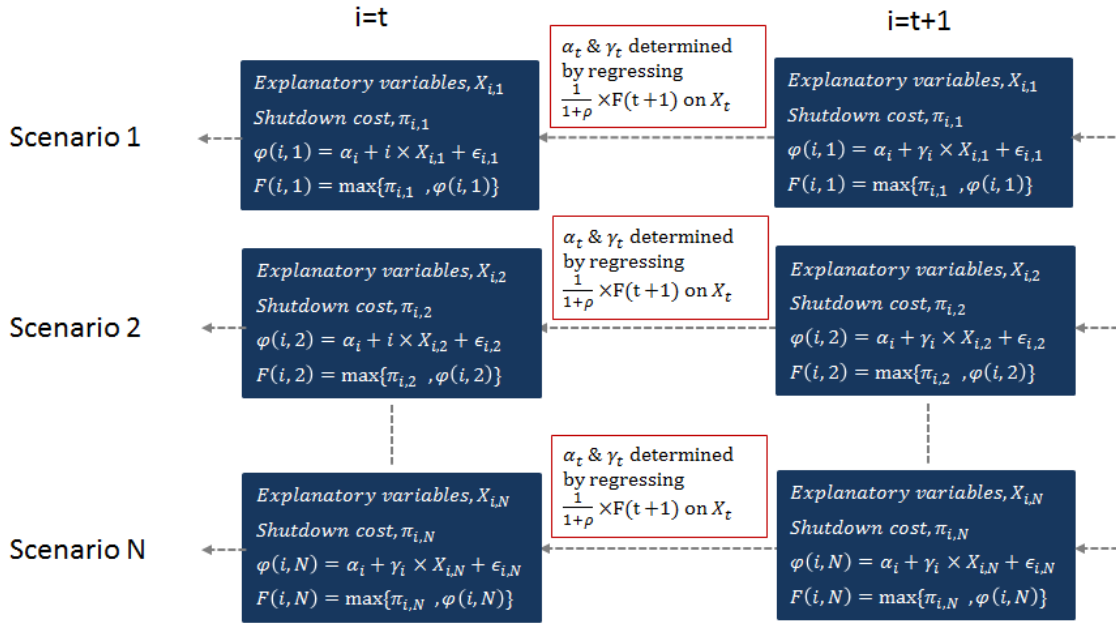


Fig. 3.2 Illustration of approximating the continuation value with the LSM method

By repeating the above procedure at every time step and for every scenario we can derive an optimal operating policy for the smelter. The continuation value, $\Phi_z(t_n, X_{t_n}(\omega))$, for any mode h is approximated by performing the multivariate regression defined by equation (3.4). Let $A_z(t, j, \omega)$ be the optimal cash flow at time j given that you are in mode z at time t along the path $\omega, (t, \omega)$. The recursive equation is then defined as:

$$A_z(t, j, \omega) = \begin{cases} \text{if } \tau(\omega) \neq t, & \begin{cases} A_z(t+1, j, \omega) & \text{for all } j = t+1, \dots \\ \Pi'_{z,z}(t, X_t(\omega)) & \text{for } j = t \end{cases} \\ \text{if } \tau(\omega) = t, & \begin{cases} A_\zeta(t+1, j, \omega) & \text{for all } j = t+1, \dots \\ \Pi'_{z,\zeta}(t, X_t(\omega)) & \text{for } j = t \end{cases} \end{cases} \quad (3.6)$$

Given that you are now at (t_n, X_{t_n}) the continuation value of mode h is:

$$\Phi_h(t_n, X_{t_n}) = \mathbb{E}_{t_n}^* \left[\sum_{i=n+1}^N \frac{1}{(1+\rho)^{t_i-t_n}} \times A(t_n, t_i, \cdot) \right]$$

The optimal policy found by the LSM method can be applied to estimate the smelter value. For each scenario, the value of the smelter is the discounted cash flows received by operating the smelter under the derived policy. Note that when calculating the estimated smelter value actual cash flows are used, as opposed to the approximated continuation values used when deriving the operational policy. After estimating the NPV of the smelter under each scenario, the smelter value is then approximated by averaging over all scenarios.

The recursive equation (3.6) is not directly intuitive, therefore a brief qualitative explanation is provided given that you are now in the state (t, X_t, z) . The main idea is to calculate the value of future payoffs net switching costs given an optimal policy at all future dates conditional on being in the current state. If t is not a stopping time and $j = t$, you get the payoff of being in mode z . When $j > t$, you get the optimal payoff according to (3.6), but with t increased by one time step. If t is a stopping time, you switch mode and get the immediate payoff from switching and the optimal payoff according to (3.6), only with t increased by one time step and with mode switched from z to ζ . In fact, the recursion works in a backward fashion, as you move down to the deepest recursion level before working your way back up.

We can now finish the formulation of a model for the smelter problem studied in this thesis by replacing parts of the equations above with problem specific variables. Let z_1 denote an operating smelter, z_2 a temporarily shut down smelter and z_3 a permanently shut down smelter.

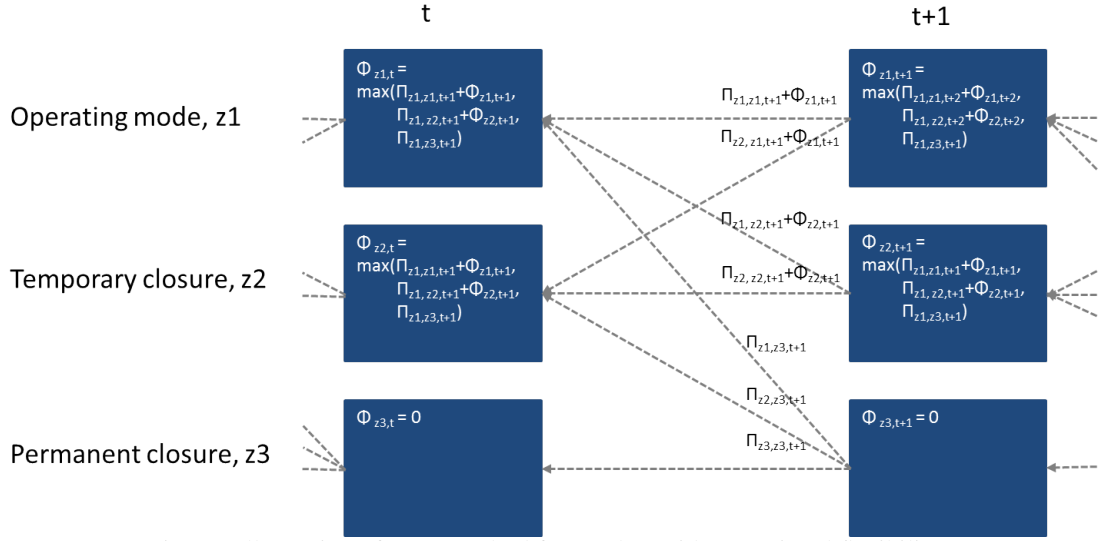


Fig. 3.3 Illustration of LSM method for smelter with operational flexibility

The payoffs, Π_z (USD/mt), when holding contract length l and contract type c fixed for the different operating modes in a given scenario s are:

$$\Pi(t, X_t, z_1) = FCF_{s,t}$$

$$\Pi(t, X_t, z_2) = -O^{temp} + (M \times y_{s,t}^1 - p_{s,t,l,c}) \times d, \quad \text{where } d = 0 \text{ if } t \bmod l = 0, \\ d = 1 \text{ otherwise}$$

$$\Pi(t, X_t, z_3) = 0$$

$$\Pi'_{z_1,z_1}(t_n, X_{t_n}(\omega)) = FCF_{s,t}$$

$$\Pi'_{z_1,z_2}(t_n, X_{t_n}(\omega)) = -(K^{temp} + O^{temp}) + (M \times y_{s,t}^1 - p_{s,t,l,c}) \times d \quad \text{where } d = 0 \text{ if } \\ t \bmod l = 0, \\ d=1 \text{ otherwise}$$

$$\Pi'_{z_1,z_3}(t_n, X_{t_n}(\omega)) = -K^{direct perm} + q_{s,t,l,c}$$

$$\Pi'_{z_2,z_3}(t_n, X_{t_n}(\omega)) = -K^{perm} + q_{s,t,l,c}$$

$$\Pi'_{z_2,z_1}(t_n, X_{t_n}(\omega)) = -K^{operate} + FCF_{s,t}$$

$$\Pi'_{z_3,z_2}(t_n, X_{t_n}(\omega)) = -\infty \quad \text{and} \quad \Pi'_{z_3,z_1}(t_n, X_{t_n}(\omega)) = -\infty$$

In addition to the payoffs defined above, we also limit the maximum number of time steps for which it is possible to stay temporarily shut down without shutting down permanently or reopening the smelter. As the smelter stays temporarily shut down, the cost of reactivating the smelter furnaces will with time increase to the point where reopening the smelter will no

longer be an option (Bastian-Pinto et al., 2013). In this thesis we have defined three years as a maximum limit to stay temporarily shut down. This means that the option to restart the smelter after a temporary shutdown will be an American option with maturity of three years, and will have a penalty equal to the cost of shutting down permanently if it is not exercised.

Equations (3.3) and (3.6), as well as the decision rule for switching mode presented above, apply to the specific smelter problem when the latter definitions of operating modes, \mathbf{Z} , and payouts, $\Pi(t, X_t, z)$ and $\Pi'_{z,h}(t_n, X_{t_n}(\omega))$, are used. We have hence derived a complete formulation of the real option problem of an aluminium smelter with temporary and permanent shutdown options, solved by using the least squares Monte Carlo method.

3.3 Operational flexibility with own power production

In the case of own power sourcing, where the aluminium producer has the option to produce the power required rather than purchase the power through external contracts, a different set of regressions is more suitable to approximate the continuation values. If the decision to temporarily or permanently shut down the smelter is undertaken, the producer will pay the shutdown cost and receive the stand-alone cash flows generated from selling the produced power on the spot market. The future stand-alone cash flows from the power asset are also uncertain and must be approximated by performing a regression, in a similar manner as for the future cash flows from the smelter. Since the produced power can be sold in the spot market, the spot price is a relevant explanatory variable for the power asset cash flows. The smelter is valued on basis of USD/mt produced aluminium while the spot electricity price is given in EUR/MWh. We therefore include the USD/EUR exchange rate as a second explanatory variable. Hence, for each scenario we get the following regression:

$$\begin{aligned} \Phi_{s,t}^{Power} = & \beta_1^t \times y_{s,t}^1 + \beta_2^t \times x_{s,t}^E + \beta_3^t \times (y_{s,t}^1)^2 + \beta_4^t \times (x_{s,t}^E)^2 + \beta_5^t \times y_{s,t}^1 \times x_{s,t}^E + \beta_6^t \times (y_{s,t}^1)^3 \\ & + \beta_7^t \times (x_{s,t}^E)^3 + \beta_8^t \times (y_{s,t}^1)^2 \times x_{s,t}^E + \beta_9^t \times y_{s,t}^1 \times (x_{s,t}^E)^2 \end{aligned} \quad (3.7)$$

The regression result, $\Phi_{s,t}^{Power}$, will be an approximation of the continuation value when only receiving the future stand-alone cash flows from the power asset after having conducted a permanent shutdown of the smelter. It is therefore comparable to the remaining contract exposure when purchasing power through external contracts. To approximate the continuation value of mode z at time t , we again have to use a regression to avoid perfect foresight. As opposed to the case with external power contracts, the continuation value of the option

when producing own power is not correlated with the remaining time left on any contract. We therefore regress on $FCF_{s,t}$, the expected value of cash flows from power production $\Phi_{s,t}^{Power}$, the spot electricity price $y_{s,t}^1$ and the USD/EUR exchange rate $x_{s,t}^E$. Hence, in the case of own power sourcing the expected continuation value of the option is approximated using the following regression:

$$\begin{aligned}
\Phi_{s,t} = & \beta_1^t \times FCF_{s,t} + \beta_2^t \times \Phi_{s,t}^{Power} + \beta_3^t \times y_{s,t}^1 + \beta_4^t \times x_{s,t}^E + \beta_5^t \times (FCF_{s,t})^2 + \beta_6^t \times (\Phi_{s,t}^{Power})^2 \\
& + \beta_7^t \times (y_{s,t}^1)^2 + \beta_8^t \times (x_{s,t}^E)^2 + \beta_9^t \times (FCF_{s,t})^3 + \beta_{10}^t \times (\Phi_{s,t}^{Power})^3 + \beta_{11}^t \times (y_{s,t}^1)^3 \\
& + \beta_{12}^t \times (x_{s,t}^E)^3 + \beta_{13}^t \times FCF_{s,t} \times \Phi_{s,t}^{Power} + \beta_{14}^t \times FCF_{s,t} \times y_{s,t}^1 + \beta_{15}^t \times FCF_{s,t} \times x_{s,t}^E \\
& + \beta_{16}^t \times \Phi_{s,t}^{Power} \times y_{s,t}^1 + \beta_{17}^t \times \Phi_{s,t}^{Power} \times x_{s,t}^E + \beta_{18}^t \times y_{s,t}^1 \times x_{s,t}^E
\end{aligned} \tag{3.8}$$

By using equation (3.6) to derive an operating policy and calculating the net present value back to $t = 0$, we can estimate the value of the smelter with own power production as described in section 3.2.

3.4 Special case: only option to shut down permanently

Having defined the option to operate the smelter with multiple operating states, it is straightforward to formulate a model for the special case of only having the option to shut down the plant permanently. When valuing the smelter with only the option to shut down permanently, we define two operating modes where z again denotes the general operating mode and $Z = \{\text{operating, permanently shutdown}\}$. The payoffs, Π_z (USD/mt), when holding contract length l and contract type c for different operating modes in a given scenario s will then be formulated as:

$$\begin{aligned}
\Pi(t, X_t, z_1) &= FCF_{s,t} \\
\Pi(t, X_t, z_2) &= 0 \\
\Pi'_{z_1, z_1}(t_n, X_{t_n}(\omega)) &= FCF_{s,t} \\
\Pi'_{z_1, z_2}(t_n, X_{t_n}(\omega)) &= -K^{direct perm} + q_{s,t,l,c} \\
\Pi'_{z_2, z_1}(t_n, X_{t_n}(\omega)) &= -\infty
\end{aligned}$$

In theory, these are the same payoffs as for the switching option described in section 3.2, but where we define an infinitely large cost to switch from operating to temporarily shut down.

Hence, we exclude the possibility to temporarily shut down. We can now derive an estimate of the smelter value by using the LSM method in the same way as described in section 3.2.

3.5 Price processes and scenario generation

The price processes adapted in this thesis are chosen according to the type of processes one of the major aluminium producers use in their models, in order to derive comparable results. The rationale behind the choice of price processes will thus not be further elaborated in this thesis. Let T' denote the number of quarters in the historical time series, and I denote the number of stochastic variables. We can then define \mathbf{H} to be a $(T' \times I)$ matrix containing historical quarterly prices for the stochastic variables. Assuming that the aluminium price, electricity prices and exchange rates follow AR(1) processes (Alexander, 2008) with an explicit mean expectation, the price process for the i th variable is then described by:

$$p_{t,i} = \alpha_i + \gamma_i \times p_{t-\Delta t,i} + \varepsilon_{t,i} \quad , \quad \varepsilon_{t,i} \sim N(0, \sigma_i^2) \text{ and i.i.d.}$$

The parameters α_i and γ_i are estimated through linear least squares regression of the time t prices on the time $t - \Delta t$ prices from \mathbf{H} .³ The residuals $\varepsilon_{t,i}$ of the regressions are used to derive the covariance matrix. Note that Δt and T' used in the simulations of time series are based quarters whereas Δt and T used in the aluminium smelter option problem are based on years. Let \mathbf{E} be the matrix containing all the residuals for the different prices, then the covariance matrix, Σ , is defined by $\Sigma = \frac{\mathbf{E}^T \times \mathbf{E}}{T' - 1}$.

Having defined the price process, we will now describe the procedure that must be undertaken to generate a scenario of prices, a procedure that is well described by Haugh (2004).

First, the main part of the scenario generation is to generate random draws, $\mathbf{L}(T', I) = \begin{pmatrix} \varepsilon_{1,1} & \cdots & \varepsilon_{1,I} \\ \vdots & \ddots & \vdots \\ \varepsilon_{T',1} & \cdots & \varepsilon_{T',I} \end{pmatrix}$ where $\varepsilon_{t,i} \sim N(0, \Sigma)$. Note that since we are generating scenarios for

³The long-term unconditional mean for the i th price process is defined by $\mu_i = \frac{\alpha_i}{(1-\gamma_i)}$, however note that this is in most cases different from the explicit expected mean. In order to derive processes that converge to explicitly stated expected long-term means the α_i s must be adjusted by assuming μ_i is fixed and solving the equation $\alpha_i = \mu_i \times (1 - \gamma_i)$. The $p_{0,i}$ s are set equal to the assumed long-term expected means of the variables. Finally, half-life of process i , h_i , is a measure of the speed of mean reversion and is given by the formula: $h_i = \frac{-\ln(2)}{\ln(\gamma_i)}$ (Mark, 2000). The interpretation of the half-life measure is that it denotes the time process i needs to halve its distance from its mean. A higher value of the half-life measure is thus equivalent to slower mean reversion.

correlated processes, the epsilons should be correlated. To do this, we first generate random variables that are normally distributed with zero mean and variance 1, this yields for each t : $L_{t,i} \sim N(0, 1)$. Thus $c_1 L_{t,1} \dots c_i L_{t,i} \sim N(0, \sigma^2)$ where $\sigma^2 = c_1^2 + \dots + c_i^2$. Then $\mathbf{CL} \sim N(0, \mathbf{C}^T \mathbf{C})$, which reduces our problem to finding \mathbf{C} such that $\mathbf{C}^T \mathbf{C} = \mathbf{\Sigma}$. The matrix \mathbf{C} is commonly referred to as the Cholesky-decomposition of $\mathbf{\Sigma}$. From linear algebra we know that a symmetric positive-definite matrix \mathbf{K} can be expressed as $\mathbf{K} = \mathbf{U}^T \mathbf{D} \mathbf{U}$ where \mathbf{U} is an upper-triangular matrix and \mathbf{D} a diagonal matrix with non-negative elements. In our problem we have that $\mathbf{\Sigma} = \mathbf{U}^T \mathbf{D} \mathbf{U}$, which yields the result $\mathbf{C} = \sqrt{\mathbf{D}} \mathbf{U}$.⁴ Thus the correlated random draws $\varepsilon_{t,i}$ are calculated by $\varepsilon(T', I) = \mathbf{CL}$. The matrix $\varepsilon(T', I)$ now represents random price movements. Using the above, the simulated prices can now be calculated by the formula $p_{i,t} = \alpha'_i + \gamma_i \times p_{i,t-1} + \varepsilon_{i,t}$ where the $\varepsilon_{i,t}$ s are the epsilons derived above.

⁴If the covariance matrix is not positive definite it can be transformed through regularization, which works if the negative eigenvalue is close to zero.

Chapter 4

Results

The model described in the previous sections has been implemented in Matlab in order to derive solutions for given parameter values (see Appendix E for a validation of the Matlab routine). Realistic parameter values have been collected from industry sources and are listed in table 4.1:

Parameter	Parameter name	Parameter value
Ca	Carbon Price	400 USD/mt
A	Alumina price	400 USD/mt
ρ	WACC	5%
Tax	Company tax rate	27%
E^L	Electrolysis cost local currency	482 USD/mt
E^U	Electrolysis cost USD	163 USD/mt
C^L	Casthouse cost local currency	257 USD/mt
C^U	Casthouse income USD	425 USD/mt
M	MWH/mt produced aluminium	14
O^{temp}	Yearly operating cost for a temporarily shutdown smelter	0 USD/mt
$K^{directperm}$	Cost of switching mode from operating to permanent shutdown	2000 USD/mt
K^{temp}	Cost of switching mode from operating to temporarily shutdown	1000 USD/mt
K^{perm}	Cost of switching mode from temporarily shutdown to permanently shutdown	1000 USD/mt
$K^{operate}$	Cost of switching mode from temporarily shutdown to operating	1000 USD/mt
μ_{XE}	Long-term expected mean of the USD/EUR exchange rate	1.3
μ_{XN}	Long-term expected mean of the USD/EUR exchange rate	0.17
μ_{Y1}	Long-term expected mean of the 1-year electricity price	40 (EUR/MWH)
μ_{Y3}	Long-term expected mean of the 3-year electricity price	40 (EUR/MWH)

Table 4.1 Parameter values

Estimation of the parameters for the different autoregressive time series is based on historical quarterly prices of the respective variables from December 2002 to June 2014 and the first values of the different time series are set equal to the respective expected long-term means. After calibrating the autoregressive processes the estimated intercepts must be adjusted to fit the expected long-term means of the respective processes, a procedure that is described in section 3.5. Estimated parameters and half-life (in terms of quarters)

of the different processes can be found in table 4.2, while figures 4.1- 4.5 show examples of 30 sample paths for the stochastic variables over 20 years. Note that since the price processes in this thesis are modelled based on price levels, the value of out of the money shutdown options may be overestimated if modelling log-level prices in fact provides a better fit to reality. This is due to the fact that the lognormal distribution is positively skewed, while a normal distribution has no skewness. A normal distribution may therefore tend to overestimate the size of the left tail of the distribution of prices compared to a lognormal distribution. Thus, the probability of incurring lower prices may be too high.

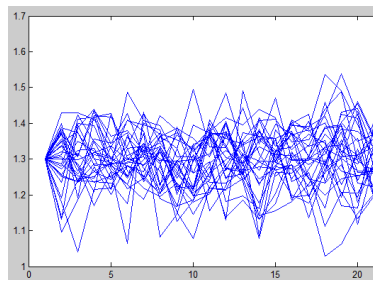


Fig. 4.1 Thirty sample paths - USD/EUR

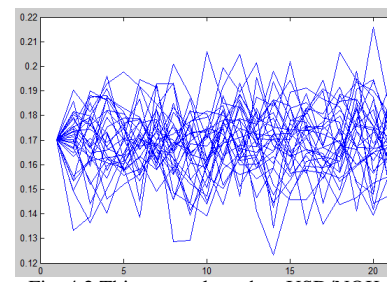


Fig. 4.2 Thirty sample paths - USD/NOK

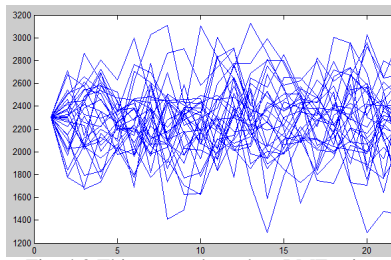


Fig. 4.3 Thirty sample paths - LME price

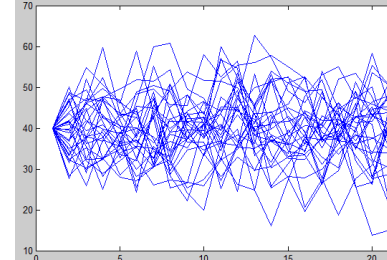


Fig. 4.4 Thirty sample paths - El. price 1-year

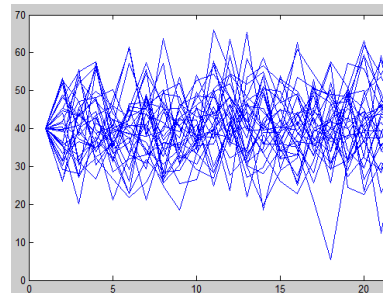


Fig. 4.5 Thirty sample paths - El. price 3-year

Stochastic variable	α_i^1	γ_i		Half-life [quarters]				
		Estimate	p-value	Estimate	Lower SE	Upper SE	Lower 95%	Upper 95%
USD/EUR	0.2714	0.791	6.27×10^{-15}	2.9600	2.1353	4.5833	1.6284	9.4278
USD/NOK	0.0346	0.797	1.42×10^{-12}	3.0489	2.0662	5.3412	1.5111	17.5125
LME price	337.5555	0.853	1.63×10^{-15}	4.3672	2.8240	8.7839	2.0270	177.2684
El. price 1-year	5.1758	0.871	1.20×10^{-15}	5.0022	3.0876	11.6599	2.1660	n.a.
El. price 3-year	11.7297	0.707	3.13×10^{-8}	1.9971	1.3627	3.3325	0.9835	8.2366

Table 4.2 Estimated parameter values for the autoregressive processes

¹This will vary dependent on the explicitly stated long-term expected mean of the respective processes

The intercepts shown in table 4.2 are fitted to the long-term expected means listed in table 4.1 and a base case of 2,300 for the long-term expected mean of the LME price. In table 4.2, we also see that there is evidence of mean reversion in the processes followed by the stochastic variables within a 95% confidence interval, except for the electricity spot price. For the latter there is only evidence of mean reversion within a 90% confidence interval. Despite this, we use the estimated coefficient for the process followed by the electricity spot price for two reasons. First, the electricity spot price only has strong impact on the cash flows of the smelter when electricity is purchased in the spot market. Secondly, as stated in section 3.5 the price processes adapted in this thesis are chosen according to the type of processes one of the major aluminium producers use in their models in order to derive comparable results and the focus in this thesis is primarily on real options modelling of the operational flexibilities of a smelter. Note that the 95% confidence interval for the LME price is wide and that one must have this in mind when doing analyses varying the speed of mean reversion. A thorough empirical study of the stochastic variables is a highly recommended further extension to this thesis.

To limit the scope of the analyses, long-term expected means of exchange rates and electricity prices are held constant throughout the model simulations. This is because the smelter value and related operational policies are more sensitive to changes in the relative changes in the aluminium price than to similar relative changes in the two former (refer to appendix C for details). Expanding the scope of the analyses is left as a recommendation for further work. Thus, in subsequent analyses, only the long-term expected mean of the LME price is varied. Finally, note that the investment cost and potential selling cost of the power asset have not been included in the smelter value estimated under own power sourcing. Therefore, the smelter value calculated in the case of own power sourcing is not directly comparable to the smelter value calculated with other contract types since the electricity cost is artificially low.

4.1 LSM valuation of smelter with operational flexibility

Varying the expected long-term mean of the LME price yields a range of different smelter values as well as different operational policies. Analysing the profitability and operational policy of the smelter under different long-term LME price expectations is of high value to management as the LME price is the only external variable directly impacting the revenues of an operating smelter. Low LME prices directly impact the profitability of the smelter and longer periods with lower LME prices have historically led to shutdowns (United States

International Trade Commission (1987), Geological Survey US (2011) & Alcoa (2014)). In figures 4.6- 4.20 we therefore do analyses where the long-term expected mean of the LME price is varied and used as independent variable in the plots.

Below are numerical solutions to the switching option problem derived by running the Matlab model. 1,000 scenarios have been used to derive the regression coefficients to be used in the approximations of continuation values while 10,000 scenarios have been used when calculating operational policies and smelter values. The value plots exhibit the same traits for different currencies when holding the contract length fixed, we thus include only the value plots for USD contracts in this section, plots for other currencies may be found in appendix B.

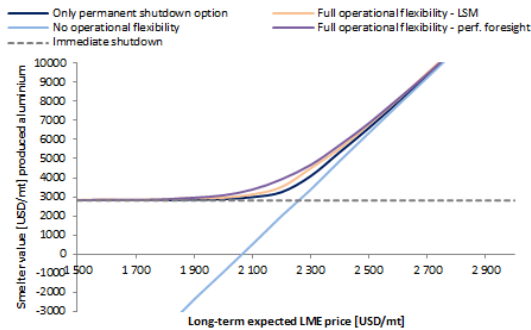


Fig. 4.6 Value plot - Own sourcing

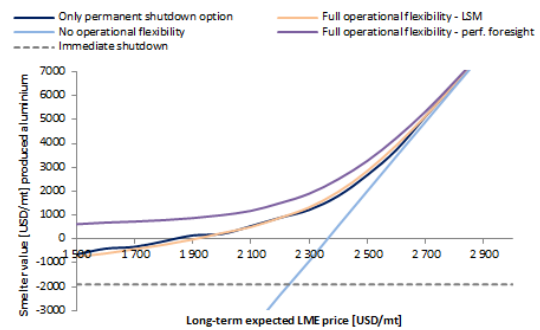


Fig. 4.7 Value plot - 20-year USD contract

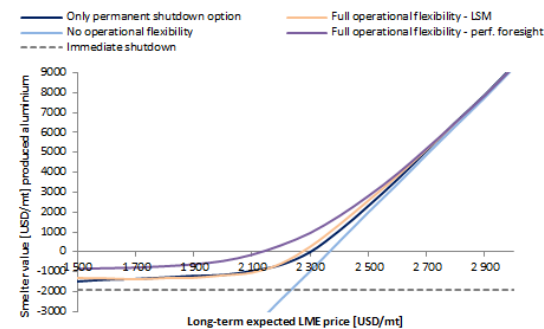


Fig. 4.8 Value plot - 10-year USD contract

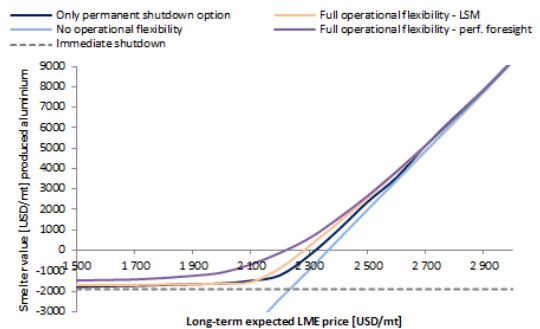


Fig. 4.9 Value plot - 5-year USD contract

Figures 4.6- 4.10 show that the smelter values with only permanent shutdown options and the smelter values with full operational flexibility, both derived by the LSM approach, mostly stay within their lower and upper bounds. The lower bounds are the smelter values when operating the whole time period. The upper bounds are the smelter values calculated under perfect foresight, and have been derived through the same steps as with the LSM approach excluding the approximation of continuation values through multivariate regression. The bounds are defined as follows:

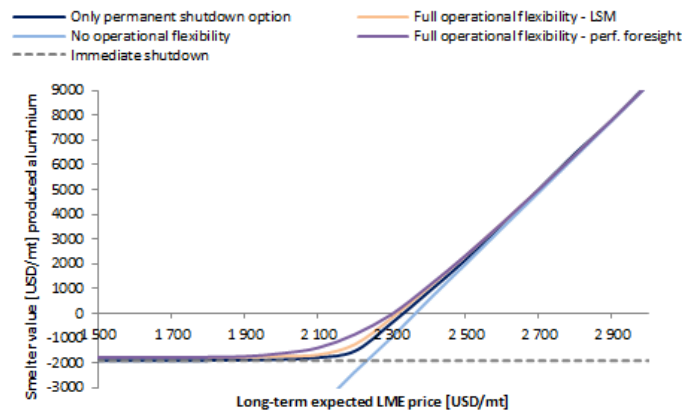


Fig. 4.10 Value plot - Spot contract

$$Value(\text{No closure}) \leq Value(\text{Full operational flex. - LSM}) < Value(\text{Full operational flex. - perf. foresight})$$

Note that if the smelter has permanent and/or temporary shutdown options another strict lower bound for the smelter value can be defined, namely the smelter value if an immediate permanent shutdown is conducted:

$$Value(\text{Imm. shutdown}) \leq Value(\text{Only permanent shutdown option}) \leq Value(\text{Full operational flex. - LSM})$$

This additional bound has also been included in figures 4.6- 4.10. As the figures show, the value of the smelter with full operational flexibility tends to marginally move out of its lower bound when the smelter value converges to the value of immediate shutdown. This happens for low expected long-term means of the LME price, and is caused by regression errors as the continuation values of temporary shutdowns may be overestimated in cases where a more optimal policy actually would be to permanently shut down right away.

Contract type	Probability of perm. shutdowns	Probability of temp. shutdowns	Avg. # of temporary shutdowns/scenario	Scenarios without shutdowns
Own sourcing	99.3 %	32.1 %	0.53	0.1 %
20-Nok	97.2 %	5.5 %	0.09	0.5 %
20-Eur	97.2 %	5.3 %	0.09	0.5 %
20-Usd	97.4 %	5.0 %	0.08	0.4 %
10-Nok	98.4 %	37.2 %	0.82	0.3 %
10-Eur	98.3 %	37.7 %	0.88	0.3 %
10-Usd	98.4 %	36.6 %	0.96	0.3 %
5-Nok	99.8 %	94.1 %	2.63	0.0 %
5-Eur	99.5 %	88.3 %	2.31	0.1 %
5-Usd	99.5 %	86.0 %	2.34	0.0 %
Spot	99.4 %	99.9 %	2.99	0.1 %

Table 4.3 Closure dynamics when $\mu_{LME} = 2,000$ USD/mt

The results shown in tables 4.3- 4.8 for three different values of μ_{LME} represent a low case, a base case and a high case, respectively. One would therefore expect very different results in terms of closure dynamics and smelter values as well as to what extent the options

Contract type	No closure	Only permanent shutdown	Full operational flex. - LSM	Full operational flex. - perf. foresight
Own sourcing	-970	2,891	2,953	3,085
20-Nok	-5,309	-337	-333	390
20-Eur	-5,310	-227	-197	522
20-Usd	-5,306	201	245	1,001
10-Nok	-5,301	-1,347	-1,302	-721
10-Eur	-5,308	-1,302	-1,299	-658
10-Usd	-5,337	-1,162	-1,141	-440
5-Nok	-5,302	-1,684	-1,690	-1,203
5-Eur	-5,297	-1,678	-1,682	-1,209
5-Usd	-5,351	-1,619	-1,629	-1,087
Spot	-5,312	-1,853	-1,729	-1,641

Table 4.4 Smelter value ranges (USD/mt produced aluminium) when $\mu_{LME} = 2,000$ USD/mt

Contract type	Probability of perm. shutdowns	Probability of temp. shutdowns	Avg. # of temporary shutdowns/scenario	Scenarios without shutdowns
Own sourcing	12.1 %	50.4 %	0.52	39.0 %
20-Nok	67.5 %	20.5 %	0.21	16.2 %
20-Eur	69.3 %	19.4 %	0.20	15.4 %
20-Usd	75.2 %	16.3 %	0.17	12.2 %
10-Nok	34.0 %	43.1 %	0.45	33.1 %
10-Eur	36.2 %	42.9 %	0.45	31.9 %
10-Usd	42.0 %	41.2 %	0.43	28.2 %
5-Nok	25.6 %	47.0 %	0.51	38.0 %
5-Eur	26.4 %	47.4 %	0.51	37.7 %
5-Usd	28.9 %	48.5 %	0.52	35.2 %
Spot	0.2 %	50.5 %	0.51	49.4 %

Table 4.5 Closure dynamics when $\mu_{LME} = 2,300$ USD/mt

Contract type	No closure	Only permanent shutdown	Full operational flex. - LSM	Full operational flex. - perf. foresight
Own sourcing	3,381	4,088	4,503	4,675
20-Nok	-930	707	791	1,324
20-Eur	-930	807	907	1,453
20-Usd	-929	1,217	1,323	1,896
10-Nok	-925	-172	98	685
10-Eur	-933	-133	128	749
10-Usd	-971	-7	258	951
5-Nok	-918	-279	104	444
5-Eur	-926	-256	132	484
5-Usd	-960	-251	200	588
Spot	-967	-387	-185	-34

Table 4.6 Smelter value ranges (USD/mt produced aluminium) when $\mu_{LME} = 2,300$ USD/mt

Contract type	Probability of perm. shutdowns	Probability of temp. shutdowns	Avg. # of temporary shutdowns/scenario	Scenarios without shutdowns
Own sourcing	4.0 %	23.4 %	0.24	72.9 %
20-Nok	12.2 %	15.3 %	0.15	73.3 %
20-Eur	14.1 %	15.3 %	0.15	71.4 %
20-Usd	20.3 %	15.9 %	0.16	64.8 %
10-Nok	12.0 %	18.1 %	0.18	72.3 %
10-Eur	12.6 %	18.4 %	0.19	71.5 %
10-Usd	14.9 %	20.0 %	0.20	68.3 %
5-Nok	11.2 %	17.0 %	0.17	73.5 %
5-Eur	11.9 %	17.5 %	0.18	72.3 %
5-Usd	14.0 %	19.7 %	0.20	69.1 %
Spot	0.1 %	20.1 %	0.20	79.7 %

Table 4.7 Closure dynamics when $\mu_{LME} = 2,600$ USD/mt

Contract type	No closure	Only permanent shutdown	Full operational flex. - LSM	Full operational flex. - perf. foresight
Own sourcing	7,810	7,968	8,094	8,128
20-Nok	3,473	3,646	3,766	3,957
20-Eur	3,473	3,676	3,798	4,019
20-Usd	3,467	3,733	3,929	4,232
10-Nok	3,448	3,625	3,757	3,820
10-Eur	3,437	3,633	3,771	3,844
10-Usd	3,408	3,645	3,817	3,932
5-Nok	3,463	3,636	3,765	3,796
5-Eur	3,453	3,645	3,778	3,813
5-Usd	3,414	3,561	3,808	3,859
Spot	3,432	3,588	3,629	3,636

Table 4.8 Smelter value ranges (USD/mt produced aluminium) when $\mu_{LME} = 2,600$ USD/mt

to permanently- and/or temporarily shut down operations are value adding. Starting with the base case where $\mu_{LME} = \$2,300$, table 4.6 shows that introducing managerial flexibility in terms of different operating modes increases the value of the smelter (USD/mt produced aluminium) from a negative number to a positive number, an increase of approximately \$1,300 – \$2,000 under different contract types, compared to having no shutdown options. Table 4.5 shows that the percentage of scenarios in which one or more temporary shutdowns have occurred is more than 40% for most of the contract types. This clearly shows that having temporary shutdown options is value adding compared to having only permanent shutdown options. In contrast, when analysing the high case in tables 4.7 and 4.8 it is evident that the value added by operational flexibility is as expected smaller, because it would much more seldom be rational to shut down operations with expectations of high LME prices. On the other hand, in a low case having the option to shut down operations adds much value, but the value added by the temporary shutdown options is however not as high as in the base case, this is because early permanent shutdowns is the most rational strategy in most scenarios. When analysing low case and base case mean aluminium prices, the tables 4.3 and 4.6 show that the value of the smelter is substantially larger for the 20-year contracts. We believe this is due to speculative positions in electricity prices and exchange rates. More precisely, the value of closing a 20-year contract in favourable market conditions is higher than closing a 10-year contract. Managers will thus wait for more favourable electricity spot prices before shutting down the smelter, hence increasing the total value of the smelter. We emphasise that this is a hypothesis that need further investigation.

Another pattern that materialises in the tables displaying smelter value ranges is that the USD contracts are always higher in value than otherwise identical contracts under full operational flexibility. A reason for this may be that all revenues for an operating smelter are in USD, so purchasing the main cost element, namely electricity, in the same currency may severely reduce currency risk. We recommend that a further investigation of this observation

should be conducted in further work.

As discussed in chapter 1 shutdowns of smelters, both temporary and permanent, may have wider negative economic impacts than for just the smelter owner. The LME price is one of the critical variables affecting the decision. It is thus of interest to study the probabilities of different types of shutdowns given different levels for the long-term expected mean of the LME price. Figures 4.11- 4.14 show the probabilities of shutdowns under different contract types, given different levels of the long-term mean LME price, holding long-term mean expectations of all other stochastic variables constant. The data series "Permanent shutdown" plots the percentage of all scenarios in which a permanent shutdown was conducted at some point in time against the long-term expected mean of the LME price, and will be denoted as the probability of permanent shutdowns. Similarly, the data series "Temporary shutdowns" plots the percentage of scenarios in which one or more temporary shutdowns occurred² and will be referred to as the probability of temporary shutdowns. Finally, the series "Operating with no shutdowns" plots the percentage of scenarios in which no types of shutdowns have occurred at any point in time and will be referred to as the probability of no shutdowns. Since the shutdown patterns exhibit the same traits for different currencies when holding the contract length fixed, we include only the USD contracts in this section. Plots for other currencies may be found in appendix B.

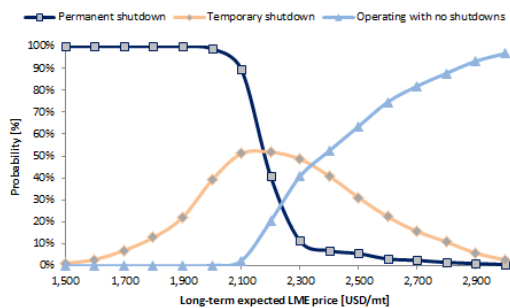


Fig. 4.11 Shutdown risk: Own sourcing

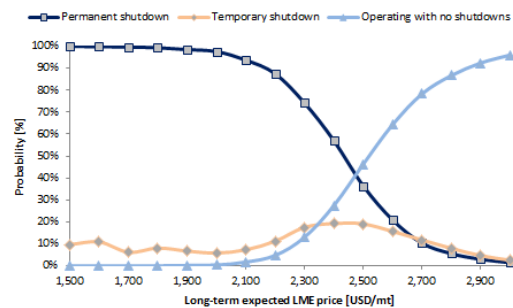


Fig. 4.12 Shutdown risk: 20-year USD contract

Although own power sourcing has different underlying dynamics than, and is not directly comparable to, the other types of power contracts, it is still of interest to do a high-level comparison of the shutdown profiles. From figure 4.11 it is evident that when the long-term expected mean of the LME price passes \$2,000 there is a sharp decrease in the percentage of scenarios in which permanent shutdowns occur, a decrease that is offset by an increasing percentage of scenarios with temporary shutdowns. That is, temporary shut-

²Note that there may be scenarios where temporary shutdowns are followed by permanent shutdowns later in time, so the two latter measures are not mutually exclusive.

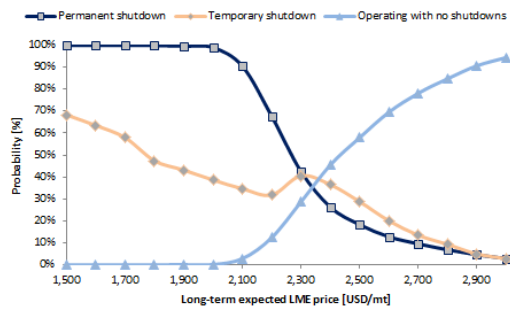


Fig. 4.13 Shutdown risk: 10-year USD contract

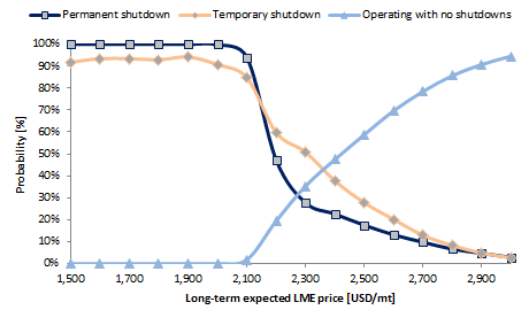


Fig. 4.14 Shutdown risk: 5-year USD contract

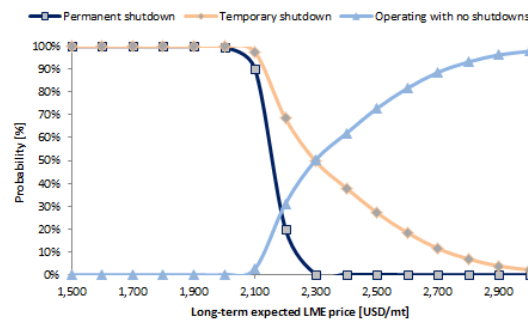


Fig. 4.15 Shutdown risk: Spot contract

downs seem to take the place of permanent shutdowns at this point with the percentage measure peaking when the long-term expected mean of the LME price is \$2,200, then slowly decreasing. In addition, even though the plots of the percentage of scenarios in which no shutdowns occur have very similar patterns across all contract types, own electricity sourcing clearly seems to be the best power input option for reducing the risk of shutdowns for lower ranges of the long-term expected LME-price. This is, as previously mentioned, because the investment cost and potential selling cost of the power asset is not included in the analysis leading to an artificially low electricity cost. However, this observation may be used in the decision-making process when determining whether to invest in a power asset. If the electricity cost from the power asset is lower than external power contracts, even when including the investment cost, the probability of shutdowns is reduced at lower levels of the expected mean LME price.

When comparing the different contract lengths in figures 4.12- 4.15, two observations are worth elaborating. First, it is clear that the shorter the contract length the sharper the decline in the probabilities of permanent shutdowns when the long-term expected mean of the LME price exceeds \$2,100. Nevertheless, the pattern of permanent shutdown probabilities is the same when comparing the different contract lengths. Interpreting the sharp declines in permanent shutdown probabilities it is clear that long-term expected means of

the LME price of approximately \$2,300 or higher, are the price levels for which it is profitable with aluminium production. Lower price levels entail having either permanent or temporary shutdowns.

Secondly, the probabilities of temporary shutdowns also follow the same patterns across different contract lengths, but only when the long-term expected mean of the LME price is greater than \$2,100. Left of that point, the shorter the contract length, the higher the ratio of temporary shutdowns. Around that point it seems that the sharper the decline in probabilities of permanent shutdowns the higher the probabilities of temporary shutdowns. This is an observation illustrating that for shorter contract lengths, having temporary shutdown options enables the aluminium producer to quickly reduce the risk of permanent shutdowns as soon as the long-term expected mean of the LME price reaches profitable levels. Having to operate the smelter when the LME price is low may not be rational compared to a permanent shutdown, even though one would expect the LME price to return to its mean level. Exercising the temporary shutdown option in such a scenario may however be more profitable than a permanent shutdown. The temporary shutdown option thus acts as a hedge against downturns in the LME price when the long-term expected mean of the LME price is right above profitable levels. Note that the region around a long-term expected mean of the LME price of around \$2,300 is a region in which the permanent shutdown options go from being in the money to out of the money. One would therefore expect that temporary shutdown options are most often exercised in this region. This pattern is observed in figures 4.12 and 4.13 where the probabilities of temporary shutdowns have local peaks.

An interesting analysis to conduct is to study how the probabilities of different shutdown types are affected by a change in the speed of mean reversion of the price process followed by the LME price. Figures 4.16- 4.20 are plots similar to figures 4.11- 4.15, but with the speed of mean reversion increased by forcing a lower γ_i on the price process that the LME price follows. Whereas the estimated γ_i of the price process followed by the LME price is 0.853 we now force it to have the value of 0.600 for the sake of this analysis, which increases the speed of mean reversion. The associated half-life then becomes 1.4 quarters as opposed to the previous 4.4 quarters.

The dotted lines in figures 4.16- 4.20 illustrate the shutdown probabilities for the case of slow mean reversion, while the thick lines show the shutdown probabilities for the case of quicker mean reversion. It is evident from the figures, that the increase in the probability of operating with no shutdowns when the long-term expected mean of the LME price increases, is greater when the speed of mean reversion is higher. This result was expected and the intuition behind it is that as long as the long-term expected mean of the LME price is at a

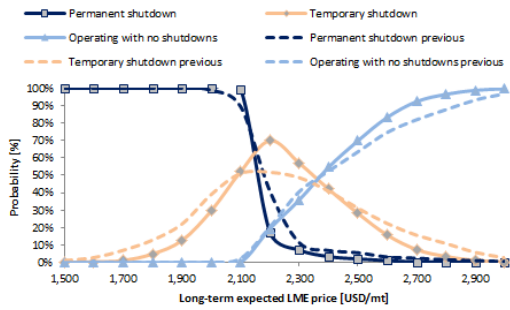


Fig. 4.16 Shutdown risk: Own sourcing

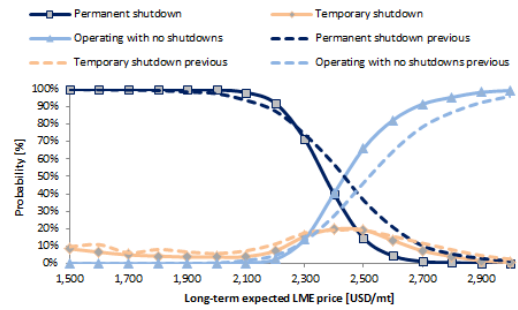


Fig. 4.17 Shutdown risk: 20-year USD contract

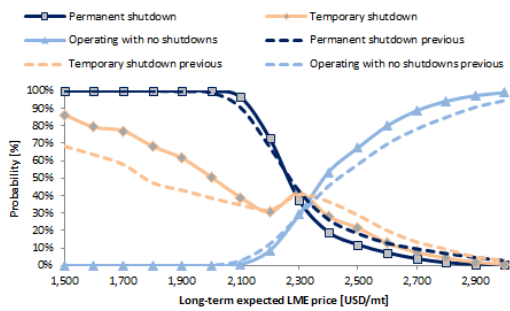


Fig. 4.18 Shutdown risk: 10-year USD contract

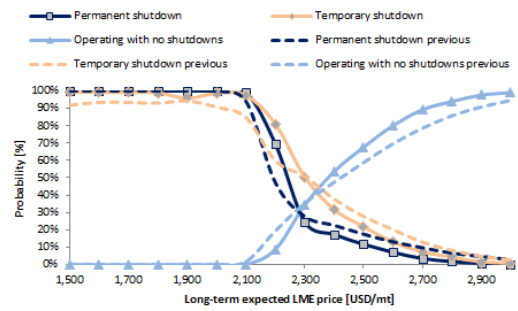


Fig. 4.19 Shutdown risk: 5-year USD contract

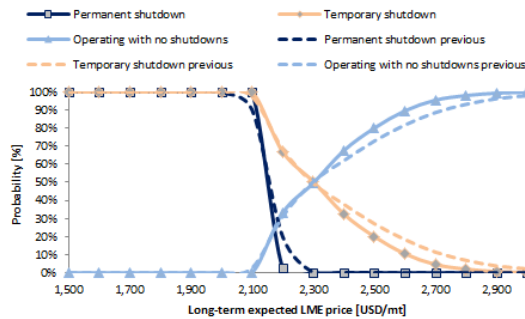


Fig. 4.20 Shutdown risk: Spot contract

profitable level the aluminium producer is less likely to benefit from shutdowns in the case of low prices, because the price will quickly revert back to profitable levels. On the other hand, in the case of slow mean reversion the price may strongly deviate from its mean for longer time periods increasing the benefits and thus probabilities of shutdowns, especially permanent ones. This can also be observed in the figures by studying the difference between the thick and dotted dark blue lines. Another observation worth to comment is that the chosen base case level of the long-term expected mean of the LME price of \$2,300 and nearby levels make up a region in which the smelter seems to go from being non-profitable to profitable illustrated by line intersections and strongly changing trends. Varying the mean reversion speed only tightens this region.

Volatility of the stochastic process followed by the LME price impacts the value of the smelter with operational flexibility and thus also the operational policy. An aluminium producer with an operating smelter is concerned about the risk of having to shut down the smelter and it is of special interest to analyse the risk of having to shut down the smelter within a shorter time horizon of 1-5 years.

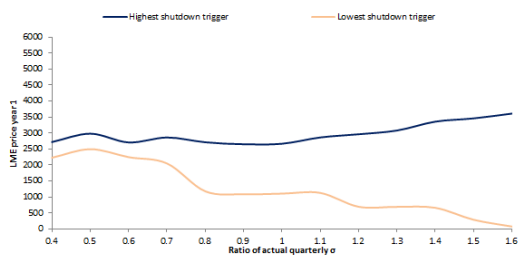


Fig. 4.21 Shutdown triggers: Own sourcing

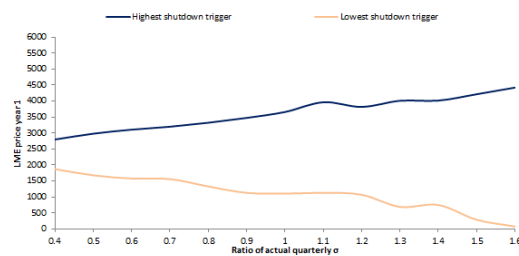


Fig. 4.22 Shutdown triggers: 20-year USD contract

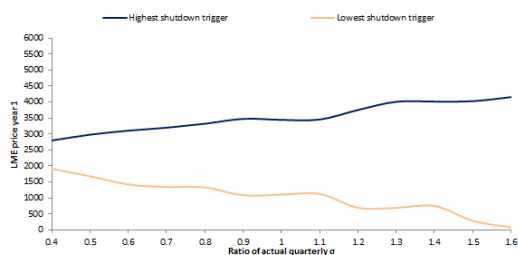


Fig. 4.23 Shutdown triggers: 10-year USD contract

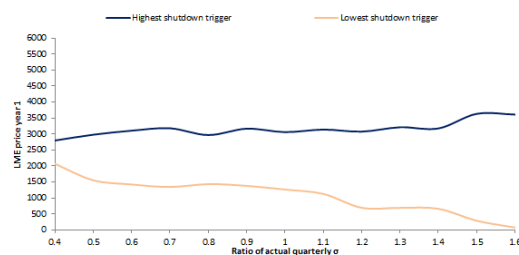


Fig. 4.24 Shutdown triggers: 5-year USD contract

Figures 4.21- 4.25 show the lowest and highest LME prices in year 1 for which a permanent or temporary shutdown was conducted within 5 years in one of the simulated scenarios, given different quarterly volatility levels. These volatility levels are expressed as multiples (0.4x – 1.6x) of the volatility of the LME price derived from the historical time series. Plots for other contract currencies can be found in appendix B. The analysis is based on the base

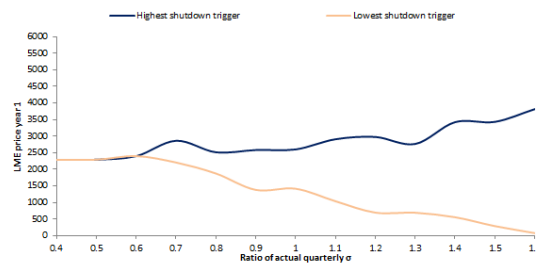


Fig. 4.25 Shutdown triggers: Spot contract

case in which the long-term expected mean of the LME price is \$2,300. The figures can help the aluminium producer assess the probability of having to shut down the smelter within the next few years. For instance if the year 1 LME price is above the highest shutdown trigger the producer can be confident that no shutdowns will be conducted the next few years. On the other hand, if the year 1 LME price is below the lowest shutdown trigger the producer has all reasons to expect shutdowns within the next few years. If the year 1 aluminium price is between the two triggers the risk picture is not that clear, but one should expect some shutdowns. The relationship between the triggers and level of volatility in the LME stochastic process followed by the LME price is evident. Higher volatility yields a greater highest shutdown trigger and smaller lowest shutdown trigger. Conversely lower volatility yields a smaller highest shutdown trigger and a greater lowest shutdown trigger.

To what extent the full optionality is value adding depends on both the volatility of the stochastic process followed by the LME price as well as the expected long-term mean of the LME price. Tables 4.9- 4.18 are sensitivity tables that show the nominal value increase in USD/mt produced aluminium of having full operational flexibility compared to not having any shutdown options and having only permanent shutdown options respectively, for different expected long-term mean LME-prices and different levels of volatility. The low, base and high cases are the same as in tables 4.3- 4.8 while volatility is set to different multiples of the actual estimated volatility. Ideally such tables would express the value increase in terms of percentage, but as the smelter value in several cases changes from a negative to a positive number, percentage as a measure would not be meaningful because percentage change is based on change in magnitude regardless of sign. The sensitivity tables exhibit approximately the same patterns for different currencies when holding the contract length fixed, we thus include only the USD contracts in this section, sensitivity tables for other currencies may be found in appendix B.

It is clear from tables 4.9- 4.18 that the value increase of having full operational flexibil-

Own sourcing

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	45	284	740
	Base	680	1,121	1,708
	<i>Low</i>	3,834	3,923	4,177

Table 4.9 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	44	127	197
	Base	432	415	468
	<i>Low</i>	48	62	112

Table 4.10 Value over only perm. shut. options

20-year USD

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	282	461	863
	Base	2,131	2,252	2,511
	<i>Low</i>	5,492	5,551	5,618

Table 4.11 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	49	195	244
	Base	32	106	145
	<i>Low</i>	42	44	25

Table 4.12 Value over only perm. shut. options

10-year USD

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	124	409	843
	Base	977	1,229	1,726
	<i>Low</i>	4,032	4,196	4,317

Table 4.13 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	431	172	312
	Base	179	266	387
	<i>Low</i>	-6	21	-4

Table 4.14 Value over only perm. shut. options

5-year USD

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	118	394	837
	Base	804	1,161	1,678
	<i>Low</i>	3,666	3,722	3,935

Table 4.15 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	12	247	273
	Base	386	452	393
	<i>Low</i>	20	-10	42

Table 4.16 Value over only perm. shut. options

Spot contract

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	21	197	593
	Base	406	782	1,286
	<i>Low</i>	3,550	3,583	3,775

Table 4.17 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	60	41	138
	Base	142	201	183
	<i>Low</i>	128	123	139

Table 4.18 Value over only perm. shut. options

ity, compared to having no shutdown options, is larger the lower the long-term expected LME price and the higher the volatility. These are reasonable results that one would expect, because for the low case of the long-term expected LME price the smelter is experiencing severe losses and a permanent shutdown would be the only rational strategy. However, when the volatility increases, having temporary shutdown options entails that the aluminium producer may exploit periods of short jumps in the LME price while at the same time limiting losses when the price is low. This pattern is evident in the sensitivity tables that show the value increase of introducing full operational flexibility when already having the option to permanently shut down the smelter. A second pattern that is evident is the fact that when comparing contract lengths, the nominal value increase of full operational flexibility versus no operational flexibility is larger the longer the contract length. This may be explained by the plots of permanent shutdowns in figures 4.12- 4.15 where one can observe that the shorter the contract length the steeper the decrease in probabilities of permanent shutdowns, hence for shorter contract lengths the option to permanently shut down is of less value. A

third pattern in the sensitivity tables that is worth noticing is that the value increase of introducing full operational flexibility when already having the option to permanently shut down operations is largest for short-term electricity contracts and in the base case of the long-term expected mean of the LME price. For the 10-year and 20-year contracts this value increase is approximately the same in the base and high case. Again, a possible explanation to this may be found by studying figures 4.12- 4.15. In the base case the long-term expected mean of the LME price is \$2,300, around this point the probabilities of permanent shutdowns have decreased drastically for shorter contract lengths. The same is observed for 10-year and 20-year contracts only for a long-term expected mean of the LME price at a somewhat higher level. In addition, the probabilities of temporary shutdowns have upward bumps for the 20-year and 10-year contracts in this region. For the 5-year and spot contracts the probabilities of temporary shutdowns are already high. Thus, it seems that the temporary shutdowns replace permanent shutdowns in these regions, but since the risk of periods with unfavourable LME prices and negative profitability is still high at this point the temporary shutdown options are actively used. That is to say that the temporary shutdowns options add more value at this point than e.g. in a high case where the risk of unfavourable LME prices is much lower. Finally, we see that in the low case the additional value of full operational flexibility when already having permanent shutdown options is sometimes negative, but small in magnitude. This is most likely due to small numerical errors. For this level of the long-term expected mean of the LME price permanent shutdowns occur at an early point in approximately all scenarios, and the value-added by temporary shutdowns is negligible and not captured in the continuation value approximations.

4.2 Accuracy of the regressions

Approximating the continuation values by regressing them on the current values of the state variables is, as described in section 3.2, the technique that is used in the LSM method when deriving operational policies in order to avoid the bias of perfect foresight. The accuracy of these regressions thus has direct impact on the final results, and it is important to do analyses of the explanatory power for different regression equations and use this as a decision basis when choosing explanatory variables. Further on, the results are sensitive to the number of scenarios chosen. As previously stated we have used 1,000 scenarios, referred to as in-sample scenarios, for estimating the regression coefficients to be used in continuation value approximations. 10,000 scenarios, referred to as out-of-sample scenarios, are used to estimate smelter values and operational policies. The rationale behind using 1,000 in-

sample scenarios is illustrated in figure 4.26, which shows a scatter plot of the smelter values under a 10-year USD power contract when varying the number of in-sample scenarios and running the model six times for each set of the number of the latter holding the number of out-of-sample scenarios fixed at 10,000. It is clear that the smelter value estimates increase with the number of in-sample scenarios, up to 1,000 in-sample scenarios. The reason for this is that the optimal operating policy is determined using approximated continuation values, while the estimated value is calculated by applying this policy and working with the actual simulated continuation values. When the number of in-sample scenarios is low, the sample set is not representable and the estimated regression parameters provide a poor fit when used for another sample. Therefore, the resulting estimated policy is far from optimal resulting in a low estimated smelter value as well as a large spread in estimated values (refer to Longstaff and Schwartz (2001) for details on convergence of the LSM method). From the figure we see that the smelter value estimates converge for 1,000 or more in-sample scenarios, hence choosing 1,000 in-sample scenarios seems sufficient.

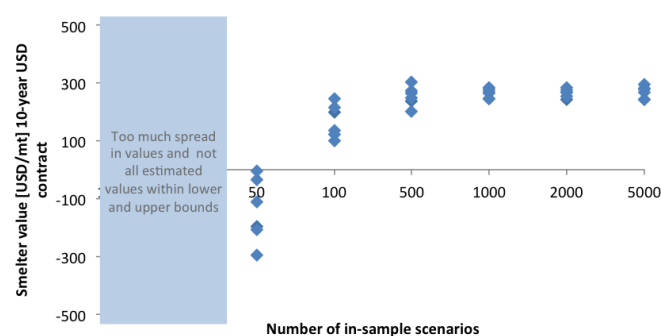


Fig. 4.26 Convergence of smelter value estimates for different number of in-sample scenarios

Table 4.19 shows the average degrees of explanatory power, R-squared, for different sets of explanatory variables on a per contract basis for the base case of long-term expected mean of the LME price. Note that since we are only interested in the approximated value, the regression coefficients are only used for this purpose and are not basis for any further analysis. This means that our goal is to maximise R-squared and not R-squared adjusted. Columns *Operating* and *Temporary* denote what continuation value the statistic is linked to. Since there are two potential continuation values, namely the two former, these must be approximated at each step in order to have the correct decision basis. Further on, since the regression in the case of own power sourcing is different from the other contract types the regression statistics from the own sourcing case are included in a separate table. We use polynomials of power 1 and 2 of the explanatory variables in addition to cross products. The variable sets considered for the cases with external power contracts are:

Contract type	Variable set 1		Variable set 2		Variable set 3		Variable set 4		Variable set 5	
	Operating	Temporary	Operating	Temporary	Operating	Temporary	Operating	Temporary	Operating	Temporary
20-NOK	0.371	0.220	0.378	0.209	0.370	0.214	0.367	0.221	-0.266	0.074
20-EUR	0.376	0.208	0.384	0.190	0.379	0.202	0.375	0.208	0.272	-0.037
20-USD	0.401	0.156	0.408	0.141	0.402	0.154	0.402	0.159	0.278	0.040
10-NOK	0.305	0.241	0.305	0.228	0.307	0.248	0.294	0.258	0.277	-0.649
10-EUR	0.308	0.228	0.305	0.217	0.309	0.242	0.296	0.252	0.277	-0.570
10-USD	0.324	0.204	0.320	0.197	0.323	0.216	0.306	0.234	0.288	-0.386
5-NOK	0.300	0.259	0.295	0.210	0.295	0.239	0.280	0.245	0.253	-1.550
5-EUR	0.305	0.249	0.296	0.199	0.297	0.235	0.282	0.237	0.251	-1.452
5-USD	0.332	0.239	0.321	0.185	0.315	0.219	0.298	0.221	0.250	-1.183
Spot	0.292	0.309	0.205	0.246	0.215	0.250	0.211	0.259	0.123	-3.806

Table 4.19 Explanatory power for different variable sets in the case of external power contracts

Contract type	Variable set 2		Variable set 3		Variable set 4		Variable set 5	
	Operating	Temporary	Operating	Temporary	Operating	Temporary	Operating	Temporary
20-NOK	0.007	-0.012	-0.001	-0.006	-0.005	0.001	-0.637	-0.146
20-EUR	0.007	-0.018	0.003	-0.006	-0.001	0.000	-0.105	-0.245
20-USD	0.007	-0.016	0.001	-0.002	0.001	0.002	-0.123	-0.116
10-NOK	0.000	-0.013	0.002	0.007	-0.012	0.016	-0.029	-0.890
10-EUR	-0.003	-0.010	0.002	0.014	-0.012	0.024	-0.030	-0.798
10-USD	-0.004	-0.008	-0.001	0.011	-0.017	0.030	-0.036	-0.590
5-NOK	-0.006	-0.049	-0.005	-0.020	-0.020	-0.014	-0.047	-1.809
5-EUR	-0.009	-0.049	-0.008	-0.014	-0.023	-0.012	-0.053	-1.700
5-USD	-0.011	-0.053	-0.017	-0.020	-0.034	-0.017	-0.082	-1.422
Spot	-0.088	-0.064	-0.077	-0.060	-0.081	-0.051	-0.170	-4.115

Table 4.20 Difference in explanatory power compared to variable set 1 in the case of external power contracts

Variable set 1:	$y_{s,t}^1, y_{s,t}^3, x_{s,t}^E, FCF_{s,t}$ and $\delta_{t,l}$
Variable set 2:	$y_{s,t}^1, x_{s,t}^E, FCF_{s,t}$ and $\delta_{t,l}$
Variable set 3:	$y_{s,t}^1, FCF_{s,t}$ and $\delta_{t,l}$
Variable set 4:	$y_{s,t}^1$ and $FCF_{s,t}$
Variable set 5:	$FCF_{s,t}$

Table 4.20 shows the difference in explanatory power with respect to variable set 1, which is the set that has been used in this thesis. From the table it is clear that for close to all contracts the variable set we are using in the model yields the highest explanatory power to approximate the continuation values, especially for the spot contract type.

In the case of own power sourcing we have defined the following variable sets:

Variable set 1':	$y_{s,t}^1, x_{s,t}^E, FCF_{s,t}$ and $\Phi_{s,t}^{Power}$
Variable set 2':	$y_{s,t}^1, FCF_{s,t}$ and $\Phi_{s,t}^{Power}$
Variable set 3':	$FCF_{s,t}$ and $\Phi_{s,t}^{Power}$
Variable set 4':	$FCF_{s,t}$

Contract type	Variable set 1'		Variable set 2'		Variable set 3'		Variable set 4'	
	Operating	Temporary	Operating	Temporary	Operating	Temporary	Operating	Temporary
Own sourcing	0.29	0.26	0.28	0.24	0.28	0.22	-3.36	0.17

Table 4.21 Explanatory power for different variable sets in the case of own power sourcing

Contract type	Variable set 2'		Variable set 3'		Variable set 4'	
	Operating	Temporary	Operating	Temporary	Operating	Temporary
Own sourcing	-0.01	-0.02	-0.01	-0.04	-3.65	-0.09

Table 4.22 Difference in explanatory power compared to variable set 1' in the case of own power sourcing

Table 4.22 shows the difference in explanatory power with respect to variable set 1', which is the set that has been used in this thesis. From the table it is clear that the variable set we are using in the model yields the highest explanatory power.

A final remark is that since papers as Gamba (2003) and Bastian-Pinto et al. (2013) do not include any statistics from the regressions performed when using the least squares Monte Carlo method, there is little basis to be used for comparison. The decision on what set of explanatory variables to use has therefore been made based on our analysis above, which shows that variable set 1 and variable set 1' yield satisfactory explanatory power compared to other variable sets for the cases of external power contracts and own sourcing respectively.

Chapter 5

Conclusion

The global market for aluminium has the last few years been characterised by low LME prices and oversupply. This has put aluminium producers in a challenging position and increased the probability of smelter shutdowns, which has wider negative economic impacts. Capturing the flexibility available for an aluminium producer when valuating smelters is therefore of high relevance in order to create a good decision base for decision makers when developing strategies to meet the current market challenges.

In this project thesis we have defined an approach to value and derive an optimal operational policy for an aluminium smelter that is exposed to five different correlated stochastic variables and which has operational flexibility in terms of permanent and temporary shutdown options. The approach entails modelling the smelter cash flows under different electricity contract schemes as well as in the case of owning a co-generation unit and simultaneously solving the two interdependent problems of maximising the smelter value and deriving the optimal operating policy. Since no analytical solution to the defined problem exists the problem was solved numerically by using a modification of the least squares Monte Carlo valuation technique described in Gamba (2003). The generic nature of the latter made the technique directly applicable to the aluminium smelter problem and we found it to be fairly robust to the choice of explanatory variables to be used when estimating the continuation values through least squares regression.

In the analysis of the results we found that adding temporary shutdown options when already having permanent shutdown options is of highest value in cases where the permanent shutdown options go from being in the money to out of the money. Exercising the temporary shutdown option in such a scenario can be more profitable than conducting a permanent shutdown, thus acting as a hedge against downturns in the LME price. In this region, the dynamics of the shutdown probabilities with respect to the long-term expected mean LME

price change the most, and the series of permanent shutdown probabilities intercept with the series of probabilities of operating without any shutdowns. Outside this region we found that under expectations of a low long-term mean of the LME price, direct permanent shutdown is a more rational strategy than temporary shutdowns, and under expectations of a high long-term mean of the LME price, no shutdowns is the most rational strategy.

When comparing the different contract lengths we found that short-term electricity contracts experienced the sharpest decrease in probability of permanent shutdowns when increasing the long-term expected mean of the LME price. In order to minimise the risk of shutdowns, own power sourcing seems to be the most attractive choice, but this result is however biased as the power asset investment cost is not included in the resulting electricity cost incurred by the smelter. For the external electricity sourcing contracts, we found that the nominal increase in the smelter value when introducing full operational flexibility compared to having no operational flexibility is larger for long-term contracts. A possible explanation to this is that when having large amounts of pre-purchased electricity, the aluminium producer can exploit peaks in the spot price for which this pre-purchased amount of electricity is sold.

The results did as expected prove to be dependent on the speed of mean reversion. A higher speed of mean reversion yields a stronger increase in the probability of operating with no shutdowns when the long-term expected mean of the LME price increases compared to a low speed of mean reversion. Thus, a higher speed of mean reversion yields a steeper decreasing curve for the probability of permanent shutdowns, as well as a more narrow region where temporary shutdowns are value adding with respect to increasing levels of the expected mean LME price.

In summary, introducing full operational flexibility to an aluminium smelter can increase the smelter value since management may earn positive profits for lower levels of the long-term expected mean LME price, while also reducing the probability of permanent shutdowns. Analysing such a smelter from a real options approach helps capture the value of these benefits, and the least squares Monte Carlo method has shown to be a suitable way of doing this.

5.1 Further extensions

There are several potential future extensions of the work in this thesis. First, the assumption that the values of all stochastic variables follow AR(1)-processes is subject to discussion. There are possibly several different dynamics for the underlying stochastic processes that

have not been captured by the AR(1)-modelling, which is clear from the wide confidence intervals in table 4.2. Another remark regarding the time series modelling is that the long-term expected means of the variables are based on input from industry sources, there is thus room for improving the model through a more thorough analysis of the long-term expected means. Therefore a complete empirical analysis of the stochastic variables should be conducted.

Secondly, the analysis was based on holding the expected long-term means of all stochastic variables except for the LME price constant. Analysing the closure dynamics and smelter value varying one or more of these could definitely be of relevance.

Recent papers such as e.g. Nadarajah et al., 2014 study how to derive tighter lower and upper bounds for the real options valuation when using LSM as well as a modification of LSM. Figures 4.6- 4.10 show that there is a gap between the upper bound and smelter value estimate when full operational flexibility is in place. Conducting an analysis on lower-and upper bounds could help assess the accuracy of the model and help further assessment of its validity. Another related extension could be to use the values derived from the in-sample scenarios as upper bound.

Finally, another assumption of the model in this thesis is that the smelter can only hold one type of electricity contract during the simulated time period. In order to increase to what extent the model can be used as a valuable decision-making tool for industry players the model should be extended to include an optimization of a portfolio of electricity contracts to be held over the simulated time period.

References

- Alcoa (2014). Alcoa Worldwide Smelting Capacity as of September 30, 2014. Available from: <http://www.alcoa.com/ingot/en/capacity.asp>. [2014-11-19].
- Alexander, C. (2008). *Market Risk Analysis, Practical Financial Econometrics*, volume 1 of Market Risk Analysis of *The Wiley Finance Series*. John Wiley & Sons, 2008, West Sussex UK.
- Areal, N., Rodrigues, A., and Armada, M. J. R. (2008). Improvements to the Least Squares Monte Carlo Option Valuation Method. Management Research Unit - University of Minho, Portugal, working paper series.
- Bastian-Pinto, C. L., Brandão, L. E. T., Raphael, R., and Ozorio, L. M. (2013). Financial Valuation of Operational Flexibilities in the Aluminium Industry using Real Option Theory. Department of Management, Ibmecc Business School, IAG Business School; 2013 Real Options Annual Conference.
- Boyle, P. P. (1977). Options: A Monte Carlo Approach. *Journal of Financial Economics*, 4:323–338.
- Brekke, K. A. and Øksendal, B. (1994). Optimal Switching in an Economic Activity under Uncertainty. *SIAM Journal on Control and Optimization*, 32(4):1021–1036.
- Carmona, R. and Ludkovski, M. (2010). Valuation of Energy Storage: An Optimal Switching Approach. *Quantitative Finance*, 10(4):359–374.
- Carriere, J. F. (1996). Valuation of the early-exercise price for options using simulations and nonparametric regression. *Insurance: Mathematics and Economics*, 19(1):19–30.
- Center for European Policy Studies and Economisti Associati (2013). Assessment of Cumulative Cost Impact For The Steel And The Aluminium Industry. Available from: http://ec.europa.eu/enterprise/sectors/metals-minerals/files/final-report-aluminium_en.pdf. [2014-12-10].
- Cortazar, G. and Schwartz, L. (1998). Monte Carlo Evaluation Model of an Undeveloped Oil Field. *Journal of Energy Finance & Development*, 3:73–84.
- Dixit, A. and Pindyck, R. (1994). *Investment Under Uncertainty*. Princeton University press, New Jersey.
- Dubal Aluminium (2014). The Primary Aluminium Production Process. Available from: <https://www.dubal.ae/who-we-are/our-production-process/aluminium-smelting.aspx>. [2014-11-14].

- Gamba, A. (2003). Real Options Valuation: A Monte Carlo Approach. Faculty of Management, University of Calgary WP No. 2002/3; EFA 2002 Berlin Meetings Presented Paper.
- Garen, J., Jepsen, C., and Scott, F. (2009). Economic Forces Shaping the Aluminum Industry. University of Kentucky, Lexington - report prepared for the Sloan Center for a Sustainable Aluminum Industry.
- Geological Survey US (2011). *Minerals Yearbook, 2008, V. 1, Metals and Minerals*. Government Printing Office, 2011.
- Harton, T. W. (2003). Virtual Smelter Modelling for Metal Flow Management. Department of Engineering Science, University of Auckland, Proceedings of the 45th Annual Conference of the ORSNZ.
- Haugh, M. (2004). The Monte Carlo Framework, Examples from Finance and Generating Correlated Random Variables. Available from: http://www.columbia.edu/~mh2078/MCS04/MCS_framework_FEegs.pdf. [2014-12-15].
- Kulatilaka, N. and Trigeorgis, L. (1994). The General Flexibility to Switch: Real Options Revisited. *International Journal of Finance*, 6:778–798.
- Lemelin, B. (2009). Mine Project Evaluation: A Real Options Approach with Least-Squares Monte Carlo Simulations. Département de Génie des Mines, de la Métallurgie et des Matériaux, Université Laval Québec.
- Longstaff, E. S. and Schwartz, F. A. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. *The Review of Financial Studies*, 14(1):113–147.
- Mark, N. C. (2000). *International Macroeconomics and Finance: Theory and Empirical Methods*. Blackwell Publishers, 2000.
- Mazières, D. and Boogert, A. (2013). A radial basis function approach to gas storage valuation. *Journal of Energy Markets*, 6(2):19–50.
- Moreno, M. and Navas, J. F. (2003). On the Robustness of Least-Squares Monte Carlo (LSM) for Pricing American Derivatives. *Review of Derivatives Research*, 6(2).
- Nadarajah, S., Margot, F., and Secomandi, N. (2014). Comparison of Least Squares Monte Carlo Methods with Applications to Energy Real Options. College of Business Administration, University of Illinois at Chicago and Tepper School of Business, Carnegie Mellon University.
- Nappi, C. (2013). The Global Aluminium Industry 40 years from 1972. Available from: http://www.world-aluminium.org/media/filer_public/2013/02/25/an_outlook_of_the_global_aluminium_industry_1972_-_present_day.pdf. [2014-12-10].
- Norsk Hydro (2012). Aluminium, *Environment and Society*. Available from: http://www.hydro.com/upload/Documents/downloadcenter/About%20aluminium/Aluminium_environment-and-society.pdf. [2014-12-11].

- Norsk Hydro (2014). Market Outlook - Capital Markets Day 2014. Available from: http://www.hydro.com/upload/Documents/Presentations/Capital%20Markets%20Day/2014/Market_CMD_2014.pdf. [2014-12-11].
- Norwegian Water Resources and Energy Directorate (2001). Generell Orientering om Konsesjonskraft. KTV – Notat nr. 53/2001.
- Rodrigues, A. and Armada, M. J. R. (2006). The Valuation of Real Options with the Least Squares Monte Carlo Simulation Method. Management Research Unit - University of Minho, Portugal, Working paper series.
- Tsitsiklis, J. N. and Roy, B. V. (2001). Regression Methods for Pricing Complex American-Style Options. *IEEE Transactions on Neural Networks*, 12(4):694–703.
- United States International Trade Commission (1987). *Certain Electrical Conductor Aluminum Redraw Rod from Venezuela*. U.S. International Trade Commission, 1987.
- U.S. Geological Survey (2013). Annual Statistics Mineral Commodity Summaries - Bauxite and Alumina, February 2014. Available from: <http://minerals.usgs.gov/minerals/pubs/commodity/bauxite/mcs-2014-bauxi.pdf>. [2014-12-11].
- Øye, H. A. and Sørli, M. (2011). Power Failure, Restart and Repair. *Aluminum International Today*, 23(2):25–27.

Appendix A

Details of the mathematical model

Definitions of help variables

Extensive model for aluminium smelters

Parameters for aluminium smelters

Ca	Carbon Price	$USD/mt\ prod.al$
A	Alumina price	$USD/mt\ prod.al$
Raw	Ca+A: raw materials	$USD/mt\ prod.al$
Tax	Company tax rate	%
E^L	Electrolysis cost local currency given today's $x_{s,0}^N$	$USD/mt\ prod.al$
E^U	Electrolysis cost USD	$USD/mt\ prod.al$
C^L	Casthouse cost local currency given today's $x_{s,0}^N$	$USD/mt\ prod.al$
C^U	Casthouse income USD	$USD/mt\ prod.al$
M	Power input per production volume	$MWh/mt\ prod.al$
μ_{X^E}	Long-term expected mean of the USD/EUR exchange rate	USD/EUR
μ_{X^N}	Long-term expected mean of the USD/NOK exchange rate	USD/NOK
μ_{LME}	Long-term expected mean of the LME price	$USD/mt\ prod.al$
μ_{Y^1}	Long-term expected mean of the 1-year electricity price	EUR/MWh
μ_{Y^3}	Long-term expected mean of the 3-year electricity price	EUR/MWh

Help variables

$$\begin{aligned}
o_{s,t} &= \frac{E^L \times x_{s,0}^E}{x_{s,t}^E} + EU + \frac{C^L \times x_{s,0}^E}{x_{s,t}^E} - CU && \forall t, \forall s \\
P_{s,t,1,EUR} &= y_{s,t-1}^1 \times M \times x_{s,t}^E && \forall t, \forall s \\
P_{s,t,5,EUR} &= y_{s,0}^3 \times M \times x_{s,t}^E && t = 1 : 5, \forall s \\
P_{s,t,6,EUR} &= y_{s,5}^3 \times M \times x_{s,t}^E && t = 6 : 10, \forall s \\
P_{s,t,11,EUR} &= y_{s,10}^3 \times M \times x_{s,t}^E && t = 11 : 15, \forall s \\
P_{s,t,16,EUR} &= y_{s,15}^3 \times M \times x_{s,t}^E && t = 16 : 20, \forall s \\
P_{s,t,1,USD} &= y_{s,0}^3 \times M \times x_{s,t}^E && t = 1 : 10, \forall s \\
P_{s,t,11,USD} &= y_{s,10}^3 \times M \times x_{s,t}^E && t = 11 : 20, \forall s \\
P_{s,t,20,USD} &= y_{s,0}^3 \times M \times x_{s,t}^E && t = 1 : 20, \forall s \\
P_{s,t,5,NOK} &= y_{s,0}^3 \times M \times \frac{x_{s,0}^E}{x_{s,0}^N} x_{s,t}^N && t = 1 : 5, \forall s \\
P_{s,t,6,NOK} &= y_{s,5}^3 \times M \times \frac{x_{s,5}^E}{x_{s,5}^N} x_{s,t}^N && t = 6 : 10, \forall s \\
P_{s,t,11,NOK} &= y_{s,10}^3 \times M \times \frac{x_{s,10}^E}{x_{s,10}^N} x_{s,t}^N && t = 11 : 15, \forall s \\
P_{s,t,16,NOK} &= y_{s,15}^3 \times M \times \frac{x_{s,15}^E}{x_{s,15}^N} x_{s,t}^N && t = 16 : 20, \forall s \\
P_{s,t,1,USD} &= y_{s,0}^3 \times M \times \frac{x_{s,0}^E}{x_{s,0}^N} x_{s,t}^N && t = 1 : 10, \forall s \\
P_{s,t,11,USD} &= y_{s,10}^3 \times M \times \frac{x_{s,10}^E}{x_{s,10}^N} x_{s,t}^N && t = 11 : 20, \forall s \\
P_{s,t,20,USD} &= y_{s,0}^3 \times M \times \frac{x_{s,0}^E}{x_{s,0}^N} x_{s,t}^N && t = 1 : 20, \forall s \\
tax_{s,t,l,c} &= PretaxCF_{s,t,l,c} * Tax, && \forall s, \forall t, \forall l, \forall c \\
q_{s,t,1,c} &= 0 && \forall s, \forall t, \forall c \\
q_{s,t,5,c} &= 0 && t = [5, 10, 15, 20], \forall s, \forall c \\
q_{s,t,10,c} &= 0 && t = [10, 20], \forall s, \forall c \\
q_{s,t,20,c} &= 0 && t = 20, \forall s, \forall c
\end{aligned}$$

$$\begin{aligned}
q_{s,t,5,c} &= \left(\sum_{i=t+1}^5 M \right) \times (y_{s,t}^3 \times x_{s,t}^E - \frac{P_{s,t,l,c}}{M}) \quad t = 1 : 4, \forall s, \forall c \\
q_{s,t,5,c} &= \left(\sum_{i=t+1}^{10} M \right) \times (y_{s,t}^3 \times x_{s,t}^E - \frac{P_{s,t,l,c}}{M}) \quad t = 6 : 9, \forall s, \forall c \\
q_{s,t,5,c} &= \left(\sum_{i=t+1}^{15} M \right) \times (y_{s,t}^3 \times x_{s,t}^E - \frac{P_{s,t,l,c}}{M}) \quad t = 11 : 14, \forall s, \forall c \\
q_{s,t,5,c} &= \left(\sum_{i=t+1}^{20} M \right) \times (y_{s,t}^3 \times x_{s,t}^E - \frac{P_{s,t,l,c}}{M}) \quad t = 16 : 19, \forall s, \forall c \\
q_{s,t,10,c} &= \left(\sum_{i=t+1}^{10} M \right) \times (y_{s,t}^3 \times x_{s,t}^E - \frac{P_{s,t,l,c}}{M}) \quad t = 1 : 9, \forall s, \forall c \\
q_{s,t,10,c} &= \left(\sum_{i=t+1}^{20} M \right) \times (y_{s,t}^3 \times x_{s,t}^E - \frac{P_{s,t,l,c}}{M}) \quad t = 11 : 19, \forall s, \forall c \\
q_{s,t,20,c} &= \left(\sum_{i=t+1}^{20} M \right) \times (y_{s,t}^3 \times x_{s,t}^E - \frac{P_{s,t,l,c}}{M}) \quad t = 1 : 19, \forall s, \forall c
\end{aligned}$$

Extensive model for power sourcing

Parameters Power Sourcing

$OPEX$	Opex + sustaining capex	NOK/MWh
P	Yearly production	MWh
G^F	Fixed grid tariff	NOK/MWh
G^R	Variable grid tariff rate of electricity spot price	NOK/MWh
Tax^P	Property tax	NOK/MWh
LME^{1976}	1976-price in year t=0	NOK/MWh
S^R	Rate of production to regulated sales	NOK/MWh
S^P	Sales price regulated sales	NOK/MWh
A	Asset value for tax purposes	NOK
D	Depreciation period	$Years$
Tax_I	Free allowance of taxation asset value	$\%$
Tax_R	Resource rent tax rate	$\%$
π	Inflation rate	$\%$
G^{1976}	Yearly growth rate of the 1976-price	$\%$

Help variables own sourcing

$g_{s,t}$	$= (G^F + G^R \times y_{s,t}^1) \times P \times x_{s,t}^N$	[grid tariff (USD/mt) at time t in scenario s]
$l_{s,t}^{1976}$	$= LME^{1976} \times (G^{1976})^t \times x_{s,t}^N$	[1976-price (USD/mt) at time t in scenario s]
$i_{s,t}^{SA}$	$= ((1 - S^R) \times y_{s,t}^1 + S^R \times S^P) \times P \times x_{s,t}^N$	[income stand-alone (USD/mt) at time t in scenario s]
$i_{s,t}^{OU}$	$= ((1 - S^R) \times l_{s,t}^{1976} + S^R \times S^P) \times P \times x_{s,t}^N$	[income own use at time t in scenario s]
$t_{s,t}^P$	$= P \times Tax^P \times x_{s,t}^N$	[property tax own sourcing (USD/mt) at time t in scenario s]
$o_{s,t}^{powerasset}$	$= P \times (OPEX + g_{s,t} + t_{s,t}^P) \times x_{s,t}^N$	[opex, grid cost and property tax own sourcing (USD/mt produced aluminium) at time t in scenario s]
$d_{s,t}$	$= \frac{P \times A \times (1 + \pi)^t}{D} \times x_{s,t}^N \times 1000$	[depreciation (USD/mt) at time t in scenario s]
$f_{s,t}$	$= P \times A \times \tau_I \times x_{s,t}^N \times 1000$	[free allowance (USD/mt)]
$m_{s,t}$	$= P \times S^R \times (S^P - y_{s,t}^1) \times M \times x_{s,t}^N$	[regulated sales margin cost USD/mt produced aluminium at time t in scenario s]
$CF_pretax_{s,t}^{OU}$	$= i_{s,t}^{OU} - OPEX \times P \times x_{s,t} - g_{s,t} - t_{s,t}^P$	[the income component of the cash flow is calculated by multiplying the yearly production with the 1976-price]
$CF_pretax_{s,t}^{SA}$	$= i_{s,t}^{SA} - opex_{s,t}^{powerassetSA} \times x_{s,t} - g_{s,t} - tax_{s,t}$	[the income component of the cash flow is calculated by multiplying the yearly production with the spot electricity price]
$rrt_{s,t}^{OU}$	$= (CF_pretax_{s,t}^{OU} - d_{s,t} - f_{s,t}) \times Tax_R$	[resource rent tax as explained later in this appendix]
$Tax_sourced_{s,t}$	$= (PretaxCF_sourced_{s,t} - d_{s,t}) \times Tax$	[corporate tax with own sourcing]
$rrt_{s,t}^{SA}$	$= (CF_pretax_{s,t}^{SA} - d_{s,t} - f_{s,t}) \times Tax_R$	[resource rent tax as explained later in this appendix, but with income component based on spot electricity price]
$Tax-SA_{s,t}$	$= (PretaxCF_SA_{s,t} - d_{s,t}) \times Tax$	[corporate tax stand-alone power asset]

Detailed explanation of cash flows with own power sourcing

Let "Regulated sales margin cost" be denoted by $m_{s,t}$ in terms of USD/mt produced aluminium. Further on, let $o_{s,t}^{powerasset}$ (defined in appendix A) be the sum of operational expenses, grid tariff and variable property tax for the power asset (USD/mt produced aluminium), then the power contract price for the smelter, $p_{s,t}$, in the case of own power sourcing is just $m_{s,t} + o_{s,t}^{powerasset}$, also in terms of USD/mt produced aluminium. An additional cost incurred by the owner of the power asset is the so-called industry specific resource rent tax, $rrt_{s,t}^{ownuse}$, which is derived from the power asset cash flows linked to the 1976 reference price. Thus, we get the following unadjusted cash flow (USD/mt produced aluminium) for the smelter which is only dependent on time and scenario, $FCF_sourced_{s,t}^{unadjusted}$, when power is sourced from an own power asset:

$$\begin{aligned}
 & l_{t,s} \\
 & \quad - Raw \\
 & \quad - o_{s,t}^{powerasset} \\
 & \quad - m_{s,t} \\
 & \quad = \underline{PretaxCF_sourced_{s,t}} \\
 & \quad - tax_sourced_{s,t} \\
 & \quad - rrt_{s,t}^{ownuse} \\
 & \quad = \underline{FCF_sourced_{s,t}^{unadjusted}}
 \end{aligned}$$

The after-tax stand-alone free cash flows for the power asset must also be calculated as these are received if the smelter is shutdown at any given point thus selling the output of the power asset externally. Let $i_{s,t}^{SA}$ be the stand-alone income (USD/mt planned produced aluminium) of the power asset, which is the sum of revenues from unregulated- and regulated power sales (see appendix A for detailed derivation). The after-tax stand-alone free cash flows can then be formulated the following way:

$$\begin{aligned} & i_{s,t}^{standalone} \\ & - opex_{s,t}^{powerasset} \\ & = \underline{PretaxCF_standalone}_{s,t} \\ & - tax_standalone_{s,t} \\ & - rrt_{s,t}^{standalone} \\ & = \underline{FCF_standalone}_{s,t}^{unadjusted} \end{aligned}$$

Appendix B

Additional result figures

Value plots

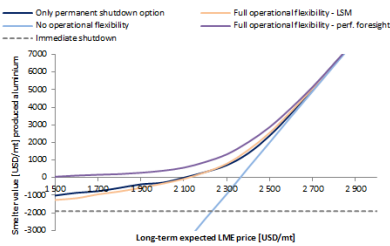


Fig. B.1 Value plot -20-year NOK contract

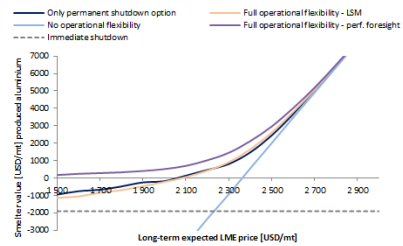


Fig. B.2 Value plot - 20-year EUR contract

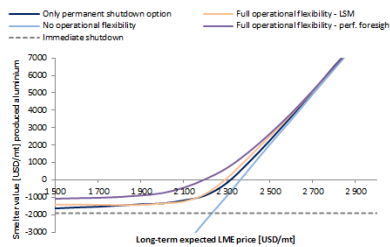


Fig. B.3 Value plot - 10-year NOK contract

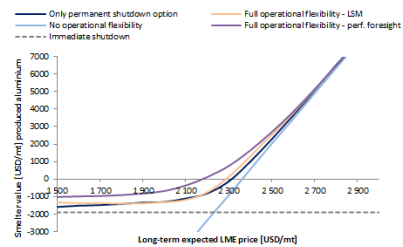


Fig. B.4 Value plot - 10-year EUR contract

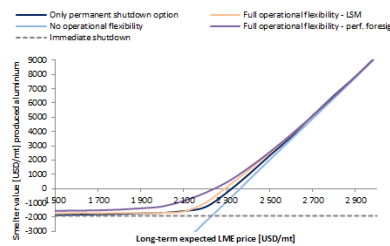


Fig. B.5 Value plot - 5-year NOK contract

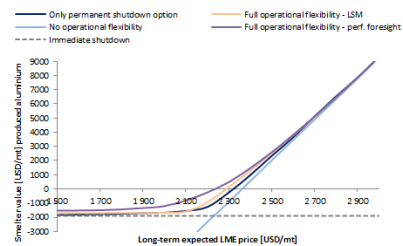


Fig. B.6 Value plot - 5-year EUR contract

Shutdown ratios

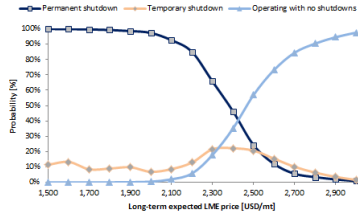


Fig. B.7 Shutdown risk: 20-year NOK contract

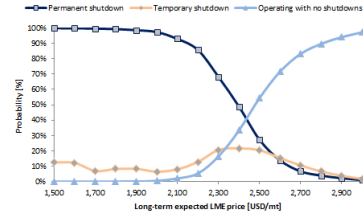


Fig. B.8 Shutdown risk: 20-year EUR contract

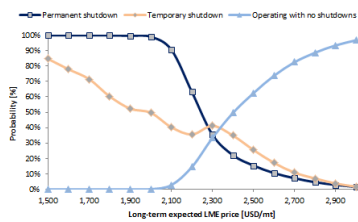


Fig. B.9 Shutdown risk: 10-year NOK contract

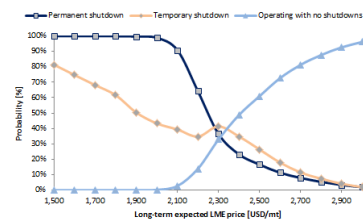


Fig. B.10 Shutdown risk: 10-year EUR contract

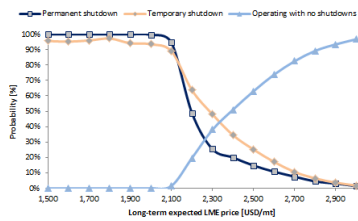


Fig. B.11 Shutdown risk: 5-year NOK contract

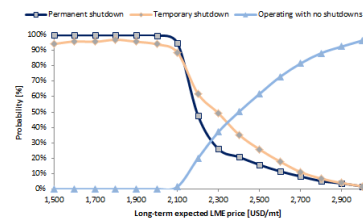


Fig. B.12 Shutdown risk: 5-year EUR contract

Shutdown triggers

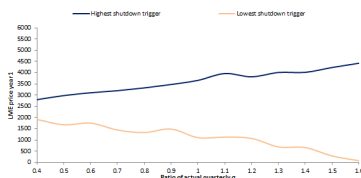


Fig. B.13 Shutdown triggers: 20-year NOK contract

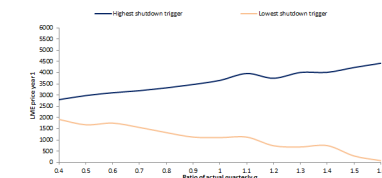


Fig. B.14 Shutdown triggers: 20-year EUR contract

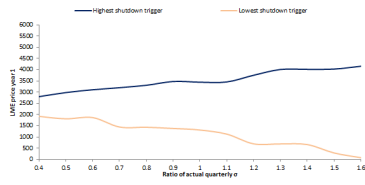


Fig. B.15 Shutdown triggers: 10-year NOK contract

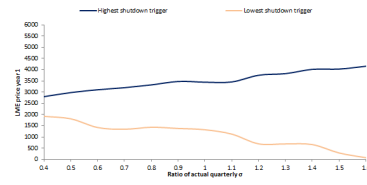


Fig. B.16 Shutdown triggers: 10-year EUR contract

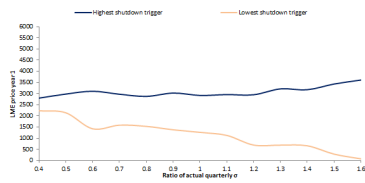


Fig. B.17 Shutdown triggers: 5-year NOK contract

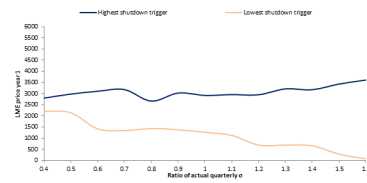


Fig. B.18 Shutdown triggers: 5-year EUR contract

Sensitivity tables

20-year NOK

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	112	294	716
	Base	1,578	1,720	2,031
	<i>Low</i>	4,918	4,976	5,064

Table B.1 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	<i>High</i>	44	120	239
	Base	60	83	127
	<i>Low</i>	10	4	-6

Table B.2 Value over only perm. shut. options

20-year EUR

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	142	325	739
	Base	1,699	1,837	2,134
	Low	5,037	5,113	5,193

Table B.3 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	43	122	234
	Base	72	100	152
	Low	22	30	27

Table B.4 Value over only perm. shut. options

10-year NOK

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	65	310	753
	Base	728	1,022	1,564
	Low	3,834	4,000	4,142

Table B.5 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	9	133	298
	Base	167	270	357
	Low	6	45	42

Table B.6 Value over only perm. shut. options

10-year EUR

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	76	334	776
	Base	782	1,061	1,595
	Low	3,886	4,009	4,185

Table B.7 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	20	138	299
	Base	181	261	353
	Low	4	3	38

Table B.8 Value over only perm. shut. options

5-year NOK

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	63	302	743
	Base	659	1,022	1,554
	Low	3,576	3,612	3,798

Table B.9 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	10	129	236
	Base	333	383	346
	Low	51	-6	16

Table B.10 Value over only perm. shut. options

5-year EUR

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	72	325	765
	Base	691	1,058	1,583
	Low	3,600	3,615	3,843

Table B.11 Value over no closure

		Quarterly volatility		
		0.5x	1.0x	1.5x
LME	High	12	133	244
	Base	343	388	357
	Low	44	-4	33

Table B.12 Value over only perm. shut. options

Appendix C

Sensitivities to expected long-term mean of the different stochastic variables

Tables of smelter value and shutdown probabilities sensitivities to the expected long-term mean of the different stochastic variables

The sensitivities in the following tables are based on the differences compared to a base case. In this base case the following assumptions are made:

- $\mu_{Y^1} = 40 \text{ EUR}/MWh$
- $\mu_{Y^3} = 40 \text{ EUR}/MWh$
- $\mu_l = \$2,300$
- $\mu_{X^E} = 1.17$
- $\mu_{X^N} = 0.17$

Here μ_l denotes the expected long-term mean of the LME price, while descriptions of the other notations are listed in table 4.1.

<i>Own sourcing</i>	Variable	Multiple	Value difference [USD/mt]	Difference prob. perm. shutdown	Difference prob. temp. shutdown	Difference prob. no shutdowns
	El. price 1-year mean	0.7x	91	-3.5 %	-13.6 %	16.4 %
	El. price 1-year mean	1.3x	85	42.1 %	7.9 %	-25.4 %
	El. price 3-year mean	0.7x	-125	1.0 %	0.5 %	-0.7 %
	El. price 3-year mean	1.3x	551	-2.2 %	0.9 %	0.8 %
	LME price mean	0.7x	-1,548	88.9 %	-46.4 %	-39.7 %
	LME price mean	1.3x	9,111	-10.5 %	-47.1 %	56.9 %
	USD/EUR mean	0.7x	-85	-6.4 %	-10.4 %	16.5 %
	USD/EUR mean	1.3x	486	45.4 %	2.7 %	-26.3 %
	USD/NOK mean	0.7x	1,275	-3.4 %	-10.0 %	13.0 %
USD/NOK mean	1.3x	-1,001	20.6 %	6.4 %	-16.5 %	

Table C.1 Own sourcing - deviations from base case when varying long-term expected means of the stochastic variables

<i>20-year USD</i>	Variable	Multiple	Value difference	Difference prob. perm. shutdown	Difference prob. temp. shutdown	Difference prob. no shutdowns
	El. price 1-year mean	0.7x	-1,409	4.2 %	-2.1 %	-2.3 %
	El. price 1-year mean	1.3x	2,786	4.8 %	-2.5 %	-2.0 %
	El. price 3-year mean	0.7x	3,430	6.4 %	-3.6 %	-3.7 %
	El. price 3-year mean	1.3x	-3,183	26.5 %	-13.0 %	-14.8 %
	LME price mean	0.7x	-1,890	31.1 %	-10.4 %	-15.9 %
	LME price mean	1.3x	7,923	-67.2 %	-17.1 %	80.0 %
	USD/EUR mean	0.7x	1,493	-48.2 %	2.1 %	44.7 %
	USD/EUR mean	1.3x	-212	25.6 %	-12.8 %	-14.8 %
	USD/NOK mean	0.7x	-30	7.7 %	-3.8 %	-4.5 %
USD/NOK mean	1.3x	44	4.9 %	-2.8 %	-2.2 %	

Table C.2 20-year USD - deviations from base case when varying long-term expected means of the stochastic variables

<i>10-year USD</i>	Variable	Multiple	Value difference	Difference prob. perm. shutdown	Difference prob. temp. shutdown	Difference prob. no shutdowns
	El. price 1-year mean	0.7x	-448	-1.2 %	0.5 %	0.4 %
	El. price 1-year mean	1.3x	1,150	-0.7 %	0.0 %	0.9 %
	El. price 3-year mean	0.7x	3,262	-1.2 %	0.7 %	0.3 %
	El. price 3-year mean	1.3x	-2,630	48.7 %	-10.8 %	-25.9 %
	LME price mean	0.7x	-1,554	56.8 %	20.5 %	-27.9 %
	LME price mean	1.3x	8,973	-40.4 %	-37.3 %	66.2 %
	USD/EUR mean	0.7x	2,475	-27.8 %	-16.0 %	36.0 %
	USD/EUR mean	1.3x	-1,048	45.7 %	-17.6 %	-25.6 %
	USD/NOK mean	0.7x	-7	-0.2 %	1.4 %	-0.5 %
USD/NOK mean	1.3x	75	-1.7 %	-1.5 %	1.6 %	

Table C.3 10-year USD - deviations from base case when varying long-term expected means of the stochastic variables

<i>10-year NOK</i>	Variable	Multiple	Value difference	Difference prob. perm. shutdown	Difference prob. temp. shutdown	Difference prob. no shutdowns
	El. price 1-year mean	0.7x	-525	-8.9 %	2.8 %	5.0 %
	El. price 1-year mean	1.3x	981	-7.8 %	1.5 %	5.3 %
	El. price 3-year mean	0.7x	3,214	-9.2 %	2.6 %	5.2 %
	El. price 3-year mean	1.3x	-3,011	49.8 %	-3.3 %	-26.0 %
	LME price mean	0.7x	-1,621	56.8 %	37.3 %	-27.9 %
	LME price mean	1.3x	8,981	-41.8 %	-38.2 %	68.4 %
	USD/EUR mean	0.7x	2,443	-30.3 %	-17.3 %	39.1 %
	USD/EUR mean	1.3x	-1,322	46.0 %	-14.7 %	-25.5 %
	USD/NOK mean	0.7x	-221	-11.1 %	4.3 %	5.9 %

Table C.4 10-year NOK - deviations from base case when varying long-term expected means of the stochastic variables

<i>10-year EUR</i>	Variable	Multiple	Value difference	Difference prob. perm. shutdown	Difference prob. temp. shutdown	Difference prob. no shutdowns
	El. price 1-year mean	0.7x	-502	-6.7 %	2.0 %	3.9 %
	El. price 1-year mean	1.3x	1,019	-6.6 %	1.3 %	4.5 %
	El. price 3-year mean	0.7x	3,224	-7.0 %	2.3 %	4.0 %
	El. price 3-year mean	1.3x	-2,920	49.8 %	-6.8 %	-26.1 %
	LME price mean	0.7x	-1,604	56.8 %	28.5 %	-27.9 %
	LME price mean	1.3x	8,979	-41.5 %	-38.0 %	67.9 %
	USD/EUR mean	0.7x	2,433	-30.0 %	-17.6 %	39.3 %
	USD/EUR mean	1.3x	-1,234	45.5 %	-17.0 %	-25.4 %
	USD/NOK mean	0.7x	-115	-6.4 %	3.4 %	3.4 %
USD/NOK mean	1.3x	-34	-7.3 %	-0.2 %	5.4 %	

Table C.5 10-year EUR - deviations from base case when varying long-term expected means of the stochastic variables

<i>5-year USD</i>	Variable	Multiple	Value difference	Difference prob. perm. shutdown	Difference prob. temp. shutdown	Difference prob. no shutdowns
	El. price 1-year mean	0.7x	-410	-0.8 %	-0.2 %	0.7 %
	El. price 1-year mean	1.3x	999	-0.5 %	0.1 %	0.3 %
	El. price 3-year mean	0.7x	3,251	-0.2 %	-0.3 %	0.2 %
	El. price 3-year mean	1.3x	-2,576	64.0 %	35.4 %	-33.5 %
	LME price mean	0.7x	-1,832	71.0 %	42.5 %	-35.0 %
	LME price mean	1.3x	9,015	-26.2 %	-45.7 %	59.3 %
	USD/EUR mean	0.7x	2,534	-13.5 %	-23.9 %	28.9 %
	USD/EUR mean	1.3x	-1,635	62.8 %	31.6 %	-33.2 %
	USD/NOK mean	0.7x	12	-0.7 %	2.4 %	-0.5 %
USD/NOK mean	1.3x	50	-0.9 %	-1.9 %	1.8 %	

Table C.6 5-year USD - deviations from base case when varying long-term expected means of the stochastic variables

<i>Spot</i>	Variable	Multiple	Value difference	Difference prob. perm. shutdown	Difference prob. temp. shutdown	Difference prob. no shutdowns
	El. price 1-year mean	0.7x	2,687	0.1 %	-23.7 %	23.5 %
	El. price 1-year mean	1.3x	-1,539	91.6 %	47.3 %	-47.1 %
	El. price 3-year mean	0.7x	-17	-0.2 %	0.2 %	-0.1 %
	El. price 3-year mean	1.3x	-12	-0.3 %	-0.3 %	0.6 %
	LME price mean	0.7x	-1,629	99.6 %	49.8 %	-49.5 %
	LME price mean	1.3x	9,301	-0.4 %	-47.9 %	48.2 %
	USD/EUR mean	0.7x	2,695	-0.3 %	-24.3 %	24.6 %
	USD/EUR mean	1.3x	-1,537	92.3 %	47.7 %	-47.5 %
	USD/NOK mean	0.7x	-52	-0.1 %	-0.1 %	0.2 %
USD/NOK mean	1.3x	-12	-0.3 %	-0.9 %	1.1 %	

Table C.7 Spot - deviations from base case when varying long-term expected means of the stochastic variables

Comments to the tables

The degree of impact from the different stochastic variables on the free cash flows of an aluminium smelter is varying due to the heterogeneous composition of cost and revenues. When analysing e.g. the smelter value or the shutdown probabilities one would ideally take into account varying expectations about all stochastic variables, but this could potentially produce results that are hard to interpret. In order to limit the scope of such analyses a natural approach is to vary expectations about only one of the stochastic variables while holding all other expectations constant. Note that all variables are still stochastic, but that the long-term expected means of the non-analysed variables are now held constant. Intuitively the stochastic variable for which to vary expectations should have large impact on the free cash flows of the smelter. First, tables C.1- C.7 clearly show that when doing the same relative change of the long-term expected mean of one of the stochastic variables, the LME expectations have the strongest impact on the free cash flows as well as on shutdown probabilities. The natural interpretation of this is that the LME price is the main source of revenues for the smelter. We therefore argue that doing analyses for different expectations of the long-term expected mean of the LME price should be of highest priority. Secondly, it is clear that the USD/EUR exchange rate also has a significant impact on the free cash flows and shutdown probabilities. This is probably because the base currency of electricity is EUR. Thus, even if entering long-term electricity contracts in other currencies than EUR, the net gain from the position is still dependent on how the USD/EUR exchange rate evolves. This is illustrated by tables C.3- C.5. The impact on free cash flows and shutdown probabilities from varying the long-term expected mean of the USD/EUR exchange rate is approximately the same for the 10-year contracts in USD, NOK and EUR. Consequently, after the LME price, doing analyses varying the long-term expected mean of the USD/EUR exchange rate should be prioritized. Finally, varying the long-term expected mean of the 3-year electricity price has a noticeable impact on the free cash flows and shutdown probabilities except for the spot contracts which intuitively are noticeably impacted by the 1-year electricity price.

Appendix D

Pseudocode for the MATLAB program

Below is the pseudocode for the most important routines of the Matlab program that was written to solve the problem described in this thesis. A few comments to the attached pseudocode:

- The code for the cases of own power sourcing is very similar to the other contract types with only a few minor differences. Therefore this has not been attached.
- The only difference between the LSM approach and perfect foresight is that one uses the actual continuation values along a path and not the approximated ones in order to derive an operating policy. The code for perfect foresight is thus not attached.
- Scenarios for the time series of the stochastic variables are generated following the procedure described in sections 3.5. Scenarios are generated on quarterly basis and in our model we then just pick the generated data from each 4th time step to be used for each respective year. The associated code is not attached.
- The Matlab program also involves a number of other functions that are used to analyse the data and write out the results presented in section 4, this is not attached.
- In order to solve the smelter problem when only having permanent shutdown options one only has to set the cost to go from operating to permanently shutdown as well as temporary shutdown opex arbitrarily large.

Main Program

```

Get input data on assumptions and historical time series for variables from external file
For Each TimeSeries Do %Calculate AR-parameters
    [Coefficients(i, :, :), Residuals(:,i)] = Regression(NominalValues(:,i), LongTermMean(i,1), ExplicitLongTermMean); %Calculate
AR-parameters
End
InSampleScenarios= call GenerateScenarios %generates scenarios for all the variable types to be used for regressions
OutSampleScenarios= call GenerateScenarios %generates scenarios for all the variable types to be used for valuations
For Each ContractType Do %calculate FCFs of smelter and RemExp at each time step for each scenario
    If ContractType="Own sourcing" Then
        [InFcfScenarios, InRemExp, InCfPowerAsset] = call ContractPower(InSampleScenarios, ContractType)
        [OutFcfScenarios, OutRemExp, OutCfPowerAsset] = call ContractPower(OutSampleScenarios, ContractType)
    Else
        [InFcfScenarios, InRemExp, InContractPrice] = call Contract(InSampleScenarios, ContractType)
        [OutFcfScenarios, OutInRemExp, OutInContractPrice] = call Contract(OutSampleScenarios, ContractType)
    End If
End For
For Each ContractType Do %calculate the average NPV of all scenarios for the different closure triggers
    AverageNpvNoClosure = call NoClosure(OutFcfScenarios, RemExp)
    AverageNpvComboLogic = call CumulativeMarginsTrigger(OutFcfScenarios, RemExp)
    AverageNpvPerfForesight = call PerfectForesight(OutFcfScenarios, RemExp)
End For
For Each ContractType Do %calculate the average NPV of all scenarios when only permanent shutdown option and store the
results
    If ContractType="Own sourcing" Then
        [CoefsContinuationOperating, CoefsContinuationTemporary, CoefsRemExp]=call
        CoefGeneratorPermShutOwnSourcing( InFcfScenarios, InSampleScenarios, InRemExp,
        InCfPowerAsset)
        [AverageNpvPermShutOwnSourcing, PolicyPermShutOwnSourcing]=call PermShutOwnSourcing
        (OutFcfScenarios, OutScenarios, CoefsContinuationOperating,
        CoefsContinuationTemporary, CoefsRemExp, OutCfPowerAsset)
    Else
        [CoefsContinuationOperating, CoefsContinuationTemporary]=call CoefGeneratorPermShut
        (InFcfScenarios, InSampleScenarios, InRemExp, InCfPowerAsset)
        [AverageNpvPermShut, PolicyPermShut]=call Permshut
        (OutFcfScenarios, OutScenarios, CoefsContinuationOperating,
        CoefsContinuationTemporary)
    End If
End For
For Each ContractType Do %calculate the average NPV of all scenarios when full operational flexibility using the LSM method and store
the results
    If ContractType="Own sourcing" Then
        [CoefsContinuationOperating, CoefsContinuationTemporary, CoefsRemExp]=call
        CoefGeneratorOwnSourcing( InFcfScenarios, InSampleScenarios, InRemExp,
        InCfPowerAsset)
        [AverageNpvPermShutOwnSourcing, PolicyPermShutOwnSourcing]=call SwitchingOwnSourcing
        (OutFcfScenarios, OutScenarios, CoefsContinuationOperating,
        CoefsContinuationTemporary, CoefsRemExp, OutCfPowerAsset)
    Else
        [CoefsContinuationOperating, CoefsContinuationTemporary]=call CoefGenerator
        (InFcfScenarios, InSampleScenarios, InRemExp, InCfPowerAsset)
        [AverageNpvPermShut, PolicyPermShut]=call Switching
        (OutFcfScenarios, OutScenarios, CoefsContinuationOperating,
        CoefsContinuationTemporary)
    End If
End For
For Each ContractType Do %calculate the average NPV of all scenarios when full operational flexibility under perfect f
oresight and store the results
    If ContractType="Own sourcing" Then
        [AverageNpvPermShutOwnSourcing, PolicyPermShutOwnSourcing]=call SwitchingOwnSourcingPerfForesight
        (OutFcfScenarios, OutScenarios, CoefsContinuationOperating,
        CoefsContinuationTemporary, CoefsRemExp, OutCfPowerAsset)
    Else
        [AverageNpvPermShut, PolicyPermShut]=call SwitchingPerfForesight
        (OutFcfScenarios, OutScenarios, CoefsContinuationOperating,
        CoefsContinuationTemporary)
    End If
End For

```

CoefGeneratorFullOperationalFlexibility

Note that 0 = operating, 1 = temporarily shutdown and 2 = permanently shutdown

Initiate parameters:

```
- NumScen = # of scenarios
- T = lifetime
- InSampleScenarios = scenarios for the short-term electricity price
- ShutdownCostOtoP = switching cost from operating to permanent shutdown
- ShutdownCostOtoT = switching cost from operating to temporary shutdown
- StartupCost = switching cost from temporary shutdown to operating
- ShutdownCostTtoP = switching cost from temporary shutdown to permanent shutdown
- ShutdownOpex = operational expenses incurred while in temporary shutdown
- TempDuration = maximum number of years in temporary shutdown
- Wacc = discount rate for the cash flows of the smelter
```

Initiate variables and arrays:

```
- ElSale = net value of selling current year's pre-purchased electricity in the spot market
- yOperating = array to store NPV of coming from an operating state at each time step
- yTemporaryShutdown = array to store NPV of coming from an operating state at each time step
- yTemporaryShutdownAdjusted = array used to temporarily store values when maximum limit of periods in temporary shutdown is exceeded
- CoefOperatingStorage = store coefficients used to approximate continuation values of being in operating mode
- CoefTemporaryStorage = store coefficients used to approximate continuation values of in temporary shutdown
- Tarray = array that holds the remaining time of a contract given time t.
- TemporaryShutdownCounter = counts the number of executive time periods in which the smelter has been temporarily shutdown
- RegressionParameters = array that temporarily stores values of explanatory variables for the multivariate regression
```

% Filling Tarray and ElSale

```
For t = 2:T
    For s = 1:NumScen,
        if mod(t-1,ContractLength)==0
            else
                Tarray(s,t) = ContractLength-mod(t-1,ContractLength);
            end
        if mod(t-1, ContractLength) == 0
            ElSale(s,t) = 0;
        else
            ElSale(s,t) = 14*InSampleScenarios(s,t)-ContractPriceUsdTon(s,t);
        end
    end
end
```

%Calculate smelter values at time T and respective terminal values. Store in index t-1.

```
For s = 1:NumScen,
    %Create terminal value estimates
    yOperatingTerminalValue=InSampleFcf(s,T)/Wacc;
    yTemporaryShutdownTerminalValue=(ElSale(s,T-1)+ShutdownOpex)/Wacc;
```

```
% Calculate values for yOperating - that is given that you at T-1 is operating, what operating mode should be chosen at T
If (InSampleFcf(s,T)+yOperatingTerminalValue> ShutdownCostOtoT+ElSale(s,T-1)+ShutdownOpex
+yTemporaryShutdownTerminalValue && InSampleFcf(s,T)+yOperatingTerminalValue>(ShutdownCostOtoP
+RemExp(s,T-1))), %Choose to remain operating
    yOperating(s,T-1,1)=(InSampleFcf(s,T)+yOperatingTerminalValue)*(1/(1+Wacc));
Else If (ShutdownCostOtoT+ElSale(s,T-1)+ShutdownOpex+yTemporaryShutdownTerminalValue>InSampleFcf(s,T)
+yOperatingTerminalValue && ShutdownCostOtoT+ElSale(s,T-1)+ShutdownOpex
+yTemporaryShutdownTerminalValue>ShutdownCostOtoP+RemExp(s,T-1)), %Choose to temporarily
shutdown
    yOperating(s,T-1,1)=(ShutdownCostOtoT+ElSale(s,T-1)
+ShutdownOpex+yTemporaryShutdownTerminalValue)*(1/(1+Wacc));
Else %Choose to permanently shutdown
    yOperating(s,T-1,1)=(ShutdownCostOtoP+RemExp(s,T-1))*(1/(1+Wacc));
End If
```

% Calculate values for yTemporaryShutdown- that is given that you at T-1 are temporarily shutdown, what operating mode should be chosen at T. Store in index t-1.

```
If (InSampleFcf(s,T)+StartupCost+yOperatingTerminalValue> ElSale(s,T-1)+ShutdownOpex
+yTemporaryShutdownTerminalValue && InSampleFcf(s,T)+StartupCost+yOperatingTerminalValue>
(ShutdownCostTtoP+RemExp(s,T-1))), % Choose to open the smelter at time T
    yTemporaryShutdown(s,T-1,1)=(InSampleFcf(s,T)+yOperatingTerminalValue)*(1/(1+Wacc));
    TemporaryShutdownCounter(s)=0;
Else If (ElSale(s,T-1)+ShutdownOpex+yTemporaryShutdownTerminalValue>InSampleFcf(s,T)+StartupCost+
yOperatingTerminalValue && ElSale(s,T-1)+ShutdownOpex+yTemporaryShutdownTerminalValue>
ShutdownCostTtoP+RemExp(s,T-1)), % Choose to remain temporarily shutdown at time T
    yTemporaryShutdown(s,T-1,1)=(ElSale(s,T-1)+ShutdownOpex+yTemporaryShutdownTerminalValue)
*(1/(1+Wacc));
    TemporaryShutdownCounter(s)=1;
Else %Shut down permanently at time T
    yTemporaryShutdown(s,T-1,1)=(ShutdownCostTtoP+RemExp(s,T-1))*(1/(1+Wacc));
    TemporaryShutdownCounter(s)=0;
End If
```

```
End For
```

t=T;

While t>2 %The backwards dynamic programming routine. Starts at t=T and works backwards.

% Regress continuation value of operating and temporary shutdown at time t on values of the explanatory variables at time t-1 and store the coefficients

```
[CoefOperating,~,statsOper]=MultiVarRegressionFiveVariables(InSampleFcf(:,t-1), InSampleScenarios(:,t-1),InSampleUsdEur
(:,t-2), InSampleNpy3(:,t-1), Tarray(:,t-1), yOperating(:,t-1,1), NumScen);
[CoefTemporary,~,statsTemp]=MultiVarRegressionFiveVariables(InSampleFcf(:,t-1), InSampleScenarios(:,t-1),InSampleUsdEur
(:,t-2), InSampleNpy3(:,t-1),Tarray(:,t-1), yTemporaryShutdown(:,t-1,1), NumScen);
```

-> Store the regression coefficients in CoefOperatingStorage and CoefTemporaryStorage

-> Create an array with the values of the explanatory variables at time t

```
For s=1:NumScen, % For each scenario determine the optimal operating mode at time t
    % Calculate approximations of the continuation values of operating and temporarily shutdown
    ContinuationOperating=(RegressionParameters(s,:)*CoefOperating);
    ContinuationTemporary=(RegressionParameters(s,:)*CoefTemporary);
```

% Calculate values for yOperating - that is given that you at t-1 is operating, what operating mode should be chosen at t.

```
If (InSampleFcf(s,t-1)+ContinuationOperating> ShutdownCostOtoT+ElSale(s,t-2)+ShutdownOpex+ContinuationTemporary
&& InSampleFcf(s,t-1)+ContinuationOperating>(ShutdownCostOtoP+RemExp(s,t-2))), %Choose to remain
operating at time t
    yOperating(s,t-2,1)=(InSampleFcf(s,t-1)+ContinuationOperating)*(1/(1+Wacc));
Else If (ShutdownCostOtoT+ElSale(s,t-2)+ShutdownOpex+ContinuationTemporary>InSampleFcf(s,t-1)
+ContinuationOperating && ShutdownCostOtoT+ElSale(s,t-2)+ShutdownOpex+ContinuationTemporary>
```



```

ShutdownCostOtoP+RemExp(s,t-2)), % Choose to temporarily shutdown at time t
    yOperating(s,t-2,1)=(ShutdownCostOtoP+ElSale(s,t-2)+ShutdownOpex+ContinuationTemporary)*(1/(1
        +Wacc));
Else %Shut down permanently at time t
    yOperating(s,t-2,1)=(ShutdownCostOtoP+RemExp(s,t-2))*(1/(1+Wacc));
End If

% Calculate values for yTemporaryShutdown- that is given that you at t-1 are temporarily shutdown, what
operating mode should be chosen at t. Store in index t-1.
If (InSampleFcf(s,t-1)+StartupCost+ContinuationOperating> ElSale(s,t-2)+ShutdownOpex+ContinuationTemporary &&
InSampleFcf(s,t-1)+StartupCost+ContinuationOperating>(ShutdownCostTtoP+RemExp(s,t-2))), %Choose to re-
open at time t
    yTemporaryShutdown(s,t-2,1)=(InSampleFcf(s,t-1)+StartupCost+ContinuationOperating)*(1/(1+Wacc));
    yTemporaryShutdown(s,t-1,2)=0;
    TemporaryShutdownCounter(s)=0;
    For i=t:T
        yTemporaryShutdown(s,i,2)=yOperating(s,i,2); %Since choose to re-open ,the optimal operating
            modes after time t correspond to yOperating
    End For
Else If (ElSale(s,t-2)+ShutdownOpex+ContinuationTemporary>InSampleFcf(s,t-1)+StartupCost+ContinuationOperating
&& ElSale(s,t-2)+ShutdownOpex+ContinuationTemporary>ShutdownCostTtoP+RemExp(s,t-2)), %Choose to remain
temporarily shutdown
    yTemporaryShutdown(s,t-2,1)=(ElSale(s,t-2)+ShutdownOpex+ContinuationTemporary)*(1/(1+Wacc));
    yTemporaryShutdown(s,t-1,2)=1;
    TemporaryShutdownCounter(s)=TemporaryShutdownCounter(s)+1;
Else %Choose to permanently shutdown at time t
    yTemporaryShutdown(s,t-2,1)=(ShutdownCostTtoP+RemExp(s,t-2))*(1/(1+Wacc));
    yTemporaryShutdown(s,t-1,2)=2;
    For i=t:T
        yTemporaryShutdown(s,i,2)=2; %Since permanent shutdown, operating mode at all times after time t is
            permanently shutdown
    End For
    TemporaryShutdownCounter(s)=0;
End If
End For

If t<T
    yTemporaryShutdownAdjusted= array to store temporary choices when solving violations
    -> Check if TemporaryShutdownCounter(s)=3 , then set Violation=1 %If so the maximum number of periods in temporary
        shutdown has been exceeded in scenario s

    If Violation==1; %If maximum number of periods in temporary shutdown exceeded then check if best to change the
        operating mode at the end of the violation period to operating or permanently shutdown. This is done
        working on with the yTemporaryShutdown array.
    -> Store values of explanatory variables at time t+2
    w=t+1
    For s=1:NumScen
        If w==21 %The last period in the violation period is the final node
            yOperatingTerminalValue=InSampleFcf(s,w)/Wacc;
            If (InSampleFcf(s,w)+StartupCost+yOperatingTerminalValue>(ShutdownCostTtoP+RemExp(s,w-1)))
                && TemporaryShutdownCounter(s)=3, %Choose whether to operate or permanently shutdown at
                    t+2 if violation in scenario
                    yTemporaryShutdownAdjusted(s,w-1,1)=(InSampleFcf(s,w)+ StartupCost
                        +yOperatingTerminalValue)*(1/(1+Wacc));
                    yTemporaryShutdownAdjusted(s,w,2)=0;
            Else If TemporaryShutdownCounter(s)==3
                yTemporaryShutdownAdjusted(s,w-1,1)=(ShutdownCostTtoP+RemExp(s,w-1))*(1/(1
                    +Wacc));
                yTemporaryShutdownAdjusted(s,w,2)=2;
            End If
        Else %The last period in the violation period is not the final node
            If TemporaryShutdownCounter(s) == 3,
                %Approximate continuation value of operating
                ContinuationOperating=(RegressionParameters(s,:)*CoefOperatingStorage(:,w+1,1));
            End If
            If(InSampleFcf(s,w)+StartupCost+ContinuationOperating>(ShutdownCostTtoP+RemExp(s,w-1)))
                && TemporaryShutdownCounter(s)=3, %Choose whether to operate or permanently shutdown at
                    t+2 if violation in scenario s
                    yTemporaryShutdownAdjusted(s,w-1,1)=(InSampleFcf(s,w)+ StartupCost+
                        ContinuationOperating)*(1/(1+Wacc));
                    yTemporaryShutdownAdjusted(s,w,2)=0;
            For i=w+1:T
                yTemporaryShutdownAdjusted(s,i,1)=yOperating(s,i,1); %Since choose to
                    re-open ,the optimal operating modes
                    after time t correspond to yOperating
                yTemporaryShutdownAdjusted(s,i,2)=yOperating(s,i,2);
            End
            Else If TemporaryShutdownCounter(s)==3
                yTemporaryShutdownAdjusted(s,w-1,1)=(ShutdownCostTtoP+RemExp(s,w-1))*(1/(1
                    +Wacc));
                yTemporaryShutdownAdjusted(s,w,2)=2;
            End If
        End If
    End For

    For i=w:-1:t %Work backwards in the violation period just like in the standard procedure to determine
        optimal control.
    -> Store the values of the explanatory variables in a matrix
    % Regress continuation value of temporary shutdown at time i on values of the explanatory variables at
    time i-1 and store the coefficients
    [CoefTemporary,~,stataAdj]=MultiVarRegressionFiveVariables(InSampleFcf(:,i-1),
        InSampleScenarios(:,i-1),InSampleUsdEur(:,i-2),InSampleNpy3(:,i-1), Tarray(:,i-1),
        yTemporaryShutdownAdjusted(:,i-1,1), NumScen);
    -> Store the regression coefficients
    For s=1:NumScen,
        ContinuationOperating=(RegressionParameters(s,:)*CoefOperatingStorage(:,i,1)); %Approximate
            continuation value
            of operating
        ContinuationTemporary=(RegressionParameters(s,:)*CoefTemporaryStorage(:,i,1)); %Approximate
            continuation value
            of temporary shutdown
    End For

```

```

% Calculate values for yTemporaryShutdown- that is given that you at i-1 are temporarily
shutdown, what operating mode should be chosen at i. Store in index i-1.
If (InSampleFcf(s,i-1)+StartupCost+ContinuationOperating> ElSale(s,i-2)
+ShutdownOpex+ContinuationTemporary && InSampleFcf(s,i-1)+ StartupCost+
ContinuationOperating>(ShutdownCostTtoP+RemExp(s,i-2))&&TemporaryShutdownCounter(s)=3),
yTemporaryShutdownAdjusted(s,i-2,1)=(InSampleFcf(s,i-1)+ StartupCost+
ContinuationOperating)*(1/(1+Wacc));
yTemporaryShutdownAdjusted(s,i-1,2)=0;
For k=i:T
yTemporaryShutdownAdjusted(s,k,1)=yOperating(s,k,1);
yTemporaryShutdownAdjusted(s,k,2)=yOperating(s,k,2);
End For
Else If (ElSale(s,i-2)+ShutdownOpex+ContinuationTemporary>InSampleFcf(s,i-1)+StartupCost+
ContinuationOperating && ElSale(s,i-2)+ShutdownOpex+ContinuationTemporary>
ShutdownCostTtoP+RemExp(s,i-2) && TemporaryShutdownCounter(s)=3),
yTemporaryShutdownAdjusted(s,i-2,1)=(ElSale(s,i-2)+ShutdownOpex+
ContinuationTemporary)*(1/(1+Wacc));
yTemporaryShutdownAdjusted(s,i-1,2)=1;
Counter(s)=Counter(s)+1; %Count new number of temporary shutdowns in the
violation period
Else If TemporaryShutdownCounter(s)=3
yTemporaryShutdownAdjusted(s,i-2,1)=(ShutdownCostTtoP+RemExp(s,i-2))*(1/(1
+Wacc));
yTemporaryShutdownAdjusted(s,i-1,2)=2;
For k=i:T
yTemporaryShutdownAdjusted(s,k,2)=2;
End For
End If
End For
End For
%Determine optimal operating mode at time t given that you at time t-1 is temporarily shutdown. Now possible to
choose temporary shutdown since removed the violation in the violation period.
i=t;
-> Store the values of the explanatory variables in a matrix
% Regress continuation value of temporary shutdown at time t on values of the explanatory variables at time t-1
and store the coefficients
[CoeffTemporary,~,statsTemp]=MultiVarRegressionFiveVariables(InSampleFcf(:,i-1), InSampleScenarios(:,i-
1),InSampleUsdEur(:,i-2),InSampleNpy3(:,i-1), Tarray(:,i-1), yTemporaryShutdownAdjusted(:,i-1,1),
NumScen);
-> Store regression coefficients
For s=1:NumScen
ContinuationOperating=(RegressionParameters(s,*)CoefOperatingStorage(:,i,1)); %Approximate continuation
value of operating
ContinuationTemporary=(RegressionParameters(s,*)CoefTemporaryStorage(:,i,1));%Approximate continuation
value of temporary shutdown
End For
If TemporaryShutdownCounter(s)=3
If (InSampleFcf(s,i-1)+StartupCost+ContinuationOperating> ElSale(s,i-2)+ShutdownOpex+
ContinuationTemporary && InSampleFcf(s,i-1)+StartupCost+ContinuationOperating>
(ShutdownCostTtoP+RemExp(s,i-2))), %Choose to re-open at time t
yTemporaryShutdown(s,i-2,1)=(InSampleFcf(s,i-1)+StartupCost+ContinuationOperating)*(1/(1
+Wacc));
yTemporaryShutdown(s,i-1,2)=0;
TemporaryShutdownCounter(s)=0;
For n=i:T
yTemporaryShutdown(s,n,2)=yOperating(s,n,2);
End For
Else If (ElSale(s,i-2)+ShutdownOpex+ContinuationTemporary>InSampleFcf(s,i-1)+StartupCost+
ContinuationOperating && ElSale(s,i-2)+ShutdownOpex+ContinuationTemporary>
ShutdownCostTtoP+RemExp(s,i-2)), %Choose to remain temporarily shutdown at time t
yTemporaryShutdown(s,i-2,1)=(ElSale(s,i-2)+ShutdownOpex+ContinuationTemporary)*(1/(1
+Wacc));
yTemporaryShutdown(s,i-1,2)=1;
TemporaryShutdownCounter(s)=Counter(s);
For p=i:T %Note that the violation adjusted values and operating modes are now stored
in yTemporaryShutdown
yTemporaryShutdown(s,p-2,1)=yTemporaryShutdownAdjusted(s,p-2,1);
yTemporaryShutdown(s,p-1,2)=yTemporaryShutdownAdjusted(s,p-1,2);
End For
Else %Choose to permanently shutdown at time t
yTemporaryShutdown(s,i-2,1)=(ShutdownCostTtoP+RemExp(s,i-2))*(1/(1+Wacc));
yTemporaryShutdown(s,i-1,2)=2;
For r=i:T
yTemporaryShutdown(s,r,2)=2;
End For
TemporaryShutdownCounter(s)=0;
End If
End If
End For
End If
End If
t=t-1; %Decrease the time step with one
-> Reset counter
End While

```

ValueWithFullOperationalFlexibility

Equal to the code for CoefGenerator only that no new regressions are conducted and out-of-sample scenarios are used. Instead the already stored regression coefficients are used to approximate continuation values by multiplying them with the out-of-sample values of the explanatory variables. In addition, the smelter value is calculated in the end based on the derived policy. The code for this is as follows:

```
%Calculate values given derived non-optimal policy
yCFs=zeros(NumScen, T); % Create array to store the calculated cash flows
For s=1:NumScen
    PermanentIndicator=0; % Indicator variable to keep track whether a permanent shutdown has already been conducted
    % Fill cash flows at time 1. We know that we come from an operating state.
    If yOperating(s,2,2)==0 %Operating at this time in this scenario
        yCFs(s,2)=Fcf(s,2);
    Else If yOperating(s,2,2)==1 %Temporarily shutdown at this time in this scenario
        yCFs(s,2)=ShutdownCostOtoT+ElSale(s,1)+ShutdownOpex;
    Else
        yCFs(s,2)=ShutdownCostOtoP+RemExp(s,1);
        PermanentIndicator=1;
    End If

    %Fill cash flows for the rest of the period. Uncertain what state we come from.
    For t=3:T
        If PermanentIndicator==0 % If no permanent shutdown has been conducted in this scenario
            If yOperating(s,t,2)==0 %Operating at this time in this scenario
                If yOperating(s,t-1,2)==0 %Come from an operating state
                    yCFs(s,t)=Fcf(s,t);
                Else If yOperating(s,t-1,2)==1 %Come from a temporarily shutdown state
                    yCFs(s,t)=Fcf(s,t)+StartupCost;
                End
            Else If yOperating(s,t,2)==1 %Temporarily shutdown at this time in this scenario
                If yOperating(s,t-1,2)==0 %Come from an operating state
                    yCFs(s,t)=ShutdownCostOtoT+ElSale(s,t-1)+ShutdownOpex;
                Else If yOperating(s,t-1,2)==1 %Come from a temporarily shutdown state
                    yCFs(s,t)=ElSale(s,t-1)+ShutdownOpex;
                End
            Else %Shutdown permanently at this time in this scenario
                If yOperating(s,t-1,2)==0 %Come from an operating state
                    yCFs(s,t)=ShutdownCostOtoP+RemExp(s,t-1);
                Else If yOperating(s,t-1,2)==1 %Come from a temporarily shutdown state
                    yCFs(s,t)=ShutdownCostTtoP+RemExp(s,t-1);
                End
                PermanentIndicator=1; %Set indicator value to true.
            End
        Else %We have already shutdown permanently
            yCFs(s,t)=0;
        End If
    End For

    %Include terminal values given state at time T if not permanently shutdown
    If PermanentIndicator==0
        If yOperating(s,T,2)==0
            yOperatingTerminalValue=Fcf(s,T)/Wacc;
            yCFs(s,T)=yCFs(s,T)+yOperatingTerminalValue;
        Else If yOperating(s,T,2)==1,
            yTemporaryShutdownTerminalValue=(ElSale(s,T-1)+ShutdownOpex)/Wacc;
            yCFs(s,T)=yCFs(s,T)+yTemporaryShutdownTerminalValue;
        End If
    End If
End For

yNPV=zeros(NumScen); %Array to store NPV

%Calculate and store NPV from each scenario
For s=1:NumScen;
    Npv=0;
    For t=1:T-1
        Npv=Npv+yCFs(s,t+1)*(1/((1+Wacc)^t)); %Calculate NPV of the cash flow given that we are now in year 0
    End For
    yNPV(s)=Npv;
End For

%Calculate average MNP across all scenarios
Npv=0;
For s = 1:NumScen,
    Npv = Npv + yNPV(s);
End For

Npv = Npv/NumScen;

%Store the previously derived policy
yPolicy=ones(NumScen,T)*7;
For s = 1:NumScen,
    For t = 1:T,
        yPolicy(s,t)=yOperating(s,t,2); %Store the optimal policy at each time step
    End For
End For
```

Appendix E

Validation of the Matlab routine

Due to the number of industry specific constraints, we applied the MATLAB code and LSM logic described in this paper on a different real option problem with the intention to validate the code. A fairly simple real option problem was found in the appendices of Lemelin (2009). The paper presents a method to value mining investments by using the LSM approach, and a simple case of timing when to produce gold from a gold mine is used as a demonstration in the appendix of the paper. As the policy differs slightly from the aluminium smelter case we evaluated the investment decision by first using the policy presented in Lemelin (2009), and later by using the valuation technique we have used in the smelter case.

The mining investment case is a single factor model where the payoff of the underlying asset, the gold mine, is explained by the price of gold. The simplified numerical example in Lemelin (2009) maximizes the expected value of extracting gold from a mine using three time steps. The gold can either be extracted in time $t + 1$ or $t + 2$ given that we are in time t . Thus, the owners have the option to shut down the mine temporarily and extract gold at a later time for a given shutdown, reopen and maintenance cost. Since there is a short time horizon, the optimal operating policy is derived by comparing the expected payoffs of the two operating modes at time t , meaning that the payoff in time $t + 1$ and the payoff in time $t + 2$ are both regressed on the same explanatory variables from time t . This differs from the aluminium smelter case where continuation values from $t + 2$ are regressed with explanatory variables from time $t + 1$, and an optimal policy is derived at each time step.

In order to properly validate the code we first value the mine by using the method in Lemelin (2009). The following parameters and gold price paths was used in order to calculate cash flows and continuation values:

By regressing the cash flows on gold prices and squared gold prices from time t we

Item	Description	Path	Year t	Year t+1	Year t+2
		1	563.33	541.81	550.83
Production rate	\$500,000 oz/year	2	336.63	469.55	512.54
Production cost	\$450/oz	3	549.68	537.65	487.7
Risk-free interest rate	3%/year	4	390.65	350.84	483.28
Cost inflation	3%/year	5	643.49	728.94	632.73
Maintenance cost	\$10M/year	6	350.03	420.12	430.79
Shutting cost	\$5M	7	634.19	804.16	748.41
Reopening cost	\$5M	8	424.45	519.34	595.31
Abandonment cost	\$0	9	597.71	700.17	615.59
		10	440.83	540.34	674.47

Table E.1 Parameter values

Table E.2 Simulated gold prices

estimated the value of both producing gold now and receive a cash flow in time $t + 1$ and delaying investments with one time step and receive a cash flow in time $t + 2$. The optimal operating policy was derived by choosing the maximum value from the two operating modes. When using the same calculation method as in Lemelin (2009), our optimal operating policy table matched the policy in table C.8 from the paper. The mine was valued at \$62.43M using the option, and \$65.13M using the actual simulated cash flows.

We now use the same approach and decision methods as applied on the aluminium smelter to value the gold mine. This implies that an optimal operating mode is decided at time $t + 2$ for each scenario. The discounted continuation values are then regressed on explanatory variables at time $t + 1$ and a new optimal operating mode is derived in time $t + 1$ from maximizing the value at that time. Thus, regressions are performed and optimal operating modes are decided for each time step rather than regressing every value back to time t at once and then make a decision. When applying this approach, our optimal operating table deviates in two scenarios compared to the table in Lemelin (2009), as illustrated below:

Path	Operating mode from Lemelin	Operating mode with smelter approach
1	Keep open	Keep open
2	Shut down	Shut down
3	Keep open	Keep open
4	Shut down	Shut down
5	Keep open	Keep open
6	Shut down	Shut down
7	Keep open	Keep open
8	Shut down	Keep open
9	Keep open	Keep open
10	Shut down	Keep open

Table E.3 Optimal policies

When calculating the value of the mine using option pricing we arrive at \$61.88M compared to \$62.43M in Lemelin (2009). Our approach seems fairly accurate despite evaluating the mine with a different approach and with fairly few price scenarios.