Norwegian University of Science and Technology NTNU

Faculty of Social Science and Technology Management

Department of Industrial Economics and Technology Management

MASTER THESIS

for

stud.techn.

Sigurd-Erik Nissen-Meyer and Espen Bråthen

Field of study	**Financial Engineering** Investering, finans og økonomistyring
Start date	15-1 2009
Title	**Evaluation of Risk Management Methods for a Hydropower Producer** Evaluering av strategier innen risikostyring for et el-kraftverk
Purpose	Study risk management methods for a hydropower producer.

Main contents:

Electricity producers operate in a market with high volatility, and the price and production volume may therefore change rapidly. Many hydropower producers will consequently have an incentive to apply risk management methods for improving the predictability of their income. Two papers will be written:

1. Evaluate a static hedging strategy with respect to:
   - Attitude towards risk
   - A no hedge strategy
   - Risk measurements: (Conditional) Value at Risk, Cash Flow at Risk, volatility

2. Evaluate a dynamic hedging strategy with respect to:
   - Attitude towards risk
   - A no hedge strategy, the static hedging strategy from paper one
   - Risk measurements: (Conditional) Value at Risk, Cash Flow at Risk, volatility

In both papers the strategies will be tested on the production portfolio of a Norwegian hydropower producer.

Tim Torvatn
Department Management

Stein-Erik Fleten
Supervisor

DECLARATION

Stud.techn. *Espen Bråthen*
Department of Industrial Economics and Technology Management

I hereby declare that I have written the above mentioned
thesis without any kind of illegal assistance

Place

Date

Signature

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PREFACE

This Master's thesis is the final assignment in the Master of Science degree program in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU) in Trondheim, Norway. The Master's thesis has been written during the spring 2009 and is a part of the specialization in Financial Engineering.

The topic of the thesis is risk management for hydropower producers in the Nordic market; more specifically financial hedging strategies using forward contracts at Nord Pool. The thesis consists of two independent academic papers. Both papers have been written with the intention of being published in an academic journal. The first paper considers static hedging strategies, which is the strategy that most producers use in practice. The paper aims to give a thorough evaluation of static hedging strategies and compare their performance to an alternative; the natural hedging strategy. This strategy, which basically means no hedging, benefits from the fact that the electricity price and the production volume of a hydropower producer are negatively correlated. Consequently, the "natural hedge" that the negative correlation inherently provides, is a costless alternative that potentially could give an acceptable downside protection of a hydropower producer's revenue. The second paper focuses on a dynamic hedging strategy and evaluates it compared to static hedging strategies and the natural hedging strategy. Previous research has indicated that dynamic hedging strategies enable much more efficient hedges than static strategies. However, these studies have primarily considered the in-sample performance. Our paper contributes by conducting an extensive out-of-sample test on actual data from a hydropower producer in Norway.

ACKNOWLEDGEMTS

First and foremost we would like to thank our supervisor Professor Stein-Erik Fleten at the Department of Industrial Economics and Technology Management at NTNU. Throughout the spring, he has been advising us in doing research and writing the thesis, which makes us very grateful. Secondly, we would also like to thank PhD Janne Kettunen, at Helsinki University of Technology/London Business School, for advices regarding the stochastic optimization model which is the basis for the second paper. Thirdly, we would like to thank Kjetil Vatne (Production Manager), Martin Frigård (Market Analyst), Jo Langøygard (Production Planner) and Marius Klausen (Production Planner) at NTE Energi AS, for providing us with data and helpful advices. We would also like to thank PhD student Johan Sollie, at the Department of Industrial Economics and Technology Management at NTNU, for assistance and advices regarding estimation of input parameters. Finally, we thank Professor Håkon Tjelmeland and Professor Håvard Rue, at the Department of Mathematical Sciences at NTNU, for assistance regarding stochastic processes.

Trondheim, June 18, 2009

Espen Bråthen

Sigurd-Erik Nissen-Meyer

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PAPER I

Evaluation of static hedging strategies for hydropower producers in the Nordic market

ABSTRACT

In this paper we develop an optimization model to derive static hedging strategies for hydropower producers. We use this model to propose strategies with different risk characteristics. Previous research has primarily considered dynamic hedging strategies; however, static hedging strategies are the common choice among practitioners because of their simplicity. Our contribution will therefore be to evaluate the performance of static hedging strategies for hydropower producers. The strategies are tested and compared with the natural hedging strategy, which means no hedging, on historical and predicted data. The results show that hedging with use of forward contracts significantly reduces the risk in terms of VaR, CVaR, and standard deviation of the revenue. Furthermore, this improvement results in just a minor reduction of the mean revenue. Our results imply that despite its simplicity, static hedging strategies are useful for hydropower producers in means of risk reduction at a small cost.

Key words: Risk management, Static hedging, Hydropower producers, Nordic electricity market, Risk premium

1 INTRODUCTION

The liberalization of the Nordic power market in the early 1990`s dramatically changed the competitive environment for hydropower producers. Before the liberalization, the electricity price was regulated by the governments. Consequently, producers did not have any incentives to hedge the electricity price. However, after the deregulation, the control of the electricity price has been removed and as a result the price variation has increased[1]. This has led to the development of a market for electricity derivatives. As a result Nord Pool, the power exchange for the Nordic countries, was established in 1993. At Nord Pool standardized derivatives, such as forwards/futures and options, are traded and enable a way for producers to manage and handle their risk exposure to the electricity price. The task of managing the risk with respect to the electricity price is however not an easy one. As mentioned above, the electricity price has high volatility and may have spikes of several orders of magnitude within a short time. This is mainly caused by the fact that electricity has very limited storage possibilities. Hydropower producers can to some extent store indirectly in water reservoirs. However, consumers can not buy electricity for storage. This implies that the cost-of-carry relationship between spot and forwards break down. In other words, the relationship between the spot and forwards is weaker than for other commodities. The electricity price therefore also experience strong seasonality.

Over the last decade there has been an increasing interest, both among practitioners and in academia, in the area of risk management for electricity producers. These have had to adapt to the new environment which the above-mentioned liberalization has caused, and in some way or another employ methods that aim to manage the new risk exposures. For a hydropower producer the electricity price and the inflow, that is how much water that flows into the reservoirs, are the two most significant determinants for the revenue. As both price and inflow experience large variations, they are also the two most important risk factors for future revenue. Previous research has primarily considered dynamic hedging strategies. Fleten, Wallace and Ziemba (2002) uses stochastic programming to find the optimal integrated production schedule and financial hedging plan for a hydropower producer. Kettunen, Salo and Bunn (2007) uses a similar approach, but take the production plan as given and focus on finding the optimal financial hedging plan. Näsäkkälä and Keppo (2005) on the other hand uses a static hedging strategy with forward contracts. This strategy is derived by minimizing

[1] Knittel and Roberts (2005)

the variance of the portfolio. I.e. it is assumed that the risk adjusted expected value of the portfolio is maximized when the portfolio variance is minimized. In this paper we will present an optimization model for deriving static hedging strategies. However, instead of minimizing the portfolio variance, the static strategies are derived by maximizing the expected revenue subject to constraints on the portfolio variance and Value-at-Risk (VaR). The static strategies will be evaluated and compared to a benchmark; the natural hedging strategy. This strategy, which basically means no hedging of the electricity price, benefits from the fact that the inflow and price are negatively correlated and thereby inherently provides a "natural hedge" of the revenue. The static hedging strategies can in short terms be explained as using forward/future contracts to sell some percentage of the expected future production. These strategies can of course include options and other derivatives, but is static in the sense that the positions are not changed as new market information becomes available. The natural hedging strategy and the static strategies will be evaluated by empirical tests on historical data, as well as by a test based on simulated price and production scenarios. The tests aim to answer the question, which of the two approaches yields the best result from the point of view of a typical hydropower producer in the Norwegian market.

The paper is structured as follows. In Section 2 we will discuss the purpose and goal of risk management from the view of a hydropower producer. In Section 3 we present the risk measures that will be used to evaluate the hedging strategies. In Section 4 we present and discuss the natural hedging strategy and the static hedging strategies. In Section 5 we present and discuss the results from the empirical tests. Section 6 concludes.

2 RISK MANGEMENT FOR HYDROPOWER PRODUCERS

In this section we will discuss different factors, aspects and considerations that have to be taken into account when employing risk management. We will do this from the point of view of a hydropower producer. To start, we need a definition of risk management. According to Krapels (2000), risk management can be defined as *the control and limitation of the risks faced by an organization due to its exposure to changes in financial and commodity markets.* This means that in order to employ risk management properly, an organization first has to identify the risk factors they face and what the exposure to these risk factors is. When the risks are identified and the amount of exposure the organization have to each of them is measured, one have to prioritize and decide how the risks should be handled and controlled.

Depending on the organization`s goal and attitude towards risk, some risks should be eliminated, some should be limited and some should be left as they are or increased. It is important to note that risk management does not imply that all risks should be eliminated, since without any risk exposure the return will be limited. However, the key of proper risk management is to be aware of all risks the organization faces and continuously measure, control and handle them in a way that is consistent with the organization goal and risk attitude.

Section 2.1 briefly presents the price and inflow risk. Section 2.2 considers the purpose of risk management and risk premiums, and Section 2.3 presents historical and predicted data of the electricity price and a hydropower producer's inflow (production volume).

2.1 Review of the risk aspects for a hydropower producer

As mentioned above, the first step when applying risk management is to identify the risks the organization faces and evaluate the exposure towards these risks. In this section we will present the electricity price risk and inflow risk, which are the two most important risks a hydropower producer faces[2].

Price risk is risk that stems from changes in the value of spot positions due to changes in the electricity spot price. For instance, if a producer has decided to sell 50% of this year's production on the spot market, the value of that 50% will change as the electricity spot price changes. The electricity spot price has high volatility[3] and will therefore have significant impact on the value of the production. The price risk is therefore one of the most important risks for a hydropower producer.

Inflow risk is risk that stems from the fact that precipitation and inflow to the water reservoirs may vary a lot from year to year. Since the production volume depends on the inflow to the reservoirs, this variation consequently causes variation and uncertainty in the future revenue. We consider inflow risk as the same as uncertainty in production volume.

[2] Kristiansen (2006)
[3] Benth, Benth and Koekebakker (2008)

2.2 Purpose of risk management and risk premium

An important consideration when deciding how the risks are going to be managed, is the hydropower producer's attitude towards risk. For a risk averse producer that wants good predictability of future revenue and needs to ensure that the revenue will be over a certain level, a risk management program with extensive use of hedging is suitable. On the other hand, for a less risk averse producer that can handle a greater standard deviation in the revenue and is able to survive a period with unusual low price and/or production, a risk management program with less hedging is needed.

Another important issue when considering hedging strategies is the risk premium[4] that is embedded into the derivatives. In the first place, one could think that a producer should have to pay a premium, which reduces the expected revenue, if derivatives are used to reduce the variability and downside risk of future revenue. However, in its most general form this argument could also be applied to the consumer side and one would get the opposite result, since the derivatives' payoff is a zero-sum game. To actually deduce what the risk premium should be is consequently not easy. Some previous research has been done on this topic. For instance, Bolinger, Wiser and Golove (2002) shows that natural gas swap prices in the US have a negative risk premium, which means that the swap prices are an overestimate of their corresponding spot prices. Bessembinder and Lemmon (2002) finds that electricity prices in the US have a negative risk premium if expected demand is low and demand variance is moderate, and a positive risk premium when expected demand is high and demand variance is high. Geman and Vasicek (2001) finds evidence of a negative risk premium in the Pennsylvania-New Jersey-Maryland electricity market for forward contracts with short time to delivery. For contracts with long time to delivery the risk premium becomes positive. These results are supported by Longstaff and Wang (2004) that finds evidence of a significant negative risk premium for contracts with short time to delivery. Benth, Benth and Koekebakker (2008) also finds evidence of a negative risk premium at Nordpool for contracts with short time to delivery and a positive risk premium for contracts with long time to delivery. Furió and Meneu (2009) analyze the Spanish power market and find presence of a negative risk premium. This is a result of higher flexibility on the supply side that leaves the demand side with higher incentives of hedging under normal market conditions.

[4] See Appendix A.1 for definition of risk premium

Krapels (2000) suggests that the positive skewness, due to spikes, in the electricity price may lead to a negative risk premium. Generally, price spikes give the consumer an incentive to pay a premium for hedging the price, while the producer wants to receive a premium since it will not benefit from the spikes if the price is hedged. Krapels (2000) supports this with an anecdote about pricing of electricity options: "It is common knowledge, however, that traders in many OTC electricity options markets have become so fearful of being physically "net short" (having agreed to deliver electricity in the future at an earlier agreed-upon price) when one of the price spikes occurs that they place extremely high standard deviation assumptions into the pricing of OTC electricity call options".

2.3 View of price risk and inflow risk

In this section we will discuss the properties of the historic data and the predicted data on the spot price and the production volume. The distributions of the historical and predicted data can be computed analytically by estimating them with a certain probability distribution or they can be estimated empirically. We have chosen to focus on the last method. We will discuss the statistical properties and show the empirical distributions in order to give a quantitative overview of the two main risks the hydropower producer faces. For the production volume the inflow is assumed to be the only varying factor, which mainly depends on the weather. It should therefore repeat itself, and historical data should consequently be representative for the future. The same can be true for prices, if the circumstances are believed to be stable. However, as the historical data is only based on past events, they will lack events yet to be seen. Therefore we will also use predicted data. For the historical data it should be noted that the statistical measurements are only calculated based on 13 observations. Lack of data may therefore be a source for noise. We will in the application of our optimization model, which is presented in Section 4.3, base our calculations on the price and production data presented in this section.

2.3.1 Historical prices and production volume

Figure 2.1 and Figure 2.2 show the annual production for a Norwegian hydropower producer and the annual average spot price, respectively, in the period 1996 to 2008. The production is adjusted for reservoirs that were acquired during the period. Both the production and the spot price have a relatively high standard deviation and are considered the two most important risk factors for a hydropower producer's future revenue. From Figure 2.2 it can be seen that the price has followed a strong upward moving trend during the period 2000 – 2008. As this trend is unlikely to continue in the long run, it implies that the historical data may not be representative for the future, and may be considered as a special case.

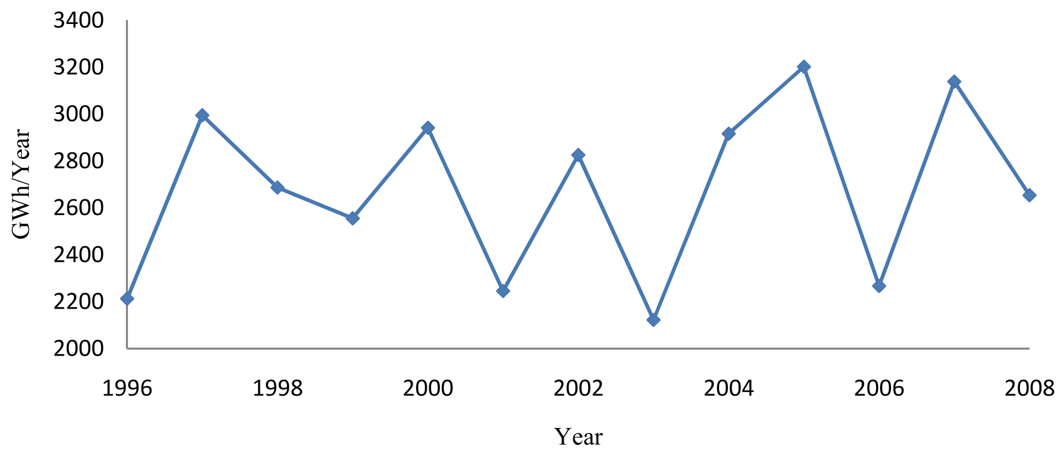

Figure 2.1 The historical annual production for the hydropower producer. The average and standard deviation are 2674 and 369 GWh/Year, respectively.

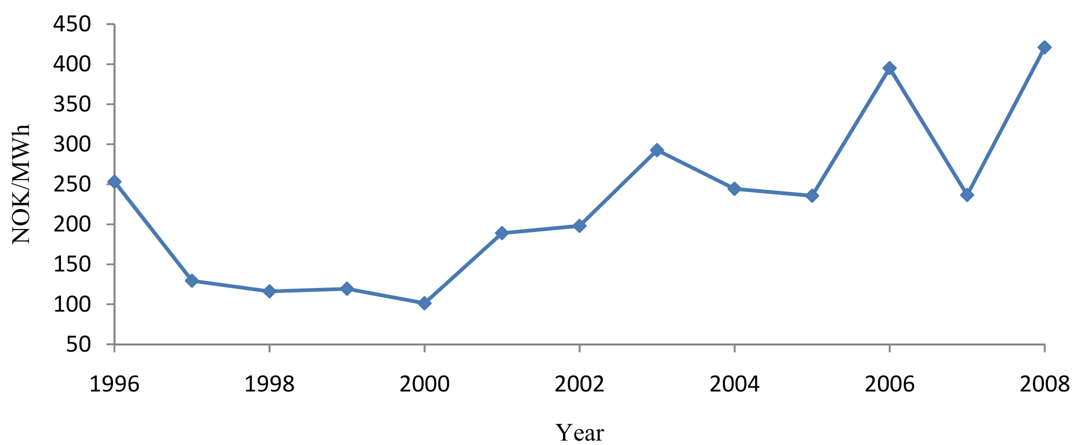

Figure 2.2 The historical average annual spot price for the hydropower producer. The average and standard deviation are 225 and 101 NOK, respectively.

Table 2.1 shows statistics for the production and spot price for the historic data. As we can see, there is a significant difference between the maximum and minimum values for both the production and spot price. Furthermore, the standard deviation is relatively high for the production, and particularly high for the spot price. It can also be seen in the table that the correlation between the spot price and production is negative. This gives a decrease in the standard deviation for the annual revenue and will be investigated further in Section 4.2

Statistics	Production	Spot price	Spot Revenue
Mean	2674	225	586
St. dev.	369	101	237
Skewness	-0.2	0.7	0.5
Kurtosis	-1.4	-0.1	-0.5
Min	2122	101	295
Max	3202	421	1064
Correlation	-0.33		

Table 2.1 Descriptive statistics on an annual basis for the historical data.

2.3.2 Predicted prices and production volume

The predicted data for 2010 consist of 70 equiprobable scenarios made from generation planning tools used by the hydropower producer and from the bottom-up electricity sector model Multi-area Power Scheduling (MPS). This is an equilibrium model frequently used for price forecasting in Scandinavia. The model was developed by SINTEF Energy Research and is described in Botnen et al. (1992) and Egeland et al. (1982).

The same results yield for the predicted data as for the historical; both the production and the spot price have relatively high standard deviations. This is shown by Figure 2.3 and Figure 2.4 which show the empirical probability distributions of the annual average spot price and the annual production, respectively.

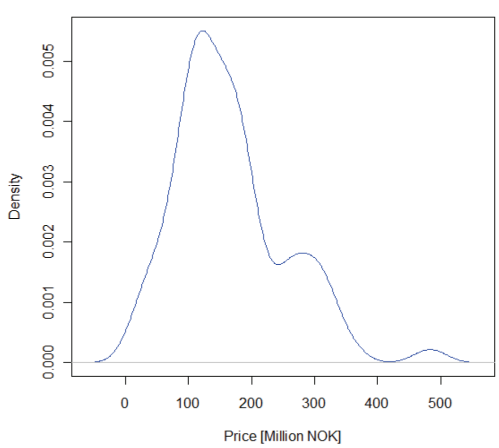

Figure 2.3 Empirical probability distribution of the forecast price.

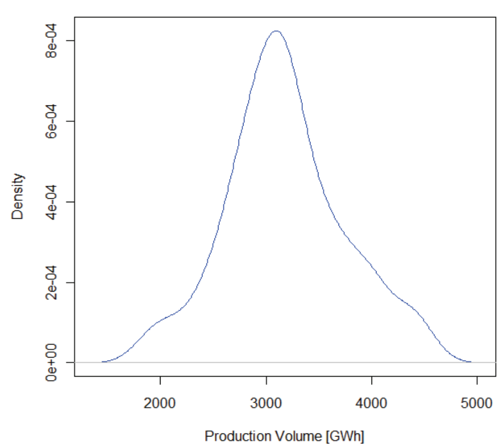

Figure 2.4 Empirical probability distribution of the forecast production volume.

Table 2.2 shows descriptive statistics for the predicted data for 2010. The values are quite similar to values for the historical data, except for the mean of the production which is about 800 GWh higher. However, this is caused by additional reservoirs that are not included in the historical data. For the spot price, the mean and standard deviation is a bit higher than for the historical data. Also, note that the correlation for the predicted data is -0.09 compared to -0.33 for the historical data.

Statistics	Production	Spot price	Spot Revenue
Mean	3421	287	1018
St. dev.	504	130	471
Skewness	0.01	1.11	1.31
Kurtosis	-0.26	1.27	2.31
Min	2180	90	329
Max	4589	720	2622
Correlation	-0.09		

Table 2.2 Descriptive statistics on an annual basis for the predicted data

3 RISK MEASUREMENT

In this section we will describe how the hydropower producer can measure its risk by standard statistical tools. These risk measurements will be used to reach an optimal hedging strategy with respect to the hydropower producer's risk aversion. The risk measurements are based on the end-of-year revenue, and are conclusive and straightforward to interpret in terms of what risk profile the hydropower producer is seeking. Note that the reliability of the risk measurements will depend on the reliability of the estimated revenue distribution. The risk measurements are calculated from empirical distributions, since the profit from the electricity market is hard to model analytically. This stems from the fact that they consist of price spikes that will violate the normality assumptions, which are often used for stocks and other underlying assets. It should be noted that these distributions consist of market risk as well as the specific hydropower producer's risk. Using derivatives from Nord Pool will therefore only secure the market risk, while the specific business risk will still be present. By measuring the end revenue, both risks will be taken into account in our analysis. In Section 3.1 we will introduce the Value at Risk[5] (VaR) technique and a modification of VaR; the Conditional Value at Risk (CVaR) for further explanation of the downside risk will be presented in

[5] See Appendix A.1 for definition of VaR, CVaR and CFaR

Section 3.2. Additionally, Cash Flow at Risk (CFaR), as described by Guth and Sepetys (2001), could have been used. However, we have chosen not to, as CFaR is just an alternative measure of VaR and will therefore provide no further information.

3.1 Value at Risk

We have chosen to focus on the VaR 10% of the end-of-year revenue for defining an acceptable threshold. The threshold is chosen by the hydropower producer, and the time horizon is based on what we believe the hydropower producer will have most benefit from focusing on, as it is coherent with the time period of most budgets and balance sheets. Additionally, we believe monthly and quarterly fluctuations will be of less importance than the end-of-year revenue, as the demand for liquidity on a shorter term will be of less importance than the liquidity on an annual term. As the VaR 10% sets the minimum possible value the revenue can obtain in a 90% interval, we believe this value is of more interest for the hydropower producer, and will stress this value in the testing. This value is crucial for defining a threshold limit, which if violated could lead to capital structure crisis and thereby higher debt yield. Even though VaR may be one of the most popular risk measurements, we believe it will be insufficient for our analysis. As the distribution of the revenue has a spiky behavior and the shape of the left tail may be thick and not monotonically decreasing, we believe the VaR should be evaluated in the context of other risk measurements as well. This is supported by Unger and Lüthi (2002). We will therefore investigate the VaR in combination with the CVaR as described in the next section.

3.2 Conditional Value at Risk (CVaR)

Conditional value at risk (CVaR) was proposed by Rockafellar and Uryasev (2002) as a measure which combines features from expected shortfall and Value at Risk. As given by its definition, CVaR will be at least as low as VaR. It will represent the expected value given that we are below the VaR limit. Therefore when solving an optimization model of the revenue, with restrictions on VaR, the CVaR may not be optimal in the view of the hydropower producer. Even though VaR is a popular statistical measurement regarding the risk taken by the companies, it may easily be misleading especially in case of heavy tails. Major shortfalls may be possible even though the VaR 10% shows a relatively high value. The CVaR method allows us to further investigate the potential shortfalls, by giving us an impression of the length of the downside tail. In other words, to get an impression of what happens if the revenue is known to be below VaR we will use CVaR. We will also use it to compare two strategies with nearly similar VaR.

4 SUGGESTED RISK MANAGEMENT STRATEGIES

In this section we will introduce the natural hedging strategy and present an optimization model for deriving static strategies. We will start with Section 4.1, explaining the derivatives at Nord Pool which are the cumber stones for the hedging strategies. We will then give an introduction to the natural hedging strategy in Section 4.2, and finally in Section 4.3 we will describe the model we use for deriving the static hedging strategies.

4.1 Securities market

There are four main types of derivatives available at Nord Pool; future contracts, forward contracts, options and contracts for difference. The future and forward contracts are somehow different from traditional future and forward contracts in the financial markets. The main difference is that they have a delivery period. The underlying is not delivered on a fixed point in time, but over a period where the payoff of the contract is calculated as the hourly difference between the forward/future price and the spot price. In this sense the Nord Pool future/forward contracts correspond to the textbook definition of swaps. The future contracts are marked to market each day prior to the delivery period. In the delivery period the payoff is calculated as the difference between the spot price and the future price on the last trading day. The future contracts have either daily or weekly delivery period. At any time there are between one and seven daily future contracts and six weekly future contracts available. The forward contracts are settled in the same manner as the future contracts, however without the mark to market settlement prior to the delivery period. Forward contracts are available with monthly, quarterly, and annual delivery periods. At any time there are six monthly, eleven quarterly and five annual contracts available. The liquidity is relatively high for both future and forward contracts, except for the daily contracts and the annual contracts with three, four and five years to delivery.

European-style call and put options with quarterly or annual forward contracts as underlying are also available. However, the liquidity of these contracts is low. Ideally option contracts could provide very efficient hedging strategies, see for instance Krapels (2000), but the low liquidity makes it hard to use them for risk management purposes in practice and would result in high transaction costs because of the bid-ask spreads.

The fourth main type of contracts on Nord Pool is contracts for difference (CFDs). These contracts are made for hedging the difference between the system price and the local area price. The forward and future contracts are settled against the system price while the hydropower producer gets the local area price when selling the production. This local area price is only equal to the system price in case of no congestion in the transmission grid. However, in reality there is often a difference between the system price and the local area price. Therefore the forward/future contracts will not eliminate all the price risk in case of a perfect hedge. The CFDs can be used to eliminate this difference and if used in combination with the forward/future contracts, a perfect hedge of the price is achievable. However, also for these contracts the liquidity is low.

As a result of liquidity and time horizon, we will use quarterly and annual forward contracts in our further analysis. This is also the common choice for risk management among hydropower producers in the Nordic market. We have left out call and put options in the electricity market because of low liquidity, and derivatives for commodities in related markets as a consequence of low correlation with the Norwegian electricity market, see for instance Gjølberg (2001).

4.2 Natural hedging strategy

The natural hedging strategy can be seen as the maximum degree of risk the hydropower producer is able to undertake, under the assumption that it is not speculating. This is a result of the fact that the natural hedging strategy is the same as not hedging at all. The strategy leads to the highest uncertainty in future revenue and highest possible shortfalls, but also the highest upside potential. The strategy will therefore be best suited for producers with a low degree of risk aversion. We have seen in Section 2.4 that both prices and volume are very volatile, but negative correlation between them, may lead to an acceptable standard deviation for the future revenue. A negative correlation should be stable and significant in order for this to be true.

The main reason for the negative correlation between price and hydropower production in the Norwegian market is that the market is regional, and 99% of the electricity production comes from hydropower. For hydropower production the most important factor for the production volume is the inflow to the reservoirs, which again depends on the precipitation. Since local precipitation is correlated with national precipitation, water shortage is often national and not just local. Additionally, most of the residential heating is done via electricity. This means that when the temperature is low, the electricity demand will increase. However, when the temperature is low, there is more likely less precipitation and inflow. Consequently, when the demand is high, the supply and production volume is likely to be limited and the electricity price rises. In years with high precipitation, it is the other way around. Supply increases due to the high precipitation and the demand decreases due to higher temperature. This again leads to a lower electricity price. To investigate this empirically, we have estimated the correlation in Table 4.1. It is done on an annual basis to avoid seasonal effects, and measured during different time periods to investigate the stability.

Period	Correlation	Period	Correlation
1996 - 1999	-0.78	1997 - 2008	-0.33
1996 - 2000	**-0.81**	1998 - 2008	-0.28
1996 - 2001	**-0.82**	1999 - 2008	-0.30
1996 - 2002	**-0.67**	2000 - 2008	-0.39
1996 - 2003	**-0.78**	2001 - 2008	-0.32
1996 - 2004	**-0.58**	2002 - 2008	-0.59
1996 - 2005	-0.36	2003 - 2008	-0.61
1996 - 2006	-0.47	2004 - 2008	**-0.86**
1996 - 2007	-0.40	2005 - 2008	**-0.89**
1996 - 2008	-0.33		

Table 4.1 Correlation (annual granularity) between price and production volume in the respective time periods. Values in bold are significant different from zero based on t-tests.

Table 4.1 shows that the correlation is relatively high for most time intervals. T-tests on the significance show that 7 out of 19 coefficients are significantly different from zero. It is important to note that the robustness of the T-tests is limited due to the low number of data points. However, if one in addition to these results consider the fundamental properties of hydropower production (which intuitively imply a negative correlation), it is tempting to conclude that a negative correlation is present under normal market conditions.

The negative correlation will reduce the standard deviation of the revenue, compared to the high standard deviation in price and volume, and is the basis for the natural hedging strategy. When investing in forward /future contracts this correlation effect will be lost, but as the price is locked for the given period, the standard deviation will only stem from the risk in volume. The standard deviation will therefore still be lower. The natural hedging strategy may however still be a good choice, depending on the risk aversion of the hydropower producer, as a result of (i) no transaction costs, (ii) no loss of revenue in means of hedging costs and (iii) utilization of the negative correlation between price and inflow. However, it should also be noted that even though the correlation has been negative historically, and there are reasons to believe that the correlation under normal circumstances will be negative in the future, there could be special events in the future where both production volume and price collapse, for instance due to a crisis in the global economy. An example is the current financial crisis, which has resulted in a reduction of both price and production. In this case the natural hedging strategy will give no protection.

The natural hedging strategy can also be used as a benchmark for other hedging strategies. In our empirical tests we will therefore compare our static hedging strategies with the natural hedging strategy. This enables an evaluation of the hedging costs compared to the standard deviation and shortfalls the producer will have in case of this no hedging method.

4.3 Static hedging strategy

We will define a static hedging strategy as a strategy where the positions are fixed for a period of time according to a predetermined scheme. The positions are consequently not adjusted as new market information becomes available. A static strategy can use all types of derivatives, and the strategy is defined by the proportions held by each of the derivatives, the derivative's time horizons, and the trading dates. Among producers in the Nordic market it is common to use contracts with quarterly and annual time horizons.

4.3.1 Model introduction

The two main goals of a static strategy is to reduce the standard deviation of future revenue for better decision and budgeting support, and to insure against major shortfalls. The degree of standard deviation reduction and protection against shortfalls are for a static strategy determined by the proportion of the production sold with forwards and the time horizon of these contracts. The main question when designing a static hedging strategy is therefore what the proportions and time horizons should be in order to meet the hydropower producer's risk preferences. In order to determine the optimal proportions and time horizons of the contracts, we chose to develop an optimization model that determines (based on input on spot price, production and forward prices) what the best proportions and time horizons are. When the model finds the proportion in form of weights of expected production, the purpose of the producer's risk management and the properties of their risk aversion are taken into account. Additionally, the model implicitly finds the optimal time horizons from the set of available forward contracts. Historical data can be used as input to determine what has historically been the best strategy. One could also use predicted data for future years as input, and in this way determine the best possible strategy for the future.

4.3.2 The model

The model aims to determine the optimal weights for a given set of forward contracts. This is done by maximizing the revenue, subject to a set of Value at Risk (VaR) constraints, a set of standard deviation constraints and a set of trading constraints. Since hydropower production costs are constant with respect to the choice of hedging strategy, maximizing the revenue is equivalent to maximizing the profit. The revenue consists of three parts. The first part is spot revenue; that is revenue from sale of the production at the spot price. This part is independent of the weights, and could therefore be omitted from the problem formulation. However, for completeness we have chosen to include it. The second part and the third part are profit/loss from the annual and quarterly forward contracts, respectively. The profit/loss over a given time period is calculated from the forward contracts that had delivery during that time period. I.e. the contracts are not marked to market prior to delivery. This corresponds to how the payoff of these contracts is settled at Nord Pool. The VaR constraints are a measure of the producer's aversion against shortfalls. The standard deviation constraints are determined by the degree of certainty the producer wants of future revenue. Additionally, we impose the trading constraints that make sure that the producer only can be short in the contracts, since long positions are considered as speculation[6].

Definitions:

T_y = Number of years into the future that can be hedged with annual forwards

T_q = Number of quarters into the future that can be hedged with quarterly forwards

N = Number of years in the test period

S = Number of future scenarios when the model is run on predicted data

VaR_α = The 1 year α percent value at risk limit

[6] Kettunen, Salo and Bunn (2007)

σ^2_{max} = Limits the variance of the annual revenue

$Q_{y,w,s}$ = The production in week *w* in year *y* in scenario *s*

$\bar{Q}$ = The annual expected production. Assumed to be constant

$P_{y,w,s}$ = The spot price in week *w* in year *y* in scenario *s*

$F^Y_{y,t}$ = Forward price in year *y* for an annual forward contract with delivery in year *y+t*

$F^Q_{y,t}$ = Forward price in year *y* for a quarterly forward contract with delivery in quarter *t* in year *y+1* if *t = 1,2,3,4* or in quarter *(t-4)* in year *y+2* if *t = 5,6,7,8*

y_i = Percentage of annual expected production that at any time should be short in annual forward contracts the i*th* year from now, i = 1,2,..,T_y

q_i = Percentage of annual expected production that at any time should be short in quarterly forward contracts the i*th* quarter next year if *i = 1,2,3,4* or the (i-4)*th* quarter two years from now if *i = 5,..,*T_q

$R_{y,s}$ = Support variable that calculates the revenue from sale of the production at the spot price in year *y* and scenario *s*

$\pi^Q_{y,s}$ = Support variable that calculates the profit/loss of the quarterly forward contracts in year *y* and scenario *s*

$\pi^Y_{y,s}$ = Support variable that calculates the profit/loss of the annual forward contracts in year *y* and scenario *s*

$\overline{Tot\_Rev_i}$ = Support variable that calculates the total expected revenue in year *y*

$z_{y,s}$ = Binary support variables used in the VaR constraints for year *y* in scenario *s*

Problem formulation:

$$Maximize \frac{1}{S}\sum_{i=1}^{N}\sum_{s=1}^{S}(R_{i,s}+\pi^Q_{i,s}+\pi^Y_{i,s})$$

Subject to

$$R_{i,s}=\sum_{w=1}^{52}P_{i,w,s}Q_{i,w,s} \qquad \forall i=1,...,N \quad s=1,...,S \tag{1}$$

$$\pi^{Y}_{1,s} = F^{Y}_{0,1} \cdot \overline{Q} \cdot y_1 - \sum_{w=1}^{52} P_{1,w,s} \cdot \overline{Q} \cdot \frac{1}{52} \cdot y_1 \qquad \forall s = 1,\ldots,S \tag{2}$$

$$\pi^{Y}_{1,s} = F^{Y}_{0,1} \cdot \overline{Q} \cdot y_i - \sum_{t=1}^{i-1} F^{Y}_{t,i-t} \cdot \overline{Q} \cdot (y_{i-t} - y_{i+1-t}) - \sum_{w=1}^{52} P_{2,w,s} \cdot \overline{Q} \cdot \frac{1}{52} \cdot y_1 \tag{3}$$

$$\forall i = 2,\ldots,(T_y - 1) \quad s = 1,\ldots,S$$

$$\pi^{Y}_{i,s} = F^{Y}_{i-T_Y,T_Y} \cdot \overline{Q} \cdot y_{T_Y} + \sum_{t=1}^{T_Y-1} F^{Y}_{i-T_Y+t,T_Y-t} \cdot \overline{Q} \cdot (y_{T_Y-t} - y_{T_Y+1-t}) - \sum_{w=1}^{52} P_{i,w,s} \cdot \overline{Q} \cdot \frac{1}{52} \cdot y_1 \tag{4}$$

$$\forall i = T_y,\ldots,N \quad s = 1,\ldots,S$$

$$\pi^{Q}_{i,s} = \sum_{t=1}^{4} \left(F^{Q}_{0,t} \cdot \overline{Q} \cdot \frac{1}{4} \cdot q_t - \sum_{w=1}^{13} P_{1,w+13\cdot(t-1),s} \cdot \overline{Q} \cdot \frac{1}{52} \cdot q_t \right) \qquad \forall s = 1,\ldots,S \tag{5}$$

$$\pi^{Q}_{i,s} = \sum_{t=1}^{4} F^{Q}_{i-2,t} \cdot \overline{Q} \cdot \frac{1}{4} \cdot q_{t+4} + F^{Q}_{i-1,t} \cdot \overline{Q} \cdot \frac{1}{4} \cdot (q_t - q_{t+4}) - \sum_{w=1}^{13} P_{i,w+13\cdot(t-1),s} \cdot \overline{Q} \cdot \frac{1}{52} \cdot q_t \tag{6}$$

$$\forall i = 2,\ldots,N \quad s = 1,\ldots,S$$

$$R_{i,s} + \pi^{Q}_{i,s} + \pi^{Y}_{i,s} \geq VaR_{\alpha} \cdot z_{i,s} \qquad \forall i = 1,\ldots,N \quad s = 1,\ldots,S \tag{7}$$

$$\sum_{i=1}^{S} z_{i,s} \geq S \cdot (1 - \alpha) \qquad \forall i = 1,\ldots,N \tag{8}$$

$$\frac{1}{S-1} \sum_{i=1}^{S} (R_{i,s} + \pi^{Q}_{i,s} + \pi^{Y}_{i,s} - \overline{\text{Tot\_Rev}_i})^2 \leq \sigma^2_{Max} \qquad \forall i = 1,\ldots,N \tag{9}$$

$$\overline{\text{Tot\_Rev}}_i = \frac{1}{S}\sum_{s=1}^{S}(R_{i,s} + \pi_{i,s}^{Q} + \pi_{i,s}^{Y}) \qquad \forall i = 1,...,N \qquad (10)$$

$$y_i \geq y_{i+1} \qquad \forall i = 1,...,T_Y - 1 \qquad (11)$$

$$q_i \geq q_{i+4} \qquad \forall i = 1,...,4 \qquad (12)$$

$$y_1 + q_i \leq 1 \qquad \forall i = 1,...,4 \qquad (13)$$

$$y_i \geq 0, q_i \geq 0 \qquad i = 1,...,T_y \qquad j = 1,...,T_q \qquad (14)$$

Explanation of the problem:

Objective: Maximize expected revenue

Subject to:

(1) Calculation of the revenue generated by spot sale of production for each year and scenario

(2) Calculation of the profit/loss from the annual forward contracts for every scenario in the first year

(3) Calculation of the profit/loss from the annual forward contracts for every scenario from the second year to the (T_y-1)th year

(4) Calculation of the profit/loss from the annual forward contracts for every scenario in the (T_y-1)th year to year N

(5) Calculation of the profit/loss from the quarterly forward contracts for every scenario in the first year

(6) Calculation of the profit/loss from the quarterly forward contracts for every scenario from the second year to year N

(7),(8) Restrict the *VaR α* of the annual revenue

(9) Restricts the variance of the revenue for each year

(10) Support constraint for (9). Calculates the mean of the total revenue for each year

(11) Ensures that annual forward contracts are not bought

(12) Ensures that quarterly forward contracts are not bought

(13) Ensures that the position in forward contracts is not greater than the expected production.

(14) Ensures that there cannot be held long positions in forwards

Solving this model[7] will return the weights y_i and q_i which completely specify the static hedging strategy by denoting the amount of annual expected production that should be short in annual and quarterly forward contracts at any point in time. In quarter i every year, $i = 1,\ldots,4$, $y_1 + q_i$ of the expected production will be short in forward contracts. y_2 and y_3 determine how much of the expected production two and three years from now, respectively, that currently should be short. The value of the objective function equals the expected total revenue during the years in consideration.

4.3.3 Model evaluation

Since the model is an optimization model, it will return the strategy that gives the highest revenue, subject to the constraints and input data. In this way the model can be used to determine a producer's hedging strategy once their risk aversion in terms of risk measurement restrictions is identified. However, it is important to note that the strategy is optimal with respect to the input data for spot price, production and forward prices. For instance, if the model is run on historical data, the model finds the strategy that historically has been the best. If one thinks that the historical data is a good prediction, the strategy might be a good choice. However, if the future is expected to differ a lot from the past, using the best historical strategy may obviously lead to poor results. Running the model on historical data is therefore best as a performance measure, and the hydropower producer's current hedging strategy can be compared to the strategies from the optimization model, which can be considered as theoretical upper limits for the years in consideration. Running the model on predicted data

[7] See Appendix A.2 for implementation in AMPL.

will possibly give a strategy that is close to optimal for the future, given that the predictions are accurate. However, as with historical data, using the strategy may lead to poor results if the predictions do not turn out to be accurate. To derive a strategy based on predicted data and also test it on historical data may therefore be a good way of stress testing the strategy and evaluate its robustness, given that the future has similar properties as the past.

Finally, it is important to be aware of what Smith and Winkler (2006) calls the optimizers curse. This is a statistical phenomenon which states that when a decision maker makes a choice among different alternatives, he is in danger of overestimating the value of the chosen alternative. The chosen alternative is therefore likely to not be optimal. In our model this may lead to an upward bias in the performance of the chosen strategy. However, the fact that the different strategies are positively correlated with respect to the input parameters will reduce the problem of optimizers curse. The spot price may lead to some optimization bias if the estimation errors of the spot price in different time periods are not correlated. I.e. if Q1 spot prices are overestimated while Q2 spot prices are underestimated, a bias towards Q2 contracts will occur. This effect will be reduced by diversifying the weights of the contracts.

5 EMPIRICAL TESTS

In this section we will investigate the performance of different static hedging strategies and the natural hedging strategy, and compare them based on the risk measurements. In Section 5.1 we will give an evaluation of the data set. Section 5.2 will give an explanation of the different strategies, and how their weights were derived. The strategies will be tested based on historical data and predicated data in Section 5.3, and finally in Section 5.4 we will compare the strategies with respect to the results of both the historical and predicted tests.

5.1 Data evaluation

The data used for the tests consist of two main categories; historical data, which is collected from the period 1998 – 2008, and predicted scenarios for year 2010. The historical data consists of weekly production, weekly spot price and historical forward prices, collected from the hydropower producer and Nord Pool. The predicted data consists of 70 scenarios for weekly production and weekly spot price for 2010. The scenarios are made by the MPS model and generation planning tools as explained in Section 2.3. For the static strategies we have used annual and quarterly contracts, as these are long-term contracts which suit the time perspective of the hydropower producer. Additionally, these contracts have high liquidity. For the historic test, the forward prices are collected 07.09 each year. For the predicted test, the data is collected 16.03.2009, corresponding to the date the prediction was made. The choice of dates should be of minor importance given an efficient market assumption, and since we want to focus on the weights in the strategies and not the timing of the sale, we have chosen to only use these dates. The weights have also been calculated on other dates, with minor differences and are therefore left out in the rest of the paper. To convert the forward prices nominated in EUR, we have used the EUR/NOK exchange rate for the corresponding date. We have however ignored the interest rate parity of the exchange rate as this is of minor importance.

5.2 Derivation of hedging strategies

Two strategies are derived by running the optimization model on the historical data. These are referred to as H1 and H2 and have VaR 10% constraints at 340 and 350 million NOK, respectively. These bounds are chosen because the hydropower producer had similar VaR 10% during the test period, and make a direct comparison between H1 and H2 and the current strategy (CS) of the hydropower producer possible. Due to the short period of only 10 data points, the standard deviation is not restricted. The weights derived for H1 and H2, in addition to the natural hedging and the hydropower producer's current strategy can be found in Table 5.1.

Constracts	Nat. Hedg.	CS	H1	H2
y1	-	0.50	0.20	0.31
y2	-	0.30	0.03	0.10
y3	-	0.20	-	-
q1	-	-	0.80	0.69
q2	-	-	0.37	-
q3	-	-	-	-
q4	-	-	-	0.23
q5	-	-	-	-
q6	-	-	-	-
q7	-	-	-	-
q8	-	-	-	0.23
Amount hedged of the current year's expected production	-	0.50	0.49	0.54

Table 5.1 Optimized weights for historic data.

The predicted strategies P1 to P7 are derived similarly to the historical strategies, but now the model is run on the predicted data. The strategies are derived by setting restrictions on VaR 10% and standard deviation from a high risk profile to a low risk profile. The restrictions and weights are shown in Table 5.2. As the predicted values reflect the hydropower producer's future expectations, the model should give a good hedging strategy complying with the restrictions.

	High Risk			Middle Risk			Low Risk	
Contracts	Nat. Hedg.	P1	P2	P3	P4	P5	P6	P7
VaR 10%	500	550	600	650	700	750	800	850
St. dev.	550	500	450	400	350	300	250	200
y1	-	-	-	-	-	-	-	-
q1	-	-	-	-	0.16	0.56	0.70	1.00
q2	-	-	-	-	-	-	-	0.82
q3	-	0.30	0.81	0.89	0.44	1.00	1.00	0.84
q4	-	-	-	0.25	0.74	0.50	0.83	0.84
Amount hedged of expected production in 2010	-	0.08	0.20	0.29	0.34	0.52	0.63	0.88

Table 5.2 Optimized weights for predicted data.

5.3 Results of the empirical tests

This section will show results from tests on the historical data from 1999 – 2008 and on the predicted data for 2010. The different strategies will be evaluated with respect to VaR, standard deviation and mean revenue/hedging cost[8]. Additionally, we will calculate CVaR for the predicted test. This is not done for the historical data as a consequence of only 10 data points. The hedging cost shows how much it will cost the hydropower producer, in means of lost revenue, to reduce volatility and secure against shortfalls.

5.3.1 Historical test

Table 5.3 shows the mean, standard deviation and VaR 10% of the annual revenue, as well as the annual hedging cost during the test period, 1999-2008. Table 5.4 shows the ranking of the different strategies for each of the measurements from Table 5.3. There is a clear relationship between the risk measures (standard deviation and VaR) and the mean revenue; the more risky a strategy is, the higher is the mean revenue. The CS strategy performs well during the test period with the second highest VaR and lowest standard deviation. However, it should be noted that this strategy clearly has the highest hedging costs, which results in the lowest mean revenue. The H1 and H2 strategies have similar values for VaR and standard deviation as CS, but much lower hedging costs. This is due to the fact that these strategies are optimized based on the data in the test period, and consequently, as emphasized in Section 4.3.3, can be considered as theoretical upper limits on the mean revenue for the given VaR values.

For the optimized strategies, P1 to P7, with predicted data as input, Table 5.4 shows that the performance is not in line with the intentions of the strategies. The ranking of the strategies seems to be somehow random and does not correspond to their underlying risk characteristics from the optimization model where the risk aversion, in terms of VaR and standard deviation, increases linearly from P1 to P7. This suggests and supports our findings in Section 2.4 that the evolution of the production and in particular the price during the time period 1999 to 2008 differs from the predicted production and price data for 2010.

There is a clear relationship between the measurements and the amount of production that is hedged. From Table 5.1 we can see that for the H1 strategy 49% of expected production is hedged, while for the H2 strategy 54% of expected production is hedged. In other words, the less risky the strategy is, the higher is the amount of hedged production. We also see that no

[8] The hedging cost of a strategy *S* is defined as the difference between the mean revenue of the natural hedging strategy and the mean revenue of *S*.

hedging is done three years prior to delivery; that is y_3 is equal 0. One possible explanation is that the spot price has increased a lot during the test period, see Figure 2.4.2, and this increase has probably not been anticipated in the forward curve; especially not in the long end. In general the strategy will therefore benefit by using the contracts in the short end of the curve. Another explanation is that in general the forward price of annual contracts tends to increase as time to delivery decreases, see for instance Benth, Benth and Koekebakker (2008). A producer therefore benefits by choosing the contracts that are closest to delivery. For the quarterly contracts we see that the Q1 contract is used most. As the case is for annual contracts, also for quarterly contracts the forward price tends to increase as the delivery date approaches[9].

We also see that the hedging costs are consistent with previous research on risk premiums. Strategies that mainly use contracts with short time to delivery, for instance H1 and H2, have either a hedging profit or just a small loss, while the strategies that use contracts with longer time to delivery have higher hedging losses. This is consistent with the findings in Benth, Benth and Koekebakker (2008), that there has historically been a negative risk premium for contracts with short time to delivery, and a positive risk premium for contracts with long time to delivery. Consequently, the producer benefits if using the quarterly contracts with shortest time to delivery.

Strategy	**Mean**	**St. dev.**	**VaR 10%**	**Cost**
Natural Hedg.	636	245	295	0
CS	591	221	347	45
H1	649	295	340	-13
H2	630	279	350	6
P1	629	236	305	7
P2	616	223	323	20
P3	608	222	328	28
P4	612	237	319	24
P5	614	256	345	22
P6	609	270	341	27
P7	620	321	285	16

Table 5.3 Statistics on an annual basis of the strategies for the historic data. All numbers are in million NOK.

[9] Benth, Benth and Koekebakker (2008)

Mean		St. dev.		VaR 10%	
H1	649	CS	221	H2	350
Natural Hedg.	636	P3	222	CS	347
H2	630	P2	223	P5	345
P1	629	P1	236	P6	341
P7	620	P4	237	H1	340
P2	616	Natural Hedg.	245	P3	328
P5	614	P5	256	P2	323
P4	612	P6	270	P4	319
P6	609	H2	279	P1	305
P3	608	H1	295	Natural Hedg.	295
CS	591	P7	321	P7	285

Table 5.4 Ranking of the strategies with respect to the statistics for the historic data. All numbers are in million NOK.

5.3.2 Predicted scenarios test

Table 5.5 shows the mean, standard deviation, VaR 10% and CVaR 10% of the predicted scenarios annual revenue, as well as the hedging cost for each strategy. Table 5.6 shows the ranking of the different strategies for each of the measurements from Table 5.5. The seven strategies optimized with respect to the predicted scenarios will be ranked according to their restrictions. So strategy P7 and P8 will be the most secured positions, but also have the lowest mean revenue and vice versa for the natural strategy and strategy P1. As expected, we get the same results as in Section 5.3.1; the in-sample optimized strategies, P1-P7 will give better results than H1, H2 and CS. However, the difference is small. For instance, the P5 strategy compared to the CS strategy has a VaR 10% 32 million NOK above and a standard deviation 7 million NOK below, but the mean is only 2 million NOK higher. The numbers are similar if P5 is compared to H1 and H2. This indicates that H1, H2 and CS seem to be relatively robust. This is also supported by the fact that the ranking of H1, H2 and CS is the same as in the historic test.

Overall the results show that the benefits by hedging are significant in terms of increasing VaR 10% and reducing standard deviation. At a maximum cost of 20 million NOK the VaR 10% can be increased from 511 to 844 million NOK, which is an increase of 65%. Similarly, the CVaR can be increased from 448 to 730 million NOK and the standard deviation decreased from 471 to 173 million NOK, which is an increase and decrease of 63%, respectively.

Strategy	Mean	St. dev.	VaR 10%	CVaR 10%	Cost
Natural Hedg.	1018	471	511	448	0
H1	1007	299	707	668	11
H2	1005	261	737	697	12
CS	1006	274	717	689	12
P1	1017	444	541	483	1
P2	1015	398	591	543	3
P3	1013	357	627	588	5
P4	1010	319	679	626	8
P5	1008	267	739	701	10
P6	1005	219	787	741	13
P7	998	173	844	730	20

Table 5.5 Statistics on an annual basis of the strategies for the predicted data. All numbers are in million NOK.

Mean		St. dev.		VaR 10%		CVaR 10%	
Natural Hedg.	1018	P7	173	P7	844	P7	730
P1	1017	P6	219	P6	787	P6	741
P2	1015	H2	261	P5	739	P5	701
P3	1013	P5	267	H2	737	H2	697
P4	1010	CS	274	CS	717	CS	689
P5	1008	H1	299	H1	707	H1	668
H1	1007	P4	319	P4	679	P4	626
CS	1006	P3	357	P3	627	P3	588
H2	1005	P2	398	P2	591	P2	543
P6	1005	P1	444	P1	541	P1	483
P7	998	Natural Hedg.	471	Natural Hedg.	511	Natural Hedg.	448

Table 5.6 Ranking of the strategies with respect to the statistics for the predicted data. All numbers are in million NOK.

In Figure 5.1, the empirical probability density functions (pdf) for predicted revenue are plotted for five strategies. The figure shows how the use of hedging can transform the pdf of the revenue. For instance the most risky strategy, the natural hedging strategy, has a wide shape with high standard deviation. On the other hand the P7 strategy has a much narrower pdf, where the downside risk, but also the upside potential is limited. The strategies P1, P5 and P7 have a hedge ratio of 8%, 52% and 88% of expected production, respectively. We can see how the pdfs are narrowed as the hedge ratio increases.

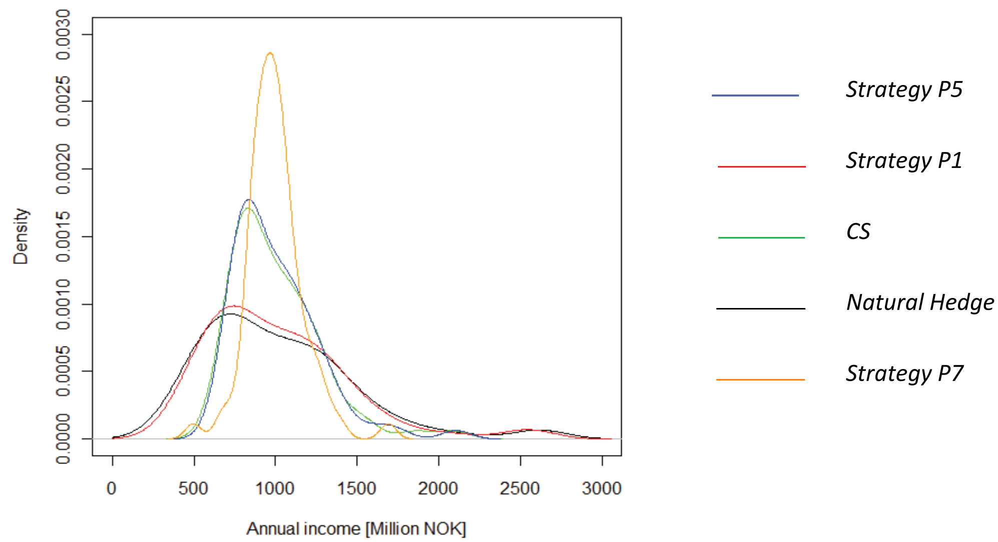

Figure 5.1 Empirical probability distribution of selected strategies.

Figure 5.2 shows the cost versus risk measures of the optimized strategies. The VaR 10% seems to follow a linear pattern with respect to its cost. The linearity indicates that the marginal risk reduction, in terms of VaR, is approximately constant for all hedging ratios. However, the CVaR 10% seems to flatten out around the cost of 13 million NOK, which indicates a marginal risk reduction in CVaR of zero above 13 million NOK. As discussed in Section 3.1, even though VaR increases as cost increases, the expected value of the revenue below the VaR 10% remains the same. Consequently, the hydropower producer will increase the insurance in the 90% secured revenue, but the tail will still suffer from the major shortfalls in the remaining 10% interval.

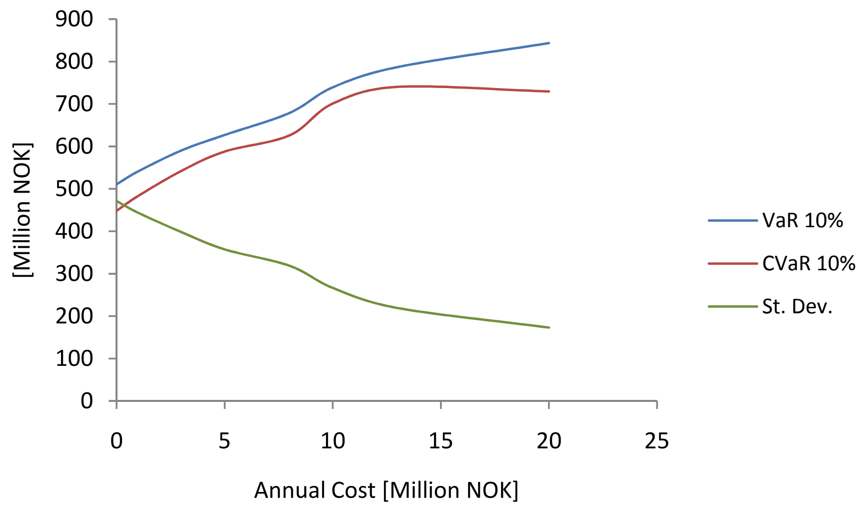

Figure 5.2 Risk measures plotted against the cost of the optimized strategies.

5.4 Performance summary

In this section we will compare the results from the historical test and the predicted test. The results have shown that static hedging is beneficial in terms of increasing the VaR and CVaR, and reducing the standard deviation of the revenue. This comes at just a minor decrease in mean revenue. The minor reduction in mean revenue is in line with the theory about risk premium, where it is shown that reducing the variability and downside risk does not necessarily have to affect the mean revenue substantially.

An important property of the strategies is how robust they are with respect to uncertainty in the input data. It is important to remember that the strategies are optimized on the distribution of the input data. However, if this distribution is not the one that will occur in the future, the strategies may not perform as expected. It was shown that the ranking of H1, H2 and CS was the same for both the historical and predicted tests, which indicates that the strategies are robust. However, this is in contrast with the results when the predicted strategies were tested on the historic data. For instance the strategy P7, which is the strategy with the highest VaR 10% constraint and lowest standard deviation for the predicted test, is the strategy with the lowest VaR 10% and highest standard deviation for the historic test. Even the natural hedging strategy, will give better results for VaR 10% and standard deviation. Additionally, as mentioned above, the ranking of the risk profile for P1 to P7 is no longer consistent with their intentional risk profiles, but seems to be random. This suggests that the strategies are not robust, and may therefore consequently fail if the predicted data turns out to be inaccurate. A possible way to compensate for this is to use a diversified version of the optimal strategies. This is done by restricting the use of each contract, such that the strategy will not only utilize a small number of contracts, but divide the risk over multiple contracts. As shown in Table 5.2 the optimal strategy mainly uses Q3 and Q4 forward contracts, because these are the contracts that give the best profit during the test period. Under an efficient market assumption none of the contracts should perform better than the other, and one should make use of several contracts in order to diversify the risk. Such diversification will also reduce optimizers bias.

Finally, it is important to note that the model in a practical setting should be rerun at certain times and the weights adjusted accordingly. Such a re-optimization is similar to the method proposed by Bjerksund, Stensland and Vagstad (2008). This allows the strategy to incorporate new information and capture changes in the distributions of the input parameters.

6 CONCLUSION

In this paper we have developed an optimization model for deriving static hedging strategies. We have used this model to propose strategies with different risk characteristics for a hydropower producer. The strategies were tested and compared with the natural hedging strategy on historical and predicted data. The results show that hedging with use of forward contracts significantly reduces the risk in terms of VaR, CVaR and standard deviation. This improvement results in just a minor reduction of the mean revenue. It is also shown that the strategies may not be that robust, as they depend on the accuracy of the input data. A solution is to diversify the strategies across the available contracts, such that the strategy does not depend only on the performance of a small number of contracts.

As a concluding remark, static hedging provides a simple strategy, which gives good results with respect to risk management for hydropower producers. However, further studies that compare the out-of-sample performance of static and dynamic hedging strategies should be conducted.

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APPENDIX

A.1 Formulas

$$VaR(V,\alpha) = F(V;\alpha) \tag{A.1.1}$$

$$CVaR(V,\alpha) = E[V \mid V \leq F(V;\alpha)] \tag{A.1.2}$$

where $F(V;\alpha)$ is the cumulative distribution of the random variable V.

$$CFaR(V,\alpha) = K - VaR(V,\alpha) \tag{A.1.3}$$

where K is an appropriate chosen constant.

Risk premium: $$(1+\pi)^{T-t} = \frac{E_t(S_T)}{F_{t,T}}(1+r)^{T-t} \tag{A.1.4}$$

where π is the risk premium, *r* is the risk free rate, $E_t(S_T)$ is the expected spot price at time *T*, and $F_{t,T}$ is the forward price at time *t* for a contract with delivery at time *T*.

A.2 Implementation of model in AMPL

The model is implemented in two similar ways. One for the historical data, presented in A.2.1 and one for the predicted data presented in A.2.2.

A.2.1 Model for historical data

```
#1 INPUT DATA
set index1;
set index2;
param yearlyTimeHorizon := 5;
param quarterlyTimeHorizon := 8;
param numberOfYears := 11;
param alpha := 0.1;
param VaR_1yr_alpha := 350;

param expectedProduction{index1};
param spot{index2};
param production{index2};
param yearlyForwards{1..numberOfYears, 1..yearlyTimeHorizon};
param quarterlyForwards{1..numberOfYears, 1..quarterlyTimeHorizon};

data AMPL_Data_0709.dat;
```

```
#2 OPTIMIZATION PROBLEM

#2.1 Decision variables
var x {1..yearlyTimeHorizon} >= 0;
var q {1..quarterlyTimeHorizon} >= 0;
var spotProfit {1..numberOfYears};
var quarterlyProfit {1..numberOfYears};
var yearlyProfit {1..numberOfYears};
var z {1..numberOfYears} binary;

#2.2 Objective function
maximize income: sum {y in 2..numberOfYears} (yearlyProfit[y] + quarterlyProfit[y]);

#2.3 Revenue constraints
subject to spotProfitCalc1999: spotProfit[2] = sum {w in 1..52} spot[w]*production[w]/1000;

subject to yearlyProfitCalc1999: yearlyProfit[2] =(yearlyForwards[1,1]*expectedProduction[1]*x[1] -
(sum {w in 1..52} spot[w]*expectedProduction[1]*x[1]/52))/1000;

subject to quarterlyProfitCalc1999: quarterlyProfit[2] = (sum{i in 1..4}
(quarterlyForwards[1,i]*expectedProduction[1]*0.25*q[i] - (sum {j in 1..13} spot[j + 13*(i-
1)]*expectedProduction[1]*q[i]/52)))/1000;

subject to spotProfitCalc2000: spotProfit[3] = sum {w in 53..104} spot[w]*production[w]/1000;

subject to yearlyProfitCalc2000: yearlyProfit[3] = (yearlyForwards[1,2]*expectedProduction[1]*x[2] +
yearlyForwards[2,1]*expectedProduction[1]*(x[1] - x[2]) - (sum {w in 53..104}
spot[w]*expectedProduction[1]*x[1]/52))/1000;

subject to quarterlyProfitCalc2000: quarterlyProfit[3] = (sum{k in 5..8}
quarterlyForwards[1,k]*expectedProduction[1]*0.25*q[k] + sum{i in 1..4}
(quarterlyForwards[2,i]*expectedProduction[1]*0.25*(q[i]-q[i+4]) - (sum {j in 1..13} spot[j + 13*(i-1) +
52]*expectedProduction[1]*q[i]/52)))/1000;

subject to spotProfitCalc2001_2008 {y in 4..numberOfYears}: spotProfit[y] = sum {w in 1..52} (spot[w +
52*(y-2)]*production[w + 52*(y-2)])/1000;

subject to yearlyProfitCalc2001_2008 {y in 4..numberOfYears}: yearlyProfit[y] = (yearlyForwards[y-
3,3]*expectedProduction[1]*x[3] + yearlyForwards[y-2,2]*expectedProduction[1]*(x[2] - x[3]) +
yearlyForwards[y-1,1]*expectedProduction[1]*(x[1] - x[2]) - (sum {w in 1..52} spot[w + 52*(y-
2)]*expectedProduction[1]*x[1]/52))/1000;

subject to quarterlyProfitCalc2001_2008 {y in 4..numberOfYears}: quarterlyProfit[y] = (sum{k in 5..8}
quarterlyForwards[y-2,k]*expectedProduction[1]*0.25*q[k] + sum{i in 1..4} (quarterlyForwards[y-
1,i]*expectedProduction[1]*0.25*(q[i]-q[i+4]) - (sum {j in 1..13} spot[j + 13*(i-1) + 52*(y-
2)]*expectedProduction[1]*q[i]/52)))/1000;

#Trading constraints
subject to onlyShortYearly {i in 1..yearlyTimeHorizon-1}: x[i] >= x[i+1];
subject to onlyShortQuarterly {i in 1..4}: q[i] >= q[i+4];
subject to notShortMoreThanEP {i in 1..4}: x[1] + q[i]<= 1;

#2.4 VaR constraints:
subject to VaR_1yr_alphaConstraint{y in 2..numberOfYears}:
        spotProfit[y] + yearlyProfit[y] + quarterlyProfit[y] >= VaR_1yr_10pr*z[y];

subject to zConstraint: sum {i in 1..numberOfYears-1} z[i] >= numberOfYears*(1-alpha);
```

A.2.2 Model for predicted data

```
#1 INPUT DATA
set index1;
set index2;
param annualTimeHorizon := 1;
param quarterlyTimeHorizon := 8;
param numberOfYears := 1;
param numberOfScenarios := 70;
param alpha := 0.1;
param VaR_1yr_alpha := 400;
param varianceLimit = 10000000;
param expectedProduction{index1};
param production2010{1..52, 1..numberOfScenarios};
param price2010{1..52, 1..numberOfScenarios};

param annualForwards{1..numberOfYears, 1..annualTimeHorizon};
param quarterlyForwards{1..numberOfYears, 1..quarterlyTimeHorizon};

data predAMPL_Data_0703.dat;

#2 OPTIMIZATION PROBLEM

#2.1 Decision variables
var x {1..annualTimeHorizon} >= 0;
var q {1..quarterlyTimeHorizon} >= 0;
var spotProfit {1..numberOfYears, 1..numberOfScenarios};
var quarterlyProfit {1..numberOfYears, 1..numberOfScenarios};
var annualProfit {1..numberOfYears, 1..numberOfScenarios};
var mean{1..numberOfYears};
var z {1..numberOfYears, 1..numberOfScenarios} binary;

#2.2 Objective function
maximize revenue: sum {y in 1..numberOfYears, s in 1..numberOfScenarios} (annualProfit[y,s] +
quarterlyProfit[y,s])/( numberOfScenarios);

#2.2 Revenue constraints
subject to spotProfitCalc2010 {s in 1..numberOfScenarios}: spotProfit[2,s] = sum {w in 1..52}
price2010[w,s]*production2010[w,s]/1000;

subject to annualProfitCalc2010 {s in 1..numberOfScenarios}: annualProfit[2,s]
=(annualForwards[1,2]*expectedProduction[1]*x[2] +
annualForwards[2,1]*expectedProduction[1]*(x[1]-x[2]) - (sum {w in 1..52}
price2010[w,s]*expectedProduction[1]*x[1]/52))/1000;

subject to quarterlyProfitCalc2010 {s in 1..numberOfScenarios}: quarterlyProfit[2,s] = (sum{i in 1..4}
(quarterlyForwards[1,i]*expectedProduction[1]*0.25*q[i+4] +
quarterlyForwards[2,i]*expectedProduction[1]*0.25*(q[i] - q[i+4])  - (sum {j in 1..13} price2010[j + 13*(i-
1),s]*expectedProduction[1]*q[i]/52)))/1000;

#2.3 Trading constraints
subject to onlyShortAnnual {i in 1..annualTimeHorizon-1}: x[i] >= x[i+1];
```

```
subject to onlyShortQuarterly {i in 1..4}: q[i] >= q[i+4];
subject to notShortMoreThanEP {i in 1..4}: x[1] + q[i]<= 1;

#2.4 VaR constraints:
subject to VaR_1yr_alphaConstraint{y in 2..numberOfYears, s in 1..numberOfScenarios}:
	spotProfit[y,s] + annualProfit[y,s] + quarterlyProfit[y,s] >= VaR_1yr_alpha*z[y,s];

subject to zConstraint: sum {i in 1..numberOfYears-1, s in 1..numberOfScenarios} z[i,s] >= (1-
alpha)*NumberOfScenarios;

#2.5 Variance constraints
subject to varianceConstraints2010: sum{s in 1..numberOfScenarios} ((spotProfit[2,s] +
annualProfit[2,s] + quarterlyProfit[2,s] - mean[2])^2)/(numberOfScenarios-1) <= varianceLimit2010;
subject to meanCalc {y in 1..numberOfYears}: mean[y] = (sum{s in 1..numberOfScenarios}
spotProfit[y,s] + annualProfit[y,s] + quarterlyProfit[y,s])/numberOfScenarios;
```

PAPER II

Evaluation of a dynamic hedging strategy for hydropower producers in the Nordic market

ABSTRACT

In this paper we use the methodology of Peterson and Stapleton (2002) to develop a stochastic optimization model for deriving a dynamic hedging strategy for hydropower producers. The approach is similar to Kettunen, Salo and Bunn (2007), but is applied on the producer side and over a longer time horizon. Previous research has primarily considered in-sample performance. Our contribution is to evaluate the ex post performance in means of risk reduction and mean revenue. The dynamic hedging strategy is compared to the natural hedging strategy, which means no hedging, and a static hedging strategy which is the common choice in practice amongst hydropower producers. Our results show that dynamic and static hedging give significant benefits in terms of increasing VaR, CVaR and the mean revenue compared to the natural hedging strategy. Additionally, the dynamic hedging strategy has significant better VaR and CVaR than the static hedging strategy, while it indicates a better performance in the mean revenue. Our results imply that dynamic hedging strategies should be considered in a practical setting.

Key words: Risk management, Dynamic hedging, Hydropower producers, Nordic electricity market, Stochastic optimization

1 INTRODUCTION

The liberalization of the Nordic power market in the early 1990`s dramatically changed the competitive environment for hydropower producers. Before the liberalization, the electricity price was regulated by the governments. Consequently, producers did not have any incentives to hedge the electricity price. However, after the deregulation, the control of the electricity price has been removed and as a result the price variation increased[1]. This has led to the development of a market for electricity derivatives. As a result Nord Pool, the power exchange for the Nordic countries, was established in 1993. At Nord Pool standardized derivatives, such as forwards/futures and options, are traded and enable a way for producers to manage and handle their risk exposure to the electricity price. The task of managing the risk with respect to the electricity price is however not an easy one. As mentioned above, the electricity price has high volatility and may have spikes of several orders of magnitude within short time. This is mainly caused by the fact that electricity has very limited storage possibilities. Hydropower producers can to some extent store indirectly in water reservoirs. However, consumers can not buy electricity for storage. This implies that the cost-of-carry relationship between spot and forwards break down. In other words, the relationship between the spot and forwards is weaker than for other commodities. The electricity price therefore also experience strong seasonality.

Over the last decade there has been an increasing interest, both in academia and among practitioners, in the area of risk management for electricity producers. These have had to adapt to the new environment which the above-mentioned liberalization has caused, and in some way or another employ methods that aim to manage the new risk exposures. For a hydropower producer the electricity price and the inflow, that is how much water that flows into the reservoirs, are the two most significant determinants for the revenue. As both the price and the inflow experience huge variations, they are also the two most important risk factors for the future revenue. Most of the approaches that producers have employed in order to manage the risk of price and inflow have been static in nature. This is mainly due to the fact that static hedging strategies are easy to carry out, require less transaction and management costs, and are able at minor hedging costs to reduce the risk of future revenue significantly[2]. The relatively good performance of static hedging strategies is however not a legitimate argument for not considering dynamic hedging strategies. Several studies have over

[1] Knittel and Roberts (2005)

[2] Fleten, Bråthen and Nissen-Meyer (2009)

the last few years indicated that dynamic hedging strategies offer significantly more efficient hedges than static strategies; for instance Fleten, Wallace and Ziemba (2002) and Kettunen Salo and Bunn (2007). The studies have also indicated that dynamic hedging strategies should be implementable and manageable from a practical point of view. Fleten, Wallace and Ziemba (2002) uses stochastic programming to find the optimal integrated production schedule and financial hedging plan for a hydropower producer. Bjerksund, Stensland and Vagstad (2008) uses deterministic dynamic programming to deduce the optimal production and financial hedging plan for a natural gas storage, and claims that similar methods could be used for electricity producers. Kettunen, Salo and Bunn (2007) proposes an approach similar to the one in Fleten, Wallace and Ziemba (2002), but take the production plan as given and focuses on finding the optimal financial hedging plan.

In this paper we use the methodology of Peterson and Stapleton (2002) to create a scenario tree of the electricity price and hydropower production. We then use an optimization approach similar to the one in Kettunen, Salo and Bunn (2007), to derive the optimal dynamic hedging strategy of a hydropower producer's revenue. Our resulting dynamic hedging strategy is then tested and compared to results of a static hedging strategy and a natural hedging strategy on the same set of data. As Kettunen, Salo and Bunn (2007) suggests for further work, the model is tested ex post; more specifically on actual data from a Nordic hydropower producer in the period January 2007 to April 2009. Additionally, as the paper also suggests, we increase the time horizon from six weeks to six months. The main purpose of this paper is to evaluate the proposed dynamic hedging strategy using actual data, and compare its ex post performance to a more well-established (among practitioners) static hedging strategy. The results are evaluated in terms of (i) the (Conditional) Value at Risk and standard deviation of the revenue and (ii) the expected/actual revenue and hedging profit/loss.

The paper is structured as follows. Section 2 presents the theory that underlies the proposed method. Section 3 gives a review of earlier research within risk management for hydropower producers. Section 4 presents and discusses our implementation of the proposed method. Section 5 evaluates the results from the empirical tests. Section 6 concludes.

2 THEORY AND ASSUMPTIONS

Peterson and Stapleton (2002) provides a methodology for modeling two or more correlated Ornstein-Uhlenbeck processes by binomial trees. As both price and production contain mean-reversion and are correlated, this methodology gives a suitable approximation for a hydropower producer's revenue. After first modeling the price and production with two binomial trees, these trees are merged into a scenario tree that captures the correlation. An optimization model that incorporates trading and risk management constraints is then applied to derive an optimal dynamic hedging strategy. This strategy consists of a set of weights that, in any node at any time step in the scenario tree, indicates the amount and type of derivatives that should be traded. In this paper we will only consider forward contracts, as these are the contracts with highest liquidity in the Nordic market. We will also assume that the length of the forward contracts' delivery period equals the length of one time period in the tree. It will however in practice be no problem to use forward contracts with different delivery periods and other derivatives, for instance options. In this section, theory for the method will be provided and in Section 4 an implementation of the method will be presented.

2.1 Binomial modeling

The production tree is first constructed independently of the price. Afterwards, the price tree is constructed to capture the correlation with the production tree. The underlying assumption of the production and price trees is that the logarithmic levels of the production and price follow Ornstein-Uhlenbeck processes. More specifically, if P_t and S_t are the production and price at time t respectively, the log-production p_t and the log-price s_t are assumed to follow mean reverting Ornstein-Uhlenbeck processes:

$$\begin{aligned} dp_t &= \kappa_p(\phi_t - p_t)dt + \sigma_p(t)dW_{1,t} \\ ds_t &= \kappa_s(\theta_t - s_t)dt + \sigma_s(t)dW_{2,t} \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} p_t &= \ln[P_t / E(P_t)] \\ s_t &= \ln[S_t / E(S_t)] \\ E[dW_{1,t}dW_{2,t}] &= \rho dt \end{aligned}$$

In equation (2.1), ϕ_t and θ_t are constants, κ_p and κ_s are the rates of mean reversion of p_t and s_t and $\sigma_p(t)$ and $\sigma_s(t)$ are the conditional standard deviations of p_t and s_t, respectively. By transforming the processes into discrete time, the following discrete time versions of equation (2.1) are obtained (see Peterson and Stapleton (2002) for details):

$$\begin{aligned} p_t &= \alpha_{p,t} + \beta_{p,t} p_{t-1} + \varepsilon_{p,t} \\ s_t &= \alpha_{s,t} + \beta_{s,t} s_{t-1} + \gamma_{s,t} p_{t-1} + \delta_{s,t} p_t + \varepsilon_{s,t} \end{aligned} \tag{2.2}$$

where

$$\alpha_{p_t} = \frac{-t\sigma_{p,t}^2 + \beta_{p,t}(t-1)\sigma_{p,t-1}^2}{2}$$

$$\alpha_{s_t} = \frac{-t\sigma_{s,t}^2 + \beta_{s,t}(t-1)\sigma_{s,t-1}^2 + \gamma_{s,t}(t-1)\sigma_{p,t-1}^2 + \delta_{s,t} t\sigma_{p,t}^2}{2}$$

$$\beta_{p,t} = 1 - \kappa_p$$

$$\beta_{s,t} = 1 - \kappa_s$$

$$\gamma_{s,t} = \frac{\rho\sigma_s(t)}{\sigma_p(t)}(-1 + \kappa_p)$$

$$\delta_{s,t} = \frac{\rho\sigma_s(t)}{\sigma_p(t)}$$

where $\sigma_{p,t}$ and $\sigma_{s,t}$ are the periodized unconditional standard deviations of p_t and s_t respectively; I.e. the standard deviation of p_t and s_t from time 0 to time t equals $\sqrt{t}\sigma_{p,t}$ and $\sqrt{t}\sigma_{s,t}$ respectively.

2.1.1 Construction of production tree

Given the discrete version of the production process in equation (2.2) and values for the expected production $E_0[P_t]$ at time 0 for each future time period *t-1* to *t*, the production tree can be constructed. The production in each node is calculated in the following way:

$$P_t(r) = E_0[P_t] u_{p_t}^{(t-r)} d_{p_t}^{(r)} \tag{2.3}$$

where

$$d_{p_t} = \frac{2}{1+\exp(2\sigma_p(t))}$$

$$u_{p_t} = 2 - d_{p_t}$$

and r is the cumulative number of down movements from time 0 to time t.

The probability of an upward move at node r at time t is given by

$$q_{p_t}(r) = \frac{\alpha_{p_t} + \beta_{p_t} p_{t-1} - (t-1-r)\ln(u_{p_t}) - (1+r)\ln(d_{p_t})}{\ln(u_{p_t}) - \ln(d_{p_t})} \qquad (2.4)$$

2.1.2 Construction of price tree

After the production tree is created, the price tree is modeled to capture the correlation with the production. The price tree can be modeled, given the discrete version of the price process in equation (2.2), risk premiums[3] Π_t for contracts with t periods to delivery, and forward prices $F_{0,t}$ at time 0 for each future time period t to $t+1$. The price in each node is then calculated in the following way:

$$S_t(r) = E_0[S_t] u_{s_t}^{(t-r)} d_{s_t}^{(r)} \qquad (2.5)$$

where

$$d_{p_t} = \frac{2}{1+\exp(2\sigma_p(t))}$$

$$u_{p_t} = 2 - d_{p_t}$$

$$E_0[S_t] = F_{0,t-1}(1+\pi_t)$$

The probability of an upward move at node r at time t is given by

$$q_{s_t}(r) = \frac{\alpha_{s,t} + \gamma_{s,t} p_{t-1} + \delta_{s,t} p_t + \beta_{p,t} s_{t-1} - (t-1-r)\ln(u_{s_t}) - (1+r)\ln(d_{s_t})}{\ln(u_{s_t}) - \ln(d_{s_t})} \qquad (2.6)$$

[3] See Appendix A.1 for definition of risk premium

2.1.3 Construction of scenario tree

The production tree and the price tree can now be combined into a path dependent scenario tree. At each time step *t*, this tree has 2^{2t} nodes. Let V_t^j be defined as

$$V_t^j = [v_0^j, v_1^j, \ldots, v_{t-1}^j] \tag{2.7}$$

$$v_i^j = \begin{cases} 0, & \textit{if } j \textit{ had a upward move at time } i \\ 1, & \textit{if } j \textit{ had a downward move at time } i \end{cases} \qquad \text{where j = P,S and i = 0…t-1}$$

Each node in the scenario tree can then be uniquely defined by a vector pair (V_t^P, V_t^S), where V_t^P and V_t^S denote the evolution of the production and price up to time *t,* respectively. Since the scenario tree is not recombining, every node at time *t* has a unique predecessor at all previous time points *s*, *s* < *t*. Let the predecessor of node (V_t^P, V_t^S) at time *s*, with *s* < *t,* be given by $B_s(V_t^P, V_t^S)$. Let $N_P(V_t^P, V_t^S)$ and $N_S(V_t^P, V_t^S)$ be the cumulative number of downward moves the production and price had from time 0 to time *t,* respectively:

$$N_P(V_t^P, V_t^S) = t - \sum_{i=0}^{t-1} v_i^P \qquad v_i^P \in V_t^P$$

$$N_S(V_t^P, V_t^S) = t - \sum_{i=0}^{t-1} v_i^S \qquad v_i^S \in V_t^S \tag{2.8}$$

The probability at time zero of reaching node (V_t^P, V_t^S) at time *t* is then given by

$$p(V_t^P, V_t^S) = \prod_{i=0}^{t-1} q_{P_i}\left(N_P(B_i(V_t^P, V_t^S))\right)^{1-v_i^P}\left[1 - q_{P_i}\left(N_P(B_i(V_t^P, V_t^S))\right)\right]^{v_i^P}$$
$$q_{S_i}\left(N_S(B_i(V_t^P, V_t^S))\right)^{1-v_i^S}\left[1 - q_{S_i}\left(N_S(B_i(V_t^P, V_t^S))\right)\right]^{v_i^S} \tag{2.9}$$

Let $P(V_t^P, V_t^S)$ and $S(V_t^P, V_t^S)$ be the production and price at node (V_t^P, V_t^S) in the scenario tree. $P(V_t^P, V_t^S)$ and $S(V_t^P, V_t^S)$ are then given by

$$P(V_t^P, V_t^S) = P_t(N_P(V_t^P, V_t^S)) \tag{2.10}$$

$$S(V_t^P, V_t^S) = S_t(N_S(V_t^P, V_t^S)) \tag{2.11}$$

where $P_t(.)$ and $S_t(.)$ are defined in equation (2.3) and (2.5) respectively.

The forward price $F_{t,T}(V_t^P, V_t^S)$ in node (V_t^P, V_t^S) at time *t* for delivery in period *T* to *T+1* is calculated as

$$F_{t,T}(V_t^P, V_t^S) = \frac{1}{p(V_t^P, V_t^S)(1+\Pi_{T-t})} \left[\sum_{\substack{\forall (V_{T+1}^P, V_{T+1}^S) | B_t(V_{T+1}^P, V_{T+1}^S) \\ = (V_t^P, V_t^S)}} S(V_{T+1}^P, V_{T+1}^S) p(V_{T+1}^P, V_{T+1}^S) \right] \quad (2.12)$$

The expected production $E[P_T | (V_t^P, V_t^S)]$ in time period *T* to *T+1*, given that we are in node (V_t^P, V_t^S) at time *t* is given by

$$E[P_T | (V_t^P, V_t^S)] = \frac{1}{p(V_t^P, V_t^S)} \left[\sum_{\forall (V_{T+1}^P, V_{T+1}^S) | B_t(V_{T+1}^P, V_{T+1}^S) = (V_t^P, V_t^S)} P(V_{T+1}^P, V_{T+1}^S) p(V_{T+1}^P, V_{T+1}^S) \right] \quad (2.13)$$

2.2 Optimization model

After the scenario tree for future price and production is constructed, the revenue in each time period *t-1* to *t* for every node at time *t* is calculated. Each such periodic revenue consists of two parts; revenue from sale of production at the spot price and profit/loss from forward contracts that have delivery during that time period. I.e. the contracts are not marked to market prior to delivery. Let $R_{t-1,t}(V_t^P, V_t^S)$ be the revenue in time period *t-1* to *t*, given that we are in node (V_t^P, V_t^S) at time *t*. The value of $R_{t-1,t}(V_t^P, V_t^S)$ is then given by

$$\begin{aligned} R_{t-1,t}(V_t^P, V_t^S) = & \underbrace{\sum_{i=0}^{t-2} \left[\left(F_{i,t-1}(B_i(V_t^P, V_t^S) - S(V_t^P, V_t^S) \right) x_{i,t-1}(B_i(V_t^P, V_t^S) \right]}_{\text{Forward profit/loss}} \\ & + \underbrace{S(V_t^P, V_t^S) P(V_t^P, V_t^S)}_{\text{Spot revenue}} \end{aligned} \quad (2.14)$$

where $x_{i,t-1}(B_i(V_t^P, V_t^S)$ is the amount shorted at time *i* of a forward contract with delivery in period *t-1* to *t*.

The cumulative revenue $R_{0,t}(V_t^P, V_t^S)$ in node (V_t^P, V_t^S) at time *t* can then be calculated as the sum of all the periodic revenues in predecessor nodes of (V_t^P, V_t^S) :

$$R_{0,t}(V_t^P, V_t^S) = \sum_{i=1}^{t} R_{i-1,i}(B_{i-1}(V_t^P, V_t^S) \tag{2.15}$$

The dynamic hedging strategy can now be derived by defining an optimization model on the scenario tree. The optimization model will typically be formulated as maximizing the expected accumulated end term revenue with respect to $x_{t,T}(V_t^P, V_t^S)$, *t = 0,..N-2, T = t+1,.., N-1* (where *N* is the number of time periods in the tree), subject to a set of constraints. The set of constraints consists of revenue constraints, trading constraints, and risk management constraints. The revenue constraints are simply given by equation (2.14) and (2.15). The trading constraints define the type of trades that are allowed and the type of trades that should be restricted. Assuming that an electricity producer is trading for hedging purposes and not speculating, these constraints will typically be; (i) in any period it is not allowed to short more than the period's expected production and (ii) it is not allowed to buy contracts, unless they are previously shorted. The risk management constraints can in general be considered as restrictions on the distribution of the end of term revenue and/or restrictions on the revenue in each period. These constraints will typically restrict the revenues' (Conditional) Value at Risk, Cash Flow at Risk and/or variance[4].

2.3 Assumptions and evaluation

Since the model is an optimization model, it will by definition return the dynamic strategy that gives the highest expected revenue, subject to the constraints and input data. In this way it is easy to use the model to determine a producer's hedging strategy once their risk aversion in terms of risk management and trading restrictions is identified. However, it is important to note that the strategy is optimal with respect to the input data for spot price, production and forward prices. If these are not accurately estimated the model performance will be limited in out-of-sample tests. Recently, there has however been published research that provides a general framework for improving the out-of-sample performance of portfolios, for details see DeMiguel et.al. (2009). Applying such methods could potentially give the model a more robust out-of-sample performance.

[4] See Appendix A.1 for definitions.

It is also important to be aware of the underlying assumption of the price and production processes in the scenario tree; that their logarithmic levels follow Ornstein-Uhlenbeck processes. Even though the Ornstein-Uhlenbeck process is able to incorporate important properties of the production and electricity price, and is widely used in financial modeling (especially for interest rates and commodities), it may fail to approximate the electricity price and production due to spikes, skewness, high kurtosis and long term uncertainty[5]. Therefore more complex stochastic processes could potentially serve as better approximations of the price and production than the Ornstein Uhlenbeck process. However, using more complex processes would involve more input parameters and thereby more estimation errors.

Another important consideration is the number of time periods in the scenario tree. The accuracy of the approximation of the Ornstein-Uhlenbeck processes is dependent on this number; the more time periods the better is the approximation. This is, as reported in Ho, Stapleton and Subrahmanyam (1994), due to the fact that if the number of time periods is small, the conditional probabilities given by equation (2.5) and (2.6) may fall outside of the natural bounds for probability (i.e. the value of $q_{p_t}(r)$ and $q_{s_t}(r)$ could be less than zero or greater than one). Furthermore, the fact that the number of nodes in the scenario tree is 2^{2t} at time *t*, limits the number of possible time steps. A solution is to restrict the values of $q_{p_t}(r)$ and $q_{s_t}(r)$ to be in the interval [0, 1]. Ho, Stapleton and Subrahmanyam (1995) investigates the accuracy of the obtained variance and mean in the tree when *N* is small and with the [0,1] restriction on $q_{p_t}(r)$ and $q_{s_t}(r)$ added. The results show that the accuracy is acceptable even for small values of *N,* and that the accuracy is not adversely affected by the [0, 1] restriction, except for large values of ρ (e.g. ρ=0.9).

3 REVIEW OF EARLIER WORK

Earlier work on stochastic programming for deriving dynamic hedging strategies for electricity producers may be divided into two main categories. The first category integrates production planning and financial hedging into one problem, and aims to find the best combination of these two. The second category considers only financial hedging and takes the production schedule as an exogenously given stochastic variable. Both categories can be considered as extensions to the approach most electricity producers are using in practice; first

[5] Benth, Benth, and Koekebakker (2008)

use a stochastic model to derive a dynamic production schedule, then use a static model to derive the hedging strategy.

Fleten, Wallace and Ziemba (2002) develops and implements a general framework within the first category. A model based on the framework is then implemented and tested on a numerical example. The results show that this approach has significant benefits compared to the approach that most electricity producers are using. The risk is reduced by 32% for the same level of expected profit, and the expected profit is increased by 1.1% for the same level of risk. These results are supported by Sen, Yu and Genc (2003), who develops a large scale stochastic model for integrated production scheduling and financial hedging, and finds that the model provides a powerful and robust method for scheduling and hedging in the wholesale electricity market. Eichorn et al. (2004) also develops a stochastic model and uses it to derive an optimal dynamic production schedule and financial hedging strategy.

Kettunen, Salo and Bunn (2007) proposes a method within the second category, i.e. they take the production schedule as given and focus on finding a dynamic hedging strategy. The model is tested on numerical examples from the point of view of a power distributor, which is an analogous problem to the one an electricity producer faces. The results show that the method yields higher expected return at a lower risk level than more conventional approaches, such as periodic optimization and static strategies.

The work above shows that stochastic programming for deriving dynamic hedging strategies is a useful method, and can give substantial improvements in the risk profile and the expected value of a hydropower producer's revenue.

4 MODEL

This section explains the implementation of the theory discussed in Section 2 in detail. Section 4.1 considers the implementation of the scenario tree. Section 4.2 describes the optimization model's constraints. Section 4.3 describes our implementation of the optimization model and Section 4.4 gives an evaluation of the implementation.

4.1 The scenario tree

First, the conditional standard deviation of the log-production is calculated assuming a discrete AR(1) process as described by Ho, Stapleton and Subrahmanyam (1994):

$$\sigma_p^2(t) = t\sigma_{p,t}^2 - \beta_{p,t}^2 (t-1)\sigma_{p,t-1}^2 \tag{4.1}$$

where $\sigma_{p,t}$ and $\beta_{p,t}$ are defined as in equation (2.2)

Similarly, the conditional standard deviation for the log-price is calculated using Peterson and Stapleton (2002) equation (21):

$$\begin{aligned}\sigma_s^2(t) = t\sigma_{s,t}^2 - [&\beta_{s,t}^2 (t-1)\sigma_{s,t-1}^2 + \gamma_{s,t}^2 (t-1)\sigma_{p,t-1}^2 + \delta_s^2 t\sigma_{p,t}^2 \\ &+ 2\beta_{s,t}^2 \gamma_{s,t}^2 \operatorname{cov}(s_{t-1}, p_{t-1}) + 2\beta_{y,t}^2 \gamma_{s,t}^2 \operatorname{cov}(s_{t-1}, p_t) + 2\delta_{s,t}^2 \gamma_{s,t}^2 \operatorname{cov}(p_{t-1}, p_t)]\end{aligned} \tag{4.2}$$

where the parameters are defined as in equation (2.2)

Given the conditional standard deviations, the nodal values for production and price are calculated by equation (2.3) and equation (2.5). Then the conditional probabilities q_{p_t} and q_{s_t} for the production and price are calculated using equation (2.4) and equation (2.7). The scenario tree is then constructed using the theory from Section 2.1.3.

4.2 Constraints

The hydropower producer in this paper is assumed to be a hedger and not a speculator. As suggested in Section 2, the trading constraints are therefore set such that the hydropower producer will at any time not sell more than expected production and not hold a total long position. Additionally, we extend the theory from Section 2 to include previously bought forward contracts, i.e. at time zero the hydropower producer has potentially an initial portfolio of forward contracts for the next five months.

The risk profile of the hydropower producer is assumed to be described by a Value at Risk (VaR) constraint at the end term revenue. As the VaR is not a complete measure for the actual risk profile[6], additional values can be taken into consideration and will be evaluated when testing the model in Section 5.

[6] Fleten, Bråthen and Nissen-Meyer (2009)

4.3 Optimization model

When the hydropower producer has given its risk profile and the modeling of the scenario tree is done, the optimization model will determine the optimal weights of the available forward contracts. The optimal weights will be determined according to the possible future scenarios the hydropower producer faces, and the weights will be readjusted at each time step. By applying the theory from Section 2.2 on the scenario tree constructed in Section 4.1 and the constraints from Section 4.2, we obtain the following maximization problem.

Definitions:

$F_{t,T}(w)$ = Forward price at time *t* for a forward with delivery in period *T* to *T+1*

$F_{t,T}^{P}$ = Forward price of a forward with delivery in period *T* to *T+1* that was shorted/bought *t* months before time zero, i.e. it is already in the portfolio.

$x_{t,T}(w)$ = Amount shorted/bought at node w, at time t, of a contract with delivery in period *T* to *T+1*

$x_{t,T}^{-}(w)$ = Amount shorted at node w, at time t, of a contract with delivery in period *T* to *T+1*

$x_{t,T}^{+}(w)$ = Amount bought at node w, at time t, of a contract with delivery in period *T* to *T+1*

$X_{i,T}$ = Amount shorted *i* months before time zero of a contract with delivery in period *T* to *T+1*

$P_t(w) =$ Production in period *t-1* to *t* if in node *w* at time *t*

$S_t(w) =$ Price in period *t-1* to *t* if in node *w* at time *t*

$p_t(w) =$ Probability of being in node *w* at time *t*

$R_{0,t}(w) =$ Cumulative revenue from time 0 to time *t* if at node *w*

N = Number of time periods

V_t = The set of possible nodes in the scenario tree at time *t*

VaR_α = α percentage VaR limit for the cumulative revenue at time N $R_{0,N}(w)$, $w \epsilon V_N$

$z(w)$ = Binary variable for the VaR constraint, $w \epsilon V_N$

α = VaR quantile

r_t = Risk free interest rate in time period *t-1* to *t*

Problem formulation:

$$Maximize \sum_{w \in V_N} p_N(w) R_{0,N}(w)$$

Subject to

$$R_{0,1}(w) = P_1(w)S_1(w) + \sum_{i=1}^{N-1}\left[F_{1,i}^P - S_1(w)\right]X_{i,1} \qquad \forall w \in V_1 \qquad (1)$$

$$\begin{aligned} R_{0,t}(w) = {} & (1+r)R_{0,t-1}(B_{t-1}(w)) + P_t(w)S_t(w) \\ & + \sum_{i=0}^{t-2}\left[F_{i,t-1}(B_i(w)) - S_t(w)\right]x_{i,t-1}(B_i(w)) \\ & + \sum_{j=1}^{N-t}\left[F_{j,t}^P - S_t(w)\right]X_{j,t} \end{aligned} \qquad \forall w \in V_t, t = 2,..,N \qquad (2)$$

$$x_{0,T}^+(w) \le \sum_{i=0}^{N-t} X_{i,T} \qquad \forall w \in V_0, T = 0,..,N-2 \qquad (3)$$

$$x_{0,N-1}^+(w) = 0 \qquad \forall w \in V_0 \qquad (4)$$

$$x_{t,T}^+(w) \le \sum_{i=0}^{t-1} x_{i,T}(B_i(w)) + \sum_{j=0}^{N-T} X_{i,T} \qquad \forall t = 1,..N-2, \; T = t+1,..,N-1, w \in V_t \qquad (5)$$

$$x_{0,T}(w) + \sum_{i=0}^{N-t} X_{i,T} \le E_0[P_{T+1}] \qquad \forall T = 1,..,N-2 \qquad (6)$$

$$x_{0,N-1}^-(w) \le E_0[P_N] \qquad \forall w \in V_0 \qquad (7)$$

$$x_{t,T}^-(w) + \sum_{i=0}^{t-1} x_{i,T}(B_i(w)) + \sum_{j=0}^{N-T} X_{i,T} \le E_t[P_{T+1}] \qquad \forall t = 1,..N-2, \; T = t+1,..,N-1, w \in V_t \qquad (8)$$

$$R_{0,N}(w) \geq VaR_{\alpha} z(w) \qquad \forall w \in V_N \tag{9}$$

$$\sum_{w \in V_N} (1 - z(w)) p_N(w) \leq \alpha \tag{10}$$

Explanation of the problem:

Maximize initial expected cumulative revenue at time *N*

Subject to:

(1): Calculates the revenue at time 1 for all nodes

(2): Calculates the cumulative revenue for all nodes at time *t*

(3), (4) and (5): No total long positions are allowed

(6), (7) and (8): No total short position exceeding the expected production in any time period

(9) and (10): Restricts the $VaR\ \alpha$ of the revenue at time *N*, $R_{0,N}(w)$, $w \epsilon V_N$

Solving this model[7] will return the weights $x_{i,t}(.)$, which indicate the positions to be held in every node in the scenario tree. The model will therefore give a strategy on how to readjust the portfolio. The optimal value of the objective function equals the initial expected cumulative revenue at the end of the time horizon of the scenario tree.

4.4 Model evaluation

The model is implemented with only six time steps. This may lead to a less accurate model because of few time steps, but should be sufficient for an evaluation of the model. In a practical setting the scenario tree should be extended. However, this comes at the cost of a more computational expensive problem. A unit increase in the number of time steps results in a growth in the scenario tree with a multiple of four, and the optimization problem may easily

[7] See Appendix A.3 for an implementation in AMPL.

become too computational complex. Scenario reductions techniques like Heitsch and Römisch (2003) may be applied to reduce the problem size.

In the implementation we use only monthly forward contracts. However, as Nord Pool also has high liquidity in quarterly and yearly contracts, these contracts may also be of interest in a practical setting. The model may then be applied as an American option pricing tree, with n steps between each trading date. If for instance quarterly contracts are used, a solution will be to simulate prices and productions with n steps between each trading date, and allow the hydropower producer to make a decision at the exercise nodes.

For measuring the risk aversion of the hydropower producer, restrictions on the VaR 10% have been implemented. It should be noted that this may not suit the hydropower producer's preferences, but the model can easily be extended to include other risk measures such as volatility, CVaR or CFaR. It is also possible to change the model to minimizing VaR or CVaR. This shows that the model is flexible with respect to different attitudes towards risk. It also allows the hydropower producer to evaluate different risk alternatives, with respect to which risk/mean revenue combination it wants. In our implementation we have, as mentioned, chosen a VaR constraint. However, if implementing the model in a practical setting, a constraint on CVaR should be used, see for instance Krokhmal, Palmquist and Uryasev (2002). This is because the problem then will be linear and therefore less computational expensive.

Since the optimization model is constrained by not being allowed to short more forwards than expected production, it will at certain time points have to buy back contracts if the expected production falls. This may often be a costly restriction and not optimal for the hydropower producer. The hydropower producer should therefore consider whether speculation should be allowed or not.

As a hydropower producer mainly faces fixed and only minor variable transactions costs, this has been neglected in the implementation. However, transactions costs can easily be implemented, for instance by the methods proposed by Boyle and Lin (1997) and Fleten, Ziemba and Wallace (2002).

5 EMPIRICAL TESTS

In this section we will present and discuss results from an out-of-sample test with the dynamic hedging strategy derived from the model implemented in Section 4. To evaluate the performance of this strategy, we also use the model to derive a static hedging strategy. Section 5.1 gives a brief description of the input data and how they are estimated. Section 5.2 describes the hedging strategies and the results from the out-of-sample test. Finally, in Section 5.3 the results are discussed.

5.1 Data description and parameter estimation

The model is tested on data from the period January 2007 to April 2009 and is re-optimized monthly with data that only would be known at that time. Input data consist of monthly forward prices, and forecasted production and price data from a hydropower producer in Norway. The forecasted data consist of historical forecasts conducted each month back to 2007. As an example, 1 June 2007 the model is run on forward prices, and production and price scenarios made at this date. Each re-optimization gives a set of weights that indicates the types and amounts of contracts that should be shorted or bought. The resulting strategy is then applied on the actual production and price data in the period.

The forecasted price and production data consist of 70 equiprobable scenarios made from generation planning tools used by the hydropower producer and from the bottom-up electricity sector model Multi-area Power Scheduling (MPS). This is an equilibrium model frequently used for price forecasting in Scandinavia. The model was developed by SINTEF Energy Research and is described in Botnen et al. (1992) and Egeland et al. (1982).

Actual production data are collected from the hydropower producer, and actual data for the price are collected from Nord Pool. Since the forward prices at Nord Pool are nominated in EUR, we convert them into NOK by using the spot NOK/EUR exchange for the same day as the forward price was collected. I.e. we have ignored the interest parity of the exchange rate, as this is of minor importance. We have used the system price (NOK) to calculate both the payoff of the forward contracts and revenue from spot sale of production. In reality a producer does not get the system price when selling production, but a local area price which might be different from the system price. However, if we use the local area price for spot revenue calculation this difference would influence and reduce the generality of the test results. The risk free interest rate has also been ignored since the model has a time length of only six months, and the potential interest rate effect would therefore be of less relevance.

The input parameters used in the dynamic model are unconditional standard deviations of the price and production, mean reversion of price and production, correlation between price and production, risk premiums on the forward contracts, and expected production in each period. All input parameters are calculated on logarithmic levels[8] except for the risk premiums and expected production which is calculated on the untransformed values. Mean reversion parameters are estimated by first applying a state space model and Kalman filter[9] to deseasonalize the data, and then fit an AR(1) process to estimate the mean reversion. The correlation between price and production is also estimated from the deseasonalized data. The risk premiums are assumed to differ across contracts and time to delivery for each contract. It is estimated by the historical average risk premium from 2004 to the date of re-optimization; see for instance Pietz (2009). Expected production in each period is calculated straightforward as the average of the production scenarios.

5.2 Test results

The model has been run in three different ways to derive three different strategies; a natural hedging strategy, a static hedging strategy and a dynamic hedging strategy. The natural hedging strategy means simply no hedging and is derived by running the model with no trades allowed. The static hedging strategy is derived by only allowing trades at time 0, and the dynamic strategy is derived by allowing trades at all time points. This allows for a direct comparison between the three different types of strategies, and an evaluation whether the derived dynamic hedging strategy will give an actual better performance. The VaR 10% limit is set to 10% over the VaR 10% limit for the natural hedging strategy for the corresponding period. I.e. the hedging strategies aim to improve the VaR 10% by 10% compared to the natural hedging strategy. For some months it is not possible to obtain an increase of 10%. In these cases the VaR 10% limit is set as high as possible. The results are presented in Table 5.1.

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[8]The logarithmic levels are calculated as $\ln(\frac{S_t}{E[S_t]})$ for the price and $\ln(\frac{P_t}{E[P_t]})$ for the production. See Section 2 for details.

[9] See Harvey (1989).

	Expected												Actual		
	Dynamic				**Static**				**Natural**				**Dynamic**	**Static**	**Natural**
	Revenue	VaR 10%	CVaR 10%	St.dev.	Revenue	VaR 10%	CVaR 10%	St.dev.	Revenue	VaR 10%	CVaR 10%	St.dev.	Revenue	Revenue	Revenue
Jan.07 - Jun.07	592	535	518	61	590	529	516	61	590	529	516	61	**377**	**377**	**378**
Feb.07 - Jul.07	426	354	338	65	423	347	326	71	423	347	326	71	**324**	**317**	**318**
Mar.07 - Aug.07	278	228	205	52	271	209	190	62	273	201	184	69	286	277	262
Apr.07 - Sep.07	225	169	158	50	212	151	143	57	208	132	119	74	306	296	272
Mai.07 - Oct.07	255	182	161	54	239	178	141	57	229	139	125	75	363	355	339
Jun.07 - Nov.07	271	199	179	61	255	191	181	59	229	148	127	85	453	446	437
Jul.07 - Dec.07	366	270	270	101	373	292	292	91	296	184	184	97	534	525	517
Aug.07 - Jan.08	469	352	320	93	443	336	307	83	325	220	196	89	668	654	635
Sep.07 - Feb.08	619	501	457	123	585	495	450	105	440	307	271	121	728	698	706
Oct.07 - Mar.08	701	562	538	111	654	515	488	107	517	378	334	122	737	697	724
Nov.07 - Apr.08	779	635	611	118	697	562	512	116	624	459	423	132	728	678	704
Dec.07 - May.08	713	612	579	80	638	556	521	72	631	503	468	104	633	583	596
Jan.08 - Jun.08	767	639	590	114	713	574	536	114	724	582	540	118	**566**	**516**	**528**
Feb.08 - Jul.08	517	362	335	139	474	308	283	139	496	329	304	139	441	405	427
Mar.08 - Aug.08	313	220	192	90	290	192	167	93	301	200	176	95	388	394	406
Apr.08 - Sep.08	341	247	214	92	309	222	186	90	313	207	179	97	336	353	391
May.08 - Oct.08	411	263	210	134	395	249	200	125	391	244	210	127	359	380	422
Jun.08 - Nov.08	613	514	510	150	603	437	383	141	584	401	355	155	**489**	513	545
Jul.08 - Dec.08	705	514	510	227	672	446	426	216	661	370	330	260	639	651	652
Aug.08 - Jan.09	722	584	545	162	721	593	557	147	559	388	328	172	734	778	732
Sep.08 - Feb.09	895	628	584	236	800	574	523	209	743	473	410	244	781	840	752
Oct.08 - Mar.09	930	687	630	183	861	645	596	168	769	548	490	196	820	893	759
Nov.08 - Apr.09	870	735	691	114	893	758	692	113	697	531	481	145	796	861	708
Average	556	434	406	114	527	407	375	109	479	340	308	124	543	543	531

Table 5.1 Expected revenue, VaR 10%, CVaR 10%, standard deviation and actual revenue during every six month period from January 2007 to April 2009 for the dynamic, static and natural hedging strategy. All numbers are in million NOK. The cases where actual revenue is below the VaR 10% is in bold.

5.2.1 Risk performance

From Table 5.1 we can see that the dynamic hedging strategy has an expected VaR 10% above the static hedging strategy in 20 out of 23 cases. The average expected VaR 10% is 434 million NOK compared to 407 million NOK for the static hedging strategy. T-statistics[10] show that the difference is statistically significant with a p-value of 0.005. The natural hedging strategy is below the dynamic and static hedging strategies' VaR 10% in all cases and 18 cases respectively. Its average VaR 10% is 340 million NOK.

If comparing actual revenues to their corresponding VaR 10%, we see that the natural and static hedging strategies are below their VaR 10% in 3 out of 23 cases (i.e. 13%) and the dynamic strategy is below its VaR 10% in four cases (i.e. 17%). This suggests that the expected VaR 10% may be relatively close to the actual VaR 10%. Table 5.1 also shows that in 19 out of 23 cases the dynamic hedging strategy has a higher CVaR 10% compared to the static hedging strategy. The average is 406 million NOK compared to 375 million NOK for the static hedging strategy. The difference is statistically significant with a p value of 0.006. Compared to the natural hedging strategy, the dynamic hedging strategy and the static hedging strategy will have a better CVaR 10% in 23 and 17 cases. Both the VaR 10% and CVaR 10% for the natural hedging strategy are significantly[10] (p-value of 0) lower than the corresponding values for the static and dynamic hedging strategy.

The static hedging strategy has a lower standard deviation than the dynamic hedging strategy in 16 out of 23 cases. Compared to the natural hedging strategy the standard deviation is lower in 20 cases. The dynamic strategy has a lower standard deviation in 18 cases compared to the natural hedge strategy. The standard deviations are 114, 109 and 124 million NOK for the dynamic, static and natural hedging strategy respectively. Even though the static hedging strategy has a lower standard deviation than the less restricted dynamic hedging strategy, it should be noted that the model does not take this criterion into consideration during the optimization. In order to compare the expected standard deviation of the hedging strategies to the deviations in actual revenue, we estimate the average standard deviation on the difference between the expected revenue and the actual revenue during the test period. The numbers are summarized in Table 5.2 below. As we can see, the expected standard deviation is close to the actual standard deviation except for the natural hedging strategy, which means that the model predicts the uncertainty of the revenue with good accuracy during this period.

[10] The values for VaR and CVaR are not perfectly independent as the six months periods are overlapping. The T-statistics will therefore overestimate the difference.

	Dynamic	Static	Natural
Actual st. dev.	117	111	147
Expected st. dev.	114	109	124
% difference	3 %	2 %	19 %

Table 5.2 The revenue's actual standard deviation compared to expected standard deviation for the dynamic, static and natural hedging strategy. All numbers are in million NOK.

5.2.2 Weights

Figure 5.1 shows the average short positions (weights) at time zero for the different contracts. All weights can be found in Table A.2.1 in the Appendix. Intuitively, one could expect that the static hedging strategy in average has higher short positions in contracts with long time to delivery. This follows from the fact that it is only at time zero it is possible for this strategy to short these contracts. For the dynamic strategy it is different. The dynamic hedging strategy will always have the opportunity to wait one or more periods before shorting the contracts, and will therefore on average undertake fewer positions at time zero for these contracts. From Figure 5.1 we see that this is also the case. For contracts with four months to delivery, the static hedging strategy clearly has a higher average short position than the dynamic hedging strategy, and a marginally higher short position for contracts with five months to delivery.
For contracts with two or three months to delivery it is the other way around. Since the static hedging strategy in average shorts more contracts with four and five months to delivery, it will in most cases have more limited opportunities, than the dynamic strategy, to short contracts with two or three months to delivery. This is because most of the expected production two and three months from now was shorted two and three months ago on contracts with four or five months to delivery. From the figure we see that this is also the case. The dynamic hedging strategy clearly has a higher average short position than the static hedging strategy in contracts with two months to delivery, and a marginally higher position in contracts with three months to delivery.

For contracts with one month to delivery, one could expect that both strategies have approximately the same average short position. This is because it is only at time zero it is possible to short these contracts, and the strategies therefore face the same problem. It is also important to be aware of the fact that the short positions depend on the risk premiums[11]. Since

[11] See table A.2.2 in the Appendix for an overview of average historical risk premiums.

the risk premiums differ across time to delivery and delivery months, there will in many cases be deviations from what one could expect from the intuition above.

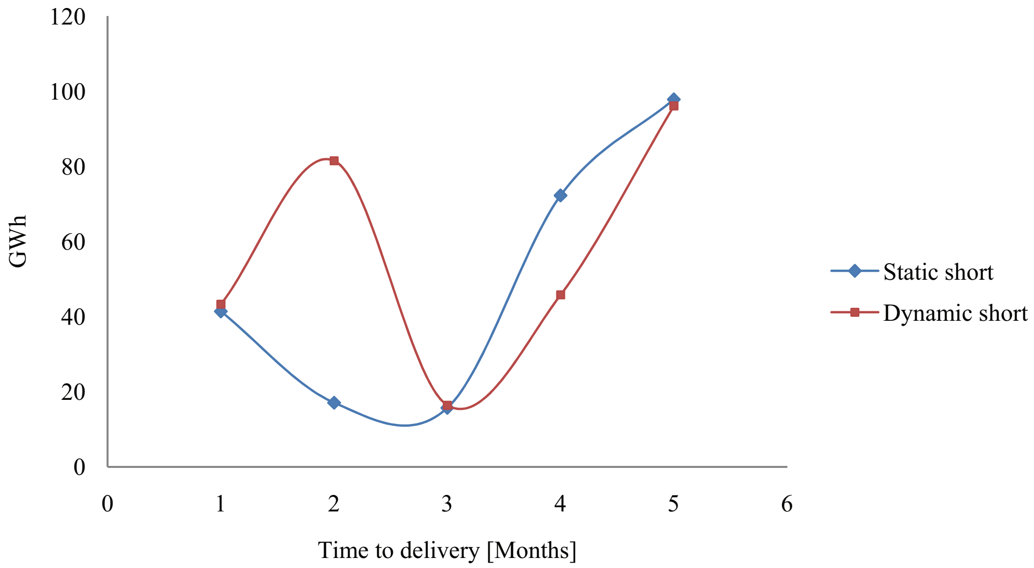

Figure 5.1 Average short positions in GWh undertaken at time zero for the dynamic and static hedging strategy.

5.2.3 Revenue and hedge profit

From Table 5.1 we see that the dynamic strategy will have higher expected revenue in all cases compared to the natural hedging strategy, and in 21 out of 23 cases compared to the static hedging strategy. The corresponding values for actual revenues are 17 and 13. The static hedging strategy will have higher expected revenue in 16 out of 23 cases compared to the natural hedging strategy and actual higher revenue in 10 cases. The average expected revenue is 556, 527 and 479 million NOK for the dynamic, static and natural hedging strategy respectively, while the actual revenue is 543, 543 and 531 million NOK. The percentage difference between expected and actual revenue is -2.2%, 3.08% and 10.77%. T-statistics[12] show that the dynamic hedging strategy has significantly higher expected revenue than both the static hedging strategy and the natural hedging strategy with p values of 0.005 and 0, respectively. The difference between the expected revenue for the dynamic hedging strategy and its actual return is shown in Figure 5.1 below. The figure confirms the good

[12] The values for expected revenue are not perfectly independent as the six months periods are overlapping. The T-statistics will therefore overestimate the difference.

approximation of the actual revenue, as the actual value fluctuates around the expected revenue with a standard deviation of 117 million NOK.

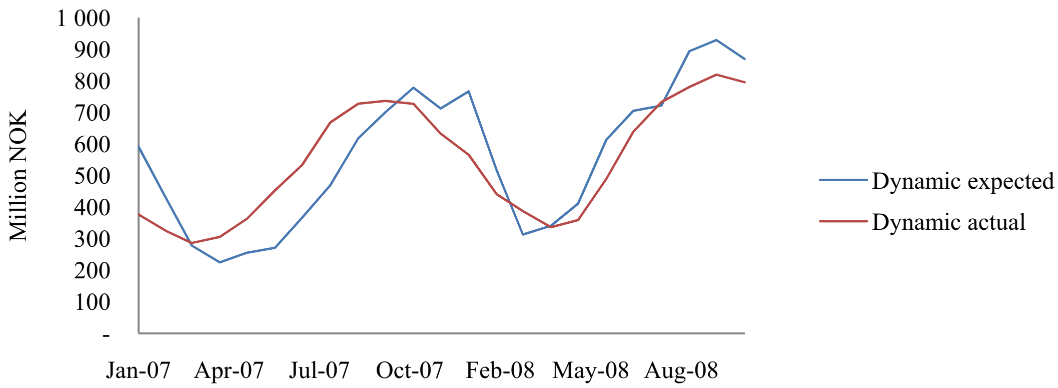

Figure 5.2 The six month expected revenue vs. six month actual revenue for the dynamic hedging strategy.

Figure 5.3 shows the monthly hedge profit for the dynamic and static hedging strategy. The average monthly hedging profits are 3.15 and 4.35 million NOK with a standard deviation of 15.93 and 18.01 million NOK for the dynamic and the static hedging strategy, respectively. This shows high variability for both hedging strategies. T-tests show that the difference in mean is not statistically significant.

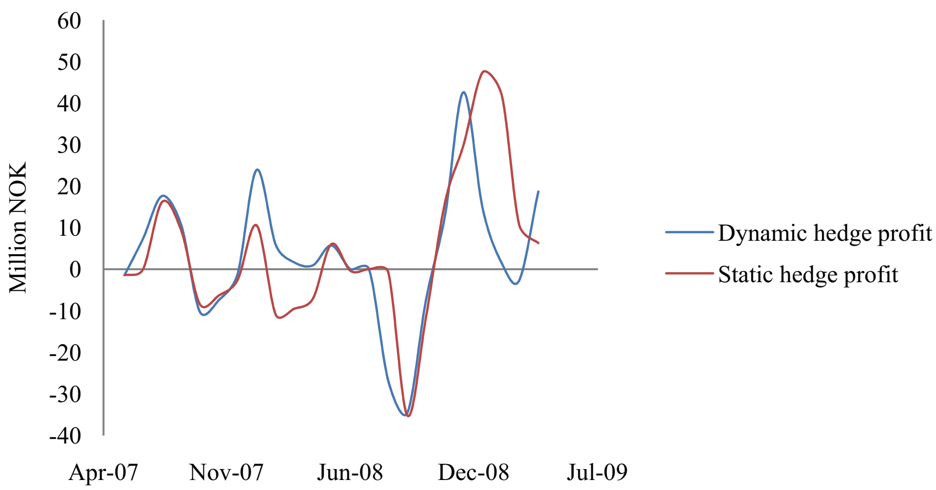

Figure 5.3 The actual monthly hedging profit/loss of the dynamic and static strategy.

5.3 Discussion

As the results shows, both the dynamic and the static hedging strategy improve the VaR 10%, CVaR 10%, standard deviation and the mean revenue compared to the natural hedging strategy. This suggests that hedging strategies with forward contracts are able to reduce the risk and simultaneously improve the revenue, both in expectation and in out-of-sample tests. Additionally, the benefits of the dynamic and static hedging strategy compared to the natural hedging strategy may also possibly increase if more derivatives are included in the model.

We also see from the results that the model is capable of approximating the revenue with good accuracy. The expected revenues are in most cases similar to the average of the actual revenues. The same holds for the expected standard deviations and expected VaR 10%. This suggests that the methodology proposed by Peterson and Stapelton (2002) is able to model the price and production processes to a satisfying degree for this application, and that the assumption of Ornstein-Uhlenbeck processes for production and price returns gives reasonable results. It should also be noted, as mentioned in Section 4, that the model accuracy will increase even more, if the binomial tree consists of more time steps.

Comparing the dynamic hedging strategy with the static hedging strategy shows that the risk profile of the dynamic hedging strategy is significantly better in terms of expected VaR 10% and CVaR 10%, while the standard deviation is lower for the static hedging strategy. However, the model has not taken this standard deviation into account when deriving the strategies and may be improved for the dynamic strategy if restricted. For the revenue, the dynamic hedging strategy has a higher expected mean while the actual mean is equal for the test period. Figure 5.4 shows the difference in actual hedging profit per month and the cumulative average difference. The period January 2007 to April 2007 is not included as there were no short positions in forwards with delivery during these four months. The dynamic hedging strategy seems to outperform the static hedging strategy from April 2007 to December 2008. However, in the following three months, the static hedging strategy has an extraordinary period. Both strategies have average positive hedging profits during these three months. However, the profits for the static strategy are 47.5, 41.8 and 10.7 million NOK compared to 14.6, 1.4 and -2.7 million NOK for the dynamic strategy. The monthly hedging profits are shown in Table A.2.3 in the Appendix.

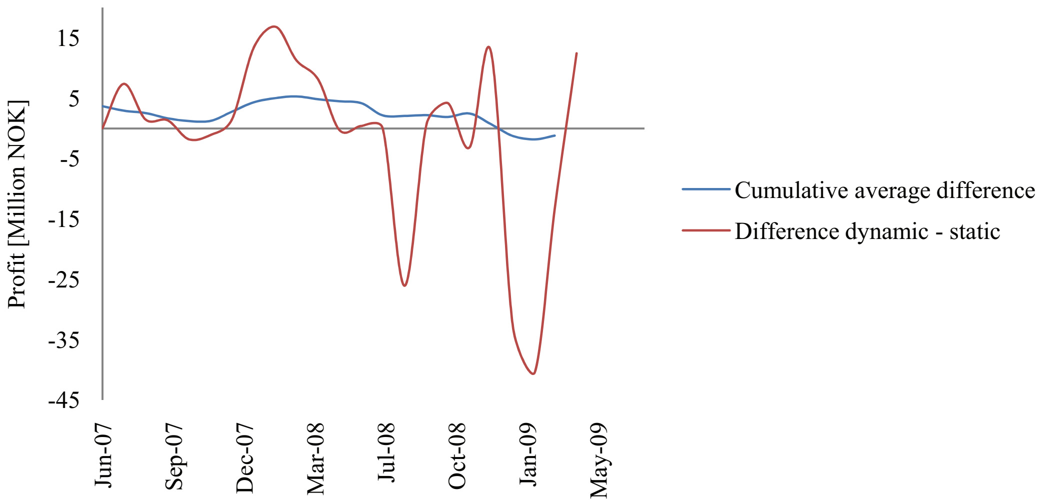

Figure 5.4 The difference and cumulative average difference in monthly hedging profit between the dynamic and static hedging strategy.

An investigation of the forward prices during the fall 2008 for contracts with delivery in January, February or March 2009, and the system price for these three months, shows the influence of the financial crisis on the electricity market. Figure 5.5 shows how the forward curves are shifted downward[13]. From the figure it is evident that the financial crisis dramatically changed the market conditions during the fall. In September 2008, the forward price for January was 613 NOK, in December the price had declined to 419 NOK, while the average system price in January ended up at 385 NOK. For February and March the numbers are similar. Under these abnormal circumstances shorting forwards with long time to delivery will give a high hedging profit.

[13] The forward prices are converted from EUR to NOK using the spot exchange rate. Note that because of the great increase in the NOK/EUR exchange rate during the fall 2008, the downward shift in terms of EUR is even higher.

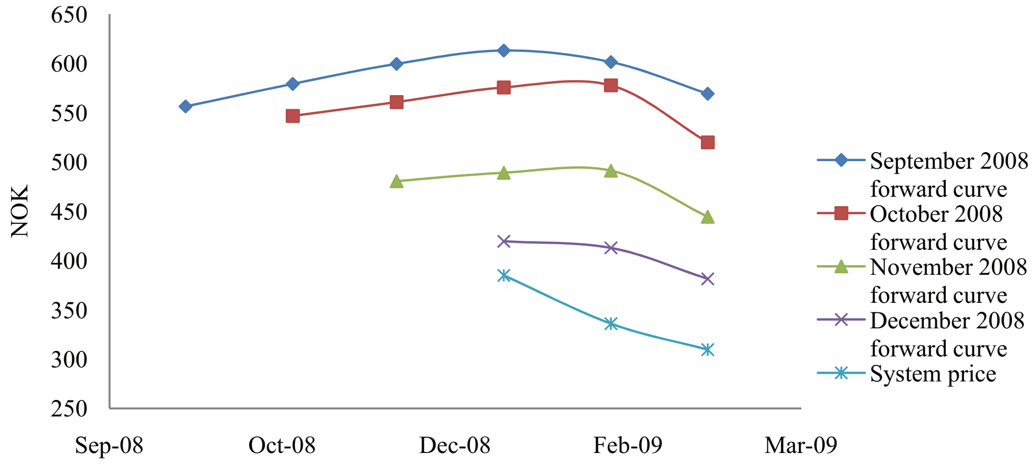

Figure 5.5 Forward curves during the fall 2008 and system price for January, February and March.

Figure 5.1 shows that the static hedging strategy in average shorts more contracts with long time to delivery than the dynamic hedging strategy. Table A.2.1 in the Appendix confirms that this is also the case for these three specific months. In other words, the static hedging strategy will have a superior hedging profit compared to the dynamic hedging strategy in these three months. In addition to a global economical crisis, large shifts in the forward curves can also be caused by an unexpected drop or jump in the inflow. An example is the winter 2003, where the forward price for a contract[14] with delivery in March, declined from 750 NOK in the beginning of January to 377 NOK in the end of February. However, including such events in a test period of only two years, would give a false impression of the average marked behavior. Consequently, if this period is excluded from our analysis, the results that will appear are shown in Table 5.3 below.

[14] Block contract with delivery in week 9 to 12

	Mean Revenue		Hedging Profit	
	Actual	**Expected**	**Mean**	**St.dev.**
Dynamic	492	493	2.13	16.92
Static	480	465	-0.33	13.77
Difference	13	28	2.46	3.15

Table 5.3 Expected six month revenue, actual six month revenue and actual monthly hedging profit for the dynamic and static hedging strategy with January 2009 – April 2009 removed. All numbers are in million NOK.

As we can see, the expected mean revenue difference remains approximately the same, which is to be expected as the dynamic hedging strategy will always perform at least as good, or better, than the static hedging strategy in expectation. Furthermore, the abnormal market conditions, that are excluded, should not significantly influence the difference in expected value, since the model is not able to capture abnormal market conditions before they occur. This is so because the future expected spot price is calculated directly from the forward prices in the market, i.e. that it assumes the market is right. Table 5.3 also shows that the dynamic hedging strategy now has an average hedging profit of 2.13 million NOK compared to -0.33 million NOK for the static hedging strategy, i.e. in the test period adjusted for the abnormal market circumstances, the dynamic hedging strategy has a higher hedging profit than the static hedging strategy in addition to a higher VaR 10% and CVaR 10%. Furthermore, Figure 5.4 shows that the cumulative average difference in hedging profit is above zero during the whole test period. It should be noted that this difference is not statistically significant; however overall our results indicate that the difference is positive.

6 CONCLUSION

The results show that dynamic and static hedging strategies give significant benefits compared to the natural hedging strategy, both with respect to the risk profile and expected mean revenue. This is in accordance with Fleten, Wallace and Ziemba (2002) and Kettunen, Salo and Bunn (2007). For the actual revenue, the dynamic hedging strategy has given the best results under normal market circumstances. However, the difference cannot be proven to be statistically significant in our test period.

The methodology used in this paper for predicting future revenue gives a good approximation of the actual data. Furthermore the model accuracy can be further improved by increasing the number of time steps, and the revenue possibly increased by including more derivatives.

Since most hydropower producers in the Nordic market today use static hedging strategies, our results indicate that there are potential benefits by considering dynamic hedging strategies. In addition to the risk reduction, the mean operating revenue and profit are also increased. Assuming an operating margin of 50%, which is typical for hydropower producers in the Nordic market, a 2.65 percentage[15] increase in revenue translates to an increase in operating profit of 5.30%. As the model also is flexible in terms of its risk restrictions, it can easily be adjusted for the specific hydropower risk aversion.

As a concluding remark, this paper contributes with an extensive out-of-sample test and provides new insight on the ex post performance of dynamic hedging strategies compared to static and natural hedging strategies. However, further research should be conducted in order to examine the effects of adding more derivatives, investigating the robustness and stability of the ex post performance and evaluate whether the ex post revenue difference between dynamic and static hedging strategy is statistically significant.

[15] $\text{Annual percentage increase} = \left(\frac{\text{Actual dynamic hedging strategy's mean revenue}}{\text{Actual static hedging strategy's mean revenue}} - 1 \right) / 50\%$

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APPENDIX

A.1 Formulas

$$VaR(V,\alpha) = F(V;\alpha) \qquad \text{(A.1.1)}$$

$$CVaR(V,\alpha) = E[V \mid V \le F(V;\alpha)] \qquad \text{(A.1.2)}$$

where $F(V;\alpha)$ is the cumulative distribution of the random variable V.

$$CFaR(V,\alpha) = K - VaR(V,\alpha) \qquad \text{(A.1.3)}$$

where K is an appropriate chosen constant.

Risk premium: $$(1+\pi)^{T-t} = \frac{E_t(S_T)}{F_{t,T}}(1+r)^{T-t} \qquad \text{(A.1.4)}$$

where π is the risk premium, *r* is the risk free rate, $E_t(S_T)$ is the expected spot price at time *T*, and $F_{t,T}$ is the forward price at time *t* for a contract with delivery at time *T*.

A.2 Tables

	Dynamic hedging strategy							Static hedging strategy						
	Total short	Total long	T1	T2	T3	T4	T5	Total short	Total long	T1	T2	T3	T4	T5
Jan.07 - Jun.07	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Feb.07 - Jul.07	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mar.07 - Aug.07	190	0	0	0	0	0	190	103	0	0	0	0	0	103
Apr.07 - Sep.07	3	0	0	0	0	3	0	245	0	0	0	0	105	140
Mai.07 - Oct.07	405	-14	210	0	-14	0	196	296	-6	210	0	-6	60	26
Jun.07 - Nov.07	249	-7	221	-7	0	28	0	186	-4	0	-4	18	165	3
Jul.07 - Dec.07	720	-86	33	208	-86	177	302	614	0	7	41	6	211	349
Aug.07 - Jan.08	116	-21	20	56	39	-21	0	201	-85	-30	22	18	-55	160
Sep.07 - Feb.08	62	-46	44	18	-46	0	0	463	-12	18	15	-12	252	177
Oct.07 - Mar.08	0	-33	-15	-18	0	0	0	512	-36	-28	-8	4	234	274
Nov.08 - Apr.08	729	0	39	353	0	337	0	365	-18	-18	5	6	83	271
Dec.07 - May.08	841	-53	81	390	-53	249	122	13	-343	13	-13	-59	-271	0
Jan.08 - Jun.08	0	-1044	-390	-284	-249	-122	0	0	-703	-405	-298	0	0	0
Feb.08 - Jul.08	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Mar.08 - Aug.08	0	0	0	0	0	0	0	73	0	73	0	0	0	0
Apr.08 - Sep.08	556	0	102	0	0	186	269	337	0	102	48	0	21	166
May.08 - Oct.08	215	-175	0	0	-6	-169	215	237	-175	-48	0	-21	-106	237
Jun.08 - Nov.08	605	0	0	87	186	44	288	304	-111	0	5	1	-111	298
Jul.08 - Dec.08	13	-160	-77	-71	-12	13	0	110	-65	-5	-60	36	42	32
Aug.08 - Jan.09	394	-128	-56	-60	-12	0	394	752	-27	159	53	-27	326	214
Sept.08 - Feb.09	97	-61	97	-22	0	-39	0	75	-94	70	-10	6	-85	0
Oct.08 - Mar.09	266	-381	-27	266	-354	0	0	342	-141	-63	-78	167	175	0
Nov.08 - Apr.09	632	0	83	298	0	0	252	632	0	63	15	145	217	193
Dec.08 - Mar.09	761	-44	76	319	217	-44	150	142	0	63	36	18	24	0
Jan.09 - Apr.09	0	-707	-319	-217	-21	-150	0	0	-809	-356	-235	-217	0	0
Feb.09 - Mai.09	0	-187	0	-187	0	0	0	0	0	0	0	0	0	0
Mar.09 - Jun.09	0	0	0	0	0	0	0	175	0	175	0	0	0	0
Apr.09 - Jul.09	788	0	165	205	0	199	219	421	0	165	220	0	36	0
Total short	7645	-	1169	2201	443	1237	2596	6597	-	1118	461	424	1952	2643
Total long	-	-3146	-884	-865	-852	-545	0	-	-2628	-953	-706	-341	-629	0

Table A.2.1 Derived weights. All numbers are in GWh. T1 denotes contracts with 1 month to delivery, T2 denotes contracts with 2 months to delivery, etc.

	Time to Delivery (Months)					
Contract	**1**	**2**	**3**	**4**	**5**	**6**
Jan	-19.6 %	-28.3 %	-27.4 %	-28.5 %	-29.2 %	-25.5 %
Feb	-5.9 %	-20.9 %	-28.8 %	-28.1 %	-29.2 %	-30.0 %
Mar	4.5 %	1.3 %	-12.8 %	-19.1 %	-18.3 %	-18.5 %
Apr	9.5 %	11.5 %	7.8 %	-6.4 %	-12.5 %	-9.5 %
May	-8.4 %	0.0 %	11.6 %	11.5 %	-6.4 %	-12.4 %
Jun	1.3 %	-2.7 %	6.3 %	19.3 %	20.3 %	4.8 %
Jul	-4.3 %	-0.8 %	-3.7 %	7.3 %	22.6 %	24.0 %
Aug	-3.9 %	-0.5 %	5.4 %	2.2 %	12.0 %	25.6 %
Sep	-7.4 %	-6.2 %	-2.7 %	3.4 %	0.0 %	7.3 %
Oct	-4.1 %	-5.1 %	-3.6 %	0.3 %	5.6 %	2.9 %
Nov	-8.0 %	-10.9 %	-13.0 %	-10.4 %	-7.5 %	0.8 %
Dec	-18.1 %	-20.4 %	-21.6 %	-22.7 %	-21.4 %	-19.0 %

Table A.2.2 Average percentage risk premiums in the period 2004-2008

	Dynamic	**Static**	**Difference**
Jun-07	-1.41	-1.41	0.00
Jul-07	7.36	0.00	7.36
Aug-07	17.66	16.21	1.46
Sep-07	10.60	9.26	1.34
Oct-07	-10.23	-8.42	-1.81
Nov-07	-7.38	-6.29	-1.09
Dec-07	-1.20	-2.64	1.45
Jan-08	23.94	10.55	13.39
Feb-08	5.84	-10.97	16.81
Mar-08	1.72	-9.56	11.29
Apr-08	0.96	-7.10	8.06
May-08	5.74	6.04	-0.30
Jun-08	0.00	-0.39	0.39
Jul-08	0.00	0.00	0.00
Aug-08	-26.79	-0.71	-26.08
Sep-08	-34.55	-35.24	0.69
Oct-08	-7.52	-11.70	4.18
Nov-08	13.00	16.13	-3.13
Dec-08	42.64	29.96	12.67
Jan-09	14.59	47.45	-32.87
Feb-09	1.42	41.81	-40.39
Mar-09	-2.68	10.75	-13.43
Apr-09	18.73	6.32	12.42
Average	3.15	4.35	-1.20
St.dev.	15.93	18.01	14.46

Table A.2.3 Actual monthly hedging profits. All numbers are in million NOK.

A.3 Implementation of model in AMPL

```
#1 INPUT DATA

#1.1 Parameters from data file:
param N;
param rho;
param rho_1;
param meanRevertPrice;
param meanRevertProd;
param Price0;
param Prod0;
param stdevPrice{0..N};
param stdevProd{0..N};
param stdevPriceC{0..N};
param stdevProdC{0..N};
param riskPrem{1..N,1..N};
param F{0..N};
param EoProd{0..N};
param EProd{t in 1..N-2, 1..2^(2*t), t2 in t+1..N-1};
param forward{t in 1..N-2, 1..2^(2*t), t2 in t+1..N-1};
param r;
param riskPremFirstMonth;
param VaR;
param VaRImp := 0.1;
param prevForward{t in 1..5, 1..(6-t)};
param shorted{t in 1..5, 1..(6-t)};

data SEP08_6.dat;

#1.2 Parameters to be calculated:
param V{t in 1..2*N, 1..2^(2*N)};
param Price{t in 1..N, 1..2^(2*N)};
param Prod{t in 1..N, 1..2^(2*N)};
param Prob{t in 1..N, 1..2^(2*N)};
param EoPrice{0..N};
param uPrice{1..N};
param dPrice{1..N};
param uProd{1..N};
param dProd{1..N};
param NuPrice{t in 1..N, 1..2^(2*N)};
param NuProd{t in 1..N, 1..2^(2*N)};

#2 CALCULATION OF PARMETERS AND CONSTRUCTION OF SCENARO TREE

#2.1 Calculation of expected price in each period:
let EoPrice[0] := Price0;
let EoPrice[1] := F[0]*(1+riskPremFirstMonth);
for{t in 2..N} {
	let EoPrice[t] := F[t-1]*(1+riskPrem[t,t]);
}

#2.2 Calculation of conditional standard deviations:
param betaPrice;
let betaPrice := 1 - meanRevertPrice;
param betaProd;
let betaProd := 1 - meanRevertProd;

let stdevProdC[0] := stdevProd[0];
```

```
let stdevProdC[1] := stdevProd[1];

for{t in 2..N} {
	let stdevProdC[t] := (stdevProd[t]^2 - betaProd^2*stdevProd[t-1]^2)^0.5;
}

let stdevPriceC[0] := stdevPrice[0];
let stdevPriceC[1] := stdevPrice[1];

param A;
param B;

for{t in 2..N} {
	let A := (stdevPrice[t]^2 - betaPrice^2*stdevPrice[t-1]^2);
	let B :=(1+((rho^2)/stdevProdC[t]^2)*((-1+meanRevertProd)^2*stdevProd[t-
1]^2+stdevProd[t]^2+2*betaPrice^2*((-1+meanRevertProd)^2*rho*stdevProd[t-1]*stdevPrice[t-
1]+rho_1*stdevPrice[t-1]*stdevProd[t])));
	let stdevPriceC[t] := (A/B)^0.5
}

#2.3 Calculation of u and d for both price and production:
for{t in 1..N} {

	let dPrice[t] := 2/(1+exp(2*stdevPriceC[t]));
	let uPrice[t] := 2-dPrice[t];
	let dProd[t] := 2/(1+exp(2*stdevProdC[t]));
	let uProd[t] := 2-dProd[t];
}

#2.4 Rescale the unconditional standard deviations to "per month":
for{t in 2..N} {
	let stdevPrice[t] := stdevPrice[t]/(t^0.5);
	let stdevProd[t] := stdevProd[t]/(t^0.5);
}

#2.5 Create scenario tree:
param b;
let b := 1;
param even;
let even := 0;

for{j in 1..2*N} {
	if even=1 then
		let even := 0;
	else
		let even := 1;

	for{i in 1..2^j} {
		if b=1 then
			let b := 0;
		else
			let b := 1;
		for{e in 1..2^(2*N-j)} {
			if even=1 then
				let V[(j+1)/2,e + (i-1)*2^(2*N-j)] := b;
			else
				let V[j/2+N,e + (i-1)*2^(2*N-j)] := b;
		}
	}
}
```

```
#2.6 Calculation of spot price and production for each scenario:
for{t in 1..N} {
	for{i in 1..2^(2*N)} {
		let NuPrice[t,i] := 0;
		let NuProd[t,i] := 0;
		for{z in 1..t} {
			let NuPrice[t,i] := NuPrice[t,i] + V[z,i];
			let NuProd[t,i] := NuProd[t,i] + V[z+N,i];
			}
		let Price[t,i] := (uPrice[t]^NuPrice[t,i])*(dPrice[t]^(t-NuPrice[t,i]))*EoPrice[t];
		let Prod[t,i] := (uProd[t]^NuProd[t,i])*(dProd[t]^(t-NuProd[t,i]))*EoProd[t];
	}
}


#2.7 Calculation of conditional probabilities:
param probPrice{t in 1..N, 1..2^(2*N)};
param alphaPrice{t in 1..N, 1..2^(2*N)};
param gammaPrice{t in 1..N, 1..2^(2*N)};
param deltaPrice{t in 1..N, 1..2^(2*N)};

param probProd{t in 1..N, 1..2^(2*N)};
param alphaProd{t in 1..N};

for{t in 1..N} {
	let alphaProd[t] := 0.5*(betaProd*(t-1)*stdevProd[t-1]^2-t*stdevProd[t]^2);
	for{i in 1..2^(2*N)} {
		let gammaPrice[t,i] := rho*(stdevPrice[t]/stdevProd[t])*(-1+meanRevertProd);
		let deltaPrice[t,i] := rho*(stdevPrice[t]/stdevProd[t]);
		let alphaPrice[t,i] := 0.5*(betaPrice*(t-1)*stdevPrice[t-1]^2-
t*stdevPrice[t]^2+gammaPrice[t,i]*(t-1)*stdevProd[t-1]^2+deltaPrice[t,i]*t*stdevProd[t]^2);

		if t=1 then {
			if(V[t,i] = 1) then {
				let probPrice[t,i] :=
(1/log(uPrice[t]/dPrice[t]))*(alphaPrice[t,i]+betaPrice*log(Price0/EoPrice[t-
1])+gammaPrice[t,i]*log(Prod0/EoProd[t-1])+deltaPrice[t,i]*log(Prod[t,i]/EoProd[t])-(t-1-(t-
NuPrice[t,i]))*log(uPrice[t])-(1+(t-NuPrice[t,i]))*log(dPrice[t]));
				}
			if(V[t+N,i] = 1) then {
				let probProd[t,i] :=
(1/log(uProd[t]/dProd[t]))*(alphaProd[t]+betaProd*log(Prod0/EoProd[t-1])-(t-1-(t-
NuProd[t,i]))*log(uProd[t])-(1+(t-NuProd[t,i]))*log(dProd[t]));
			}
		}

		else {
			if(V[t,i] = 1) then {
				let probPrice[t,i] :=
(1/log(uPrice[t]/dPrice[t]))*(alphaPrice[t,i]+betaPrice*log(Price[t-1,i]/EoPrice[t-
1])+gammaPrice[t,i]*log(Prod[t-1,i]/EoProd[t-1])+deltaPrice[t,i]*log(Prod[t,i]/EoProd[t])-(t-1-(t-
NuPrice[t,i]))*log(uPrice[t])-(1+(t-NuPrice[t,i]))*log(dPrice[t]));

				if(probPrice[t,i] < 0) then {
					let probPrice[t,i] := 0;
				}
				if(probPrice[t,i] > 1) then {
					let probPrice[t,i] := 1;
				}
```

```
                        }

                        if(V[t+N,i] = 1) then {
                                let probProd[t,i] :=
(1/log(uProd[t]/dProd[t]))*(alphaProd[t]+betaProd*log(Prod[t-1,i]/EoProd[t-1])-(t-1-(t-
NuProd[t,i]))*log(uProd[t])-(1+(t-NuProd[t,i]))*log(dProd[t]));
                                if(probProd[t,i] < 0) then {
                                        let probProd[t,i] := 0;
                                }
                                if(probProd[t,i] > 1) then {
                                        let probProd[t,i] := 1;
                                }
                        }
                }
        }

        for{i in 1..2^(2*N)} {
                if(V[t,i] = 0) then {
                        let probPrice[t,i] := 1-probPrice[t,i+(2^(2*N))/(2*2^(2*(t-1)))];
                        }
                if(V[t+N,i] = 0) then {
                        let probProd[t,i] := 1-probProd[t,i+0.5*(2^(2*N))/(2*2^(2*(t-1)))];
                }
        }
}

#2.8 Calculation of cummulative probabilities:
for{t in 1..N} {
        for{i in 1..2^(2*N)} {
                let Prob[t,i] := 1;
                for{j in 1..t}{
                        let Prob[t,i] := Prob[t,i]*probPrice[j,i]*probProd[j,i];
                }
        }
}

#2.9 Rescale mean in case of devatioans due to the the constraints on probability:
param treeMeanProd{1..N};
param treeMeanPrice{1..N};

for{t in 1..N} {
        let treeMeanProd[t] := 0;
        let treeMeanPrice[t] := 0;

        for{s in 1..2^(2*N)} {
                let treeMeanProd[t] := treeMeanProd[t] + Prob[t,s]*Prod[t,s];
                let treeMeanPrice[t] := treeMeanPrice[t] + Prob[t,s]*Price[t,s];
        }
        let treeMeanProd[t] := treeMeanProd[t]/(2^(2*(N-t)));
        let treeMeanPrice[t] := treeMeanPrice[t]/(2^(2*(N-t)));
}
for{t in 1..N} {
        for{s in 1..2^(2*N)} {
                let Prod[t,s] := Prod[t,s]*EoProd[t]/treeMeanProd[t];
                let Price[t,s] := Price[t,s]*EoPrice[t]/treeMeanPrice[t];
        }
}
for{t in 1..N} {
        let treeMeanProd[t] := 0;
        let treeMeanPrice[t] := 0;
```

```
        for{s in 1..2^(2*N)} {
                let treeMeanProd[t] := treeMeanProd[t] + Prob[t,s]*Prod[t,s];
                let treeMeanPrice[t] := treeMeanPrice[t] + Prob[t,s]*Price[t,s];
        }
        let treeMeanProd[t] := treeMeanProd[t]/(2^(2*(N-t)));
        let treeMeanPrice[t] := treeMeanPrice[t]/(2^(2*(N-t)));
}

#2.10 Calculation of future forward prices:
for{t in 1..N-2} {
        for{s in 1..2^(2*t)} {
                for{t1 in t+1..N-1} {
                        let forward[t,s,t1] := 0;
                        for{fs in 1..2^(2*(t1+1-t))}{
                                if(Prob[t,1+(s-1)*2^(2*(N-t))] != 0) then {
                                        let forward[t,s,t1] := forward[t,s,t1] + Price[t1+1, 1+(s-
1)*2^(2*(N-t))+(fs-1)*2^(2*(N-(t1+1)))]*Prob[t1+1, 1+(s-1)*2^(2*(N-t))+(fs-1)*2^(2*(N-
(t1+1)))]/((1+riskPrem[t1-t,t1])*Prob[t,1+(s-1)*2^(2*(N-t))]);
                                }
                        }
                }
        }
}

#2.11 Calculation of future expected production:
for{t in 1..N-2} {
        for{s in 1..2^(2*t)} {
                for{t1 in t+1..N-1} {
                        let EProd[t,s,t1] := 0;
                        for{fs in 1..2^(2*(t1+1-t))}{
                                if(Prob[t,1+(s-1)*2^(2*(N-t))] != 0) then {
                                        let EProd[t,s,t1] := EProd[t,s,t1] + Prod[t1+1, 1+(s-1)*2^(2*(N-
t))+(fs-1)*2^(2*(N-(t1+1)))]*Prob[t1+1, 1+(s-1)*2^(2*(N-t))+(fs-1)*2^(2*(N-(t1+1)))]/(Prob[t,1+(s-
1)*2^(2*(N-t))]);
                                }
                        }
                }
        }
}


#3 OPTIMIZATION PROBLEM

#3.1 Decision variables:
var R{t in 0..N, 1..2^(2*t)};
var x{t in 0..N-2, 1..2^(2*t), t1 in t+1..N-1};
var xP{t in 0..N-2, 1..2^(2*t), t1 in t+1..N-1} >= 0;
var xM{t in 0..N-2, 1..2^(2*t), t1 in t+1..N-1} >= 0;
var z {1..2^(2*N)} binary;

#3.2 Objective function:
maximize income: sum{s in 1..2^(2*N)}(Prob[N,s]*R[N,s]);

#3.3 Constraints:

#3.3.1 Revenue constraints:
subject to initialRevnueConstraint{s in 1..4}:
R[1,s] = Prod[1,1 + (s-1)*2^(2*(N-1))]*Price[1,1 + (s-1)*2^(2*(N-1))]/1000 + sum{m in
1..5}(prevForward[m,1] - Price[1,1 + (s-1)*2^(2*(N-1))])*shorted[m,1]/1000;
```

```
subject to RevenueConstraint{t in 2..N,s in 1..2^(2*t)}:
R[t,s] = (1+r)*R[t-1,1+floor((s-0.1)/4)] + Prod[t,1 + (s-1)*2^(2*(N-t))]*Price[t,1 + (s-1)*2^(2*(N-t))]/1000 +
(F[t-1] - Price[t,1 + (s-1)*2^(2*(N-t))])*(xM[0,1,t-1] - xP[0,1,t-1])/1000 + (1/1000)*sum{i in 1..(t-
2)}((forward[i,1 + floor((s-0.1)/4^(t-i)),t-1] - Price[t,1 + (s-1)*2^(2*(N-t))])*(xM[i,1+floor((s-0.1)/4^(t-i)),t-1]
- xP[i,1+floor((s-0.1)/4^(t-i)),t-1])) + sum{tt in 1..(6-t)} (prevForward[tt,t] - Price[t,1 + (s-1)*2^(2*(N-
t))])*shorted[tt,t]/1000;

#3.3.2 Trading constraints:
subject to notLongAtTime0a{t1 in 1..N-2}: xP[0,1,t1] <= sum{tb in 1..(N-(t1+1))} shorted[tb,t1+1];

subject to notLongAtTime0b: xP[0,1,5] = 0;

subject to notLong2{t in 1..N-2, s in 1..2^(2*t), t1 in (t+1)..N-1}:
xP[t,s,t1] <= sum{tp in 0..t-1} (xM[tp,1+floor((s-0.1)/4^(t-tp)),t1] - xP[tp,1+floor((s-0.1)/4^(t-tp)),t1]) +
sum{tb in 1..(N-(t1+1))} shorted[tb,t1+1];

subject to noShortMoreThanEPatTime0a{t in 1..N-2}:
x[0,1,t] + sum{tb in 1..(N-(t+1))} shorted[tb,t+1] <= EoProd[t+1];

subject to noShortMoreThanEPatTime0b: xM[0,1,5] <= EoProd[6];

subject to noShortMoreThanEPatTimeN{t in 1..N-2, s in 1..2^(2*t), t1 in t+1..N-1 : Prob[t,1+(s-
1)*2^(2*(N-t))] != 0}:
xM[t,s,t1] + sum{tp in 0..t-1} (xM[tp, 1+floor((s-0.1)/4^(t-tp)),t1] - xP[tp, 1+floor((s-0.1)/4^(t-tp)),t1]) +
sum{tb in 1..(N-(t1+1))} shorted[tb,t1+1] <= EProd[t,s,t1];

subject to xConstraint{t in 0..N-2, s in 1..2^(2*t), t1 in t+1..N-1}: x[t,s,t1] = xM[t,s,t1] - xP[t,s,t1];

#3.3.3 Risk management (VaR) constraints:
subject to valueAtRisk{s in 1..2^(2*N)}:
R[N,s] >= VaR*(1+VaRImp)*z[s];
subject to setAlpha{s in 1..2^(2*N)}:
sum{i in 1..2^(2*N)} (1-z[i])*Prob[N,i] <= 0.1;
```