Norwegian University of Science and Technology
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Faculty of Social Science and Technology Management

Department of Industrial Economics and Technology Management

MASTER THESIS

for

STUD.TECHN. Mari Bjørnsgard
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Field of study	**Accounting and Finance** Investering, finans og økonomistyring
Start date	15.01.2007
Title	**Case Studies in Hydropower Scheduling** Casestudier i driftsplanlegging, vasskraft
Purpose	Electricity companies owning storage hydroelectric plants have a complex task of scheduling the release of water from reservoirs. This thesis will develop a stochastic decision model for this problem.

Main contents:

1. Gather and present data from Norwegian hydropower producers, and other relevant data.

2. Develop an optimization model for hydroelectric scheduling subject to price and inflow uncertainty.

Department Management

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MASTEROPPGAVE

VÅR semester 2007

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ERKLÆRING

Jeg erklærer herved på ære og samvittighet at jeg har utført ovennevnte hovedoppgave selv og uten noen som helst ulovlig hjelp

Trondheim — Sted

07.06.2007 — dato

Mari Bjørnsgard — Signatur

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Preface

This master thesis is prepared at the Norwegian University of Science and Technology, Department of Industrial Economics and Technology Management during spring 2007.

We would like to thank our teaching supervisor, Associate Professor Stein-Erik Fleten for good support and constructive feedback. Additionally, we would like to thank the power producers which have provided useful data.

Trondheim, June 8^{th} 2007

Mari Bjørnsgard

Linn Kristin Hauge

Abstract

This thesis considers long-term production planning for hydro power plants. The production planning problem power producers face is when to release water from reservoirs with the objective of maximizing the profit, regarding uncertainty in future inflow and prices. This must be done without violating reservoir restrictions or other constraints. We formulate the production planning problem as a deterministic equivalent of a stochastic model and solve it using linear programming. The model is implemented in the computer programs Matlab, Scenred and Mosel Xpress. Based on an event tree describing future states of price and inflow, the expected discounted income is maximized.

Two stochastic models describing the spot price dynamics are applied; a logarithmic one- and two-factor price model. The parameters in these models are found using historical spot and forward prices. Additionally, one-factor models describing hydrologic inflow dynamics are used.

The optimization model is tested for six single station systems with one main reservoir. A price taking producer is assumed, local prices are disregarded and the power efficiency is assumed to be constant. Due to random events, the starting point and the period the model is tested for are important for how well the model behaves.

Running the model forward in time, the value of a two-factor price model compared to a one-factor price model is found. The results show that the two-factor price model has most value for power plants with low seasonal dependent inflow and low utilization time. Further on, the value of a stochastic price model compared to a deterministic price model is found. Power plants with high inflow variation have larger value of a stochastic solution compared to power plants with lower inflow variations.

With the purpose of comparing the model recommendations to the actual production, the model is back-tested. It is recommended by the model to discharge more water from reservoirs during the fall 2006 than what was actually done for all six power plants. 2006 had abnormal high spot prices, so the back-test is also performed for the period spring 2005 - spring 2006, with more similar results between the modeled and actual outcome.

Contents

Chapter 1

Introduction

Hydro power producers have a complex task when determining their production strategy. Water can be stored in reservoirs or discharged, generating electricity. The optimal decision depends on two stochastic variables, inflow and electricity price. In this thesis a stochastic optimization model is developed and tested for six power plants. The model maximizes the market value of hydro production. Future inflow and price are modeled as stochastic processes using one- and two-factor models describing the expectation and volatility. Fan scenarios are generated and subsequently made into a scenario tree. Finally, an optimal production strategy based on the scenario tree is found.

Factor models explaining price dynamics are well-known and generally accepted. The same applies to the optimization model presented. It is also chosen to model inflow using factor models. Consideration of which models to use are out of the scope of this thesis. The main purpose in this thesis is to collect data for real power plants with the object of investigating which power plant properties are decisive for how usable the model is.

The stochastic programming model is solved as a deterministic equivalent using linear programming (LP). Advantages of this approach, compared to a stochastic dynamic programming approach[1], is the possibility to employ a detailed topology description. Hence, individual reservoir plans can be found. Further more, in this approach price and inflow models are easily replaced if better descriptions are developed. A disadvantage is that a more aggregated time description than in SDP needs to be employed.

The model is tested forward and backward in time. To make comparisons

[1] Stochastic dynamic programming is in widespread use, for example in the model EOPS (Fosso et al., 2006)

easier, the testing is done for power plants with only one reservoir. Further on, pumping and seasonal dependent restrictions on reservoir level and water flow are disregarded. Main results are the value of a two-factor model solution compared to a one-factor model solution for each power plant. The value of a stochastic solution for the different power plants are also investigated. Finally, the model is back-tested, comparing the actual production for all the power plants to the production strategy proposed by the model.

This report is divided into nine chapters. Chapter two gives the background for the problem. Information about hydrologic inflow and the power market are presented. The next chapter looks into aspects of stochastic programming and introduces scenario trees. One- and two-factor models are presented. Additionally, the variance in optimal solution, the detection of a lower bound and the estimation of the value of a stochastic solution are described.

The fourth chapter describes the spot price and inflow models employed. Information about parameter stability and correlation is given. The optimization model applied is introduced in chapter five.

Data from six hydropower producers are gathered and presented in chapter six. Chapter seven gives the results from running the model both forward and backward in time for the six power plants. The value of a stochastic solution and the value of a two-factor model compared to an one-factor model are discussed. The two last chapters provides a conclusion and suggestions for further work.

Chapter 2

Hydropower Production in the Nordic Market

2.1 Hydropower Production Planning

The production decision every power station with reservoirs face is when to release water stored in reservoirs. Fosso, Gjengedal, Haugstad and Mo (2006) and Flatabø, Haugstad and Mo (2002) describe the production planning problem. Water is a free resource with no cost, but it is a limited resource with alternative future use and hence it has value. This is the so-called water value, which refers to the marginal value of having one more unit of water.

A hydro power plant with reservoir can be considered a complex derivative on the spot electricity price, viewed as a real option on future energy production. The producer has the option to postpone production, waiting for more favorable price conditions or more information about future inflow or other conditions affecting the value of future production. The water value represents the exercise price. Basic real option theory, presented by among others Trigeorgis (1996), states that a project with flexibility is more valuable than a project without flexibility. Optimization theory, presented in for example Rardin (2000), reach the same conclusion. The objective value can never increase when imposing tighter constraints on the solution. Hence, a plant with storage capacity will always be more valuable than a plant without storage possibilities since the water balance constraint is less strict.

Production decisions will depend on reservoir level, future inflow, future electricity prices and present electricity price. Future electricity prices and water inflow are unknown. This makes production planning a stochastic problem. Local restrictions, license conditions, start and stop costs and non-linear connection between water usage and power production will also affect production decisions. To be able to solve this complex problem it is often decomposed into three parts; long term, medium term and short term scheduling.

In long term planning, an optimal hydro power scheduling strategy is found. The time horizon is 1-5 years ahead, depending among other factors on the size of the reservoir. The EMPS model, a market analysis forecasting future prices, is widely used in Norway and Sweden. This model is only employed by the largest power producers in the market. Smaller power producers use the EOPS model for local analysis. The price forecast from the EMPS model represents the external market in the EOPS model, assuming the company has no influence on the market price. Both market price and local inflow are stochastic parameters in this model.

Long term planning imposes boundary conditions on the more detailed medium and short term scheduling. The three scheduling levels can be coupled in different ways. Valuing the water at one level and using this as a recourse price at the next level is a flexible and usable method. Another possibility is to decide the size of the reservoir at the end of one period at one level, using this as a restriction at the next level. This is a simple, but not flexible procedure.

2.1.1 Degree of regulation and utilization time

The degree of regulation affects the horizon of hydro power production planning. It determines how far into the future water value calculations are needed for a power system. The measurement is given as the reservoir capacity in fraction of yearly inflow:

$$R = \frac{M_{max}}{I} \tag{2.1}$$

M_{max} is the maximum reservoir level and I is yearly inflow. The water value equals zero for the entire reservoir in the spring flood period, given a system with low degree of regulation where losses due to spring flood always occur. Hence, planning beyond the next spring flood is not necessary. Given a system with higher degree of regulation, the planning horizon increase (Fosso et al., 2006).

Another parameter describing the power plant is the utilization time. This factor measures the size of the reservoir compared to the power capacity.

$$U = \frac{M_{max}}{P_{max}} \tag{2.2}$$

P_max is power capacity. It describes the time spent to empty a full reservoir when running the generator at full power. High utilization time gives the power plant little flexibility. A utilization time at for instance 6000 hours is a considerable long time period, since one year consists of 8736 hours. A 1000 hour long utilization gives large flexibility.

2.2 Hydrological Inflow

In hydro power production, hydrologic inflow uncertainty is a notable aspect. The purpose of predicting inflow from a reservoir is to find the optimum schedule of water discharge to the reservoir. Inflow prediction will often be of most significance to systems with low degree of regulation, because their reservoirs more frequently arrive at a critical level. The prediction will always be uncertain, but not necessarily symmetric. Hence, the optimum reservoir discharge should not be decided based on the most probable inflow outcome, but on the entire set of opportunities. For further details on this issue, the reader is referred to Killingtveit and Sælthun (2005) and Dingman (2002).

In Norway, many metering stations with long observation series of water flow are distributed all over the country. Based on these series, inflow from most drainage basins can be computed. This gives a good starting point for the description of inflow as a stochastic process. For lack of better knowledge, the expected value of future inflow is assumed to approximately equal the mean of former observed inflows. Principally, the same assumption is applicable for the inflow standard error (Fosso et al., 2006).

2.3 The Nordic Power Market

Power producers operating in the Nordic power market have to obey concession laws, but can otherwise maximize profits based on uncertain future power prices (Fosso et al., 2006). They are exposed to competition and are free to trade in an open market.

Nord Pool is a market place for purchase and sale of electricity where Norway, Sweden, Denmark and Finland are participating. Information about Nord Pool is provided at www.nordpool.no. Trade at the Nord Pool market includes both physical and financially trading. The physical market consists of a balancing market and a spot market. The spot market and the financially market will be briefly commented in this section.

2.3.1 The Elspot market

The Elspot market is trading electricity with physical delivery the next day. Producers report how much they are willing to supply given different prices for every hour the next day. Purchasers report their demands in the same way. Aggregating the demand and supply curves, a market equilibrium is found. This gives the optimum quantum and the system price, which is expected to reflect the margin cost of power assuming no grid bottlenecks. It is also the reference price for the financial market. Capacity constraints in the distribution grid will lead to different local prices between areas divided by the constraint whenever the maximum amount of power is being transferred. Normally, there are three such price areas in Norway: The southern, middle and northern part (Nord Pool, 2007).

2.3.2 The financial market

The market of power derivatives consists of Nord Pool's financial market and the market of bilateral contracts. The financial market is trading daily and weekly futures and monthly, quarterly and yearly forwards. Other financial products are also traded (Nord Pool, 2007).

The future and forward market contains information about expected price developments, which can be useful in hydro power production planning (Fosso et al., 2006). Futures have mark-to-market settlement and a time horizon of 8-9 weeks. Forwards are traded up to five years in advance, and the settlement is accumulated after the last delivery day. The contracts are standardized (1 MW base load) and they are settled against the system price in the Elspot market (Nord Pool, 2007).

Haug (2005) explains that the value of a forward contract at Nord Pool is determined from the risk adjusted expected average system price development during a certain period. These properties are actually describing a swap contract, not a forward contract. Forward contracts are actually valued as the expected risk adjusted price for a certain time point of delivery. The settlement is calculated differently for swaps and forwards. Swaps accumulate the settlement during a period of time. On the contrary, forwards do only have settlement at the decided delivery point. Hence, swaps can be explained as a strip of forwards, so swaps in the power market can then be turned in to strips of daily forwards on electricity. However, to avoid confusion, common practice is followed in this thesis by naming the swaps traded at Nord Pool as futures and forwards.

Fundamental finance, presented in for example McDonald (2003), states that the forward contract price can be determined as

$$F_{0,T} = S_0 \times e^{(r_f - \delta)T} \tag{2.3}$$

provided a complete market without arbitrage opportunities. S_0 is the present system price, r_f is the risk free discounting rate, δ is a lease rate, also called convenience yield, and T is the time to maturity.

The valuation of forward contracts is partly arbitrage pricing due to the fact that hydro power producers are able to store water. Haug (2005) points out that owing to the fact that the possibilities of hydro storing still are limited, a correct pricing of forwards also involves expectations about physical relations such as future temperatures, precipitation and snow melting.

2.3.3 Electricity properties

Lucia and Schwartz (2001) study the price properties of energy traded at Nord Pool. Electricity is a special commodity due to its highly limited possibilities for storing and transportation. These aspects contribute to almost non-existing arbitrage opportunities in the electricity market. The non-storability of electricity makes electricity delivered at different times and different dates considered as distinct commodities. The price is dependent on the supply and demand at the specified time, which varies between seasons, weekends and weekdays. The study finds evidence of properties such as mean reversion, positive skewness and excess kurtosis of energy prices at Nord Pool.

2.3.4 Assumptions about the power market

When modeling power plants in subsequent chapters, some assumptions are done regarding the power market and risk adjustment.

- **Completeness and no-arbitrage:** Risk-adjusted price and inflow processes are used. The market price of risk (McDonald 2003) is assumed to be zero for inflow and it is assumed to be fully reflected in futures and forward prices regarding price uncertainty. This means that maximizing expected revenues is equivalent to maximizing the market value of the hydro power resources. This is in line with Fleten and Wallace (2003).
- **Market power:** The Nordic power market is assumed to be well functioning. Hence, the power producers are price takers, and price is modeled as exogenous. Hjalmarsson (2000) carries out an econometric

study of market power at Nord Pool on a system level, where the hypothesis of long-term and short-term market power is rejected. Theory about market power generally, and in hydropower economics specially, can be further explored in Schotter (2002) and Førsund (2007).

- **Local prices:** Local prices are not taken into account. Price models foreseeing the system price is found. By disregarding local prices, the same spot price model can be used for each company analysed. Besides, on a weekly basis the area prices tends to be pretty similar, as displayed in figure 2.1.

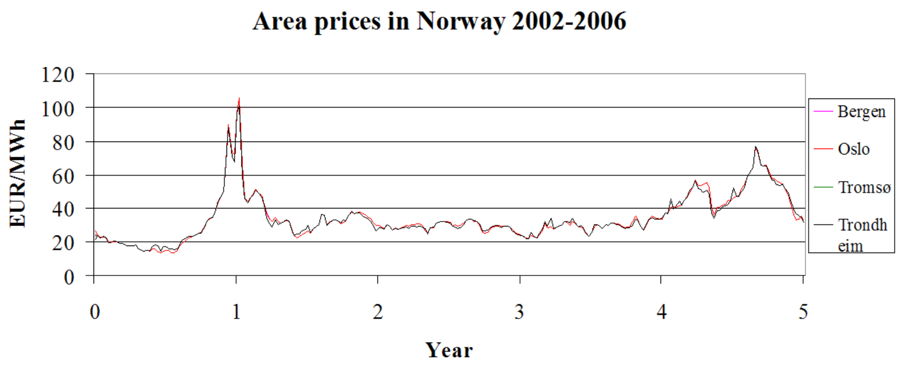

Figure 2.1: Weekly observed area prices in Norway measured in EUR/MWh in the period 2002 - 2006.

Chapter 3

Stochastic Models

3.1 Stochastic Programming

This section deals with the differences between deterministic and stochastic programming. For more literature on these topics, the reader is referred to Rardin (1998) and Fosso et al. (2006).

In deterministic programming all parameters are assumed to be certain. The model will find an optimal solution given fixed start, end and framework conditions. In reality many factors are unknown and uncertain. Scenario optimization will consider risk due to uncertain parameters, though assuming the parameters to be known and certain in each scenario. Therefore limits will be exploited, which might have consequences if the future turns out to be different than expected.

Advantages of a deterministic model are easily interpreted results and short time needed to find a solution compared to a stochastic model. If time horizon is short and the parameter uncertainty is small, a deterministic model will do well. The character of consequences an unforeseen occurrence can give should also be considered when deciding what model to use.

In reality the future is unknown and parameters are affected by future events. Stochastic programming takes this into consideration when finding an optimal solution. For each uncertain parameter, a probability distribution describing possible future outcomes is needed. These parameters are called stochastic variables and the uncertainty distributions are inputs to the stochastic model.

Finding these probability distributions is a complicated and time consuming task. Many stochastic variables will make the optimization model complex. Hence, the model must be simplified by treating less vital stochastic parameters deterministic to achieve an acceptable solution time. Consideration regarding complexity versus solution time must be done for each stochastic factor with the purpose of the model as an important aspect.

Several solution algorithms solving stochastic models exist. Reducing the original problem to an equivalent deterministic problem is a simple and straightforward method. This is done by expressing the stochastic parameters with discrete probability distributions. Possible outcomes of the stochastic variables, scenarios, must be generated to solve the model in this way.

3.1.1 Scenario tree

In stochastic programming it is often convenient to represent stochastic variables in a scenario tree. Given the variables' probability distributions and the correlations, a set of fan scenarios can be found. From these fan scenarios, the scenario tree is made.

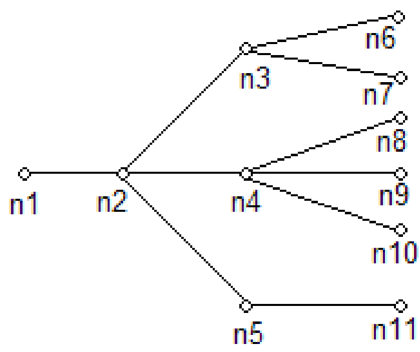

Figure 3.1: Scenario tree with eleven nodes describing possible future states of the stochastic variables.

An example of a scenario tree is shown in figure 3.1. This is a scenario tree with four time steps, six scenarios and eleven nodes. At each point in time, the nodes in the tree represent the possible states the stochastic variables can have. The number of nodes at each point in time grows as the time horizon expands due to increasing uncertainty.

Heitsch and Römisch (2005) have developed two methods for generating scenario trees from sets of fan scenarios, one forward and one backward method.

By deleting and bundling scenarios, a scenario tree is made. The forward method starts at the first and ends at the final point in time. The most representative nodes are selected at each point in time. Figure 3.2 gives an illustration of forward tree construction.

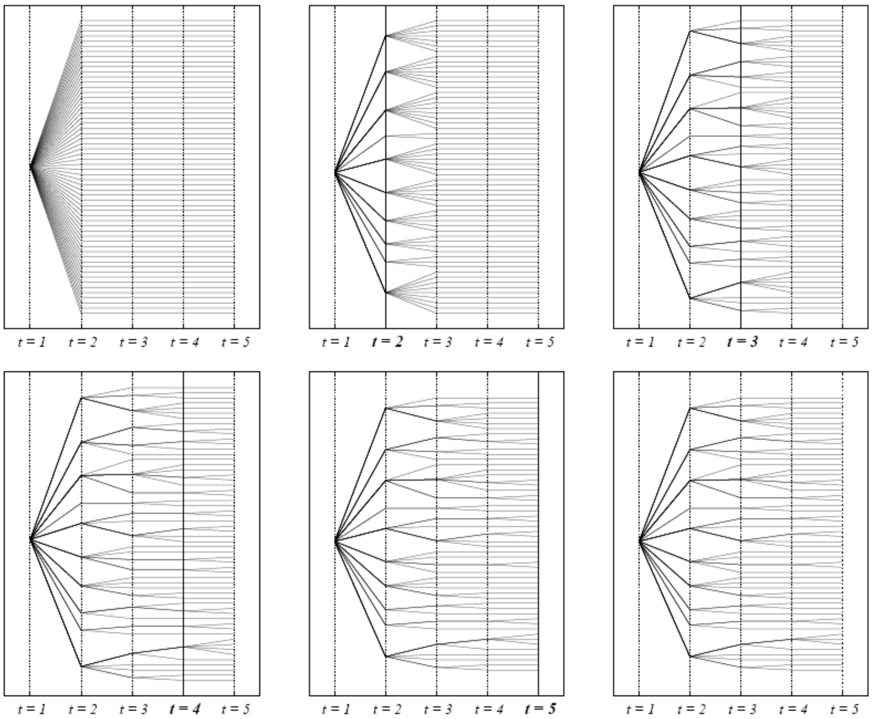

Figure 3.2: Forward tree generation by bundling and deleting scenarios (Heitsch and Römisch, 2005).

Scenred, a C++ based program made by Heitsch, constructs scenario trees from sets of scenarios. It uses the methods described in (Heitsch and Römisch, 2005). The program is used to generate scenario trees describing spot price and water inflow in this thesis.

Three parameters have to be set when running Scenred; relative probabilistic tolerance ϵ_p, relative filtration tolerance ϵ_f and a tree construction parameter q. The relative probabilistic tolerance is used to measure the distances between the original and the approximated probability distributions whereas the filtration tolerance measures the filtration or information distance. The construction parameter q affects the tolerances at each branching point. More information about the parameters can be found in Heitsch and Römisch (2005).

3.2 Stochastic Models for Uncertainty

Good price and inflow models are crucial in hydro power production planning(Fosso et al., 2006). Lucia and Schwartz (2001) and Schwartz and Smith (2000) discuss models for spot system price dynamics and valuation of spot price derivatives. The models describe spot price behaviour using two components: A predictable deterministic function capturing spot price cycles and trends and a stochastic component following a continuous time diffusion process. This is a so-called factor model. Factor models have a defined number of stochastic elements, each with a particular distribution.

A factor model represents future expectation and uncertainty. Models with more factors are better to represent variance structure than models with fewer.

3.2.1 One-factor model

The one-factor model represents the stochastic prosess G_t by

$$G_t = f(t) + \chi_t \tag{3.1}$$

where f(t) is a deterministic time function and χ_t is a stochastic process given by

$$d\chi_t = -\kappa\chi_t dt + \sigma dZ_\chi \tag{3.2}$$

$\kappa > 0$, $\chi(0) = \chi_0$ and $d\chi$ represents an increment to a standard Brownian Motion Z_χ. χ_t is the only source of uncertainty in this model. χ_t follows a stationary mean-reverting process, an Ornstein-Uhlenbeck process with a zero long-run mean and a mean reverting factor κ. The expected value is:

$$E_0(G_T) = f_T + (G_0 - f_0)e^{-\kappa T} + \alpha(1 - e^{-\kappa T}) \tag{3.3}$$

One-factor model based on the logarithmic value
Applying the one-factor model, the stochastic process of the logarithmic value is

$$lnG_t = f(t) + \chi_t \tag{3.4}$$

f(t) and χ_t have the same properties as earlier mentioned. The stochastic process is log normally distributed, and its expected value is

$$E_0(G_T) = exp(f(T)+(lnG_0-f(0))e^{-\kappa T}+\alpha(1-e^{-\kappa T})+\frac{\sigma^2}{4\kappa}(1-e^{-2\kappa T})) \tag{3.5}$$

Finding a risk-adjusted model, a risk-adjusted stochastic process with the form

$$d\chi_t = (-\kappa\chi_t - \lambda_\chi)\, dt + \sigma_\chi dZ_\chi \tag{3.6}$$

must be applied.

Applying this model for the spot price and given an interest rate independent of the spot price, the forward price will be equal to the expected spot price in a risk adjusted case. The difference between the forward price and the true expected spot price is the risk premium.

$$F_0(G_0, T) = E_0(G_T) \tag{3.7}$$

Both spot prices and water inflow are given in discrete time. Hence, the model parameters have to be estimated using discrete parameters.

$$\chi_t = \chi_{t-1} \times e^{-\kappa \Delta t} - \frac{\lambda_\chi}{\kappa}(1 - e^{-2\kappa \Delta t}) + u_t \tag{3.8}$$

Here u_t is the model error. The parameters in f(t), κ, λ_χ and χ_0 needs to be estimated (Lucia and Schwartz, 2000).

Variance in a one-factor model

The stochastic process 3.8 has a variance (Dias 2007).

$$Var[x_t] = Var[u_t] = \sigma_\chi^2 \Delta t \tag{3.9}$$

By estimating the variance of u_t from empirical data, σ_χ is found. The variance can also be expressed dependent on κ, but since κ is calibrated towards the forward price, this simpler way of expressing the variance is applied. A variance independent of κ will not be affected by calibration.

3.2.2 Two-factor model

By decomposing the stochastic part into two factors, the logarithmic model is extended to

$$lnG_t = f(t) + \chi_t + \xi_t \tag{3.10}$$

In this model, the χ_t- term tries to capture short time deviations whereas ξ_t is the long term equilibrium level. Short-run deviations (temporary deviations resulting from unusual weather, supply disruption etc for a price model) are assumed to follow the risk-adjusted Ornstein-Uhlenbeck process reverting toward $\frac{\lambda_\chi}{\kappa}$, shown in equation 3.6. The equilibrium level is assumed to follow a Brownian motion process

$$d_{\xi t} = \mu_\xi dt + \sigma_\xi dz_\xi \tag{3.11}$$

Changes in the equilibrium level represent changes expected to persist. dz_χ and dz_ξ are correlated increments of standard Brownian motion processes

$$dz_\chi \times dz_\xi = \rho_{\chi\xi} \tag{3.12}$$

ξ_t and χ_t are jointly normally distributed with covariance matrix

$$Cov[\chi_t, \xi_t] = \begin{bmatrix} (1-e^{-2\kappa T})\frac{\sigma^2}{2\kappa} & (1-e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} \\ (1-e^{-2\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} & \sigma_\xi^2 t \end{bmatrix} \quad (3.13)$$

The variance of χ_t is here expressed dependent on κ and not in the simplified way explained in the previous section. This is because the variance is estimated simultaneously as the risik-adjusted model parameters. Hence, calibration of κ is not necessary.

The future value in log form is

$$ln(F_T, 0) = ln(G_T) = e^{-\kappa T}\chi_0 + \xi_0 + A(T) \quad (3.14)$$

where

$$A(T) = f(t)+\mu_\xi T-(1-e^{-\kappa T})\frac{\lambda_\chi}{\kappa}+\frac{1}{2}((1-e^{-2\kappa T})\frac{\sigma^2}{2\kappa}+\sigma_\xi^2 T+2(1-e^{-\kappa T})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}) \quad (3.15)$$

3.2.3 Deterministic part

To implement the previous general models, the deterministic term f(t) must be specified. Seasonal time variations could be incorporated using a cosinus function, and the deterministic component becomes

$$f(t) = \alpha + \gamma cos((t+\tau)\frac{2\pi}{52}) \quad (3.16)$$

α, γ and τ are parameters that must be estimated (Lucia and Schwartz, 2000).

3.2.4 Forward prices between two points in time

Forward prices (swaps) at Nord Pool are defined for a period of time, $F_0(P_0, T_1, T_2)$, with T_1 being the starting point and T_2 the ending point. Given the expected forward price for one point in time, equation 3.7, the expected price over a period can be found using the definition (Lucia and Schwartz, 2000), (Koekebakker and Ollmar, 2005)

$$F_0(P_0, T_1, T_2) = \frac{\int_{T_1}^{T_2} e^{-rT} F_0(G_0, T) dT}{\int_{T_1}^{T_2} e^{-rT} dT} \quad (3.17)$$

where r is the risk adjusted interest rate. In this way the fact that time to maturity is varying over the time period will be taken into account. The varying price expectation over the period and the effect this has on the expectation for the total period will then be allowed for.

3.3 Kalman Filter

A Kalman filter is used to estimate the parameters in the two factor spot price model. An introduction to the Kalman filter is given by Bishop and Welch (2006). Arnold et al (2005) describes the filter as a recursive algorithm that produces estimates of a time series of unobservable variables, the state variables, using a related but observable time series of variables. A set of mathematical equations estimates the state variables in a way that minimizes the mean of the squared error.

Harvey(1989) considers time series models and the Kalman filter. The Kalman filter gives a way to estimate unknown parameters in a model. When estimating the state variables, the parameters are assumed to be known. The likelihood of the observations given a set of parameters can be found (Harvey, 1989). Rerunning the Kalman filter with better estimates of the parameters until the likelihood function converges to a level will give a parameter estimate (Lucia and Schwartz, 2000).

The set of equations which estimates the state variables when running the Kalman filter are described in Schwartz and Smith (2000). A measurement equation which states the model definition of the forward prices is

$$y_t = d_t + F_t^{'} x_t + v_t \qquad t = 1, ..., n_t \tag{3.18}$$

where

$$y_t = [lnF_{T_1}, ..., lnF_{T_n}] \tag{3.19}$$

$$d_t = [A(T_1), ..., A(T_n)] \tag{3.20}$$

$$F_t = \left[e^{-\kappa T_1} 1, ..., e^{-\kappa T_n} 1\right] \tag{3.21}$$

The model these equations are based on assume forwards given at a single point in time. In the following section these equations will be modified so the Kalman filter can be applied to estimate parameters based on forwards given with a starting point and an ending point.

3.4 Value of Stochastic Solution

In this thesis, a stochastic optimization model is developed. It will be examined to which extent this model is superior compared to a deterministic two factor model. This is done by applying the principles of the value of stochastic solution, discussed by Wallace (1999).

The value of stochastic solution (VSS) is found by solving the corresponding deterministic model and comparing the expected objective value of the stochastic and deterministic model respectively. VSS measures the expected increase in objective value from solving the stochastic version of the model rather than the deterministic one. Hence, it is a measurement of how important it is to explicitly consider uncertainty by solving a stochastic model. VSS can be found using the following formula:

$$VSS = ESS - EMV \tag{3.22}$$

EMV is expected objective value of the mean value solution (deterministic problem) evaluated over all scenarios and ESS is expected objective value of stochastic solution.

In a multistage case, like the one in this thesis, the definition of VSS requires that the deterministic problem is solved repeatedly in all nodes of the scenario tree. The objective is to achieve a fair comparison. Deterministic models are resolved as new information is available so comparing the stochastic solution to its root node solution would underestimate the strengths of the deterministic model.

Following the mindset of VSS, the value of a two-factor compared to a one-factor price model can be found. This is done subtracting the expected objective value when a one-factor price model is applied from the expected objective value when using a two-factor price model. This is a fair comparison, because both of the models are stochastic.

3.5 Variance in Optimal Value

Rerunning the stochastic model will give different results due to the fact that scenarios are generated by random variables. To be able to say something about the uncertainty of the optimal solution found, the variance in optimal value is calculated.

Shapiro and Philpott(2007) explains how lower and upper bounds for the optimal solution can be found statistically. It is assumed that the optimal solution have a student t-distribution. When comparing two expected values a lower value of the difference given $100\,(1-\alpha)$ percent certainty is given by Walpole et al. (2002).

$$d_0 = \overline{v}_2 - \overline{v}_1 - t_{\alpha,\nu}\sqrt{\frac{s_2^2}{m_2} + \frac{s_1^2}{m_1}} \tag{3.23}$$

$\overline{v}_2$ and $\overline{v}_1$ being the average values, s_1 and s_2 the estimated standard deviation and m_1 and m_2 number of times the optimal solutions are found. $t_{\alpha,\nu}$ is the value of the t-distribution with ν degrees of freedom giving a $100\,(1-\alpha)$ percent lower bound. ν defined as

$$\nu = \frac{(s_1^2/m_1 + s_2^2/m_2)^2}{(s_1^2/m_1)^2/(m_1-1) + (s_2^2/m_2)^2/(m_2-1)} \tag{3.24}$$

Chapter 4

Models in This Paper

In this thesis two models are used to capture spot price dynamics, a two-factor model and a one-factor model. Both models are based on the log spot price. Additionally, one-factor models capturing hydro inflow dynamics are applied. All these models are estimated based on discrete data. Since the two-factor price model estimated using a Kalman filter is continuous, the one-factor models are also continuously employed. The model parameters are estimated from discrete data and used to generate discrete scenarios.

4.1 Inflow Model

The inflow model is based on the one-factor model presented in section 3.2.1. Parameters are estimated using the least squares method, based on weekly historical inflow given from each power plant respectively. Some of the available inflow series are long (i.e.100 years) and others are shorter. The entire length of the inflow series are used, except from series exceeding thirty years. This is due to the fact that the last observed inflow values are more likely to occur again (Killingtveit and Sælthun, 1995). The inflow models will be subject to a more thorough discussion in chapter 6, where they are presented together with the power plants they belong to.

4.2 Spot Price Models

As explained in section 2.3.1, both a system price, representing the total market with no restrictions, and local prices due to capacity constraints in the distribution grid exist. Local prices, not system prices, affect electricity company's decisions. The markets view of future prices is reflected in forward prices, but the forward price is connected to the system price, not local prices. Forward prices may still be useful when finding a price model explaining future local prices because they contain information of expected fluctuations. The easiest method to find a local price model by using forward prices, is to find a constant ratio or a constant difference between the system price and the local price. Multiplying each forward by this ratio or adding the constant and replacing the system spot price by the local spot price, will lead to a model representing the local price. More information about this is found in Haldrup and Nielsen (2006).

Local prices are disregarded in this thesis, but it is important to be aware of this simplification. Two stochastic models describing the spot price are used, a one-factor and a two-factor model, both logarithmic. The models are described in chapter 3.2. The parameters of one two-factor and 23 one-factor models, describing the price dynamics given at different points in time, are found.

4.2.1 Price data description

Using the ln value of spot prices, the parameters in the one-factor model are found. The spot prices are observed weekly at Nord Pool during the time span 2002 - 2006, totally 261 observations. The same spot prices combined with spot price derivatives are used when finding the parameters in the two-factor model. 14 spot price derivatives are used at each time step, including six monthly, five quarterly or seasonal and three yearly forwards. Several price data are given in NOK/MWh at Nord Pool. Weekly exchange rates from DNB are used to translate the price data into EUR/MWh.

In this subsection properties of the total spot price data set will be looked into. The spot price development throughout the sample period is displayed in figure 4.1. The mean value of the spot price is 34,12 EUR/MWh and the standard deviation is 13,56. The lowest and highest observed spot price is 14,543 and 103,65 EUR/MWh, respectively.

The spot price data set has a positive skewness of 1,784. Positive skewness indicates that it is more likely to experience spot prices higher than the mean value, compared to a normal distribution. The excess kurtosis is 4,770, meaning that very high or very low spot prices are more likely to occur compared to a normal distribution. All data discussed are shown in table 4.1.

Applying the observed skewness and kurtosis in a Bera-Jarque test, it can be detected whether the spot price and the ln of the spot price is normally

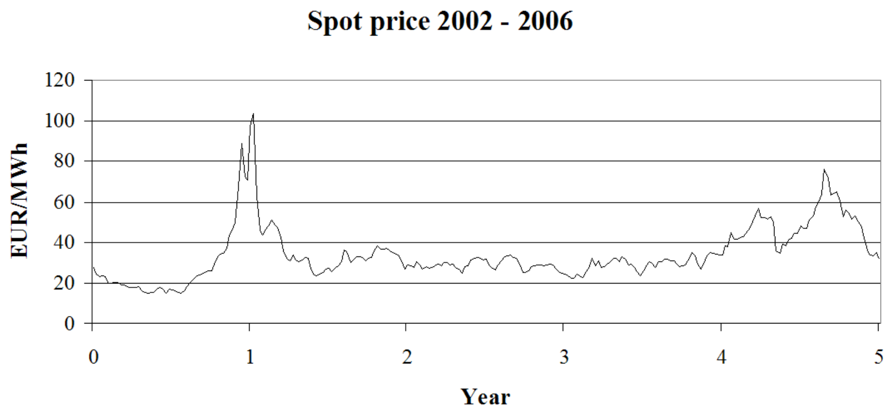

Figure 4.1: Weekly observed spot prices measured in EUR/MWh during the period 2002-2006

Table 4.1: Properties of the price data

Price data			
Mean value	34,12 EUR/MWh	σ	13,56%
Max value	103,65 EUR/MWh	Skewness	1,784
Min value	14,54 EUR/Mwh	Excess kurt.	4,770

distributed or not. In this case, normality is rejected for both spot prices and ln spot prices on a 5 % significance level.

As earlier mentioned, the future and forward market contains information about expected price development. This information should be incorporated in the price models in such a way that the expectations are reflected. In figure 4.2 the futures and forwards incorporated in the one-factor price models are displayed. The future and forward price structures are observed every fourth week in the period spring 2005 until the end of 2006. Each observation consists of a set of selected futures closing prices listed on that date. The selection is done in such a way that the future/forward with the smallest time dissolution always are used at each point in time, i.e. if there exist four weekly futures and one monthly forward covering the same time period, the four weekly futures are choosed to represent the term structure. In the figure each observation date is indicated by a new curve starting at that point in time.

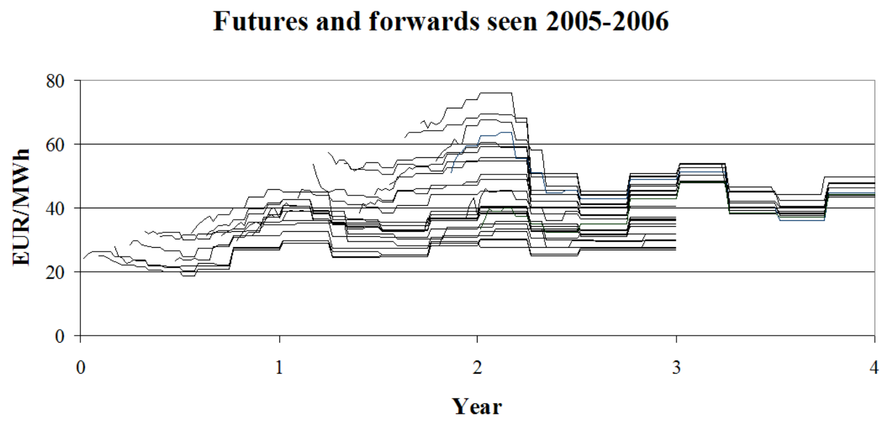

Figure 4.2: The term structure evoulution of selected futures and forward prices from January 2005 to December 2008 observed every fourth week from January 2005 to December 2006. For any curve, the turning points indicate a different price corresponding to a subsequent maturity. The length of the following flat part represents the length of the delivery period.

4.2.2 One-factor price model

One-factor price models (see equation 3.4), are needed for the analysis later on for every fourth week from spring 2005 until the end of 2006. The information available at every point in time is used to find the best model parameters possible. Parameters are first estimated based on historical prices using the least squares method where the stochastic term in every time step is dependent of the residual in the previous time step.

Figure 4.3 show how the model made for week one 2007 fits the historical prices. This is an in-sample graph. With the purpose of showing an out-of sample graph, the price model found for week 52 2005 is used to describe the spot dynamics in 2006. This can be viewed in figure 4.4. Comparing the price model to the actual price for both the in-sample and out-of sample graph, it is observed that both the in-sample and out-sample price model is considerably lower than the actual price in 2006. This could partly be explained by the fact that this year was very special when it comes to spot prices.

The price parameters need to be adjusted by the price of risk to arrive at the risk-adjusted process. Eydeland and Wolyniec (2003) describe how to obtain the risk-adjusted parameters by recovering them from the prices of liquidly traded products. In this thesis, the one-factor price model is calibrated by

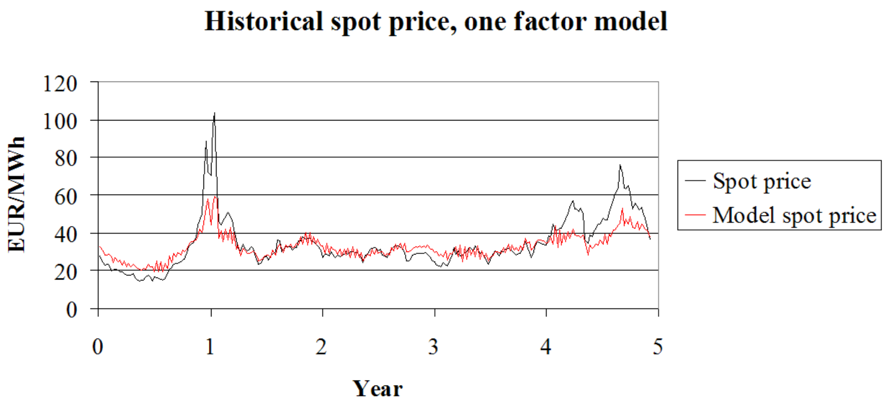

Figure 4.3: The actual weekly spot price plotted against the model estimates (fitted). The model estimation is based on spot price data for the period 2002 - 2006, hence it is in-sample.

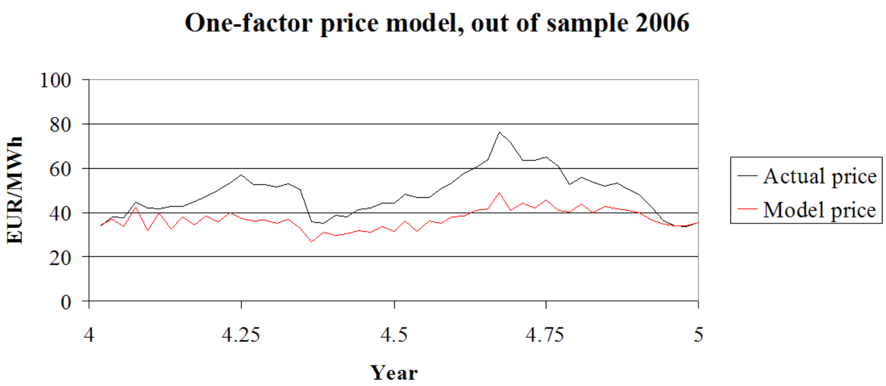

Figure 4.4: The actual weekly spot price plotted against the model estimates (fitted) for the year 2006. The model estimation is based on spot price data for the period 2002 - 2005, so the model is out-of sample.

changing the parameters κ and λ with the purpose of fitting the prevailing forward curve. The expected forward price should be similar to the actual forward curve. This is done using the least squares method. Parameter λ is not part of the original model, so its value is zero previous to calibration. Every model is assumed to have the same variance, found at the end of the time period. This assumption is made to let the variance be determined by the longest data series employed and to simplify work. Figure 4.5 shows the calibrated expected forward price compared to the forward curve as it is seen in week 52 2006. The forward curve in this figure is in fact one of the curves presented in figure 4.2, and it is found in the same manner as described in section 4.2.1.

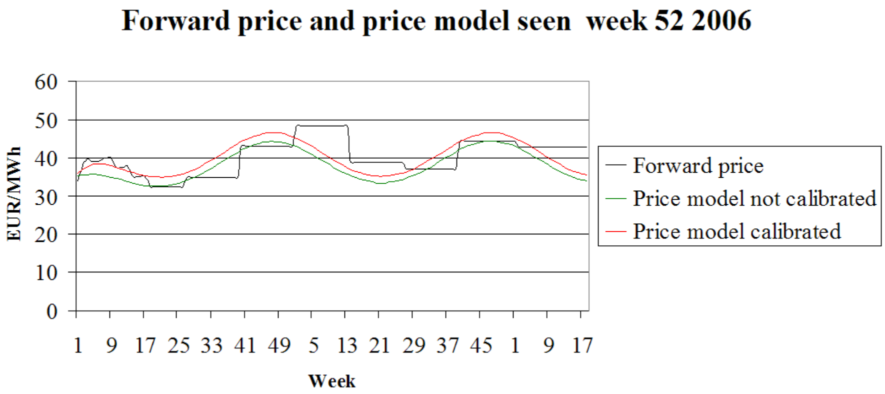

Figure 4.5: The one factor price model with data known in week 52 2006 calibrated to the forward curve as it is seen at that point in time. The calibration is done using the least squares method.

One-factor price model validation and parameter stability

The parameter stability of the one-factor model found in week 52 2006 will be evaluated. Studying the graphs in figure 4.3, the estimated spot price seems to follow the actual spot price fairly well. However, the estimated spot price does not capture the peaks in the actual spot price. To check the validation of the one-factor price models, the parameter stability is checked. Additionally, some tests of the residuals in the model are performed. A good model should have normally distributed residuals with a zero mean, none autocorrelation and time-independent variance. These tests are performed in PCgive.

The purpose of a parameter stability test is to detect whether the parameters are constant for the entire sample period or not. This is done by splitting the data set into two sub-periods and hence comparing the parameter results for each sub-period to the entire data set-parameters. The absolute value of the deviation for each sub-period parameter compared to the entire data set parameter is shown in percentage in table 4.2. It is observed that the deviations are small for α and τ, meaning that the price level and time of seasonal variation in price are relatively stable. The deviation is high for γ, exceeding 100%. γ explains seasonal fluctuations, explaining to which extent the price is seasonally dependent.

The residuals in the one-factor model have a mean of -0,00086, which approximates to 0. Normality is rejected on a 5 % significance level. Figure

Table 4.2: Parameter stability one-factor price model found in week 52 2006. The parameters estimated using two sub-periods are compared to the parameters found when using the entire sample. The percentage deviations are displayed.

Parameter stability one-factor model		
Parameter	**First Sample**	**Second Sample**
α	2,84%	2,78%
γ	125,88%	137,11%
τ	5,30%	10,07%
κ	15,84%	8,59%

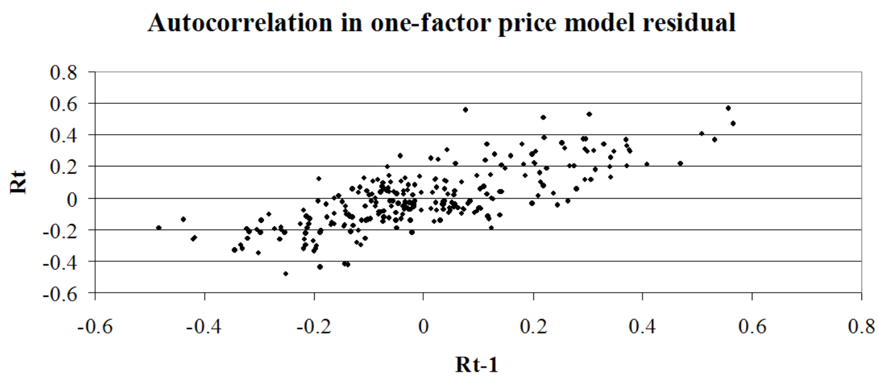

Figure 4.6: Autocorrelation in the one-factor price model residuals. The price model is based on historical data for the period 2002-2006.

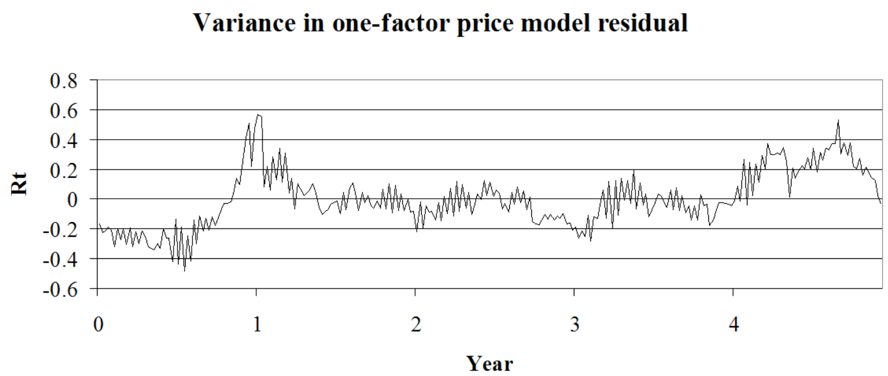

Figure 4.7: Variance in the one-factor price model residuals. The price model is based on historical data for the period 2002-2006.

4.6 indicates a positive autocorrelation in the residuals. This is confirmed by carrying out a formal Durbin-Watson test, which reveals a positive autocorrelation on a 5 % significance level. Figure 4.7 displays the residuals development against time, indicating time independent variance. Summed up, these findings indicate that the price model is not perfect.

4.2.3 Two-factor price model

The parameters of the two-factor price model are estimated using a Kalman filter. The parameter estimation is based on historical futures and forwards traded at Nord Pool. These are given over a period of time. The Kalman filter equations (3.20) and (3.21) must therefore be modified by using equation (4.1).

$$d'_t = \frac{\int_{T_1}^{T_2} e^{-rT} d_t dT}{\int_{T_1}^{T_2} e^{-rT} dT} \tag{4.1}$$

$$F'_t = \frac{\int_{T_1}^{T_2} e^{-rT} F_t dT}{\int_{T_1}^{T_2} e^{-rT} dT} \tag{4.2}$$

The computer files running the Kalman filter are developed based on files Alstad and Foss made in 2004. Running the Kalman filter using weekly observations of the spot price and 14 forward prices for five years, 2002-2006, on a 1,60 GHz Intel Celeron CPU with 1024 MB RAM, 1785 seconds are used. To test the Kalman filter, scenarios generated with the estimated parameters are used as input to run the Kalman filter once more. A large number of scenarios should be used, to represent the price dynamic well. Unfortunately the Kalman filter used does not manage to find parameters if the number of price scenarios applied are larger than 80. The filter then uses 7,2 hours running and the parameters estimated are considerably different from the original values. In order to use this method as a proper test for the model, the filter have to handle more scenarios. Even so, the result still indicates an unstable parameter estimation by this Kalman filter. This coincides with tests done by varying the original input file.

The expectation of the two-factor price model found using parameters from the Kalman filter is to low compared to the forward curve. Therefore, the two-factor model is calibrated to the forward curve to obtain a better and more realistic expectation.

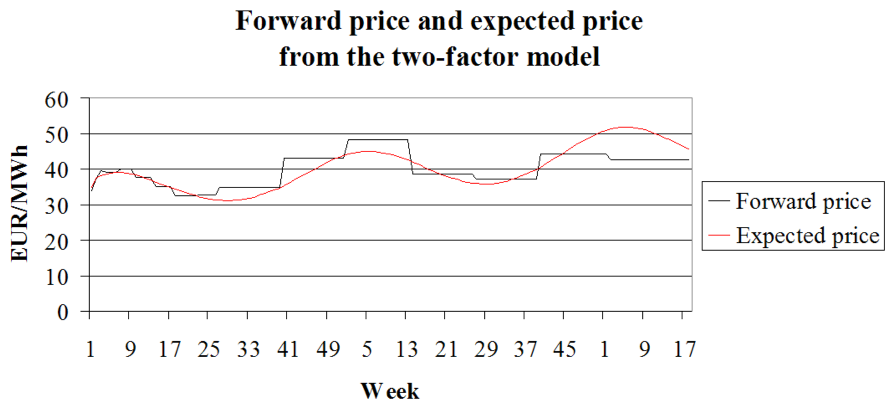

Figure 4.8: Two-factor price model adjusted to the forward price curve.

Two-factor price model validation and parameter stability
It is not performed a model parameter stability test for the two-factor price model, because the Kalman filter applied does not seem to work optimal. Normality in the two-factor model residuals are rejected on a 5 % significance level. Performing a Durbin-Watson test, it is also detected autocorrelation on the same significance level. However, choosing model parameters carefully, the model is good enough for its purpose in this thesis.

4.3 Correlation

The spot price at Nord Pool is strongly affected by water inflow to hydro power reservoirs in Norway because hydro power contributes to a great amount of the total power production. Limited inflow leads to lower reservoirs and subsequently higher system prices. Scarce inflows do often occur nationally as well as regionally at the same time, so the spot price and inflow in each area will be correlated. The stochastic elements of the price and inflow models should reflect this. It could be argued that the spot price is correlated to the reservoir level, not the inflow. However, this is difficult to model and therefore not the approach taken in this thesis. Correlation between spot price and inflow is used.

From the stochastic variable time series, the correlation between the factors can be found. For each time series the normal distributed variable N(0,1) explaining the error is found. Then the correlation between these variables are estimated. This is done for each power plant inflow and for both one-factor and two-factor price models.

Generating correlated random variables

When the models are used to generate price and inflow scenarios, a number of uncorrelated normal distributed random variables are first generated. Correlated random variables are found from the uncorrelated variables and the pairwise correlations. In this thesis it is at most three random variables. The correlated normal-distributed variables, Z_1, Z_2 and Z_3 are found from these equations:

$$Z_1 = \epsilon_1 \tag{4.3}$$

$$Z_2 = \rho_{1,2}\epsilon_1 + \epsilon_2\sqrt{1-\rho_{1,2}^2} \tag{4.4}$$

$$Z_3 = \rho_{3,1}\epsilon_1 + \frac{(\rho_{3,2}-\rho_{2,1}\rho_{3,1})\epsilon_2}{\sqrt{1-\rho_{2,1}^2}} + \sqrt{1-\rho_{3,1}^2 - \frac{(\rho_{3,2}-\rho_{2,1}\rho_{3,1})^2}{1-\rho_{2,1}}}\epsilon_3 \tag{4.5}$$

ϵ_1, ϵ_2 and ϵ_3 are independent and distributed as N(0,1) McDonald (2006).

Chapter 5

Optimization Model

5.1 Long Term Hydropower Production Planning

In long-term hydropower planning, the goal is to find the best scheduling strategy, considering both future prices and inflow. The producers want to maximize their profit by utilizing the water at best possible point in time.

The power efficiency in hydro power stations are not constant. The amount of water needed to produce one kWh depends on what power the power station is running at. In this model the power produced is calculated on a weekly basis. Therefore, the output in each hour is not known, so the ratio between water current and electricity produced is assumed to be constant. In the optimization model this will be expressed through a constant efficiency coefficient. This is a common used assumption in long-term hydro power planning, done by among others Wallace and Fleten (2002).

One of the advantages of the model employed is that it considerates topology and connection between several power plants. Even so, all the power plants analysed in this thesis are assumed to be uncomplicated and isolated. Pumping of water as well as seasonal dependent restrictions are disregarded. The main point here is to compare several hydro power plants, and the simplification of the plants will ease the analysis. Notwithstanding, the model can without much effort be extended to include multiple connected power plants. The models applied are based on Winnem(2006) and Pedersen (2006).

At the ending point of the analysis, water stored in reservoirs will have no value in this model. To achieve an optimal reservoir level at the end of the planning horizon, a constraint on the final reservoir is set. It is chosen to set a minimum final reservoir level in order to prevent the reservoir to be emptied at the ending point. The size of this minimum level depends on what time of the year the end of the planning period is. Power plants with a lot of spring inflow compared to the reservoir size will empty their reservoirs prior to spring. During the fall most power plants should seek to retain a high reservoir level to prepare for the winter production. An alternative method to achieve an optimal reservoir level at the end of the planning horizon, is to give the water a value at the final stage. This requires a method of valuing the water.

5.2 Deterministic Model

In a deterministic model all parameters are assumed to be certain and known. The deterministic model optimizing water discharge and storage in hydro power scheduling will treat electricity price and water inflow as certain in all periods of the planning horizon.

Set:
A: set of planning periods A=(0,1,...,T)

Index:
t: index for time period

Parameters:
Π_t: electricity price in time period t
ψ_t: water inflow in time period t
η: efficiency coefficient
M_{max}: maximum reservoir level
M_{min}: minimum reservoir level
M_0: initial reservoir level
M_T: minimum reservoir level at the end of the planning period
Q_{max}: maximum flow of water
r: interest rate

Variables:
V_0: present value of total production in the planning period
m_t: ending reservoir level at time period t
l_t: loss of water due to flood in time period t
p_t: produced energy in time period t
q_t: water flow in time period t

Objective function:

$$V_0 = \max_{q_t, m_t, l_t} \sum_{t=0}^{T} \frac{\Pi_t}{(1+r)^t} \times p_t \tag{5.1}$$

Restrictions:

$$p_t = \eta \times q_t \qquad , t \in A \tag{5.2}$$

$$m_t - m_{t-1} + q_t + l_t = \psi_t \qquad , t \in A \tag{5.3}$$

$$M_{min} \leq m_t \leq M_{max} \qquad , t \in A \tag{5.4}$$

$$m_0 = M_0 \tag{5.5}$$

$$m_T \geq M_T \tag{5.6}$$

$$q_t \leq Q_{max} \qquad , t \in A \tag{5.7}$$

$$q_t, l_t \geq 0 \qquad , t \in A \tag{5.8}$$

The objective function 5.1 is the sum of the discounted income in each time period. Equation 5.2 states that the energy production depends linearly on the water flow, by the efficiency. In reality this linearity does not exist, and the production will depend nonlinearly on both the power and the reservoir level. Restriction 5.3 gives the reservoir balance; the difference between the reservoir level in two time periods is equal to the net inflow during this time. The reservoir level has to be larger than minimum level and less than maximum level in each time period, given by restriction 5.4. Both initial and minimum final reservoir level is given, respectively equation 5.5 and 5.6. The water flow has an upper level stated by restriction 5.7. Equation 5.8 states that neither loss of water due to flood nor water flow can be negative.

5.3 Stochastic Model

A stochastic model takes uncertainties into account. This model has stochastic representation of both spot price and water inflow. The objective function 5.9 is the expected sum of the discounted income.

Objective function:

$$V_t = \max_{q_t, m_t, l_t} E\left[\sum_{t=0}^{T} \frac{\Pi_t}{(1+r)^t} \times p_t\right] \tag{5.9}$$

The constraints for the stochastic model is equal to the restrictions in the deterministic case, only with stochastic variables.

5.4 Deterministic Equivalent

Assuming that the stochastic variables can be described by discrete probability distributions, the stochastic model can be simplified and approximately described by a deterministic equivalent. The stochasticity is represented by a scenario tree. The model can then be solved using standard linear programming, no algorithms are necessary.

New index:
n: node index

New parameters:
N: number of nodes
n_T: nodes in time period T
t_n: point in time node n
$\alpha(n,k)$: index of node prevailing node n in time period t-k
P_n: probability that the state in node n will occur

Objectiv function:

$$V_0 = \max_{q_n, m_n, l_n} \sum_{n \in N} P_n \times \frac{\Pi_n}{(1+r)^{t_n}} \times p_n \tag{5.10}$$

Restrictions:

$$p_n = \eta \times q_n \qquad , n \in N \tag{5.11}$$

$$m_n - m_{\alpha(n,1)} + q_n + l_n = \psi_n \qquad , n \in N \tag{5.12}$$

$$M_{min} \leq m_n \leq M_{max} \qquad , n \in N \tag{5.13}$$

$$m_1 = M_0 \tag{5.14}$$

$$m_n \geq M_T \qquad , n \in n_T \tag{5.15}$$

$$q_n \leq Q_{max} \qquad , n \in N \tag{5.16}$$

$$q_n, l_n \geq 0 \qquad , n \in N \tag{5.17}$$

The objective function 5.10 is the sum of the discounted income in each possible state multiplied by its probability, i.e. the sum of the expected discounted income. Restriction 5.12 states that the reservoir level in each node n depends on the level in the predecessor node $\alpha(n,1)$.

5.5 Water Value

Water value is the economic value one extra unit water will give, i.e. the marginal value of having one more unit of water. Disregarding the reservoir constraint, it will be profitable to produce electricity if the price is higher than the water value and profitable to save the water in the opposite case. The value depends on both future prices and inflow which are stochastic variables not known. Given a scenario representation, as the deterministic equivalent in chapter 5.4 employs, each inflow and price scenario will generate a water value scenario. Both the expected value and the different possible scenarios with their probabilities can be worth looking into.

A dual variable reflects the rate of change in primal optimal value per unit increase in the right-hand-side value of the corresponding constraint (Rardin, 1998). The dual variable of the reservoir constraint, equation 5.12, will give the rate of change in the objective value per unit increase in the reservoir level. That is the marginal value of increasing the reservoir level by one unit, i.e. the water value.

As mentioned in chapter 2.1 a method to connect long term and medium term planning is valuing the water. Hence the water value found in long term planning can be made use of in medium term planning (Fosso et al., 2006).

Chapter 6

Presentation of Hydro Power Plants

The deterministic equivalent of the optimization model presented in chapter 5.4 is applied for six power plants. In this chapter, the power plants modeled are presented. Special properties for each power plant are highlighted. Single station systems with one main reservoir are modeled with the objective of simplifying the comparison between the plants. However, the optimization model is easily extensible to include multiple reservoirs. In this presentation and the analysis following in the next chapter proportions, not real values, are used. This is done to avoid descriptions of sensitive information about the power plants. The characteristics of the inflow to each power plant are also studied and displayed in graphs. To avoid too long time series, these graphs do only show the inflow for the last ten years. In some inflow graphs, the modeled inflow has a few negative values. In reality, this could result from high evaporation or measuring errors. It is not dealt with negative inflows in this paper, so the few negative inflow values are set to zero when estimating model parameters.

6.1 Power Plant Location and Properties

Figure 6.1 indicates where the power plants are located. They are randomly distributed in Norway and will therefore experience dissimilar climate which influence the inflow.

Table 6.1 gives an overview of some of the power plant properties, ranking different plants from 1 to 6 for different properties. 1 represents the highest value of the specific property among the plants studied, and 6 represents the

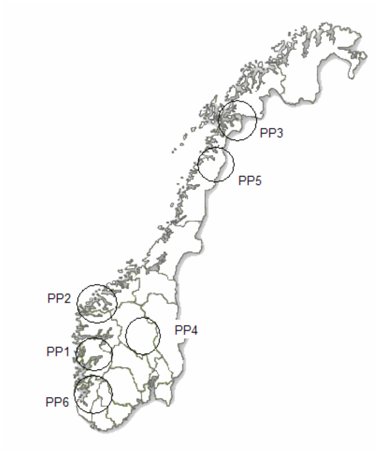

Figure 6.1: Location of the six power plants analysed

lowest value. The mean, μ and the standard deviation, σ, for the inflow to all power plants are found in PcGive. μ is equal to the α parameter found in the one-factor inflow models.

Table 6.1: Power plant ranking from 1 to 6 where 1 indicates the highest value observed.

Power plant ranking						
Power Plant	**1**	**2**	**3**	**4**	**5**	**6**
Average annual inflow	5	6	4	2	3	1
Inflow standard deviation	5	6	4	2	3	1
Max reservoir	4	6	5	3	2	1
Max production output	6	5	4	2	3	1
Average annual production	6	5	4	2	3	1
Efficiency coefficient	6	4	1	5	3	2
Degree of regulation	2	6	5	4	1	3
Utilization time	2	6	5	4	1	3
Seasonal dependence: γ / μ	2	6	4	5	1	3

6.2 Power Plant 1

The inflow data belonging to power plant 1 consists of weekly observations in the period 1977-2006. Inflow for the last ten years is displayed in figure 6.2, together with the modeled inflow. Mean inflow to this plant and inflow standard deviation is the second smallest observed among the six plants studied.

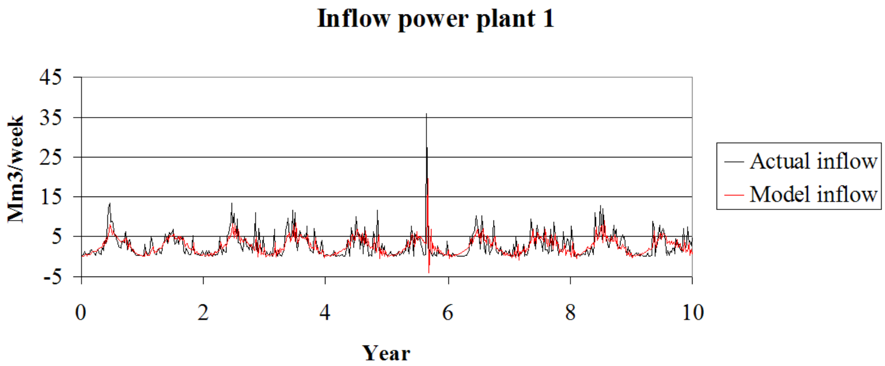

Figure 6.2: The actual weekly inflow during the period 1997-2006 to power plant 1 plotted against the model estimates (fitted).

The inflow is seasonal dependent, but without extreme fluctuations. Reservoir size is in the middle, and max production output, annual production and efficiency have the smallest values observed among the six plants. The degree of regulation is 1,22, meaning that the reservoir can store more water than the annual inflow. A utilization time at 4250 hours makes this power plant relatively flexible, especially considering the high degree of regulation.

6.3 Power Plant 2

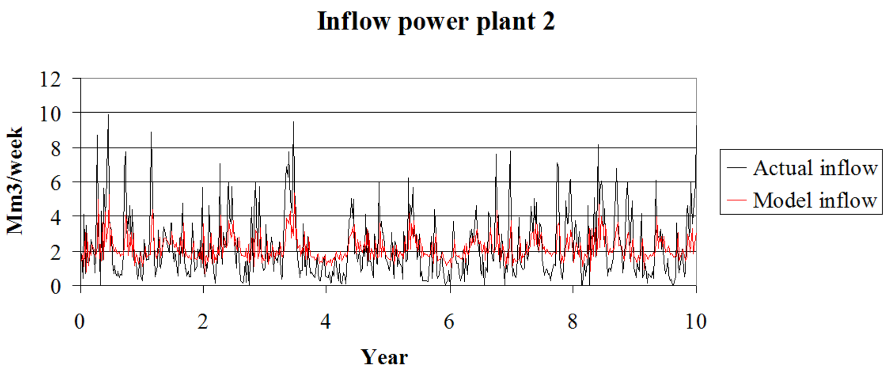

Figure 6.3: The actual weekly inflow during the period 1997-2006 to power plant 2 plotted against the model estimates (fitted)

The inflow data for power plant 2 is made up of weekly observations in period 1981-2006, where the last ten years inflow is displayed in figure 6.3. Annual inflow, standard deviation, inflow mean and max reservoir level is the lowest among the cases studied in this paper. It has the least seasonal dependent inflow, which can be seen both from the sketch in figure 6.4 and the ratio between γ and μ. Average annual production and max production output is second lowest of all cases. Additionally, the degree of regulation is 0,37, which makes this power plant the least regulated plant. The utilization time is 1620 hours, the shortest of these six power plants.

6.4 Power Plant 3

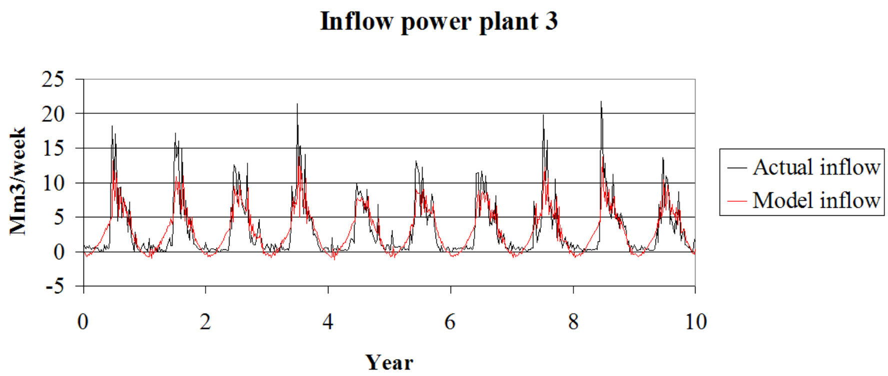

Figure 6.4: The actual weekly inflow during the period 1997-2006 to power plant 3 plotted against the model estimates (fitted)

Power plant 3 has weekly inflow observations for 1983-2006. A inflow graph is displayed in figure 6.4. This power plant ends up in the middle of the studied power plants when it comes to mean inflow, mean annual inflow, inflow standard deviation, max production output and annual production. It is the power plant with most seasonal dependent inflow. The reservoir is the second smallest one among the six plants. The degree of regulation amounts to 0,61 and the utilization time 2190 hours.

6.5 Power Plant 4

Inflow to power plant 4 is observed weekly in the period 1990-2006. Inflow for the last ten years is displayed in figure 6.5. This power plant has the second highest values when it comes to mean inflow, inflow standard deviation, max production output and annual production among the six power plants. It is the power plant with largest peaks in inflow. The max reservoir level is

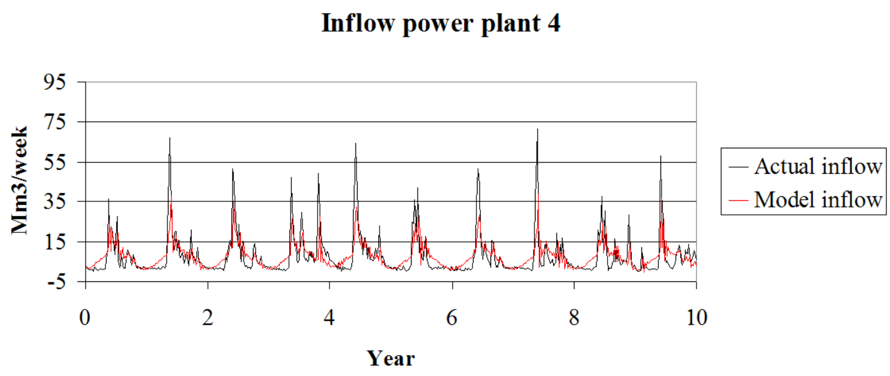

Figure 6.5: The actual weekly inflow during the period 1997-2006 to power plant 4 plotted against the model estimates (fitted)

in the middle of the studied cases. The degree of regulation is 0.65 and the utilization time 1630 hours.

6.6 Power Plant 5

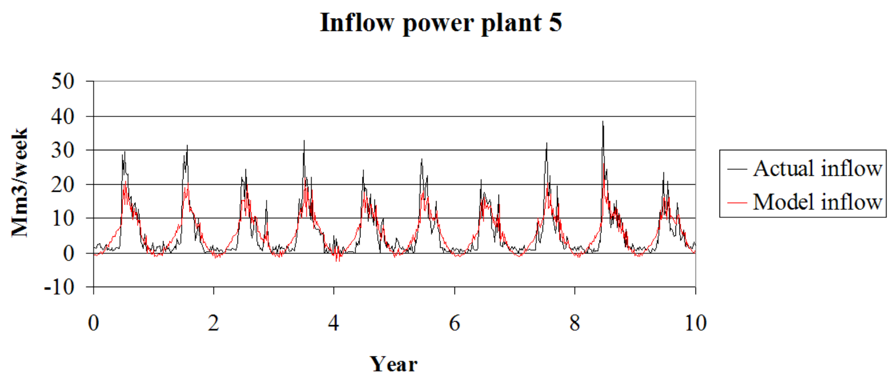

Figure 6.6: The actual weekly inflow during the period 1997-2006 to power plant 5 plotted against the model estimates (fitted)

Power plant 5 has weekly observed inflow from 1984 until 2006. An inflow graph is displayed in figure 6.6. Mean inflow, standard deviation, max production output and annual production of this power plant is in the middle of the studied cases. It has the second most seasonal dependent inflow, very similar to the inflow power plant 3 experiences. The reservoir is the second highest and the degree of regulation is 1,67, which is the highest of the power plants studied. A utilization time equal 5200 hours, the highest of these power plants, is not surprising considering the size of the reservoir.

6.7 Power Plant 6

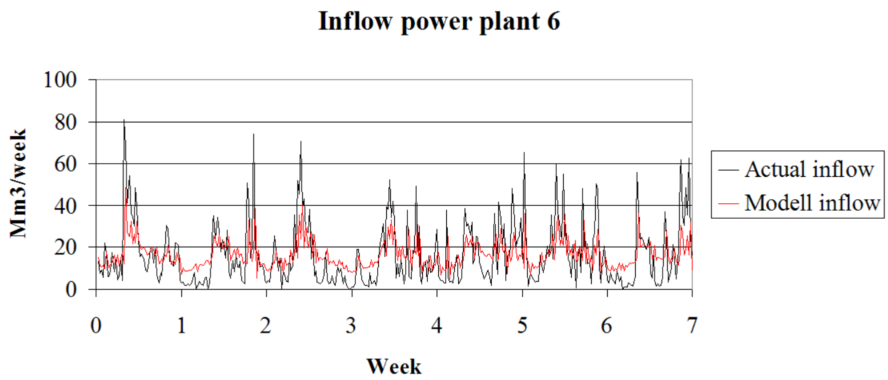

Figure 6.7: The actual weekly inflow during the period 2000-2006 to power plant 6 plotted against the model estimates (fitted)

Inflow data belonging to power plant 6 is weekly observed in the period 2000-2006, the shortest available time series of inflow data. It is displayed in figure 6.7. This plant has the second least seasonal dependent inflow. It varies a lot over the year, but not with the seasonal characteristics seen for other power plants. This is the power plant with highest inflow mean, inflow standard deviation, max reservoir level, max production and annual inflow among the six power plants studied. The efficiency is the second highest of the studied plants. A degree of regulation at 0,7 and a utilization time equal 4137 hours makes this an average power plant with respect to these parameters.

6.8 Inflow Model Performance

The inflow model parameters belonging to the different power plants are estimated based on inflow data series with varying lengths. This section attempts to indicate how good the inflow models are. Inspecting the model inflows compared to actual inflows by pure sight, it is found that the models inflow do not capture the peaks in the actual inflows. To check the validation of the inflow models, the same tests as described in section 4.2.2 are performed.

First, the parameter stability for each power plant is checked. For each power plant, the inflow series are divided into two subparts with equal length. Subsequently, the parameters estimated from the subparts are compared to the parameters estimated from the total inflow series. The absolute percentage deviation is shown in table 6.2. Parameter α, which is a constant describing the level of the inflow, deviates between 0,06 and 7,09 %. Power plant 6 has the highest deviation. γ explains to which extent the inflow is dependent of seasonality. The deviation of this parameter varies between 2,19 and 101,72 %. Power plant 6 has the highest γ-deviation as well. Excluding plant 6, the highest difference becomes 12,76 %. τ describes seasonality, and has deviations between 1,1 % and 21,66%. Eight of the τ-deviations are under 2 %, so this parameter seems to be pretty stable. Power plant 6 has the highest difference in τ. Finally, κ, representing the speed of mean reversion, has deviations between 1,76 and 31,09 %, where power plant 1 has the highest deviation.

An explanation to the high deviations for several parameters in the inflow model to power plant 6 may be the short length of the inflow series belonging to this plant. Parameter γ is the least stable parameter, which is natural due to the fact that this power plant do not have any clear seasonal dependence in its inflow. Power plant 2, the other power plant with little seasonal variation, has also high deviation for parameter γ.

Performing a Bera-Jarque test on the residual R_t, normality is rejected. According to Durbin-Watson tests, it is not found evidence of autocorrelation in the residuals for five of the six power plants. A typical plot of residuals with no autocorrelation is shown in figure 6.8. This plot is from the inflow model of power plant 3. Power plant 5 has positive autocorrelation. Graphs of the residuals against time indicate time dependent variation for all the power plants. This is due to the fact that the model does not capture the inflow peaks. A typical plot is shown in figure 6.9. This plot is also based on the inflow model of power plant 3.

Summed up, the inflow model parameters seem to be fairly stable, excluding some extreme cases in power plant 6. Except from power plant 5, there is no sign of positive autocorrelation in the stochastic terms. This is positive indications when it comes to model validation. On the other hand, the residuals are not normally distributed and time dependent variation is indicated, so the inflow model could be better in this respect.

Table 6.2: Parameter stability, one-factor inflow model found in week 52 2006. The parameters estimated using two sub-periods are compared to the parameters found when using the entire sample. The percentage deviations are displayed.

Inflow Paramater Stability					
		α	γ	τ	κ
Power plant 1	First Sample	0,12%	2,37%	1,01%	31,09%
	Second Sample	0,10%	2,19%	1,07%	25,16%
Power plant 2	First Sample	0,06%	12,76%	3,55%	6,57%
	Second Sample	0,08%	11,01%	4,50%	5,97%
Power plant 3	First Sample	5,46%	2,58%	1,34%	3,29%
	Second Sample	5,41%	2,86%	1,26%	2,87%
Power plant 4	First Sample	4,73%	4,95%	0,67%	7,58%
	Second Sample	4,35%	4,42%	0,61%	10,38%
Power plant 5	First Sample	3,42%	2,76%	0,31%	4,45%
	Second Sample	3,21%	2,65%	0,25%	5,99%
Power plant 6	First Sample	7,09%	101,72%	5,89%	1,76%
	Second Sample	4,88%	36,55%	21,66%	6,08%

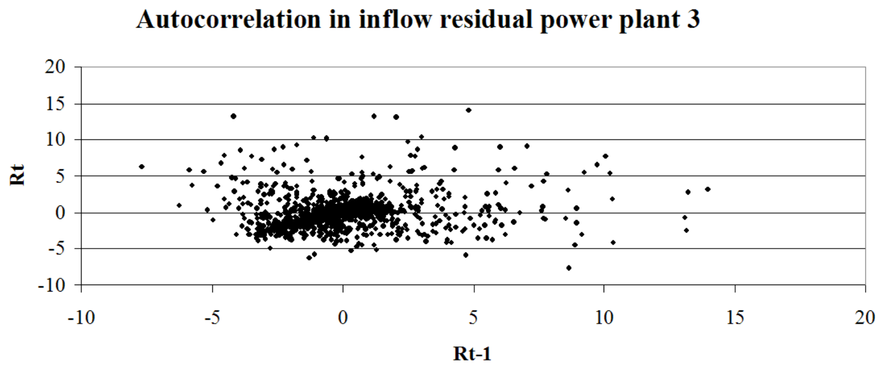

Figure 6.8: Autocorrelation in inflow residual, power plant 3. The inflow model is based on historical inflow data.

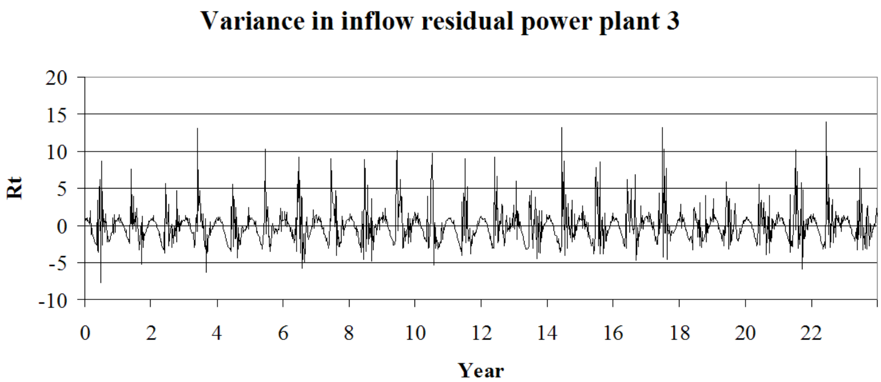

Figure 6.9: Variance in inflow residual, power plant 3. The inflow model is based on historical inflow data.

Chapter 7

Analysis

The deterministic equivalent model described in chapter 5.4 is implemented in optimization software Mosel Xpress, see Xpress MP reference manual. The computer programs Matlab and Scenred are used to generate an input file to Xpress. First Matlab generates a number of fan-scenarios for the stochastic variables inflow and price. Scenred then reduces these to a scenario tree. An example of a scenario tree from Scenred is shown i figure 7.1. The final input file with all input parameters is made in Matlab.

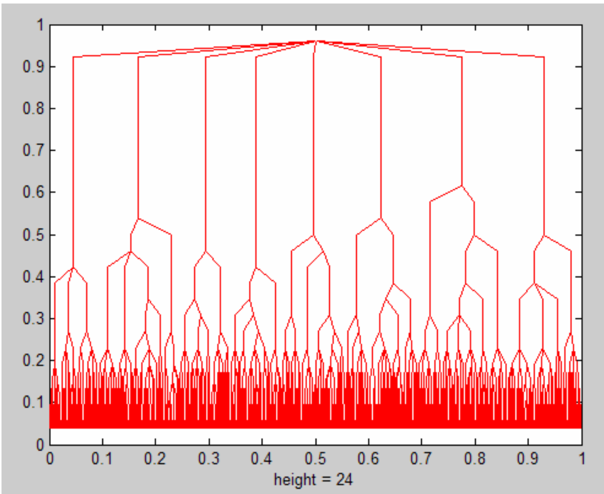

Figure 7.1: Scenario tree made by Scenred.

Two Matlab files, Scenred.m and xpress.m and the Scenred file are run at a 1,60 GHz Intel Celeron mobile CPU with 1024 MB RAM. Xpress is run at 2,4 GHz Intel Celeron P4 CPU with 512 MB RAM. In table 7.1 the time spent when running each of the files, number of nodes and scenarios in the scenario tree and number of simplex iterations are shown for power plant five. The time horizon is 121 weeks and the three Scenred parameters described in chapter 3.1.1 are set to ϵ_p = 0,80, ϵ_f = 0,85 and q = 0,65. Time spent varies every time the model is run, but the table indicates the level.

Table 7.1: Time spent running the files, number of scenarios and nodes in the event tree and number of simplex iterations

File	**Program**	**250 sc.**	**1000 sc.**	**5000 sc.**	**8000 sc.**
Scenred.m	Matlab	3 sec	4 sec	8 sec	11 sec
Scenred	Scenred	2 sec	11 sec	743 sec	2618 sec
Xpressdat.m	Matlab	4 sec	6 sec	42 sec	69 sec
Vannprod 1.0fhm	Xpress	0,2 sec	1,2 sec	13,8 sec	16,4 sec
Number of scenarios in event tree		157	597	2855	3883
Number of nodes in event tree		1108	3136	12266	16519
Simplex iterations when optimizing		3917	11195	42303	56193

All parameters describing the stochastic variables, time horizon, period classification and power plant description are gathered in one excel sheet. The models described in chapter 4 are implemented and explain future values of inflow and price. These are easily interchangeable if better models describing possible future values of spot price and inflow are available.

Program codes, examples of input files and files where the parameters describing the stochastic variables are enclosed on a cd.

The model is run both forward and backward in time for the six power plants introduced in chapter 6. Small time steps close to present time and longer time steps longer off makes the analysis more detailed in the most important time periods. It also reduces the total number of periods, which lowers the time spent running the model. Time steps with different lengths are employed by using both weekly, monthly and quarterly periods. A continuously compounded risk-neutral interest rate of 4 % is set.

Maximum production level is set equal to the power capacity multiplied by number of hours in each period. Minimal production is set equal to zero. Normally a power plant will run at a power which gives best possible efficiency. It will also be varied more often than once a week. Generators often have a lower production level given that they are running, but can also be switched off and produce zero. Costs connected to starting and stopping the generator are also normal. To be able to include these aspects, an hourly production level is needed. In this model the time steps are weeks and the goal is to find a long-term strategy for the water reservoir. The simplifications are therefore acceptable.

It will either be profitable to produce at maximum level or to save the water for later use in each period, i.e. the spot price is either higher or lower than the water value. This is why the model will give a "bang bang" production strategy, maximum or minimum production, as long as it satisfies reservoir constraints. The efficiency varies in the area of possible power ratings, often with a best possible level at the middle and reducing towards maximum and minimum power ratings. Average efficiencies are used when testing the model. For most power plants this will be higher than the efficiency at maximum power. The varying efficiency in reality can lead to a dissimilar total production over the analysis period, even though the reservoir level is equal both at the beginning and the end of the period.

Power plants can have restrictions on reservoir level and water flow at specific time periods. The same applies to restrictions on how fast the water flow can change. From the information gathered from the power producers, no such restrictions exist for any of the six power plants. This is assumed to be true.

7.1 Testing the Model Forward in Time

The model is run forward in time, assuming that the present time is January 1st 2007. All information available at this date is assumed known, nothing more. This is the realistic situation every power producer experiences. The same time horizon is set for every power plant to make it easier to compare the results. In the model water stored in reservoirs at the end of the analysis horizon have no value. To prevent this from affecting the outcome too much, the ending point of the analysis is set in the spring. Due to spring flood, the water in reservoirs in the spring normally has approximately zero value for one-year reservoirs. This is because spilled water do not contribute to income for the power plant.

The power plants included here are located in different areas with dissimilar climate. To be able to have the same ending point of the analysis, the final reservoir level is set to be at least the average level for the last seven years, 2000-2006, at this date. Without this constraint, all reservoirs including the two multi-annual, would have been emptied at the end of the planning horizon. The average value is used because this contains information of what is normally expected to be the correct reservoir level at this point in time. The power plant with the largest degree of regulation decides how long the time horizon should be. April 30th 2009 is set as the ending point of this analysis, two years and four months ahead. Thus the time horizon consists of 121 weeks divided in 25 periods; eight weekly, twelve monthly and five quarterly.

A start level of the reservoir equal to the average reservoir level in week 52 the last seven years is set. It is considered to set the initial reservoir level equal to the actual reservoir level in week 52 2006. However, since the main point of this analysis is to compare the power plants, the comparison should not be affected by circumstances affecting the initial reservoir levels that are special for this particular year.

The model is run for both a one-factor and a two-factor price model. A one-factor inflow model is used in both cases. Models and parameter estimations are explained in chapter 4.

7.1.1 Variance in optimal value

Rerunning the model will give different results due to the fact that fan-scenarios are generated by random variables. To be able to say something about the uncertainty of the optimal solution found, twenty optimal values for 250 and 1000 generated fan scenarios for both a one-factor and a two-factor price model for power plant 5. Due to the time consumed, only 10 optimal values are found for 5000 and 8000 generated fan scenarios. Average values and standard deviations are shown in table 7.2 and 7.3.

Table 7.2: Average value and standard deviation for the optimal objective value for different number of generated fan scenarios when a one-factor price model is used.

Number of fan scenarios	**250**	**1000**	**250**	**1000**
Average optimal value	105863050	103955650	103300700	104719000
Standard deviation	2816423	1467708	1044386	2970083

Table 7.3: Average value and standard deviation for the optimal objective value for different number of generated fan scenarios, when a two-factor price model is used.

Number of fan scenarios	**250**	**1000**	**5000**	**8000**
Average optimal value	112017400	110556150	111008700	112041300
Standard deviation	4035357	2472748	1065490	1500504

There is a considerable higher standard deviation when 250 fan scenarios are generated, compared to 1000 scenarios for both of the two price models. Increasing the number of generated fan-scenarios from 1000 to 5000 also reduces the variance. However, it is surprising to see that by generating 8000 fan scenarios, the standard deviation for the objective value increases when using both a one-factor and a two-factor price model. This is due to random events and shows that the number of runs should be increased to give a better representation of how the uncertainty varies with number of generated fan scenarios. Since this is a time consuming task it is not done. Figure 7.2 and 7.3 displays the objective values found. It shows both how the variance decreases by increasing number of generated fan scenarios and how random events can occur. More generated scenarios gives a better representation of the probability distribution of the stochastic variables and less variation in the optimal solution is therefore found when running the model several times.

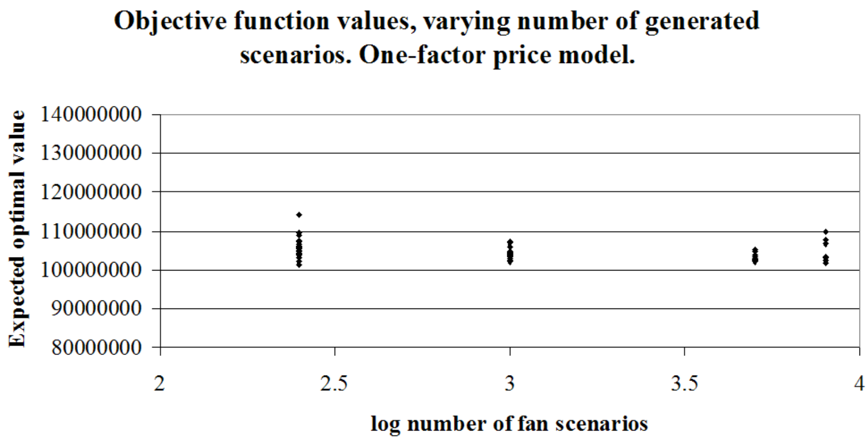

Figure 7.2: Optimal objective value by running the model with different numbers of generated fan scenarios.

Taking variance and the time consumed running the model for different number of generated fan scenarios into account, it is chosen to generate 1000 scenarios when testing the model forward in time. An average of the optimal value from the twenty runs is used in the analysis. The case closest to the average value is regarded when describing reservoir level and production.

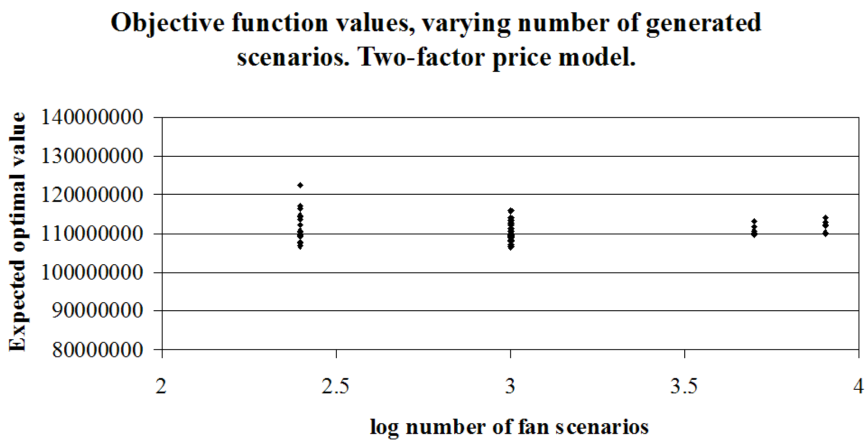

Figure 7.3: Optimal objective value by running the model with different numbers of generated fan scenarios.

7.1.2 Value of stochastic solution

The value of a stochastic compared to a deterministic model is found by comparing the optimal values generated by the two models. To achieve a fair comparison between a stochastic and a deterministic solution, the deterministic case has to be solved in all nodes of the event tree. Since the scenario trees generated here have about 500 nodes, this is a time consuming task. The main goal here is to compare different power plants. To be able to compare the six power plants, the deterministic and stochastic optimal values at the root nodes are found. Since the comparison is not fair, as explained in chapter 3.4, the real values are less. Owing to the facts that the same simplifications are done for all power plants, and only power plant specific differences exist, the percentage increase will show for which power plants a stochastic solution has most value. A two-factor price model is used when calculating the stochastic solution forward in time. The forward curve is used as the deterministic price. For the stochastic case, the model is run 20 times. The average increase in value between the stochastic and the deterministic case and the inflow standard deviation divided by the expected inflow of each power plant are given in table 7.4.

The values in table 7.4 contains little information besides comparing the power plants. Looking at the table, it can be seen that the power plants with largest inflow variation divided by the mean also have the largest value of a stochastic solution. This result is not surprising, since the advantage of the stochastic solution is that it takes variation into account when planning the production. Power plants with less variation compared to the expected

Table 7.4: Percentage expected income increase from deterministic to stochastic model, root node solutions, and inflow standard deviation divided by expected inflow for each power plant.

Value of a stochastic solution		Inflow: σ / μ
Power plant 1	27 %	1,09
Power plant 2	20 %	0,84
Power plant 3	29 %	1,36
Power plant 4	37 %	1,52
Power plant 5	28 %	1,33
Power plant 6	22 %	0,92

value will therefore benefit less of including variation. Degree of regulation, utilization time and the power plant capacity do not seem to affect the value of a stochastic solution.

Value of stochastic solution is further discussed in chapter 7.2.3, by comparing a stochastic and a deterministic model run backward in time.

7.1.3 Value of two-factor model

As explained in chapter 3.4, the value of a two-factor compared to a one-factor price model can be found by comparing the optimal values generated using the two models. Table 7.5 shows the outcome of comparing the average optimal solution of twenty model runs and a lower bound of the value. The lower bound is found by assuming that both one- and two-factor optimal values are student t-distributed and using equation (3.23). A level of significance equal one % is set, meaning that it is only one % likely that the true value is lower than the lower value set, given that the assumptions made are correct.

The two-factor price model gives on average from 6,8 to 9,7 % increase in optimal value. The objective value of power plant 2 and 6 increase most by using a two-factor price model. These plants have the least inflow standard deviation divided by the expected inflow and have least value of a stochastic solution. The fact that a two-factor model have most value for these power plants is surprising, since the advantage of the two-factor model is that it gives a better representation of the price uncertainty.

Power plant 2 has the smallest degree of regulation. A small degree of regulation means that the reservoir is small compared to the annual inflow so the storing capacity is low. In this case the reservoir has only capacity

Table 7.5: Percentage increase from one-factor to two-factor price model, the ratio between the seasonal dependency factor γ, the inflow average μ and the utilization time for each power plant.

Value of a two-factor price model			**Inflow**	
Power plant	**Average**	**Lower bound**	γ / μ	**Utilization time**
Power plant 1	7,1 %	5,5 %	0,89	4250
Power plant 2	9,7 %	8,5 %	0,20	1620
Power plant 3	6,8 %	5,7 %	1,37	2190
Power plant 4	7,8 %	6,4 %	0,86	1630
Power plant 5	6,0 %	4,8 %	1,32	5200
Power plant 6	9,1 %	7,7 %	0,28	4140

to store 37 % of the annual inflow. Power plant 5 has the largest degree of regulation, and is also the power plant the two-factor model gives least increase in value. This is also an unexpected result, since the power plant with large degree of regulation has more opportunity to exploit variations in the spot price.

The reason for these unexpected results can be explained by other power plant properties. A better representation of the price has most value for power plants with low seasonal dependent inflow. This can be seen in table 7.5 from the ratio between γ and μ. Power plants with seasonal independent inflow appears to have more opportunity to exploit information of price variations, since the consumption of water is more flexible when water continuously inflows.

The table also shows the utilization time for each power plant, which represents another kind of flexibility, as explained in section 2.1.1. Power plant 2 has the lowest and number 5 the highest utilization time. Apart from power plant 6, the power plants with low utilization times have more value of a two-factor price model than the power plants with high utilization times. The advantage of a two-factor price model is that it gives information of long-term changes in the spot price. A low utilization time means that the reservoir can be emptied within a short period of time. Power plants with this possibility will depend on information of long-term changes in the spot price to find the optimal production strategy. A high utilization time means that the power plant do not have as much flexibility in deciding when to discharge the stored water, since it will take a long time to empty it. These power plants will therefore have less value of information of long-term changes in the spot price.

The lower bound gives the value which the value of a two-factor price model is higher than with 99 % certainty. It is given as a percentage increase from the average optimal objective value using a one-factor price model and varies between 5,5 and 8,5 %, high increases in value.

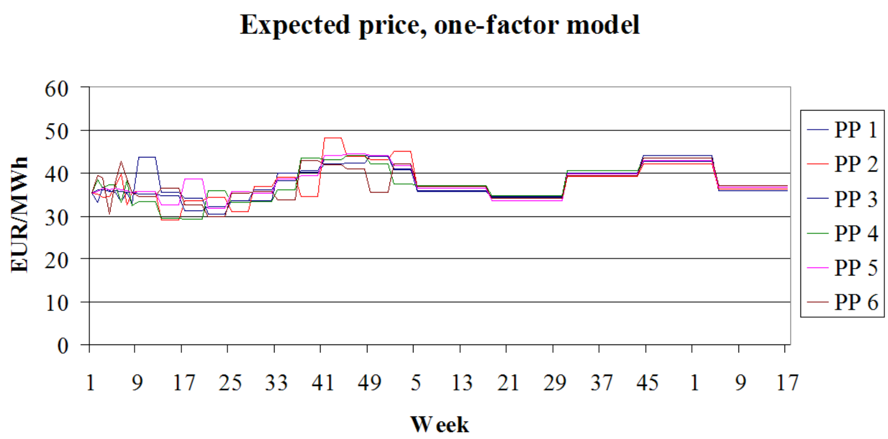

Figure 7.4: Expected spotprice for the one-factor price model from the scenario tree for all six power plants.

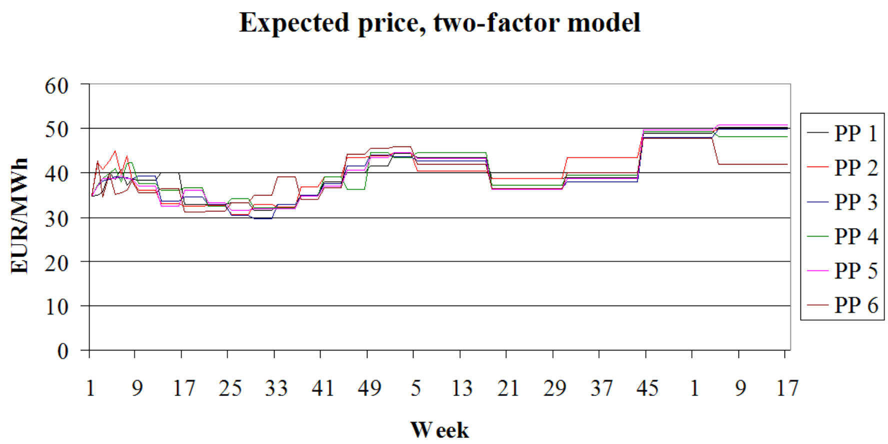

Figure 7.5: Expected spotprice for the two-factor price model from the scenario tree for all six power plants.

To confirm that these results not are affected by dissimilar price expectations, the expectation and variation of the one-factor and two-factor price models are compared. The theoretical expectations are shown in chapter 4.2. Figure 7.4 and 7.5 shows the expected price given by the scenario tree for all six power plants, using a one-factor and two-factor price model, respectively. The expectations varies between the power plants, but are quite similar for the one- and two-factor model. They are adjusted towards the same forward curve. However, the long-term part in the two-factor model gives a price increase over time. The two-factor price expectation is lower at the beginning of the planning period and higher at the end compared to the one-factor price expectation. Therefore, the value of a two-factor model should not be much affected by dissimilar price expectations.

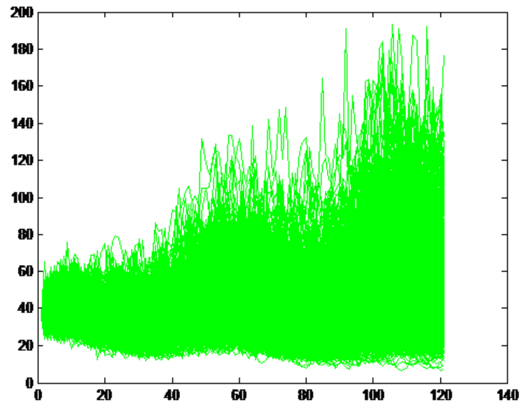

Figure 7.6: Price fan scenarios generated when using a two-factor model.

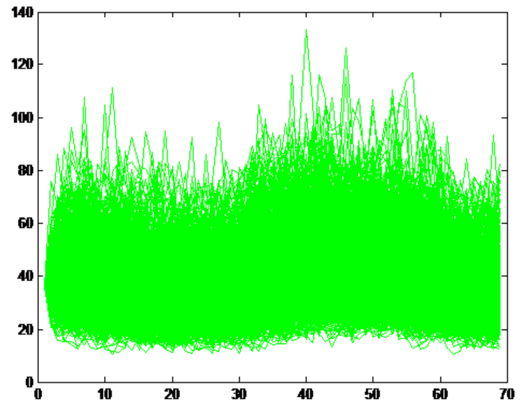

Figure 7.7: Price fan scenarios generated when using a one-factor model.

A two-factor model will represent uncertainty better since it includes two stochastic terms expressing long-term and short-term variations. Even so, the two models describe the same variation. The size of the parameters, attached in appendix 2, show that the random variables generated when scenarios are found are similar for the two models. In the one-factor model, the ln spot price depends on a random variable with standard deviation 0,1907, whereas the two-factor model depends on two random variables with standard deviation 0,1629 and 0,0419, with correlation -0,6. I.e. the models have equivalent variations. Scenarios generated are sketched in figure 7.6 and 7.7.

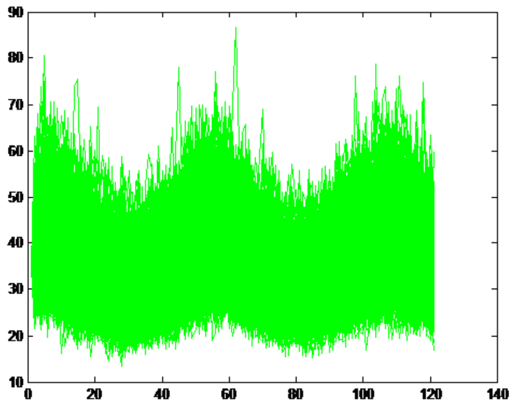

Figure 7.8: Price fan scenarios generated when using a two-factor model simplified to a one-factor.

The difference between the one-factor and two-factor model is one stochastic variable. By removing the stochastic term ξ from the two-factor model, a one-factor model remains. This one-factor model will represent the price worse than both the original two-factor model and the estimated one-factor model. To test if the increase in value between the two price models is true, the optimal objective value given by the simplified two-factor price model is found. For all six power plants, this gives a lower value than both the original one- and two factor model. Values are cited in appendix 3. Price scenarios generated when using this simplification are shown in figure 7.8.

7.1.4 Scenario example

A scenario tree like the one shown in figure 7.1 describes possible future inflow and price states. Given this, an optimal production strategy is then found. To show how varying the scenarios can be, two examples for power plant 3 will be described. The two-factor price model is used. Inflow, price and recommended reservoir level for both scenarios are shown in figure 7.9, 7.10 and 7.11.

When looking at these two scenarios it is important to keep in mind that the scenarios are not assumed to be deterministic when the optimal production plan is generated. Several states are possible in the future from each of the nodes in the scenario tree. The optimal production strategy given these possible future outcomes are generated in each node.

Inflow in the two scenarios are not that different, but the prices are very dissimilar and the optimal production level considerably different. This shows to which extent the optimal reservoir level is affected by possible future outcomes of price and inflow.

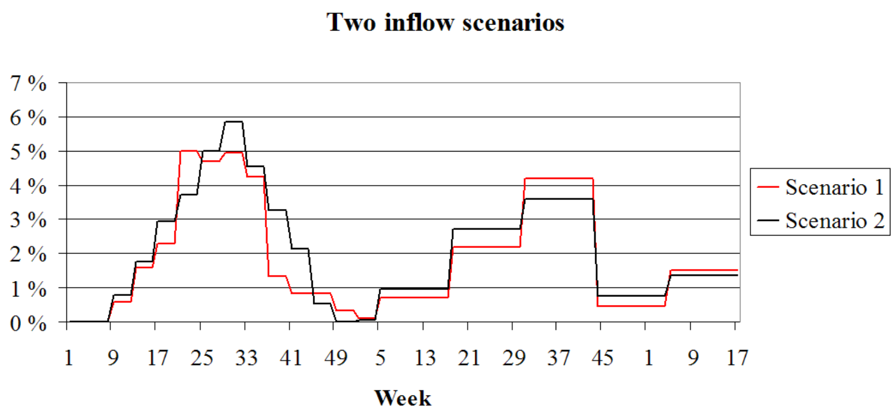

Figure 7.9: Two inflow scenarios for power plant 3

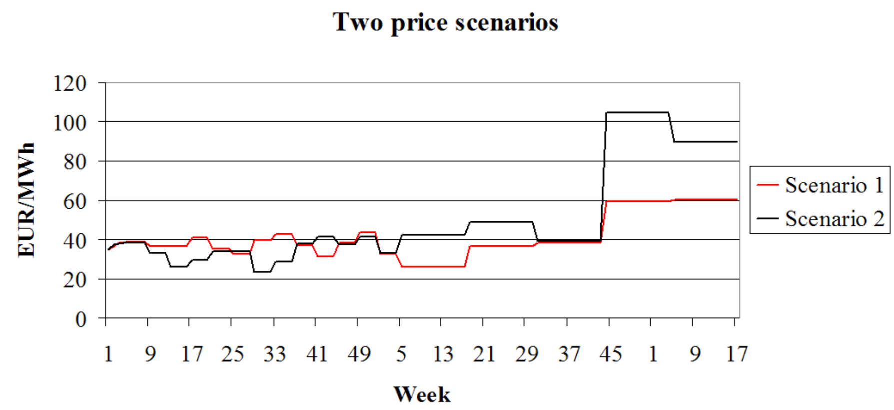

Figure 7.10: Two spot price scenarios for power plant 3.

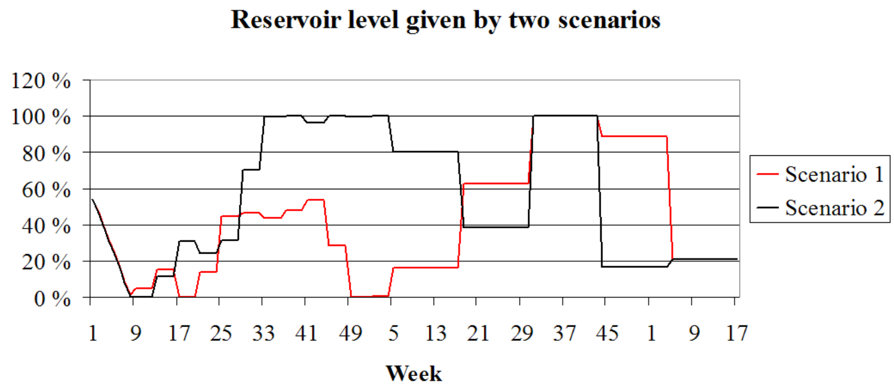

Figure 7.11: Optimal reservoir level for the two scenarios for power plant 3.

7.2 Backtesting the Model

In reality the long term planning model will be run repeatedly whenever new information, like weather forecasts and forward prices are known. To be able to compare the optimal production strategy given by the model to the actual strategy carried out by the power producers, the model is run for six power plants every fourth week over a period of time. The information available at each point in time is taken into consideration. Since the inflow model is based on historical data and the fact that information from one single year have almost no effect on the model parameters, the same models which are used when analysing forward in time are also applied when back-testing the model. The spot price fluctuates much more than inflow. Information about the future is also available through the forward price. A new price model is therefore found for each time the model is run. Due to problems calculating the parameters in the two-factor model, one-factor price models are employed. Chapter 4 explains how model parameters are found and calibrated.

The reservoir level and production the model recommends are compared to the actual situation in six different power plants. To find the scenario closest to what happened, the absolute value of the deviation between actual value and scenario value for both inflow and price are found. Since the deviation is a lot higher in value for inflow compared to the price, the deviations for each week are divided by the average value for the specific week the last six years for both price and inflow. The scenario with least total deviation rate decides what to produce the next four weeks. It is important to keep the earlier mentioned shortcomings and assumptions in mind when reading this section.

7.2.1 Backtesting the model for year 2006

The actual reservoir level in week 52 2005 is used as the initial level for every power plant. April 30th 2008 is set as the end of the planning horizon. In this way the planning period always ends in the spring and a constant minimum ending reservoir level is used. This level is set according to average level as in the previous chapter.

250 scenarios are first generated and then reduced to a scenario tree. For the cases tested here this gives from four to eight scenarios the first four weeks. Four possible scenarios are few, especially considering that there are two stochastic variables expressed. More scenarios would give scenarios closer to the actual incident and thereby a better strategy. By increasing the number of scenarios initially generated or decreasing the reduction to the scenario tree, more scenarios the first weeks can be evaluated in the optimization model. However, since the uncertainty increases forward in time and a scenario tree is made, the number of nodes has to increase considerably to obtain more scenarios the first four weeks. It is rather prioritized to run the model for several power plants, even with few scenarios, to explore for which type of power plant the model is best functioning.

In 2006 the spot price did not follow the normal fluctuations. The price had a peak in the fall and an abnormal high level throughout the year. This can be seen in figure 7.12.

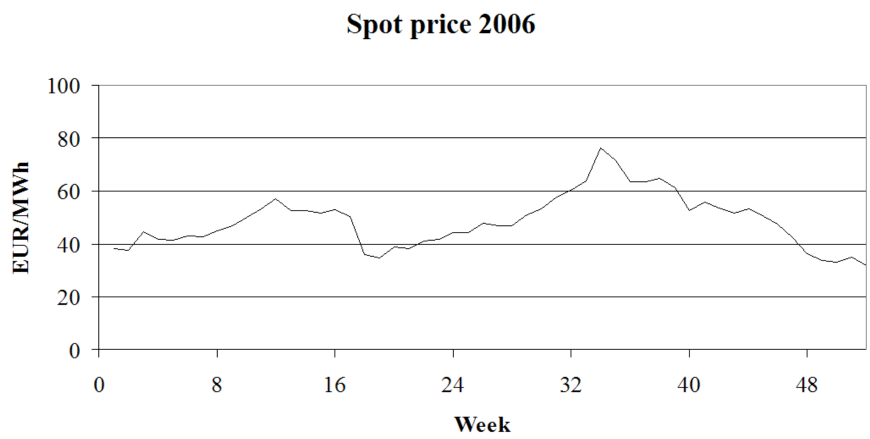

Figure 7.12: Average weekly spot price for 2006.

Reservoir level 2006
Reservoir level for all six power plants for the year 2006 is sketched in figure 7.13 to figure 7.18. The actual level and the model recommendations are shown.

Mutually for all power plants, the reservoir is first reduced towards the spring. The model recommendation follows the actual reservoir level fairly well. After reaching the lowest reservoir level, all reservoirs starts to fill up, similar to the actual development. In the fall and early winter the model recommends to produce more than what was actually done for all six power plants.

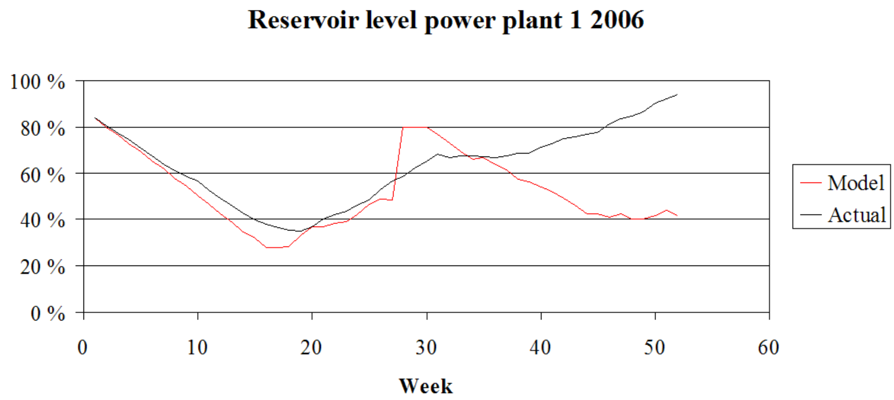

Figure 7.13: Actual reservoir level and the model recommendation for power plant 1 when the model is run every fourth week in 2006.

When optimizing, a lowest level of the final reservoir is set one year and four months forward in time from the ending reservoir in this analysis. This can affect the outcome for power plants with high degree of regulation, power plant 1 and 5 in this analysis.

Power plant 1 has a degree of regulation of 1,22, which gives great possibility to vary the reservoir. In reality the power plant had almost no production after week 17. The reservoir was filled. This is surprising since the price was abnormally high this year.

Power plant 2 is first emptied, then refilled. From week 35 the model recommends to produce at full capacity, due to high prices. At the end of the year it is recommended to produce less and start refilling the reservoir. It is possible to decrease and increase the reservoir quickly because it has inflow throughout the year and low utilization time. In reality a nearly constant

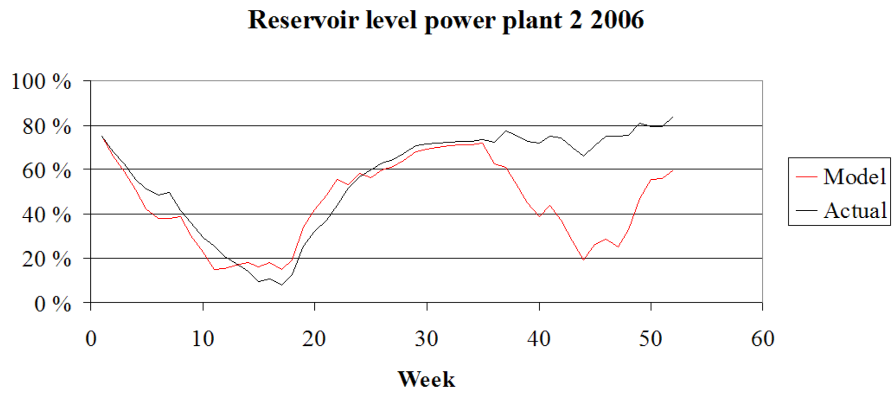

Figure 7.14: Actual reservoir level and the model recommendation for power plant 2 when the model is run every fourth week in 2006.

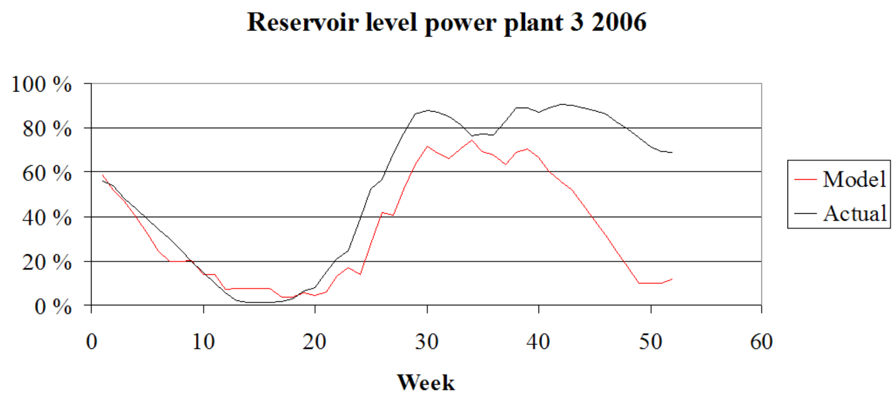

Figure 7.15: Actual reservoir level and the model recommendation for power plant 3 when the model is run every fourth week in 2006.

reservoir level was maintained from week 35 and throughout the year.

Power plant 3 and 5 do not produce anything the last four weeks, according to the model, when the price is low. Since the inflow is almost zero, the reservoir level keeps stable. There is full production during the fall price peak for both these power plants.

Due to less inflow than expected, power plant 4 has nearly no production during the beginning of the price peak. As for the other power plants, the reservoir level is reduced from week 35.

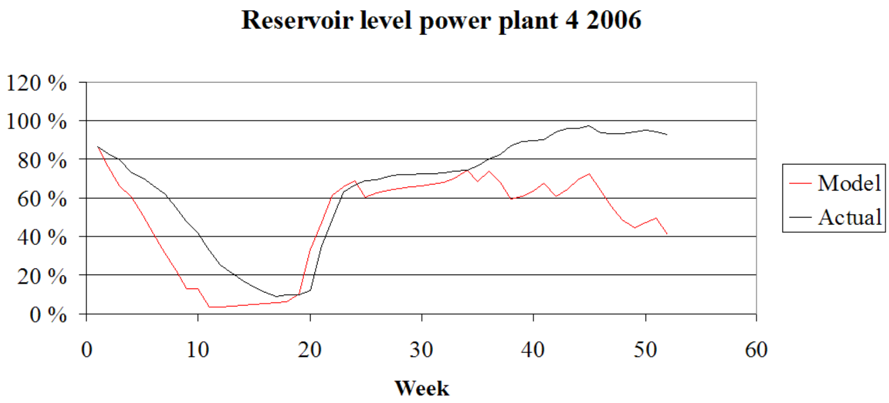

Figure 7.16: Actual reservoir level and the model recommendation for power plant 4 when the model is run every fourth week in 2006.

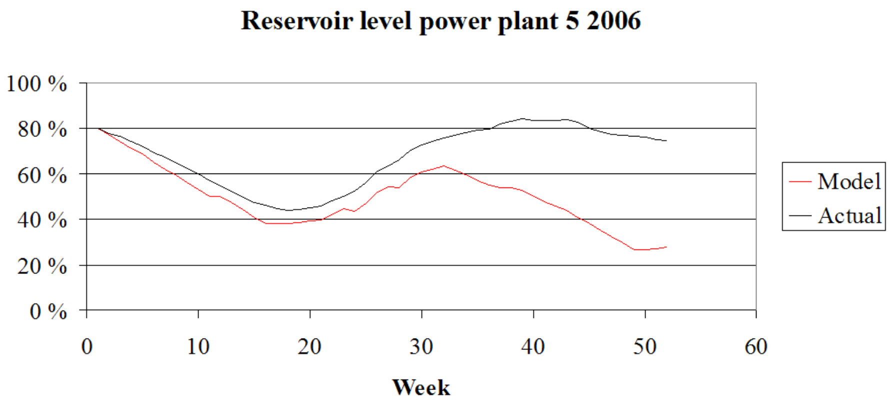

Figure 7.17: Actual reservoir level and the model recommendation for power plant 5 when the model is run every fourth week in 2006.

The power plant with the largest production capacity of these six, power plant 6, produces during the whole year both in reality and in the model recommendation. The real production is smoother than the max-min production strategy the model recommends. During some weeks in the fall the model recommends to consume water, but in reality it was stored.

The reason why the model and the actual production plan is so different for each of the six power plants can be found in forward prices and the price model expectations. Different production strategies are in particular found the four last times the model is run, week 36, 40, 44 and 48. The forward prices seen at each of these points in time are displayed in figure 7.19.

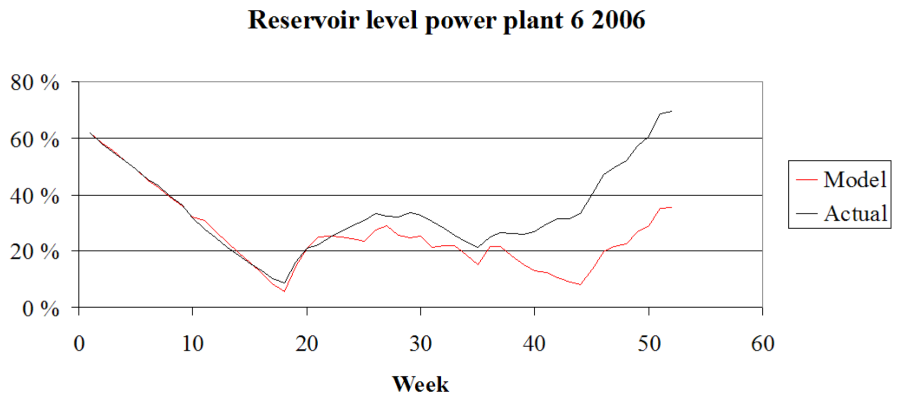

Figure 7.18: Actual reservoir level and the model recommendation for power plant 6 when the model is run every fourth week in 2006.

Forward prices during the price peak period show that high prices are expected to persist throughout the winter, subsequently normalizing at a higher level than earlier. This explains why producers save the stored water instead of consuming it during what proved to be a price peak.

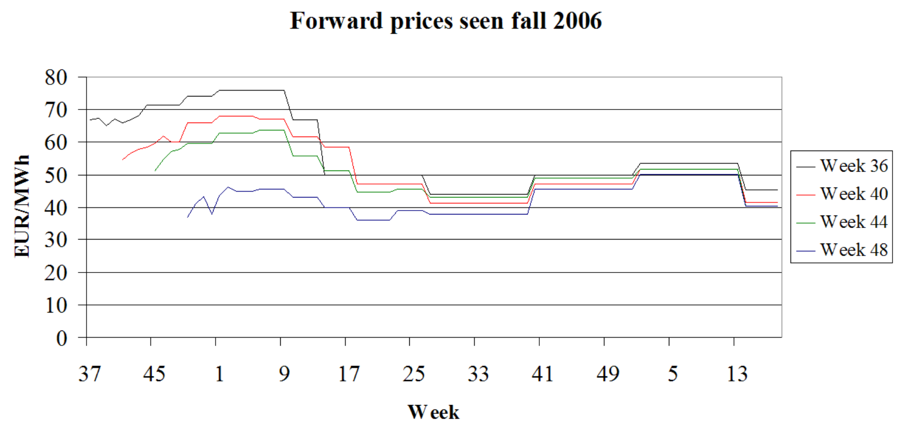

Figure 7.19: Forward prices seen in week 36, 40, 44 and 48 2006. These are extracted from figure 4.2.

Since the price model is calibrated towards the forward price at each point in time, it is surprising to see that the model advises to reduce the reservoir level at all of the power plants during the fall. To understand this, the price model has to be evaluated. In figure 7.20 the forward price in week 40 and the price model previous and after calibration are sketched. As the figure shows, the price model does not capture long term changes expected by the forward prices. Based on historical prices, the model expects the price to be lower than the forward price during the winter. Similar price models compared to forward prices are generated for week 36 and 44, leading to the reservoir levels displayed in the figures above.

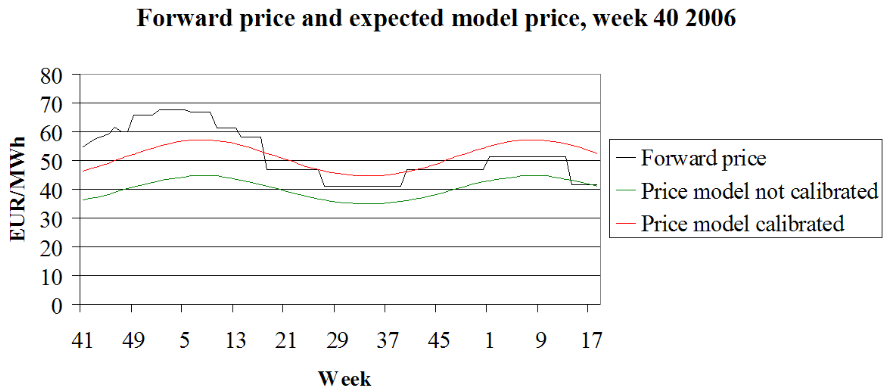

Figure 7.20: Forward price and the expected price given by the one-factor price model seen in week 40 2006.

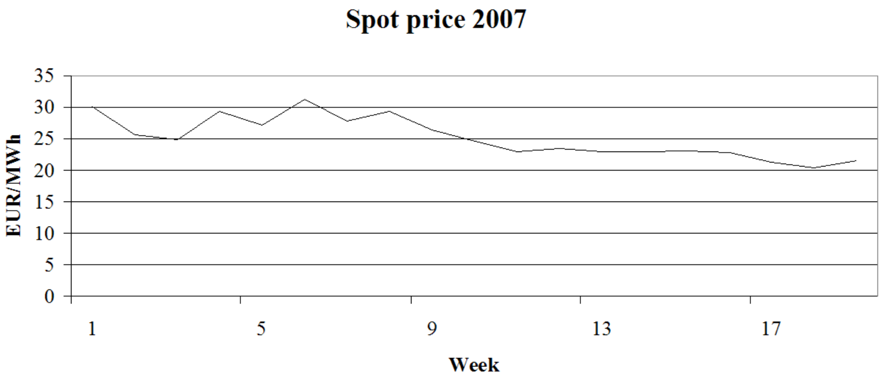

Figure 7.21: Average weekly spot price the first 19 weeks of 2007.

To be able to say if the modeled or the actual production plan is the better, reservoir level and production plan until spring 2007 should be compared. Only information until year 2006 is available, so this is not possible. Yet the actual spot price can be found. It is sketched in figure 7.21. Even with spot prices only known for the first 19 weeks at present, this still gives a basis for valuing the strategic decision made by the producers and the recommendation of this model. Due to spring flood it is the available prices that are of importance.

The actual price in the first weeks of 2007 is lower than any of the observed forward prices from 2006 predicted. All weekly prices are also lower than any of the weekly prices in 2006. As a result of this, the producers would benefit from consuming water during fall 2006 instead of saving it for later that winter when prices were lower. This could not be known at the time the decision had to be made, but it shows that the one-factor models applied in this analysis are in fact better predictions of future prices than the forward prices in this particular case. This is accidental, since the intention is that the price model should reflect the forward price.

Income in 2006

Multiplying the production each week by the average weekly price and the discounting factor, the total discounted income for 2006 is found. This is done for each of the six power plants for both the actual production and the production recommended by the model. The model production resulted in a higher income in all six cases, but also a higher total production. This is possible due to the fact that this analysis is done based on a calendar year, not a production year and that the reservoir level at the beginning and the end of the year are dissimilar. Production and income increase are shown in table 7.6.

Table 7.6: Production and income increase between the reality and the model recommendations in 2006.

Production and income increase		
Power plant	Production increase	Income increase
Power plant 1	168,5 %	183,4 %
Power plant 2	14,3 %	24,0 %
Power plant 3	37,0 %	37,3 %
Power plant 4	42,1 %	41,8 %
Power plant 5	67,7 %	79,2 %
Power plant 6	27,8 %	29,1 %

To be able to compare the two production alternatives, the average price for the production is found. In this way a comparison can be done without considering the reservoir level at the end of the year. As can be seen in table 7.7, the model gives a higher average price for all power plants except number 3 and 4, where the average prices are equal. Power plant 4 did not produce during the beginning of the price peak due to less inflow than expected. This can imply that inflow should have less and the price more effect on what scenario to choose than the 50/50 weight applied. The actual production of power plant 3 is well distributed over the period and it produces much during periods with high prices.

Table 7.7: Average price for the production in 2006, in reality and recommended by the model.

Average price in 2006			
Power plant	Actual	This model	Percentage increase
Power plant 1	47,2	49,8	5,7 %
Power plant 2	44,4	48,1	8,3 %
Power plant 3	49,4	49,3	-0,2 %
Power plant 4	45,8	45,8	0,0 %
Power plant 5	47,3	50,6	7,0 %
Power plant 6	46,9	47,4	1,1 %

The average prices imply that the model gives a good production strategy and an improved allocation of the water over the year than the actual case. By running more scenarios, both initially and in the scenario tree, the model should give an even better solution. However it is important to remember the simplifications done when looking at these results and the fact that coincident accidental events caused these results.

To make the incomes comparable, they are adjusted for dissimilar ending reservoirs. The income given by the production based on the model recommendation is adjusted by subtracting the loss in expected future income due to a lower ending reservoir. This adjusted income is then comparable to the actual income in 2006. The model is run forward in time from the end of 2006 for both actual and recommended ending reservoir for power plant 1 and 2. These are the power plants with least and most difference between actual and recommended total production during 2006. For power plant 1 and 2 the adjusted incomes are almost equal to the actual income.

The outcome of this analysis is highly dependent on the price during the comparison period and the expected future price. The fact that the adjusted recommended and actual income is approximately equal for power plant 1 and 2 makes sense, since the forward price at this point in time is quite equal the price during the period compared.

7.2.2 Backtesting the model from spring 2005 to spring 2006

Due to abnormal inflow and price developments in 2006, it is chosen to study how the one-factor model performs in a more normal year. The model is run for a production year, rather than a calendar year, because this is the most realistic situation for power producers. Week 17 2005 is set as the starting point of the analysis and the model is rerun each fourth week until week 16 2006. The spot price development for the test period is displayed in figure 7.22. For 2005, the spot price varies between about 22 and up to 35 EUR/MWh. This is more normally than spot prices approaching 80 EUR/MWh, observed in 2006. The curvature of the spot price in 2005 is also reasonable, with prices decreasing towards summer, subsequently increasing during the fall and winter. However, in the beginning of 2006 the spot price rise abruptly, ending at a peak of 56,8 EUR/MWh in week 12.

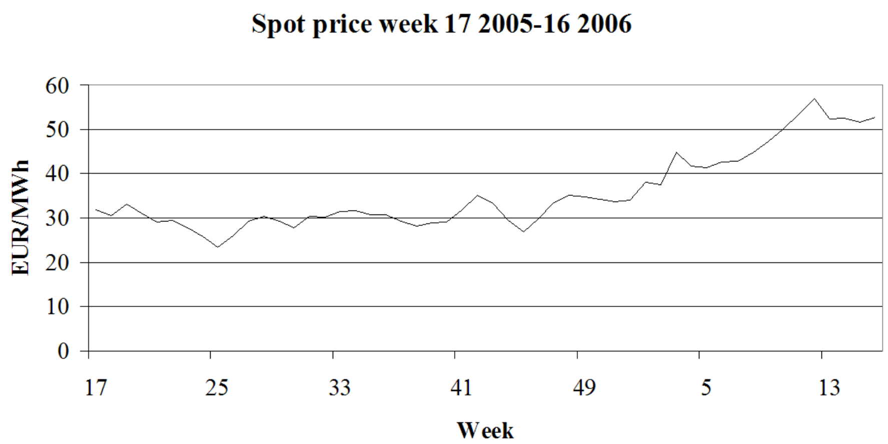

Figure 7.22: Average weekly spot price in the period week 17 2005 - week 16 2006.

The optimal model production strategy for the six power plants are compared to the actual production in 2005. This is done in the same manner as explained in section 7.2.1. The inflow for the test period are foreseen using the same models as when analysing forward in time. New price model parameters are found at each time-step the model is rerun, based on available spot, future and forward price information at that point of time. The actual reservoir level in week 17 2005 is used as initial reservoir level for each power plant. The planning horizon ends at April 29, 2007. The number of fan scenarios generated is increased to 1000, which reduces to about 500 when the scenario tree is found. This gives four to eight scenarios the first four weeks for the six plants studied.

Reservoir level spring 2005 - spring 2006

Actual and modeled reservoir levels for the six power plants during the planning period are shown in figure 7.23 to figure 7.28. All reservoirs are recharged throughout the spring and summer months. So far, the modeled reservoirs seems to follow the actual reservoirs movements rather well. The reservoirs are all pretty high and stable from week 33 to 49. Subsequently, the reservoirs are discharged.

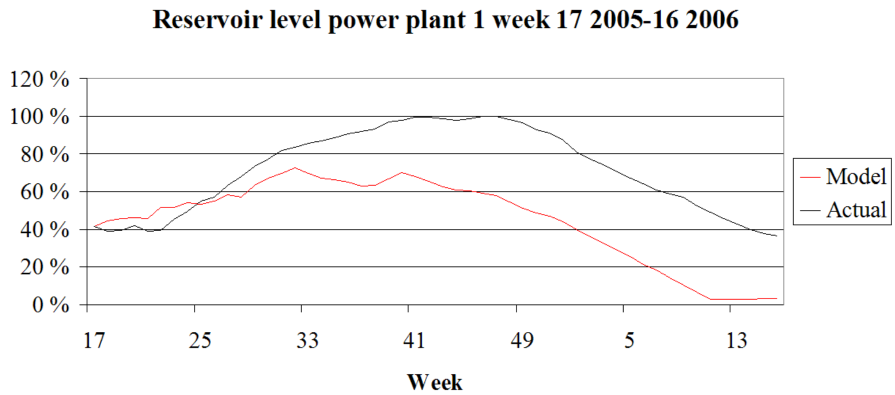

Figure 7.23: Actual reservoir level and the model recommendation for power plant 1 when the model is run every fourth week in in the period week 17 2005 - week 16 2006

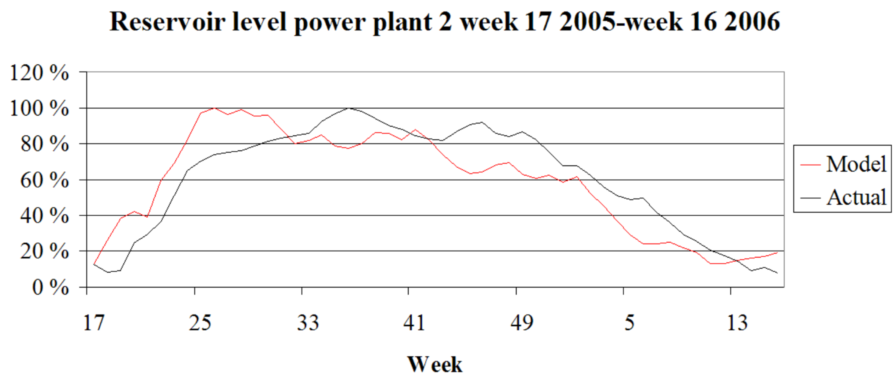

Figure 7.24: Actual reservoir level and the model recommendation for power plant 2 when the model is run every fourth week in the period week 17 2005 - 16 2006

As explained in section 7.2.1, outcomes for power plants with large degree of regulation can be affected by the minimum final reservoir, which is set at April 29th 2007. This concerns power plant 1 and 5. The model recommends to discharge these plants much more than what is actually done.

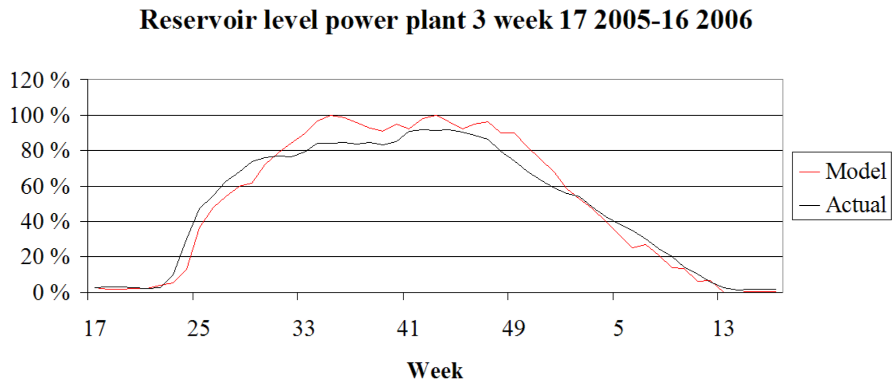

Figure 7.25: Actual reservoir level and the model recommendation for power plant 2 when the model is run every fourth week in the period week 17 2005 - 16 2006

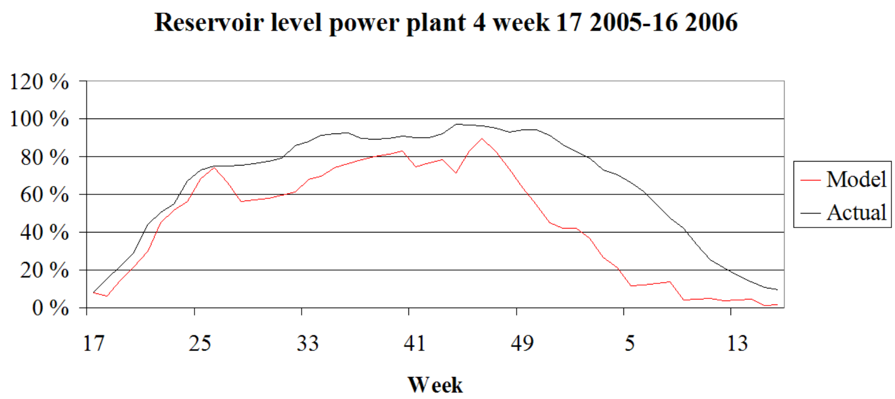

Figure 7.26: Actual reservoir level and the model recommendation for power plant 4 when the model is run every fourth week in the period week 17 2005 - 16 2006

From about week 41, the reservoir level recommendation is ca 25 % lower than the reservoir level maintained in reality for power plant 5. The difference is even higher for power plant 1, where the model recommends a reservoir level about 40 % lower than the actual reservoir level from week 44 and until the spring flood. These extensive discharges must be evaluated in context with price expectations. Figure 7.29 shows price expectations observed during the fall 2005 and spring 2006. The price level is expected to decline, so reservoir discharging makes sense.

Power plant 2, 3, 4 and 6 have model recommendations more similar to the

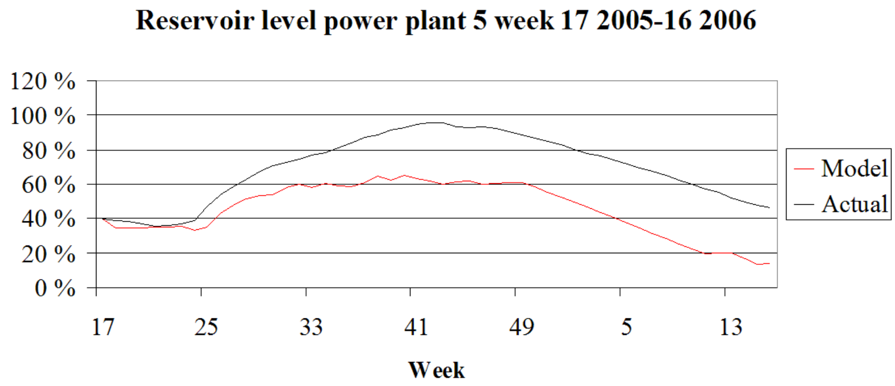

Figure 7.27: Actual reservoir level and the model recommendation for power plant 5 when the model is run every fourth week in the period week 17 2005 - 16 2006

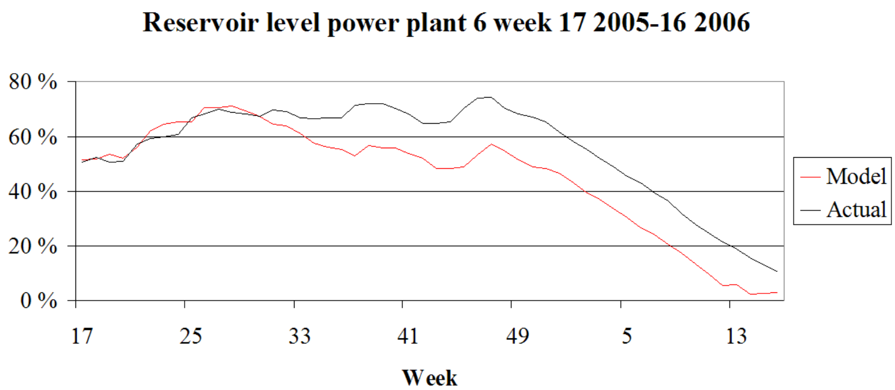

Figure 7.28: Actual reservoir level and the model recommendation for power plant 6 when the model is run every fourth week in the period week 17 2005 - 16 2006

actual reservoir level. Even so, the recommended reservoir levels tend to be lower than the actual reservoir levels for these plants as well. However, in week 16 2006 both the recommended and the actual reservoir levels end up quite close to each other for these plants, they are almost emptied. This is because the water value is close to zero throughout the period spring flood is expected to occur for power plants with low degree of regulation.

The reservoir level of power plant 6 is 50 % at the starting point, which is much higher than the reservoir levels of the other plants with regulation degree less than one. Power plant 6 has the least seasonal dependent inflow

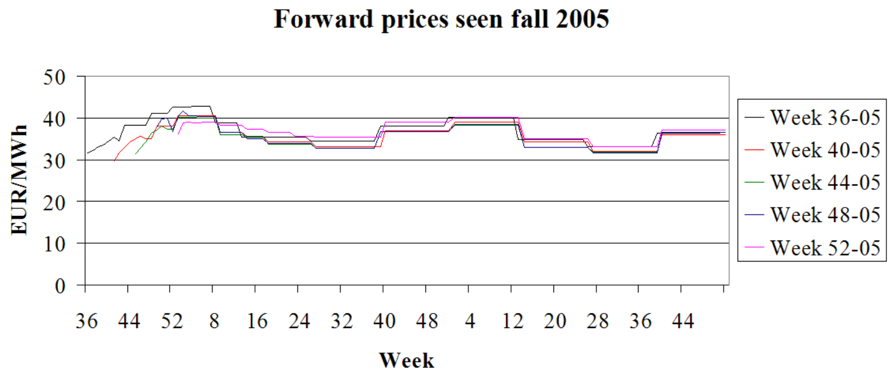

Figure 7.29: Forward prices seen in week 36, 40, 44, 48 and 52 2005. These are extracted from figure 4.2.

among the plants studied, so the reservoir could to some extent be recharged throughout the entire year if it is desirable. The modeled reservoir is totally empty at the ending point of the test period, whereas the actual reservoir never is below 10 %.

The modeled reservoir in power plant 3 follows the actual reservoir very closely, and it is the only plant where the model recommends to store more water than what is actually done. This plant has the most seasonal dependent inflow which gives less flexibility. This may contribute to the very similar production strategies experienced here.

Income in spring 2005 - spring 2006
The total discounted incomes for the period spring 2005 - spring 2006 are found in the same manner as for 2006 for the six power plants. Both actual production and production recommended by the model are also found. The percentage changes in production and income are displayed in figure 7.8. In most cases, the production increases. The incomes increase as well, but not quite as much as the productions. For power plant 3, both production and income are slightly reduced. For power plant 4, a production increase is recommended by the model. Even so, the income decreases more than the production increase.

The model suggests to increase the production in power plant 1 and 5 by more than 35 %. This is possible because these plants have large reservoirs and high degrees of regulation. Power plant 2 and 6 are recommended to produce somewhat more than what is actually done. The modeled and actual reservoirs ends up at the same level at the ending point for these plants, so the production increase recommended by the model is probably due to

Table 7.8: Production and income increase between actual and this model in the period week 17 2005 - 16 2006

Production and income increase week 17 2005 - 16 2006		
Power plant	Production increase	Income increase
Power plant 1	38,6 %	22,7 %
Power plant 2	10,9 %	6,0 %
Power plant 3	-2,4 %	-4,1 %
Power plant 4	5,3 %	-5,8 %
Power plant 5	36,3 %	21,1 %
Power plant 6	5,6 %	1,8 %

the constant efficiency simplification.

The average price for the actual and modeled production is shown in figure 7.9. When comparing the average price for the actual and modeled production, it is observed that these values are pretty similar. If the modeled production plan was applied, the average price for the production would decrease for all six power plants. Due to all simplifications done in the model, and the fact that the model only is run every fourth week, it is worth noticing that the recommended strategies end up so closely to the real production strategies.

Table 7.9: Average price for the production in the period week 17 2005 - 16 2006, actual and given by this model

Average price in week 17.05 - 16.06			
Power plant	Actual	This model	Percentage change
Power plant 1	37,5	33,2	-11,5 %
Power plant 2	34,6	33,0	-4,4 %
Power plant 3	34,4	33,8	-1,8 %
Power plant 4	38,5	34,4	-10,6 %
Power plant 5	39,7	35,3	-11,1 %
Power plant 6	34,6	33,4	-3,6 %

7.2.3 Value of stochastic solution

To compare the stochastic and deterministic model in a fair way, both are run every fourth week from spring 2005 until spring 2006 for power plant 3 and 4. By summing up the optimal expected income for the first four weeks the 13 times the model is run, the total expected value of the deterministic model is actually better than the total expected value of the stochastic model for both power plants. This looks like an error, but comparing the two cases it can be seen that the expected production given by the 13 runs is very different. The stochastic model has a higher expected production in the spring and summer and lower in the winter compared to the deterministic model. The risk of overflow causes this. The deterministic model has a certain reservoir inflow and the power producers can therefore produce less in time periods with low expected price and more in periods with high expected price.

The deterministic model does of course not reflect the true situation. In reality uncertainty is a fact and allowing for this will increase expected income. Every time the two models are run, the stochastic model has a larger value for the total time period, but not necessarily for the first four weeks. This method of comparing a stochastic towards a deterministic model does not work. Running the deterministic model in every node in the event tree of the stochastic modellation is the best method of comparing the two models. However, as earlier mentioned, this is time consuming for a event tree with numerous nodes and it is therefore not done.

Chapter 8

Conclusion

A long-term planning optimization model is run for six different power plants both forward and backward in time. This is a deterministic equivalent of a stochastic model, taking the uncertainty of hydrologic inflow and electricity price into account.

The results from seeing forward in time are used to compare the six power plants regarding the value of stochastic solution and the value of a two-factor compared to a one-factor price model. Power plants with most volatility in hydrologic inflow have highest value of a stochastic solution. This makes sense, since the advantage of the stochastic model is that it regards the uncertainty of the stochastic variables.

A two-factor price model represents price uncertainty better than a one-factor model, by including a stochastic term for long-term changes. Surprisingly, the two power plants with most value of a stochastic solution have least value of a two-factor model. The expected optimal value of the production seeing two years and four months ahead, increases between 6,0 and 9,7 % when applying a two-factor model. The optimal value for power plants with low seasonal dependent inflow increase more in value than power plants with high seasonal dependent inflow. A more continuous inflow gives more flexibility to exploit price information, for example during the winter when inflow for power plants with highly seasonal dependent inflow is close to zero.

Other power plant properties affecting the value of a two-factor model are the degree of regulation and the utilization time. Power plants with high degree of regulation are less affected by a two-factor model than power plants with low degree of regulation. Except for power plant 6, the optimal value increase more for power plants with low utilization time compared to a higher utilization time when a two-factor model is used. A short utilization time means that the power plant has high power capacity compared to the reservoir volume and great possibility to discharge the stored water over a short period of time. Information of long-term changes in the spot price is therefore more important.

From the back-testing it is observed that the price expectation is of great importance for the production strategy. How well the model follows the forward price and if the forward prices seen at each decision point reflects the true spot prices are important aspects when it comes to how well functioning the model is.

For the year 2006 this model recommends to discharge more of the stored water compared to what was actually done. This turned out to be a wise decision because the price was consistently lower during the first four months of 2007 compared to the price in 2006. The forward price predicted that the price would stay high, something the one-factor model applied here did not capture.

A back-test for the period spring 2005 to spring 2006 is also performed. Reservoir levels recommended by the model is much similar to the actual levels, except for power plant 2 and 5. These plants have high degree of regulation and can therefore decrease the reservoir level. The model recommendation would give a production plan worse than the actual production plan for all power plants this period. This is expected since the model is run only every fourth week, having few scenarios the first weeks and no possibility to change the production if unexpected happenings occurred.

The results show that maximizing total expected future value of the production and taking the uncertainty of price and inflow into account gives an applicable production strategy if the stochastic parameters are modeled well. Applying a price model including long-term changes is valuable in this context.

Chapter 9

Further Work

There are many aspects that can be further developed.

- Information is available for seven other power plants. The optimization model can be tested for those and statistical methods can be applied when analysing the outcomes.
- Available information can be used to test both this optimization model over a longer period of time, as well as other hydro power planning models. Other stochastic models describing price and inflow can also be applied.
- The value of a two-factor model can be further analysed. A more stable Kalman filter should be developed in order to give a better evaluation of the estimated parameters.
- Water values can be further examined by comparing the values given by this model to values given by other long-term planning models.
- The model can be further developed by giving a water value at the end of the planning horizon instead of a minimum reservoir level and by introducing an efficiency that is not constant.

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Appendix 1:

Program sketch and overview

The program consists of four parts. First matlab generates scenarios for the stochastic variables, using defined models. Scenred is employed to make an event tree based on the scenarios. Matlab files can then prepare the input file for Xpress, consisting of both the scenario tree and power plant specific information. Xpress runs the optimization model based on this information and gives the optimal production plan.

Figure 1 shows the first three steps and the different files applied in each step. Colors are used to distinguish files in different programs. All parameters and technical information used are gathered in one excel file.

All files and more information about each file, are enclosed on a CD.

Table 1: Color representing different computer programs and files in the program sketch

Program	**Colour**
Excel	Red
Matlab	Blue
Data file	Black

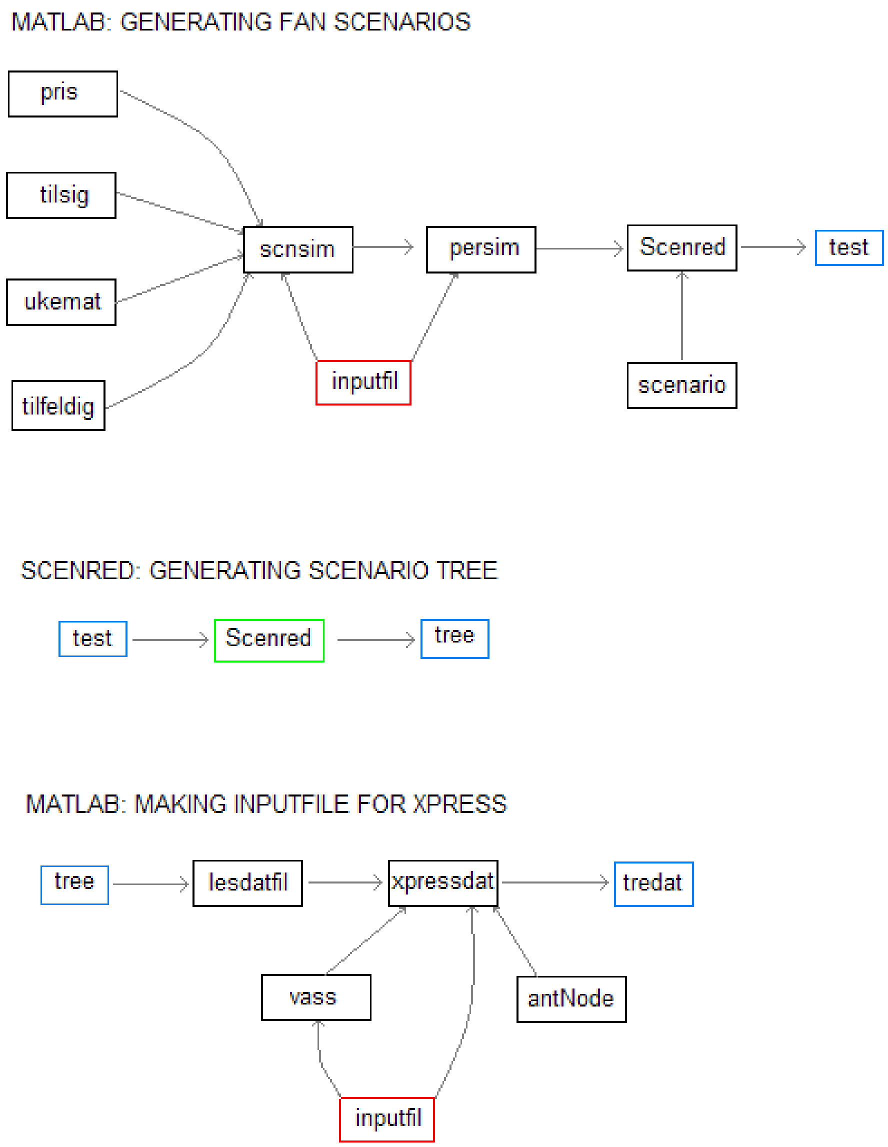

Figure 1: Program sketch

Appendix 2:

Price model parameters

Several models describing the stochastic variables are found: One inflow model for each power plant, one-factor price models for every fourth week from spring 2005 until the end of 2006 and one two-factor price model with information given January 1st 2007. Table 2 shows the parameter values of one inflow model and the two price models estimated from the information available at the end of 2006.

Table 2: Parameter values for models made with available data in week 52 2006

	Price model		**Inflow model**
Parameters	**Two-factor**	**One-factor**	**One-factor**
α	4,662	3,446	2,797
γ	0,147	0,141	2,502
τ	-0,074	-0,909	-0,555
κ	38,094	9,223	44,442
λ	43,695	-1,125	0
σ_χ	1,174	1,375	22,808
μ	0,094	0	0
σ_ξ	0,302	0	0
$\rho_{\chi,\xi}$	-0,614	0	0

Appendix 3:

Optimal values forward in time

The optimal objective values given in table 3 are the average values when running the model 20 times for each power plant. Three different price models are applied. From the results it can be seen that a two-factor price model gives better results for all power plants compared to a one-factor model. A two-factor model can be simplified into a one-factor model by deleting the stochastic term ξ. The optimal objective value will then be lower than both original models for all six power plants.

Table 3: Average optimal values given by different price models

Average optimal values, different price models			
Power plant	**Two-factor**	**One-factor**	**Two-factor as one**
Power plant 1	26972885	25190645	20555470
Power plant 2	35210195	32108585	26655790
Power plant 3	60324110	56502785	45653640
Power plant 4	127206550	118026700	95873390
Power plant 5	110556150	103955650	83741270
Power plant 6	312633650	286637250	237996700