Anvendt Økonomi og Optimering

TIØ4500

Fordypningsprosjekt

Bidding in Elbas

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Høst 2017

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Preface

This report is written as a project report in the course TIØ4500 Managerial Economics and Operations Research, Specialization Project at the Department of Industrial Economics and Technology Management at the Norwegian University of Science and Technology(NTNU).

We would like to thank our supervisors Professor Stein-Erik Fleten, Professor and Director CenSES Asgeir Tomasgard, PhD Candidate Ellen Krohn Aasgård at NTNU for their valuable guidance and helpful feedback. Their knowledge and interest in the problem have been of great importance.

We would also like to thank TrønderEnergi (TE) for providing data to perform a realistic market analysis. A special thanks is given to Gunnar Aronsen from TrønderEnergi for his guidance and way of providing insights into industrial challenges.

Trondheim, December 19^{th}, 2017

Summary

The volumes traded in the intraday market, Elbas, have been steadily increasing over the last years. An increased proportion of renewable energy sources in the energy mix require more flexibility in terms of power trades close to the production hour. To adhere to this development and maximize profits, market participants need to better understand the stochastic process of Elbas prices.

This study investigates the parameters describing this process, based on a well established assumption of correlation between spot prices and Elbas trade levels. Spot prices in one hour are strongly correlated with the price in the previous hour, making Markov a suitable process for price modelling. The main task in the study is to investigate development of a suitable transition matrix that provides a forecasted process description that is correct in expectation.

A multistage optimization problem is presented to describe the essensials concerning the decision problem that market participants face when bidding in Elbas. The problem is solved by dynamic programming, where it is limited to entail the pricing and timing question. Decision support is provided through a contingency plan, developed to describe how a specified state should result in a specific action in terms of bidding. Input parameters describing the future price process are given by the transition matrix where the Markovian property is the underlying driver.

While the optimization problem is quite trivial with the assumptions made in this study, it provides a basis for further investigation. In addition, the stochastic process of Elbas prices is investigated thoroughly, which is the main objective of this study.

The unconventional way of approaching the modelling challenge has been developed in the interface between people with academic and industrial knowledge. Challenges were described along with potential drivers and triggers, and relevant simplification were made in cooperation to avoid drifting too far away from the real challenge.

1 Introduction

Over the last few decades, the Nordic and Baltic countries have coupled their energy markets into a common, deregulated market operated by Nord Pool. Their objective is to obtain a free market between market areas to increase efficiency and liquidity, as well as to create a more secure power supply. The increasing amount of renewable energy sources, specifically the increasing share of wind power, makes the topic of modelling and investigating electricity market prices central. Moreover, M.M. Belsnes (2015) stated that: "The behaviour of the market is expected to become much more volatile due to the transition toward more renewable power production in the energy systems." Along with the increased market volatility the intraday electricity market, Elbas, has increased its importance in terms of more frequent trades and larger volumes traded (Chap: 2). The changing structure indicates the necessity of investigating new bidding strategies. It is essential for power producers to adhere to these changes to remain competitive. The main motivation for the study is therefore to be able to be ahead of market changes by providing intraday market participants with updated methods to model price processes that provides a good insight into relevant movements.

The scope of this study is to investigate how historical data can be useful to build optimal bidding strategies in Elbas that fully, or to a greater sense than today, take advantage of the market depth to increase profitability. *Is there an undiscovered willingness to trade at certain points in time, such that when discovered it can increase flexibility and remove dead weight losses?*.

As a contribution to the area of research, a stochastic model is built and discussed thoroughly, and the scope of this study is to form as well as to test and reflect upon the model's strengths and weaknesses. An important part of this study has been to understand the link between market mechanisms, by being able to consider the price processes free from externalities, with close to white noise. In addition, the stochastic model is put in context with an optimization model.

There exists a lot of research and literature on the topic, but the major contribution of this project is that firstly, it links Elbas prices to spot prices based on the correlation between them. Secondly, historical capacity data has received a lot of attention when investigating the existing unutilized potential to trade. These perspectives together with the stochastic, discrete model and the optimization program contributes to the field, in a way that has not been performed before. Due to the hole in the literature, we have extended our literature search to fields outside the electricity market to find relevant information.

Even though the gain from strategic positioning in Elbas is expectedly larger in the future, as the intraday marked becomes more volatile, it is also highly relevant today. The intraday market provides the ability to correct the current situation due to deviations from expected situations by adjusting imbalances between production and consumption. Insight into the complexity can create room for better bidding that is adapted to the relevant market structure, namely a more volatile market. The research question is elaborated on under Chapter 4, and has the following essence:

Is it possible to understand and model the mechanisms in the market, namely the market depth and price process dependencies, from historical data, in order to solve a relevant dynamic optimization program that can add value to a market participants trading portfolio?

Moreover, the purpose of this study is to shed light on the changing market structure and try to provide decision support to market participants. With data from our industrial partner: the Norwegian power producer TrønderEnergi (Sec: 2.2.1), the analysis is made from TE's perspective, but can be generalized. From TrønderEnergi's perspective a lot of the motivation deal with the ability to provide bids that fully take advantage of the market depth, or the potential to decrease costs by avoiding start and stop costs related to power production, or to obtain power balance and in general try to avoid participation in the regulating market. While they aim at balancing most of their production in advance, it may be beneficial to correct their bidding through participation in the intraday market to better take advantage of opportunities.

In order for the reader to understand the context of the problem relevant background information is introduces in Chapter 2. A reader who is already familiar with this topic can skip this section. A literature review in Chapter 3 is provided to shed the light on how the challenges that market participants face have been approached before, in addition to relevant literature from other fields. Afterwards, the problem description is elaborated on in Chapter 4, before the problem formulation in Chapter 5 provides assumptions and simplifications (Sec: 5.2.1) made followed by an in depth analysis (Sec: 5.2) to get an understanding of the characteristics of the Elbas market in order to develop a suitable model of the price process. Afterwards, an optimization problem (Sec: 5.3 is formulated to provide decision support in some of the decisions a market participant make in Elbas. The model output is illustrated by a few instances in the computational study in Chapter 6, to show how its output is utilized in a contingency plan. Lastly, a discussion along with concluding remarks are presented in Chapter 7, before an explanation of how the findings are useful for future research is provided in Chapter 8. This chapter is important considering this study creates the basis for a more comprehensive study.

2 The Power Market and Power Production

2.1 The Power Market

As any other commodity, electricity may be sold and bought. The power that is transmitted and utilized on different levels of the electricity grid - including the central, regional and distributional levels - has often gone through a process of being traded on the wholesale market, i.e. between producers and suppliers, brokers, large industrial companies and other large market agents. The Nordic and Baltic countries have *deregulated* their power markets, and *coupled* them into a common market facilitated by Nord Pool (Wangensteen, 2012). 380 companies from 20 different countries trade under Nord Pool's operation (NordPool, 2016), where market clearing, settlement and services in day-ahead and intraday markets are some of their responsibilities.

In deregulated electricity markets, the power price is determined by demand and supply. The objective is to obtain something close to a *perfect market*, which will maximize social surplus. With free flow of power between countries, the dispatch of power production at facilities with different associated costs will assure that the best price for society is obtained, across country borders. Areas with overproduction will be able to sell power to areas with deficits. If there are no transmission and distribution constraints, the power price will be equal for all participants according to basic economic theory, which is the objective of coupled markets (Wangensteen, 2012).

Integrating markets across country borders, assures a diversity in the power sources supplying the grid, which assures a better *security of supply.* Relying too much on a single or few energy sources, the supply will be highly sensitive to changes in weather conditions, fuel prices or other factors essential for that specific energy source. With a combination of energy sources and geographical placements, the total supply is less affected by single variations. In addition, an increased number of market participants makes the market closer to a perfect one as the market clearing gets more efficient. According to NordPool (2017a), the development over the last years has shown increased production volumes and transmission capacities, and even more diverse energy sources supplying the grid. There is nothing indicating that this development will stop during the years to come.

As a commercial company participating in the power market as a power producer, it is fundamental to understand such changes in the market. Reviewing ones strategies and procedures to better adapt is important to stay competitive in a dynamic market environment.

2.1.1 Bidding Areas

Even though the market is coupled between the Nordic and Baltic countries, transmission capacities restricts the volumes traded and results in *congestion*. In practice, it means that bottlenecks in the transmission system creates market imperfections, and free flow between producers and consumers are not possible. Hence, *market areas* are constructed such that each market area functions as a market place with a common power price, whereas power trades between the market areas may be affected by congestion. The market areas in the Nordic and Baltic power market are illustrated in Figure 1.

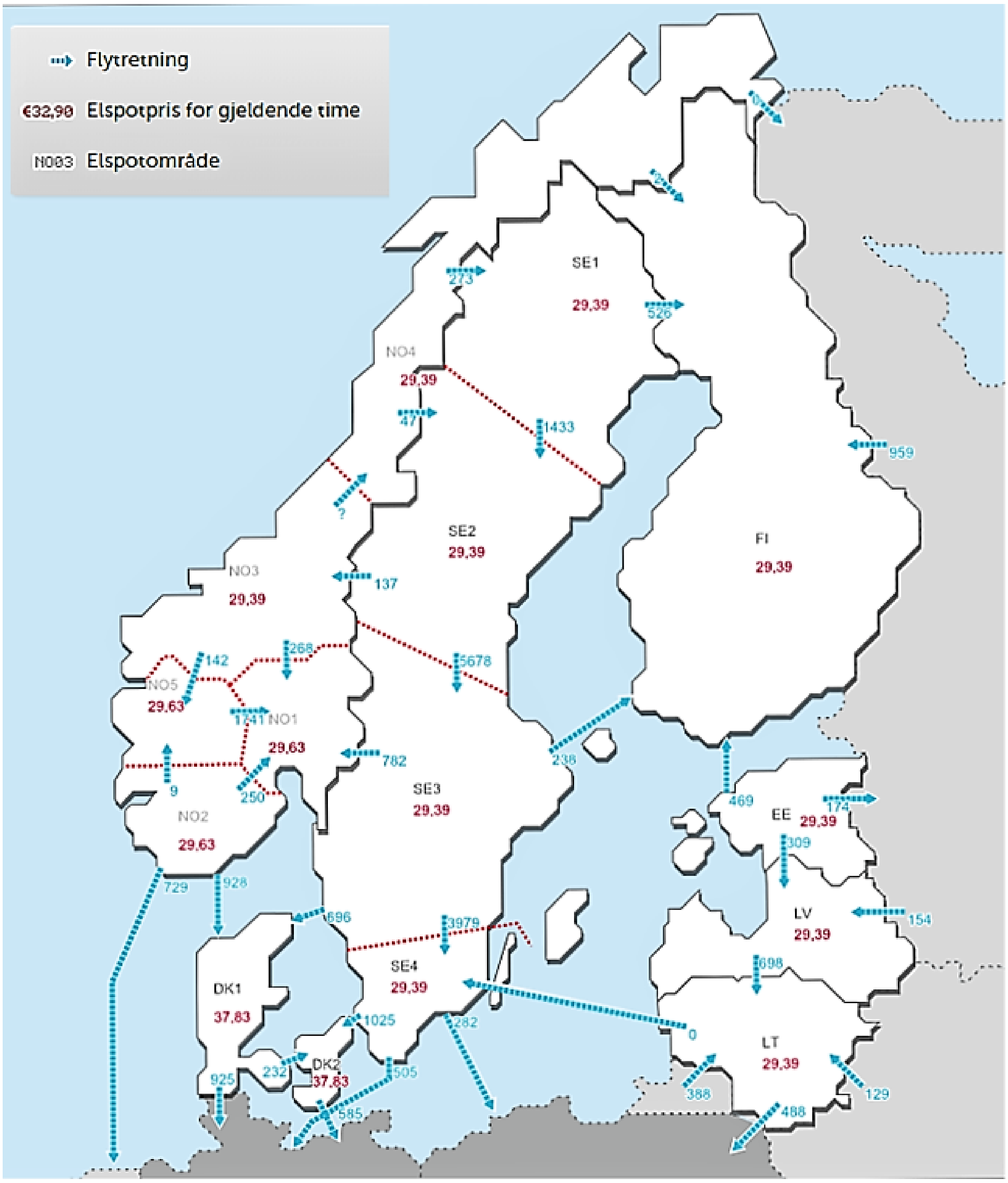

Figure 1: A map showing market areas and related spot prices and power flows. Snapshot from Statnett (2017), illustrating the 15^{th} of November 2017, at 23:59.

Even though all market areas are not all directly connected to each other, the Nord Pool power markets are constructed such that the net amount of energy exported/imported are correct according to the trades each market area has committed to. As an example, the market area NO3 is directly connected to NO1, NO4, NO5 and SE2 only, but as long as there is capacity from NO3 to any of those areas, participants in NO3 also have the opportunity to trade within all market areas with available capacities from there. Hence, in practice, all market areas in Nord Pool are connected.

For the scope of this report, an *available area* means that there exists at least one path, with available capacity on all consecutive lines, for the trade in question.

As long as there is transmission capacity, power will flow from a low price area to a high price area. This results in equal power prices in different market areas at times with no congestion. Considering the snapshot in figure 1, multiple market areas have an equal energy price of 29.39 $[\frac{EUR}{MWh}]$. However, if again evaluating NO3's position, two directly connected market areas have a higher power price that NO3. The power flows *from* NO3 *in direction to* NO1 and NO5. This indicates that the transfer capacity in that direction are fully utilized and restricts the power flow.

The available capacity between areas will vary as new power trade commitments between areas are done, and there are times where line faults occur. Hence, the power price in each area will vary according to which other areas it has available transmission capacity to at any moment in time. A *system price* refers to the power price that would have been if there was unlimited transfer capacities, and no congestion.

2.1.2 Roles and Responsibilities in the Power market

To better understand how the different types of power markets works, it is useful to know what roles and responsibilities the participants in the power market has.

Production companies are responsible for selling active power to the market, while the grid companies operates, maintains and invest in the electricity network. Both of these may act as a supplier to the end user. While these market participants have certain responsibilities, the end users are independent in the sense that they control their own consumption pattern. While they may be affected through pricing of power, there is no direct control of their consumption behavior.

This leads to an important responsibility of balancing production and demand at any point in time. The security of supply is maintained by a system operator (SO). In the Nordic market, four transmission system operators (TSO's) has the joint responsibility as system operators and maintaining the transmission grid (Wangensteen, 2012). The power exchange is facilitated by a market operator. In the Nordic power market, Nord Pool is responsible for both financial and physical trade in the day-ahead and intraday markets. Hence, Nord Pool plays an important role related to congestion management between bidding areas, while it is the SO's responsibilty to set the transfer limits. This indicates closely related tasks between the TSOs and Nord Pool.

2.1.3 Power markets of Interest

The energy market is complex in its structure, where power to be delivered/produced in a certain *production hour* may be traded several times in several markets prior to its production. The most important power markets for the scope of this report are facilitated by Nord Pool, including the *day-ahead market*, referred to as the *Spot market*, and the *Intraday market*, referred to as *Elbas*.

The final *balance settlement* is done in the *Regulating Power market*, which is not covered in this report. However, a brief introduction to how the final balance settlement is done is described, so as to better understand the dynamics of power trade affected by the *balance agreement* of supply/demand balance in the grid.

As already stated, the Spot market is a market for selling and buying power the day before the actual production hour. Elbas provides an opportunity to regulate ones commitments up until one hour prior to production. All imbalance remaining between demand and supply in the grid are then regulated at the actual time of production in the Regulating Power market. An overview of this is shown in figure 2, as a time line up until the production hour. A further description of what this means in practice is included in the following paragraphs.

All hours are given by *standard time*, which is the Central European Time (CET). It is also utilizing a 24 hour clock. The production hour h represents all production hours during day 1 from hour $1 - 24$.

The Spot Market

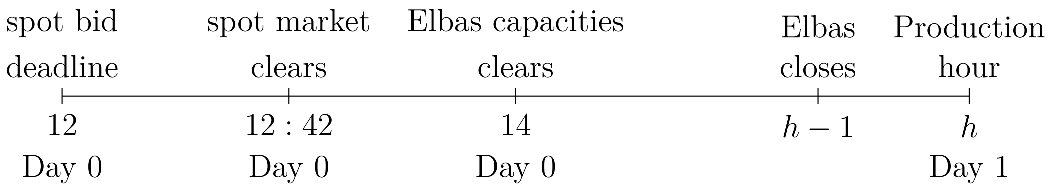

Figure 2: Time line overview of day-ahead and intraday power markets

Most of the power traded in Nord Pool is traded the day ahead of production, in the Spot marked. In 2016, the total volume traded in the Spot market was 391 TWh, which amounts to more than 77% of the total volume traded by Nord Pool and exceeds 98% of the total volume if excluding the UK day-ahead market (NordPool, 2016).

The day ahead of production, the deadline for submitting bids in the spot market is at CET 12. It is a sealed bid auction where short term contracts of purchases and sales for each production hour the following day is committed. Each participant in the spot market delivers a bid curve to the market operator Nord Pool, containing what amount of energy they are willing to buy and sell given specific price ranges. Nord Pool matches all bids concerning the same production hour, evaluating the intersection between willingness to buy and sell in the market throughout the following day (NordPool, 2017c). They create 24 market crosses as illustrated in figure 3, one for each production hour.

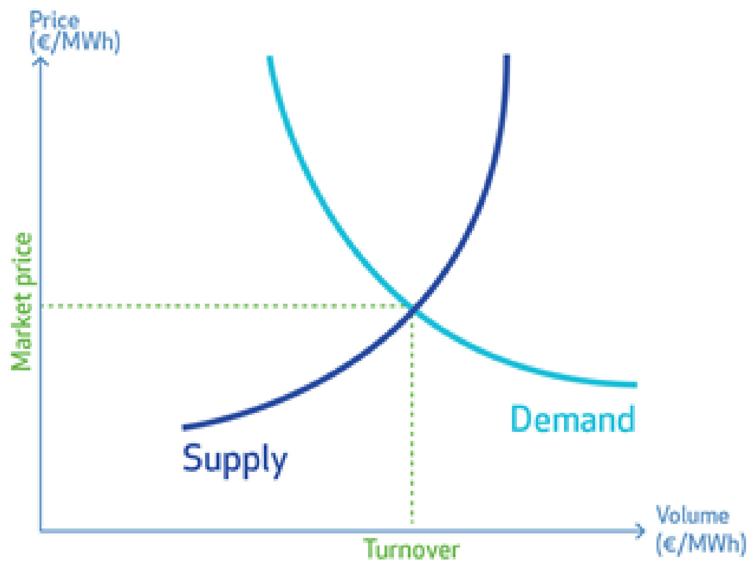

Figure 3: an illustartion of how the spot hourly prices are set by where the accumulated bid curves for buy price and sell price meet. Snapshot from NordPool (2017c)

Profits are calculated as volume dispatched by a market participant times the market price in the given hour (Boomsma et al., 2014).

When the spot market clears at some point after 12:42, the hourly spot prices and trades for the following day are announced simultaneously to the market. Due to congestion in the grid, these prices will differ between market areas (Wangensteen, 2012).

Elbas

Unlike the spot market, Elbas does not have a market clearing. It is a first-come, first-served market. The best prices come first and the participants pay as bid. If a bid has not been accepted by any counterpart when the market closes one hour prior to production, there will be no trade.

To ease the reading of this report, a specification of terms should be done concerning *time* in the intraday market. As already described, a production hour refers to the point in time where the physical delivery of power is to happen. However, in Elbas, this commitment may have been done at any point in time prior to the production hour. For the scope of this report, the term *trade hour* will refer to the time when the commitment was made.

The objective of a intraday market is to help secure the balance between supply and demand of electricity each hour. Even though the spot market's clearing was done to ensure this balance, uncertainties concerning production capacities and demand results in imbalance as the production hour is closing in. In addition, the participants has an opportunity to rather buy or sell their own commitments from the spot dispatch, if new information indicates a possibility to increase profits. The participant still has to carry out its commitments from the spot trade, but they can trade in Elbas to fill their commitments in a different way than initially planned.

An example might be if power prices in one area falls as a result of unforeseen wind conditions providing a higher power supply than expected. A power producer might want to buy power from this area in order to fill committed demand, rather than producing themselves, as the market price is assumed to be low. However, the volumes traded in Elbas are restricted by the remaining transfer capacities after the spot clearing, and trades can only be done with participants located in available market areas.

In general, both the volumes and frequency of trades in Elbas are characterized by the tradition of Elbas being a platform to create balance in the grid, and the liquidity is considered low. In 2015 the number of trades was 285 385, and the traded volume was 5795 GWh (Botnen Holm, 2017). In the period from March 2012 to February 2013, the number of trades was 190 533 and the amount traded was 3624 GWh. That is a power increment of almost 60 % and almost 50 % more trades (Scharff and Amelin, 2015). Due to an increasing share of renewable energy sources in the energy mix, more uncertainties concerning production capacities occur. Both wind and solar power are hard to store, and therefore only have a value at the time of occurrence. As the weather is uncertain, the flexibility of trade that Elbas offers is getting more important to facilitate the renewable development. Hence, the volumes traded in Elbas are expected to increase over the years as further renewable development continues (Conseil, 2015). Unlike the spot market, new trades happens continuously up until the production hour. These trades contain information about the willingness to pay for consuming or producing energy given a specific production hour. Data concerning a trade is referred to as *ticker data*. The challenge is that the low frequency of trades in Elbas makes it hard to understand the dynamics in the market, even with updated information. Up until now, predicting Elbas bids has not been investigated by many market participants, as accepted bids are observed to be relatively random with large variations. The small volumes has not made Elbas an attractive market to invest time and resources to investigate the market dynamics further, and a marginal cost pricing is often applied. It means that the bid price is given by the marginal cost of producing or consuming, and no premium is gained. New information is provided when trades are performed, so information occurs as *events*, rather than at clock based times. But what information do these trades hold? Are they representative clearing prices? The accepted bids holds information in that they represent a price where both a buyer and a seller was willing to trade. However, the depth in the market is not observable when few bids are present, and it is not possible to know what the *highest* price the buyer would have accepted, or the *lowest* price the seller would have accepted. However, with a development as mentioned above, it is predicted that Elbas will become more important, and greater volumes will be traded intraday. Such a development will contribute to the Elbas market's liquidity, and may ease the prediction of price developments.

It is not mandatory to participate in Elbas, but any imbalance between each participant's commitments to the grid and actual outcome must be regulated accordingly in the Regulating Power market. In other words, if the potential to profit in Elbas is observed by a participant, it can *choose* to act on it to earn a premium, or wait for the imbalance to be evened out by the TSO.

The Regulating Power Market

An imbalance of electrical supply and consumption results in frequency changes in the voltage supply, which reduces the power quality for consumers, and may even cause severe damage to equipment. As the power system is continuously exposed to uncertain factors that may disturb an earlier planned balance of supply and demand, there must be sufficient *reserves* in the power system to ensure balance at the time of production and consumption.

While participating in Elbas is not required, all participants in the wholesale market of electrical power in Norway are *obliged* to sign a balance agreement with Statnett. Statnett is the TSO in Norway, and has the authority to settle the final balance in the Regulating Power market. The agreement ensures that all parties shall plan and provide hourly balance, so no participant can *plan* to be unbalanced as a mean of financial gain (Statnett).

Tariffs for up regulation and down regulation are set by the TSO, where a two-price mechanism works as an incentive to ensure balance. It means that the participant should always be worse off settling their balance in the Regulating power market rather than plan their production and consumption sufficiently ahead. The two-price mechanism is presented in Table 1.

	Upward regulation	Downward regulation
Production deficit	Pay BM price	Pay spot price
Production surplus	Receive spot price	Receive BM price

Table 1: The two-price mechanism in the Regulating Power Market (Engmark and Sandven, 2017)

The participants register their BM-bids to the TSO, but the BM-price is determined *after* the production hour according to the actual regulations carried out.

The alternative to the two-price mechanism is that all participants either pay or receive the balancing market price. However, this may cause wrong incentives so that a participant may be better off by *not* seeking balance ahead.

2.2 Power Production and Resource Management

Electricity cannot be stored in large quantities after it is produced, and in fact not even all energy sources can be stored. Wind, for instance, must be utilized at the same time as it appears. Water in reservoirs, coal or gas may be stored, but have costs related to start-ups. There are different factors that affects the production costs, uncertainties and alternative gain when evaluating power production from different technologies and energy sources.

In general however, power producers face some sort of costs related to production and facilities, which may include:

- *Fixed costs* concerning e.g basic storage and maintenance, minimum staff etc.
- *Semi fixed costs* e.g units must start or stop, extra staff requirements etc.
- *Marginal costs* appearing as a direct effect of each unit of increased production, e.g resource consumption, efficiency of power production plant etc.

In a power trade situation, a trade-off between marginal cost and semi fixed costs will be evaluated to sort out which trades that potentially may result in a financial gain. In addition, when today's production affects the future possibilities to gain a profit, an *alternative cost* of production is often added to the marginal cost, as a mean of measuring what potential gain would be obtained in the future, if there is no production today.

For the scope of this report, a hydro power producer is base of analysis, hence evaluating costs in this report will be according to hydro power production. A brief introduction to hydro power production is therefore presented to ease the reading of the following analysis.

However, any power producer with costs associated to the production as described above, should find an equivalent analysis applicable.

2.2.1 TrønderEnergi and Hydro Power Production

The industrial partner of this report is TrønderEnergi, who is a power producer and a member in Nord Pool, located in the market area NO3. They produce about 2,1 TWh per year, most of it from hydro power production and about 200 GWh from wind power (TrønderEnergi, 2017). This is only a fraction of the power traded in Nord Pool in 2016 505 TWh (NordPool, 2017b). TrønderEnergi have models that consider their whole portfolio of power production plants, evaluating production plans according to uncertain factors such as water inflow and power prices.

Hydro power is a well established and renewable energy source. According to Statkraft (2017), hydro power amounts to 99 percent of the total power production in Norway, while the same number in the world is approximately 17 percent. An important advantage of hydro power is the flexibility of time of production (Statkraft, 2009). A reservoir acts as a natural storage of energy, where water can be used for production now or stored to satisfy demand later on. As there is no costs associated to water, the alternative cost of producing the water is the marginal cost of production. The alternative cost is referred to as *the water value*, and occurs since the water resource is *scarce*. The water can be utilized in different ways, and to determine what way will profit the most is a decision of insecurity, especially concerning future inflow and power prices.

It might be difficult to understand how there is a marginal cost related to the water in the reservoir, as the water is for free and is a result of the weather and inflow over time. However, the water is a restricted resource, hence there is a opportunity to earn more or less money if choosing to hold back some water in anticipation of higher prices in the future. The short run marginal cost of hydro power is considered to be the *opportunity cost* of each kWh the water is capable of producing. As a hydro power producer, knowing the water value of the water in a reservoir is therefor important in all aspects involving energy trades.

The problem of dispatching the water in the most profitable way on the longer term basis has been thoroughly investigated in the literature, and is not within the scope of this report. However, one of the results from such a long term optimization model is that the value of water in the reservoir is determined. More precisely, the water value is the marginal cost per kWh of stored water (Faanes et al., 2016), and will differ between reservoirs due to parameters such as inflow or characteristics specific to the hydro systems. If a reservoir is likely to overflow, the water value will be close to zero, while if the reservoir is about to run empty, the water value will increase rapidly. This indicates that the water value is highly dependent on the operation of a hydro power plant, as the reservoir level will decrease whenever there is production over some extent in time. For the scope of this report however, the water value is considered constant throughout a specific day.

There are also some common parameters for all reservoirs within the same area that affects the water value, such as expected demand or market prices (Faanes et al., 2016). If it is expected high demand and high prices, the water value will typically increase.

In addition to the opportunity cost of water, an important cost aspect for hydro producers are how the production schedule affects the number of production plants required. As an example, when trading in the electricity markets, a hydro power producer may be willing to produce at a relatively low marginal gain, if starting or stopping of plants would otherwise be required. This is a question of trade off between marginal cost and semi fixed costs.

3 Literature review

The purpose of this chapter is to give an overview of relevant literature in the fields of analysis in this report, as well as to present how this study differs from established literature within the field of bidding strategies in an intraday power market.

3.1 Overview literature review

First, topics concerning market behaviour and power trade will be presented. As will be further elaborated, the spot market dynamics are highly relevant to understand the intraday market, and thus literature within both these markets are of relevance to bidding strategies in Elbas. In the second section, tools for data analysis and stochastic modelling are presented through relevant literature. Large amounts of historical data concerning power trades are available in this study, and therefore literature regarding large data handling is referred to, to strengthen the robustness. The last section of this chapter will focus on optimization tools and how to apply data analysis in a dynamic program.

3.2 Market Participants' Behaviour

When considering a market participants' behaviour, and suggesting how they should bid to optimize their portfolio, it is crucial to know what type of market participant they are, and hence how their actions will play a role in the market as a whole. A price-taker will not be assumed to have a large impact on the market as a whole, whilst a price-makers behaviour can affect other players' decisions. In the spot market the number of participants is large, and most participants, including TrønderEnergi, are price-takers. Even though one can argue that TrønderEnergi is as a price-taker in the spot market, the same assumption is more questionable in the Elbas market, seeing that there are a lot fewer participants and the overall volumes traded in Elbas are small compared to the ones in spot (Boomsma et al., 2014).

The sensitivity related to all trades performed in Elbas, due to the relatively low liquidity, is crucial knowledge to have when forecasting prices in the Elbas market. The entire volume that is to be traded is often small (this might be relaxed in the future), and thus, the actions of each participant have a larger impact. The supply-demand situation is percent wise entirely changed for that production hour. This enhances the relevance of the option problem, see Section 3.6.3, and hence an important factor in the problem modelling. If a participant covers an imbalance in a production hour, by bidding at a certain point in time, the same imbalance cannot be corrected for by another participant - or themselves. In this sense, a participant do not only have to consider everyone else's behaviour, but also try not to block opportunities for itself. By trading at a point in time, that affect later opportunities negatively.

3.3 The Electricity Market

A quotation from Wen and Kumar (2001) presents the basis for this report:

"Theoretically, in perfectly competitive market, suppliers should bid at, or very close to, their marginal production costs to maximize returns. However, the electricity market is not perfectly competitive, and power suppliers may seek to benefit by bidding a price higher than marginal production cost."

From this part, it will become clear that many studies are related to bidding in the spot market, where modelling the market and scheduling ones resources have been thoroughly investigated. However, the gap in the literature is related to the intraday market.

In this section, we will shed light on other closely related or possible approaches to the modelling and optimization challenge that have been evaluated, as well as how parts of the problem have been solved similarly before in relation to the same or other topics. The purpose is to argue the approach in this report.

3.3.1 Bidding in the Elbas market

When determining a bidding strategy in the Elbas market, it is crucial to know that different participants may have different motivations to join the market. In a survey among Swedish balance responsible parties (Pogosjan and Winberg, 2013), the main motivation for participation was the possibility to perform a reduction in imbalance costs in the Regulating power market (Sec: 2.1.3). Other motivational factors contain the possibility to optimize own production schedules to avoid unnecessary start/stop costs of generators. A third motivation is that intraday trading opens for a more flexible system, where one participant can offer flexibility in production to other participants who are willing to pay for the power, since their own production cost is higher than that of buying from another participant (Scharff and Amelin, 2015).

Botnen Holm (2017) developed a model of the intraday price by utilizing regression analysis. The spot price and the regulation power price were used as determinants, as both could be used to explain intraday price. Variations in the determinants impact were found to be related to seasonal variations, time periods within a market session and different price areas. The model was found to have an overall good prediction ability, but struggled when prices were extremely high or low (Botnen Holm, 2017).

The literature on Elbas prices is less extensive, but is modelled using the autoregressive model ARMAX by Boomsma et al. (2014). The study evaluates sequential electricity markets, including the day-ahead and the intraday market, and solves the problem as a two stage stochastic program. In our study, we want to evaluate all possible trade hours as an individual bidding decision, and the number of stages in such a model will increase the complexity rapidly.

The potential in the market, i.e. what bids one can get accepted and profit the most from in Elbas, can be approximated in different ways. This is not discussed a lot in the literature. In xx the balancing price is forecasted by performing analyses and modelling historical data from the Elbas market. The drawback with this method is that the full potential might not have been utilized. If a bid is accepted it does not mean a higher bid would not have been accepted. The problem can be approached if data with bid matrices are available and one can see what bids were not accepted. In that case one always have an upper bound (for selling, opposite for buying) of the market electricity price potential at that given time.

3.3.2 Relation between spot prices and Elbas prices

Though bidding in Elbas and modelling of the intraday market dynamics have not been as thoroughly covered as Spot in the literature, there are strong indications that Elbas prices are strongly related to spot prices for a certain production hour. The relationship between the two market's prices has been investigated in multiple studies, and on interesting question is to what degree Elbas prices the following day may be extrapolated by the spot market clearing.

Faria and Fleten (2011) found a high correlation between spot prices and the average Elbas prices from historical data. In the case where there is no capacity between price areas, there can not be any power transmission, and the Elbas prices observed by each agent are correlated with the spot prices in their own area. Olsson and Söder (2008) and Klæboe et al. (2015) model the balancing price directly including correlation with the spot price. Skytte (1999) also finds that the balancing price can be explained by the spot price, whilst Jaehnert et al. (2009) indicate no correlation between the spot and the balancing prices.

One of the reasons for this deviation is that the design of the balancing market varies between different countries. Another argument against correlation is based on area dependency, hence which markets are available at a time. In this project the market have been divided into markets that have, or do not have capacity on the transmission lines to trade with NO3. To use the assumption of correlation, the underlying spot price could not be the system price (2.1.3), or the price in NO3, but rather the min(buy) and the max(sell) prices of all available areas. The availability is as seen from N03, where TrønderEnergi is located.

3.4 Spot Price Modelling

Spot price modelling is a well covered area in the literature, and different methods are utilized. As we will see, a range of econometric or statistical models have been suggested in the literature.

Benth et al. (2012) perform a comparison between 3 different continuous-time methods for electricity price modelling: 1) the jump-diffusion model, 2) the threshold model and 3) the factor model. In conclusion they found that in the two first, the mean reversion parameter utilized is not able to distinguish between spikes and base signal. However, the latter is able to do so, but has a drawback since the variability in the paths is not captured leading to an underestimation of the deviation of the base signal.

Weron et al. (2003) also utilizes the jump-diffusion model, whilst Bakke et al. (2016) investigate a regime switching model applied to the spot market.

A common factor for many of the methods utilized for spot price modelling is that they possess a crucial property, namely the Markov property. Therefore, more related literature where this property is of great importance is presented and explained under the Section 3.4.2. For instance, we will discuss variants of the jump-diffusion and the regime switching model further.

Weron and Misiorek (2008) compares the accuracy of 12 time series methods for short-term (day-ahead) spot price forecasting in auction-type electricity markets, with the conclusion that they perform well under different market conditions. Depending on the data set and the goal of the analysis different approaches are suitable.

In this study, the availability of historical data is relatively good, having hourly price data from 1^{st} of January 2013 to the 30^{th} of August 2017. The challenge of determining what model fits the data set, and hence also the estimation of parameters in the particular model, is based on testing different empirically found parameters - and hence, differs from the standard parametric approach (Pflug and Pichler, 2016). The large amount of data justifies using empirical data as a robust basis (Sec: 5.2).

Pflug and Pichler (2016) describe this as the non-parametric approach to scenario generation. One of the main arguments to build the stochastic process on actual data sets, rather than on process distributions created to fit the data set is that some information can get lost in the extra step. The process could describe the most likely cases very well, but struggle to reproduce spikes or abnormalities. When including all data in the process of forming a for instances an empirically obtained transition matrix (Sec: 3.4.2), all information is utilized.

3.4.1 Seasonality and Spikes

Seasonality can be handled in different ways. Some authors use sinusoidal functions (Janczura and Weron, 2010), others use wavelet smoothing (Nowotarski et al., 2013), which is another possibility that is less periodic and less sensitive to outliers. The downside of this method is that it gives a very good in-sample (IS) fit to the data, but has a poor forecasting ability. Nowotarski et al. (2013) also make use of dummy variables, for the different months, weeks or hours of the day, to adjust for seasonality.

In this project, the major seasonal observations from the data set are repeating patterns after 24 hours. This is discussed further in Chapter 5 in Section 5.2.

Further on spikes, i.e. prices that take on a value much higher or lower than the real mean of the data set, have to be handled. The upside of underestimating their influence is that on average the model is expected to do well, the downside is that the model may not be able to reproduce the data set, when sampling. One important insight for this study, is that if the state space in the model is too small, and cannot capture the whole, real state space the model is not very good and will collapse if it is to handle infeasible input parameters. If a state space for the spot price is modelled as S$\in$ $[\gamma_-, \gamma_+]$ and a realized price turns out to be $\gamma_+ + \alpha$, the state is infeasible, and hence the model will collapse if trying to predict a future based on a realization it does not acknowledge.

In this paper, the challenge of covering all possible, feasible outcomes is handled by dividing the state space into sub spaces, where the outer sub spaces are given a large span - in order to cover spikes. One weakness of this type of modelling is that all observations within a state are considered equal, and are assumed to have the value of the mean of that subspace, based on the distribution of real data points within the state. Therefore, there is no way of knowing if the price is in state i, is it closer to the upper or the lower limit of the sub space? More accurate time series model can help describe this, but then the challenge of over-fitting must be handled carefully. For this project, the modelling have been limited to consider what state the price is in, and assumes that is has the value of the sub space mean price.

3.4.2 Markov Process

A well known process in the literature is the Markov process. To introduce the process central notation is presented. The definition of the state space in this study will be introduced in section 5. For now, simplified notation is used. The state space S denotes the space of the Markov Chain:

$$s_1, ..., \overline{S}, \quad \in \mathcal{S}. \tag{1}$$

- $P_{ij}(t)$: The probability that the process changes from state s_i to s_j when time changes from t to t+1.
- $P_{ii}(t)$: The probability that the process remains in state s_i when the time changes from t to t+1.

There exist many different processes that rely on the Markov property. According to Kirkwood (2015), it is defined as in Equation 2:

$$P(X_t = s_{i_t} | X_{t-1} = s_{i_{t-1}}, ..., X_1 = s_{i_1}, X_0 = s_{i_0}) = P(X_t = s_{i_t} | X_{t-1} = s_{i_{t-1}}) \tag{2}$$

for any $s_{i_0}, s_{i_1}, ..., s_{i_{t-1}}, s_{i_t} \in S$. $\mathcal{T}$ is a countable index set, here describing the time. X_t is a random variable that gives the state of the process for each time $t \in \mathcal{T}$.

The Markov process has been utilized in different studies to model price development. Numerous stochastic processes that possess the Markov property exist. The Brownian and geometric Brownian motion are investigated by Barlow (2002), but it was concluded that neither were a good fit for the spot price. In stead jump models were looked at as independent Poisson processes used to describe movements between two states in a Hidden Markov Model (HMM). When there are two processes of which only one can be observed, and the other one (of interest) is underlying or hidden, a HMM can be of use, as also performed by Sasikumar and Abdullah (2016). Barlow (2002) concluded that working with jump models is difficult, and that other methods should be investigated first. Deng (2000) and Bakke et al. (2016) also make use of multi-state Markov chains in a regime switching model. In the first, a continuous-time Markov chain entails an arrival density based on a Poisson process. In the latter, each different state is associated with a price process utilized within the regime.

Economic fluctuations and stock prices have received attention in business cycle analysis. For example, a Markov regime switching model is also exploited by Lanne and Lütkepohl (2008), in order to identify shocks in co-integrated structural vector autoregressions and investigate different identification schemes for bivariate systems. The stock price and electrcity price are similar in the way that they are both depending on many external factors. However, the relevant factors may differ. This way of modelling was not utilized, and we rather consider the spot price the observable state and the state of interest. In another study, Markov parameters are also used to describe the discontinuous behaviour of the balancing market prices (Olsson and Söder, 2008).

Kongelf and Overrein (2017) model the regulating markets using quantile autoregression to provide probabilistic forecasts for the market prices. They propose to model the regulating prices, not only by using one, but 24 individual models under the argument that all 24 hours during the day have different statistical properties. Even tough they consider *regulating prices* whilst the subject of this study are the Elbas prices the idea of splitting the hours is relevant. For the purpose of this study, this is considered a seasonality challenge and commented on under Section 5.2.

Olsson and Söder (2008) propose a method based on seasonal autoregressive integrating moving average (SARIMA) and Markov processes. The combination of these makes this study highly relevant, as this study will discuss similar connections in later sections.

The Markov property is hence a property that many before have associated with electricity price processes. An important note to make is that in this study the scope is to model well the average behaviour, not the spikes in particular.

In accordance with mentioned literature, we assume that the Markov property is one that is important when describing electricity prices. As elaborated in Section 5.2, there is clearly correlation between the spot price data included in this study. In particular between each step and the previous. Hence, it is desirable to utilize a stochastic process for modelling that can account for conditional properties in time series.

If a process is Markovian, and the current state is known in addition to a process one is able to predict the likelihood of the process ending up in all other states in the next step. In this study we investigate the case of forming a transition matrix, not only with two or three different states as in the literature referred to above, but with $\overline{S}$ different states. The motivation for this is that this way the process can be entirely based on previous empirical transitions. A transition matrix, often denoted P, describes how the system is likely to transition between all different states, and typically take the form shown in Equation 3:

$$P = \begin{array}{c} \\ 1 \\ . \\ n \end{array} \begin{array}{c} \begin{array}{ccc} 1 & ... & n \end{array} \\ \begin{pmatrix} P_{1,1} & ... & P_{1,n} \\ . & & . \\ P_{n,1} & ... & P_{n,n} \end{pmatrix} \end{array} \tag{3}$$

A Markov chain have many interesting properties that are of interest to investigate when working with a Markov process. Among the most relevant are the equilibrium state along with the criteria that must be fulfilled in order for a process to have one unique equilibrium. Firstly, the process must be irreducible. It means that all states must *communicate*, which is equivalent to saying that all states are accessible from all the other states (Kirkwood, 2015), see Equation 4.

$$P^n(i,j) > 0, \qquad\qquad P^m(j,i) > 0 \tag{4}$$

This also means that there cannot be multiple recurrent equivalence classes, since that would lead to multiple equilibria. In practice it makes no sense to have a recurrent class potentially with an absorbing state because the interpretation would be that once the price enters a state it remains here forever. If this was the case it would have been easy to predict, at least once this absorbing state would have been entered. In a larger sense one could discuss if the change in prices due to inflation and other factors could exclude sets of prices, but this is outside the scope of this study.

It is not required that they are accessible in every step, but for some step m and n that can be different. Moreover it is also required that the process is aperiodic. Aperiodicity is related to what the time between possible return to a state. If four states are linked, and the process can only move in one direction, the process has a greatest common divisor of the process is 2, and hence it has a period of 2. However, if the greatest common divisor is 1 we deal with a aperiodic process. In the analysis it will be utilized that if a Markov chain is aperiodic and irreducible it will, at least in the finite case which is relevant here, have a unique equilibrium (Kirkwood, 2015).

The equilibrium of a transition matrix, P, is given by Equation 5:

$$\lim_{t \to \inf} P \tag{5}$$

or by solving Equation 6 for $\hat{\Pi}$:

$$\hat{\mathbf{\Pi}} P = \hat{\mathbf{\Pi}} \tag{6}$$

Which is equivalent to raising P to a sufficiently high number. Another useful property is the expected value of the discrete random process. The equation for computing this is shown in Equation 7:

$$E[X] = \sum_i n_i P(X = n_i) \tag{7}$$

, where X is a discrete random variable; where the range of X is countable set of real numbers n_1, n_2,.... $P(X = n_i)$ in Equation 7 and 8 denotes the probability of $X = n_i$. Equation 7 holds as long as Equation 8 converges:

$$\sum_i |n_i| P(X = n_i) \tag{8}$$

These properties and equations are commonly discussed in literature and utilized for analytic purposes. They play an important role when determining how good the model in Section 5.2 is. Here, the same notation as Kirkwood (2015) utilizes is used.

There are many challenges when determining a robust Markov transition matrix to describe the process (Wiesemann et al., 2013). An analysis have been performed in order to choose a Markov transition matrix that is representative for simulation.

This article aims to model a Markov transition matrix that can model the development of the market potential as a function of the best spot prices available (i.e. with capacity to transfer at least the desired volume in the given direction). When referring to the *best* spot price, it can refer either to the highest, which is the best price to sell for, or the lowest referring to the best price to buy bower for.

3.5 Adjusting the Data

A common topic when dealing with empirical data, as in this study, is the need to extract the behaviour of the underlying process of investigation, and hence removing externalities caused by factors that have little or no correlation with the process. To isolate a process is often a tough task, but it is frequently discussed in the literature.

3.5.1 Discretization of Price Levels

The state space of the power price is infinitely large, if disregarding the technical upper and lower limits. Prices can take on all continuous values. To ease the complexity of the problem, a division of the state space in a reasonable manner, i.e. a reasonable number of states and a suitable size for each state, is performed.

The issue of defining states to use in a Markov process can be solved in different ways. The feasible area can be split in a suitable number of classes with an appropriate size. Zhou (2015) forecasts stock price using weighted Markov transition matrix where the state blocks were determined according to the mean value $\overline{X}$ and *mean square error* (MSE) of the sample. The upper and lower states were outside the mean plus or minus the mean square error of the sample respectively, whilst the blocks in between were separated by an equal and absolute price.

Bally and Pagès (2003a) provide comprehensive descriptions on how to perform an optimal quantization. They propose a new grid method which is based on recursive stochastic algorithms that are again based on simulation. In the article the scope is to solve an optimal stopping problem. In a separate study, Bally and Pagès (2003b) also consider the challenge of quantization for optimal stopping problems, and widely discuss statistical error induced by the Monte Carlo estimation.

These articles are relevant because they deal with the challenge of dividing the state space in an optimal manner, so that the division can be used for further modelling.

Monte Carlo (MC) simulation will be utilized later in this study, in combination with, and in order to evaluate a set of Markov processes. Löndorf et al. (2013) uses MC simulation to define a set of sample reservoir states, and hence utilize the results to obtain an optimal decision policy. This motivate us to use simulation to evaluate our model in Chapter 5.

3.5.2 Transforming Data

Data transformation is the process of determining a mathematical function to each point in a data set. The reasoning behind the application of transformation is for instance to make statistical tests and deriving of estimates more convenient. Another reasoning can be to improve interpretability of graphs. Transformation is also utilized in this study, to be able to work with data between zero and one. Nowotarski and Weron (2018) discuss the topic of transformation, when studying discuss the matter of electricity price forecasting. They suggest using probability integral transformation (PIT) when working with empirical data. This method is based on considering the probability integral, or the cumulative distribution function (cdf) of a process. If the process follows a known distribution the cdf can be derived from known expressions, otherwise an approach is to split the whole solution space into small intervals of desired solution count the occurences of all outcomes within to obtain the empirical cdf. Based on the cdf, the process can be transformed into numbers between zero and one. An important note to make here, is that the cumulative distribution is discrete and only given for the empirical data. In order to transform prices back through an inverse transformation, a continuous function is needed. A possible approach could be using linear interpolation between points. However, it is inconvenient, and Nowotarski and Weron (2018) propose different procedures. Different distribution fits will be discussed in Section 5.2. The purpose of the defining a continuous cdf is to use it to transform, but also be able to inverse transform the numbers at a later point. Equation 9 denotes the relationship between the continuous cdf of F_x and data points Y. Y will later take the values of price points from empirical data, with a fitted curve F_x.

$$F_x(X) = Y \tag{9}$$

As will be seen in this report, the Kernel Normal distribution, which is also suggested by Nowotarski and Weron (2018), is utilized to find a continuous cdf (Sec: 5.2).

3.6 Bidding Strategy Optimization Models

Different approaches have been evaluated in the literature when bidding in the spot market. While Boomsma et al. (2014) emphasizes a sequential bidding strategy across multiple power markets, most literature like Aasgård et al. (2017) and Anderson and Philpott (2002) focus on bidding strategies in the spot market alone. However, the latter articles also consider operational aspects. Another article considers *intraday* decisions for hydro storage systems, integrating bidding and storage decisions in the formulation (Löndorf et al., 2013). A multistage problem is solved, utilizing *stochastic dual dynamic programming* (SDDP), but with some ideas from *approximate dynamic programming* (ADP) so that the model does not require stagewise independence of the stochastic process.

What distinguishes this study from those mentioned above, is that this study evaluates intraday bidding as a sequential decision problem in it self, from the intraday market opens and up until each production hour.

Firstly, only a fraction of the papers investigating bidding in the electricity market looks at Elbas - they more often describe spot price bidding. Secondly, the spot price is seldom modelled as a Markov process where the transition matrices are built directly from counting transitions in data sets.

This project aims at formulating a Markov process model, with discrete time and prices. The time resolution is hourly, and the price discretization as described in Section 5.2. In order to from the model different approaches are investigated to determine a suitable model. The tuning of the transition parameters is performed using analysis of MSE and hit percent, see Section 5.2.

3.6.1 Dynamic programming

Different optimization models may be considered beneficial for application when constructing a bidding strategy in the electricity market.

An important aspect is the uncertainty involved due to stochastic price fluctuations in the intraday electricity market. According to Dixit and Pindyck (1994), dynamic programming is a powerful tool when treating multistage problems with uncertainty, and decision variables at each stage. It breaks the problem down so that each decision x_t involves only two components, with the objective to obtain the optimal *policy* for decision making. This is stated by *Bellman's principle of optimality* (Bellman, 1957):

"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

In this case, the bidding value at a time t represents the first component, and the expected value of all future states and corresponding decisions, represents the second component of consideration. The time steps t corresponds to the stages in a dynamic program, the state S_t of the system in each stage is given by price developments, and the probability that $S_{t+1} = s'$ given the current state is given by $\mathbb{P}(S_{t+1} = s'|S_t, x_t)$.

The value recursion of a state is then given by Equation 10, which is also referred to as the *Bellman equation*.

$$V_t(S_t) = min(C_t(S_t, x_t) + \sum_{s' \in \mathcal{S}} \mathbb{P}(S_{t+1} = s'|S_t, x_t)V_{t+1}(s')) \qquad (10)$$

This study deals with a multistage problem with few state variables and a repeating structure. In addition it is argued in Section 3.4.2 that the Markov property holds for the investigated process, and thus DP is a suitable model.

3.6.2 The Curses of Dimensionality

Though dynamic programming is suitable when there is a multistage sequential decision problem, Powell (2011) emphasizes the three curses of dimensionality, which are the most common reasons why dynamic programming cannot be used. The sizes of the state space, outcome space and the action space all contributes to increased complexity of a dynamic problem. In this report, the state space is handled as discrete levels as described in section 6.4, divided into a finite number of levels sufficient to describe the overall market situation. The outcome space, which is all possible outcomes of future information not known at the time of computation, will in this problem only concern new information about the market price. Hence, the outcome space is handled the same way as the state space, divided into a finite number of discrete price levels. The action space is binary as in the optimal stopping problem, which is described further in Section (3.6.3). There is a *bid* or *no bid* decision, which largely decreases the action space compared to an asset valuation problem or the resource allocation problem (Powell, 2011).

Notice how the number of stages is *not* included in the curses of dimensionality in dynamic programming. This is a great advantage compared to other stochastic optimization methods, at least as long as it is possible to contain the other dimensions within reasonable limits (Dixit and Pindyck, 1994).

Another advantage is that the output is of the form of a policy, suited to construct a *contingency plan*. It is easier to simulate when a policy is obtained, thus evaluation of the output from a dynamic program is easier to evaluate than output from optimization over a scenario tree. Even if the structure of the problem has limited flexibility, and an optimal policy hard to compete, there are heuristic approaches to obtain a policy. Wu et al. evaluates the performance of a *rolling intrinsic* (RI) policy, based on a theoretical analysis of seasonal energy storage options. They also adjust certain prices before applying the RA policy, which recovers almost 70% of the value loss of the RI policy, and refers to this method as the *price-adjusted rolling intrinsic* (PARI) policy.

As the *main focus* in this study is to analyze historical data to understand market dynamics, rather than development of an optimization model, the complexity of the dynamic program in this study is limited. However, future research should emphasize to develop the optimization model so it better fits the complexity of the problem to be solved. This is further elaborated in Chapter 8.

3.6.3 Optimal Stopping

Garnier and Madlener (2015) discuss how optimal timing of trades in the intraday electricity market is important to decrease risk. Both postponing a bid for too long, and immediate trade after the spot market clearing, are related to risk. Firstly, when a production hour closes in, the market tightens up. Power producers with uncertain production capacity needs to settle their imbalance shortly before delivery. More flexible energy sources often has a start-up or shut-down time related to the power plants, which is longer that the time to production. Hence, a participant postponing their trades for too long, face a risk of not getting their bids accepted by a counterpart. On the other hand, too far away from the time of production, there are similar uncertainties as faced in the spot market, and the probability that the market participant trades wrong volumes increase.

Optimal stopping is mentioned by Dixit and Pindyck (1994) as a class within dynamic programming problems particularly important within investment analysis. It is a dynamic problem, concerning what time to take a specific action, where the action space is binary in each node; either stopping to gain the termination value or continuation to the next period. There will be a similar choice to be made in the next period. Their applications are similar to the bidding strategy problem within the scope of this report. Dixit and Pindyck (1994) assumes a certain structure of the future development, so that there exists a optimal cutoff price level p^*, where termination is optimal on the one side and continuation is optimal on the other side.

4 Problem Description

The purpose of this project is to analyze historical price data, in order to create a discrete, stochastic model that can be utilized to form a bidding strategy and provide decision support in the intraday electricity market - Elbas. The focus is centralized around understanding the market dynamics and discovering the willingness to trade. It is suggested that willingness to trade can be modelled making use of price asymmetry between market areas that participants want to benefit from, along with information about bottlenecks. The latter describes which of the market areas a participant actually can expect to trade with, which also indicates the price levels available for trade. In addition, the stochastic model is put into an optimization context, to illustrate how the bidding problem is considered a sequential decision problem with an option value of waiting. However, the dynamic program presented is more of a conceptual base, and should be subject for further research before the model completely covers the real bidding situation in Elbas.

Each day, there are 24 different production hours, which can be regarded as products subject for trade. Any participant can bid in the Elbas market, both to sell and buy, on all of these products. The bid contain information about *volume* and *price*, and can be given at any *time* after the opening of Elbas at CET 14 the day ahead of production, and before the closing of Elbas one hour prior to each specific production hour. If no counterpart accepts the bid, there will be no trade.

The optimal time of trade and price level must be determined for each product to maximize profit.

A final decision support should be related to price, timing and volume, but some simplifications have been made with respect to volume and rationing (Sec: 5.3.1). In this study, the volume is determined after the spot clearing, and can differ between the different products. However, the total volume of a product is to be traded in one single trade.

Power prices change continuously, and therefore the optimal bidding problem is nontrivial for participants seeking to maximize profit. Imperfections in the market creates an opportunity to profit from premiums - excess returns beyond covering the marginal cost of production. This potential is the basis for this study, along with the growing liquidity and increasing interest in the intraday market. The stochastic process of Elbas prices, from the market opens and up until the production hour, describes the actual market depth, and what price levels a counterpart is willing to accept. Correct anticipation of the current potential in the market and future prices, is fundamental to be able to determine when to bid at at what price level.

For an optimization model to provide reliable results, the stochastic price model must be sufficiently precise.

Literature paying interest to the hourly prices in the day-ahead market have established that autocorrelation is a property of the electricity market (Graf von Luckner et al., 2017). In addition, correlation between Elbas and spot prices is pointed out (Sec: 3.3.2). This creates the basis of this study, since if one have knowledge of spot prices, one implicitly have some information about the Elbas prices.

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5 Problem Formulation

This chapter entails the analysis and most of the results from this study. Initially, necessary notation is listed, and explained further when used in the text. In smaller examples used along the notation is introduced separately for convenience.

Section 5.2 is a market analysis resulting in a price process model, describing the willingness to trade in Elbas. More specifically, the output is a transition matrix, that constitutes the basis of a process grid. This grid is investigated by a dynamic program in Section 5.3. Both sections are introduced by listing important assumptions.

Figure 4 provides a more visual outline of the structure of this chapter.

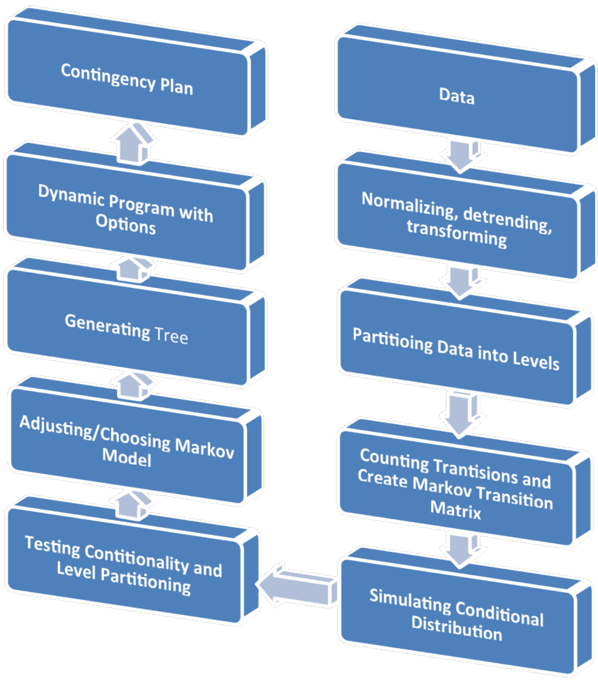


Figure 4: Problem Approach

5.1 Notation

Indices

t	Time step
h	Production hour
s	State
u	Price level

Sets

$\mathcal{T}$	Set of time steps t
$\mathcal{H}$	Set of production hours h
$\mathcal{S}$	Set of states s
$\mathcal{U}$	Set of price levels u
$\mathcal{L}$	Set of possible paths with historical memory of length L
$\mathcal{X}$	Set of possible decisions

Parameters

$\overline{H}$	Number of production hours
$\overline{S}$	Number of states
$\overline{U}$	Number of price levels
L	Number of stages included in path memory
$\overline{T}$	Number of time steps t
$\mathbb{P}_{s',s}$	Transition probability from $s' = S_{t,h}$ to $s = S_{t+1,h}$

Variables

$u_{t,h}$ Price level in time step t concerning production hour h

$S_{t,h}$ State in time step t concerning production hour h

$W_{t,h}$ Exogenous information arrived between time step $t-1$ and t

Decision Variables

$x_{t,h}$ Decision in time step t concerning production hour h

$X^{\pi}(S_{t,h})$ Decision Policy in state $s = S_{t,h}$

5.2 Price Process Analysis and Modelling

The scope of this section is to model the willingness to sell and buy power in Elbas, as seen from TrønderEnergi's, i.e. NO3's, perspective, based on the first assumption listed below. The analysis can easily be generalized to yield for other market participants, by letting parameters depend on the participant of interest. In all places where NO3 is used, another area could have been utilized instead. Some simplifications and assumptions are listed below, and will be explained further where considered useful.

5.2.1 Assumptions and simplifications

Simplifications

1. Some areas in the market are excluded from the analysis, due to lack of data.

2. It is more important to model average behaviour well, than spikes.

3. Time and state spaces are discretized.

Assumptions

1. There exist an unutilized trading potential that is desirable for market participants to know.

2. Spot prices and Elbas prices are correlated.

3. Availability between areas require the existence of at least one path between them which have minimum 50 [MW] available capacity.

4. *Short-run marginal cost* (SRMC) of coal is an external factor with great impact on electricity market price. Normalization of spot prices to SRMC of coal is here assumed an informative measure of the spot price process.

5. Spot prices have the Markov property.

The electricity market situation is complex. To be able to model price behaviour the market situation is to be investigated thoroughly. Capacity data indicates bottlenecks in the market and is therefore an important part in the analysis. In accordance with assumption number 2, listed in above, and as discussed in Chapter 3, the spot price process will be broken down and used as a basis to find the potential in the Elbas market. Moreover, Elbas trades are included in the analysis to provide an understanding of the liquidity. In addition, trading patterns will be discussed and can become interesting knowledge when creating bidding strategies in the Elbas market. The last data set utilized is the SRMC of coal. This will be explained further when relevant. The format of the information is explained to give the reader insight to the procedure. Common for capacity data and spot price data is that it is revealed the day ahead, when spot prices clear.

The time period investigated is from 01.01.2013 to 30.08.2017, which corresponds to 40872 hours.

The market price in Elbas varies, and is argued to be correlated with the spot price (Sec: 3.3.2). This assumption (2) is the basis for investigating the stochastic *spot prices* and their development over time. Observing the spot price and being able to predict it will, along with the assumption of correlation, make us able to better predict the Elbas potential.

5.2.2 Capacity Data

The format of the capacity data is shown in Table 2.

Date	Alias	Hour1	Hour2	Hour3A	Hour3B	Hour4	...	Hour24	Sum
06.03.2017	$NO2\_NO5$	200,0	200,0	200,0	-	200,0	...	200,1	16408,7
06.03.2017	$NO3\_NO1$	0,0	0,0	0,0	-	0,0	...	0,0	0,0
06.03.2017	$DK2\_DK1$	600,0	600,0	533,8	-	600,0	...	625,5	23483,0
...	...	...	...	...	...	...	...	...	...

Table 2: Capacity Data Format

One excel file per week during the period is investigated. All areas in the market are associated with an area name as introduced in Chapter 2, Figure 1. *Alias* in Table 2 refers to line connections between two market areas, and the available transmission capacities are listed for each production hour, here as *Hour 1*, *Hour 2*, ...,*Hour 24*. From this arbitrarily chosen file, we see, that the capacity on the line $NO2\_NO5$ is very stable during all hours on this specific day. Hour 3B is dedicated a separate column. This is to handle when the clock is adjusted one hour back or forth in local time. However, when working with data, time is converted into *standard time* CET (Sec: 2.1.3) to avoid jumps in time, and rather model it as continuous. The last column summarizes the capacity between two areas during a day. The capacity is listed with units $[\frac{EUR}{MWh}]$.

A note to make is that even tough capacity data in Table 2 is known the day ahead, it is not constant. As soon as a trade in Elbas is realized, the capacities are adjusted correspondingly. I.e. The values only indicate the start characteristic of the system when the spot market clears, but changes depending on Elbas trades made throughout the day in. The interpretation of this is that if there is no available capacity for power flow from one market area to the other, trades increasing the power flow will not be possible. The value in the corresponding row $From \quad Area\_To \quad Area$ in Table 2 would show zero. See for instance capacity from NO3 to NO1 in any hour during the 6^{th} of March 2017. The areas are not available to each other in this direction, hence NO1 is not an available *selling* market for NO3. However, it is possible that the opposite capacity is non-zero, i.e. that NO1 can export to NO3.

An exemplification is if one area produces a lot more power than expected during a production hour, for example due to special wind conditions, but have already utilized all capacity to export energy, the price in this area will be lowered. Another area would not be able to buy power at this low price, since the markets are not available to each other to trade in that direction. This bring us into the spot price description in the different areas.

5.2.3 Spot Price Data

The information was gathered from files in the format seen from Table 3.

Date	Alias	Hour1	Hour2	Hour3A	Hour3B	Hour4	...	Hour24	Avg.
06.03.2017	System price	30,32	29,61	28,85	-	28,08	...	30,97	32,99
06.03.2017	Oslo	30,47	30,30	29,46	-	29,54	...	31,16	32,96
06.03.2017	Troms	24,87	24,86	24,85	-	24,85	...	24,45	25,60
06.03.2017	Copenhagen	30,40	29,04	27,99	-	25,35	...	31,16	33,01
06.03.2017	Tallin	30,40	29,04	27,99	-	26,58	...	31,16	36,89
06.03.2017	Tallin	30,40	29,04	27,99	-	26,58	...	31,16	36,89
...	...	...	...	...	...	...	...	...	...

Table 3: Spot Data Format

This information is also revealed after spot clearing the day ahead, but the price is assigned to cities rather than market areas. However, all cities within the same market area will have the same spot price. The *system price* describes the overall price in the system if electricity could float independent of capacity and physical constraints. It can be interpreted as an overall ideal price in a perfect market. Due to bottlenecks, the price of electricity differs between areas. This is interesting, because it can be taken advantage of when trading in the electricity market. The information of interest is firstly, what are the *highest* and *lowest* market area prices that are available to the area of interest? For instance, if the price in DK1 is very low, TrønderEnergi would like to know if there is any available capacity to import power from DK1.

To model the market situation, two things are of special interest:

1. What is the *lowest* price that a participant can buy for, given that there is *import* capacity?

2. What is the *highest* price that others are willing to pay for the power, given that there is *export* capacity?

We start by determining these price processes from historical data, because we want to perform analysis on these extreme prices, to map and later simulate potential trading opportunities over time.

5.2.4 Elbas Trading Data

Availability of historical trading data, i.e. *ticker-data*, in the Elbas market makes it possible to also study the nature of this price process. As pointed out in previous sections, and elaborated on here: the number of trades in the Elbas market is relatively low. We therefore stick to the assumption (2) of modelling the spot price due to correlation with Elbas, but find it natural to also take a look at the current Elbas situation.

In Figure 5 the number of trades involving each of the 24 products, during the respective years are counted and illustrated.

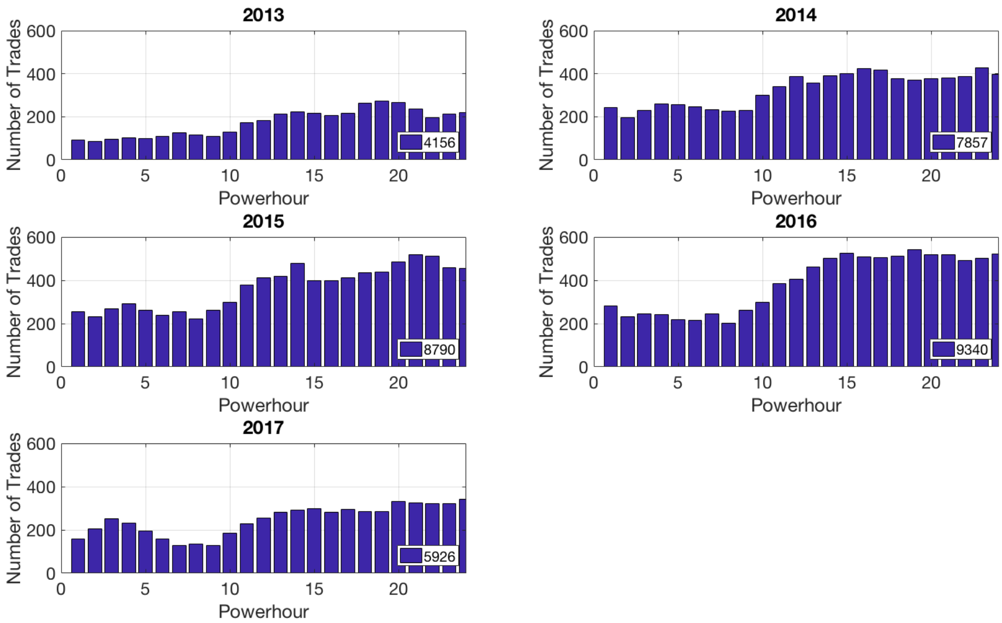


Figure 5: Accumulated number of trades related to each product (1-24) for the years 2013-2017. NB: 2017 only covers the period up to the 30^{th} of August.

The number in the lower, right corner represented the total number of trades in that year, summing all the production hours. Some observations are made: firstly, the trend is that the number of trades increases from the first production hour onwards with some variations. The total number of trades during the year increases, from 4156 in 2013 to 7857 in 2014, 8790 in 2015 and 9340 in 2016. Note, however, that 2017 is not representable for this comparison, since data was only available until the start of the researh: the 30^{th} of August 2017. This illustration does not consider the *volumes* traded, nor the *price* -

only the *number* of trades associated with each production hour.

Figure 6 - 9 provide more information about the bid *price* and the *time* of the trades. Each production hour is investigated separately. We recall that different products are associated with each hour, and the time allowed to bid is from the Elbas opens until one (or in some cases two) hours before the production hour. From Figure 6 - 9 it can be observed that as time approaches the production hour, more trades are realized. It is also noted that some of the products are traded already around 12 PM the day before. The number of trades (along the right y-axis) are accumulated numbers in the time period from the 1^{st} of January 2016 until the 30^{th} of August 2017. The prices of the trades can be interpreted along the left y-axis, along with the blue dots. It can be noted that the prices in the first 1-7, and the last 18-24 production hours have less variation in price, i.e. are less spread out, than in the production hours in between. The y-axis have a static length in all subplots, to make them easily comparable.

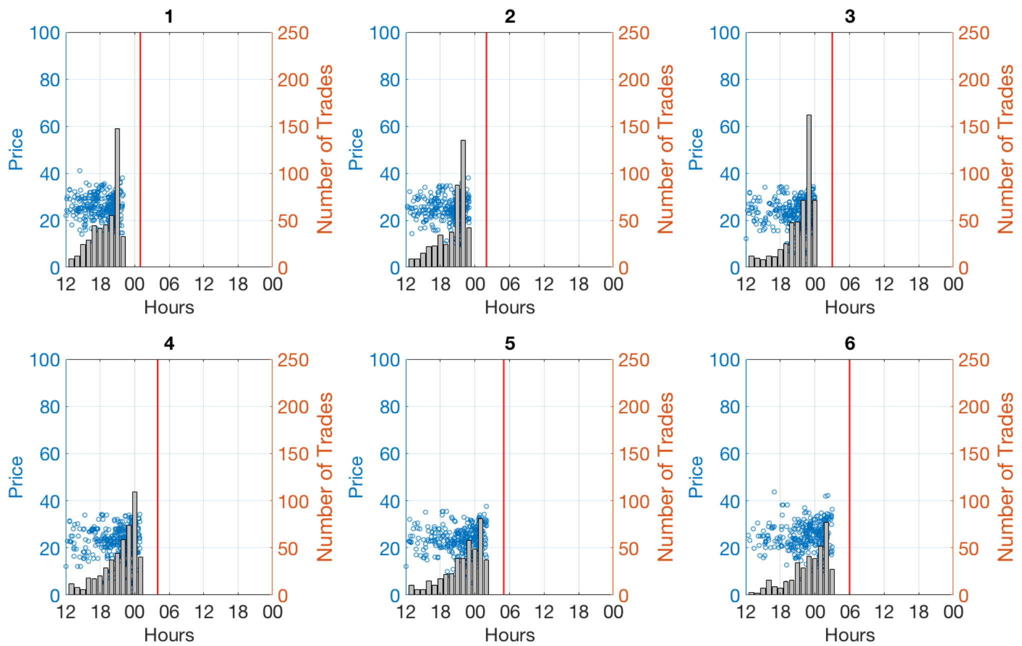


Figure 6: One subplot associated with one production hour (1-6). Scatter plot shows the relationship between price and time of trade, bars show the number of realized trades during a bid hour. Accumulated data per production hour from the 1^{st} of Jan 2016 until the 30^{th} of August 2017.

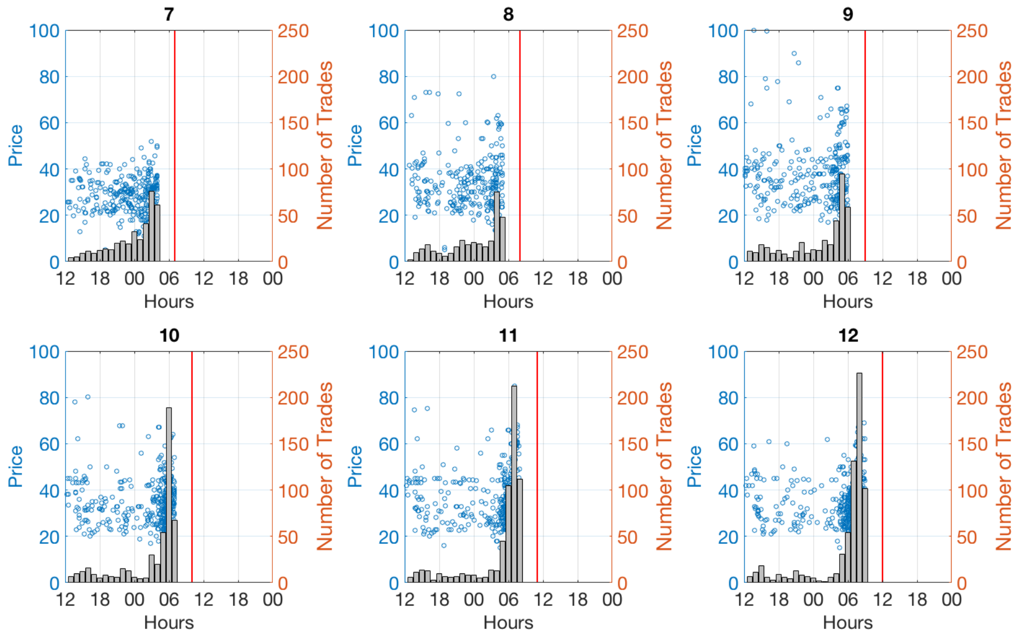


Figure 7: As in Figure 6 - One subplot associated with one production hour (7-12).

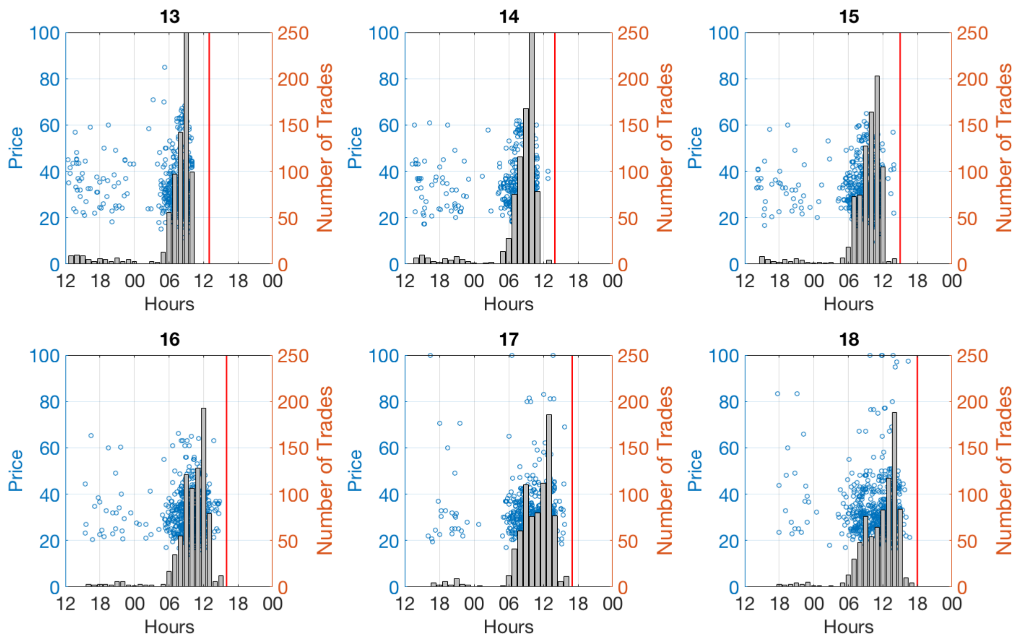


Figure 8: As in Figure 6 - One subplot associated with one production hour (13-18).

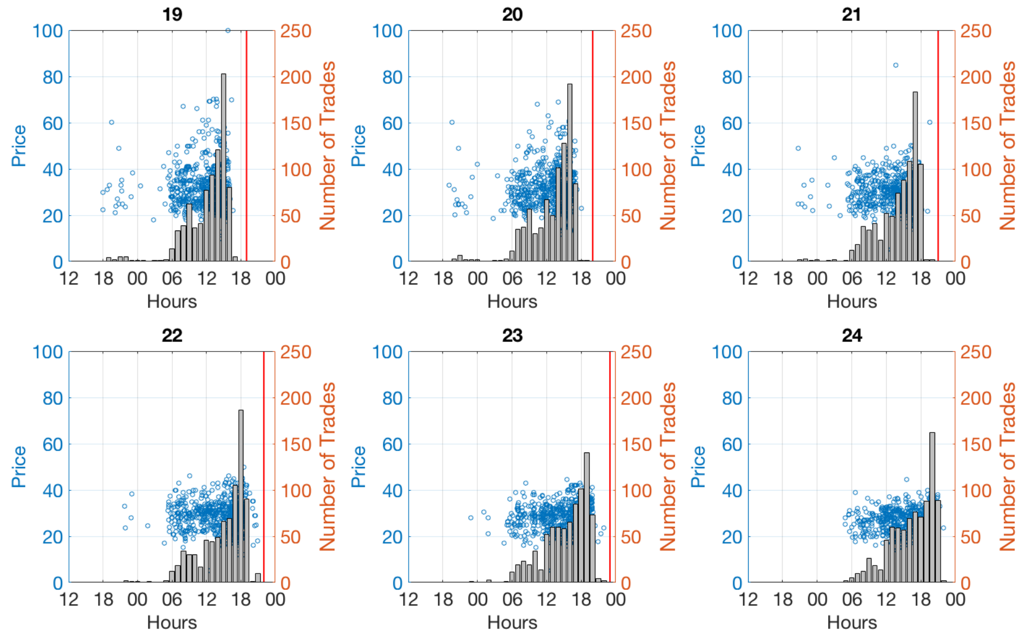


Figure 9: As in Figure 6 - One subplot associated with one production hour (19-24).

In the first 9 products (except from product 3), the accumulated number of trades during the bidding hours never exceed 60. For all of the following (except from product 23), the number of trades exceed 60 for one or more bid hours.

There might be different reasons for these behavioural patterns. The different degree of price variations can be reasoned by that in the first 1-7 hours the production hour is quite close to the point where information about spot prices and Elbas-capacity were revealed. These products have the shortest bidding time horizon, and relatively seen a short time for unexpected twists to influence the expected price. Bidders tend to have a more similar bidding behaviour, than for the products further into the future. Towards the last production hours the price variation is again lower. One possible reason could be that this is the furthest away one could be from the revelation of the spot price of that day. If the insecurity is too large, it can have the effect that no participant believes enough in their models too deviate from the spot price. They might also bid according to their own MC rather than trying to get a premium.

Some other observations are that the number of trades during the night is very low, and the opposite during work hours. As soon as new information about the future is revealed, s.a. when spot prices are cleared for the day ahead, models can be calibrated and run. During a work day, when most resources are normally put into bidding, it could be expected to see a wide set of trade process go through since this is when less safe bids could be suggested - namely the ones to provide premiums. Numerous factors can be mentioned to explain the patterns, and no single explanation can be utilized (at this point) to predict the future behavior. The amount of trades is generally low, but with a growing liquidity in the market the patterns can be investigated further.

5.2.5 Investigating the Willingness to Trade Over the Past 4.5 Years

The next step is to utilize the information from the data, in order to model the willingness to trade in the investigated period. With NO3 as the reference market area, the information is combined in this manner:

1. All available market areas at a given point in time are listed for *selling* and *buying*
2. The maximum and minimum prices are extracted to determine the potential to sell and buy in every hour

Figure 10 is a snapshot of the capacity data, in accordance with assumption 3 listed in the beginning of the section, as seen from NO3. The market areas are illustrated as nodes. The nodes are connected by directed edges (connecting arrows) showing what direction, if any, there is available capacity. The weights on the edges represent the available capacity, in MW, on the respective transmission lines when the Elbas market opens. The assumption refers to the lower capacity limit that is implemented, removing edges with less than 50 MW capacity from the digraph. The limit is set for practical reasons, since smaller volumes are seldom traded. This parameter can be chosen to fit the characteristics of any market participant.

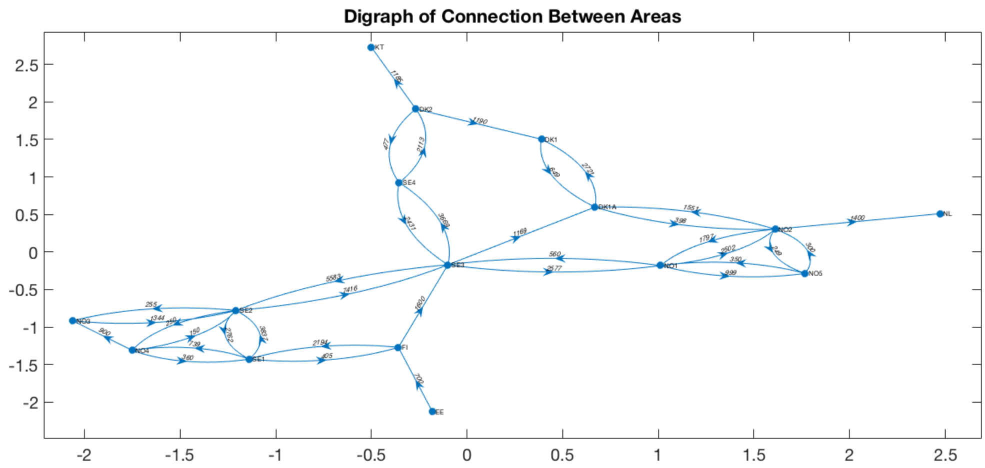


Figure 10: Digraph showing the market situation during the 7^{th} production hour on 28^{th} of July 2013 (arbitrarily chosen).

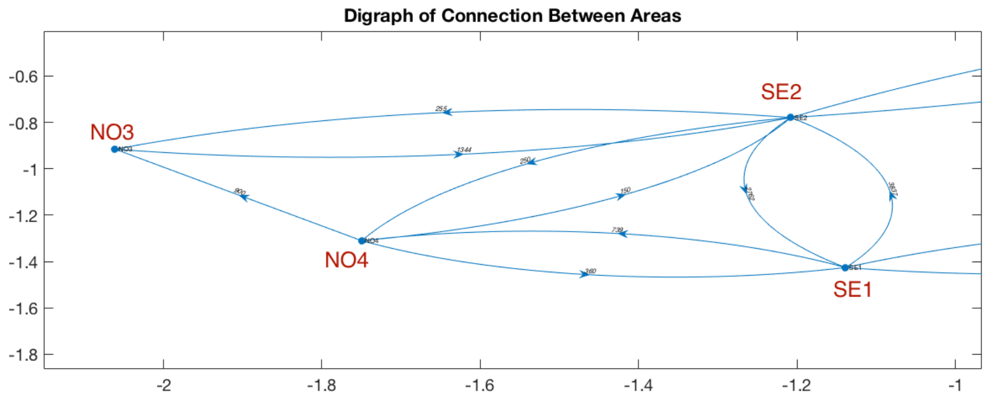


Figure 11: A Close up from Figure 10, Showing the Capacities on Connecting Transmission lines.

The digraphs in Figure 10 represents capacities in one specific hour. For analytic purposes a script runs to achieve this information for all hours in the sample data set. Figure 11 is a zoomed-in version of Figure 10, made for illustration purposes.

With knowledge of availability, we now consider what areas - among the available - have the most attractive prices. Attractive here refers to high for *sales* prices and low for *buy* prices. The potential to sell and buy is different, and is based on the available capacity to export and import respectively. To investigate the potential to *sell*, an inbuilt function in MATLAB - *shorthestpath* - is utilized, and results in a number of available areas depending on the information from the capacity table. The area associated with the highest price is noted, and the price is put into a time series. However, the areas available to *buy* from might be different. These are obtained by running the same function, only now switching the start and stop node. A similar time series is created.

To summarize, we have that evaluating the area price in all available areas, a maximum(export) and minimum(import) price is found through all hours in the investigated period, which represent the best trade prices available for NO3. This lead to information of potential willingness to trade at during the 4.5 years, organized in two separate time series with evenly (hourly) separated prices. The time series representing the *buy* prices are given by the minimum available prices, while the potential *sell* prices are given by the maximum available prices. Figure 12 illustrates the upper and lower price potentials from historical data from January 2017. The system price is illustrated in the same plot, to show how bottlenecks affect the prices in the areas. The creation of the two time series is performed on the entire data set. The reason that Figure 12 only illustrates one month, is that a longer period would be at the expense of readability, and the essence is illustrated in this figure.

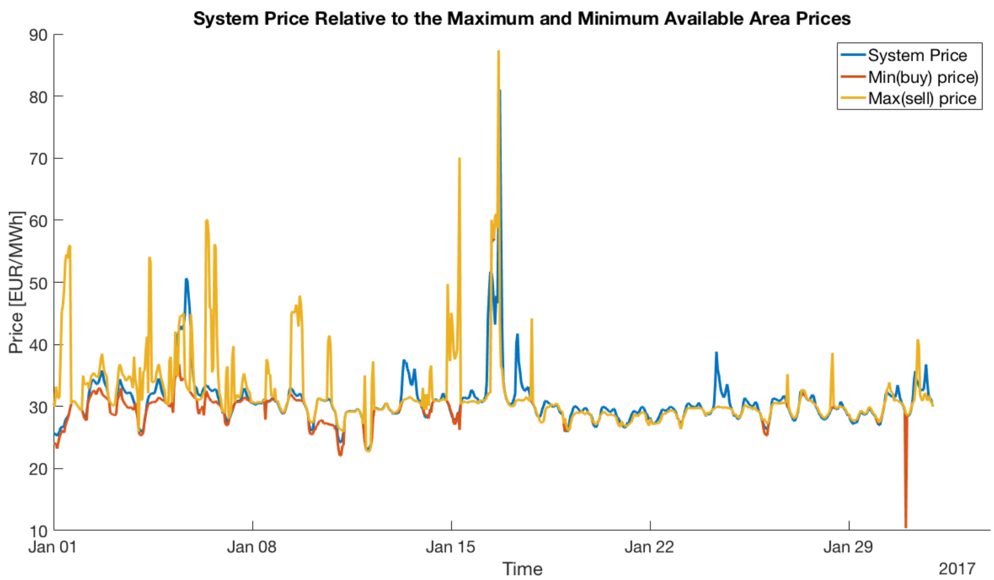


Figure 12: Shows variations in the highest and lowest available price areas at a given time, throughout January 2017.

It is important to note here, that these are all day-ahead *spot prices*, and does not provide an upper or lower bound for the historical Elbas prices. I.e. the Elbas prices *could* lay above maximum or below minimum spot prices. However, the convenience of the spot price information is that it is correlated with Elbas prices as stated in assumption (3).

5.2.6 Time Series Analysis - Correlation Within the Data Set

Some tests to investigate auto-correlations were run, and the results can be seen from the figures 13, 14, 15 and 16. All figures consider auto-correlation of time series with the length of a week, i.e 7 days or 168 hours.

Figure 13 illustrates the auto-correlation in the *buy* prices, while Figure 14 illustrates the same for *sales* prices. The auto-correlation in a time series indicates how the time series is correlated with it self at different time lags. The first line in each figure, indicating the correlation with a time lag of 0, is equal to 1 for both *buy* prices and *sell* prices, since every price point in the time series is fully correlated with itself. Then the auto-correlation decreases as the time lag increases, before increasing to a local maximum 24 hours later. The same pattern repeats itself, indicating a seasonal effect where equal production hours have greater correlation than other price values in between. Further discussion about seasonal effects in Section 5.2.10. Though both *buy* prices and *sell* prices shows the same seasonal pattern, the value of correlation in *sell* prices is significantly lower than for the *buy* prices.

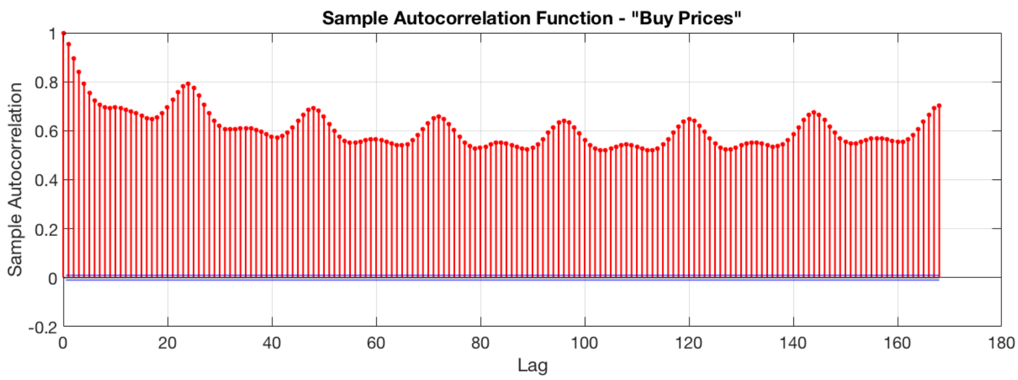


Figure 13: Illustrates the auto-correlation between prices at time lags up until a week, in the time series of *buy* prices.

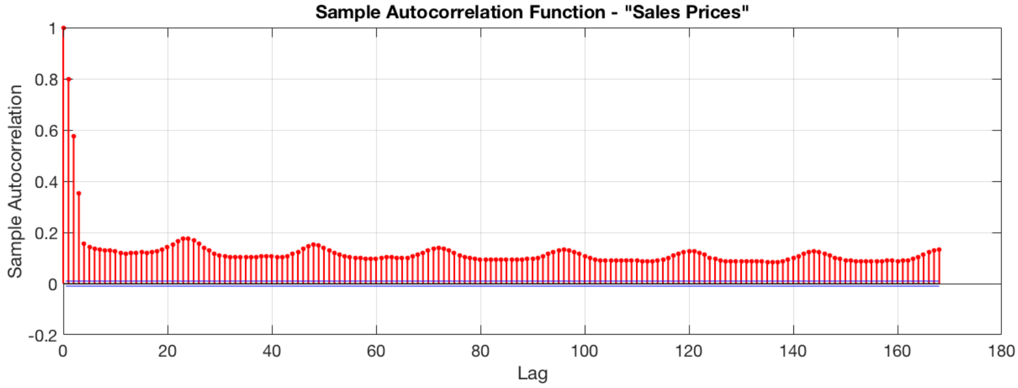


Figure 14: Illustrates the auto-correlation between prices at time lags up until a week, in the time series of *sales* prices.

However, while these plots indicates that there is auto-correlation in the time series, it does not control for correlation at shorter time lags, in between the points in question. The partial auto-correlation is a measure of auto-correlation where the correlation of all shorter time lags are adjusted for. These plots are illustrated for *buy* prices and *sell* prices in figures 15 and 16 respectively. While the *auto-correlation* values were relatively large for the time series of *buy* prices, the *partial auto-correlation* values indicates that there is a huge difference in correlation when the time lag increases from 1 to 2 hours. A way to interpret this is that the prices at greater time lags rather has a high auto-correlation to the price in question due to the large correlation to the values in between, all of them with a time lag of 1 between each price. For *sales* prices, we observe something similar, but the partial auto-correlation does not decrease as rapidly as in the time series of *buy* prices.

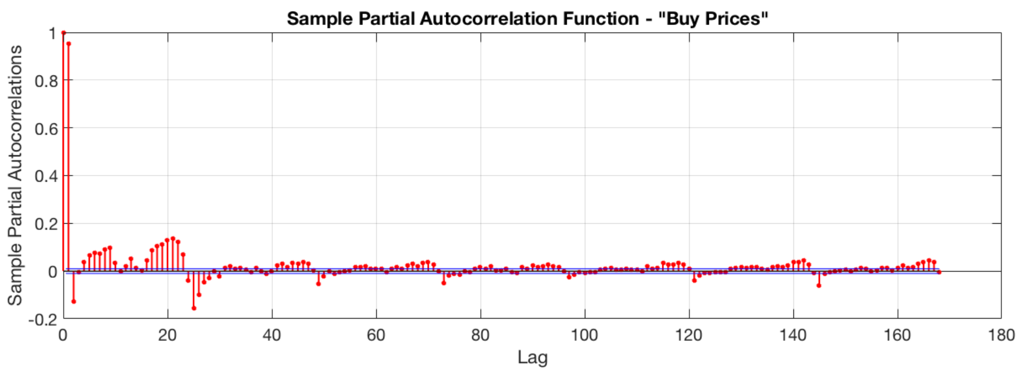


Figure 15: Illustrates the partial auto-correlation between prices at time lags up until a week, in the time series of *buy prices*.

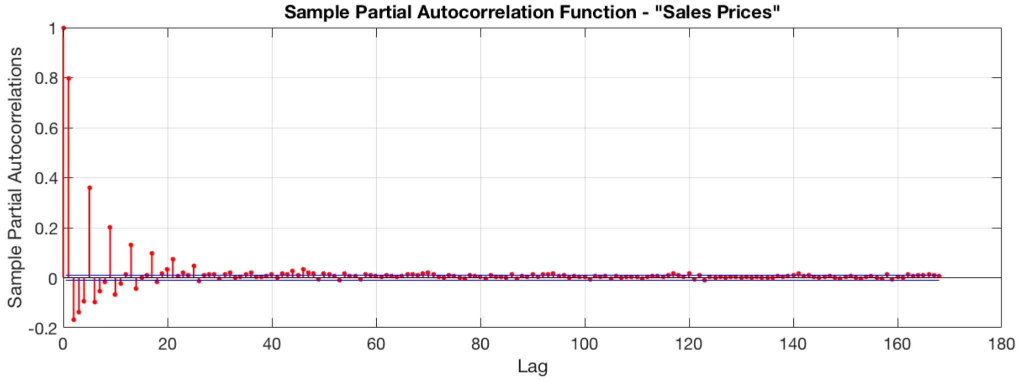


Figure 16: Illustrates the partial auto-correlation between prices at time lags up until a week, in the time series of *sales prices*.

An interesting observation is the correlation between every point in the time series, and the previous point. This correlation is observed to be quite a lot larger than correlations with all other points, with a larger time lag.

5.2.7 Modelling Seasonality - Daily Variations

By investigating the Figures 14 and 13 it becomes clear that there are some seasonal variations in the data set. The plot shows how the whole data set performs over time, if accumulated into 180 hours. From the more than 40 000 hours in the data set, it can be of little meaning to plot over a one year period, because 1 year periods have only happened 4.5 times during the period. To take from this plot is 1) Numerous 180-hour periods fit into the data set, so the information is based on a lot of data, and 2) the repeating pattern every 24 hours, pointing towards daily trends. Moreover, the daily price variations are looked into, before we discuss how to account for this in the model.

The price is normally lower in the morning and during the night when electricity demanded is relatively low. The price normally peaks around 8-9 AM in the morning when people prepare for work and school, and again when they return to cook dinner around 5 PM, see Figure 17. In the figure all days during 2016 are averaged for 12 different spot price areas, to illustrate daily variations. In addition, the system price is plotted in the upper left corner. The figure shows a pattern, but also that the pattern in different areas differ slightly. The prices are the actual spot prices, not as *seen* from NO3 (see Section

5.2.2). The major discovery, is that one would expect a product during peak hours to be more expensive than other products. The reason is that a higher demand results in a supply-demand relationship where the willingness to pay increases to fulfill demand, unless supply increases simultaneously. These daily variations must be accounted for in the model. There are several ways to try to handle daily variations when modelling, as mentioned in Section 3.4.1.

Since the willingness to pay for electricity often varies with the product type (we recall that electricity in each hour during the day represent different products, see Chapter 2), it is important to have a way to differ between modelling willingness to pay for "4 AM-products" relative to "8 AM-products" - when the willingness to pay is higher on average. In other words, some product prices are more likely to be in high ranges of the price state space, for example night product prices are seldom high.

This is taken into the model by setting the initial willingness to pay for products in Elbas equal to the Spot price for the production hour. In an average day, spot prices will peak around certain hours. Considering the 24 hours with different spot prices, each actual spot price correspond to the initial Elbas price forecast for electricity in those hours in the model, based on a clearing the day ahead. Moving forward in time, the price stays the same unless information is added. The relevant information to update the current state would specifically be trades of the same product. These propose new information about the current situation. The technicalities are specified under Section 5.3.

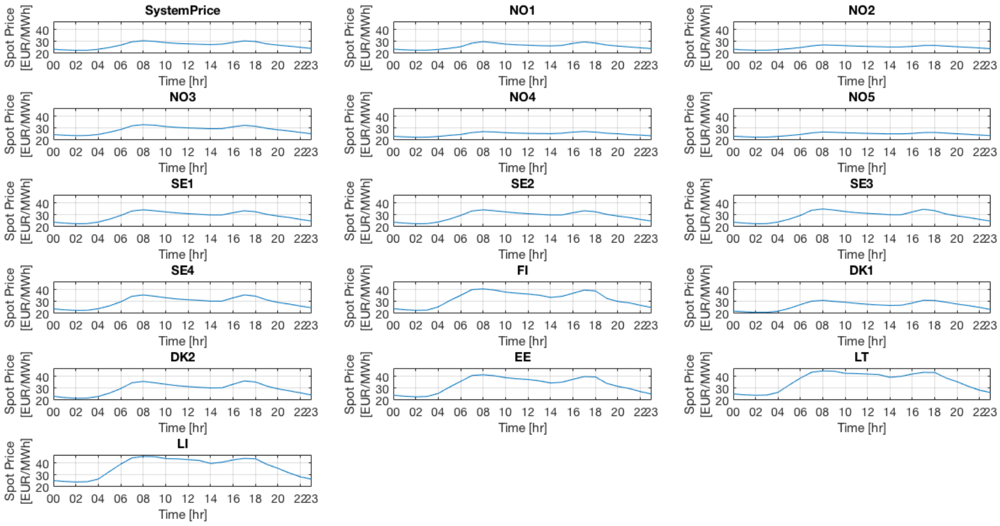


Figure 17: Average Daily Variations in Spot Prices in Different Areas.

A product belonging to peak hours, initialized with a high price, and the opposite for non-peak products.

5.2.8 The Model: A Markov Process

The chosen model is build on the Markov property, in accordance with assumption 5 in the list in the beginning of the section. This, due to the results from the auto-correlation tests in Section 5.2.6, that revealed a large correlation between one data point and the previous, together with the fact that a lot of literature (Chap: 3) suggests the use of a Markov model, within the field of electricity prices. We recall from Section 3.4.2, that in a Markov process it is enough to have information about the current state and a transition matrix in order to determine the probabilities of the next movement.

Moreover, due to large amounts of empirical data, an empirical analysis is chosen. The stochastic process is one where the transitions between states is described by a transition matrix, P (Sec: 3.4.2). P will be formed on the basis of counting transitions from one time step to the next. However, the creation of P can be performed in different ways. We later define some cases, where each will result in a different matrix representing the same Markov process. In this section we aim to form the most suitable matrix P, which later will be used as input to a dynamic program.

To evaluate the different matrices a Monte Carlo simulations are run. Here, paths will be generated based on the P matrix, and compared to large sets of OOS data. In addition a long term evaluation of the P matrix is performed, to find out how well the model fits the OOS data. Both methods are explained in depth later, and only presented to give a context to the proceeding analysis.

When simulating (Sec: 5.2.10), a goal is to simulate realizations of the process that are correct in expectation. In other words, if the number of scenario estimates goes towards infinity or a sufficiently high number, the simulation should *not* have a systematic error. This is discussed further in Section 5.2.10.

5.2.9 Adjusting the Data Set

At this point, the data set has the format of two price vectors, with the maximum available *sales* prices and minimum available *buy* prices as described in Section 5.2.

Before starting the analysis, it is beneficial to adjust the data set for some variations in order to make it easier to work with and analyze. Different approaches can be used to get rid of external variations, and the goal is to transform the data into having close to white noise. Trends and variations driven by externalities will be removed in a manner that allow us to transform it back later. This task does not have one unique way of being performed, but in this study three adjustments are made.

1. Normalization of data to a common price driver
2. Linear detrending
3. Probability integral transformation

Normalizing the Data Set

The first step is to, in accordance with assumption 4 in the list in the beginning of the chapter, normalize the spot prices by dividing on the SRMC of coal at that specific point in time. The arguments of this approach is that the water value is a function of all other alternative energy sources in the market. In a market, the power price is a function of the product mixes that constitutes the market products. When calibrating a model it is optimal, but is non-trivial, to calibrate for all effects that have an impact on the price. Moreover, other approaches could have been utilized giving another result. To only calibrate for coal has some weaknesses, but is on the other hand a reasonable place to start, since the SRMC has a great impact on the market price. A point to notice when this operation is performed, is that the relationship between spot price and SRMC collapses from time to time, so that the SRMC does not fully cover the adjustment needed to get a constant mean throughout the period. The reason is is that in some periods there are enormous amounts of unregulated production of wind and water power, which leads to a spot price noticeably below the SRMC-price.

The *sales* and *buy* prices are plotted in Figures 18 and 19 respectively. The mean and variance for each year is calculated and shown here, and will be discussed further at a later point. Two lines of degree one and two are fitted to the data set using the inbuilt function *polyfit* in MATLAB. These show regressions, and it is easily noted there is a downward sloping trend in the mean of the normalized prices from 2013 to 2017.

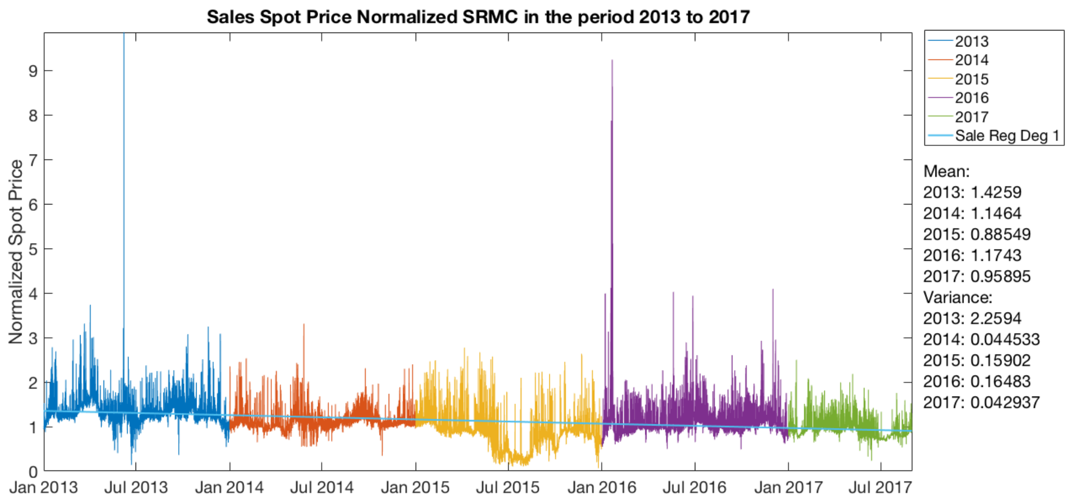


Figure 18: Hourly *sales* spot price normalized for the respective SRMC of coal. Data from the 1st of January 2013 to the 30th of August 2017.

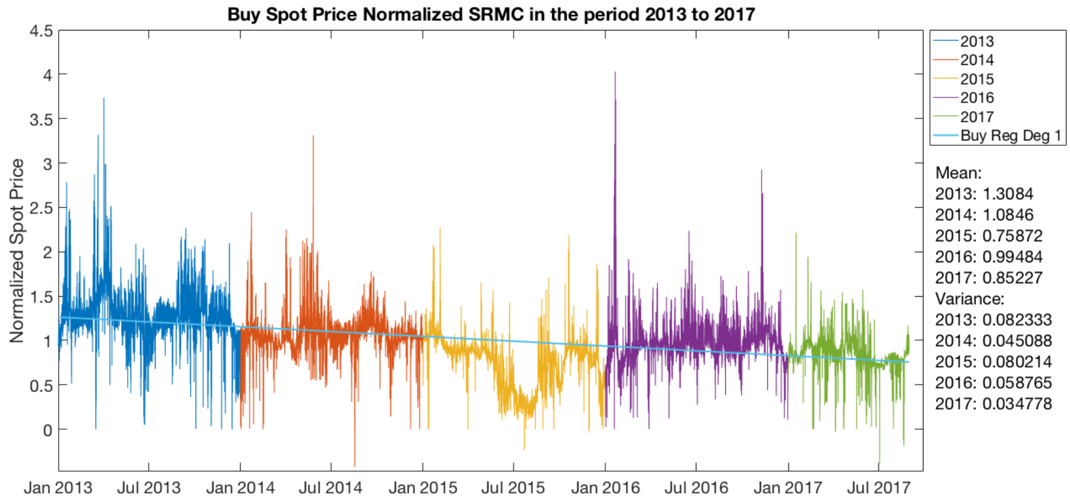


Figure 19: Hourly *buy* spot price normalized for the respective SRMC of coal. Data from the 1^{st} of January 2013 to the 30^{th} of August 2017.

After normalizing the spot prices and finding that the data set still does not have the desired properties, additional operations to remove external factors are investigated and performed. Firstly, a linear trend is removed.

Linear Detrending

As can be seen from Figure 18 and 19 in the previous section (Sec: 5.2.9), there is a trend in the data set. The trend is downward sloping. The goal of the normalizing is to extract all external noise, but as expected, more factors than coal must be adjusted for to remove noise.

One reasons for this downward sloping trend can be the increasing supply of RES, s.a. wind power (Chap: 2). With increasing supply, the prices will decrease if everything else is held constant. How the development of RES will evolve is hard to say, and not the scope of this project. It is here assumed that the trend will continue in the same downward sloping manner. In reality the wind power supply, or supply of other renewable energy sources, could increase remarkably, leaving the price process with a steeper downward sloping trend. On the other hand, demand could increase, or the supply stagnate resulting in the spot price trend flattening out.

The next step is to detrend the data set in accordance with the above argument of a linear trend. The detrending consists of subtracting the expression for the linear regression from the data set. The function expression $y = ax+b$, where a is the regression slope and b is the offset or the constant term, is stored for use in the end of the process when the trend is to be added again.

For the example case, the regression is found to have the values shown in Table 4:

Slope (a)	Constant (b)	R
$1.097082 \cdot 10^{-0.5}$	-0.166303	1.353899

Table 4: Linear trend for example case.

, where R denotes regression values for each of the matrix rows.

After finding and adjusting for the trend it is dersirable to comment on the associated confidence interval. To determine this, we want to investigate if the data set is normally distributed, because then it is straightforward to determine the confidence interval. To determine the fit to the normal distribution a QQ-plot is made, and can be seen from Figure 20.

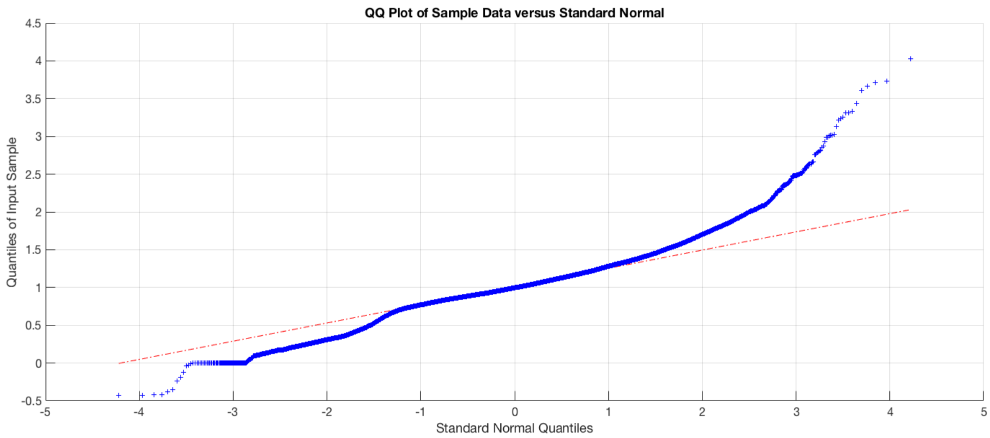


Figure 20: QQ Plot of Normalized Data Set - Buy Prices

Observing the plot in Figure 20 it is noted that in the middle, the points fall along the red line, but the points deviate in both extremities. Based on the shape of the points it can be concluded that the data set has the properties of a heavy-tailed normally distributed data set (Ford, 2015).

The properties from the normal fit are showin in Table 5:

$\underline{\mu}$	μ	$\overline{\mu}$	$\underline{\sigma}$	σ	$\overline{\sigma}$
1.1222	1.12974	1.13729	0.772891	0.77819	0.783562

Table 5: Parameters for normal distribution fit

, where μ is the mean and σ is the standard deviation. $\underline{\mu}$ and $\overline{\mu}$ are the 95 % confidence interval for μ and $\underline{\sigma}$ and $\overline{\sigma}$ are the 95 % confidence interval for σ.

Figure 21 shows the normalized data set before and after detrending. The downward sloping trend is adjusted for, as well as shifted around 0.

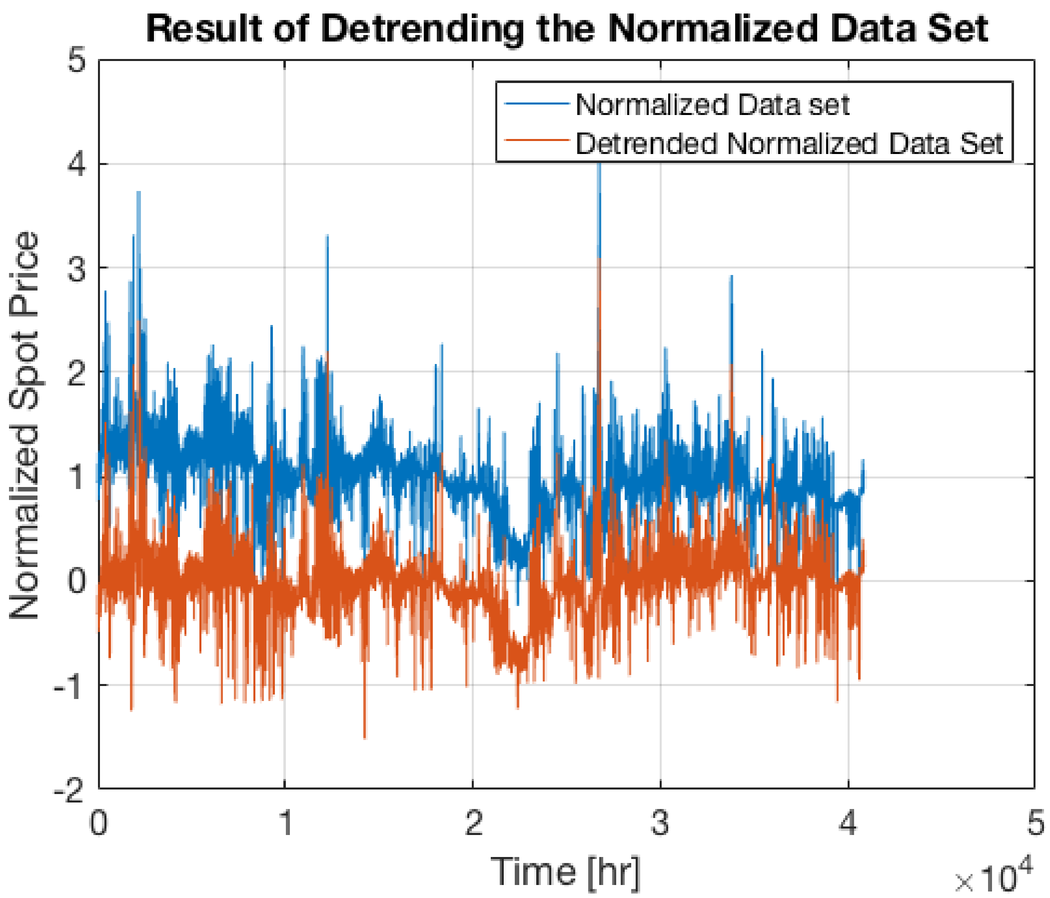


Figure 21: Illustrates the Normalized Data Set vs. the Detrended Normalized Data Set

Probability Integral Transformation

The last adjustment of the data set is to perform a PI transformation. In order to perform the transformation we must find the distribution of data points.

Firstly, all prices during the entire period from 2013 to 2017 are sorted by size, and thereafter plotted. The result can be seen from Figure 22, which shows that the major part of the prices are in the middle, whilst the number of really high or low prices are small in comparison.

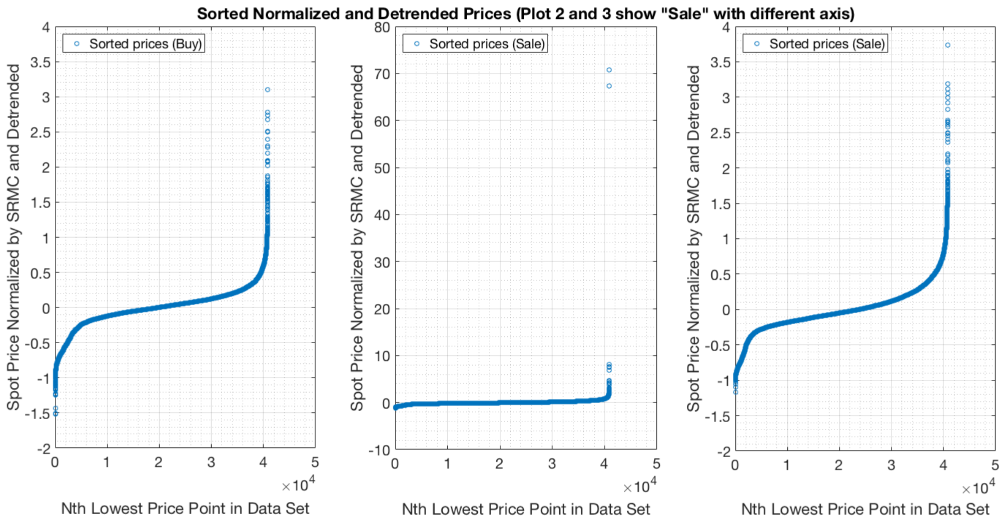


Figure 22: Plot of the sorted normalized price data.

Moreover, the empirical probability density function (pdf) is plotted as the red curve in Figure 23.

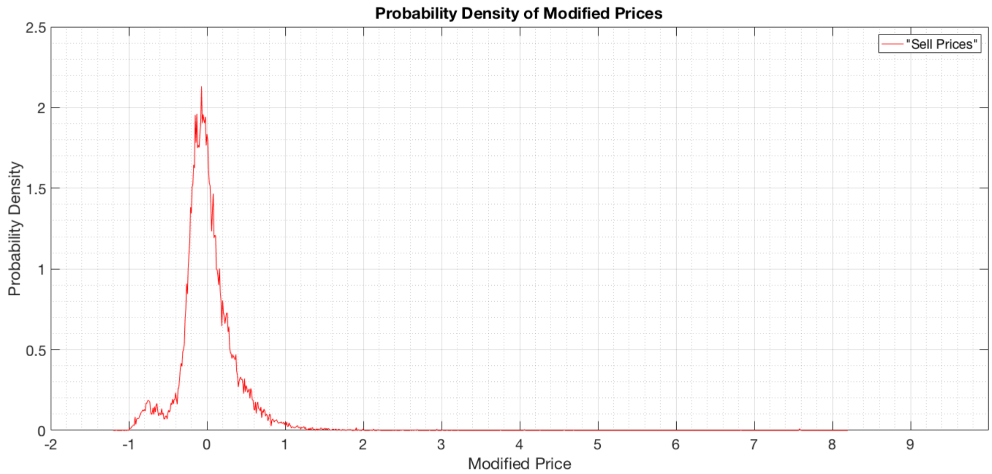


Figure 23: Probability Density Function.

The empirical cumulative distribution can be seen from the scatter plot in Figure 24.

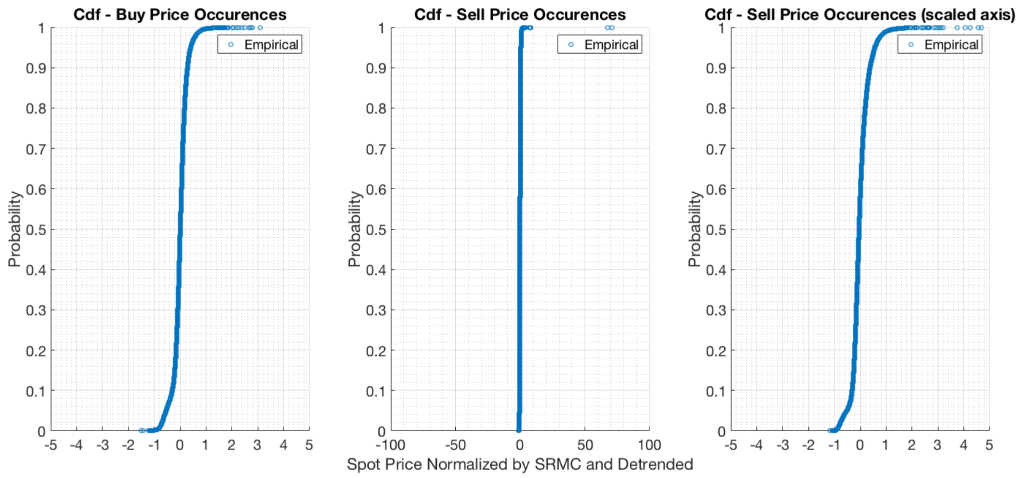


Figure 24: Cumulative Distribution Function. Illustrates the Probability of a Price Having a Lower Value than a Given Value Along the x-axis. Note that subplot 3 is a zoomed-in version of subplot 2.

Subplot 3 in Figure 24 is a zoomed-in version of subplot 2, and is brought to illustrate that if the long upper tail is cut around normalized *sales* price 8, it has a similar shape as the *buy* price curve. It is to be noted that the probability that the normalized price is below 0.02 for instance is very low, whilst the probability of a price being below 4 is close to 1, both for *sales* and *buy* case.

One issue with empirically generated cumulative distributions, such as in Figure 24, is that is can be problematic when transforming back, since it's continuous function is unknown. Transforming a set of data points into probabilities between zero and one is straightforward based on the empirical knowledge. The problem, however, occurs when inverse transforming from a discrete function where data points may be off the defined areas of the curve. To solve this, a continuous function had to be estimated. As discussed in Chapter 4, and suggested by Nowotarski and Weron (2018) a fit of the empirical cdf must be found. In order to find a curve the approach was to examine the normal distribution, once again. A similar investigation as discussed under the section of detrending the data set. However, for the purpose of this operation we conclude that the normal distribution is not a suitable model. The data is not a time series for this consideration, but rather a probability snapshot of the likelyhood associated to different prices, regardless of time. We aim to have a curve that follows the real cdf, with deviations close to zero. It is not of a high priority that the distribution must be very common, or have standard properties. The aim is an accurate fit. A shift in the estimated curve relative to the real distribution would provide critical shifts which would be a source of error. The most important is that the inverse transformation *is* in fact the inverse process of the transformation and not a source of error.

The Kernel distribution, is investigated. In Figure 25 the Normal Kernel distribution fit (black line) is illustrated along with the empirical pdf (red line). Here with a bandwidth of 0.0281. The bandwidth is a measure of the smoothness of the resulting density curve. It is out of the scope for this study to elaborate further on this.

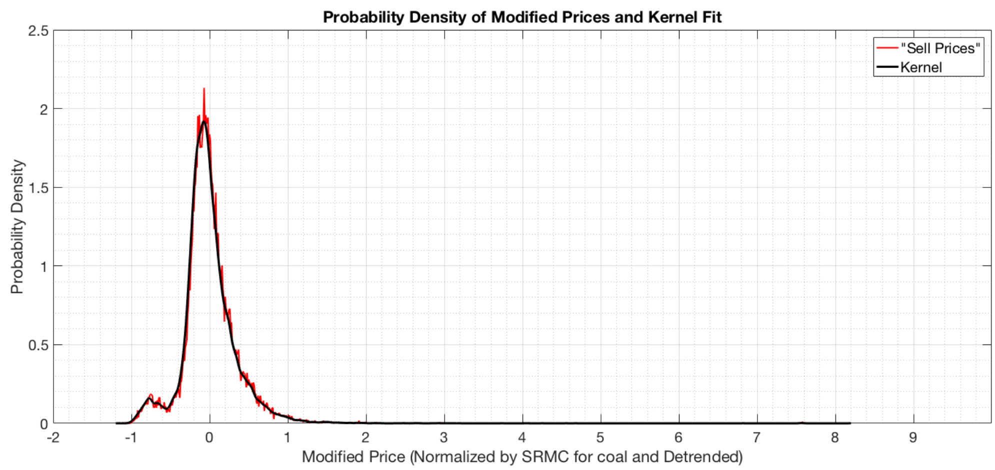


Figure 25: Probability Density Function. Empirical and Kernel Normal Fit

The fit was a lot better than that of the normal distribution, which can be seen from Figure 26.

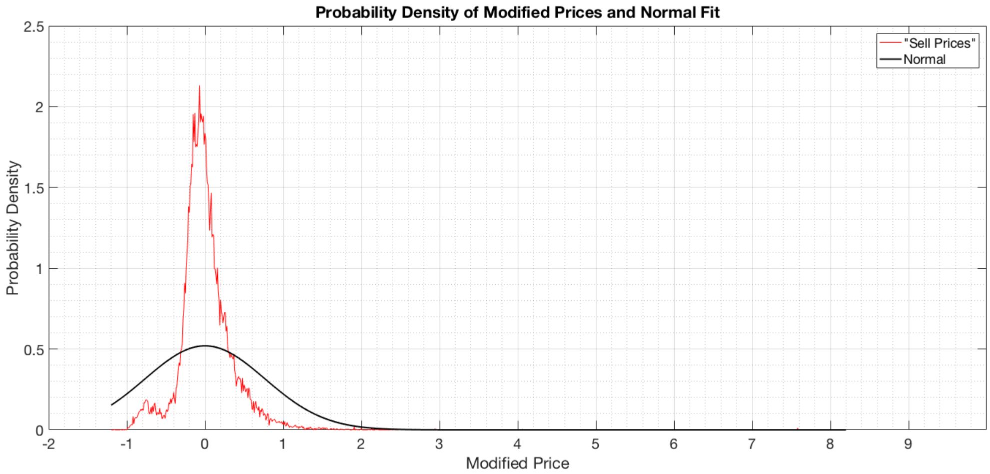


Figure 26: Probability Density Function. Empirical and Normal Fit

The cdf associated with the Kernel distribution is found for both *sales* and *buy* prices and plotted in Figure 27

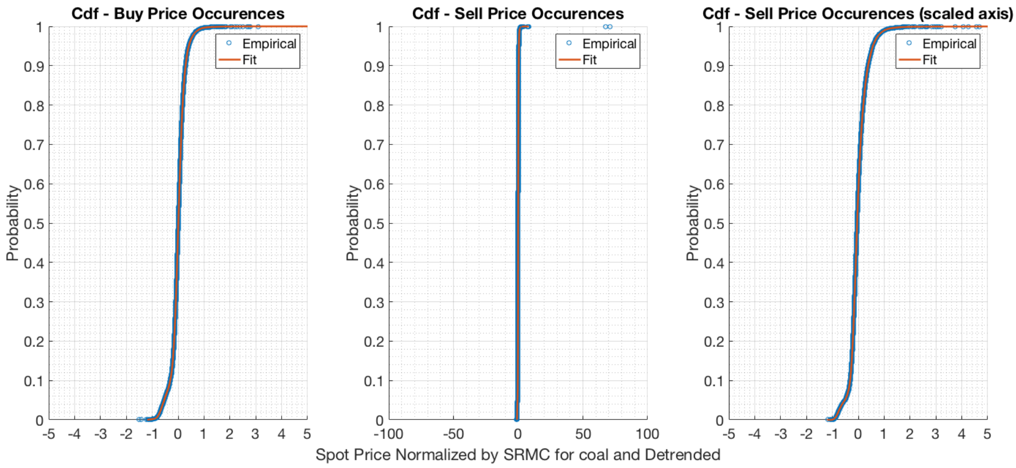


Figure 27: Cumulative Distribution Function. Illustrates the Probability of a Price Having a Lower Value than a Given Value Along the x-axis. Note that subplot 3 is a zoomed-in version of subplot 2.

An alternative could have been to form the cumulative distribution function by drawing lines between the discrete points in the empirical cumulative distribution, assuming linear interpolation between known data. Creating a polynomial fit based on minimizing the mean squared error based on regression among known points was also investigated, but the degree had to be very high to be able to fit the curves, which made this less practical.

An argument to using the Kernel estimation to fit the curve rather than for example a linear interpolation is the simplicity of utilizing it along with it's good fit. Different variants of the Kernel distribution exist. Here, the utilized version is the *Normal* variant. We choose to not go into depth of the properties of this distribution, other than what is of most importance: namely, that it's cdf is easy to obtain from the pdf to thereafter use it as the basis for the inverse transformation, which solves the challenge discussed.

In Figure 24 the long tail in the middle curve tells us that the ratio of spot price divided by the SRMC of coal is extremely high (spikes) at some points in time.

With this information, all prices were transformed, resulting in values between 0 and 1. The transformed time series can be seen for *sales* and *buy* prices in Figures 28 and 29 respectively.

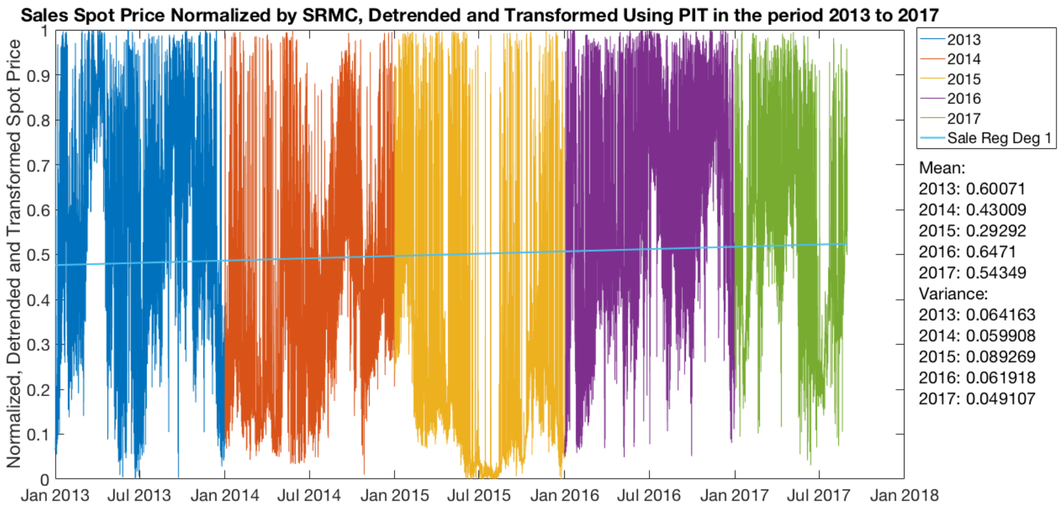


Figure 28: Hourly *sales* spot prices after normalization by nespective SRMC, detrending and transformation using PIT. Data from the 1^{st} of January 2013 to the 30^{th} of August 2017.

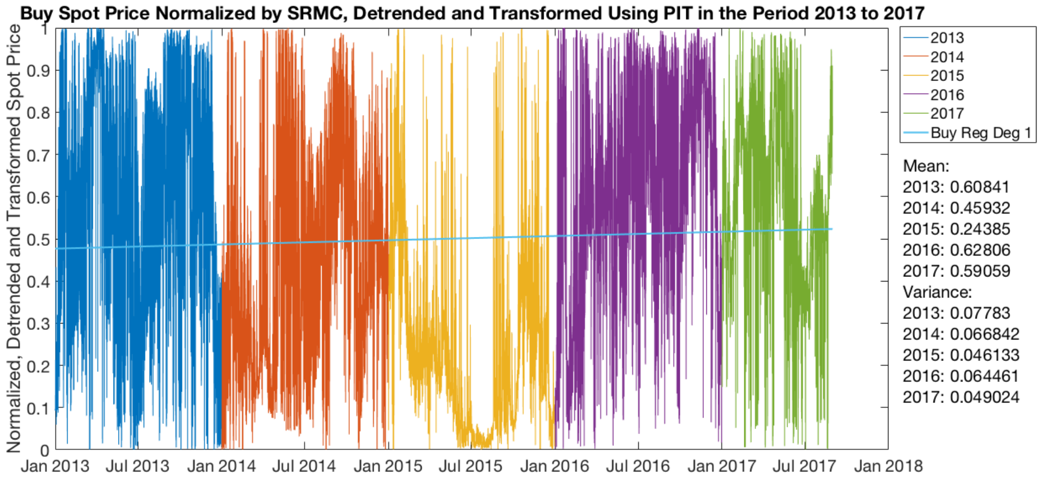


Figure 29: Hourly *buy* spot prices after normalization by respective SRMC, detrending and transformation using PIT. Data from the 1^{st} of January 2013 to the 30^{th} of August 2017

If we compare Figure 28 and 29 to Figure 18 and 19 we note that the mean of 2015 is still the absolute lowest, which is to expect. Moreover we note that the spikes no longer exist, and that the entirs process now lies between the values zero to one. Whilst the ratio of the second largest to the largest variance for *sales* prices in Figure 18 is 7.2 %, the same ratio from the transformed and detrended prices in Figure 28 is 72 %.

All these adjustments to the data set entailing normalizing, detrending and PI transformation have been performed on all years 2013-2017. The data set is now closer to having white noise.

5.2.10 Determine The Transition Matrix

The data set is now more convenient to proceed with. The next step is to decide the process model parameters in order to form the transition matrix. When a data set is to be modelled as a Markov processes, there are different approaches to develop the transition matrix. They all rely on the Markovian property, stating that all the information about the past, is stored in the previous state, which can be formulated mathematically as in Equation 2.

Some of the questions to answer are:

1. What is the most logical way to divide the state space? I.e. how many levels, $\overline{U}$, is it best to have, and by what method should the state space be split?

2. How far back, i.e. how many hours, should the step path dependency, L, be modelled to account for in the transition matrix? Does the model improve if more than one previous step is considered when moving forward?

3. What data period is most suitable to make up the IS and OOS data space?

In order not no confuse the reader we emphasize that the number of levels describes how many parts the whole price state space is split into. Two different level sizes, $\overline{U} = 5$ and 7, are investigated. The trade-off to be ware of when investigating $\overline{U}$ is that it must be large enough to provide information which is not too general, and at the same time small enough to avoid an uncontrollable dimension size 3.6.2. The question of *where* to put the limits between the levels in the price state space will be discussed shorty.

The path dependency describes how many trailing numbers that *make up a state*. In a standard Markov process this is 1. We extend the state concept to possibly include 2,3,4,5 or 6 trailing steps, so that when accounting for the previous extended state it really entails information about more of the history. We investigate if this makes a difference through analysis. A transition matrix with path dependency 3, has the format shown in Figure 5.2.10:

$$\text{[H] P} = \begin{array}{c c} & \begin{array}{cccc} 111 & 112 & ... & 555 \end{array} \\ \begin{array}{c} 111 \\ 112 \\ ... \\ 555 \end{array} & \begin{pmatrix} P_{111,111} & P_{111,112} & ... & P_{111,555} \\ P_{112,111} & P_{112,112} & ... & P_{112,555} \\ ... & ... & ... & ... \\ P_{555,111} & P_{555,112} & ... & P_{555,555} \end{pmatrix} \end{array}$$

Subsections of the data set will be taken out as IS data, leaving the rest as OOS data to test the simulation results on. Due to the constant price fluctuations, the prediction will differ depending on the in sample material. Removing the trend decreased the time dependencies in the time series. The practical interpretation is that whether we choose to model our price process based on data from 2013-2014, or any other combination of years within the period the noise is not supposed to have a great impact. In order to build a robust model, the transition matrix is based on data from a long period. We have chosen to proceed the analysis with two different time slots as IS. Two, because it can show the difference in results due to input variables, and no more cases due to time constraints. The variants tested are, 2013-2016 and 2014-2017, with the complements 2017 and 2013 as OOS respectively.

All of these questions have been approached in this section. Throughout the analysis more modelling challenges will be discussed. Figure 6 shows all the different combinations of the levels, path dependencies, in sample and out of sample combinations investigated.

Levels	Path Dependencies	Type of Trade	In Sample Year	Out Of Sample
5	1-6	Sell	2013-2016	Random in 2017
5	1-6	Buy	2013-2016	Random in 2017
5	1-6	Sell	2014-2017	Random in 2013
5	1-6	Buy	2014-2017	Random in 2013
7	1-4	Sell	2013-2016	Random in 2017
7	1-4	Buy	2013-2016	Random in 2017
7	1-4	Sell	2014-2017	Random in 2013
7	1-4	Buy	2014-2017	Random in 2013

Table 6: Overview of the Different Combinations of Input to Determine the Transition Matrix

Separate tests are run for the different combinations of input data: 36 different for level 5 and 24 cases for level 7. Note that path dependencies above 4 are not tested for 7 levels because of the quickly growing transition matrix that increases in accordance with Equation11:

$$\overline{S} = \overline{U}^{L} \tag{11}$$

Throughout the analysis, one of the cases is used to exemplify the procedure. Whenever it is referred to the *example case*, it is referred to the case in Table 7:

$\overline{U}$	L	$\overline{S}$	IS years	OOS
5	1	$5^1 = 1$	2013-2016	2017

Table 7: Example Case

The procedure also primarily discusses *Sales* prices, but the procedure is the exact same for *Buy* prices. In the end of this section, results for all cases will be presented.

Firstly, the chosen IS period is split into levels with equally many data points in each. I.e. the limits between the levels are such that the probability of being in each level is equally large, namely $\frac{1}{\overline{U}}$. The exact limits are set as the average between the upper point in the lower level and the lower point in the upper level.

Based on the limits made for IS data, the OOS data is organized into $\overline{U}$ levels. The data, both IS and OOS are organized into a table we refer to as *State Table*, which contains data for the entire period. The number of points within each level is now not by definition equal anymore. However, if the fraction of points in the OOS period are split exactly the same way as in the in IS data this would still be the case. This will be more discussed below, when discussing limiting probabilities. The purpose of converting OOS data, not only IS data, is to obtain values in the same format for comparison and testing. State Table creates the basis for the simulation. The share of points within each level is shown in Figure 30, for the example case.

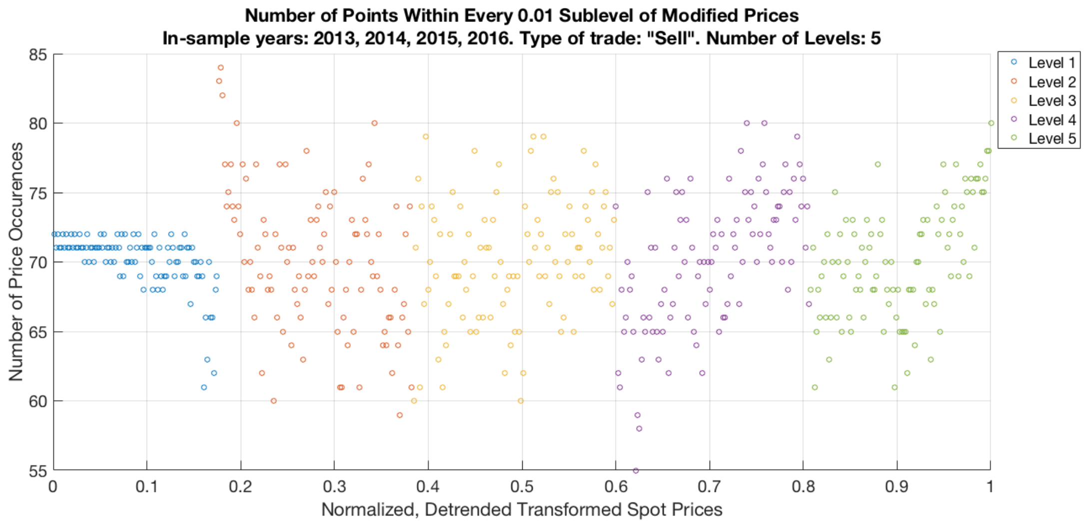
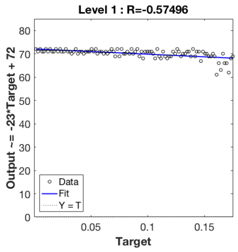
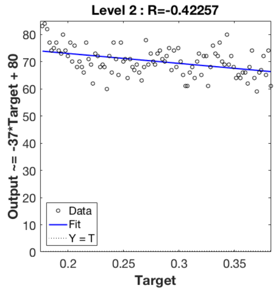


Figure 30: Scatter plot Showing Point Spread Within Each Level of the Normalized, Detrended and Transformed Price State Space. Here, In Sample Years are 2013-2016.

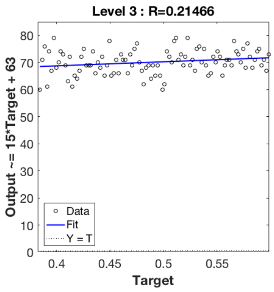
In order to determine the mean of each level, a probability distribution is fitted to the data set within. The A triangular distribution, i.e. a linear regression, is utilized. Other distributions could have been utilized. However, it is convenient for practical, modelling purposes that all levels follow the same distribution. The fitted curves for each level in the example case, are illustrated in Figure 31.



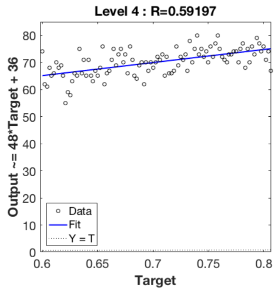
(a) Level 1



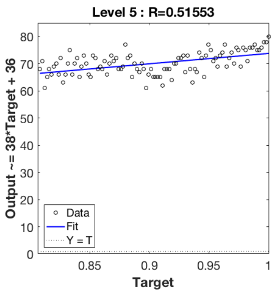
(b) Level 2



(c) Level 3



(d) Level 4



(e) Level 5

Figure 31: Regression of Normalized, Detrended and Transformed Price Points Within Level 1-5

We note that the data set has become more statistically easy to work with. Figure 40 in Appendix A shows how the data set looks if it is split into states after normalization, but *before* detrending and performance of PI transformation.

Methods to Evaluate P

There are certain properties that must be investigated when using a Markov process. In a broad sense, it is interesting to know how well the P matrix is suited to imitate the OOS data.

Are the simulated process and the OOS process having the same first raw moment, or mean? Do they have the same expected value if the processes are repeated, and if not, is the deviation large or insignificant? This is where the importance of all previous operations to remove externalities and noise are useful.

The goal is to achieve a process that does not deviate significantly from the OOS process' expected value, i.e. the P-matrix with the least error when forecasting OOS data. In order to comment on the deviations two methods are utilized. The first is related to analytic properties of the transition matrix, P, whilst the second deals with simulation.

Method 1: Long Term Behaviour

The P-matrix contains a lot of information about the process. As we recall from the literature study 3.4.2, the Markov chain has a unique equilibrium if the process is irreducible and aperiodic. We consider the P-matrix of our example case:

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{matrix} \quad 1 & \quad\quad 2 & \quad\quad 3 & \quad\quad 4 & \quad\quad 5 \end{matrix} \\ \begin{pmatrix} 0.8900 & 0.0750 & 0.0131 & 0.0108 & 0.0110 \\ 0.0819 & 0.7720 & 0.1066 & 0.0245 & 0.0150 \\ 0.0096 & 0.1241 & 0.7100 & 0.1220 & 0.0344 \\ 0.0076 & 0.0150 & 0.1449 & 0.7077 & 0.1248 \\ 0.0108 & 0.0140 & 0.0254 & 0.1349 & 0.8149 \end{pmatrix} \end{array} \tag{12}$$

It can easily be seen that the chain is irreducible, since all P(i,j) > 0, in accordance with Equation 4. Moreover, it is also aperiodic since the greatest common divisor equals 1, which can be directly concluded from the fact that all states communicate in the first P-matrix.

In this case it is designed to be:

$$\hat{\mathbf{\Pi}} = [\hat{\Pi}_1 \quad \hat{\Pi}_2 \quad \hat{\Pi}_3 \quad \hat{\Pi}_4 \quad \hat{\Pi}_5] = [0.2 \quad 0.2 \quad 0.2 \quad 0.2 \quad 0.2] \tag{13}$$

, where all $\hat{\Pi}_i's$ are equal to $\frac{1}{U} = \frac{1}{5}$ for the example case. This means that after a long time one expect 20 % of the data points to be within the different states.

Until now, only the IS transition matrix determined earlier in this section has been investigated. Now, we want to experiment with the properties of the OOS transition matrix P_{OOS}. This matrix is built by counting transition between states in the OOS price process, given that the prices are divided into states in accordance with the *IS limits*. Does P_{OOS} have the a similar long term distribution to P?

The matrix obtained is:

$$P_{OOS} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0.7266 & 0.2031 & 0.0547 & 0.0156 & 0 \\ 0.0196 & 0.8859 & 0.0709 & 0.0162 & 0.0074 \\ 0.0011 & 0.0661 & 0.7982 & 0.1056 & 0.0289 \\ 0.0020 & 0.0094 & 0.1454 & 0.7185 & 0.1247 \\ 0.0011 & 0.0099 & 0.0386 & 0.2252 & 0.7252 \end{pmatrix} \end{array} \tag{14}$$

, with the equilibrium:

$$\hat{\boldsymbol{\Pi}} = [\hat{\Pi}_1 \quad \hat{\Pi}_2 \quad \hat{\Pi}_3 \quad \hat{\Pi}_4 \quad \hat{\Pi}_5] = [0.0220 \quad 0.2539 \quad 0.3106 \quad 0.2573 \quad 0.1562] \tag{15}$$

The desired long term behaviour of these two matrices is desired to be similar, in order for the processes to tend towards the same mean. Before detrending, the problem of simulating with 2013-2016 as IS and 2017 as OOS was that the largest amount of price points in 2017 were located in the bottom levels, due to the downward sloping trend. However, now we note that this is not the case anymore. On the contrary level one is now the one with the lowest $\hat{\Pi}$. In addition, we observe that the vast majority of the price points are located in the middle level and decreases toward the ends. Figure 32 illustrates the difference in the long run probabilities of the different state.

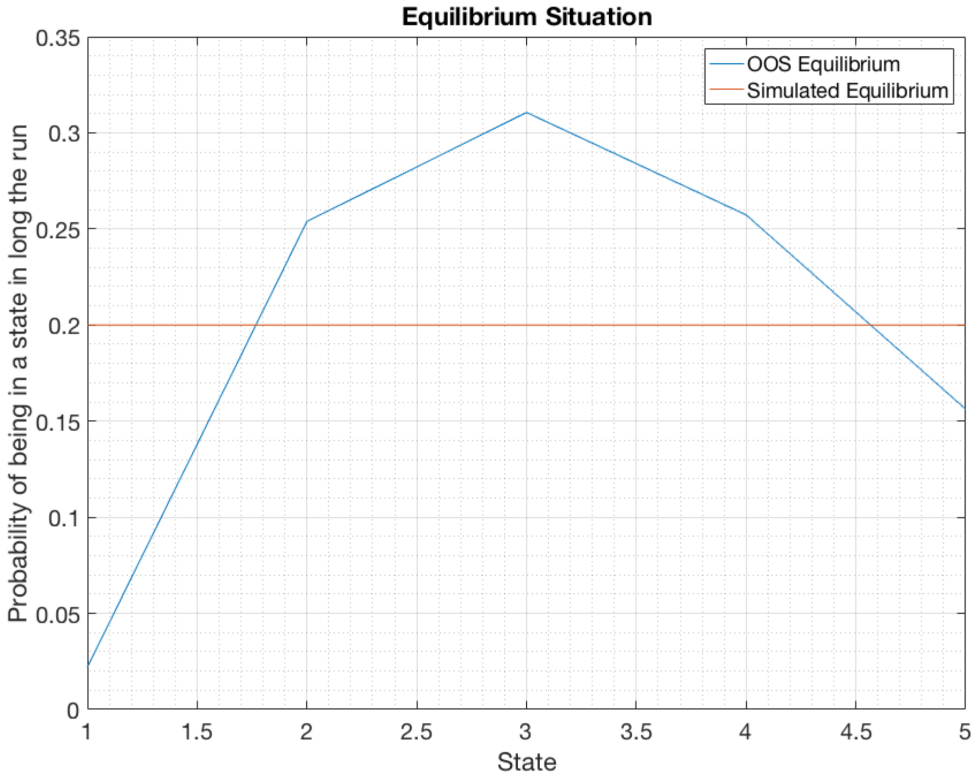


Figure 32: Plot showing the long run probabilities of being in the different states.

From this we conclude that the OOS process is more symmetric that in the case of no detrending, which is among the desired properties. However, the middle weight is too large and the tails too light. In conclusion, the OOS process will, in accordance with what is desired, tend to be in the middle level in the long run. It will in a too little degree tend towards the ends, especially downwards against the lower states. This means that the P matrix that simulates the process has a greater tendency to pull the process towards the ends.

We calculate the expected long run state in both cases, using Equation 7 for the expected long term state, knowing that Equation 8 holds, since the process is defined by 5 states in the example case (Tab: 7):

$$E[X] = \sum_i s_i P(X = s_i) = 1\hat{\Pi}_1 + 2\hat{\Pi}_2 + 3\hat{\Pi}_3 + 4\hat{\Pi}_4 + 5\hat{\Pi}_5 \tag{16}$$

P(X=s_i) denotes the probability of being in a state. Here, equal to $\hat{\Pi}_i$, since equilibrium is reached. The long term expected state is obtained for both P and P_{OOS} in Equation 17 and 18.

$$E[X_{IS/simulated}] = 1 \cdot 0.2 + 2 \cdot 0.2 + 3 \cdot 0.2 + 4 \cdot 0.2 + 5 \cdot 0.2 = 3 \tag{17}$$

$$E[X_{OOS}] = 1 \cdot 0.0220 + 2 \cdot 0.2539 + 3 \cdot 0.3106 + 4 \cdot 0.2573 + 5 \cdot 0.1562 = 3.2718 \tag{18}$$

The expected states are not equal in the two cases, but more similar than if no externalities were removed from the data set. The equilibrium state of the process in this case was found for comparison, and can be seen from Figure 42 in Appendix B. An interesting note to make is that before the data adjustment, utilizing the same method to find the transition matrix resulted in an expectation below that of the model. It is desirable that they are equal, but as we see from the calculation in Equation 18, it is now actually above! This means that the adjustments have over corrected the desired property, resulting in a offset in the opposite direction. Note the small slope in regression lines in Figure 28 and 29. We believe this came from the PI transformation, and might have colored the later procedure. However, the symmetry seen from Figure 32 is a desirable property that was non-existing in 42 in Appendix B.

Method 2: Monte Carlo Simulation

The other way of testing what matrix to go for, is based on the utilization of Monte Carlo simulation. In addition to focusing on the long term behaviour, it is interesting to evaluate how the process behaves in the first hours, namely a long time before equilibrium is reached. The model will be used to look at a few hours ahead, mostly 33 hours, which is the longest possible time between spot clearing and a production hour.

Based on the transition matrix and on an initial state, sample paths are to be simulated. In practice, this means that when the spot price clears the day ahead for all product hours it is possible to simulate a sample path of the price process.

The input to the Monte Carlo simulation is depending on the what case that is analyzed, as presented in Table 6. It is repeated here for convenience: In sample years, path dependency, number of levels, type of trade. Simulation is run for all combinations listed in the table. Some new parameters were also crucial in simulation, namely:

- *K: Length of Simulation Period*
- *N_s: Number of Scenarios*
- *N_i: Number of Iterations*

Based on the case, the associated *State Table* is retrieved, along with the case specific transition matrix. The MC procedure is described in Algorithm 1.

Algorithm 1 MONTE CARLO SIMULATION

```
procedure MC Procedure Continued
    P ← Transition matrix
    L ← Path dependency
    U ← Number of levels
    S ← Number of states
    K ← Simulation period length in hours
    i ← Row Index in P
    R ← Random Number
    nScenarios ← Number of scenarios
loop:
    for scenario = 1 : nScenarios do
        realizationVec = zeros(1, K + L − 1)
        for element = 1 : L do
            realizationVec(1, element) = basisPath(element)
        for step = 1 : L + 1 : K + L − 1 do
            R = rand();
            lastStateVec = (realizationVec(1, step − L : step − 1));
            i = 0;
            toThePower = 0;
            for elem = L : −1 : 1 do
                if elem == L then
                    i = i + lastStateVec(elem);
                else
                    i = i + (lastStateVec(elem) − 1) ∗ U^toThePower;
                toThePower = toThePower + 1;
            rowOfInterest = i;
            stateAccumProbVec = zeros(1, U);
            for col = 1 : S do
                if col == 1 then
                    stateAccumProbVec(col) = P(rowOfInterest, col);
                else
                    stateAccumProbVec(col) = stateAccumProbVec(col − 1) + P(rowOfInterest, col);
            nodeAdd = false;
            for nextColState = 1 : U^L do
                if R < stateAccumProbVec(nextColState) then
                    newNode = str2double(Ptabell.Properties.VariableNamesnextColState(end));
                    realizationVec(1, step) = newNode;
                    realizationVecPrice(1, step + 1 − L) = meanPriceInLevel(newNode);
                    nodeAdd = true;
                    break;
        realizationVec = realizationVec(nPath : end);
```

In Algorithm 1, N_s realizations of the process are simulated. The process is elaborated below.

BasisPath refers to the history that plays a role in determining the state, and only contains the initial state when the path dependency is one 1. *realizationVec* refers to the state vector of realizations that is built, whist *realizationVecPrice* corresponds to the state vector, only here all elements are set equal to the mean of the level (line 38). The purpose of this vector will be elaborated later.

In the analysis, a time sequence of desired length, K data points , is drawn at random from a uniform distribution associated with all the OOS data. Thereafter, a set of $L-1$ points in front of this (chronologically in the time series) are brought to help describe the initial state, and correspond to *BasisPath* in Algorithm 1. A loop was run $K-1$ number of times, building the simulated sequence, *realizationVec*, step wise. The first state is set equal to that of the OOS sequence. For each step following, a random number is drawn from a uniform distribution, see line 16 in Algorithm 1. This was performed using the *rand* function in MATLAB. Based on what state the system is in, the associated row in the P-matrix is accumulated and compared element wise with the uniform random number,R, in order to determine the state in the next step (Line 35 in Algorithm 1). By accumulating all entries in a row we get 1, since P is a stochastic matrix.

The simulation results in a possible sample scenario, and it is interesting to know how similar this simulation is compared to the randomly chosen OOS sequence. In Figure 33, a vector containing states of 33 trailing steps is chosen from the OOS data and plotted together with the simulated scenario. The two upper subplots are the simulated paths. The upper, left subplot is measured in states. As will be elaborated soon, the upper right version is the exact same realization. The only difference is that it is converted back to modified prices in accordance with the triangular mean of the level, found utilizing regression as seen from Figure 31.

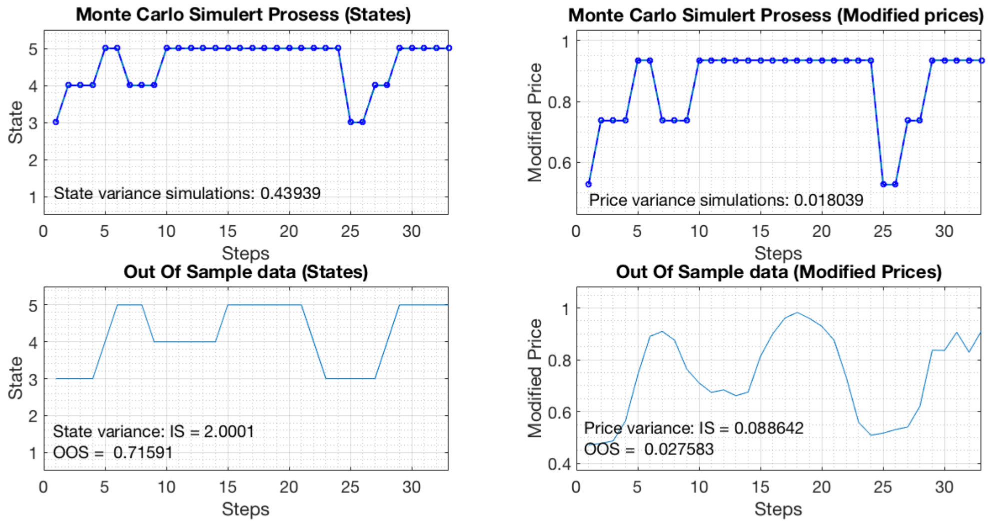


Figure 33: Monte Carlo Simulation with one Scenario.

Among the lower subplots, the left illustrates the OOS price path measured in states, determining the initial state (3 here) for the simulation. The right, however, is the underlying modified price path of the lower left subplot, before it is converted into states.

Figure 34 illustrates results from the Monte Carlo Simulation, similar to the one in Figure 33. Only here, the number of scenarios is $N_s = 10$, and the initial state is 4. The blue, dotted line in the upper simulation subplots represent the average state of all the scenarios. It is found by accumulating all scenario sequences element wise, i.e. for all hours, and finding the mean. We recall that the two lower subplots represent the same price path of the OOS sample.

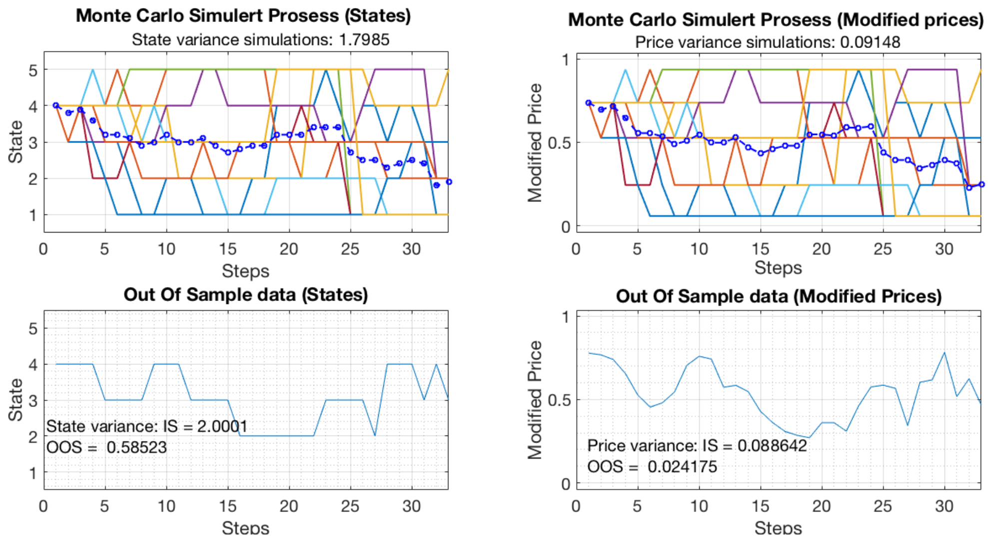


Figure 34: Monte Carlo Simulation with 10 Scenarios.

When a set of simulated scenarios exist, the next step is to determine two things. The hit percent (HP) and the mean square error (MSE). Both are defined below, in the context of this analysis:

$$x_{sim} = [x_{sim,1}, x_{sim,2}, x_{sim,3}, x_{sim,4}, x_{sim,5}...] \tag{19}$$

$$x_{OOS} = [x_{OOS,1}, x_{OOS,2}, x_{OOS,3}, x_{OOS,4}, x_{OOS,5}...] \tag{20}$$

$$MSE = \sum_{i=1}^{K} (|x_{sim,i} - x_{OOS,i}|)^2 \tag{21}$$

$$HP = \frac{\sum_{i=1}^{K} \delta_i}{K}, \qquad \delta_i = \begin{cases} 1 & \text{if } x_{sim,i} = x_{OOS,i} \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

, where K is the number of steps in the sequence x_{sim}, and δ_i is a binary variable which takes the value 1 if $x_{sim,1} = x_{OOS,1}$.

After determining the error between one simulations and the OOS vector, the result can give some information. However, one scenario carries little process information alone, so more scenarios are run to be able to determine the tendency.

Moreover, when picking *one* random OOS sequence, it is not desirable that the error should be zero and hence HP 100 %, the OOS set entails numerous sequences. Imagine two sequences from OOS both begin with initial state 2, and then differs in the trailing states. This means that the MSE would have to be larger than 0 for one sequence in order to be zero for another. It is clear that this is not desirable. In stead, what we seek is a model that behaves well (Low MSE) for a set, N_i of different sequences uniformly chosen from OOS. Thus, for a set N_i of sequences a set of N_s scenarios are run.

Performing this for all different 1-6 path lengths when $N_s = 100$, $N_i = 15$, the result is as illustrated

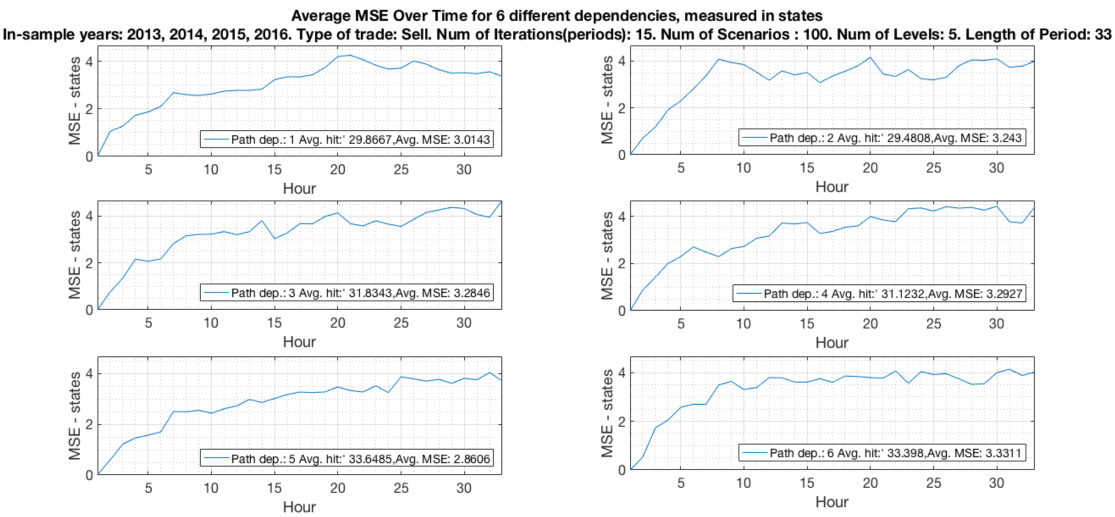


Figure 35: Overview of how the average mean square error (MSE) develops through the period of 33 time steps, measured in states.

The interpretation of these is that the error always begins in 0 due to a common initial state in the first step. This can be thought of at the willingness to pay in Elbas in the first hour, interpreted as a known fact. Since spot prices just cleared, there is no reason to expect the willingness to deviate from this, before any new information arrives. On average, the error grows quickly in the beginning, and then stabilizes around 3 to 4. We recall that the state space is discrete and takes the values of all integers from 1 to $\overline{U}$. A mean squared error of 4 equals a mean error of 2 states. This result tell us that the simulation process on avrage miss with less than 2 states, and a quite a lot less in the first hours where the initial state provides good conditional information.

When determining the error in simulation for the different P-matrices one can compare solutions without converting from states to prices. However, to get a more clear picture of how good the best solution is, conversion into known units take place.

Firstly, we recall that the mean was found for each level (Fig: 31), given the assumption of a triangular distribution that was fitted by regression within each level. In the simulated sequence, the prices are converted from states to the associated mean of the state. The OOS sequence is replaced by its previous variant, the one before conversion into states. A new comparison is made between these values, which are still modified prices (normalized, detrended and PI transformed). This type of comparison, for other random instances, is shown in Figure 36:

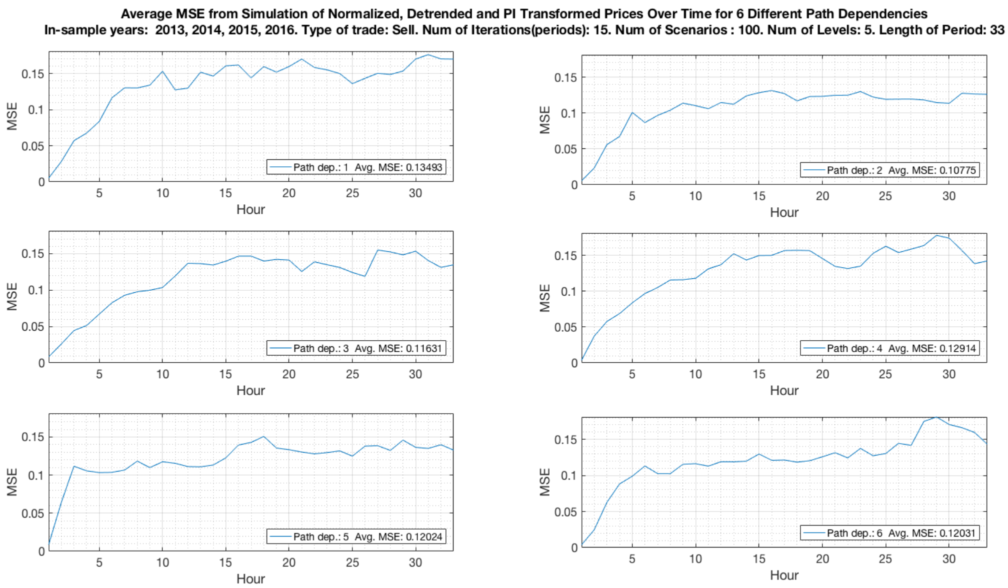


Figure 36: Overview of how the average mean square error (MSE) develops through the period of 33 time steps for modified prices.

In this case, measuring HP would be a bad measure, since the probability of an exact hit without the grouping of prices into states is extremely low, and hence only MSE is measured. In stead of inserting one figure per case presented in Table 6, the numerical results are collected in Table 8 and 9:

Levels	Path. Dep	OOS	Type of trade	MSE	Var(IS)	Var(OOS)	Var(sample)
5	1	rnd 2017	Sell	0.123840	0.088641	0.020515	0.08881
5	2	rnd 2017	Sell	0.123881	0.088641	0.022217	0.084107
5	3	rnd 2017	Sell	0.117618	0.088641	0.019405	0.086934
5	4	rnd 2017	Sell	0.124707	0.088641	0.026787	0.085931
5	5	rnd 2017	Sell	0.117587	0.088641	0.017363	0.083502
5	6	rnd 2017	Sell	0.122203	0.088641	0.031331	0.083525
5	1	rnd 1017	Buy	0.112237	0.087436	0.008861	0.078263
5	2	rnd 2017	Buy	0.113701	0.087436	0.016464	0.076627
5	3	rnd 2017	Buy	0.117702	0.087436	0.025114	0.078728
5	4	rnd 2017	Buy	0.127094	0.087436	0.018917	0.078436
5	5	rnd 2017	Buy	0.125096	0.087436	0.013894	0.076271
5	6	rnd 2017	Buy	0.111836	0.087436	0.018237	0.073635
5	1	rnd 2013	Sell	0.127117	0.085046	0.022782	0.084706
5	2	rnd 2013	Sell	0.139145	0.085046	0.041874	0.080158
5	3	rnd 2013	Sell	0.142635	0.085046	0.022064	0.084494
5	4	rnd 2013	Sell	0.151904	0.085046	0.037078	0.080297
5	5	rnd 2013	Sell	0.136513	0.085046	0.026879	0.084020
5	6	rnd 2013	Sell	0.143071	0.085046	0.024820	0.080405
5	1	rnd 2013	Buy	0.110162	0.080759	0.030188	0.063348
5	2	rnd 2013	Buy	0.118829	0.080759	0.026863	0.061973
5	3	rnd 2013	Buy	0.132139	0.080759	0.032720	0.064917
5	4	rnd 2013	Buy	0.124271	0.080759	0.044003	0.063583
5	5	rnd 2013	Buy	0.130622	0.080759	0.026559	0.062047
5	6	rnd 2013	Buy	0.119398	0.080759	0.042666	0.059892

Table 8: Results from Monte Carlo simulations. 5 levels, 100 scenarios, 15 iterations on samples of length 33 hours

Levels	Path. Dep	OOS	Type of trade	MSE	Var(IS)	Var(OOS)	Var(sample)
7	1	rnd 2017	Sell	0.130180	0.088641	0.021809	0.083226
7	2	rnd 2017	Sell	0.115152	0.088641	0.026803	0.084756
7	3	rnd 2017	Sell	0.123961	0.088641	0.025013	0.084120
7	4	rnd 2017	Sell	0.120676	0.088641	0.016348	0.085128
7	1	rnd 1017	Buy	0.102934	0.087436	0.018993	0.073092
7	2	rnd 2017	Buy	0.123987	0.087436	0.019788	0.074989
7	3	rnd 2017	Buy	0.124819	0.087436	0.026547	0.075630
7	4	rnd 2017	Buy	0.117026	0.087436	0.026836	0.075056
7	1	rnd 2013	Sell	0.134122	0.085046	0.027150	0.077113
7	2	rnd 2013	Sell	0.117267	0.085046	0.035275	0.074654
7	3	rnd 2013	Sell	0.127726	0.085046	0.033462	0.077408
7	4	rnd 2013	Sell	0.128242	0.085046	0.042823	0.072064
7	1	rnd 2013	Buy	0.133707	0.080759	0.038030	0.062635
7	2	rnd 2013	Buy	0.133647	0.080759	0.033027	0.063993
7	3	rnd 2013	Buy	0.148202	0.080759	0.036356	0.064408
7	4	rnd 2013	Buy	0.119737	0.080759	0.034399	0.063887

Table 9: Results from Monte Carlo simulations. 7 levels, 100 scenarios, 15 iterations on samples of length 33 hours

From Table 8 and 9 we see that adding more steps into the path dependency, L, does not provide a clear decay in the MSE. This result must be seen in relation with Figure 15 and 16, showing the partial correlation between steps for *Buy* and *Sale* respectively. The largest correlation is between each point ant the previous. Further back the correlation is remarkably lower, and hence it the gain of including them in the path is expected to be small. This assertion is now also supported by simulation.

To comment further on the simulation, variance in sequences has been looked into. A common challenge when making a stochastic model of a process is that when IS data is used to form a model, this model can end up having a variance that is too small to be able to imitate the new process. This is specially if the new process has properties that the IS data did not have. In this study, spikes will never be produced. However, the distribution of the points is modelled to be equal in all levels.

We shortly discuss variance in the different data sets, and how MC is related to variance.

The variance was calculated using the formula in Equation 23:

$$S^2_{N-1} = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \hat{x})^2 \tag{23}$$

, where x_i denotes the i different x values in the sequence of N elements, and $\hat{x}$ is the sample mean.

The variance in the IS data is of importance because it determines, in cooperation with the framework of the Markov model, the variance in the sampled sequences. The reason why many data points are kept within the IS data, rather than a shorter period, is that the variance over time is desirable to know. There is a lot of value in having a large data set to model from. One can also argue that smaller IS set would be interesting to look at. Due to time constraints, this was not performed in this study, but it could have been an interesting analysis to carry out. For instance, it could be to use the last month, or weeks, and updating thus update the P matrix more frequently.

We note that the variance in the in sample data is the largest in all cases. Further we recall that this was run for 100 scenarios, and hence the variance in the in sample data must be commented on. It can never be larger than that of the IS data, because the stochastic model does not give increased variance to the sampled data - rather the opposite. However, the more scenarios are run, the larger the variance is, because one sequence can only achieve a certain variance. It is therefore common to simulate multiple scenarios in order to increase sample variance and capture the entire OOS realization space. This is discussed further under Chapter 8

The model is created to handle spikes when happening, but it will never predict that they happen. The scope of the model is to model well the cases that happen frequently, excluding spikes. However, it is important that the model do not collapse if spikes do happen. If the spot price explodes to a high price, or the SRMC of coal drops so the normalized price explodes, the information into the model will be interpreted as being in the upper state, and handled accordingly. The same goes for extremely low prices, only interpreted as going into the lower level. If the model was able to predict that a price at a point is high, it will calculate expected values as if the maximum price is the mean of the upper level. Therefore the default action if price information reveals prices above mean of the upper price level is to sell (or to buy if the price drops below the mean of the lowest level). The model could never see a better solution in the future. This will be elaborated on in Section 5.3.

The Chosen P-matrix

At this point, with several candidates of the transition matrix, the next step was to determine which one to use. The criteria for a good transition matrix is that is can generate processes that is right on expectation, meaning it does not have a systematic error. In addition variance have been investigated.

Till now, we have removed externalities from the data set to make it a better basis to develop an accurate price model, followed by comparing the proposed models. We choose to proceed with the matrix in Equation 12 from the example case in Table 7. Firstly, due to the results from the MC simulation, revealing that all models obtained by the current testing, have quite similar MSE. The results from this testing have not converged to the case where one model always was better than the others. The reliability of the results would have been higher if more simulations were run on larger instances. In this case more statistical measures could be utilized to see if on method was significantly better than the others. This is commented under the Chapter 8. Choosing not to exclude any model based on MC, made the choice depend on the restrictions from curses of dimensionality (Sec: 3.6.2). Therefore, the example case is the chosen case due to its small dimensions, both the smallest number of levels and the shortest path dependency. With respect to IS and OOS data the MSE results did not converge to favour any modelling case, hence the most intuitive, i.e. the chronological case is chosen, as in the example case. As we have seen the P-matrix has an improved ability to predict the OOS data after the data adjustments, and hence closer to the goal of an equal expected value or first raw moment.

As seen from Section 5.2.10), the diagonal in the transition matrix contains relatively large probabilities, meaning that when the Elbas price is modelled to be in a state, it is most likely to continue staying in this state. Figure 37 illustrates the transition probabilities for the exemplified case. This is an important insight. Since the simulation period is only 33 hours - not a year for instance, where the initial state has a smaller impact on the latter states, it is crucial to have a precise initialization. Based on this insight, a goal for the model is that it not only considers daily seasonality, but also accounts for real time day ahead values, in cases where prices may be shifted due to some external factors. This implicitly says that if spot prices are higher than normal, then Elbas prices are likely to be so too. Again, we refer to the assumption of correlation (see Section 3.3.2) between spot and Elbas prices.

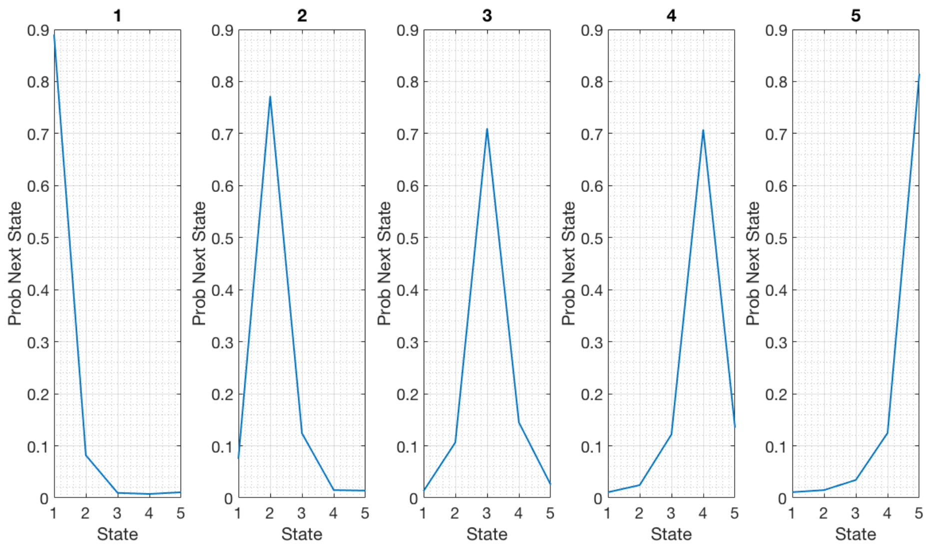


Figure 37: Illustrates the probabilities of transitioning from a state the next.

Transforming back

The next step is to explain the procedure of converting the modified prices back into the unit $[\frac{EUR}{MWh}]$. It is not only interesting to get the final bidding price in these units, but to investigate the MSE as well. In this preliminary study we explain how such inverse transformation and reversed operations work, without carrying it out. At this stage, the gain is considered limited. Since the prices are normalized, detrended and PI transformed they have to be converted back in the opposite order.

We introduce this notation for the reversed operations:

- p: Price in $[\frac{EUR}{MWh}]$
- p_N: Normalized price p
- p_{ND}:Normalized and detrended price p
- p_{NDP}: Normalized, detrended PI-transformed price p
- c_{SRMC_t}: SRMC of coal at time t

Inverse PIT

The modified price is between 0 and 1 after the process due to the cdf-transformation, and to convert it back, the cdf must be utilized. An inverse transformation based on the cdf obtained by fitting a Normal Kernel distribution is performed.

Hence, the modified price, p_{NDP}, is transformed back using:

$$p_{ND} = F_x^{-1}(p_{NDT}) \tag{24}$$

As seen, a fitted cdf is non-trivial to obtain. If a good fit for the curve is obtained one can transform numbers back by solving the inverse, i.e. inserting p_{NDP} to obtain p_{ND}.

Adding the trend

The trend that was subtracted must be added to the data point. This is based on the assumption of a downward sloping linear trend also in the future (Sec: 5.2.9).

Thus, the trend with the above mentioned properties is added to the new data points.

$$p_N = p_{ND} + (a \cdot t + b) \tag{25}$$

, where a denotes the slope of the trend and b the offset as found in Table 4. We also recall that under the assumption of a normal distribution, the confidence interval was given by Table 5. This confidence interval is however not very accurate, as it depends firstly on the fit of the normal distribution, and secondly on independence between the random points, which we have proved false by the auto-correlation tests. However, finding a more accurate confidence interval is not considered the scope here. We only point out that confidence interval is an interesting measure when investigating trends.

De-normalizing

The final operation is to divide the price on the yielding SRMC of coal. When normalizing in section 5.2.9, the $c_{SRMC_{t'}}$ yielding at the historical point in time, t' of the price p was utilized.

$$p = p_N \cdot c_{SRMC_{t''}} \tag{26}$$

, where t" is the point in time of the price p obtained from the model.

By creating a script that would perform all these reversed operations one could be able to see the values and the associated price accuracy in $[\frac{EUR}{MWh}]$ again. For this preliminary study the scope has been on determining the best method (which only needed comparison of methods in the modified price space), and little effort has been put into performing the reversed transformations and operations. This is mostly due to time constraints. However, two small instances have been tested to show the procedure (Chap: 6).

When using known distributions to describe a process that does not follow it exactly, the fit of the model is an important topic depending on the purpose of the operation. In this article the fit of the normal distribution is mentioned twice. Firstly, when considering detrending, where it is only used to help describe the confidence interval of a regression line. Here, we note that if a comprehensive analysis is to be carried out - another method must be used to determine the confidence interval. Secondly, our data is again compared to the normal distribution when considering the PI transformation. At this point a more accurate fit of the whole pdf, and cdf, is desired and hence the normal distribution is considered too rough. The problem of overfitting in the transformation is not considered a problem, since the goal here is to actually describe the curve as it is, believing it carries information when predicting in the future.

Both the long term behaviour and the short term behaviour have now been analyzed and provided important insights. It can provide a false impression of the fit of the model to only consider one of these. We now proceed to the next section, utilizing the final choice of the model, associated with the example case from Table 7.

5.3 Solution model and implementation

The scope of this section is to present the optimization model and explain how the bidding problem is considered a sequential decision problem with an option value of waiting. A reminder of the problem to be solved by the optimization model is presented briefly.

A market participant in the Nordic power markets may choose to trade in the intraday market, Elbas. It is a first come, first served market, where any participant is free to accept only those bids that create the most profit. This results in a risk of no trade, and an optimal timing of trades and correct bid levels are essential to gain the full potential in the market. In addition, trading correct volumes to ensure consumption/production balance is beneficial to limit costs related to the Regulating power market. Hence, new information occurring during the planning horizon must be taken into consideration when bid decisions are done.

Each day, a market participant faces the opportunity to trade in Elbas. Each trade affects the participant's commitment of supply in a specific *production hour*. Hence, we may regard each production hour as a single product, where there are 24 different products that can be traded each day. Production hour 1 represents the hour of power supply from CET 00:00 to 01:00, while production hour 24 refers to the hour of supply from CET 23:00-00:00. On the other hand, the trade, or commitment, may have happened at any time between the market opening and one hour prior to the production hour in question. A time line illustrated this concept in Section 2.1.3, Figure 2. The time of which a trade is done, is referred to as the *trade hour*, where trade hour 0 represents the time of which the intraday market opens.

As the concepts of time in the problem is complex, Table 10 connects the different terms related to time in the problem. Note that time in real time given by CET, is actually a time interval of *one hour* starting at the time stated in the table. The first row indicates how these concepts relate to the structure of the optimization problem, where the stages of which a decision is to be made corresponds to the trade hours. However, the term *time step* is sometimes used interchangeably with stage, only because the stages in the model represents steps in time.

Stage/time step	0	1	2	...	10	11	...	21	22	...	33	
Trade Hour	0	1	2	...	10	11	...	21	22	...	33	
Production Hour						1	...	11	12	...	23	24
CET	14	15	16	...	23	00	...	10	11	...	22	23

Table 10: Corresponding stages/time steps, trade hours, production hours and start clock hour in CET.

When the spot market clears, and Elbas capacities are announced, the initial state of the system is given by the information of price levels available at that time, namely the spot prices. These represent the willingness to trade for each production hour at the time when the spot market closes, and are set as the initial state of the system at *stage 0*. As the time goes, new trades occur. These are *exogenous information*, not known before the events of trade. The state of the system is updated for each stage according to the trades during the past trade hour, and represents the new willingess to trade in the coming trade hour. Hence, the bidding decision to make, is weather or not to bid at this level during the next trade hour. Bidding in a trade hour affects the possibility of bidding in following trade hours, and one might want to wait in anticipation of higher prices in the future.

As Elbas closes one hour prior to the production hour, not all products are available for trade throughout the whole period. The last possible trade hour for production hour 1 corresponds to stage 10, while trades concerning production hour 24 may be done at all stages up until stage 33 (Table 10).

5.3.1 Assumptions

Some assumptions are made to solve the problem. All of them are listed below, followed by an elaboration when necessary.

1. The power producer has a specified amount of power to trade each production hour

2. The total volume for each production hour must be sold in one single trade
3. Trades concerning different production hours are independent of each other
4. If the power producer bids according to the model, a counterpart will for sure accept.
5. The power producer has a upper/lower limit for the trade price that is lower than the worst expected payoff
6. Constant marginal cost over the optimization horizon.

Assumptions number 1-3 concerns resource allocation, and are not necessarily correct for the real bidding problem that power producers face in the intraday market. However, the decision to base the dynamic model on these assumptions is done in accordance with the industrial partner. This is a main perspective in Chapter 8, where future research is discussed.

Assumption number 4 assures no uncertainties in the *contribution function*, which will be introduced in Section 5.3.2.

Assumption number 5 is not really an assumption, but rather a simplification due to the objective of this study. A power producer will only accept a trade if there is an opportunity to profit. Hence, a power producer will only *sell* power at a price level greater than the consequently marginal cost of production, and eventual semi fixed costs related to start ups etc. Correspondingly, the power producer will only *buy* power if the alternative cost of production is higher than the trade price. In addition, due to the production plan, these alternative costs will differ between production hours. However, since the dynamic program in this study is more a way of illustrating the use of the stochastic model developed in Section 5.2, and a subject for future research, exact limits of trade has not been further investigated. We find it sufficient that the model is constructed so that such a limit easily can be applied once that is of interest.

Assumption number 6 is mostly related to hydro power production, where the production pattern during a day affects the water value. However, even in the case of hydro power production, the volumes traded in Elbas are rarely big enough to affect the water reservoir level, hence the water value stays approximately constant.

Important consequences of the assumptions are evaluated in Chapter 7, and possible extensions for future research are further discussed in Chapter 8.

5.3.2 Mathematical model

This section aims at introducing the mathematical model, as well as to describe some of the notation introduced in Section 5.1. The notation used in this study is based on that of Powell (2011), with some exceptions.

Note that due to the resource related assumptions (Sec: 5.3.1), the total problem becomes 48 independent problems, as there are one *sales* problem and one *buy* problem to be solved for each of the 24 different products (production hours).

State Variables

$S_{t,h}$ State variable in time step t concerning production hour h

$$S_{t,h} \in \mathcal{S} \tag{27}$$

The state variable $S_{t,h}$ contains all information required to make a decision x_t at time t, as well as computing how the system evolves over time approaching production hour h. The state contains historical information about the price levels $u_{t',h} \in \mathcal{U}$ for the L most recent time steps t', concerning a given production hour h.

$$S_{t,h} = [u_{t-L,h} \quad ... \quad u_{t-1,h} \quad u_{t,h}] \tag{28}$$

The dimensions of state space $\mathcal{S}$ is determined by the length L of historical memory and the $\overline{U}$ different price levels, given by $\overline{U}^L$ or Equation 11.

Decision Variables

$x_{t,h}$ Decision in time step t concerning production hour h

$$x_{t,h} \in \mathcal{X} \tag{29}$$

The decision variables are the part of the problem that is possible to control, the bidding decisions. The decision space is binary for a given time step t concerning a given production hour h, containing the option to bid at time t or to wait until time $t+1$ to do another evaluation.

$$\mathcal{X} = [0, 1] \tag{30}$$

$$x_{t,h} = \begin{cases} 1 & \text{if decision is to bid for hour } h \text{ in time step } t \\ 0 & \text{otherwise} \end{cases}$$

The decision $x_{t,h}$ is determined by a *policy* $X^{\pi}(S_{t,h})$, mapping a certain state at time step t, concerning production hour h, to a specific action. The optimal policy is utilized to construct a contingency plan in Chapter 6.

Exogenous Information Process

$W_{t,h}$ Exogenous information

The exogenous information concerns how the willingness to trade has developed since time step $t-1$, and are updated price levels for each production hour $h \in \mathcal{H}$. The information is based on ticker data (trades), where bids accepted between time step $t-1$ and t, concerning production hour h, represents the price level $u_{t,h}$. Hence, the exogenous information $W_{t,h}$ has an outcome space defined by $\mathcal{U}$.

$$W_{t,h} \in \mathcal{U} \tag{31}$$

Transition Function

S^M Transition function

The transition function describes how the system evolves through time from one state $S_{t,h}$ to state $S_{t+1,h}$. The state $S_{t+1,h}$ is described by the previous state due to historical information, as well as the updated price data $u_{t,h}$ determined by the exogenous information $W_{t+1,h} \in \mathcal{U}$. The decision $x_{t,h}$ does not affect the state, as the state is described by a sequence of price levels only.

$$S_{t+1,h} = S^M(S_{t,h}, W_{t+1}) \tag{32}$$

Regarding a specific production hour h' and time step t', the system state will evolve to the next time step $t'+1$ like this:

$$S_{t',h'} = [u_{t'-L,h'} \quad \dots \quad u_{t'-2,h'} \quad u_{t'-1,h'} \quad u_{t',h'}] \tag{33}$$

$$S_{t'+1,h'} = [u_{t'+1-L,h'} \quad \dots \quad u_{t'-1,h'} \quad u_{t',h'} \quad u_{t'+1,h'}] \tag{34}$$

We observe how the new state $S_{t'+1,h'}$ only contains historical information about the L most recent price levels.

With the underlying Markov property assumption (2), the probability that state $S_{t'+1,h'} = s$ depends on state $S_{t',h'} = s'$ only.

$$\mathbb{P}(S_{t'+1,h'} = s|s') = \mathbb{P}_{s',s} \tag{35}$$

The Contribution and Objective Function

The *contribution function* is the direct contribution to the objective value by making a decision $x_{t,h}$ when in state $S_{t,h}$.

$$C_{t,h}(S_{t,h}, x_{t,h}) = u_{t,h}x_{t,h} \tag{36}$$

This implies that the contribution at time step t is equal to the bid for production hour h.

The assumption that bidding in the Elbas market at a price level determined by the dynamic program for sure will result in an accepted trade, makes the contribution function deterministic. However, this might be an invalid simplification, and further research should be explored in future studies.

Since the solution concerning each production hour is completely independent of the solution of other production hours, the total contribution at time t can be expressed as:

$$C_t = \sum_{h \in \mathcal{H}} C_{t,h}(S_{t,h}, x_{t,h}) \tag{37}$$

Since the contribution for any time step t is dependent on decisions made at every time step $t \in \mathcal{T}$, a backward recourse function must be calculated at each time step, estimating the *value* $V_{t,h}$ of being in state $S_{t,h}$.

$$V_{t,h}(S_{t,h}) = \max_{x_{t,h} \in \mathcal{X}} (C_{t,h}(S_{t,h}, x_{t,h}) + \mathbf{E}^{W_{t+1,h}}[V_{t+1,h}(S_{t+1,h}|S_{t,h}, x_{t,h}, W_{t+1})]) \tag{38}$$

The associated *objective function* should find the optimal policy $X^{\pi}(S_{t,h})_{\pi \in \Pi}$ so that the contribution from all time steps $t \in \mathcal{T}$ is maximized. It is given by Bellman's equation:

$$V^*_{0,h} = \max_{\pi \in \Pi} \mathbf{E}^{\pi}\{\sum_{t \in \mathcal{T}} C^{\pi}_{t,h}(S_{t,h}, X^{\pi}_{t,h}(S_{t,h}))|S_{0,h}\} \tag{39}$$

We can write the total expected contribution for all production hours when choosing the optimal policy π as:

$$V^*_0 = \sum_{h \in \mathcal{H}} V^*_{0,h} \tag{40}$$

5.3.3 Solution method

Flytskjema, pseudokode, stegvis beskrivelse av modell

6 Computational Study

The purpose of this chapter is to present an overview of the results. It provides a link between the two previous sections (Sec: 5.2 and Sec: 5.3) from Chapter 5. The stochastic Markov process (Sec: 5.2) in the fromat of a transition matrix has the purpose of predicting how prices are likely to move between different levels in the future. This chapter mainly focuses on describing how the matrix is utilized as input to a dynamic program, which is the method utilized to obtain a contingency plan. The input and output will be discussed shortly.

The reader should be reminded of how trade hour and production hour corresponds according to the time line in Table 10. As the Elbas market closes one hour prior to production, the last possible trade hours will be as stated in Table 11.

Production Hour	Last Trade Hour	CET time
1	10	23-00
12	21	10-11
24	33	22-23

Table 11: Last possible trade hours for production hour 1, 12 and 24

6.1 General Results from the Dynamic Program

The computational study is done for the case of *selling* power i the Elbas market. Figure 38 illustrates the results from the dynamic program, where the value of being in each state $1-5$ is plotted for each trade hour in the optimization horizon. The plot shows have the *value of being in a state* changes as the production hour gets closer in time. The value of being in a state at a point in time is the best outcome of bidding in Elbas at that price level, or the value of waiting. The value of waiting is computed using the transition matrix in Equation 12.

When the last possible trade hour has passed, the value is set to zero.

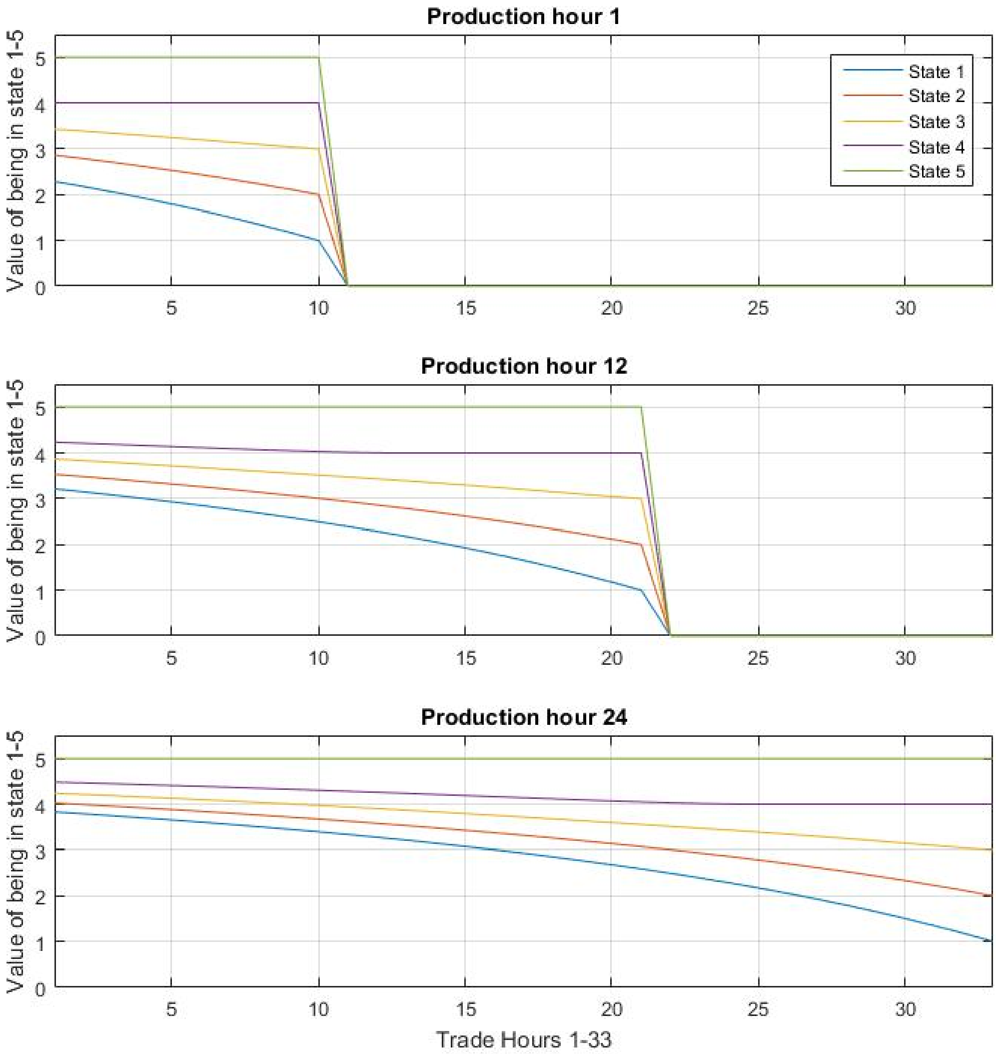


Figure 38: For the specific production hours 1, 12 and 24, the value of being in states 1-5 in each trade hour

The cut-off trade hour where it is more profitable to bid in the market rather than wait, given a state s, is listed in table 12.

	Production Hour 1	Production Hour 12	Production Hour 24
State 1	10	21	33
State 2	10	21	33
State 3	10	21	33
State 4	2	13	25
State 5	1	1	1

Table 12: The cut-off trade hour where bidding is optimal in each state

Due to the transition probabilities, the dynamic program never finds it optimal to bid in states $1-3$, unless it is the last possible trade hour. Being in state 5 will always result in a decision to bid as it is the highest possible value. However, the value of being in state 4 varies over time in such a way that there exists a optimal time of bidding, at trade hour 2, 13 and 25 respectively. Before these trade hours, the model *expects* a better payoff later and suggests to wait.

6.2 The Contingency Plan

The usage of the contingency plan is easy. The plan functions as a reference, or a recipe that will provide decision support (sell/do not sell or buy/do not buy) for a given product at a given time, given the current price situation.

Initially, before any Elbas trades become available, but after spot prices clear, the initial state is set to the state associated with the spot price. As time goes and the process makes steps in direction of the production hour, the state remains equal until new trades occurs. New trades are information that changes the situation, so that the current state must be updated. As the price process moves step wise between states in the state space, the program helps solving the optimal stopping problem.

In this section, the contingency plan is illustrated for production hour 1. We want to know what the optimal time is to sell a volume of at least 50 MW, and starts by finding the spot price for the 1^{st} production hour, CET 01:00. We know that Elbas closes one hour before the production hour, namely at CET 00:00 (trade hour 10). Standing at the beginning of the time horizon at CET 14:00, the decision of trading during the coming hour up until CET 15:00 (trade hour 1) depends on the spot price only as there has not been any trades in Elbas yet.

We present how the user would use the contingency plan as decision support in two different cases. The cases are two sample realizations of a price process, given the same initial state.

The initial state is given by the spot price for that specific production hour. After the initial state, a sequence of states follows. The sequence represents information received about the actual updated situation, and could have been achieved by considering ticker-data from Elbas trades. Since this is only for illustrational purposes the following sequence, $\hat{v_i}$, is fictional.

All future states as determined by Elbas trades that occurred during the *previous* trade hour, and are not known until entry of each trade hour.

- Initial state: 2
  - Case 1: $\hat{v_1}$ = [2 2 2 3 3 3 **4**...]
  - Case 2: $\hat{v_1}$ = [3 3 3 3 2 2 2 2 **2**]

How to use the plan is illustrated in figure 39. The black and green line illustrates case 1 and case 2 respectively. Each node is marked with a decision $= \{0, 1\}$, which is the decision to wait or bid. The line on top of the figure states the trade hour in question.

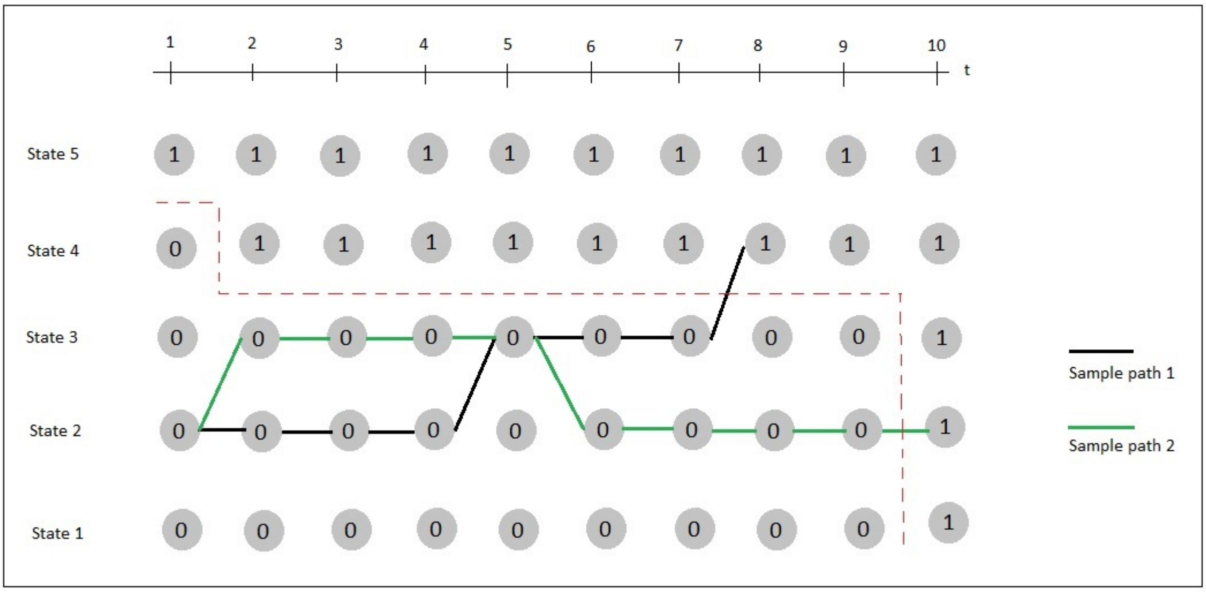


Figure 39: How the contingency plan works, illustrated for production hour 1.

Hence, in case 1, a bid would have been done in trade hour 8 at a price according to state 4. In case 2, a bid would have been done at trade hour 10 at a price according to state 2.

Notice that the red line shows the cut-off value between bidding and waiting for each trade hour. Hence, the figure may as well represent the *lowest* bid the power producer would have accepted in the market in each trade hour. In this case, only a price level corresponding to state 5 should be accepted in the first trade hour. During trade hours 2-9, the price level should be at least 4 to accept a trade. The last possible trade hour, all price levels should be accepted. Note that if involving a lower limit for the bid level, being in a state corresponding to a lower level than the limit would never return a decision to bid.

In order to convert from states back to prices, the state must be seen in the context of time. I.e. when did the cases take place? For instance, the SRMC of coal associated with the data must be used, and the correct point in time with respect to adding the trend back. The PI transformation, however, is independent of time, and does not require the instance to be put into a time context.

A strength of the contingency plan is that it does not have to be updated before the P matrix is outdated. The only changes in strategy for the user are related to the impact of received information, which can differ between products and days.

7 Concluding Remarks

In this study, parameters describing the stochastic process of Elbas trades has been determined. In addition, an optimization program has been developed to illustrate how the stochastic model can be used to determine optimal bidding strategies in the intraday electricity market. The purpose is to better understand the dynamics affecting intraday trade prices, so that an optimization model can compute a bidding strategy that maximizes profit for a market participant. A thorough analysis of empirical data has been conducted, and a discrete stochastic Markov process is suggested to describe the price development in Elbas over a desired time horizon. The transition probabilities determined from the analysis are utilized as input in a dynamic program, where an optimal bidding policy is computed.

The Elbas price process is modelled using a Markov process. The basis is firstly, the fact that Elbas prices are correlated with spot prices - and hence modelling spot prices well will give a good indication of Elbas prices. Secondly, due to the price correlation between one step and the previous indicates that a well formulated transition matrix can describe the process well.

The Markov model both entails strengths and weaknesses: From the limiting probabilities of the chosen Markov transition matrix it is found that the model manages to some degree to be correct in expectation, i.e. 1^{st} raw moment, which is a goal for this preliminary study. However, reliable measures of the performance in the second moment are more cumbersome to find. It is found that by increasing the number of scenarios the probability of capturing the future behaviour increases, as typical for simulation models where the number of states is low the matrix is irreducible, i.e. it is possible to move from each state to all the others.

One important result regarding the first few hours of simulation is that the deviation from the randomly chosen set of OOS sequences is smallest, due to the information considered true at the beginning of the first step, when spot prices are cleared and information symmetric among market participants. As seen from Figure 35 and 36 the MSE is relatively low in the first hours, before it converges to an average MSE. Larger instances would provide a smoother slope, but the tendency is clear. Of course, the MSE is bounded by the nature of the state division, and would not approach infinity, but rather converge. The accumulated MSE would, however, approach infinity, if an infinitely long simulation was run. An explanation to the slope, and that the MSE does not reach steady state quicker must be seen in relation with Figure 37. It is revealed that the tendency to return to a state in the following step is much higher than that of switching. In a short time horizon, the best guess of forecasting is therefore to predict staying in the same state, which was also the case of the OOS data, which can be read from the relatively large diagonal elements in P_{OOS} in Equation 14.

By performing adjustments of the data set, we have shown that it is statistically easier to work with the data set in order to predict future price processes. The investigated models turned out to be close to equally suitable by the chosen measures. This in turn indicates that other tests could have been utilized to separate the behaviour of the transition matrices more.

In this report, the assumption of a correlation between spot prices and Elbas prices is underlying. As mentioned, research points in this direction, but it havent been investigated further. Due to it's major impact on the result it is important to ask what error this assumption may result in. If the assumption has a systematic offset, it could be handled by adjusting for this error. I however, the prices are correlated to different degree at different points in time it would be more difficult to adjust for. However, this error is based on the absence of the assumption, and is only mentioned as a point to be aware of.

Another important point to conclude with, is that the model will not collapse if spikes occur in reality, but it will not either be able to predict that spikes will happen.

The chosen P matrix that is utilized to form a grid in the dynamic program creates the basis for how good support the optimization tool will be able to provide. If the there is a tendency in the transition matrix to overestimate or underestimate future prices, this will result in offsets in the DP, and hence could be critical for the user. We here refer to the introductory sentence with the essence that an optimization model will never be accurate unless the stochastic price model is sufficiently precise. In practice, a method like this must always be seen in the light of insights from the field.

The dynamic program developed in this study is more of a conceptual beginning approach, rather than a fully representative model of the problem in question. The main focus is to illustrate how parameters from the stochastic model can be implemented in an optimization environment, and shed light on the advantages and challenges by utilizing dynamic programming for this purpose.

The main advantage of a dynamic program is that a policy is computed. The policy maps a certain state in time to a specific action, which makes the model easier to test, i.e. with simulation. It also provides the opportunity to build a contingency plan, which is intuitive to understand and to follow by market participants. The model is not required to rerun every time exogenous information is revealed. That is important from an industrial point of view, as time constraints in the bidding decision environment require quick updates from the decision support. In addition, a strength of the dynamic programming approach in multistage decision problems, is that it breaks the problem into smaller sub problems. Hence, a planning horizon of 33 stages, as in the intraday market, is possible to handle within the dynamic programming frame work.

However, some assumptions regarding resource allocation and independencies between production hours were made in the optimization model in this study, which does not represent the real bid decision problem in Elbas. These assumptions were made to decrease the complexity of the dynamic program, because an important challenge occurs when the dimensions of either the state space, action space or the outcome space increases. As the optimization model in this study is mainly for illustration purposes, indicating how a contingency plan can be constructed utilizing a Markov process description of Elbas prices, these complex aspects of the problem were ignored. An important result of this study is thus the comprehension of the complexity a realistic model will have to handle.

The output from the dynamic program shows that only if the system is in state 4, there will be a question of optimal stopping in *sales* bids. Being in the upper most state (state 5) will always result in a bidding decision, as there are no possibility of a higher price in the future. On the other hand, if the system is in the middle state or below (states 1-3), a decision to bid will only occur in the last possible trade hour. This result is equal for all production hours. It seems that the transition probabilities tends to anticipate an above middle state future price level, which corresponds to the discussions in Section

5.2.10. This is a trivial result. However, with an increased complexity of the dynamic program, the solution becomes non-trivial and the value of the optimization framework increases.

It is worth to mention that if there had been a lower limit for which price the participant is willing to accept (5.3.1), some of the lower states would never result in a bid.

Testing the optimization problem on historical data does not seem fruitful as long as the model is this preliminary, and not representative for the real problem. However, even if testing on historical ticker-data, the low frequency of trades i the past makes it hard to get a god representative result. This study aims at modelling a future market situation with larger volumes and increased frequency of trades.

The over all contributions to TE from this study is, firstly, that the field is being investigated and light is shed onto an area where rules of thumb yield. A market analysis is performed, with the goal of supporting TE in decision making. In this preliminary study, the basic ideas from data handling and analysis to an optimization tool are all brought to give a holistic overview of the challenge. Since this study defines the basis for a thesis within the same topic, the practical contribution is expected to become larger.

8 Future Research

This preliminary report gives insights into the electricity market and price processes. In addition it provides knowledge making it more clear what future research could be interesting to carry out. This report will be followed by a thesis. Numerous paths can be investigated more into depth. Some of those considered most interesting, in order to make the study more realistic, are presented. The first ones consider analysis. More specifically, what other cases of input data, testing and weighting methods to investigate.

1. Could it be beneficial to model the different production hours by individual transition matrices, building on the idea used on regulating markets by Kongelf and Overrein (2017)?

2. It could be interesting to experiment further with the path dependency. What would happen if a new step not only depends on the L previous time steps, but was also depends on the same production hour, one day ahead?

3. Experimenting with the length of the IS data. Hence, the upside of using many years as IS is that more information lies within, whilst the downside is that the data from far ago might not be *as representative* for the future data as the most recent. Weighting methods to increase the importance of most recent data could be a suitable way to account for both factors. However, that is not investigated in this study. Another option to investigate is to design a more *light* or *flexible* P matrix in the sense that it is based only on shorter periods of time.

4. Weighting may be carried out in different ways. Since an empirical approach is utilized in this study, a problem can be that abnormal behaviour over a long time might have a greater impact on the data set than desirable. For instance, it can predict abnormal behaviour to reoccur too often in the future or drag the mean up or down. This is typically if a seldom external event characterizes a large share of the available data. An example is the very high coal prices in 2015, that can be seen from Figure 18 and 19, which illustrates deviation from all other years. This resulted in very low *normalized* spot prices over a long time horizon. There is not automatically a reason to say that every fourth year for instance, a similar period

will occur. Offsets, due to different reasons, will have an impact on the predictions. Some of these variations are important to capture other external deviation, but a way to avoid that the impact of these parts of the data set are *too* large, is to adjust the data set by putting less weight on the information from the abnormal period. This approach requires domain knowledge, and must be performed carefully. An important note here, is that this model focuses on the average behaviour.

5. Carrying out a more comprehensive MC simulation, where comparison of results is more based on statistic measures. There exist a lot of literature on the field of simulation, so a more in depth search to find a suitable way to more properly evaluate the simulated output could probably be gained from. In a future research it could be interesting to see if one can comment further on the strengths and weaknesses of the model.

6. In relation to the preliminary MC simulation, it could have been interesting here to investigate the average number of sufficient scenarios to, in general, be able to cover the possible outcomes of the OOS dataset, with some certainty. A possible formulation to investigate is for instance, how many scenarios must be simulated to be sure that the OOS data is covered in 95 % of the realizations?

7. Another way to experiment with the simulation is to collect all OOS data sequences that start with the same state, and then find the average of these and compare to a set of simulated sequences. This method would remove some variance and is referred to as the method of common random numbers (Chen et al. (2008)). It could be a more comprehensive and interesting way to measure MSE as well, whilst still making many scenarios to simulate each sequence (which is done now). However, the strength of the present study is that the OOS sequence is *chosen at random*, and the analysis is performed on *multiple iterations* (one OOS set at the time) all with a set of scenarios of samples.

Moreover, regarding the optimization method utilized in this study, there are some main considerations that should be evaluated in future research. They include resource allocation, uncertainties in the optimal stopping problem and testing of output.

There are different ways of defining the resource allocation problem that a power producer with some sort of flexibility faces in their bidding optimization.

1. The first one, which is the assumption made in this study, is that each production hour is assigned a specified amount of power to buy or sell to the Elbas market. There is to happen only one trade for each production hour, or no trade at all.

2. A second definition is to say that there may be several trades involved for each production hour, so that not all of the volume is to be sold at once. Such an extension would increase the decision space, as the volume traded is to be determined as well as the binary bid/no bid decision. The model would also increase its state space, as a resource variable would be introduced.

3. A third definition of the resource allocation, is to have a total volume that should be traded during the day, but not necessarily bounded by any specific production hour. Hence, while this study considers 48 isolated optimal stopping problems, such an extension would make the 48 solutions dependent of each other. I.e a trade committing to increased production in production hour three, affects the opportunity to produce in all other production hours as well. The dimensions of such a model are huge compared to the model in this study, and measures must be done to control this.

4. An aspects not concerning resource allocation, but rather uncertainty and risk, is that a bid according to the model output in this study for sure will result in an accepted trade with a counter part. The possibility that the bid will not be accepted should be taken into consideration, as Elbas bids are volatile even with increased frequency of trades. There is no market clearing, so the risk of bidding wrong is that there might not be a trade at all. How does the market participant relate to risk? Risk aversion would lead to a more aggressive bidding approach, so that the risk of no trade is limited, but on the other hand, the expected profit decreases. This is not covered in this study, but is an interesting perspective to the timing problem.

5. The last perspective for future research presented in this study is related to testing of the output from the dynamic program. As stated in Chapter 7, no tests have been run on the optimal policy that the optimization method provides. A challenge is to find reasonable ways to test this, as the data provided from historical trades

are affected by a market of low liquidity. A possibility is to investigate comparable markets liquidity, and evaluate whether testing on historical data from there might be of relevance. However, a broader study of literature concerning i.e the German intraday market, must then be carried out.

In other words, there are many ways to broaden the study and also make it more comprehensive and in depth. Since the goal is to provide decision support through optimization, to help TE and other market participants strategize in Elbas, the most effort in the master thesis will be on making an optimization tool that will be able to account for the right parameters and provide support in a realistic environment.

.1 Appendix A

Figure 40 shows how the data set looked if it was split into states after normalizing, but *before* detrending and performance of PI transformation.

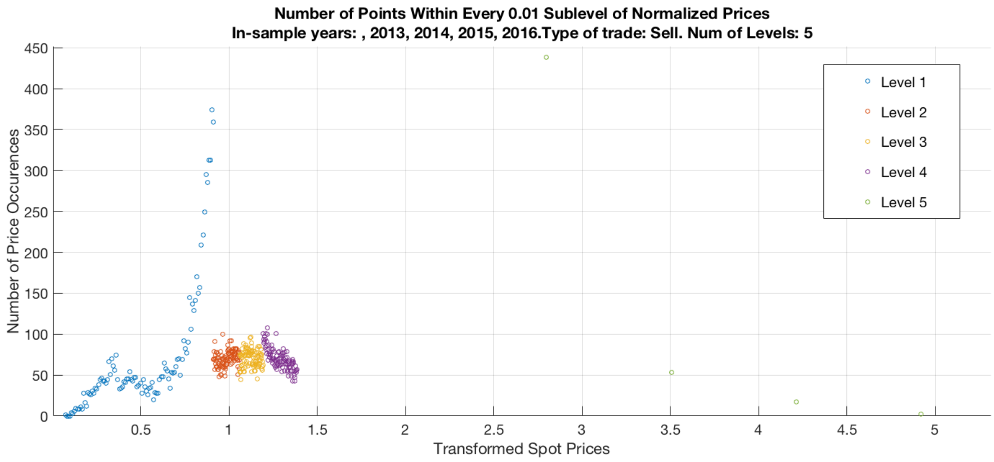
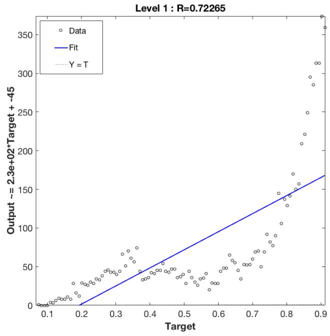
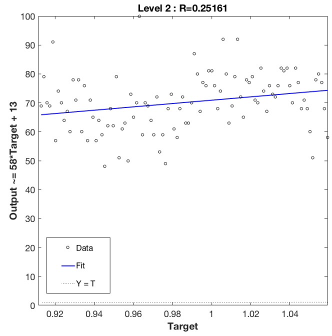


Figure 40: Scatter plot Showing Point Spread Within Each Level of the Normalized Price State Space. Here, In Sample Years are 2013-2016.

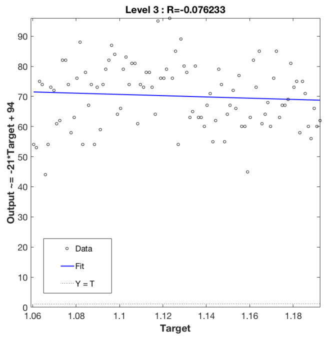
The points belonging to state 5 are green, and most of them are located either too far to the right or too high above to be in the illustration in Figure 40. The y-axis would have to take the value 7000 and the x-axis 75. Figure 41 shows the linear regression utilized to determine the mean of each level under the simplification of triangular distributed points within each level.



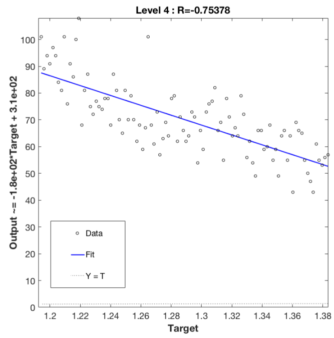
(a) Level 1



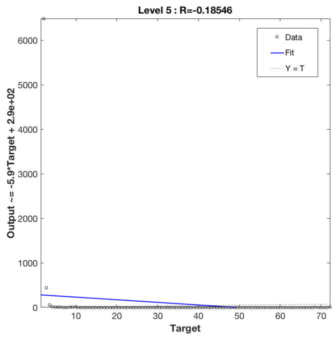
(b) Level 2



(c) Level 3



(d) Level 4



(e) Level 5

Figure 41: Regression of Normalized Price Points Within Level 1-5

Note specifically, how low the slope is in level 5 in Figure 41, compared to that of Figure 31.

.2 Appendix B

Figure 42, can be compared with Figure 32. By using the method described for Figure 32, we here observe that without removing the trend the P matrix based on IS data would not have a similar expected value to the OOS data. The results from removing the trend are therefore proved meaningful.

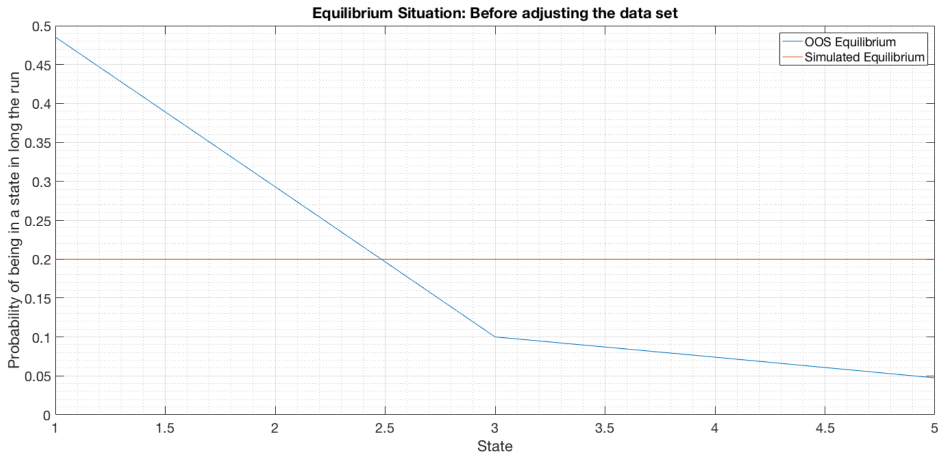


Figure 42: Plot showing the long run probabilities of being in the different states. Here, before removing trend and performing transformation on data set

.3 Appendix C

The implementation of the optimization model in Xpress Mosel. The file *basicParameters.txt* contains the basic parameters of the model. The output is read to the file *output.txt*, and further evaluated in Matlab.

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