Spot Market Bidding Taking the Balancing Power Market Into Account

Specialization Project

TIØ4500

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Abstract

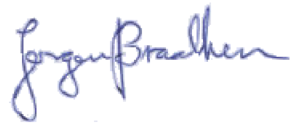
This paper discusses whether a hydropower producer in the Nordic region should take the Balancing Power Market into account in the spot bidding decisions. A stochastic programming model for the coordinated bidding problem is developed and tested for a Norwegian watercourse during the fall of 2012. Further, the paper analyses in which situations a coordinated bidding model may outperform a sequential model. The paper concludes that a coordinated model primarily is useful in situation in which the deviation between the spot price and the marginal water value is small. However, the model needs more testing before a final conclusion can be made. The deterministic equivalent of the stochastic programming problem is implemented, which resulted in very long running times. An appropriate solution method needs to be found.

Preface

This paper is written as a specialization project for the Master of Science degree at the Norwegian University of Science and Technology (NTNU), department of Industrial Economics and Technology Management within the field of Managerial Economics and Operations Research, TIØ4500. First and foremost, we would like to thank our supervisor, Professor Stein-Erik Fleten for helpful assistance throughout the semester, and PhD candidate Gro Klæboe at Department of Electric Power Engineering at NTNU for hours with valuable discussions. We would also like to thank Lars Thore Wibe Aarrestad and the good folks at Powel AS Smart Generation, and Pål Otto Eide and Knut-Harald Bakke at Norsk Hydro ASA for the great collaboration.

Trondheim, December 14, 2012

Anders Lund Eriksrud



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1 Introduction

A recent study from the European Network of Transmission System Operators for Electricity (ENTSO-E) concludes that the frequency quality in the Nordic region is unsatisfactory and has declined in recent years [5]. The amount of imbalances in the system is expected to increase as a result of several new interconnectors and increasing shares of non-flexible renewable power generation. Figure 1.1 illustrates that there is a distinct relationship between the transmission capacity out of the Nordic region and frequency imbalances. Consequently, the markets for balancing power will be growing and more important in the years to come. Sale of balancing power through the interconnectors to Europe is also introduced and expected to be growing, which is another indication that the turnover in the balancing markets will increase [25]. The Norwegian power system consists of a large share of reservoir hydropower, which is well suited for delivering balancing power.

Software for decision support in the spot market is well developed and used by most power producers in Norway. There is however no commercial software available for multimarket bidding, i.e., taking all physical markets into account in the bidding decisions. Currently, the quality of the bidding across markets is largely dependent on the experience of the production planner. Some work ([8] and [6]) has been done within the field of multimarket bidding for the Nordic markets, but no study has so far considered bidding strategies across the spot market and the Balancing Market in a producer perspective.

This paper investigates whether a producer should take the Balancing Market into account in the spot bidding phase. Section 2 gives an introduction to the Nordic power markets and the scheduling models used by Nordic hydropower producers. A stochastic programming model for coordinated bidding across Elspot and the Nordic Balancing Market is developed in Section 3. Section 4 presents a methodology for generating scenarios for the uncertain data inflow, spot price, Balancing Market volumes and premiums. The coordinated model is tested for a Norwegian watercourse during the fall of 2012 in Section 5. Further, a discussion of when the coordinated model is beneficial is conducted. The paper concludes that a sequential model will be sufficient only if the spot price deviates largely from the water values, and that the coordinated model may be useful in the majority of the time. The problem needs to be studied in more detail, and future work is suggested in Section 6.

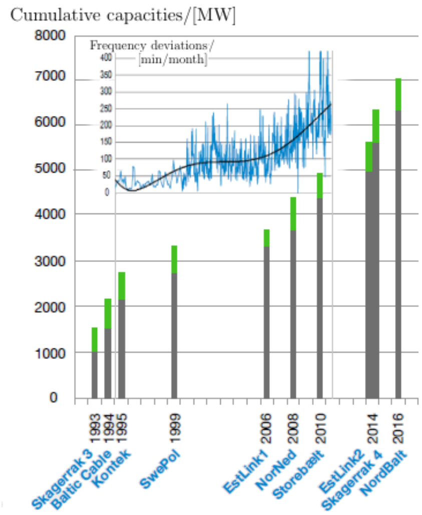


Figure 1.1: Cumulative interconnector capacities out of the Nordic region (columns) compared to frequency deviations (line) in minutes outside the region $[49.9, 50.1]$ Hz per month. The green areas represent new capacity [25].

2 The Nordic Electricity Markets and Institutional Background

This section gives an overview of the different power markets in the Nordic region and hydropower scheduling models. The focus in this paper is on the spot market and the Nordic Balancing Market, which are presented in detail. Moreover, scheduling models commonly used by hydropower producers in the Nordic region are described.

2.1 Electricity Markets

The power markets in northern Europe have been in constant development in the past decades. The power sector has changed from being publicly regulated to a market based industry. Nord Pool was established in 1993 as an exchange for the Norwegian market. In 1996 the exchange was extended to include Sweden, and thus became the world's first multinational exchange for electricity [32].

Table 2.1 shows an overview over the current Nordic electricity markets and trading routines are illustrated in Figure 2.1. The spot market, Elspot, is organized by Nord Pool Spot (NPS). This is the largest market place for purchase and sale of physical electricity in the world. In 2011, 73 % of all power in the Nordic region was traded on NPS [24]. The remaining is traded in Bilateral contracts. Section 2.2 describes Elspot in detail.

Market Place	Physical trade	Financial trade
Nord Pool Spot (NPS)	Elspot Elbas	
Transmission System Operators (TSOs)	Primary reserve (FNR and FDR) Secondary reserve (FRR) Tertiary reserve (Balancing Market)	
Nasdaq OMX Commodities		Futures Forwards Options Contracts for Difference (CfD)
Bilateral	Full delivery Load factor contracts Spot (cap and floor), etc.	Forwards Options , etc.

Table 2.1: Overview of the Nordic electricity markets.

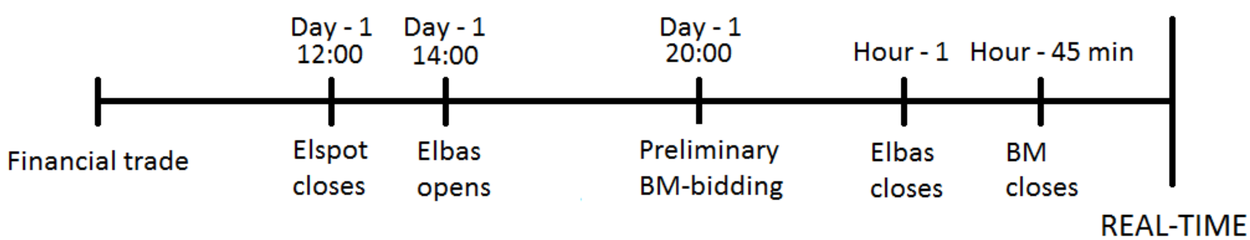


Figure 2.1: Trading routines in the Nordic electricity markets (markets for primary and secondary reserves are not included).

NPS also organizes the continuous intraday market Elbas. It provides the opportunity for trading power across the Nordic region, Germany and Estonia. Total Elbas turnover in 2011 was 2.7 TWh. The total Elbas volumes traded yearly in Norway are very small, and represent about 0.1 % of the Elspot turnover. Elbas opens at 14:00, following the closing of the Elspot auction, and trading capacities available for the next day are published. Trades in Elbas are allowed up to one hour before real-time[1], which gives the participants the opportunity to adjust for imbalances if production and consumption schedules deviate from the volume committed in Elspot. Prices are set based on a first-come, first-served principle [24].

The Transmission System Operators (TSOs) are responsible for the grid stability and the power balance in the system. Imbalances caused by deviations from the production plans and load forecasts must be leveled out in order to maintain the instant balance in the system at any point of time. Hence, the TSOs need access to balancing power, normally divided into primary, secondary and tertiary reserves.

The primary reserve is automatic and activated at low frequency deviations to ensure instantaneous power balance. The procurement procedures varies across the Nordic countries. The Norwegian TSO Statnett requires a basic delivery from producers with generators larger than 10 MVA. Additional needs are procured in established markets (FNR and FDR) [28]. The price is set as the marginal price for each price area.

If the primary reserve is unable to handle the deviations, the secondary reserve will be activated. The Nordic TSOs have decided to introduce a common secondary reserve solution from 2013, called Frequency Restoration Reserves (FRR). Statnett has signed agreements with five large producers, but procedures for bidding and financial settlements are not yet set [29].

Large deviations require activation of the tertiary reserve in the Nordic Balancing Market. This balancing does not provide instantaneous power, but is activated within 15 minutes. The Nordic Balancing Market is described in detail in Section 2.3.

The participants are also able to trade electricity derivatives in the financial markets, operated by Nasdaq OMX Commodities [4]. Producers use the financial markets for price hedging and risk management. The system price (explained in Section 2.2) is underlying the financial derivatives, and there is no physical delivery in the financial contracts. The financial market has a time horizon up to six years ahead. There is also an instrument for hedging against price differences between price areas, called Contracts for Difference (CfD).

[1] Statnett has decided to change the gate closure for the Elbas market in Norway from two hours before real-time to one hour before real-time from February 26, 2013.

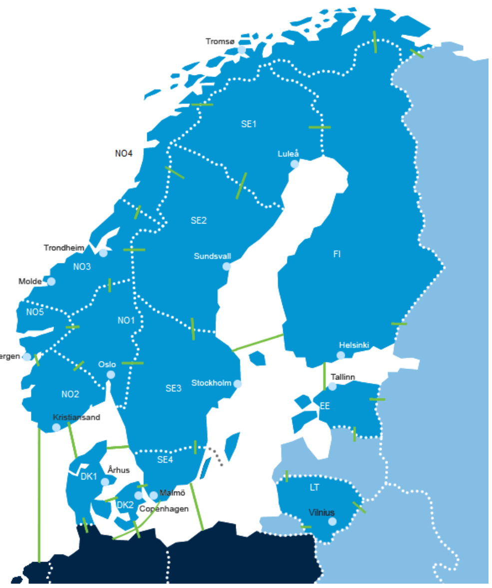


Figure 2.2: NPS price areas October 2012 [24].

2.2 The Spot Market

The spot market Elspot is a day-ahead auction in which hourly power contracts are traded daily for physical delivery. Total turnover in 2011 was 294.4 TWh. The Elspot market includes Norway, Sweden, Finland, Denmark, Estonia and Lithuania [24]. In order to handle grid congestions, the Nordic exchange area is divided into price areas with individual area prices, as shown in Figure 2.2. Absence of bottlenecks results in equal prices for all areas. The system price is an artificial price, calculated for the entire Nordic Region as if there were no transmission limits.

Prior to 12:00 noon, producers and consumers submit bids for selling and buying electricity for the next day, that is, the next 12-36 hours. NPS then calculates the area prices for each hour and area. The prices are normally published between 12:30 and 12:45. Figure 2.3 shows historical Elspot prices in the Norwegian price area NO2. Three different types of bids can be submitted.

- **Single hourly bids** represents the largest share of the Elspot trading. The participants specify the purchase and sales volumes for each hour and choose between a price dependent and a price independent bid. The minimum requirement for a single hourly bid is two pricepoints, at the minimum price (-€200) and the maximum price (€2000). A price dependent single hourly bid may consist of up to 62 pricepoints in addition to the upper and lower pricepoints. Furthermore, the bidding curve

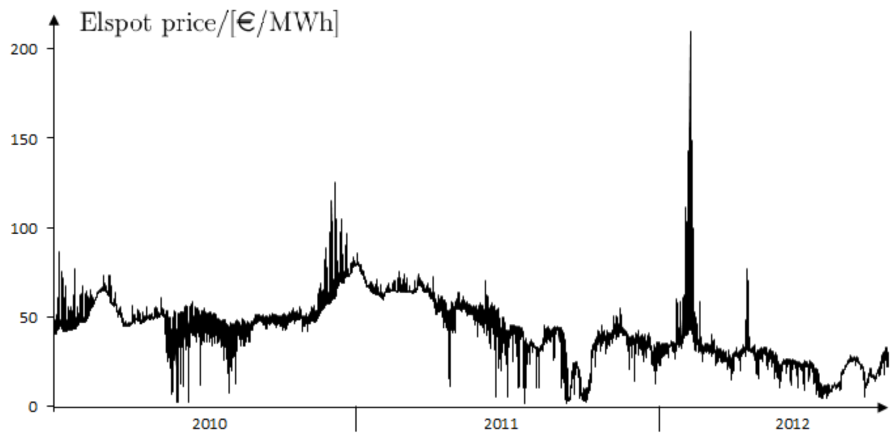


Figure 2.3: Elspot prices in NO2, 01.01.2010 - 10.03.2012 [24].

must be non-decreasing. A participant that submits price dependent bids accept that NPS will make a linear interpolation of volumes between each adjacent pair of submitted pricepoints.

- **Block bids** give the participants the opportunity to set an all-or-nothing condition for a set of at least three consecutive hours, called a *block*. A producer's block bid is knocked down if the average Elspot price for the applicable hours exceeds the bid price[2]. The block bid is useful for producers with high costs associated with starting and stopping production.

- **Flexible hourly bids** is a single hourly sales bid to which the participant sets a fixed price and volume, without specifying the applicable hour. The bid is knocked down at most once if the Elspot price exceeds the bid price for an hour during the bidding day. Flexible bids only apply to consumers.

Using market power is forbidden, the sales bids must represent the marginal cost of production. The marginal cost of hydropower is discussed in Section 2.4.

2.3 The Balancing Power Market

The common Nordic Balancing Market (BM) serves as a tool for the TSOs to ensure balance in the power system. There are three possible balancing states. The first is the situation where production equals consumption and no balancing power is needed.

[2]If including the block bid results in lower Elspot prices, such that the block bid will not be accepted with the new Elspot price, the block bid might not be knocked down although the average price is higher than the block bid price.

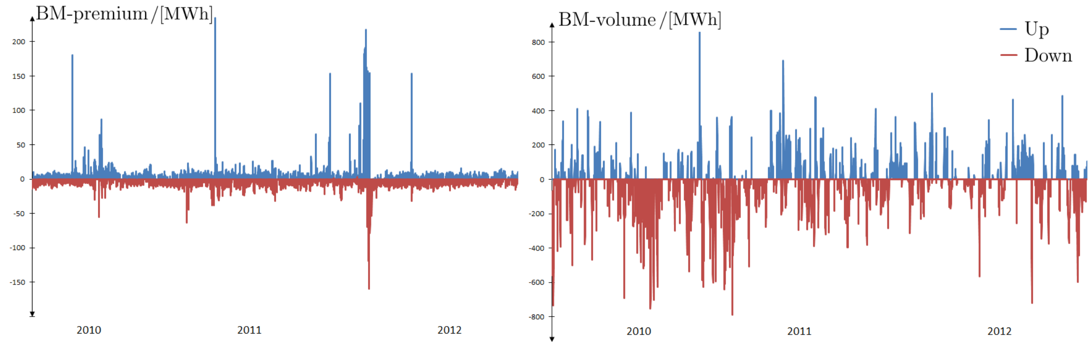


Figure 2.4: BM-premiums (left) and BM-volumes (right) in NO2, 01.01.2010 - 10.03.2012 [24].

The second state is when consumption exceeds production. Upward balancing (BM↑) is needed and the TSO asks a producer to increase production or a consumer to decrease consumption. The last state is downward balancing (BM↓), when consumption exceeds production. The TSO asks a producer to decrease production or a consumer to increase consumption. The difference between the Elspot price and the BM-price is defined as the BM-premium. Historical BM-premiums and BM-volumes in the Norwegian price area NO2 are shown in Figure 2.4. Both the volumes and the premiums are very volatile, with several spikes, especially in BM↑.

Bids in the Balancing Market are submitted for each watercourse to the local TSO after the Elspot market has closed and until 45 minutes before real-time. There is a preliminary deadline for BM-bidding at 20:00 the day before the respective hour. Separate bids for upward and downward balancing are given and the quantity must be activated within a 15 minutes notice. The bids must have a duration of one hour or more. In Norway, each bid (in NOK/MWh) must be integer divisible by 5 and have a minimum quantity of 25 MW. The minimum bidding quantity for small producers is 10 MW [26].

The bids are rated in merit order as illustrated in Figure 2.5. The lowest upward balancing bids and highest downward balancing bids are knocked down, and all participants receive the same price. Each participant can submit several separate bids. The price setting in the Balancing Market ought to be socio-economic efficient, that is, it is forbidden to abuse market power. Statnett may suspend bids that are not representing the marginal cost of production, and use the current Elspot price. In situations with congestions or errors, Statnett has the opportunity to choose bids without following the merit order. This is called custom balancing[3].

[3] *NO:* spesialregulering.

	Hours with upward balancing	Hours with downward balancing
Surplus imbalance	Elspot price	BM↓ price
Deficit imbalance	BM↑ price	Elspot price

Table 2.2: Imbalance prices for producers as defined by Statnett.

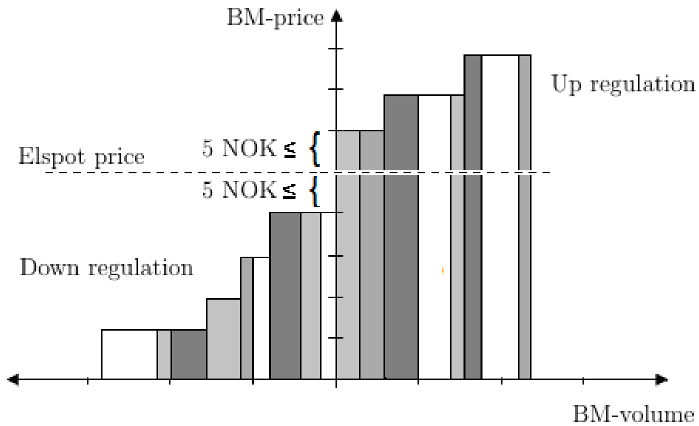


Figure 2.5: Bids in merit order for the Balancing Market.

Participants that does not meet their obligations in the markets must pay the imbalance charges, which is the worst price of Elspot price and the BM-price. Table 2.2 shows the imbalance prices given by Statnett.

Statnett has an option market that is used to ensure sufficient volumes in the Balancing Market, in which the participants receive an option price for committing to bid a certain volume into the Balancing Market. The option market is used when the load is high during winter [27].

2.4 The Value of Water

The marginal cost of hydropower is very small, because the water comes for free. However, pricing hydropower to its marginal cost is not a good idea, water is a scarce resource, and might be stored for later use. Thus, the alternative cost of the water should be considered, that is, the income from using the water in a subsequent time period. The future income of a marginal unit of water is not known, due to uncertainty in future prices and inflow. If the reservoir is full, a marginal unit of stored water will probably be spilled, hence the value is zero. If the reservoir is almost empty, the water could be stored to a period with high prices, and the value of water is high. The expected future income of a marginal unit of water is called the marginal *water value*. From the discussion above, it should be clear

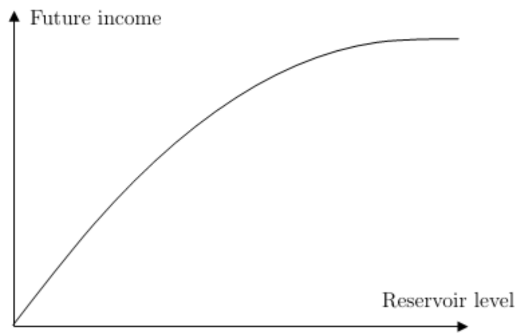


Figure 2.6: Reservoir-future income curve.

that the water value is a function of future prices, future inflow and the current reservoir level. It should also be clear that the marginal water value is decreasing with increasing reservoir levels, yielding a concave[4] reservoir-future income curve, as shown in Figure 2.6.

When start-up costs are not considered, the total operating expenses from a hydropower plant could be assumed constant in the long run. The only relevant cost is thus the marginal water value. A power plant should produce if the water value is lower than the price.

The water value could be estimated in different ways. In [7], a combination of prices on futures and forwards for electricity is used for the water value calculations. If the reservoir is empty, it is assumed that 50 % of the inflow will be sold at the current price of a certain future and the remaining 50 % will be sold at a certain forward price. Interpolating linearly to the point of a full reservoir with a marginal water value of zero gives an estimate of the water value curve. The use of more fundamental methodologies is however more common in Norway. The marginal water values are estimated using long term optimization models, as explained in the next section.

2.5 Scheduling Hierarchy

An overview of the generation hierarchy could be found in [9]. Water is a scarce resource, thus, the use of water tomorrow affects the future opportunity to produce. In order to make the optimal decision for tomorrow, all subsequent periods should therefore be taken into account. However, deciding production for all time periods to come is impossible. Further, developing detailed production plans for long planning horizons results in high computational times. Simplifications are therefore needed in order to schedule production for periods far from tomorrow. Figure 2.7 shows the hierarchy of scheduling models used

[4]The marginal water value is the derivative of the future income with respect to the reservoir level. A non-increasing derivative proves a concave curve.

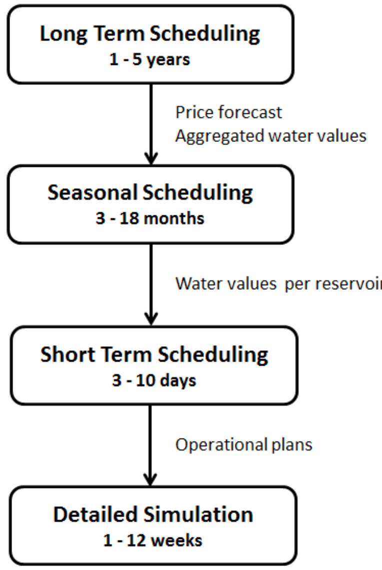


Figure 2.7: Scheduling hierarchy used by hydropower producers in the Nordic area.

by most hydropower producers in the Nordic area. Sintef Energy Research has developed the different models described below.

First, long term models are run, based on stochastic dynamic programming. The EMPS model is a multi area long term model, in which the total system is divided into separate areas. Within each area, all hydropower production is aggregated to one plant with a single reservoir. Water values for each area are calculated. The marginal water value is estimated as the shadow price of the reservoir balance constraint. The objective is to minimize costs or maximize social welfare, when thermal generation, imports and exports etc. are taken into account. When the marginal water values are calculated, a detailed optimization within each area is done with a time step of one week. Several heuristics are used in the area optimization, which to some extent rely on user experience. The EOPS model is a one-area model, in which a producer could model its own water course with a market description with prices from the EMPS model. The stochasticity in the EMPS model is described simulating the system with historical inflow series. The generated price scenarios from the EMPS model and the historical inflow series are the stochastic input data to the EOPS model.

In order to estimate the marginal water values for each reservoir, a seasonal model is run. The seasonal model is a multiscenario deterministic optimization model, in which the initial and the final reservoir levels are given. The model is run for several initial reservoir levels. The objective is to maximize future income. The marginal water values are

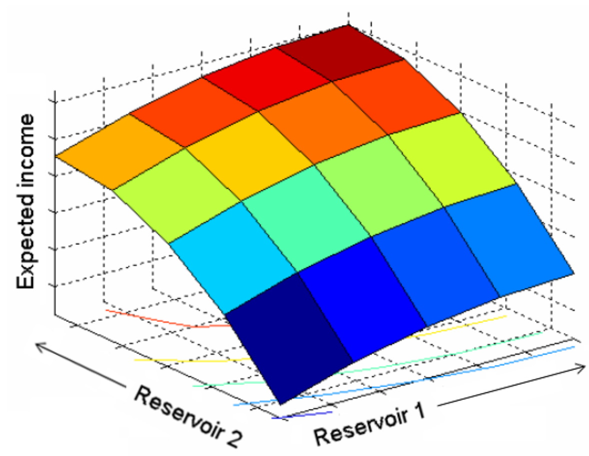


Figure 2.8: Illustration of future income curve with two reservoirs and 16 cuts.

estimated as the expected shadow prices of the reservoir balance in the first period. Now, a cut k is defined, with a corresponding reservoir level L_{kj}, future income F_k and marginal water value V_{kj} for each reservoir j in the set of reservoirs $\mathcal{J}$. Thus, an approximation of the future income v for any reservoir level l_j could be given as

$$v_j \approx F_k - V_{jk}(L_{jk} - l_j), \qquad \forall j \in \mathcal{J} \tag{2.1}$$

This approximation is only valid in a small region around the initial reservoir level L_{jk}. Because the future income curve is assumed to be concave, creating a set of cuts $\mathcal{K}$ with different initial reservoir levels, gives a better approximation of the future income

$$v \leq F_k - \sum_{j \in \mathcal{J}} V_{jk}(L_{jk} - l_j), \qquad \forall k \in \mathcal{K} \tag{2.2}$$

Note that this is an over-approximation of the concave future income curve. The cuts are illustrated in Figure 2.8 with two reservoirs.

The water value cuts can be used in the short term model as the value of the reservoirs in the end of the planning horizon. Short term scheduling is described in the next section. Because the short term model includes simplifications of the physical system, a detailed simulation is often used to evaluate the results, taking non-linearities etc. into account.

2.6 Short Term Production Scheduling

In Norway, the Short Term Hydropower Optimization (SHOP) model, developed by Sintef Energy Research is commonly used for short term scheduling. This is a deterministic model. Below is a general description of the short term hydropower scheduling problem.

The power production w equals the potential energy stored in the released water per unit time multiplied by the overall efficiency when the head is constant

$$w = \eta^g \eta^t q \gamma H^{\text{eff}} \tag{2.3}$$

where η^g is the efficiency of the generator, η^t is the efficiency of the turbine, q is the discharge, γ is the specific gravity of water and H^{eff} is the effective head, i.e., the head less the head loss. The head loss is described as αq^2, where α is a constant. The efficiencies η^g and η^t are also dependent on q, thus the production-discharge curve is non-linear. The non-linear curve can be linearized with a set of cuts $\mathcal{F}$, each cut $f \in \mathcal{F}$ has a constant marginal production E_{jf} and a constant $\hat{E}_{jf}$ that describe a linear cut for each generator j in the set of generators $\mathcal{J}$ at time t in the set of time periods $\mathcal{T}$

$$w_{jt} \leq E_{jf} q_{jt} + \hat{E}_{jf}, \qquad \forall f \in \mathcal{F}, j \in \mathcal{J}, t \in \mathcal{T} \tag{2.4}$$

Starting a generator results in increased maintenance costs [17], and should be taken into account. Binary variables u_{jt} are needed for modeling whether generator[5] $j \in \mathcal{J}$ is in operation at time $t \in \mathcal{T}$ ($u_{jt} = 1$) or not ($u_{jt} = 0$). If C_j denotes the start-up cost for generator $j \in \mathcal{J}$, the continuous term o_{jt} should be subtracted from the objective function with the following constraints

$$o_{jt} \geq C_j(u_{jt} - u_{j(t-1)}), \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{2.5}$$

$$\underline{W}_j u_{jt} \leq w_{jt} \leq \overline{W}_j u_{jt}, \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{2.6}$$

$$o_{jt} \geq 0, \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{2.7}$$

$$u_{jt} \in \{0, 1\}, \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{2.8}$$

where $\underline{W}_j$ and $\overline{W}_j$ are the lower and upper limits on the production from reservoir $j \in \mathcal{J}$, respectively. (2.5) forces o_{jt} to equal C_j if there is a start-up at time $t \in \mathcal{T}$, whereas o_{jt} will be minimized to 0 if there is no start-up [3].

Further, the reservoir balance is

$$l_{jt} = l_{j(t-1)} - q_{jt} - r_{jt} + \kappa_{jt} + \sum_{\psi \in \mathcal{G}_j} (q_{\psi(t-D_{\psi j})} + r_{\psi(t-D_{\psi})}), \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{2.9}$$

where l_{jt} denotes the reservoir level, κ_{jt} the inflow and r_{jt} the spill for reservoir $j \in \mathcal{J}$ at time $t \in \mathcal{T}$. The set of reservoirs for which discharge and spill end up in reservoir $j \in \mathcal{J}$ is denoted by $\mathcal{G}_j$, and $D_{\psi j}$ is the time delay from reservoir ψ to reservoir j.

Each reservoir level has an upper bound $\overline{L}_j$ and a lower bound $\underline{L}_j$

$$\underline{L}_j \leq l_{jt} \leq \overline{L}_j, \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{2.10}$$

[5]It is assumed that each reservoir $j \in \mathcal{J}$ is connected to one station with one generator.

The discharge from reservoir $j \in \mathcal{J}$ has upper and lower bounds $\overline{Q}_j$ and $\underline{Q}_j$, respectively.

$$\underline{Q}_j \leq q_{jt} \leq \overline{Q}_j, \qquad \forall j \in \mathcal{J}, t \in \mathcal{T} \tag{2.11}$$

The objective is to maximize profits z from the spot market, with spot price ρ_t at time $t \in \mathcal{T}$, less start-up costs plus the value of the end reservoir, represented by v

$$\max z = \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} \rho_t w_{jt} - \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} o_{jt} + v \tag{2.12}$$

3 A Stochastic Programming Model for the Coordinated Bidding Problem

The following section presents a stochastic model for the coordinated bidding problem. First, market modeling is discussed. Then, the entire mathematical model with underlying assumptions are presented.

3.1 Modeling the Markets

There has thus far been little research on multimarket bidding for a producer in the Nordic electricity market. [8] presents a bidding model for a retailer, taking the balancing market into account. In [6] a model for coordination between the spot market and the Elbas market for a price taking producer is developed. There is done some research in markets with similar structure as the Nordic market. [21] and [31] model the producer as a price setter in the Balancing Market by estimating of the residual demand curve. For markets with a high share of flexible power production, there is no reason to argue for market power in the Balancing Market if there is no market power in the spot market. The producer in this paper is modeled as a price taker in both the spot market and the Balancing Market.

Market m in the set of markets $\mathcal{M}$ is defined such that

$$m \in \mathcal{M} \triangleq \begin{cases} 1 & \text{for the spot market} \\ 2 & \text{for BM}\uparrow \\ 3 & \text{for BM}\downarrow \end{cases} \tag{3.1}$$

The clearing of the spot market is done by interpolation of the bids, as explained in Section 2.2. Letting $\mathcal{T}^B$ denote the set of time periods with bidding, $\mathcal{S}$ the set of scenarios and $\mathcal{I}_1$ the set of bidpoints, this can be expressed mathematically as

$$\rho_{1ts} = P_{1i} + \frac{P_{1(i+1)} - P_{1i}}{x_{(i+1)t} - x_{it}} (y_{1ts} - x_{it}) \qquad \text{if } x_{it} \leq y_{1ts} < x_{(i+1)t}, \qquad \forall i \in \mathcal{I}_1, t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.2}$$

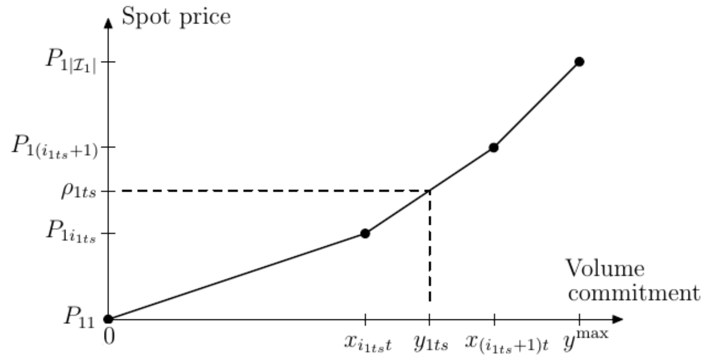


Figure 3.1: Interpolation of spot clearing price and volume.

where ρ_{1ts} denotes the spot clearing price at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$, P_{1i} is the predefined pricepoint at bidpoint $i \in \mathcal{I}_1$, x_{it} the bid volume for bidpoint $i \in \mathcal{I}_1$ at time $t \in \mathcal{T}^B$ and y_{1ts} the volume commitment for the single hourly bids in spot market at time $t \in \mathcal{T}^B$ in scenario $s \in \mathcal{S}$. (3.2) can also be written on the form

$$y_{1ts} = \frac{\rho_{1ts} - P_{1i}}{P_{1(i+1)} - P_{1i}} x_{(i+1)t} + \frac{P_{1(i+1)} - \rho_{1ts}}{P_{1(i+1)} - P_{1i}} x_{it} \qquad \text{if } P_{1i} \leq \rho_{1ts} < P_{(i+1)t}, \qquad \forall i \in \mathcal{I}_1, t \in \mathcal{T}^B \tag{3.3}$$

Block bids are not not modeled in this paper. Further, let i_{mts} be defined as

$$i_{mts} \triangleq \begin{cases} \max\left\{i \in \mathcal{I}_m \;\middle|\; P_{mi} \leq \rho_{mts}\right\}, & \text{if } \rho_{mts} \geq P_{m1} \\ 0, & \text{if } \rho_{mts} < P_{m1} \end{cases}, \quad \forall m \in \mathcal{M}, t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.4}$$

For BM↓, the prices ρ_{3ts} are negative, because trading in BM↓ represents "buying back" power that is already committed in the spot market. The bid prices must be non-decreasing, i.e., $P_{mi} \leq P_{m(i+1)}$ $\forall m \in \mathcal{M}, i \in \mathcal{I}_m$. For the spot market, i in (3.3) can now be substituted by i_{1ts}. Figure 3.1 shows how the interpolation is done.

The definition (3.4) is also valid for the Balancing Market. Because a bid b_{mits} in market $m \in \{2, 3\}$ for pricepoint $i \in \mathcal{I}_m$ at time $t \in \mathcal{T}^B$ in scenario $s \in \mathcal{S}$ is either knocked down or rejected, the clearing volume y_{mts} can be expressed as

$$y_{mts} = \sum_{i=1}^{i_{mts}} b_{mits}, \qquad \forall m \in \{2, 3\}, t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.5}$$

All BM-bids that have bid prices less than the clearing price are knocked down.

In order to avoid corner solutions, i.e., overallocation to the Balancing Market, there is done an analysis to forecast the BM-volumes ν_{mts} in each market $m \in \{2, 3\}$ at time

$t \in \mathcal{T}^B$ in scenario $s \in \mathcal{S}$. The total volume in the market represents an upper bound of the clearing volume of each participant, i.e.,

$$y_{mts} \leq \nu_{mts}, \qquad \forall m \in \{2,3\}, t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.6}$$

Moreover, the producer might experience imbalances in some scenarios when the sum of production w_{jts} does not equal the total volume commitments y_{mts}, expressed as

$$y_{1ts} + y_{2ts} - y_{3ts} = \sum_{j \in \mathcal{J}} w_{jts} + \Delta w_{ts}^{-} - \Delta w_{ts}^{+}, \qquad \forall t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.7}$$

where Δw_{ts}^{-} denotes the deficit imbalance and Δw_{ts}^{+} denotes the surplus imbalance at time $t \in \mathcal{T}^B$ in scenario $s \in \mathcal{S}$. The TSO requires that there is no expected imbalance when bidding into the spot market. This can be expressed as

$$\sum_{s \in \mathcal{S}} \pi_s (\Delta w_{ts}^{+} - \Delta w_{ts}^{-}) = 0, \qquad \forall t \in \mathcal{T}^B \tag{3.8}$$

where π_s denotes the probability of scenario $s \in \mathcal{S}$. An alternative formulation of (3.8) is including the BM-commitments $(+y_{2ts} - y_{3ts})$. This results in unbiased spot bidding, meaning that the spot commitment equals the expected production for each hour. However, constraining the BM-commitments such that the expected upward balancing equals the expected downward balancing will restrict the Balancing Market flexibility significantly. In periods with low spot prices compared to the marginal water value, a producer will tend to bid small volumes into the spot market, which gives little flexibility in BM↓ and high flexibility in BM↑. Thus, on expectation, the producer will probably commit to higher BM↑-volumes than BM↓-volumes. The situation in which the spot price exceeds the marginal water value will result in the opposite. The producer prefers to bid high volumes into the spot market, yielding higher flexibility in BM↓ than in BM↑.

Further, as explained in Section 2.3, surplus imbalances are compensated with the lowest of the spot price and BM↓-price and deficit imbalances are penalized with the highest of the spot price and the BM↑-price. In order to avoid big imbalances in both directions, a factor of 1.2 is multiplied with the imbalance prices, which gives imbalance costs σ_{ts}^{+} and σ_{ts}^{-} for time $t \in \mathcal{T}^B$ and scenario $s \in \mathcal{S}$[6]

$$\sigma_{ts}^{-} \triangleq 1.2 \max\{\rho_{1ts},\, \rho_{2ts}\}, \quad \sigma_{ts}^{+} \triangleq \frac{1}{1.2} \min\{\rho_{1ts},\, -\rho_{3ts}\}, \quad \forall t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.9}$$

and

$$\sum_{t \in \mathcal{T}^B} \sum_{s \in \mathcal{S}} \left(\sigma_{ts}^{+} \Delta w_{ts}^{+} - \sigma_{ts}^{-} \Delta w_{ts}^{-} \right) \tag{3.10}$$

[6] ρ_{3ts} is defined as negative.

is added to the objective function. The additional penalty factor of 1.2 is used in order for the producer to act risk averse with respect to imbalances. The TSOs are expecting the participants to act in a way that does not jeopardize the system reliability. Although (3.8) avoids expected imbalances, it may still allow for large imbalances in both directions. This kind of behavior will be sanctioned, because the TSO rather considers the total imbalance than the sum of surplus imbalances less the sum of deficit imbalances. If a producer acts in a way that causes large imbalances in both directions over time, it may risk its concession to produce. Therefore, a rational producer will have a certain risk aversion when it comes to imbalances. The factor 1.2 is set ad hoc, investigating the correctness of the magnitude of this factor is beyond the scope of this paper.

The BM-volumes are also bounded by the available capacity for balancing

$$y_{2ts} \leq \sum_{j \in \mathcal{J}} \overline{W}_j - y_{1ts}, \qquad \forall t \in \mathcal{T}^B \tag{3.11}$$

$$y_{3ts} \leq y_{1ts}, \qquad \forall t \in \mathcal{T}^B \tag{3.12}$$

in order to avoid scenarios with both imbalances and BM-volume sales at the same time.

3.2 Problem Formulation

The entire mathematical model is presented below based on the stochastic programming framework [1]. The underlying assumptions for the model are

- The watercourse consists of two reservoirs in cascade, each with a station containing one generator, as illustrated in Figure 3.2. The discharge and spill from reservoir 1 end up in reservoir 2
- Water value curves and production-discharge curves are concave
- The head is constant
- The producer does not participate in any other physical market than the spot market and the Balancing Market
- The producer acts as a price taker in both markets
- Custom balancing is not used
- Block bids are not used

The list of sets, data and variables is presented below. Rounded capital letters ($\mathcal{A}, \mathcal{B}, \mathcal{C}$ etc.) denote sets, greek letters denote uncertain data, capital roman letters denote deterministic data and lower case roman letters denote variables and indices.

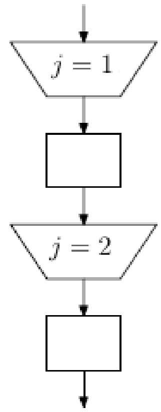


Figure 3.2: An illustration of the modeled watercourse.

Sets

$\mathcal{F}$	Production curve points
$\mathcal{I}_m$	Bid points for market $m \in \mathcal{M}$
$\mathcal{J}$	Reservoirs
$\mathcal{K}$	Cuts in the water value curve
$\mathcal{M}$	Markets
$\mathcal{S}$	Scenarios
$\mathcal{T}$	Time periods (hours)
$\mathcal{T}^B \subset \mathcal{T}$	Time periods with bidding

Data

κ_{jts}	Inflow to reservoir $j \in \mathcal{J}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$
ν_{mts}	Volume in market $m \in \{2, 3\}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$
π_s	Probability of scenario $s \in \mathcal{S}$
ρ_{mts}	Price in market $m \in \mathcal{M}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$ ($\rho_{3ts} \leq 0$)
$\sigma^+_{ts}, \sigma^-_{ts}$	Penalty of impalance at time $t \in \mathcal{T}^B$ in scenario $s \in \mathcal{S}$
C_j	Start up cost reservoir $j \in \mathcal{J}$
D	Time delay between reservoirs
$E_{jf}, \hat{E}_{jf}$	Constants for the Production-Discharge Curve
F_k	Expected future income for cut $k \in \mathcal{K}$
$\underline{L}_j, \overline{L}_j$	Minimum and maximum reservoir level for reservoir $j \in \mathcal{J}$, respectively
L_{jk}	Reservoir level for reservoir $j \in \mathcal{J}$ and cut $k \in \mathcal{K}$
P_{mi}	Price point $i \in \mathcal{I}_m$ for market $m \in \mathcal{M}$
$\underline{Q}_j, \overline{Q}_j$	Minimum and maximum discharge for reservoir $j \in \mathcal{J}$, respectively
V_{jk}	Marginal Water value for reservoir $j \in \mathcal{J}$ for cut $k \in \mathcal{K}$
$\underline{W}_j, \overline{W}_j$	Minimum and maximum production for reservoir $j \in \mathcal{J}$, respectively

Variables

b_{mits}	Bidding volume in market $m \in \{2,3\}$ for bidpoint $i \in \mathcal{I}$ at time $t \in \mathcal{T}$
l_{jts}	Reservoir level in reservoir $j \in \mathcal{J}$ at the end of period $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$
o_{jts}	=1 if production from reservoir $j \in \mathcal{J}$ starts at time $t \in \mathcal{T}^B$ in scenario $s \in \mathcal{S}$, 0 otherwise
q_{jts}	Water discharge from reservoir $j \in \mathcal{J}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$
r_{jts}	Spill from reservoir $j \in \mathcal{J}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$
u_{jts}	=1 if there is production from reservoir $j \in \mathcal{J}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$, 0 otherwise
v_s	Water value of all reservoirs in scenario $s \in \mathcal{S}$ at the end of the planning period
w_{jts}	Generation from reservoir $j \in \mathcal{J}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$
$\Delta w_{ts}^+, \Delta w_{ts}^-$	Generation imbalance (surplus and deficit) at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$
x_{it}	Bidding volume in spot market for bidpoint $i \in \mathcal{I}$ at time $t \in \mathcal{T}$ (first stage decision)
y_{mts}	Volume commitment in market $m \in \mathcal{M}$ at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$

$$\max \ z = \sum_{s\in\mathcal{S}} \pi_s \left(\sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \rho_{mts} y_{mts} + v_s - \sum_{j\in\mathcal{J}} \sum_{t\in\mathcal{T}} o_{jts} + \sum_{t\in\mathcal{T}^B} (\sigma_{ts}^+ \Delta w_{ts}^+ - \sigma_{ts}^- \Delta w_{ts}^-) \right) \tag{3.13}$$

s.t.

$$x_{it} \leq x_{(i+1)t}, \qquad \forall i \in \mathcal{I}_1, t \in \mathcal{T}^B \tag{3.14}$$

$$y_{1ts} = \frac{\rho_{1ts} - P_{1i_{1ts}}}{P_{1(i_{1ts}+1)} - P_{1i_{1ts}}} x_{(i_{1ts}+1)t} + \frac{P_{1(i_{1ts}+1)} - \rho_{1ts}}{P_{1(i_{1ts}+1)} - P_{1i_{1ts}}} x_{i_{1ts}t}, \qquad \forall t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.15}$$

$$y_{mts} = \sum_{i=1}^{i_{mts}} b_{mits}, \qquad \forall m \in \{2,3\}, t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.16}$$

$$y_{1ts} + y_{2ts} - y_{3ts} = \sum_{j\in\mathcal{J}} w_{jts} + \Delta w_{ts}^- - \Delta w_{ts}^+, \qquad \forall t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.17}$$

$$\sum_{s\in\mathcal{S}} \pi_s (\Delta w_{ts}^+ - \Delta w_{ts}^-) = 0, \qquad \forall t \in \mathcal{T}^B \tag{3.18}$$

$$y_{2ts} \leq \sum_{j\in\mathcal{J}} \overline{W}_j - y_{1ts}, \quad \forall t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.19}$$

$$y_{3ts} \leq y_{1ts}, \quad \forall t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.20}$$

$$y_{1ts} = \sum_{j\in\mathcal{J}} w_{jts}, \quad \forall t \in \mathcal{T} \setminus \mathcal{T}^B, s \in \mathcal{S} \tag{3.21}$$

$$v_s \leq F_k - \sum_{j\in\mathcal{J}} V_{jk}(L_{jk} - l_{j|\mathcal{T}|s}), \quad \forall k \in \mathcal{K}, s \in \mathcal{S} \tag{3.22}$$

$$w_{jts} \leq E_{jf} q_{jts} + \hat{E}_{jf}, \quad \forall j \in \mathcal{J}, f \in \mathcal{F}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.23}$$

$$o_{jts} \geq C_j(u_{jts} - u_{j(t-1)s}), \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.24}$$

$$\underline{W}_j u_{jts} \leq w_{jts} \leq \overline{W}_j u_{jts}, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.25}$$

$$l_{1ts} - l_{1(t-1)s} + q_{1ts} + r_{1ts} = \kappa_{1ts}, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.26}$$

$$l_{2ts} - l_{2(t-1)s} + q_{2ts} - r_{1ts} + r_{2ts} - q_{1(t-D)s} = \kappa_{2ts}, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.27}$$

$$0 \leq y_{mts} \leq \nu_{mts}, \quad \forall m \in \{2,3\}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.28}$$

$$\underline{Q}_j \leq q_{jts} \leq \overline{Q}_j, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.29}$$

$$\underline{L}_j \leq l_{jts} \leq \overline{L}_j, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.30}$$

$$o_{jts}, r_{jts} \geq 0, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.31}$$

$$\Delta w^-_{ts}, \Delta w^+_{ts} \leq 0,, \quad \forall t \in \mathcal{T}^B, s \in \mathcal{S} \tag{3.32}$$

$$u_{jts} \in \{0,1\}, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.33}$$

The objective function (3.13) maximizes expected future profits, that is, the sum of revenues from the three markets and the value of stored water, less start-up costs and penalties of imbalances. (3.14) forces the spot bids to be non-decreasing. (3.15)-(3.20) describe the market for each scenario $s \in \mathcal{S}$, as presented in Section 3.1. (3.21) assigns the commitment in the spot market to the generators after the bidding period. No imbalances are allowed after the bidding period. (3.22) is the water value constraint for each scenario, presented in Section 2.5. (3.23)-(3.27) model the physical system for each scenario, as presented in Section 2.6. Finally, (3.28)-(3.33) give the domains for the variables.

In addition, non-anticipativity constraints [1] are added. For a given set of information ζ in the set of non-anticipativity sets $\mathcal{Z}$, the same decision must be taken. This can be modeled the following way for each variable (here denoted χ_{ts})

$$\chi_{ts} = \chi_{t\zeta}, \quad \forall t \in \mathcal{T}_\zeta, s \in \mathcal{S}_\zeta, \zeta \in \mathcal{Z} \tag{3.34}$$

where $\chi_{t\zeta}$ is the decision done with the information set $\zeta \in \mathcal{Z}$, containing scenarios $s \in \mathcal{S}_\zeta$ and time periods $t \in \mathcal{T}_\zeta$. In order to model the stages correctly, an artificial day for the spot bidding can be added, see Figure 4.5.

The first bidpoint in the spot market (at -€200) is fixed to 0, i.e., $x_{1t} = 0,\ \forall t \in \mathcal{T}^B$, and the last bidpoint (at €2000) is fixed to maximum production, i.e., $x_{|\mathcal{I}_1|t} = \sum_{j \in \mathcal{J}} \overline{W}_j,\ \forall t \in \mathcal{T}^B$, according to the NPS guidelines [24].

The minimum bidding volume requirement in the Balancing Market can be accounted for by adding binary variables a_{mits}, which are equal to 1 if bidpoint $i \in \mathcal{I}_m$ is active, and 0 otherwise.

$$\min\{25, \nu_{mts}\} a_{mits} \leq b_{mits} \leq \nu_{mts} a_{mits}, \qquad \forall m \in \{2,3\}, i \in \mathcal{I}, t \in \mathcal{T}^B, s \in \mathcal{S} \qquad (3.35)$$

$$a_{mits} \in \{0,1\}, \qquad \forall m \in \{2,3\}, i \in \mathcal{I}, t \in \mathcal{T}^B, s \in \mathcal{S} \qquad (3.36)$$

If $\nu_{mts} > 25\,\text{MW}$, each active bid $b_{mits} > 0$ will be in the region $[25\,\text{MW}, \nu_{mts}]$ for market $m \in \{2,3\}$ at time $t \in \mathcal{T}^B$ in scenario $s \in \mathcal{S}$. However, if $\nu_{mts} \leq 25\,\text{MW}$, the bid volume must equal the total market volume ν_{mts} or 0.

4 Scenario Generation

This section presents a methodology for generating scenarios for the inflow, spot price, BM-volumes and BM-premiums, used in the case study in Section 5. For all parameter estimations, a training period of eight weeks is used, as found appropriate in [30]. The set of hours in the training period is denoted $\mathcal{T}^T$. All parameter estimations are done in the statistical software R [22], applying the maximum likelihood method. The scenarios are generated for the period 10.04.2012 - 10.07.2012 for the Norwegian price area NO2. The data source for historical spot prices, BM-premiums and BM-volumes is [24].

4.1 Spot Prices

Price forecasting in the spot market is well developed, and is not presented in this paper. There are two important methodologies, statistical models and fundamental models. An overview of statistical models can be found in [34]. In the Nordic Market, the EMPS model, described in Section 2.5, is commonly used for price forecasting. The future and forward markets are also relevant, because they give the market value of future exchange of electricity.

The scenario generation for spot prices in this paper is based on a deterministic forecast ρ_t^D, shown in Figure 4.1. The price has typically two peaks during the day, one in the morning and one in the afternoon. Night hours are expected to have significantly lower prices than day hours. Further, the last two days (6.10-7.10) constitute a weekend, and the expected daily price decreases during the period.

In order to describe the uncertainty in the spot price forecast, an error term ϵ_{ts} is added.

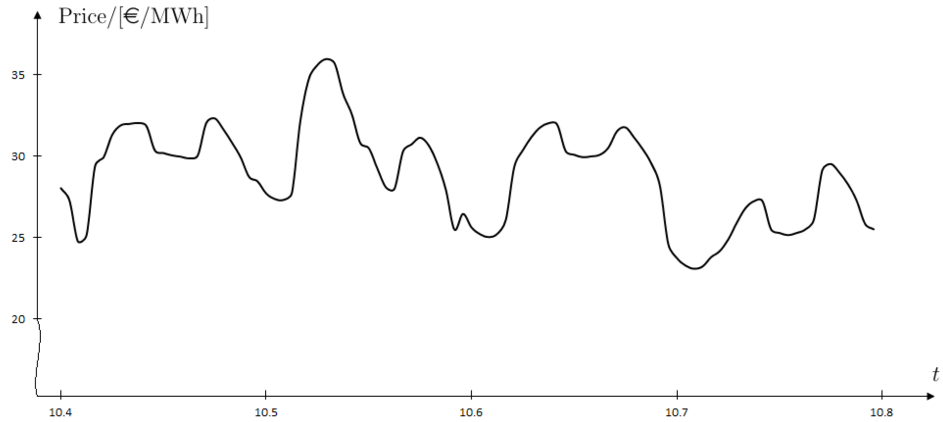


Figure 4.1: Deterministic spot forecast for the planning horizon.

The error term for the first hour is assumed to be white noise, where the variance $\sigma^2_{\epsilon_1}$ is estimated from historical forecast errors, i.e.,

$$\rho_{1s} = \rho^D_1 + \epsilon_{1s}, \qquad \forall s \in \mathcal{S} \tag{4.1}$$

$$\epsilon_{1s} \sim \mathcal{N}(0, \sigma^2_{\epsilon_1}), \qquad \forall s \in \mathcal{S} \tag{4.2}$$

The error term in the first hour is modeled as independent of the previous error terms, in order to keep the expectation of the generated scenarios equal to the deterministic forecast.

The spot prices for each day (24 hours) are revealed at the same time. Thus, the hourly error is not a time series. However, because the error terms are highly autocorrelated, the subsequent error terms are modeled with a seasonal autoregressive (AR) model. The seasonality comes from high correlation with the same hour the day before. The model for the forecast error is

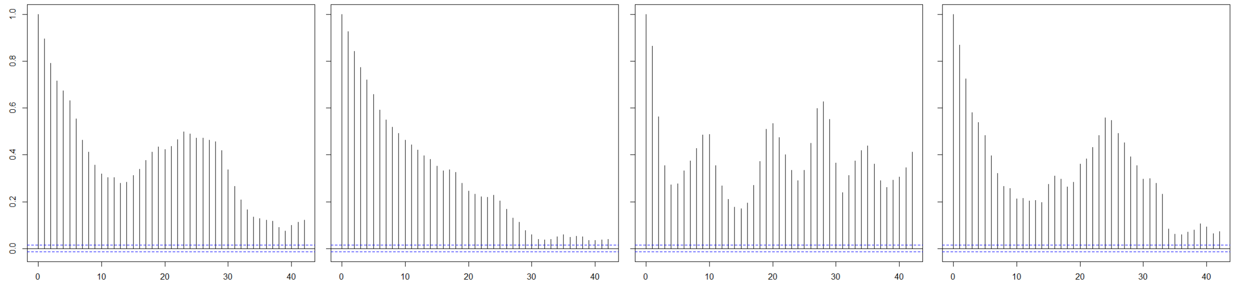
$$\epsilon^{\mathfrak{a}}_t \triangleq \epsilon_t - \mathfrak{a}^\epsilon = \Theta^\epsilon_1 \epsilon^{\mathfrak{a}}_{(t-1)} + \Theta^\epsilon_2 \epsilon^{\mathfrak{a}}_{(t-2)} + \Theta^\epsilon_{24} \epsilon^{\mathfrak{a}}_{(t-24)} + \omega^\epsilon_t, \qquad \forall t \in \mathcal{T}^T \tag{4.3}$$

where $\mathfrak{a}^\epsilon$ is the mean of $\epsilon^{\mathfrak{a}}_t$, Θ^ϵ_1, Θ^ϵ_2 and Θ^ϵ_{24} are the parameters from the AR-model, and ω^ϵ_t is the residual at time $t \in \mathcal{T}^T$.

In the scenario generation, one of the residuals ω^ϵ_t from the training period is drawn randomly for each scenario $s \in \mathcal{S}$. The random residual ω^ϵ_{ts} is added to the deterministic forecast

$$\epsilon^{\mathfrak{a}}_{ts} = \Theta^\epsilon_1 \epsilon^{\mathfrak{a}}_{(t-1)s} + \Theta^\epsilon_2 \epsilon^{\mathfrak{a}}_{(t-2)s} + \Theta^\epsilon_{24} \epsilon^{\mathfrak{a}}_{(t-24)s}, \qquad \forall t \in \mathcal{T} \setminus \{1\}, s \in \mathcal{S} \tag{4.4}$$

$$\rho_{ts} = \rho^F_t + \epsilon^{\mathfrak{a}}_{ts} + \mathfrak{a}^\epsilon + \omega^\epsilon_{ts}, \qquad \forall t \in \mathcal{T} \setminus \{1\}, s \in \mathcal{S} \tag{4.5}$$



a *b* *c* *d*

Figure 4.2: Correlograms for BM↑-volume (*a*), BM↓-volume (*b*), BM↑-premium (*c*) and BM↓-premium (*d*) for NO2, 08.18.2010 - 10.03.2012.

4.2 Balancing Market Volumes and Premiums

Price forecasting in the Nordic Balancing Market (BM) is not well studied. Some work has been done. In [23] an econometric model for the BM-premiums is developed with BM-volumes and the spot price as input signals, but the paper does not discuss uncertainty. In [19] a Markov switching state model combined with a Seasonal Autoregressive Integrated Moving Average (SARIMA) model for the BM-premiums is presented. Further work on the BM-premiums is done in [18] and extended in [2], in which wind power uncertainty is taken into account, and non-linear time series models are used. In [13] a SARIMA model is used for the BM-state, and the BM-volumes are fitted to Generalized Extreme Value (GEV) distributions. The price model described below is inspired of [19], and extended by forecasting the BM-volumes and model the dependencies between the BM-volumes and the BM-premiums.

Figure 4.2 shows the correlograms for the BM-volumes and BM-premiums. The autocorrelations are high for the first hours, and there is a tendency of seasonality, because the data is often correlated with the same hour the day before. Because of the high autocorrelations for lags up to three hours, time series models are used for modeling the BM-volumes and BM-premiums with orders up to 3. The seasonality is not taken into account.

A discrete Markov switching model [11] is used to model the state of the system in the

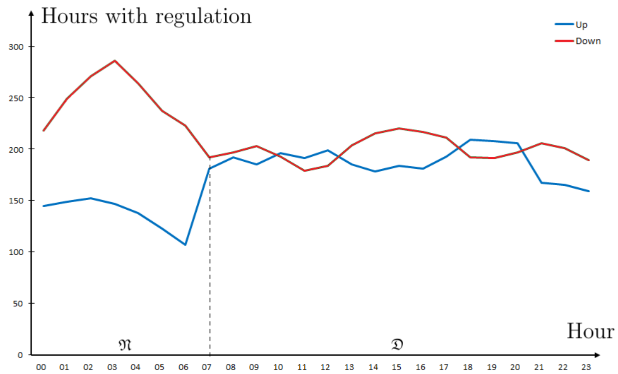


Figure 4.3: Number of hours with balancing in both directions for 2011 for NO2.

price area. There are three possible states $\mathfrak{s}_t$ for time $t \in \mathcal{T}^T$ defined[7]

$$\mathfrak{s}_t \triangleq \begin{cases} 2 & \text{if } \nu_{2t} > 0 \\ 3 & \text{if } \nu_{3t} > 0 \\ 1 & \text{otherwise} \end{cases} \qquad \forall t \in \mathcal{T}^T$$

where ν_{mt} is the BM-volume in market $m \in \{2, 3\}$ at time $t \in \mathcal{T}^T$.

In about 2.5 % of the time there is a premium in the BM-market without any BM-volume in the price area NO2. In these hours, there is export of BM-volume out of NO2. This is not taken into account in this paper, which represents a minor simplification. One could allow a limited export volume based on experience to account for this opportunity.

The transition probability $\pi_{\mathfrak{ij}}$, is defined as the probability of moving from state $\mathfrak{i}$ to state $\mathfrak{j}$. Figure 4.3 shows that there are more downward balancing during night hours and more upward balancing during day hours. Therefore, the transition probability for each state is estimated separately for night hours (hour 1 through 7) and day hours (hours 8 through 24). Because the BM-states are observable explicitly, the estimations are done in the following way

$$\pi_{\mathfrak{ij}}^{\mathfrak{T}} = \frac{\left| \left\{ t \in \mathcal{T}^{\mathfrak{T}} \,|\, \mathfrak{s}_{(t-1)} = \mathfrak{i} \cap \mathfrak{s}_t = \mathfrak{j} \right\} \right|}{\left| \left\{ t \in \mathcal{T}^{\mathfrak{T}} \,|\, \mathfrak{s}_{(t-1)} = \mathfrak{i} \right\} \right|}, \qquad \forall \mathfrak{i}, \mathfrak{j} \in \{1, 2, 3\}, \mathfrak{T} \in \{\mathfrak{N}, \mathfrak{D}\} \qquad (4.6)$$

where $\pi_{\mathfrak{ij}}^{\mathfrak{N}}$ and $\pi_{\mathfrak{ij}}^{\mathfrak{D}}$ denote the night and day transition probabilities, respectively, and $\mathcal{T}^{\mathfrak{N}}$ represents the night hours in the training period and $\mathcal{T}^{\mathfrak{D}}$ represents the day hours. The notation $|\cdot|$ denotes the number of entries in a set.

[7] In the case with both upward balancing and downward balancing, the state with the highest volume is chosen. Hours with both upward and downward balancing counts for about 0.2 % of the hours in the period 08.18.2010 - 10.03.2012, hence the simplification should not cause major errors.

Upward balancing counts for 22.2 % of the hours and downward balancing 27.6 % of the hours in the training period. The estimates of π are

$$\pi^{\mathfrak{N}} = \begin{pmatrix} 0.77 & 0.09 & 0.14 \\ 0.26 & 0.70 & 0.04 \\ 0.22 & 0.02 & 0.76 \end{pmatrix}, \qquad \pi^{\mathfrak{D}} = \begin{pmatrix} 0.84 & 0.08 & 0.08 \\ 0.18 & 0.80 & 0.02 \\ 0.15 & 0.01 & 0.84 \end{pmatrix}$$

The BM-volumes have a lower bound at zero. A common method when modeling non-negative stochastic variables is logarithmic transformation [16]. Let ν_{mt} be the observed BM-volume in market $m \in \{2,3\}$ at time $t \in \mathcal{T}^T$. The log-transformed variable is denoted $\hat{\nu}_{mt}$, and only defined for strictly positive volumes

$$\hat{\nu}_{mt} \triangleq \begin{cases} \ln(\nu_{mt}), & \text{if } \nu_{mt} > 0 \\ \text{Not defined}, & \text{otherwise} \end{cases}, \qquad \forall m \in \{2,3\}, t \in \mathcal{T}^T \tag{4.7}$$

Further, let $\mathfrak{a}_m^{\hat{\nu}}$ denote the mean (average) of $\hat{\nu}_{mt}$, and

$$\hat{\nu}_{mt}^{\mathfrak{a}} \triangleq \hat{\nu}_{mt} - \mathfrak{a}_m^{\hat{\nu}} \qquad \forall m \in \{2,3\}, t \in \mathcal{T}^T \tag{4.8}$$

An Autoregressive Moving Average ARMA($\mathfrak{p},\mathfrak{q}$) model is used for $\hat{\nu}_{mt}^{\mathfrak{a}}$ in each market $m \in \{2,3\}$. $\mathfrak{p}$ denotes the order of the Autoregressive terms and $\mathfrak{q}$ denotes the order of the Moving Average terms. The model identification is done by minimizing Akaike's Information Criterion (AIC) [33]. For BM↑ ($m = 2$), $\mathfrak{p} = 3$ and $\mathfrak{q} = 2$ is chosen and $\mathfrak{p} = 2$ and $\mathfrak{q} = 1$ is chosen for BM↓ ($m = 3$). When estimating the parameters, the majority of the data points are missing, due to no BM-volumes. Thus, this is a missing value problem, and hours without data are not considered in the parameter estimation. However, hours with balancing often come in clusters, the parameter estimation is therefore possible. The BM-volume models can be written

$$\hat{\nu}_{2t}^{\mathfrak{a}} = \Theta_1^{\nu 2}\hat{\nu}_{2(t-1)}^{\mathfrak{a}} + \Theta_2^{\nu 2}\hat{\nu}_{2(t-2)}^{\mathfrak{a}} + \Theta_3^{\nu 2}\hat{\nu}_{2(t-3)}^{\mathfrak{a}} + \Phi_1^{\nu 2}\omega_{2(t-1)}^{\nu 2} + \Phi_2^{\nu 2}\omega_{2(t-2)}^{\nu 2} + \omega_{2t}^{\nu 2}, \quad \forall t \in \mathcal{T}^T \tag{4.9}$$

$$\hat{\nu}_{3t}^{\mathfrak{a}} = \Theta_1^{\nu 3}\hat{\nu}_{3(t-1)}^{\mathfrak{a}} + \Theta_2^{\nu 3}\hat{\nu}_{3(t-2)}^{\mathfrak{a}} + \Phi_1^{\nu 3}\omega_{(t-1)}^{\nu 3}\omega_{3t}^{\nu}, \quad \forall t \in \mathcal{T}^T \tag{4.10}$$

where Θ^{ν} and Φ^{ν} are the parameters in the models and ω_{mt}^{ν} is the residual in market $m \in \{2,3\}$ at time $t \in \mathcal{T}^T$. The numerical results are summarized in Table 4.1.

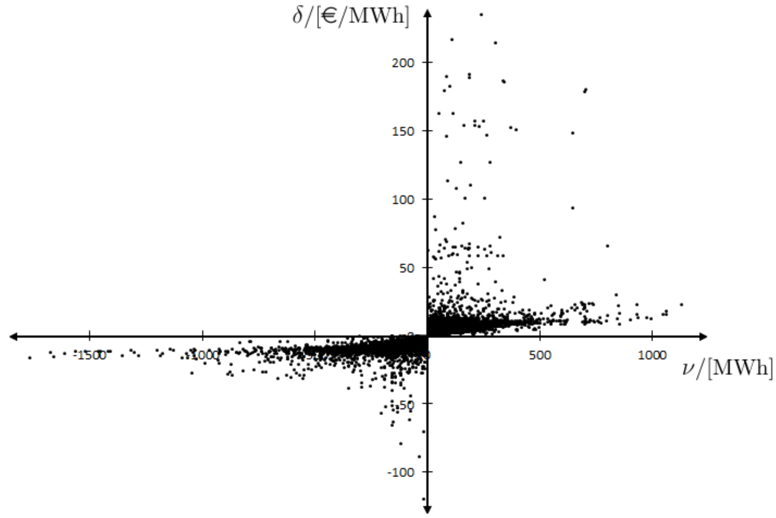


Figure 4.4: Scatterplot for the BM-volumes against the BM-premiums in NO2, 01.01.2010 - 10.03.2012.

For each time step in each scenario the BM-volume ν_{mts} is calculated as

$$\hat{\nu}^{\mathfrak{a}}_{2ts} = \Theta^{\nu 2}_1 \hat{\nu}^{\mathfrak{a}}_{2(t-1)s} + \Theta^{\nu 2}_2 \hat{\nu}^{\mathfrak{a}}_{2(t-2)s} + \Theta^{\nu 2}_3 \hat{\nu}^{\mathfrak{a}}_{2(t-3)s} + \Phi^{\nu 2}_1 \omega^{\nu 2}_{2(t-1)s} + \Phi^{\nu 2}_2 \omega^{\nu 2}_{2(t-2)s}, \quad \forall t \in \mathcal{T}^T, s \in \mathcal{S} \tag{4.11}$$

$$\hat{\nu}^{\mathfrak{a}}_{3ts} = \Theta^{\nu 3}_1 \hat{\nu}^{\mathfrak{a}}_{3(t-1)s} + \Theta^{\nu 3}_2 \hat{\nu}^{\mathfrak{a}}_{3(t-2)s} + \Phi^{\nu 3}_1 \omega^{\nu 3}_{(t-1)s}, \quad \forall t \in \mathcal{T}^T, s \in \mathcal{S} \tag{4.12}$$

$$\nu_{mts} = \exp\left(\hat{\nu}^{\mathfrak{a}}_{mts} + \mathfrak{a}^{\hat{\nu}}_m + \omega^{\nu}_{mts} + \frac{(\sigma^{\nu}_m)^2}{2}\right), \quad \forall m \in \{2,3\}, t \in \mathcal{T}^T, s \in \mathcal{S} \tag{4.13}$$

where $(\sigma^{\nu}_m)^2$ is the variance of the residuals for market $m \in \{2,3\}$ and the residuals ω^{ν}_{mts} are drawn randomly from the residuals in the training period. The term $\frac{(\sigma^{\nu}_m)^2}{2}$ is added in order to generate unbiased scenarios [16]. The simulation is done from the last data points, i.e., the last three adjacent hours with BM↑-volumes in the training period and the last two adjacent hours with BM↓-volumes.

The BM-premium δ_{mt} in market $m \in \{2,3\}$ at time $t \in \mathcal{T}^T$, is defined as the difference between the BM-price and the spot price (ρ_{1t}) for upward balancing, and the difference between the spot price and the BM-price for downward balancing, that is

$$\delta_{2t} \triangleq \begin{cases} \rho_{2t} - \rho_{1t} & \text{if } \mathfrak{s}_t = 2 \\ \text{Not defined} & \text{otherwise} \end{cases}, \qquad \forall t \in \mathcal{T}^T \tag{4.14}$$

	$\mathfrak{a}$	Θ_1	Θ_2	Θ_3	Φ_1	Φ_2	Γ	σ^2
$\hat{\nu}_{2t}^{\mathfrak{a}}$	4.40	1.67	-1.65	0.77	-1.00	1.00	NA	0.42
$\hat{\nu}_{3t}^{\mathfrak{a}}$	4.34	-0.10	0.47	NA	0.76	NA	NA	0.56
$\hat{\delta}_{2t}^{\mathfrak{a}}$	1.67	NA	NA	NA	1.06	0.58	0.02	0.10
$\hat{\delta}_{3t}^{\mathfrak{a}}$	1.85	NA	NA	NA	0.75	0.35	0.01	0.09

Table 4.1: Model parameters for the log-transformed BM-volumes and BM-premiums.

and

$$\delta_{3t} \triangleq \begin{cases} \rho_{1t} - \rho_{3t} & \text{if } \mathfrak{s}_t = 3 \\ \text{Not defined} & \text{otherwise} \end{cases}, \qquad \forall t \in \mathcal{T}^T \tag{4.15}$$

Note that, by construction, $\delta_{mt} > 0$ whenever δ_{mt} is defined. Thus, the premium is a non-negative stochastic variable, as the BM-volume. A log-transformation is therefore used also for the premiums.

Let $\hat{\delta}_{mt} \triangleq \ln(\delta_{mt})\ \forall m \in \{2,3\}, t \in \mathcal{T}^T$ and $\mathfrak{a}_m^{\hat{\delta}}$ be the mean of $\hat{\delta}_{mt}$. Further, let $\hat{\delta}_{mt}^{\mathfrak{a}} \triangleq \hat{\delta}_{mt} - \mathfrak{a}_m^{\hat{\delta}}\ \forall m \in \{2,3\}, t \in \mathcal{T}^T$. A Moving Average model of order 2 (MA(2)) is found to generate unbiased scenarios for $\hat{\delta}_{mt}^{\mathfrak{a}}$ with the BM-volume as external input. Figure 4.4 shows that there is a correlation between the BM-premium and the BM-volume. The model is also tested with the spot price as an external input, however, the correlation is not significant.

$$\hat{\delta}_{mt}^{\mathfrak{a}} = \Phi_1^{\delta m}\omega_{m(t-1)}^{\delta} + \Phi_2^{\delta m}\omega_{m(t-2)}^{\delta} + \Gamma^m \ln(\nu_{mt}) + \omega_{mt}^{\delta}, \quad \forall m \in \{2,3\}, t \in \mathcal{T}^T, s \in \mathcal{S} \tag{4.16}$$

where $\Phi^{\delta m}$ and Γ^m are the parameters in the models and ω_{mt}^{δ} denotes the residuals in market $m \in \{2,3\}$ at time $t \in \mathcal{T}^T$. The numerical results are summarized in Table 4.1.

For each time step in each scenario the BM-premium δ_{mts} is calculated as

$$\hat{\delta}_{mts}^{\mathfrak{a}} = \Phi_1^{\delta m}\omega_{m(t-1)s}^{\delta} + \Phi_2^{\delta m}\omega_{m(t-2)s}^{\delta} + \Gamma^m \ln(\nu_{mts}), \quad \forall m \in \{2,3\}, t \in \mathcal{T}^B \tag{4.17}$$

$$\delta_{mts} = \exp\left(\hat{\delta}_{mts}^{\mathfrak{a}} + \mathfrak{a}_m^{\hat{\delta}} + \omega_{mts}^{\delta} + \frac{(\sigma_m^{\delta})^2}{2}\right), \quad \forall m \in \{2,3\}, t \in \mathcal{T}^B \tag{4.18}$$

where $(\sigma_m^{\delta})^2$ is the variance of the residuals in market $m \in \{2,3\}$ and the residuals ω_{mts}^{δ} are drawn randomly from the residuals in the training period.

The simulations are done from the last two adjacent hours in the training period with volumes in market market $m \in \{2,3\}$.

4.3 Scenario Generation

The procedure below generates 3000 individual scenarios, and is implemented in Matlab. Table 4.2 shows the descriptive statistics for the input data. The characteristics of the

	Mean	Standard deviation	Min	Max
ρ_{1t}^D [€/MWh]	28.86	2.98	23.08	35.97
ν_{2t} [€/MWh]	118.59	94.78	6	502
ν_{3t} [€/MWh]	114.97	109.42	2	719
δ_{2t} [€/MWh]	5.41	2.93	0.21	15.12
δ_{3t} [€/MWh]	5.6	2.92	0.01	15.81
κ_{1t} [m^3/s]	7.52	2.20	4.02	14.11
κ_{2t} [m^3/s]	9.05	4.79	4.76	28.90

Table 4.2: Descriptive statistics, BM-data is for hours with balancing volumes.

BM↑ and BM↓ data are quite similar. The ranges and the standard deviations for the BM-volumes are very high, indicating the high uncertainty associated with these variables. The volatilities in the BM-premiums are somewhat smaller than for the BM-volumes, still noteworthy high. The table also shows that the uncertainties in the spot market and inflows are significantly smaller compared to the Balancing Market data.

For each scenario $s \in \{1, \dots, 3000\}$ **do**
  Pick one inflow scenario randomly
  Calculate the spot price for hour $t = 1$ ρ_{11s} using (4.1)-(4.2)
  For each hour $t \in \mathcal{T} \setminus \{1\}$ **do**
    Calculate the spot price ρ_{1ts} using (4.4)-(4.5)
  End do
  For each hour $t \in \mathcal{T}^B$ **do**
    Determine the BM-state $\mathfrak{s}_{ts}$ randomly using the probabilities found in (4.6)
    For each market $m \in \{2, 3\}$ **do**
      Calculate the BM-volume using (4.11)-(4.13)
      Calculate the BM-premium using (4.17)-(4.18)
      If $\mathfrak{s}_{ts} = m$ **do**
        Set ν_{mts} equal to the calculated BM-volume
        Set δ_{mts} equal to the calculated BM-premium
      Else do
        Set $\nu_{mts} := 0$ and $\delta_{mts} := 0$
      End if
    End do
  End do
End do

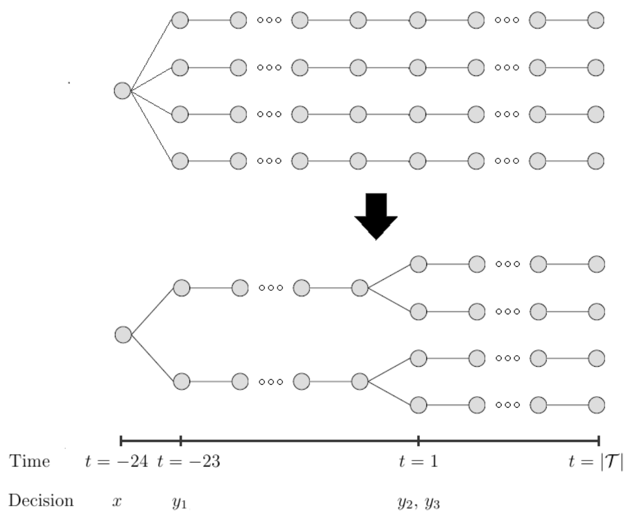


Figure 4.5: Illustration of scenario fan before scenario reduction (upper) and scenario tree after scenario reduction (lower). The figure shows when the decisions for the bidding period are done.

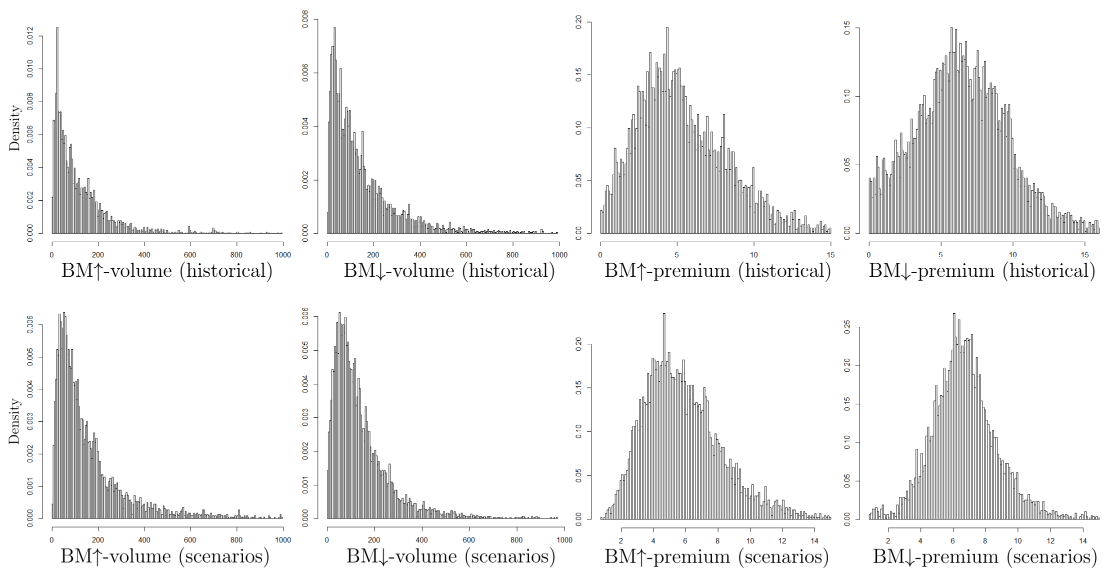


Figure 4.6: Distributions of BM-volumes (left) and BM-premiums (right) for historical data (upper) and scenario generated data (lower), the historical data is for the period 08.18.2010 - 10.03.2012.

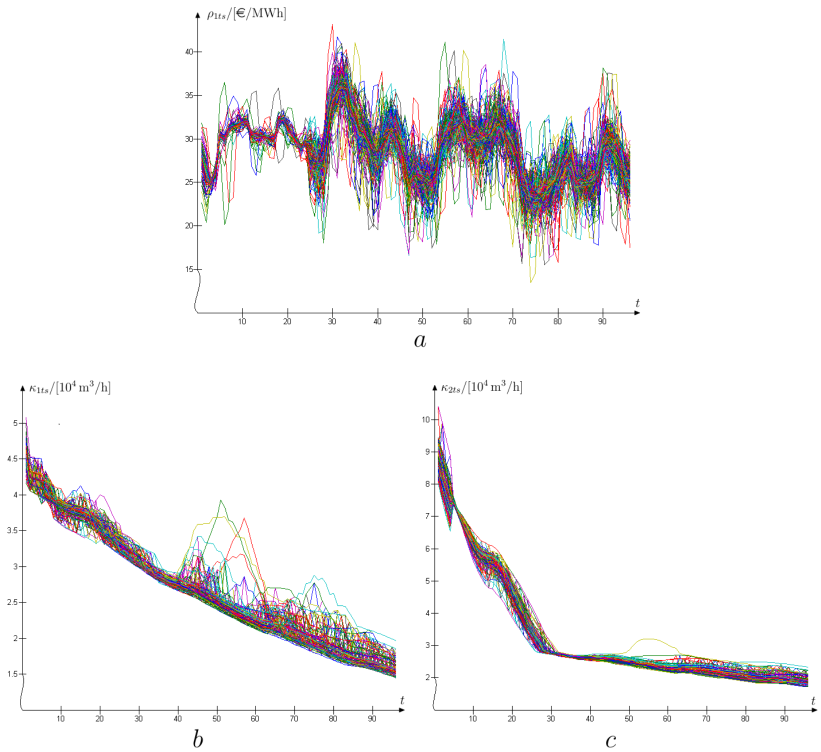


Figure 4.7: Generated spot price scenarios (a) and inflow scenarios for reservoir 1 (b) and reservoir 2 (c).

This procedure generates a scenario fan, as illustrated in the upper tree of Figure 4.5. In order to model the uncertainty correctly, one should reveal the uncertain parameters gradually. The methodology presented in [12] is used for scenario reduction and for generating a scenario tree. A reduction percentage of 0.2 is used, which resulted in 801 scenarios. A higher reduction percentage yielded fewer scenarios, but tended to remove too many hours with balancing. The means and variances of the generated scenarios were checked to be approximately equal to the observed means and variances. Figure 4.6 shows the distributions of BM-volumes and BM-premiums for the historical data and in the generated scenarios. The kurtosis for the BM↓-premium is slightly higher for the scenarios. Still, the scenarios for all stochastic data represent the historical distributions sufficiently.

Figure 4.7a shows the generated spot price scenarios. The variance is increasing from the second stage (first day) to the last stage (day 2 through 4). 50 scenarios for the inflow are input to the scenario generation. Figures 4.7b-c show the generated inflow scenarios κ_{jts} for reservoir $j \in \mathcal{J}$, at time $t \in \mathcal{T}$ in scenario $s \in \mathcal{S}$. The expected inflow is decreasing during the planning horizon. The variance is not substantially increasing.

Because this paper is mainly concerned with the uncertainty in the Balancing Market, three stages are modeled. The first stage contains one period, where the decision x_{it} is done for each hour in the bidding period. The next stage contains an artificial day (24 hours), for modeling the spot market correctly. It consists of spot clearing prices for the next day, and is used because the spot prices are revealed the day before operation, whereas the BM-volumes, BM-premiums and inflows are revealed in real-time. The decisions y_{1ts} $\forall t \in \mathcal{T}^B, s \in \mathcal{S}$ are done in the second stage. The rest of the time periods are contained in the last stage. In the three stage scenario tree, all information about BM-volumes, BM-premiums, inflows and spot prices for subsequent days are revealed at the same time. This is a simplification. However, the scope of this paper is to model how a producer should bid into the spot market the first day without knowing the BM-prices and volumes. This is modeled correctly in the scenario tree. Figure 4.5 illustrates how the scenario tree is constructed. The spot volumes for the artificial day corresponds to the commitment in the first day, i.e., $y_{1(t-24)s}$ is the commitment at time $t \in \mathcal{T}^B$.

5 Case Study

The case study presented in this section discusses whether a hydropower producer should account for the Balancing Market in the spot bidding phase. The performance from the coordinated model developed in Section 3 is disussed. A sequential model that suggests spot bids without considering the Balancing Market, then it bids into the Balancing Market with fixed spot commitments is proposed as a benchmark to the coordinated model. Finally, an analysis of when the coordinated model may outperform the sequential model is done. The modeled watercourse is located in the Norwegian price area NO2 and operated by Norsk Hydro ASA. The study is done for the fall of 2012 with high initial reservoir levels.

5.1 Case Description

There is no evidence of systematic use of market power in the Nordic spot market [10]. NO2 is the largest price area in Norway, corresponding to 29 % of the total production in 2011, consisting mainly of reservoir hydropower [24]. The producer is therefore assumed to act as a price taker in the spot market. The number of suppliers in the Balancing Market is the same as in the spot market, the fundamental assumptions about market power should hence be the same in the two markets, despite the low liquidity in the Balancing Market. The producer is therefore assumed to act as a price taker in the Balancing Market.

The case study consists of two reservoirs in cascade, each connected to a power station, as shown in Figure 5.1. The upper reservoir is large ($178 \cdot 10^6\,\mathrm{m}^3$), whereas the lower reservoir is small ($1.6 \cdot 10^6\,\mathrm{m}^3$), with little flexibility. The risk of spill is significant for the lower reservoir. The upper reservoir is an aggregation of three reservoirs, as shown in the

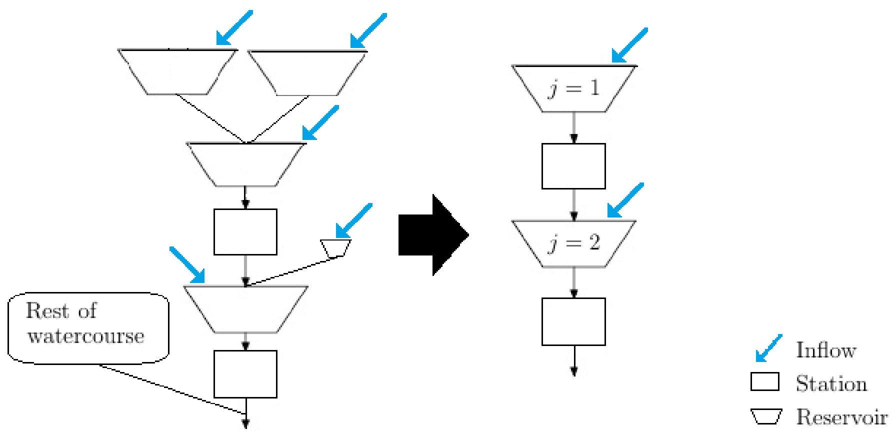


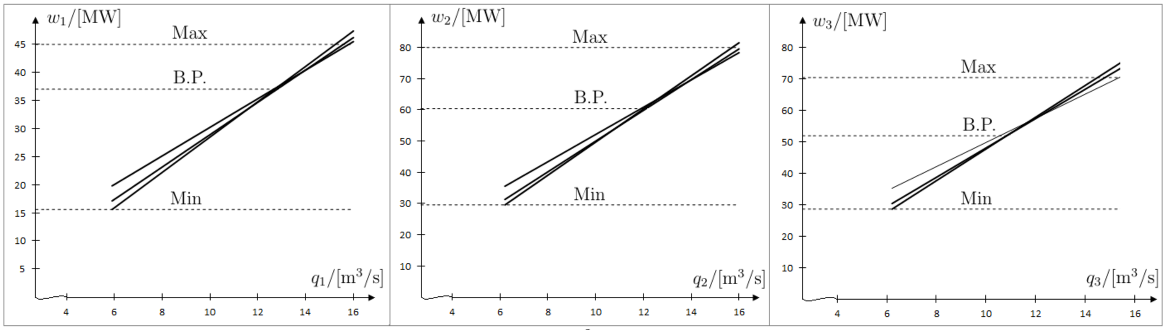
Figure 5.1: Illustration of watercourse in reality (left) and simplified (right).

figure. The marginal water values for reservoir 1 are calculated as the volume weighted average of the individual water values. The differences in the marginal water values are however small. Further, the two stations is a part of a larger watercourse, but none of the discharges are affected by the rest of the watercourse. There are no restrictions on the discharges and no time delay between the reservoirs.

The head is assumed to be constant, which is a minor simplification in the short term scheduling process. Reservoir 1 can in reality be kept at approximately constant head by controlling the upper two reservoirs. Head changes for reservoir 2 are small. Thus, the constant head assumption will not cause major errors.

The production from each generator is modeled with three linear cuts, according to (2.4). There is one generator (Figure 5.2*a*) in the upper station, with a maximum production of 45 MW. The lower station contains two generators (Figure 5.2*b*-*c*) with a total production capacity of 150 MW. In order to model start-up costs for each generator in the lower station, the following heuristic is used. The second generator is assumed to operate only if the first generator is operating. The production from the first generator is loaded as if it is operating alone (with no head loss from running the second generator). The production from the second generator is calculated as the additional production when the first generator is operating at the best point level. Lastly, the maximum production from both generators is calculated. Because of the increasing head loss when loading the second generator, its efficiency is substantially lower compared to the efficiency of the first generator. The second generator will therefore never operate when the first generator is not operating.

The model is run with a planning horizon of four days, from October 4, 2012 through



a *b* *c*

Figure 5.2: Production-Discharge curves for generator 1 (a), generator 2 (b) and generator 3 (c), with maximum production (Max), best-point production (B.P.) and minimum production (Min)

	Large model	**Simplified model**
Constraints (presolved)	1 226 229	495 717
Variables (presolved)	971 511	404 403
Running time (s)	748 818	399 792

Table 5.1: Statistics from running model the large model and the simplified model.

October 7, 2012. Seven water value cuts are implemented, valid for the end of October 7. The period has high reservoir levels, and one should hence anticipate high production rates. The currency used in the model is Euro (€). 24 bidpoints from 0 to €110 with a spacing of €5 are used, in addition to one bidpoint at €1999. The bidding constraint (3.16) is relaxed, such that the BM-production decisions are done in the last stage. Moreover, the model is not implemented with the minimum volume restriction in the Balancing Market (3.35)-(3.36), in order to limit the number of binary variables.

5.2 Results

The model is run with 801 scenarios, generated from the procedure described in Section 4.3. The deterministic equivalent of the three-stage model presented in Section 3.2 is implemented in Xpress MP [20] on an Intel Core i7 3.40 GHz processor with 16 GB RAM. Optimality was proven in the first node of the Branch-and-Bound tree. Table 5.1 shows the statistics from solving the model. The running time of approximately eight days is way too long for the model to be implemented for decision support, an appropriate solution method needs to be implemented, see Section 6.1. The idea is rather to explore whether the Balancing Market should be taken into account in the spot bidding phase.

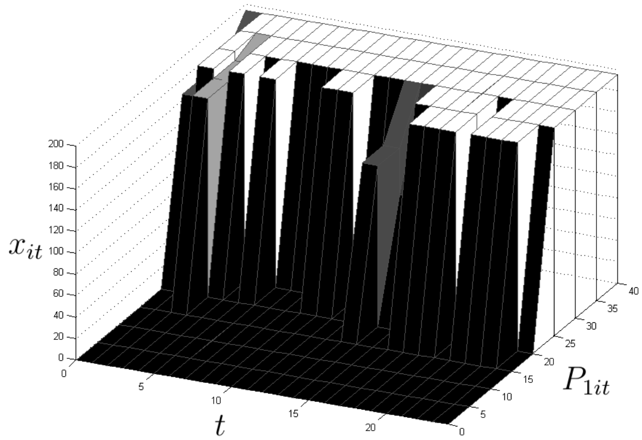


Figure 5.3: Bidding curve for the spot market.

The spot bidding curve is shown in Figure 5.3. Due to the high reservoir levels, the model tends to produce at maximum when the price reaches a certain level. Therefore, few bidpoints are active. There are however a few hours with more active bidpoints. This is caused by different marginal water values in the reservoirs, and the breakpoints in the production-discharge curves.

Figure 5.4 shows how the reservoir levels develop in the solution. Reservoir 1 is significantly reduced in the period. The reservoir level in reservoir 2 is expected to increase. This is a small reservoir. The solution tends to allocate water from the upper reservoir (1) to the lower reservoir (2), because the difference in water value is lower than the expected prices. Many of the scenarios result in maximum reservoir levels for reservoir 2. The model does not account for the risk of spill after the planning period, and sees only the marginal water values. This is a weakness of the coupling between the scheduling models, and a producer will not allow for full reservoirs if the expected inflows after the planning period are high.

There are imbalances[8] for about 4.1 % of the hours, and there is no spill in any scenario. Because the minimum volume requirement in the Balancing Market is relaxed, the results give many hours with very small volumes in the Balancing Market.

The solution gives BM↓-commitments ($y_{3ts} \geq 5\,\mathrm{MW}$) in 84 % of the hours with available BM↓-volumes, corresponding to 23.2 % of all bidding hours in every scenario. This indicates that BM↓ is used almost always when possible, and is reasonable because the expected spot prices lie just above the marginal water values, and bidding high volumes into the spot market gives high flexibility in BM↓.

[8] Only imbalances over 2 MW are counted.

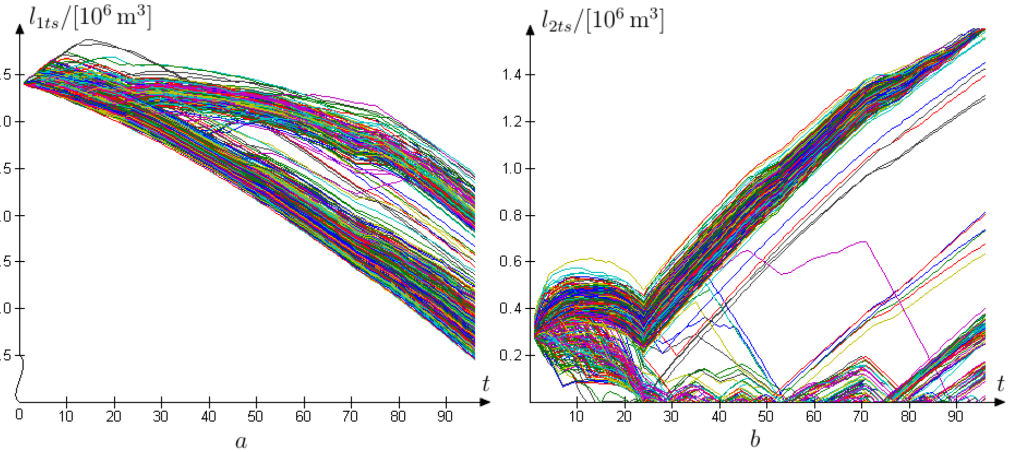


Figure 5.4: Results for the reservoir levels in the different scenarios for reservoir 1 (a) and reservoir 2 (b).

There are BM↑-commitments ($y_{2ts} > 5\,\mathrm{MW}$) in about 5 % of the hours with available BM↑-volumes, corresponding to about 1.1 % of the all hours. Bidding high volumes into the spot market reduces the flexibility in BM↑. Due to high reservoir levels, the model is not willing to risk lower total production in order achieve high flexibility in BM↑.

The objective value and the revenues in the different markets are presented in Table 5.2. The results show that the objective function value is quite stable. The standard deviation in the objective function is below 2 % of the spot revenues in the planning horizon. Furthermore, the spot volume commitment is very high in almost all scenarios for the bidding period, which is reasonable, both because the spot prices decrease during the period and because the BM↓ opportunity drives the spot volumes up.

In order to reduce the running time, the model is also run in a simplified version. The number of bidpoints is reduced to seven, each day after the bidding period has been divided into six periods containing four hours and start-up costs are only implemented for the bidding day. The running time and the numbers of variables and constraints are significantly reduced, as shown in Table 5.1. Still, the running time is way too long to be used for operational decision support. The spot bidding curve for the first day is almost identical to the result from the original model. There is a small bias to bid smaller volumes into the spot market the first day, but this counts for only 0.2 MW on average. Thus, the bid curve from the simplified model gives more or less the same results as the original model.

The value of stochastic solution [1] is not investigated in this paper, because using expected values for the Balancing Market will not make sense. If the BM-volume is known at the

		Coordinated model	
		Mean	**St.dev**
Planning horizon	Objective value/[€]	7 929 195	6 840
	Spot revenue/[€]	392 545	9 973
	Spot volume/[MWh]	13 526	336
	Reservoir level $j = 1/[10^6\,\mathrm{m}^3]$	167.752	0.506
	Reservoir level $j = 2/[10^6\,\mathrm{m}^3]$	1.527	0.518
Bidding period	Spot revenue/[€]	139 573	1 240
	Spot volume/[MWh]	4 652	39
	BM↑-revenue/[€]	66	706
	BM↑-volume/[MWh]	2.19	24.71
	BM↓-revenue/[€]	-2 619	3 681
	BM↓-volume/[MWh]	125.40	186.29
	Reservoir level $j = 1/[10^6\,\mathrm{m}^3]$	170.026	0.127
	Reservoir level $j = 2/[10^6\,\mathrm{m}^3]$	0.305	0.115

Table 5.2: Results for the coordinated model.

time of spot bidding, the BM-volume will always be assigned before the spot volume, because the price in the Balancing Market by construction is higher than the spot price. The value of the stochastic solution for the spot bidding problem is studied in [7].

5.3 Comparison With Sequential Model

In order to evaluate whether the Balancing Market should be taken into account in the spot bidding phase, the model can also be run in sequence. That is, the stochastic model is first run without the Balancing Market opportunity (fixing $b_{mt} = 0\ \forall m \in \{2,3\}, t \in \mathcal{T}^B$), which gives spot bids and spot clearing prices and volumes. Then, the Balancing Market model can be run, which is the same model with fixed spot commitments y_{1ts} for the first day ($t \in \mathcal{T}^B$). Because the last model starts in the third and last stage, this is a deterministic model for each scenario. It is left to future research to compare the preformance of the sequential model with the results from the coordinated model. However, a discussion of when the coordinated model will be beneficial is conducted below.

5.4 Discussion

The coordinated model developed in this paper represents an extension of the available decision support tools commercially available today. This section discusses in which situations such an extension might be helpful for the bidding decisions, and when a sequential model is sufficient.

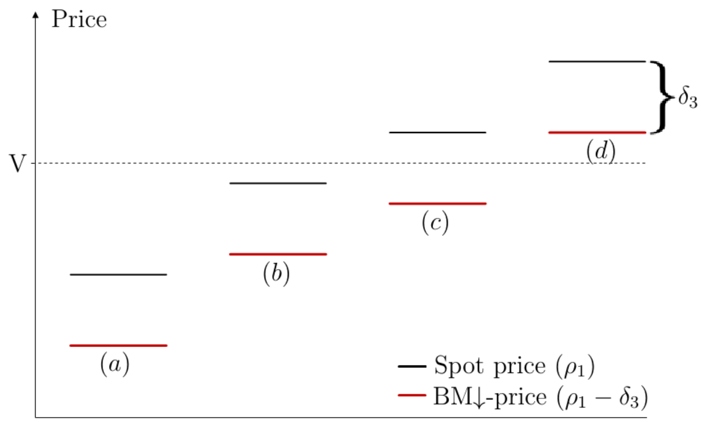


Figure 5.5: Illustration of possible states for BM↓.

Considering an hour in state 3 (downward balancing), the profit per unit production is the difference between the spot price ρ_1 and the marginal water value V if BM↓ is not used, and the BM-premium δ_3 if BM↓ is used. The model seeks to maximize profits, choosing between no bidding (zero profit), only spot bidding or spot bidding and BM↓ bidding. There are four possible states for the BM-premium, illustrated in Figure 5.5[9].

(a) $\rho_1 \ll V$:	It is not profitable to bid into any of the two markets, The producer will not risk not being commited to BM↓-volumes.
(b) $\rho_1 < V$:	It is profitable to bid into both markets if one expects high volumes in BM↓, and not profitable to bid into any market if one expects low volumes in BM↓.
(c) $\rho_1 > V$ and $\delta_3 > (\rho_1 - V)$:	It is profitable to bid into the spot market and one should bid into BM↓ whenever possible.
(d) $\rho_1 > V$ and $\delta_3 < (\rho_1 - V)$:	It is profitable to bid into spot market and not into BM↓, because it yields smaller profits than the pure spot profit.

The high spot volumes and BM↓-volumes indicate that the system in the case study is often in state (b) or (c). If the system is often in state (b), taking the Balancing Market into account will tend to increase the bidding volume in the spot market.

[9]Note that the water value is dependent on the production level, which complicates the analysis, but is taken care of in the optimization model.

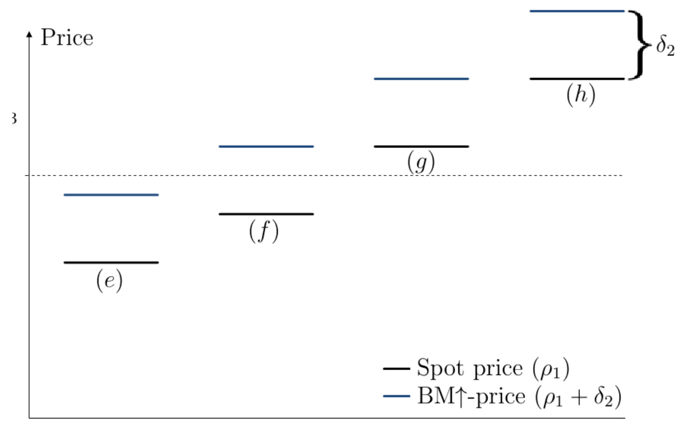


Figure 5.6: Illustration of possible states for BM↑.

For an hour in state 2 (upward balancing), the profit per unit production equals the spot price less the marginal water value if the spot market is used and BM↑ is not used. If BM↑ is used, the profit is the BM↑-price less the marginal water value. A similar analysis as done above can be carried out for BM↑. The four possible states are described below, and illustrated in Figure 5.6.

(*e*) $\rho_1 < V$ and $(\rho_1 + \delta_2) < V$: It is not profitable to bid into any of the two markets.

(*f*) $\rho_1 < V$ and $(\rho_1 + \delta_2) \geq V$: It is not profitable to bid into the spot market, and it is profitable to bid into BM↑

(*g*) $\rho_1 > V$ and $(\rho_1 + \delta_2) > V$: It is profitable to bid into both markets. If one anticipates high volumes in BM↑, one might be willing to reduce the spot volume to achieve higher volumes in BM↑.

(*h*) $\rho_1 \gg V$ and $(\rho_1 + \delta_2) \gg V$: It is very profitable to bid into the spot market, and the producer will not risk reducing its total commitments in order to obtain flexibility in BM↑

The high spot volumes and the fact that BM↑ is rarely used, indicate that the system in the case study is often in state (*b*) or (*c*) for downward balancing, and (*g*) for upward balancing. Thus, the spot price is slightly above the marginal water value, and the model choses to achieve flexibility in BM↓ rather than in BM↑. A rational decision would therefore be to bid higher volumes into the spot market in a coordinated model compared to a sequential model.

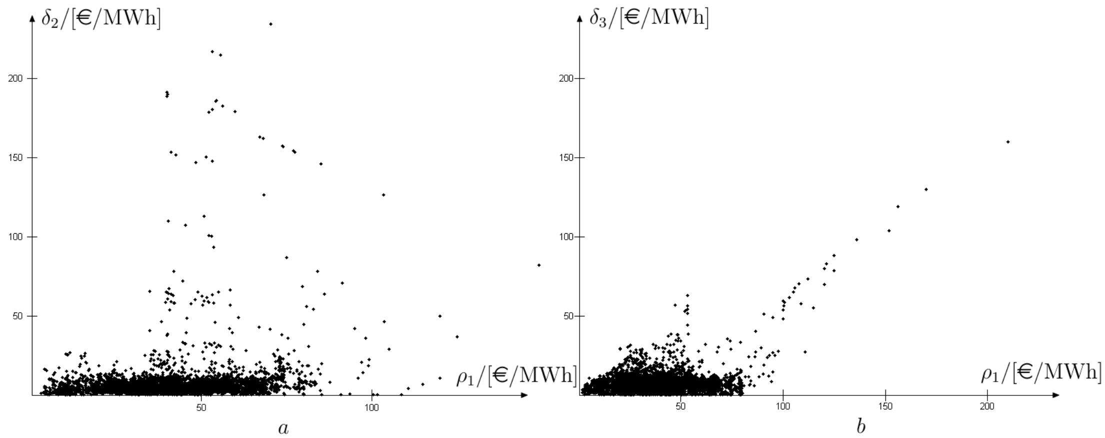


Figure 5.7: Scatterplots for the spot price against the BM↑-premium (a) and the BM↓-premium (b) in NO2, 08.18.2010 - 10.03.2012.

There are complicating factors to the analyses above. First, the marginal water values are dependent on the production level, both because of the non-constant efficiencies, and because the marginal water value is dependent on the total use of water in the planning horizon. Furthermore, although the balancing premiums had no significant correlations with the spot price in the training period, there are correlations in the long run. Figure 5.7 shows scatterplots of the BM-premiums against the spot price. The correlations with the spot price are 0.14 for the BM↑-premium and 0.18 for the BM↓-premium. The figure shows that there is no trivial dependency between the BM↑-premium and the spot price, but when the spot price exceeds a certain level (about €35/MWh), the expected premium increases. The dependency between the spot price and the BM↓-premium is somewhat linear in the long run. The fact that the correlation for BM↑ is smaller may be explained by the option market for BM↑-volumes that is active when the expected spot prices are high (during winter). The options ensure sufficient BM↑-volumes at any point of time, yielding no incentive to bid BM↑-prices that are substantially higher than the marginal water value for committed producers. When the spot price is high, the producers will not bid volumes in BM↓ for premiums that are smaller than the difference between the spot price and the marginal water value. This difference is expected to increase with increasing spot prices. Thus, the state (d) is in general unlikely, if the marginal water values for all producers in a price area are strongly correlated. Therefore, it is likely that the amount of time the coordinated model is not useful is further limited.

Summing up, if the spot price is very low ((a) and (e)) or very high ((d) and (h)) compared to the marginal water value, the Balancing Market will not be used, and needs not be accounted for in the spot bidding phase. If the spot price is near the marginal water value, it may be profitable to include the BM-flexibility in the spot bidding decisions. Moreover, data from the Nordic market indicates that state (h) is unlikely.

6 Conclusion and Future Work

The coordinated bidding model developed in this paper may prove to outperform a sequential model unless there is a big difference between the spot price and the marginal water value, compared to the expected premium in the Balancing Market. Market data indicates that the coordinated model will be beneficial in the majority of the time the producer is considering bidding volumes into the spot market.

6.1 Future Work

The case presented in this paper examines a bidding period of four days in October 2012, with high reservoir levels and prices slightly above the marginal water values. To gain more knowledge about when it is profitable to account for the Balancing Market in the spot bidding phase, the model should be tested for more cases. It is especially interesting to model different seasons, with different characteristics in the markets. Modeling more complicated watercourses could give further knowledge about the problem. Furthermore, the performance of the coordinated model should be compared to results from the sequential model, to gain further insight about when the developed model is favorable.

The long running times indicate that an appropriate solution method needs to be investigated. The L-Shaped Method [1] or Linear Decision Rules [15] can be tested. Both require continuous problem formulations, start-up costs and minimum balancing volume bids can hence not be modeled correctly with these methods. Other solution methods can be found in [14].

Producers also consider if they should participate in other physical power markets like Elbas, the balancing option market and the primary and secondary reserve markets. A study that includes all physical markets is left to future research.

The volatility in the spot price varies substantially over the day. This varying volatility is not taken into account in the scenario generation in this paper. Neither is seasonality. The use of more advanced time series models can account for these effects, and thus describe the uncertainty better.

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