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Coordinated Bidding in the Day-Ahead Market and the Balancing Market using Stochastic Programming

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PROJECT REPORT
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Preface

This report is written as a specialization project in the field of Applied Economics and Operations Research at the Norwegian University of Science and Technology (NTNU).

We would like to thank our supervisors Professor Stein-Erik Fleten and PhD cancidate Ellen Krohn Aasgård, both from the Department of Industrial Economics and Technology Management at NTNU, for guidance and valuable discussions during the project.

We would also like to express our gratitude to our industrial partner, providing us with historical data and knowledge regarding the hydropower field.

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Abstract

With the increasing share of non-dispatchable energy in the electricity markets, balancing and ancillary service markets are becoming more important. The decision making process for a power producer is becoming more complex, as the bidding problem expands to taking sequential markets into account when bidding in the day-ahead market. This report looks at the bidding problem of a Norwegian hydropower producer, bidding in the day-ahead market and the balancing market. The objective is to construct a coordinated bidding model, and evaluate the behavior by comparing it to a strategy of considering one market at a time. The deterministic equivalent of a three-stage stochastic mixed-integer programming model is implemented, in which the uncertain parameters are the day-ahead and balancing market prices. The model is based on a node-variable formulation. The possible outcomes for the market prices are generated by a tool developed at NTNU. The model optimizes the bid curves for both markets, as well as the optimal allocation of the production.

A case study of the problem is conducted with realistic input data from an industrial partner. The results show that a coordinated model will slightly outperform a sequential strategy, with qualitatively different bid curves. The gains are however not significant. The bid curves and production schedule are compared to those of our industrial partner, which results in higher profits while using the coordinated model. The bidding behavior and production schedule are quite different, as the model uses the balancing market more actively. The model could indeed be used for daily planning for a hydropower producer, as the run time of the model is quite low. However, a final conclusion of whether it is profitable to use a coordinated model cannot be made until the model is tested over a longer time horizon.

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Chapter 1

Introduction

The requirement of balancing supply and demand at each point in the grid, makes bidding into the electricity market a complex process. To prevent the issue of frequency deviations in the system, the system operators use multi-settlement markets with planning horizons of different lengths. The idea is for the markets further away from the operating time, settle the majority of the demand for the predicted load, while markets closer to the operating hour correct for smaller deviations (Jiang and Powell, 2015).

Today, most of the electricity is traded in the day-ahead market by far. The importance of real time balancing and ancillary service markets is however increasing. A closer connected European system and larger share of intermittent, renewable energy sources has increased the imbalances in the market (Statnett, a). Such energy resources, often referred to as non-dispatchable sources, are less predictable and flexible than other sources of energy. The increasing volume in the balancing and ancillary service markets provides opportunities for producers. As market prices in the day-ahead market is expected to decrease, producers can divide their capacity among several markets to maximize revenue.

Most producers today, considers the markets in a sequential order according to the time of the respective market clearing. This means that the available capacity for balancing and ancillary service markets, are reduced by commitments in the day-ahead market. An alternative approach is to take upcoming trading opportunities closer to the operating hour into account, when placing bids in the day-ahead market. This would mean to hold back capacity in the day-ahead market to be able to offer upward regulation in the balancing market, or to put forward capacity in the day-ahead market to be able to offer downward regulation in the balancing market. The big question is whether a such coordinated bidding strategy would be profitable for a hydropower producer or not (Boomsma et al., 2014).

The project is structured into eight chapters. Chapter 2 begins with the fundamentals of hydropower production, an introduction to the different energy markets, and a short description of the scheduling problem for a hydropower producer. In Chapter 3 we present electricity market bidding in the literature, with a focus on the stochastic bidding problem for a hydropower producer without market power. Chapter 4 gives a description of our problem, including assumptions and simplifications that have been made for the purpose of this report. The mathematical formulation of the problem is presented in Chapter 5, in which the structure, notation and constraints are explained. We formulate the deterministic equivalent of a three-stage stochastic mixed integer programming model, using a node-variable formulation. The possible outcomes for the second-stage and third-stage nodes, are generated as described in Chapter 6. Chapter 7 presents a computational case study, in which the performance of the model is compared to a sequential bidding strategy and the behavior of an industrial partner. Concluding remarks and potential future research is presented in Chapter 8.

Chapter 2

Background

In this chapter, we provide an overview of the background which should be understood before considering the bidding problem for a hydropower producer. Section 2.1 introduces the basics of hydropower production, and important characteristics of the operation. In Section 2.2, we describe the different electricity markets. The markets considered in this report are the day-ahead market and the balancing market, and it is thus important to understand the market rules for these markets. Section 2.3 gives an introduction to the scheduling problem for a hydropower producer, as it is intimately connected to the bidding problem.

2.1 Hydropower Production

The principle of the production of hydropower is using the energy of flowing water to produce electricity. The water flows through the turbine in the power station, and the potential energy which was stored in the water is converted to electrical energy in the generator. The amount of power produced from a hydropower plant varies with the volume of dispatched water, and the vertical change in elevation from the reservoir to the downstream level, called the head. The possibility of storing water in reservoirs, combined with relatively low startup costs compared to other energy sources, makes hydropower a highly flexible source of energy (Statnett, a). Hydropower is applied to as both base-load and peak-load demand, as it has the ability to start and stop quickly. However, frequent starts and stops are not desired by hydropower producers, as it tears on the turbines.

Hydropower has a low resource cost, as the water in the reservoirs comes for free. Inflow is water that comes into the reservoir from rain, snow and other rivers, and is uncertain due to weather conditions. The amount of water in the reservoirs may thus be a scarce resource in periods with low inflow, so a hydropower producer always needs to look at the alternative cost of using the water later. This marginal opportunity cost of the water is referred to as the water value, and indicates the marginal increase of income from a unit increase of water in the reservoir. The water value depends on the volume in the reservoir, and is highest when the water level is low and there is a shortage of resources. If the water level is high, and there is risk of spillage, the water value approaches zero. Hydropower producers base their bids on this marginal cost (Steeger et al., 2014). The water value is calculated from scheduling models described in Section 2.3.

2.2 The Electricity Markets

The majority of Norwegian hydropower is sold in the common Nordic power market, Nord Pool Spot, which is cleared day-ahead of operation. However, incidents causing an imbalance between supply and demand may occur between the closing of the day-ahead market, and the delivery. Such incidents may be a nuclear power plant that stops operating, or an unexpected high wind production. The instantaneous imbalances are regulated by something called primary reserves. If these imbalances last for longer than a couple of minutes, the secondary reserves will kick in and thus free the primary reserves. Both of these reserves are activated automatically due to changes in the frequency of the system. The last kind of reserves is the tertiary reserves, from now on referred to as the balancing market. These reserves will free the secondary reserves and are activated manually by the transmission system operator (TSO) (Statnett, c).

The Day-ahead Market

For the day-ahead market, bids must be placed before 12 pm the day before the day of operation. The hydropower producer must submit the amount of energy it is willing to deliver and to what price (NordPoolSpot). The two types of bids which are most common, are hourly bids and block bids. When using hourly bids, the producer needs to decide a set of volume-price points for each hour of the operating day. The bids which are submitted, forms a piecewise linear function, referred to as the bid curve. As for block bids, the producer bids how much it is willing to produce for a block of several consecutive hours, based on the mean price of the hours in the block. The bid curve must be submitted as a stepwise constant function. Block bids leads to a commitment for all the hours in the block, or no commitment at all.

The market price is determined from the market equilibrium between supply and demand. The prices for the next operating day are announced at 12:42 pm, and the commitments are delivered hour by hour as from midnight. Different area prices are established due to variations in available transmission capacity. Regional market conditions, like congestion and bottlenecks in the network, is reflected in the area prices. Figure 2.1 illustrates the five price areas in Norway today (Statnett, d).

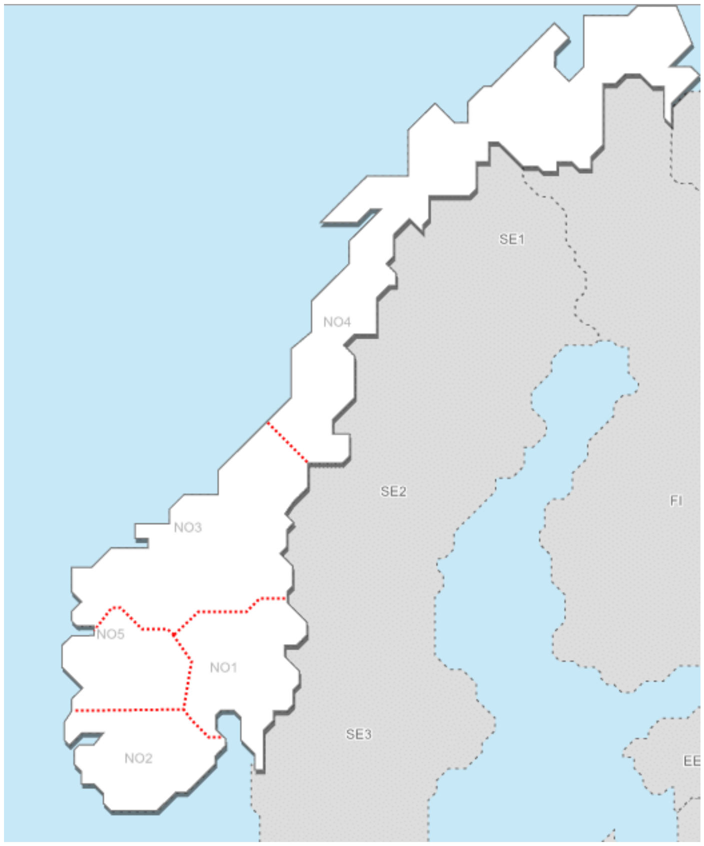


Figure 2.1: The price areas in Norway

The Balancing Market

The balancing market has different states depending on the balance in the system. If the demand in the system exceeds the available supply, the TSO may ask a producer to increase its production. Likewise, the TSO can ask a producer to decrease production if the demand in the system is lower than the planned production. This is referred to as upward and downward balancing, respectively. If the system already is in balance, there is no regulation. The market usually has a dominating direction, either upward or downward. This results in one price for upward regulation and another for downward regulation. The producers can submit bids for both directions, but they cannot commit volume for both in the same hour. Statnett requires that the bid curve is stepwise constant, instead of piecewise linear as in the hourly bids in the day-ahead market. There are several market rules for bidding in the balancing market. The lowest possible price to bid for upward balancing is 5 NOK above the day-ahead price in the area, and the highest possible price to bid for downward balancing is 5 NOK below the day-ahead price in the area. The bid prices always have to be divisible by 5. The lowest volume a producer can bid in the balancing market is 10 MW in either direction, and the bids are submitted for each group of power stations. Note that Statnett can choose to only use parts of a committed volume, if balancing is not needed the entire hour.

The prices in the balancing market is based on approving the best bids first, which means to accept the cheapest resources first (Statnett, b). The prices for upward regulation are always higher than the day-ahead prices, as all of the cheaper bids are committed in the day-ahead market. The opposite goes for downward balancing. If there is no balancing volume in one or both directions, the balancing market price is equal to the area spot price in the day-ahead market. The prices and volumes are announced on Nord Pool's website one hour after the hour of operation has finished.

Preliminary bids for the balancing market needs to be submitted by 9.30 pm the day before operation, but can be changed up to 45 minutes before the hour of operation. This makes the balancing market only available to balancing responsible parties (Boomsma et al., 2014), meaning that production can quickly be adjusted. The time line representing the closure of the different markets is shown in Figure 2.2.

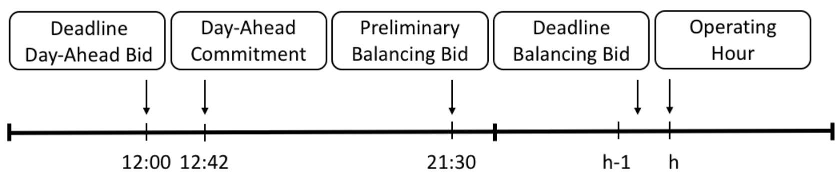


Figure 2.2: Time line of the clearing of markets

2.3 Hydropower Scheduling

The scheduling problem can be defined as optimal use of available generation resources in such a way that all relevant constraints are satisfied. After the deregulation of the power markets in the 1990's, the formal objective of hydropower scheduling changed. The objective went from minimization of costs given an expected demand, to maximization of profits given an expected market price. Due to the unique flexibility of hydropower, the hydropower producers has the possibility to schedule the production based on predicted market prices.

Uncertainty in market prices, inflow and demand makes hydropower scheduling a non-trivial task. Market prices and inflow are negatively correlated, as hydropower has a strong position in the Nordic system. Inflow and demand however, are seldom well aligned. Demand is highest in the winter when there is little inflow. This gives considerable variations in market prices and big fluctuations in the revenues for the hydropower producer. The changing hydro conditions may also lead to the need for volumes being exchanged through import or export.

Scheduling models are usually classified according to their time horizon, and may be long-term, medium-term or short-term models. Decision support software is needed due to the size and complexity of the scheduling problem. Most of the largest participants in the Nordic system uses a long-term optimization and simulation model called EMPS (EFI's Multi-area Power-market Simulator). In deregulated markets, the model's main application is the forecast of future day-ahead prices for all hydrological years. The price forecast is often used as input in another scheduling model that is widely used among producers, called the EOPS (One-area Power-market Simulator). This model may perform medium-term scheduling, where it calculates the marginal water values that can be used in the short-term scheduling. The time horizon of the short-term scheduling spans from one day up to a week with a time resolution of one hour or shorter, and involves the physical operation of the hydropower plant. That is, establishing a production plan with day-ahead commitments and real-time balancing.

Chapter 3

Electricity Market Bidding in the Literature

In this chapter, different approaches and solution methods for the problem of optimal bidding in electricity markets are presented. Section 3.1 covers some of the different variants of the bidding problem that has been introduced in the literature, and how our model relates to these variants. Different solution methods covered in the literature, are presented in Section 3.2. A literature survey on the subject of optimal bidding strategies for hydropower producers can be found in Steeger et al. (2014).

3.1 Modeling

The shift in the electricity market towards deregulation and competition lead to new challenges for producers of energy, as they could supply electricity based on profitability (Ventosa et al., 2005). The need of new decision-making models which was adapted to this new context was thus a consequence. Along with the implementation of the deregulation process, auctions were introduced to increase the competition and transparency of the electricity procurement process (Maurer and Barroso, 2011).

Ventosa et al. (2005) presents three categories of electricity market models found in the literature: equilibrium models, simulation models and optimization models. Equilibrium models represent the overall market behaviour, by including several producers and the competition between them. Models based on simulation can handle more complex assumptions than the equilibrium models, as they don't use exact mathematical modeling. Both equilibrium and simulation models are well suited for long-term planning, while optimization models are best suited for short-term problems such as models involving risk management, unit commitment, or strategic bidding (Ventosa et al., 2005). They can handle more details, and are more computationally tractable. Optimization models in this context, considers profit maximization for a single producer, taking into account operational constraints. However, optimization models may suffer from the optimizer's curse, which leads the value estimates for the recommended action to be biased (Smith and Winkler, 2006).

As the problem of optimal bidding is the one considered in this report, an optimization model is chosen. As opposed to thermal power and other non-dispatchable energy forms, hydropower production is more flexible due to the possibility of storing water in reservoirs. Because of this flexibility, and the characteristics of the different energy markets, there are many opportunities for when and where a hydropower producer can sell its power. It is thus important for producers to have a thorough bidding strategy. Fleten and Kristoffersen (2007) point out that while production has been optimized using decision models, bidding has rested on the skills of the operating engineers. The bidding problem is a short-term planning problem which can be formulated as a unit commitment problem, including which turbines to run and volumes to dispatch for all turbines at each time step (Wallace and Fleten, 2003). Due to the inclusion of binary variables to model if turbines are on or off, the problem is classified as a mixed integer programming (MIP) model.

The Price Taker Assumption

The optimization models considering the bidding problem can be classified depending on how the market prices are modeled: either as exogenous information, or as a function of the single producer's decisions (Ventosa et al., 2005). If using the first approach, it is assumed that the producer does not have large enough market-share to affect the prices in the market through their bidding process. The second approach, models a producer with market power. This is often referred to as the strategic bidding, and a literature survey on the subject can be found in David and Wen (2000). Fleten and Kristoffersen (2007) denotes these to formulations as a producer which is a price taker and a price maker, respectively. Anderson and Philpott (2002) formulates the problem of constructing optimal bid curves for a price maker. They formulate a non-linear control problem in which they represent the uncertain demand and behaviour of competitors through the use of a so-called market distribution function. Furthermore they derive optimality conditions for the bid curves. According to Faria and Fleten (2011), optimal control approaches makes it difficult to handle complex constraints and multiple state variables. Neame et al. (1999) on the other hand, considers the bidding problem for a price taker in which the bid curves is required to be stepwise constant. In this report, we model the producer as a price taker in the same way as Fleten and Kristoffersen (2007).

Handling Uncertainty

The formulations of the bidding problem covered in the literature can also be distinguished by how uncertainty in the input parameters are handled. As decision-making in electricity markets is subject to a lot of uncertainty, stochastic programming is often used due to its ability to characterize the uncertainty and to derive informed decisions (Conejo et al., 2010). Stochastic programming is defined as mathematical programming problems that includes the incorporation of uncertainty in one or several parameters (King and Wallace, 2012). Birge and Louveaux (2011) describes the basics of stochastic programming, while stochastic programming in energy is presented by Wallace and Fleten (2003). Barroso and Conejo (2006) looks at electricity markets specifically, and considers several stochastic programming models for decision-making under uncertainty in that context. While stochastic programs often are used to include the uncertainty electricity market models, deterministic short-term models are often used in practice. The Short-term Hydro Operation Planning (SHOP) model (Flatabø et al., 2002) is a deterministic optimization tool for short-term hydropower planning. Gross and Finlay (1996) use a deterministic approach, in which they introduce a globally optimal bidding strategy by bidding at cost and maximum capacity. Fleten and Kristoffersen (2007) and Belsnes et al. (2016) includes uncertainty in their optimization models and analyzes the results compared to a deterministic strategy. Both conclude that a stochastic approach is beneficial. According to Wallace and Fleten (2003), a stochastic programming approach is meaningful due to the irreversible nature of investments and decisions in electricity markets. Uncertainty is included in the model presented in this report, and the bidding problem is thus formulated as a stochastic mixed integer programming (SMIP) model. Some choose to include both price uncertainty and uncertainty in inflow, such as Braathen and Eriksrud (2013). In the same way as Fleten and Kristoffersen (2007), we cover the bidding problem for a price-taking hydropower producer, which explicitly includes price uncertainty.

Bidding Into Multiple Markets

The optimization models for the bidding problem can further be divided into single-market and multi-market models. Fleten and Kristoffersen (2007) consider the day-ahead market only, so their model can be characterized as a single-market model. As the number of electricity markets which are established increases, it has been more important to consider the sequential structure of the markets. Single-market models does not consider the opportunity in subsequent markets. The focus of this report, is to consider the bidding problem when participating in several sequential markets, which is referred to as coordinated bidding. Our model is thus a multi-market model, like presented by (Faria and Fleten, 2011; Braathen and Eriksrud, 2013; Boomsma et al., 2014; Klæboe, 2015; Kårstad et al., 2016; Kongelf and

Overrein, 2017).

Faria and Fleten (2011) considers the bidding problem for the day-ahead market and the intraday Elbas market. Kårstad et al. (2016) explores bidding in the primary reserve market and the day-ahead market, while Kongelf and Overrein (2017) in addition includes the balancing market. In Olsson (2005), Boomsma et al. (2014), Klæboe (2015), and Braathen and Eriksrud (2013), the day-ahead market bidding is considered together with balancing market bidding. These are also the markets considered in this report.

Faria and Fleten (2011) finds no significant effect on profits by using coordinated bidding compared to a sequential bidding strategy. Neither does Klæboe (2015), who concludes that coordinated bidding will have limited gain due to limitations on the volume in the balancing market. Braathen and Eriksrud (2013) concludes that more case testing has to be done before they can say if there is a gain of coordinated bidding. Boomsma et al. (2014) finds that coordinated bidding gives a significant gain. In the latter, the bidding problem is modelled as a multi-stage stochastic problem, where balancing market decisions are made hour-ahead. We have rather modeled the balancing market decisions as day-ahead. Our problem is thus modelled as a three-stage problem in a similar matter as Kongelf and Overrein (2017), but we look at two, instead of three sequential markets.

3.2 Solution Methods

Different solution techniques have been used for decision-making under uncertainty in electricity markets. Hydropower producers have the alternative of storing water for later use, so a time coupling is introduced by the use of state variables and dynamic programming (Steeger et al., 2014). In the literature of hydropower planning, the stochastic dynamic programming (SDP) approach is thus often used (Wallace and Fleten, 2003). When using SDP, the problem is solved by recursively maximizing profits in the current and future stages. King and Wallace (2012) emphasizes that the SDP method is limited by the amount of stochastics it is able to handle due to the curse of dimensionality, which means that the problem explodes in size when the number of scenarios increases. The method can neither handle complicated constraints or problem definitions. It is often necessary to aggregate or decompose the problems before solving them, in order to use these methods Wallace and Fleten (2003). Stage-wise decomposition is proven to fail as a solution technique to the bidding optimization problem (Klæboe, 2015). Kårstad et al. (2016) uses an approach which exploits the structure of the SMIP by using scenario-wise decomposition methods. However, their conclusion is that such methods are not valuable for solving the problem to optimality.

Stochastic dual dynamic programming (SDDP) is another solution method, that combines the solutions from SDP with multi-stage Benders decomposition (Pereira and Pinto, 1991). Using this method, one does not have to explore all future states. Existing long-term and medium-term planning models used in practice, use a combination of SDDP and SDP (Flatabø et al., 2002). Gjelsvik et al. (1999) introduces a hybrid of SDP and SDDP. According to Steeger et al. (2014), there is a consensus in the research environment that the hybrid SDP/SDDP is the best solution approach for a price-taker hydroproducer. Löhndorf et al. (2013) introduce a solution method they refer to as approximate dual dynamic programming (ADDP), which is an integration of SDP and approximate dynamic programming (ADP). ADP is also used by Jiang and Powell (2015), but they employ a convergent ADP algorithm which does not require any knowledge of the distribution function.

In this report, we have chosen to solve the three-stage SMIP formulated as its deterministic equivalent in a similar matter as Fleten and Kristoffersen (2007) and Boomsma et al. (2014). Solution approaches for MIP problems are thus applicable, and exact solutions are achievable (Zhu, 2006).

Chapter 4

Problem Description

In this chapter, the problem of optimal multi-market bidding for a hydropower producer is presented. The markets are modeled as a day-ahead market and a balancing market, with market rules corresponding to the Norwegian electricity markets. The objective is to maximize the profit by participating in both markets, where the alternative of bidding into the balancing market appears closer to the operating hour. The problem consists of two parts: deciding the optimal sets of bids for the markets, and allocating the available resources in an optimal way. Section 4.1 presents the bidding problem, while the problem of optimal production allocation is presented in Section 4.2. The model developed to solve the problem is formulated as a three-stage stochastic mixed-integer program. The structure of the problem is shown in Figure 4.1. The squares represent the decisions that must be made, and the circles indicate new information.

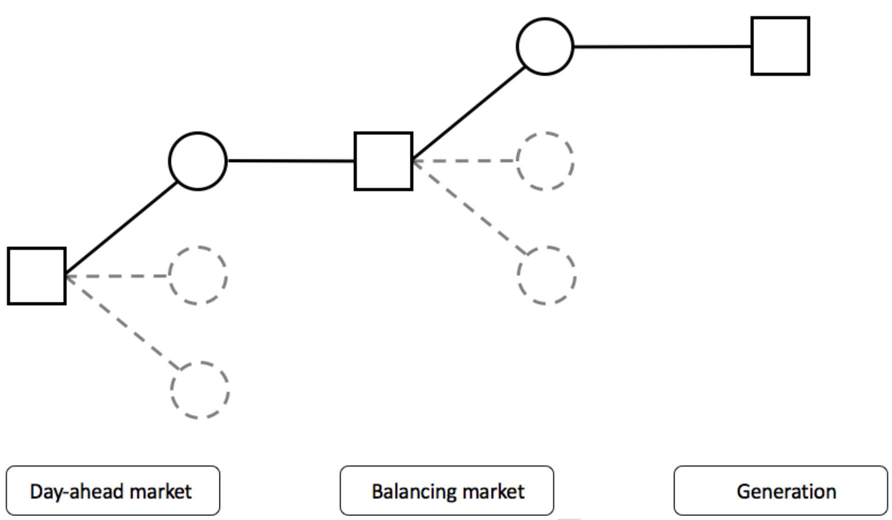


Figure 4.1: Decision tree representing the structure of the problem

4.1 The Bidding Problem

Both day-ahead and balancing market prices are unknown when making bids in the day-ahead market, which makes bidding in the day-ahead market the decisions of the first stage. The producers submit bids as volume-price points, which indicate how much power they are willing to sell at a certain market price. However, no bids above maximum capacity are approved by Nord Pool. An hourly bid curve is modeled as a piecewise linear function for the day-ahead market, according to the market rules. The obligated volume is then found by interpolation between the volume-price points. For block bids, the bid curve is modeled as a stepwise constant function. Block bids are only included as an expanded model. As the bid volumes in the day-ahead market are the first-stage decisions, the corresponding bid prices are fixed, to maintain the linear form of the problem. According to the market rules, the bid curve must be monotonic increasing. The bid curve is thus formed by increasing volume and price points. When the day-ahead market prices becomes known, the volume the producer is obligated to commit in the day-ahead market can be determined.

The bid curves for the balancing market is the decisions of the second stage. This is the bids representing the willingness of both upward and downward regulation. Statnett require the bids for this market to be higher than 10MW (Statnett, e). There is also an upper bound for each producer, as it is not possible to regulate down more than already committed in the day-ahead market, or to regulate up more than the available capacity of the portfolio. To maintain the linear form of the problem the price points are fixed, in the same manner as in the day-ahead market. The market rules states that the price points must be an integer and divisible with five. Preliminary bids need to be submitted before 9:30 pm the day ahead, but the bids can be changed until 45 minutes before the operating hour. However, in this report we assume that the producer does not change the preliminary bids, after submitting them the day before the day of operation. With this assumption, it is possible to formulate the problem with only three stages.

The commitments in the balancing market appear close to delivery. As the purpose of the balancing market is to balance the supply and demand, it is not sure whether the commitment is needed for an entire hour. However, considering the size and the run time of the model, we assume that all accepted bids are committed for the entire hour. The direction of regulation and the size of the commitment depends on the need of balance in the grid. The volume available is also strongly correlated with the price, as the price is determined from the last accepted bid. If there is a big unbalance in the system, and thus high demand in the balancing market, the prices will be good for the producers willing to regulate up or down according to this unbalance. For this report, we assume that the volume and price are not dependent. The volume a producer is able to sell in the balancing market, is however restricted by a deterministic market share, which is a percentage of the capacity in the producer's portfolio.

At the hour of operation, the producer is informed about commitments in the balancing market. When both markets are cleared, the producer knows how much to produce in the given hour. This is the final stage of the problem. The profit from selling the committed amount of power, is dependent of the realized market prices. In this problem, only the day-ahead market and the balancing market are included, meaning that the volume the producer commits in these markets must equal their total production.

4.2 The Production Allocation

The objective of trading in the different energy markets is, as mentioned, to maximize the profit from the production. As the water stored in the reservoirs either can be produced today or in the future, the water value is included in the objective as a possible future profit. This term gives a value to the water in the reservoirs, dependent of the amount of water in storage at the end of the planning horizon. Due to limitations of the storage capacity in the reservoirs, water above the highest regulated level is assumed lost. It is unwanted for the producer to lose water, thus spill is penalized in the objective function.

There may be one or more power stations connected to reservoirs in the water system. Each power station may contain one or more turbines, with a given minimum and maximum capacity. In hours where the turbines stops, instead of producing at minimum capacity, there is a cost related to turning the turbines back on. It is assumed that this is the only cost of hydropower production, in addition to the marginal cost of the water. This implies that costs such as feed-in fees, are neglected for the purpose of this report.

When planning the production, the efficiency curve of the turbines must be considered. The efficiency curve decides how much power a turbine can produce from a given amount of discharge. As the water level in the reservoirs change, the efficiency curve also change, due to head effects. When the water level in the reservoirs are low, more water is necessary to obtain the same amount of power. As the power production is dependent of both the water level and the discharge, the problem will be non-linear. Large storage capacity and small head variation during a short-term horizon, makes it sufficient to model the power output only by its dependency on the discharge (Catalão et al., 2010). The power-discharge curve is thus modeled as a piece-wise linear curve, divided into segments with different slopes.

The water level in the reservoirs is held within the lowest regulated water level and the highest regulated water level. There may be several factors affecting the water level. Due to the short-term time horizon of the problem, it is assumed that inflow into the reservoirs can be modeled as deterministic values. Uncertainty in the inflow will not affect the problem when using such short time horizon. However, an increase in reservoir volume can also be in the form of discharge, spill, or bypass from other reservoirs. Bypass is controlled discharge that goes from a higher reservoir to a lower reservoir, without producing power by going through the power station. There is a limit on how much bypass the producer can send, as with discharge. The connections between the reservoirs, determines if a given reservoir receives discharge, spill, or bypass from a reservoir higher in the system.

Chapter 5

Mathematical Model

The mathematical formulation of the three-stage stochastic mixed-integer program for coordinated sequential bidding in the day-ahead market and the balancing market is presented. The formulation is based on the work of Fleten and Kristoffersen (2007) and Kongelf and Overrein (2017). Section 5.1 explains the three-stage structure of the problem, and why a node-variable formulation is chosen. In Section 5.2, all sets, indices, parameters and variables in the model are presented. The formulation of the bidding process in the day-ahead market and balancing market is introduced in Section 5.3, while the formulation of the hydropower operation is presented in Section 5.4. The final objective function and domain constraints are presented in Section 5.5. Section 5.6 describes an expansion which includes the option of block bids. Modeling assumptions are discussed as the formulation is presented. The entire model presentation can be found in Appendix A.

5.1 Structure of the Problem

When mathematically formulating a stochastic programming problem, two different types of formulations can be used depending on how the variables are defined (Conejo et al., 2010). In a scenario-variable formulation the variables are associated with scenarios. This formulation is well suited for decomposition, as it presents an exploitable structure. In a node-variable formulation however, the variables are associated with decision points rather than scenarios. This gives a more compact formulation which is well suited for a direct solution approach (Conejo et al., 2010). In a stochastic program it is important to always include non-anticipativity constraints (Birge and Louveaux, 2011). Such constraints ensure that all decision variables with the same realizations up to a certain node, are indeed identical. In a node-variable formulation the non-anticipativity of the decisions are implicitly taken into account through the formulation (Conejo et al., 2010). Since our problem is restricted to three stages, a node-variable formulation is chosen. Using this formulation for more stages may create confusion, due to the increasing number of subscripts for the decision variables.

The uncertainty in the second and third stage of the problem, stems from the market prices in the day-ahead and balancing market, respectively. The set of possible outcomes for the nodes in the second stage is denoted $\mathcal{S}$, while the set of the possible outcomes for the third stage is denoted $\mathcal{C}$. The third-stage nodes are in this problem equal to the number of scenarios, as they represent the leaves of the scenario tree. We introduce a subset $\mathcal{C}_s \subset \mathcal{C}$, which represents the possible third-stage outcomes c given a second-stage outcome s. A scenario in this problem is thus defined as a combination of two nodes given an outcome $s \in \mathcal{S}$ and $c \in \mathcal{C}_s$. This gives us a total of $|\mathcal{S}| \cdot |\mathcal{C}_s|$ scenarios. How the possible outcomes are generated is presented in Chapter 6.

Figure 5.1 illustrates the three stages of the problem, along with corresponding decision variables. The first-stage decisions are the bid curves, u^S_{hk}, for the day-ahead market. These are the here-and-now decisions. In the second stage the uncertain day-ahead market price, ρ^S_{hs}, is realized. The committed volume, w^S_{hs}, in the day-ahead market is calculated based on this realization. The second-stage decisions are the bid curves for the balancing market, u^R_{hskm}. When going into the third stage, we get information about the realization of the uncertain balancing market prices, ρ^R_{hscm}. Again, the realization of the market prices decides the committed volume, w^R_{hscm}, in the balancing markets. The third-stage of the problem is the producer's production allocation, which seeks an optimal production schedule, x_{thsc}. The notation is explained in Section 5.2.

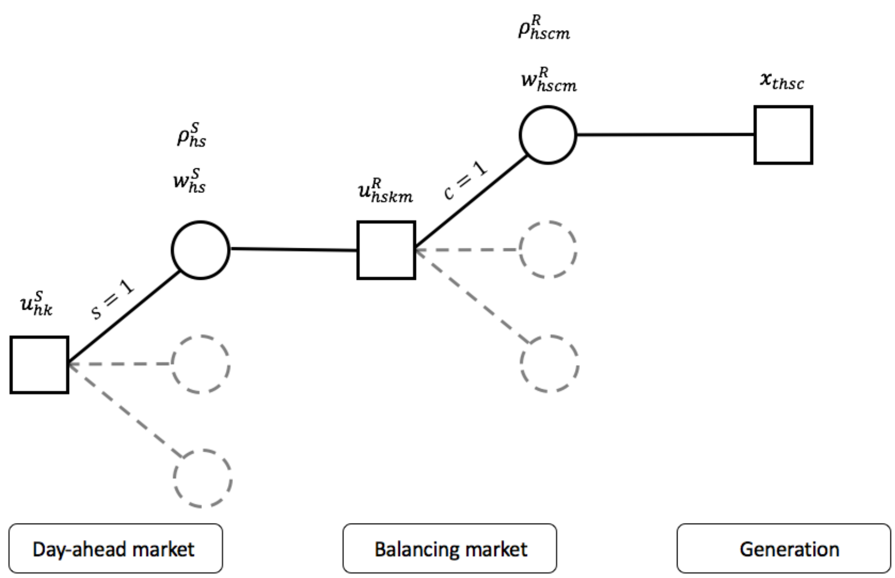


Figure 5.1: Decision tree for the coordinated bidding problem with the day-ahead and the balancing market

5.2 Notation

The notation for the sets, indices, parameters and variables used in the model are presented in this section. We use capital calligraphic letters for sets and lower-case letters for indices. The stochastic parameters are represented by the greek letters π and ρ, while upper-case roman letters are used for the deterministic parameters. Variables are presented by lower-case letters with the indices as subscript. Binary variables are represented by the greek letter δ and the letter y. Superscript S and R, represent the day-ahead and balancing market, respectively.

Sets & Indices

$\mathcal{T}$	Set of turbines, indexed by t
$\mathcal{T}_r$	Set of turbines in a given reservoir r , indexed by t
$\mathcal{H}$	Set of hours, indexed by h
$\mathcal{H}^B$	Set of bid hours $\mathcal{H}^B \subset \mathcal{H}$, indexed by h
$\mathcal{I}$	Set of discharge segments, indexed by i
$\mathcal{R}$	Set of reservoirs, indexed by r
$\mathcal{S}$	Set of second-stage outcomes for the day-ahead price, indexed by s
$\mathcal{C}$	Set of third-stage outcomes for the balancing price, indexed by c
$\mathcal{C}_s$	Set of third-stage outcomes, $\mathcal{C}_s \subset \mathcal{C}$, given the second-stage outcome s, indexed by c
$\mathcal{K}$	Set of price points, indexed by k
$\mathcal{M}$	Set of balancing markets, indexed by m

Stochastic Parameters

π_s	Probability of the second-stage outcome$s$
π_{sc}	Probability of the third-stage outcome c, given second-stage outcome s
ρ^S_{hs}	Day-ahead market price in hour h and outcome s
ρ^R_{hscm}	Balancing market price in hour h in outcomes s and c in market m

Deterministic Parameters

P_k^S	Price in bid point k for the day-ahead market
P_{km}^R	Price in bid point k for balancing market m
C_t	Unit start cost of turbine t
R_{ti}	Resource usage of turbine t in segment i
$\overline{V}_r$	Maximum volume of reservoir r
$\underline{V}_r$	Minimum volume of reservoir r
V_r^0	Initial volume for reservoir r
$\overline{Q}_t$	Maximum capacity of turbine t
$\underline{Q}_t$	Minimum capacity of turbine t
$\overline{D}_{ti}$	Maximum discharge for turbine t and segment i
$\underline{D}_t$	Minimum discharge for turbine t if the turbine is producing
$\overline{B}_r$	Maximum bypass of reservoir r
$\underline{B}_r$	Minimum bypass of reservoir r
I_{hr}	Inflow to reservoir r in hour h
W_r	Water value in reservoir r at the end of the period
$\delta_t^{T_0}$	Initial condition (on/off) for turbine t
$M_{r'r}^D$	Discharge matrix from reservoir r' to reservoir r
$M_{r'r}^S$	Spill matrix from reservoir r' to reservoir r
$M_{r'r}^B$	Bypass matrix from reservoir r' to reservoir r
E	Energy equivalent for the water
Z	Unit penalty for spill
$\underline{R}$	Minimum bid volume for the balancing market
S_m	Share of total capacity which can be committed in market m
U_m	Market index, $= 1$ for market $m = 1$, and -1 for market $m = 2$

Variables

u^S_{hk}	Bid curve volume for the day-ahead market in hour h and price point k
u^R_{hskm}	Bid curve volume for the balancing market in hour h, price point k, outcome s and market m
w^S_{hs}	Committed volume in the day-ahead market in hour h and outcome s
w^R_{hscm}	Committed volume in the balancing market in hour h, outcomes s and c, and market m
x_{thsc}	Produced power from turbine t in hour h, for outcomes s and c
d_{thsc}	Discharge from turbine t in hour h, for outcomes s and c
d^I_{thisc}	Segment discharge in segment i from turbine t in hour h, for outcomes s and c
v_{hrsc}	Volume in hour h in reservoir r and outcomes s and c
s_{hrsc}	Spill in hour h from reservoir r and outcomes s and c
b_{hrsc}	Bypass in hour h from reservoir r and outcomes s and c

Binary variables

$$\delta^R_{hscm} = \begin{cases} 1 & \text{if there is committed volume in hour } h\text{, in outcomes } s \text{ and } c\text{, in market } m \\ 0 & \text{otherwise} \end{cases}$$

$$\delta^T_{thsc} = \begin{cases} 1 & \text{if turbine } t \text{ is on in hour } h\text{, in outcomes } s \text{ and } c \\ 0 & \text{otherwise} \end{cases}$$

$$y_{thsc} = \begin{cases} 1 & \text{if turbine } t \text{ starts in hour } h\text{, in outcomes } s \text{ and } c \\ 0 & \text{otherwise} \end{cases}$$

5.3 Modeling the Markets

As explained above, the clearing of the markets takes place sequentially, so the commitment in the first market affects the possibilities of commitment in subsequent markets. We assume that the producer is risk neutral, so the risk of not being able to deliver the committed volume is not taken into account. Neither is the risk of not being dispatched in the balancing markets. All the variables and constraints directly connected to the bidding process are given for a set of bid hours, $\mathcal{H}^B \subset \mathcal{H}$, which is a subset of all the hours in the planning period, $\mathcal{H}$.

The Day-ahead Market

For the general model presented in this section, only single hourly bids are considered. The alternative of including block bids are considered in Section 7.4. A bid curve is constructed for each hour, h, of the bidding period. To keep the linear form of the problem, we choose to handle hourly bids in the same matter as Fleten and Kristoffersen (2007). The bid prices are fixated, so that for a given point $k \in \mathcal{K}$ we have a fixed price, P_k^S. It is thus only the bidding volumes that needs to be decided. For each hour $h \in \mathcal{H}^B$, we have a set of volume-price points (u_{hk}^S, P_k^S). The bid curve is the piecewise linear fitting between these points. A given market price, ρ_{hs}^S, is corresponding to a committed volume, w_{hs}^S, as illustrated in Figure 5.2.

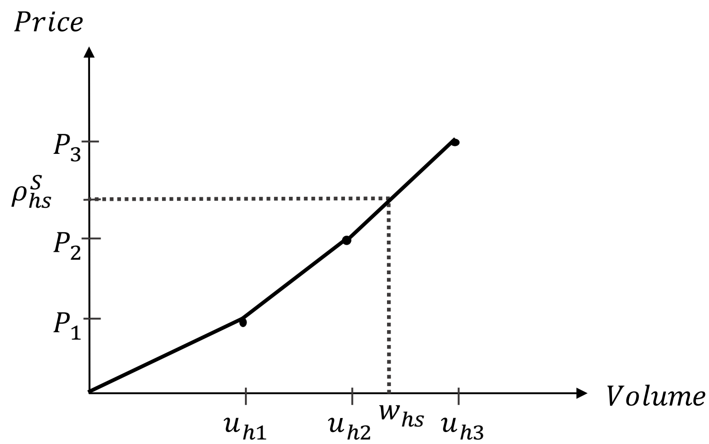


Figure 5.2: Bid curve in the day-ahead market

The committed volume, w_{hs}^S, can be found using linear interpolation between the volume-price points (u_{hk}^S, P_k^S). As introduced by Fleten and Kristoffersen (2007), we express the bid curve in terms of volumes (5.1).

$$w_{hs}^S = \frac{\rho_{hs}^S - P_{k-1}^S}{P_k^S - P_{k-1}^S} u_{hk}^S + \frac{P_k^S - \rho_{hs}^S}{P_k^S - P_{k-1}^S} u_{h,k-1}^S \quad \text{if } P_{k-1}^S \leq \rho_{hs}^S < P_k^S, \quad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K} \setminus \{1\} \tag{5.1}$$

Due to the market rule of monotonic increasing bid curves, we include the requirement of an upward sloping curve as a constraint in the model. This is done in (5.2), by requiring that the bid volume increases with the price points. The producer can not bid a volume above the total capacity of the system. To ensure this, an upper bound is set to be the sum of the maximum capacity, $\overline{Q}_t$, for all turbines, $t \in \mathcal{T}$, in the system. This is enforced by including (5.3).

$$u_{hk}^S \geq u_{h,k-1}^S \qquad h \in \mathcal{H}^B, k \in \mathcal{K} \setminus \{1\} \tag{5.2}$$

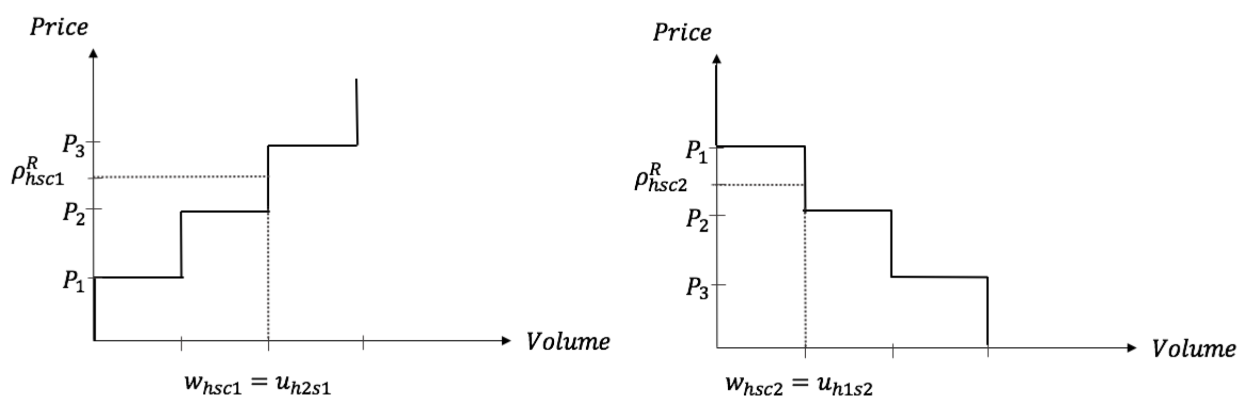
$$u^S_{hk} \leq \sum_{t \in \mathcal{T}} \overline{Q}_t \qquad h \in \mathcal{H}^B, k \in \mathcal{K} \tag{5.3}$$

The Balancing Market

There are three possible outcomes for the balancing market; upward regulation, downward regulation and no regulation. We define upward and downward regulation as two separate markets, as there are different prices, volumes and somewhat different rules in each case. A set of markets $\mathcal{M}$ is thus introduced:

$$m \in \mathcal{M} = \left\{ \begin{array}{ll} 1 & \text{upward regulation} \\ 2 & \text{downward regulation} \end{array} \right\}$$

We assume that the bid prices are fixed in the same way as for the day-ahead market, and use the same discretization into price points, $k \in \mathcal{K}$. Notice that the price points, P^R_{km}, are different for each market, $m \in \mathcal{M}$, as they go in opposite directions. For upward regulation the producer wants to bid high volumes for high prices, while for downward regulation the producer bids high volumes for low prices. The bid curves in the balancing market are modeled to be stepwise constant, rather than piece-wise linear, as illustrated in Figure 5.3.



(a) Upward regulation (m=1) **(b)** Downward regulation (m=2)

Figure 5.3: Examples of bid curves in the balancing market

As the figure illustrates, a realization of the balancing market price, ρ^R_{hscm}, corresponds to a committed volume, w^R_{hscm}. We assume that if the market price equals the fixed price, P^R_{km}, in point k, the committed volume equals the bid volume, u^R_{hskm}, in the corresponding point. The bid curves for upward and downward regulation are expressed in (5.4) and (5.5), respectively.

$$w^R_{hsc1} = u^R_{hs,k-1,1} \qquad \text{if } P^R_{k-1,1} \leq \rho^R_{hsc1} < P^R_{k1}, \quad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, k \in \mathcal{K} \setminus \{1\} \tag{5.4}$$

$$w^R_{hsc2} = u^R_{hs,k-1,2} \qquad \text{if } P^R_{k-1,2} \geq \rho^R_{hsc2} > P^R_{k2}, \quad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, k \in \mathcal{K} \setminus \{1\} \tag{5.5}$$

For the balancing market we have the requirement of upward sloping bid curve for upward regulation, and downward sloping bid curve for downward regulation. This requirement translates into increasing bid volumes with increasing points k, just as for the day-ahead market, as expressed in (5.6).

$$u^R_{hskm} \geq u^R_{hs,k-1,m} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K} \setminus \{1\}, m \in \mathcal{M} \tag{5.6}$$

Due to the fact that the day-ahead market is cleared first, the bid volume for the balancing market is restricted by the commitment in the day-ahead market. The volume which is available for upward regulation, is the total capacity of the system subtracted by the volume already reserved in the day-ahead market, as expressed by (5.7). For downward regulation, one can only regulate down the volume already committed in the day-ahead market (5.8).

$$u^R_{hsk1} \leq \sum_{t \in \mathcal{T}} \overline{Q}_t - w^S_{hs} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K} \tag{5.7}$$

$$u^R_{hsk2} \leq w^S_{hs} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K} \tag{5.8}$$

In order to provide upward or downward regulation, there is a lower limit, $\underline{R}$, for the volume a producer is required to commit (Statnett, e). This requirement is enforced by (5.9). The binary variable, δ^R_{hscm}, equals one if the producer commits any volume in market m, and zero otherwise. We also use this indicator to restrict the maximum commitment possible in the balancing market (5.10). The producer can not commit volume for both upward and downward regulation at the same time, even though there might be a chance that there exists a volume in both markets. This limitation is expressed by (5.11).

$$w^R_{hscm} \geq \underline{R}\delta^R_{hscm} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, m \in \mathcal{M} \tag{5.9}$$

$$w^R_{hscm} \leq \sum_{t \in \mathcal{T}} \overline{Q}_t \delta^R_{hscm} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, m \in \mathcal{M} \tag{5.10}$$

$$\sum_{m \in \mathcal{M}} \delta^R_{hscm} \leq 1 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.11}$$

Due to the low liquidity of the balancing market, there needs to be a limit on how much a producer can sell in this market (5.12). We assume that the total amount of power which can be committed over all bid hours $h \in \mathcal{H}^B$, is the maximum capacity of the system, multiplied with the number of hours in the bidding period, and a percentage market share, S_m.

$$\sum_{h \in \mathcal{H}^B} w^R_{hscm} \leq \sum_{t \in \mathcal{T}} \overline{Q}_t |\mathcal{H}^B| S_m \qquad s \in \mathcal{S}, c \in \mathcal{C}_s, m \in \mathcal{M} \tag{5.12}$$

Finally, the sum of the committed volume in all markets needs to equal to the sum of the production, x_{thsc}, over all turbines $t \in \mathcal{T}$, in the water system. This is expressed by (5.13). The market indicator, U_m, is positive one for upward regulation, and negative one for downward regulation. Downward regulation implies buying back volume, and thus gives an opposite contribution compared to upward regulation.

$$w^S_{hs} + \sum_{m \in \mathcal{M}} U_m w^R_{hscm} = \sum_{t \in \mathcal{T}} x_{thsc} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.13}$$

5.4 Modeling the Operation

For all the variables and constraints connected to the operation of the hydropower station, we look at all hours in the planning horizon, $h \in \mathcal{H}$. A turbine t has an upper and lower bound of how much power it can produce while running. The production, x_{thsc}, is thus limited by the maximum capacity, $\overline{Q}_t$, and minimum capacity, $\underline{Q}_t$ of the turbine (5.14). This is of course only relevant when the turbine is running, so we multiply the boundaries by the binary variable, δ^T_{thsc}, indicating if the turbine is on or off.

$$\underline{Q}_t \delta^T_{thsc} \leq x_{thsc} \leq \overline{Q}_t \delta^T_{thsc} \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.14}$$

We introduce another binary variable, y_{thsc}, which equals one if turbine t is turned on in hour h, and zero otherwise. The start/stop constraints are for the general case given by (5.15), and by (5.16) for the first hour. An initial condition for the turbines, $\delta_t^{T_0}$, is thus needed as an input parameter.

$$\delta_{thsc}^T - \delta_{t,h-1,sc}^T \leq y_{thsc} \qquad t \in \mathcal{T}, h \in \mathcal{H} \setminus \{1\}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.15}$$
$$\delta_{t1sc}^T - \delta_t^{T_0} \leq y_{t1sc} \qquad t \in \mathcal{T}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.16}$$

Production is modeled to be per unit, so head loss effects of running multiple units are not included in the model. The effects of varying water level for the production is also neglected. The production-discharge curve is modelled as a piecewise linear concave function. The production is thus represented by a linear function (5.17), in which the slope, R_{ti}, is given for each turbine $t \in \mathcal{T}$, and line segment $i \in \mathcal{I}$. The constant term, $\underline{Q}_t \delta_{thsc}^T$, sets the production to minimum capacity if the turbine is on. The segment discharge, d_{thisc}^I, has an upper limit for each segment, given by (5.18). As the function is concave, the discharge in segment i will be at maximum before the discharge in segment $i+1$ gets a value. The total discharge, d_{thsc}, for each turbine t equals the sum of the minimum discharge, $\underline{D}_t$, and the sum of all segment discharges, d_{thisc}^I, as expressed by (5.19).

$$x_{thsc} = \underline{Q}_t \delta_{thsc}^T + \sum_{i \in \mathcal{I}} R_{ti} d_{thisc}^I \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.17}$$
$$d_{thisc}^I \leq \overline{D}_{ti} \qquad t \in \mathcal{T}, h \in \mathcal{H}, i \in \mathcal{I}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.18}$$
$$d_{thsc} = \underline{D}_t \delta_{thsc}^T + \sum_{i \in \mathcal{I}} d_{thisc}^I \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.19}$$

Changes in the reservoir volume, v_{hrsc}, is given by a water balance. The general case is given by (5.20), and for the first hour by (5.21). In the first hour, the initial volume is given by V_r^0. The reservoir volume increases with inflow, I_{hr}, and decreases with discharge, d_{thsc}, spill, s_{hrsc}, and bypass b_{hrsc}, from the reservoir. This is illustrated in Figure 5.4. Reservoirs are connected through the use of indicator matrices, which indicate if there is a connection between reservoirs r' and r. We distinguish between the discharge matrix, $M_{r'r}^D$, the spill matrix, $M_{r'r}^S$, and the bypass matrix, $M_{r'r}^B$.

$$v_{hrsc} = v_{h-1,rsc} + I_{hr} - \sum_{t \in \mathcal{T}_r} d_{thsc} - s_{hrsc} - b_{hrsc} + \sum_{r' \in \mathcal{R}} \Big(M^D_{r'r} \sum_{t \in \mathcal{T}_r} d_{thsc} + M^S_{r'r} s_{hr'sc} + M^B_{r'r} b_{hr'sc} \Big)$$
$$h \in \mathcal{H} \setminus \{1\}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s \quad (5.20)$$

$$v_{1rsc} = V^0_r + I_{1r} - \sum_{t \in \mathcal{T}_r} d_{t1sc} - s_{1rsc} - b_{1rsc} + \sum_{r' \in \mathcal{R}} \Big(M^D_{r'r} \sum_{t \in \mathcal{T}_r} d_{t1sc} + M^S_{r'r} s_{1r'sc} + M^B_{r'r} b_{hr'sc} \Big)$$
$$r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s \quad (5.21)$$

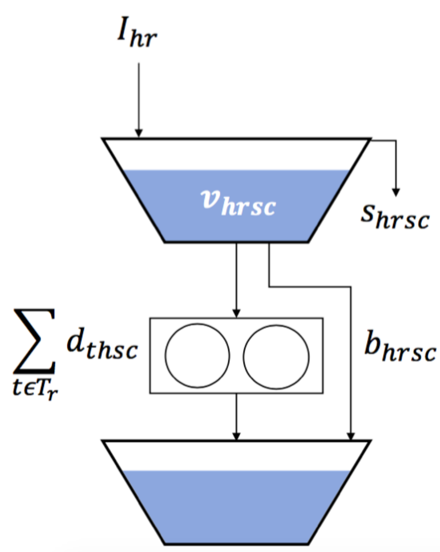


Figure 5.4: Variables included in the water balance

The volume available for discharge in a reservoir, is limited by the existing reservoir volume, v_{hrsc}, subtracted by the minimum requirement for the reservoir, $\underline{V}_r$, as expressed by (5.22). The reservoir volume and bypass has upper and lower limits as expressed by (5.23) and (5.24), respectively.

$$\sum_{t \in \mathcal{T}_r} d_{thsc} \leq v_{hrsc} - \underline{V}_r \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s \qquad (5.22)$$

$$\underline{V}_r \leq v_{hrsc} \leq \overline{V}_r \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.23}$$

$$\underline{B}_r \leq b_{hrsc} \leq \overline{B}_r \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.24}$$

5.5 Modeling the Objective Function

The objective of the bidding problem is to maximize the profits from bidding in the day-ahead market and the balancing markets, and is expressed by (5.25). As the problem is formulated as the deterministic equivalent of the stochastic program, the probability of each second-stage outcome, π_s, and the conditional probability of each third-stage outcome, π_{sc}, is included in the objective function. Furthermore, we need to sum over all possible second-stage outcomes, $s \in \mathcal{S}$, and third-stage outcomes, $c \in \mathcal{C}_s$.

$$\begin{aligned}\max \sum_{s \in \mathcal{S}} \pi_s \Bigg(\sum_{h \in \mathcal{H}^B} \rho^S_{hs} w^S_{hs} + \sum_{c \in \mathcal{C}_s} \pi_{sc} \Big(\sum_{h \in \mathcal{H}^B} \sum_{m \in \mathcal{M}} U_m \rho^R_{hscm} w^R_{hscm} + \sum_{t \in \mathcal{T}} \big(\sum_{h \in \mathcal{H} \setminus \{\mathcal{H}^B\}} \rho^S_{hs} x_{thsc} \\ - \sum_{h \in \mathcal{H}} C_t y_{thsc} \big) + \sum_{r \in \mathcal{R}} \big(\; W_r E \; (v_{|\mathcal{H}|rsc} - \underline{V}_r) - \sum_{h \in \mathcal{H}} Z s_{hrsc} \big) \Big) \Bigg)\end{aligned} \tag{5.25}$$

There are two sources of income in the bidding period, $h \in \mathcal{H}^B$. The income from the day-ahead market is expressed as the day-ahead market price, ρ^S_{hs}, multiplied with the committed volume, w^S_{hs}. The second income comes from regulating upwards and thus selling an additional volume, w^R_{hsc1}, at a market price, ρ^R_{hsc1}, higher than the day-ahead market price. Regulating downwards leads to a cost, as the producer is buying back a certain volume, w^R_{hsc2}, at a market price, ρ^R_{hsc1}, lower than the day-ahead market price. The opposite directions for the upward and downward regulation, is expressed by the market indicator, U_m. For the rest of the planning horizon, $h \in \mathcal{H} \setminus \{\mathcal{H}^B\}$, the income is modeled as the day-ahead market price, ρ^S_{hs}, multiplied with the production, x_{thsc}, in the period.

The cost of production for a given hour h is determined by the unit start-up costs, C_t, multiplied with the binary variable, y_{thsc}, indicating if the unit is started in hour h. A unit penalty, Z, is introduced to penalize spill in the objective function. In addition, the opportunity cost of the water is taken into account by including the water value, W_r, for the reservoirs, $r \in \mathcal{R}$. The water value is multiplied by the total volume available for production at the end of the horizon, $v_{|\mathcal{H}|rsc} - \underline{V}_r$, and the energy coefficient, E.

The domain restrictions of the variables are given by (5.26)–(5.39). Notice that the variables representing the committed volume for upward regulation (5.29) and downward regulation (5.30), only exists if the balancing market price is higher and lower than the day-ahead price, respectively.

$$u^S_{hk} \geq 0 \qquad h \in \mathcal{H}^B, k \in \mathcal{K} \tag{5.26}$$
$$w^S_{hs} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S} \tag{5.27}$$
$$u^R_{hskm} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K}, m \in \mathcal{M} \tag{5.28}$$
$$w^R_{hsc1} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s \mid \rho^R_{hsc1} > \rho^S_{hs} \tag{5.29}$$
$$w^R_{hsc2} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s \mid \rho^R_{hsc2} < \rho^S_{hs} \tag{5.30}$$
$$\delta^R_{hscm} \in [0, 1] \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, m \in \mathcal{M} \tag{5.31}$$
$$x_{thsc} \geq 0 \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.32}$$
$$d_{thsc} \geq 0 \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.33}$$
$$\delta^T_{thsc} \in [0, 1] \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.34}$$
$$y_{thsc} \in [0, 1] \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.35}$$
$$d^I_{thisc} \geq 0 \qquad t \in \mathcal{T}, h \in \mathcal{H}, i \in \mathcal{I}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.36}$$
$$v_{hrsc} \geq 0 \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.37}$$
$$s_{hrsc} \geq 0 \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.38}$$
$$b_{hrsc} \geq 0 \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s \tag{5.39}$$

5.6 Expanded Model

To include block bids for the day-ahead market, a few more constraints must be added to the model presented in the previous sections. First, a bid curve for the block bids must be constructed. This is modeled as a stepwise constant function in the same manner as in the balancing market, and expressed by (5.40). An indexed set is used to divide the bid hours into blocks. This is defined as the set $b \in \mathcal{B}_h$, telling us if hour h is included in block b. In this report, three blocks are used, separating the bidding period into a night, day, and evening block.

$$w^B_{bs} = u_{b,k-1} \quad \text{if } P_{k-1} \leq \overline{\rho}_{bs} < P_k, \quad h \in \mathcal{H}^B, k \in \mathcal{K} \setminus \{1\}, s \in \mathcal{S}, b \in \mathcal{B}_h \tag{5.40}$$

The block price, $\overline{\rho}_{bs}$, represents the mean day-ahead price for the hours included in block b. The volume points in the bid curve must be increasing, due to the market rules of monotonic increasing bid curves (5.41).

$$u^B_{b,k} \geq u^B_{b,k-1} \qquad h \in \mathcal{H}^B, k \in \mathcal{K} \setminus \{1\}, b \in \mathcal{B}_h \tag{5.41}$$

As it now is possible to bid both hourly and in blocks in the day-ahead market, the sum of these two bids can not exceed the capacity of the watercourse. This is expressed by (5.42).

$$\sum_{b \in \mathcal{B}_h} u^B_{bk} + u^S_{hk} \leq \sum_{t \in \mathcal{T}} \overline{Q}_t \delta^T_{thsc} \quad h \in \mathcal{H}^B, k \in \mathcal{K} \tag{5.42}$$

To prevent getting a commitment lower than the minimum capacity, $\underline{Q}_t$, a lower bound for the block bids is included in constraint (5.43).

$$\sum_{b \in B_h} u^B_{bk} \geq \sum_{t \in T} \underline{Q}_t \delta^T_{thsc} \quad h \in \mathcal{H}^B, k \in \mathcal{K}, s \in \mathcal{S}, c \in \mathcal{C} \tag{5.43}$$

Finally, the revenue from the block commitments must be included in the objective function. As the committed volume from block bids, w^B_{bs}, represents the commitment in hour h in a given block b, the commitments from all hours in all blocks must be added to the objective function. The revenue contribution of this commitment is presented in equation (5.44).

$$\sum_{s \in \mathcal{S}} \sum_{h \in \mathcal{H}^B} \sum_{b \in \mathcal{B}_h} \overline{\rho}_{bs} w^B_{bs} \tag{5.44}$$

The expanded model only introduces new non-negative continuous variables.

$$u^B_{bk} \geq 0 \qquad h \in \mathcal{H}^B, k \in \mathcal{K}, b \in \mathcal{B}_h \tag{5.45}$$

$$w^B_{bs} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, b \in \mathcal{B}_h \tag{5.46}$$

Chapter 6

Scenario Tree Generation

In order to generate a scenario tree for the stochastic parameters introduced in Section 5.2, a scenario tree generation method is needed. As explained in Section 5.1, we use a node-variable formulation for the stochastic program, instead of a scenario-variable formulation. We thus need to generate the possible outcomes for the second-stage nodes, and the possible outcomes for the third-stage nodes, given an outcome in the second-stage. The term scenario, is thus not used for the separate outcomes for these nodes. Throughout this section, we will in any case refer to the process as scenario generation.

Section 6.1 explains the procedure of generating the outcomes. The scenario generation method by Kaut (2017) is chosen in this report, due to available forecast data from a hydropower producer. The scenario generation tool is hereby referred to as ScenGen. Section 6.2 measures the stability of the method, and how many scenarios that are needed to ensure a stable solution.

6.1 Procedure

Most of the standard scenario generation methods today require a certain amount of input data, and is eliminated for the purpose of this report. Kaut (2017) introduces a method for generating scenarios, using the combination of historical forecasts and market prices as input. A hydropower producer typically has a single forecast for the future values of the day-ahead prices, which is updated every day of the working week. The forecasts are often made for at least ten days forward, to include weekend variations. The forecasting process is based on the market equilibrium between supply and demand, in which forecasts for the supply and demand are either bought or self-made by the producer. To our knowledge, no hydropower producers today makes forecasts for the balancing market prices, as there currently does not exist such tools. Thus, the forecast for the balancing market prices, is for simplicity set equal to the forecast for the day-ahead market prices.

The historical forecasts for the day-ahead prices used in ScenGen, stretches over a period of over two years, from March 10th, 2015 to October 20th, 2017. Each forecast is made for the next 13 days, that is, the next 312 hours. During weekends and holidays, the particular hydropower producer does not make forecasts, so a simplification is used. If a day is missing forecasts, the forecasts for the previous day moved 24 hours forward, is used. To maintain the same vector size for all days, the forecast for hours 288–312 is set equal to the forecast for hours 264–287. This procedure is repeated if there are consecutive days missing forecast.

To calculate the corresponding forecast errors, the historical day-ahead prices for the same period are used as input to ScenGen. Historical market prices for upward and downward regulation is also used to calculate the forecast errors for the balancing market. In addition to a historical period with forecasts, ScenGen needs a forecast for the next 13 days in order to generate a scenario tree for the future values.

As explained in Section 5.1, this is a three-stage stochastic problem. The number of branches for the second-stage and third-stage are used as input in ScenGen, such that we obtain $|\mathcal{S}|\cdot|\mathcal{C}_s|$ number of scenarios. The branching is chosen to start at the start of the bidding period, such that the period before is deterministic and equal to the input. We obtain $|\mathcal{C}_s|$ different balancing market outcomes for each day-ahead market outcome s, but the outcomes themselves are not dependent on the actual price in the day-ahead market given by s. This leads to prices for upward and downward regulation that often is lower and higher than the day-ahead market price, respectively. In reality, this would mean having a negative regulating volume. The scenarios are thus processed, so that if the upward/downward regulating price is lower/higher than the day-ahead price, it is set equal to the day-ahead price in that hour.

6.2 Evaluation

Kaut and Wallace (2007) points out that the focus when evaluating scenario generation methods is the practical performance, not the theoretical properties. The goal is not necessarily to search for a discretization of a distribution that is good in a statistical sense, but one that leads to good decisions in the model. The method is good if it performs well in a real-life problem, independent on how well the method approximates the true distribution. However, it is important that it is the model, and not the discretization of probability distributions, that drives the results (King and Wallace, 2012). The scenario generation method should in other words not affect the solution of running the model. In our case, the quality of the scenarios also depends on the forecasting method as well as the method for the scenario generation, as the method is based on external forecasts.

A common requirement of a scenario generation method is stability, which implies that we should get approximately the same optimal objective function value if the problem is solved several times, with different scenario trees generated from the same input. This is called in-sample stability, since the solutions are evaluated on the sample (tree) they came form (Kaut and Wallace, 2007). The requirement of in-sample stability is a test of internal consistency, and indicate that it should not matter which scenario tree you use (King and Wallace, 2012). The scenario generation method presented in this report is not itself random, which means that the exact same scenario tree will be produced each time it is run with the same data. According to Kaut and Wallace (2007), the stability requirement is not an issue for scenario generation methods that do not include any randomness.

However, there is one source of instability, which is the number of scenarios. The necessary number of scenarios can be found by checking for when the optimal objective function value does not change with change in the number of scenarios. The model is run with five different numbers of scenarios, where each of the instances are run ten times. The combinations used in the test is 10, 20, 60, 80 and 100 day-ahead market outcomes, with 10 balancing market outcomes for each day-ahead outcome. This gives a total range from 100–1000 scenarios. From Figure 6.1, it can be seen that the objective function value stabilizes around 800 scenarios.

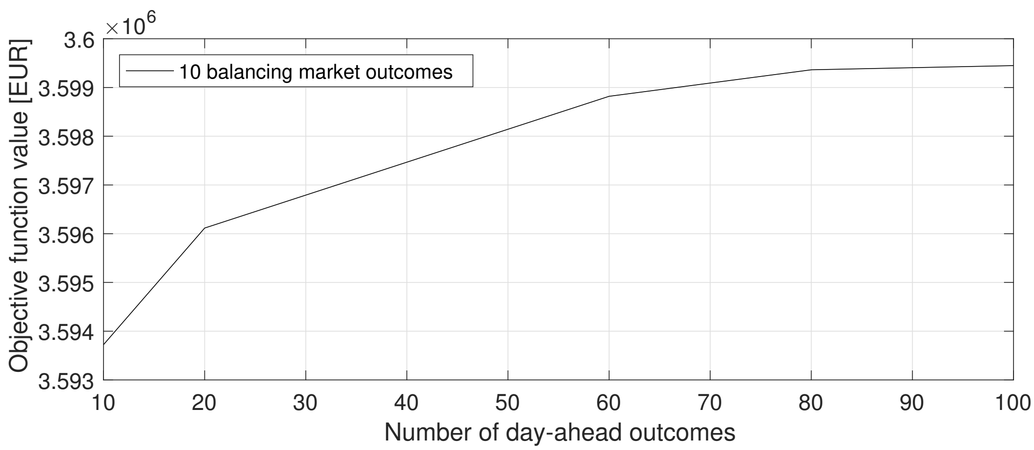


Figure 6.1: Stability of the objective function value for different scenario tree sizes

As mentioned above, in-sample stability can not be tested for this method by running the optimization model with many different trees generated from the same data. The procedure for deterministic scenario generation procedures presented in King and Wallace (2012), suggests to run the procedure with trees of slightly different sizes as an alternative way of testing for in-sample stability. For the scenario generation method chosen in this report, the duration of the period for historical forecast errors is slightly adjusted to generate different scenario sets and in that way test the in-sample stability. As the results from Table 6.1 illustrates, the objective value never deviates much from the mean. The run time increases with the number of scenarios, from around 37 seconds for 100 scenarios to 729 seconds for 800 scenarios. All tests are done for 10 balancing market outcomes for each day-ahead outcome.

Table 6.1: In-sample stability test run for 10 scenario sets based on different sizes of historical data

	10 day-ahead scenarios			**80 day-ahead scenarios**		
Scenario set	Objective function	Difference from mean	Runtime in seconds	Objective function	Difference from mean	Runtime in seconds
1	3 593 724	-0.007%	35.5	3 599 363	-0.011%	783.9
2	3 593 770	-0.006%	39.9	3 599 356	-0.011%	713.4
3	3 593 657	-0.009%	31.7	3 599 580	-0.005%	684.0
4	3 593 723	-0.007%	39.7	3 599 554	-0.005%	687.1
5	3 593 726	-0.007%	40.0	3 599 873	0.003%	725.4
6	3 594 320	0.010%	40.1	3 599 802	0.001%	716.9
7	3 594 239	0.007%	32.6	3 599 889	0.004%	705.3
8	3 594 157	0.005%	53.6	3 599 876	0.004%	680.0
9	3 594 191	0.006%	29.9	3 600 130	0.011%	718.7
10	3 594 226	0.007%	26.2	3 600 074	0.009%	874.8
Mean	3 593 973		37.0	3 599 750		729.0

Chapter 7

Computational Case Study

A case study has been conducted to analyze and measure the performance of the coordinated model presented in Chapter 5. The set up of the case study is presented in detail in Section 7.1. The model performance is measured by running the model with input data provided by a hydropower producer. This data is used throughout the study, and is based on thirteen days in the start of January 2017. The performance of the coordinated model is measured by how well it behaves compared to a separate bidding strategy with the same input data. This analysis estimates the gain of planning for both markets combined, instead of planning separately. The results are presented in Section 7.2. The behaviour of the model is compared to the historical schedule from the hydropower producer mentioned above, in Section 7.3. The results of running the model when adding the option of using block bids is presented in Section 7.4. Finally, the results from the case study are discussed in Section 7.5.

The decision models are implemented in Xpress-IVE 64 bit using Xpress-Mosel, and solved with Xpress-Optimizer on computers with an Intel Core i7-7700 3.60GHz CPU and 32.0 GB installed RAM. The scenario generation tool presented in Chapter 6 is provided by NTNU.

7.1 Case Description

The case study is based on a watercourse owned by a hydropower producer located in the NO2 price area in the southern parts of Norway. The hydropower producer has provided us with technical and historical data for the system, and is hereby referred to as our industrial partner. The watercourse consists of four reservoirs and one power station. Figure 7.1 illustrates the typology of the system.

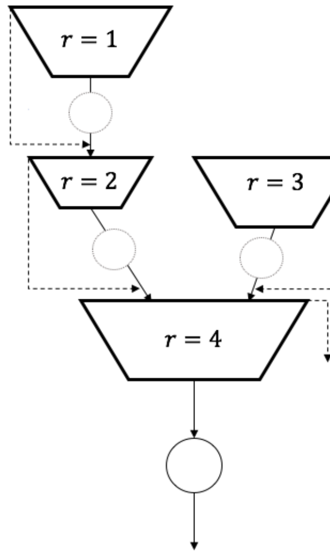


Figure 7.1: Typology of the watercourse used in the case study

The power station below reservoir four contains one turbine, with a given efficiency curve and capacity. The remaining reservoirs are not connected to power stations, so these are modeled as dummy stations with zero efficiency and zero capacity. The model does not distinguish between discharge and bypass for the dummy stations, as the discharge through the station gives no production. There is no possibility for bypass in reservoir four, so maximum bypass is set to zero for all reservoirs. If spill occurs in the upper reservoirs, it is led to the reservoir below, while spill in the lowest reservoir is lost from the system. The maximum discharge for each of the upper reservoirs, is set equal to the discharge in the situation with wide open sluicegates and a reservoir volume at 50% of highest regulated water level. As illustrated by Figure 7.2, the industrial partner uses four fixed power production points. The turbine connected to reservoir four has a minimum and maximum capacity of 16 MW and 80 MW, and a best efficiency production at 60 MW. The corresponding segment discharge for each production point is given by the production-discharge curve as illustrated in Figure 7.2. As we neglect the effect of varying water level, the curve is based on the historical average water level for reservoir four, instead of using different curves for each level.

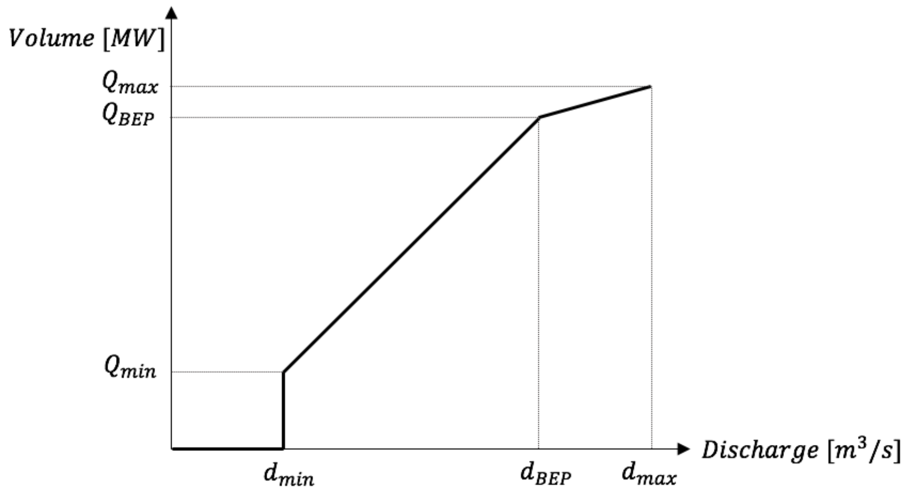


Figure 7.2: P-Q curve at the historical average water level

The planning horizon is set to 312 hours, which equals a total of 13 days. This horizon was chosen based on available data from the industrial partner. The bidding period includes hours 25–48, as the first 24 hours are considered deterministic with a production plan that was determined the day prior to the first day in the planning horizon. The bid curves for the day-ahead and balancing market is thus modeled for day two only.

The data used in the case study is collected from the period from January 9, 2017 to January 21, 2017. The bid curves are thus determined for Tuesday, January 10, 2017. The data includes all input for the model, such as initial reservoir volume, inflow, water values, forecasts, and historical prices. Inflow is modeled as a deterministic parameter, so historical inflow is used. The industrial partner assume that it is reasonable to use the same water value for all the reservoirs in the system. As the watercourse only contains one power station connected to reservoir four, and both spill and discharge from higher reservoirs ends up here, the assumption is considered to be reasonable for the case study as well. The system is flexible, and can easily transfer water from higher reservoirs. The reservoirs could have been merged for the case study, as all the water ends up in the producing reservoir, but due to available data for each reservoir and reasonable run times, each reservoir is kept separate.

For the construction of the bid curves, the fixed price points used in the case study is not based on historical data. While we use fixed price points, the industrial partner uses fixed volume points with varying price points. They normally use six price points including the compulsory extreme values. The number of price points in the case study is chosen somewhat arbitrarily. As the number of price points increases the computational time substantially, we have decided that ten price points are sufficient for the purpose of this case study. Using the required upper and lower limit of -500 and 3000 EUR for the day-ahead market as extreme points for all markets, we cover the most extreme outcomes of the market prices.

The remaining price points are set in the range from 0–40 EUR as illustrated in Table 7.1, with small intervals to cover the values that occur most often. The historical average for the day-ahead, upward regulation, and downward regulation the last two years is 24, 26, and 22 EUR, respectively. The range of price points in the case study is thus a reasonable assumption.

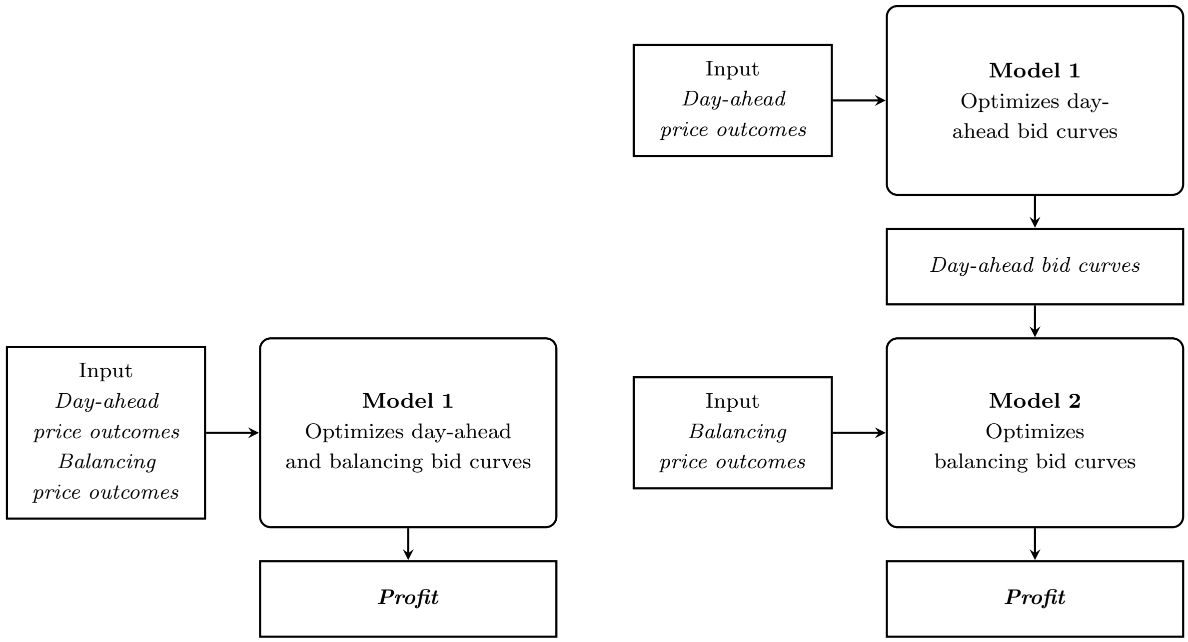
The possible outcomes of the market prices are generated as explained in Section 6.1. The volume available in the markets are not generated, however. For the less liquid balancing market, there is not enough demand in the market to sell as much of the total capacity. As discussed in Chapter 5, we limit the volume available for commitment in the balancing market by defining a deterministic parameter, the market share. In the case study, this parameter is chosen somewhat arbitrarily to 25%. We consider 25% a reasonable assumption, as producers sell most of their capacity in the day-ahead market.

Table 7.1: Price points used for the different markets in the case study

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Day-ahead and balancing (up)	-500	0	10	15	20	25	30	35	40	3000
Balancing (down)	3000	40	35	30	25	20	15	10	0	-500

The objective of the case study is to estimate the gains from using a coordinated bidding strategy, rather than a sequential strategy which is mostly used by hydropower producers today. The models are compared when using only the uncertain outcomes, but also when including the realizations of the market prices the given test day. These two different cases gives the need for two case study frameworks.

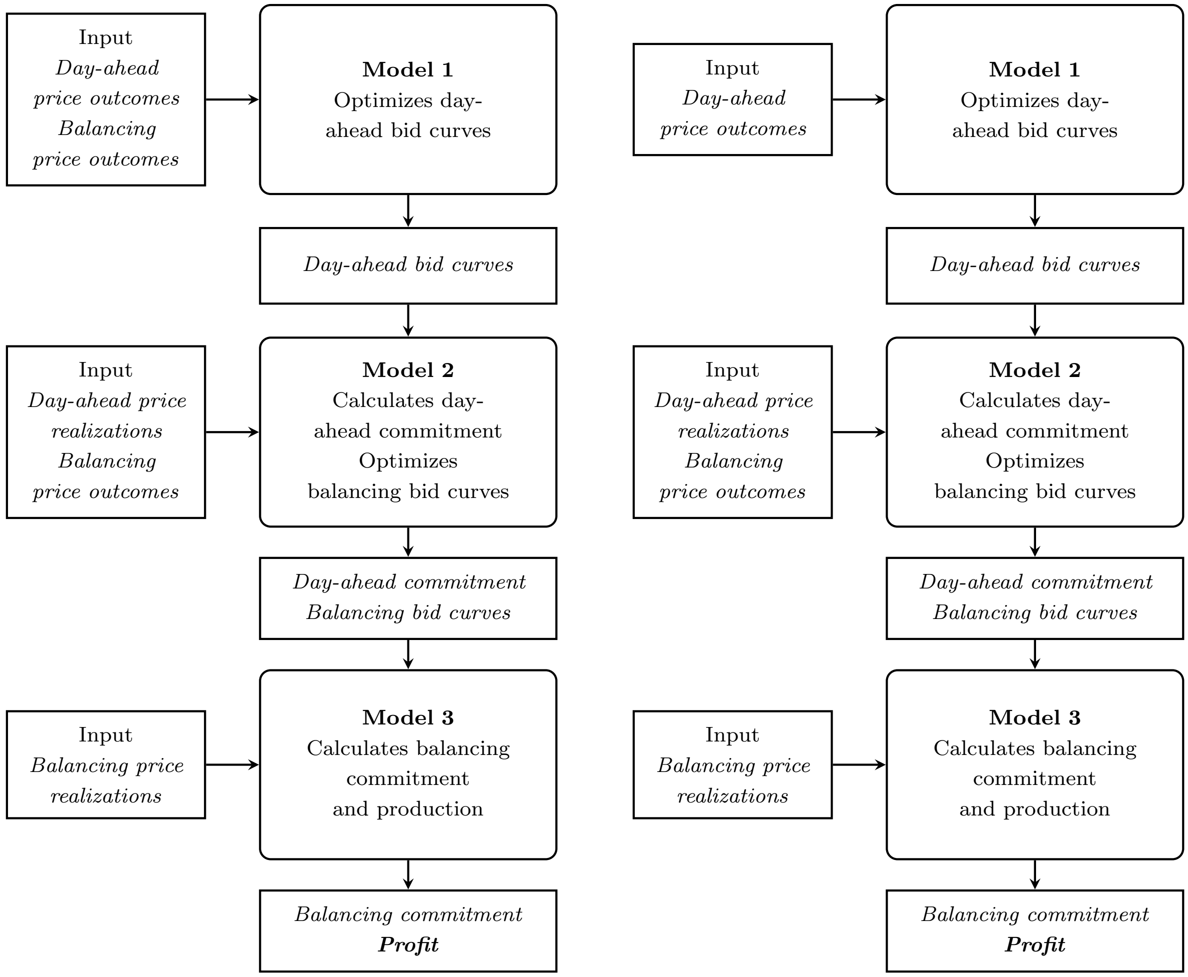
In the first part of the case study, the coordinated model is compared to a sequential strategy when using uncertain market prices. We run a sequential model, which optimizes the bid curves for the day-ahead market without considering the possible outcomes of the balancing market. The bid curves from the sequential model are used as input in the coordinated model presented in Chapter 5, but with the variables representing the bid curve volumes for the day-ahead market as fixed. The results from using this sequential framework, is compared to the results from the coordinated model alone. For the coordinated framework, all bid curves are optimized simultaneously based on the possible outcomes of all markets. The frameworks for the two different approaches are illustrated in Figure 7.3.



(a) Coordinated **(b)** Sequential

Figure 7.3: Case study framework for uncertain market prices

For the second part of the case study, we analyze how the coordinated and sequential strategies performs when the market prices are realized. The analysis is based on the framework presented in Kongelf and Overrein (2017). In this framework the models are run three times, one for each of the stages in the decision tree. The frameworks are illustrated in Figure 7.4. First, the coordinated model presented in Chapter 5 is run with the possible outcomes for the day-ahead and the balancing market prices. This model returns the bid curve for the day-ahead market. The bid curve is used as input in a second model, along with the realized day-ahead prices for the bidding period. The realization of the balancing market prices are still uncertain. The second model then returns the committed volume for the day-ahead market, and bid curves for the balancing market. Finally, a deterministic production allocation model is run. The input is the realized balancing market prices for the bidding period, in addition to the committed volume for the day-ahead market and bid curves for the balancing market. The committed volume for the balancing market can now be calculated, and the production is allocated. The output from the final run is the total profit for the coordinated model with realized market prices. The same procedure is used for the sequential framework. The difference is the input of the first model, as the sequential model calculates the bid curves for the day-ahead market without considering the possible outcomes of the balancing market.



(a) Coordinated **(b)** Sequential

Figure 7.4: Case study framework for realized market prices

7.2 Comparison with Sequential Model

The coordinated bidding model is compared to the corresponding sequential bidding model. Identical input data is given for the two models and they are run for the same time period. The only disparity between the models is the number of markets considered at the same time. The results from running both models are analyzed, followed by a discussion of the performance of the model. First, the two bidding strategies are tested for a general case with uncertain market prices, before they are tested for the realization of the market prices for the given test day.

Uncertain day-ahead and balancing market prices

For the general case with uncertain prices, the case study framework presented in Figure 7.3 is used. The results from the coordinated and the sequential frameworks are presented in Table 7.2. The table presents the objective function value for both models, in addition to the values for all the revenues and costs the objective consists of. It is evident that the coordinated model achieves a higher objective function value, due to higher revenue in the upward balancing market and higher water value at the end of the horizon. The sequential model bids more aggressively towards the day-ahead market as illustrated by the committed volumes in Table 7.3, and thus has a higher revenue in this market. However, a lower commitment in the day-ahead market gives better opportunities for future upward regulation. The coordinated model regulates up over 25% more volume in the balancing market, compared to the sequential model. Both models choose to regulate down a considerable amount in the balancing market. This is due to the level of the possible outcomes for down regulation in the balancing market is low, and in most cases lower than the water value. The coordinated model buys back approximately the same volume as the sequential model, but has a lower cost for down regulation.

Table 7.2: Comparison of the objective function value and revenues/costs when running models with uncertain market prices

	Objective	Day-ahead (bid hours)	Day-ahead (other hours)	Balancing (Up)	Balancing (Down)	Start-up	Water value
Coordinated [EUR]	3 599 605	43 032	527 371	4 378	-3 149	-1 282	3 029 255
Sequential [EUR]	3 599 433	45 745	528 246	3 461	-3 239	-1 276	3 026 496

Table 7.3: Expected volume commitments in the bidding hours with uncertain market prices

	Total	Day-ahead	Balancing (Up)	Balancing (Down)
Coordinated [MWh]	1631	1367	123	141
Sequential [MWh]	1692	1457	96	139

A common measure of model performance is the obtained average price, which is calculated by dividing the total revenue by the production in the corresponding period. It is desirable to produce at high prices in the day-ahead market, and to utilize the cases with high premiums in the balancing market. The results are presented in Table 7.4. In the bid hours period, when there exists opportunities in the balancing market, the coordinated model obtain average prices that are 0.9% higher than for the sequential model. Outside the bidding period, both models obtain the same average price per MWh, although they obtained different day-ahead revenues for the same hours.

Table 7.4: Obtained average price per MWh when running models with uncertain market prices

	Bid hours	Other hours
Coordinated [EUR/MWh]	32.8	34.9
Sequential [EUR/MWh]	32.5	34.9
Gain [%]	0.9	0.0

Realized day-ahead and balancing market prices

In the following analysis, the coordinated and the sequential model are run for the test day January 10, 2017, to compare the performance for the models for a given realization of the market prices. Both the coordinated and the sequential model is run in three parts for this analysis, as illustrated by the case study framework in section 7.1.

The results from the models are presented in Table 7.5. From the table, we can observe that the coordinated model earns less during bid hours, due to committing less volume in the day-ahead market and by buying back more water for downward regulation. However, the water value at the end of the horizon is quite larger for the coordinated model, which eventually leads to a higher objective function value by using this model. The improvement in the objective value is however minimal, and the values are practically the same. Thus, for this given day, there is not much extra profit to gain by using the coordinated model. If the realizations of the market prices had been different, the outcome of the test could also have been different. Notice that there was no upward regulation this day.

Table 7.5: Comparison of the objective function value and revenues/costs when running models with realized market prices

	Objective	Day-ahead (bid hours)	Day-ahead (other hours)	Balancing (Up)	Balancing (Down)	Start-up	Water value
Coordinated [EUR]	3 572 322	45 121	684 862	-	- 3 238	- 325	2 845 902
Sequential [EUR]	3 572 097	47 280	684 862	-	- 328	- 325	2 840 609

When running the models with the realized market prices, the coordinated model still obtains a higher average price per MWh for the bidding period compared to the sequential model, as illustrated by Table 7.6. The coordinated model is thus slightly better at adjusting the production according to market prices. The obtained average price per MWh for the other hours, is still equal for both models.

Table 7.6: Obtained average price per MWh when running models with realized market prices

	Bid hours	**Other hours**
Coordinated [EUR]	30.8	32.0
Sequential [EUR]	30.4	32.0
Gain [%]	1.3	0.0

Although obtained average price per MWh is a common performance measure, it does not take into account the value of the water which is used. The profit contribution from the day-ahead and the upward balancing market, can be calculated by multiplying the committed value with the difference between the market price and the water value (7.1). For downward regulation, the premium can be calculated by subtracting the market price from the water value (7.2). Furthermore, the profits for each hour can be summarized over all bid hours to obtain the total profit.

$$Profit = (Realized\ market\ price - Water\ value) \cdot Committed\ volume \tag{7.1}$$

$$Profit = (Water\ value - Realized\ market\ price) \cdot Committed\ volume \tag{7.2}$$

The results for both models are summarized in Table 7.7. The coordinated model has lower profits in the day-ahead market, but catches up with the sequential model by having higher profits in the balancing market. The gain in total profits by using the coordinated model is 8.5%, which indicates that the coordinated model is better at distributing the water which is available.

Table 7.7: Obtained profits when running models with realized market prices

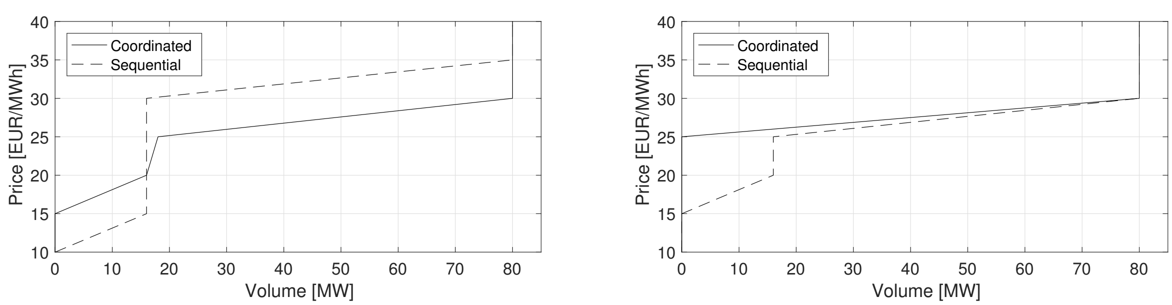
	Total	**Day-ahead**	**Balancing (Up)**	**Balancing (Down)**
Coordinated [EUR/MWh]	2745	1971	0	775
Sequential [EUR/MWh]	2530	2447	0	83
Gain [%]	8.5	-19.5	0.0	838.2

This performance measure is however very sensitive to the water values used in the case study. Table 7.8 illustrates the gain in obtained profits, when setting the water value to 1 EUR/MWh below and above the historical water value. The results shows that the coordinated model does not obtain a higher profit if the water values are increased or decreased by 1 EUR. However, notice that the gain for downward regulation is positive for the case of a higher water value.

Table 7.8: Percentage gain in obtained profits by using the coordinated model for different water values

	Sum	**Day-ahead**	**Balancing (Up)**	**Balancing (Down)**
Historical	8.5	-19.5	0.0	838.2
Below	-3.6	-4.2	0.0	-0.4
Above	-1.3	-32.3	0.0	181.7

Examples of optimal bid curves for both models in the day-ahead market is illustrated in Figure 7.5. We consider one hour during the night, and one hour during the day, due to different market prices. It can be observed from 7.5a that in hour 3, the coordinated model is willing to supply a large volume at lower prices than the sequential model. This is due to the fact that the coordinated model sees a premium for downward regulation in the night hours, and can thus bid more aggressively in the day-ahead market. Figure 7.5b illustrates the bid curves for hour 13, in which it can be observed that the coordinated model require somewhat higher prices in the day-ahead market. The model expects no premium for downward regulation during these hours, and carry out a more careful bidding behaviour during these hours. The bidding curves are thus qualitatively different in the coordinated case.



(a) Hour 3 **(b)** Hour 13

Figure 7.5: Comparison of optimal bid curves for the day-ahead market

When running the models with realized market prices for the given day, it is worth noticing that the realized premiums for the upward balancing market was very low compared to the expected outcomes. The model reserved capacity to be able to regulate upwards, but was not able to regulate as much as expected due to the low realization of the market prices. The model is thus quite sensitive to the generation of the possible outcomes. The mismatch between the realizations of the balancing prices and the expected outcomes affects the gain of the coordinated model.

7.3 Comparison with Industrial Partner

In this section, historical data provided by the industrial partner is compared to the results from running the coordinated model with realized market prices, according to the framework presented in Figure 7.4. The industrial partner use a sequential bidding strategy, but also include the experience and skills from the working engineers in the bidding process. The analysis in this section is selected based on the data available from the industrial partner.

Table 7.9 presents the actual production schedule for the hydropower plant January 10, 2017, along with the historical area price. The production schedule suggested by the coordinated model is also presented. The results indicates that the coordinated model is bidding more aggressively than the industrial partner actually did the given day, which is as expected. The coordinated model calculate bid curves for the day-ahead market while taking into account the expected premiums between the day-ahead and the balancing market prices. Looking at expected values, the premiums for both upward and downward balancing seems to be highest during the night in, hours 2–6. The model thus recommends to produce during these hours to keep the flexibility to regulate up or down, without having to consider start-up costs. The production schedule illustrated in Table 7.9, is however conditional of the actual realizations of the prices. The premium for upward balancing turned out to be almost zero the given day, while the premium for downward balancing was around 4 EUR for hours 1–5 and 7–12. With the coordinated model, the committed volume in the day-ahead market was high in these hours, to be able to regulate down to minimum capacity during hours with positive premium. It is worth noting that the industrial partner takes feed-in fees into account when bidding, which leads to different bidding prices for day and night.

The comparison of the production schedule for the industrial partner and the schedule calculated from the coordinated model, shows clear differences. The most obvious difference is the choice of no production or minimum production. The area price is very close to the water value in the hours when the coordinated model chooses to produce at minimum. As soon as the area price exceeds the water value, the production increases from minimum production, to a point between best efficiency point and maximum production. While the industrial partner operates at three fixed production points as described in Section 7.1, the coordinated model can produce at all levels between minimum and maximum capacity. This opens up for a more flexible production schedule. The production is often at maximum and minimum capacity, while the best efficiency point of 60 MW, never occurs for the coordinated model.

Table 7.9: Comparison of production schedule for industrial partner and coordinated model

Bid hour	Area price [EUR/MWh]	Industrial partner [MWh]	Coordinated model [MWh]
1	27.7	0	16
2	27.6	0	16
3	27.4	0	16
4	27.4	0	16
5	27.8	0	16
6	28.9	0	69
7	29.8	60	77
8	31.0	80	80
9	31.3	80	34
10	31.2	80	32
11	31.1	80	39
12	31.1	80	71
13	31.0	80	80
14	31.0	80	80
15	31.0	80	80
16	31.0	80	80
17	31.2	80	80
18	31.2	80	80
19	31.0	80	80
20	30.5	80	80
21	29.8	60	77
22	29.6	60	73
23	29.2	0	69
24	28.1	0	20

The fixed production points for the industrial partner is also evident in the bid curves for the balancing market, illustrated in Table 7.10. The bidding volume is mostly such that the turbine is regulated up 20 MW from best efficiency point to maximum capacity, or from zero production to maximum capacity, which ultimately requires a higher price. Equivalently for downward regulation, the industrial partner regulates down 20 MW from maximum capacity to best efficiency point, or 64 MW from maximum to minimum capacity. Table 7.10 also includes the bidding curves calculated by the coordinated model, using the bid curves for the day-ahead market as input along with the realized day-ahead prices. It can be seen that the bidding volumes calculated by the coordinated model are lower than the ones for our industrial partner, and the bids are accepted at lower prices. As we have modeled the production points as varying, this is as expected. The prices at which to bid are decided such that the total profit is maximized. The industrial partner is more cautious when bidding in the balancing market, which leads to their demand for better prices.

Table 7.10: Comparison of bid curves for the balancing market

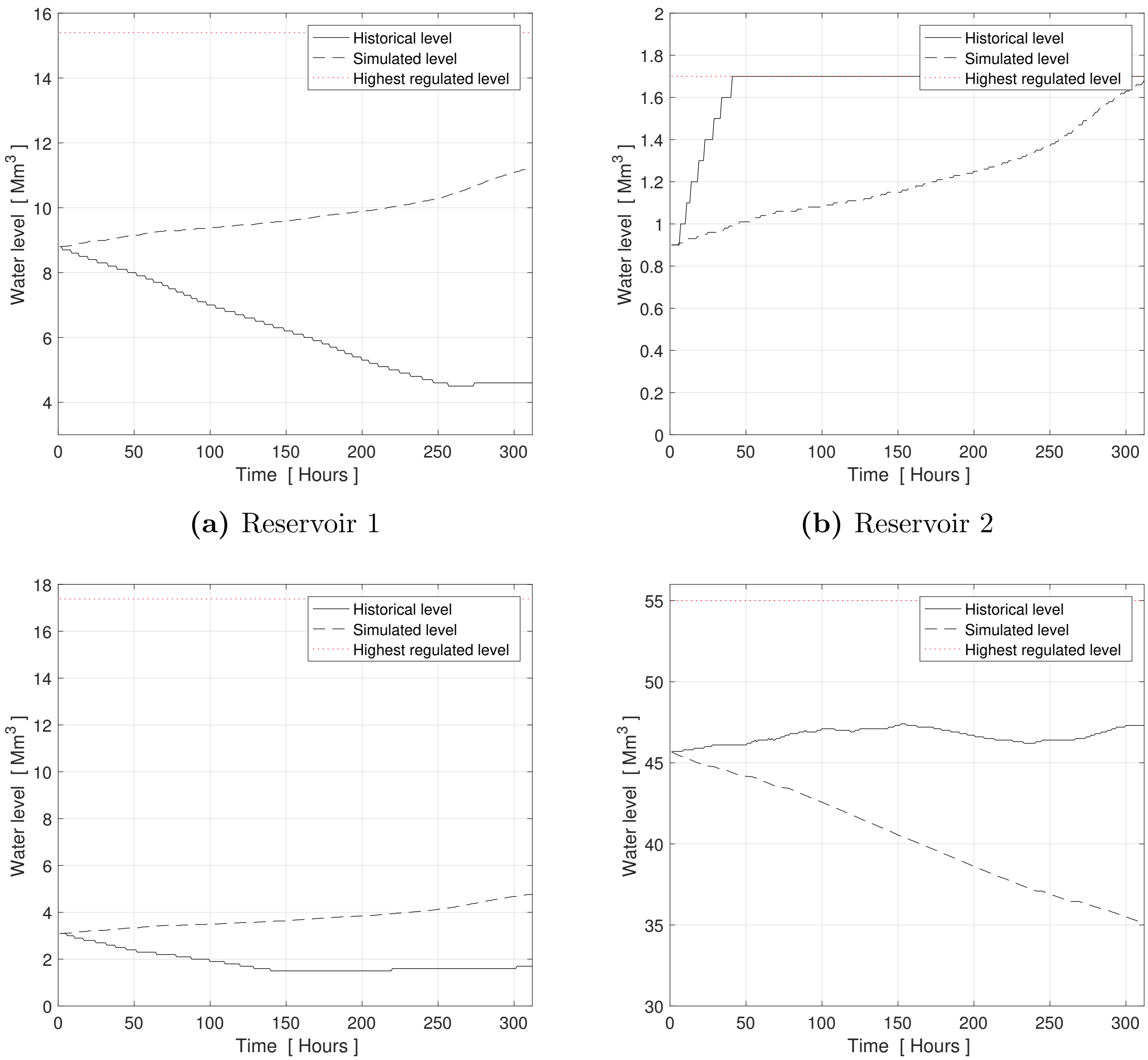
(a) Upward regulation

	Industrial partner		Coordinated model		
	Price points [EUR/MWh]		Price points [EUR/MWh]		
Bid hour	33	47	30	35	40
1	0	80	33	33	33
2	0	80	36	36	36
3	0	80	32	32	32
4	0	80	47	47	47
5	0	80	33	33	33
6	0	80	11	11	11
7	20	20	0	0	0
8	0	0	0	0	0
9	0	0	0	46	46
10	0	0	0	48	48
11	0	0	0	41	41
12	0	0	0	0	0
13	0	0	0	0	0
14	0	0	0	0	0
15	0	0	0	0	0
16	0	0	0	0	0
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	0	0	0	0	0
21	20	20	0	0	0
22	20	20	0	0	7
23	0	80	11	11	11
24	0	80	60	60	60

(b) Downward regulation

	Industrial partner		Coordinated model			
	Price points [EUR/MWh]		Price points [EUR/MWh]			
Bid hour	28	27	30	25	20	15
1	0	0	0	31	47	47
2	0	0	0	28	44	44
3	0	0	0	32	48	48
4	0	0	0	17	33	33
5	0	0	0	31	47	47
6	0	0	0	53	69	69
7	0	44	0	0	77	77
8	20	64	0	64	64	80
9	20	64	0	10	10	34
10	20	64	0	0	0	32
11	20	64	0	0	0	39
12	20	64	0	0	0	71
13	20	64	0	0	0	80
14	20	64	0	0	0	80
15	20	64	0	10	10	80
16	20	64	0	10	10	80
17	20	64	38	38	38	80
18	20	64	10	10	10	80
19	20	64	0	10	10	80
20	20	64	0	10	10	80
21	0	44	0	77	77	77
22	0	44	0	73	73	73
23	0	0	0	69	69	69
24	0	0	0	0	20	20

Figure 7.6 illustrates the development in the reservoir levels based on historical data and simulated levels from the coordinated model, for the whole planning horizon of 13 days. Keep in mind that the values calculated by the coordinated model is run one time only for the whole period, so the illustrated reservoir volumes are an expectation. It varies whether the industrial partner or the coordinated model runs the reservoirs lower. In reservoir one, which is the upper-most reservoir, the water level calculated by the model is substantially higher than the actual historical level. For the lowest reservoir, which is connected to the turbine, the water level calculated by the model is a lot lower than what our industrial partner had.



(a) Reservoir 1

(b) Reservoir 2

(c) Reservoir 3

(d) Reservoir 4

Figure 7.6: Simulated water level in the reservoirs compared to historical level

7.4 Comparison with Model Allowing Block Bids

Block bids are not included as an option for the coordinated model tested so far in the case study. As for the industrial partner, block bids are frequently used in their bidding strategy. It is thus interesting to investigate how the coordinated model behaves when allowing block bids, as introduced in Section 5.6.

A coordinated model allowing block bids in the day-ahead market is tested with realized market prices. The case study framework presented in Figure 7.4 is thus used. The option of block bids are only allowed during the bid hours, in the same way as the option of regulating up and down in the balancing market. The results of running the model is presented in Table 7.11, in which the results of running the model without blocks also are included. When comparing the results from the models, a small gain in profits is found by including block bids.

Table 7.11: Revenue when allowing for block bids in the bid hours compared to the model with only hourly bids in the bid hours

	Objective	Day-ahead Hourly (bid hours)	Day-ahead With block (bid hours)	Day-ahead (other hours)	Balancing (Up)	Balancing (Down)	Start-up	Water value
Block bids [EUR]	3 572 581	29 549	18 223	684 862	-	-3 817	-325	2 844 090
Hourly bids only [EUR]	3 572 322	45 121	-	684 862	-	-3 238	-325	2 845 902

The total production volume, and thus also the total revenue in the day-ahead market, increases when including the option of block bids. Block bids makes sure that the producer gets dispatched for all the hours in the block, if the average market price is high enough. We have not included this risk aspect for the producer, but it is worth noting.

For the given test day, block bids were frequently used in addition to hourly bids. Figure 7.7 illustrates the commitments from block bids only, for the three blocks included. The model commits a block of minimum capacity during the night hours. This prevents the producer from not being dispatched if the price in one night hour gets too low. In this way, the producer is sure to either produce the whole night without stopping, or produce nothing. Hence, maximum one start is necessary.

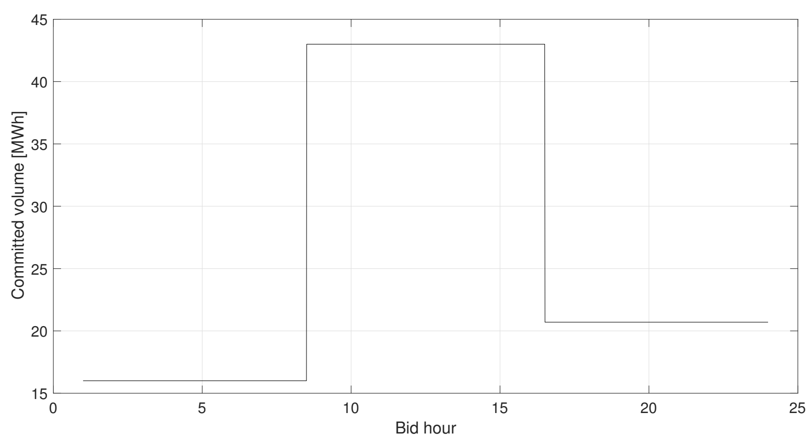


Figure 7.7: Production commitments from block bids in the bidding period

7.5 Discussion

The purpose of the analysis performed in this chapter is to assess the coordinated model, with a focus of how it behaves compared to a sequential bidding strategy. Our results shows that the coordinated model developed outperforms sequential bidding as one would predict, but that the improvements are marginal. The performance measurements carried out in the previous sections considers only one specific day, and so the results may have been different if the models were tested for different periods. As a deterministic equivalent is used, the quality of the stochastic input may affect the results. The generation of the outcomes have not been the main objective of this report. The focus is thus on the behaviour of the mathematical models, and not so much on the quantitative output.

The fraction of starts and stops of the turbine is one of the largest differences when comparing the behaviour of the coordinated model and the industrial partner. In addition to production in more hours than the industrial partner, the model is also using more water from the reservoirs. The largest differences are found in the lower reservoir closest to the power station. The model does not care about how much water that is left after the planning horizon. As the reservoir levels are quite high, the model wants to exploit the available water in the lowest reservoir. This is the reservoir in which there is highest risk of spillage, and as head effects are not taken into account, there is no problem with having a low water level in this reservoir. The industrial partner takes head effects into account, and would rather keep the water level in the lower reservoir high and send water from the upper reservoirs. As the inflow is modeled as deterministic, the model knows exactly how much water that can be kept in the reservoirs while still avoiding spill. However, the variation in the reservoir level for reservoir four turned out to be larger than expected. The assumption that the power output depends on the discharge only, and not the head effects, may not be as good as predicted. We have also assumed that the power-discharge from the historical average water level can be used, for all water levels. This affects the results, as in reality the producer accounts for the changing power-discharge curve as the reservoir level varies. The coordinated model also seems to be less sensitive to the efficiency curve than the industrial partner, as best point efficiency is seldom used. This leads to a higher water usage to extract the same amount of power, which may be why the coordinated model uses more water than the industrial partner. The industrial partner produces only at minimum capacity, best efficiency point, or maximum capacity.

Using a coordinated strategy seems to lead to a more aggressive bidding behaviour in the balancing market. A coordinated bidding strategy gains from reserving capacity for upward regulation, as it predicts the possibility of high premiums for the balancing market. For the given test day, with low premiums for upward regulation, the coordinated model did not regulate up any volume as planned. There was thus free capacity the model did not bid into the day-ahead market, due to an expectation of better possibilities in the balancing market for upward regulation. The realized prices for the day-ahead market and downward regulation, was however quite good. With average day-ahead prices above the water value, it was also easy for the sequential model to gain high profits from both the day-ahead market and downward regulation, without the use of coordinated bidding.

The obtained profits from running the coordinated model are proven to be quite sensitive to the water value. The coordinated model gives a gain while using the historical water value, but is outperformed by the sequential model with both a lower and a higher water value. There is thus a fine balance for when the coordinated model is profitable. With day-ahead prices far above or below the water value it may not be worthwhile to plan for the balancing market in the day-ahead bidding. If the water value is near the day-ahead price, it may however be profitable to use a coordinated bidding strategy.

The scenario generation tool presented in Section 6, does not generate possible outcomes for the available volume in the balancing market. The balancing market prices should ideally be directly connected to the available volume in the market. The deterministic market share parameter introduced in Chapter 5, may thus affect the results of the coordinated model. The chosen market share value of 25% was probably too high for the given test day, as the realized volumes in the balancing market were quite low. However, a sensitivity analysis has been done to investigate the results of changing the value of parameter. The analysis proves that the change in estimated gain is approximately zero, and thus the value of the market share does not have a large impact on the results of the coordinated model.

Chapter 8

Concluding Remarks

In this report, a three-stage stochastic programming model for the coordinated bidding problem in two sequential markets has been proposed. The two markets are modeled as the day-ahead market and the balancing market. The short-term production scheduling is included in the model as the third-stage decisions. We have put most effort in the formulation and implementation of the problem, including the technical input data from an industrial partner. A tool for generating possible market price outcomes based on historical forecast errors have been used, and so the modeling of the market price outcomes has not been the focus of this report. Our main objective has been to develop a coordinated bidding model, and comparing its behavior to a sequential bidding strategy.

8.1 Conclusion

The results from the case study indicates that the coordinated bidding model which is proposed, will indeed outperform a sequential strategy, given the tested input data. The coordinated model seems to bid more aggressively towards the balancing market. When including the opportunity of block bids, the model is however exploiting more of the opportunities in the day-ahead market.

The obtained price per volume produced for the coordinated model is higher when testing both strategies with uncertain values, but also for a given test day with realized prices. However, analyzing the results using different water values than the historical, indicates that the coordinated model is very sensitive to the estimation of the water values. If there is a large difference between the water value and the market prices, a coordinated strategy may no longer outperform a sequential strategy.

In the computational case study, it was revealed that the reservoir level for the lower reservoir varies more than predicted. This could affect the results as the head varies more than predicted. However, the storage capacity is large for the lower reservoir, and the reservoir is far from empty during the period.

The assumptions and simplifications for the model makes us able to develop a linear model, and solve it with a direct solution method. The model behaves reasonable compared to the behavior of our industrial partner, and the run time is short enough for daily use. However, several improvements to both scenario generation and the modelling can be done in the future to receive more realistic and significant results.

8.2 Future Research

To get a clear understanding of the possible gains of using a coordinated strategy, the proposed coordinated model should be tested for a longer period. Testing the model for only one day is not representative for the performance of the model over time. To be able to answer the question of whether the model is giving a significant gain for a hydropower producer, a simulation of the model should be conducted. This could be done by using the end state from the model as the starting state of the next day, and run for several consecutive days. The simulation model should be tested for different seasons. Furthermore, the gains should also be estimated when planning for a more complicated water system. As well as a better validation of the model, the simulation is important to evaluate the choice of modeling assumptions and simplifications in the model. Choices such as neglecting head effects, and the sensitivity of the water value must be investigated further.

To our knowledge, choosing to use fixed volume points on the bid curve and let the price points be variables, have not been evaluated before. As this is the practice of our industrial partner, it could be interesting to include this in the mathematical model to estimate the possible gains.

The focus of this report has been the correct behavior of the model, not the generation of correct input values for the stochastic parameters. The scenario generation method used, is not optimal for generating outcomes for the balancing market prices. The tool does not capture the dependency of the day-ahead and balancing market prices, nor the coupling between volumes and prices. Forecasts for the balancing market should be made in future work, and the coupling of the market price and available volume should be included. This would make the quantitative output more comparable to the industrial partner.

Appendix A

Model presentation

$$\max \sum_{s\in\mathcal{S}} \pi_s \Bigg(\sum_{h\in\mathcal{H}^B} \rho^S_{hs} w^S_{hs} + \sum_{c\in\mathcal{C}_s} \pi_{sc} \Big(\sum_{h\in\mathcal{H}^B} \sum_{m\in\mathcal{M}} U_m \rho^R_{hscm} w^R_{hscm} + \sum_{t\in\mathcal{T}} \big(\sum_{h\in\mathcal{H}\setminus\{\mathcal{H}^B\}} \rho^S_{hs} x_{thsc} - \sum_{h\in\mathcal{H}} C_t y_{thsc} \big) + \sum_{r\in\mathcal{R}} \big(W_r E\ (v_{|\mathcal{H}|rsc} - \underline{V}_r) - \sum_{h\in\mathcal{H}} Z s_{hrsc} \big) \Big) \Bigg)$$

s.t.

$$w^S_{hs} = \frac{\rho^S_{hs} - P^S_{k-1}}{P^S_k - P^S_{k-1}} u^S_{hk} + \frac{P^S_k - \rho^S_{hs}}{P^S_k - P^S_{k-1}} u^S_{h,k-1}$$

$$\text{if } P^S_{k-1} \le \rho^S_{hs} < P^S_k, \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K} \setminus \{1\}$$

$$u^S_{hk} \ge u^S_{h,k-1} \qquad h \in \mathcal{H}^B, k \in \mathcal{K} \setminus \{1\}$$

$$u^S_{hk} \le \sum_{t\in\mathcal{T}} \overline{Q}_t \qquad h \in \mathcal{H}^B, k \in \mathcal{K}$$

$$w^R_{hsc1} = u^R_{hs,k-1,1} \qquad \text{if } P^R_{k-1,1} \le \rho^R_{hsc1} < P^R_{k1}, \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, k \in \mathcal{K} \setminus \{1\}$$

$$w^R_{hsc2} = u^R_{hs,k-1,2} \qquad \text{if } P^R_{k-1,2} \ge \rho^R_{hsc2} > P^R_{k2}, \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, k \in \mathcal{K} \setminus \{1\}$$

$$u^R_{hskm} \ge u^R_{hs,k-1,m} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K} \setminus \{1\}, m \in \mathcal{M}$$

$$u^R_{hsk1} \le \sum_{t\in\mathcal{T}} \overline{Q}_t - w^S_{hs} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K}$$

$$u^R_{hsk2} \leq w^S_{hs} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K}$$

$$w^R_{hscm} \geq \underline{R}\delta^R_{hscm} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, m \in \mathcal{M}$$

$$w^R_{hscm} \leq \sum_{t \in \mathcal{T}} \overline{Q}_t \delta^R_{hscm} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, m \in \mathcal{M}$$

$$\sum_{m \in \mathcal{M}} \delta^R_{hscm} \leq 1 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$\sum_{h \in \mathcal{H}^B} w^R_{hscm} \leq \sum_{t \in \mathcal{T}} \overline{Q}_t |\mathcal{H}^B| S_m \qquad s \in \mathcal{S}, c \in \mathcal{C}_s, m \in \mathcal{M}$$

$$w^S_{hs} + \sum_{m \in \mathcal{M}} U_m w^R_{hscm} = \sum_{t \in \mathcal{T}} x_{thsc} \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$\underline{Q}_t \delta^T_{thsc} \leq x_{thsc} \leq \overline{Q}_t \delta^T_{thsc} \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$\delta^T_{thsc} - \delta^T_{t,h-1,sc} \leq y_{thsc} \qquad t \in \mathcal{T}, h \in \mathcal{H} \setminus \{1\}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$\delta^T_{t1sc} - \delta^{T_0}_t \leq y_{t1sc} \qquad t \in \mathcal{T}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$x_{thsc} = \underline{Q}_t \delta^T_{thsc} + \sum_{i \in \mathcal{I}} R_{ti} d^I_{thisc} \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$d^I_{thisc} \leq \overline{D}_{ti} \qquad t \in \mathcal{T}, h \in \mathcal{H}, i \in \mathcal{I}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$d_{thsc} = \underline{D}_t \delta^T_{thsc} + \sum_{i \in \mathcal{I}} d^I_{thisc} \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$v_{hrsc} = v_{h-1,rsc} + I_{hr} - \sum_{t \in \mathcal{T}_r} d_{thsc} - s_{hrsc} - b_{hrsc} +$$

$$\sum_{r' \in \mathcal{R}} \left(M^D_{r'r} \sum_{t \in \mathcal{T}_r} d_{thsc} + M^S_{r'r} s_{hr'sc} + M^B_{r'r} b_{hr'sc} \right) \qquad h \in \mathcal{H} \setminus \{1\}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$v_{1rsc} = V^0_r + I_{1r} - \sum_{t \in \mathcal{T}_r} d_{t1sc} - s_{1rsc} - b_{1rsc} +$$

$$\sum_{r' \in \mathcal{R}} \left(M^D_{r'r} \sum_{t \in \mathcal{T}_r} d_{t1sc} + M^S_{r'r} s_{1r'sc} + M^B_{r'r} b_{hr'sc} \right) \qquad r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$$\sum_{t \in \mathcal{T}_r} d_{thsc} \leq v_{hrsc} - \underline{V}_r \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s$$

$\underline{V}_r \leq v_{hrsc} \leq \overline{V}_r \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$\underline{B}_r \leq b_{hrsc} \leq \overline{B}_r \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$u^S_{hk} \geq 0 \qquad h \in \mathcal{H}^B, k \in \mathcal{K}$

$w^S_{hs} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}$

$u^R_{hskm} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, k \in \mathcal{K}, m \in \mathcal{M}$

$w^R_{hsc1} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s \mid \rho^R_{hsc1} > \rho^S_{hs}$

$w^R_{hsc2} \geq 0 \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s \mid \rho^R_{hsc2} < \rho^S_{hs}$

$\delta^R_{hscm} \in [0, 1] \qquad h \in \mathcal{H}^B, s \in \mathcal{S}, c \in \mathcal{C}_s, m \in \mathcal{M}$

$x_{thsc} \geq 0 \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$d_{thsc} \geq 0 \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$\delta^T_{thsc} \in [0, 1] \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$y_{thsc} \in [0, 1] \qquad t \in \mathcal{T}, h \in \mathcal{H}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$d^I_{thisc} \geq 0 \qquad t \in \mathcal{T}, h \in \mathcal{H}, i \in \mathcal{I}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$v_{hrsc} \geq 0 \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$s_{hrsc} \geq 0 \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s$

$b_{hrsc} \geq 0 \qquad h \in \mathcal{H}, r \in \mathcal{R}, s \in \mathcal{S}, c \in \mathcal{C}_s$

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