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DEPARTMENT OF INDUSTRIAL ECONOMICS AND

TECHNOLOGY MANAGEMENT

VALUATION OF GAS STORAGE

A REAL OPTIONS APPROACH

BY

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Preface

This thesis was written the spring term of 2003, and is the final thesis at the Department of Industrial Economics and Technology Management, Section for Managerial Economics and Operations Research, at the Norwegian University of Science and Technology.

Valuation of a natural gas storage facility is a complex and exciting challenge, with minor work previously done on the topic. It has been a great learning experience, and I have gained valuable insight in financial theory, and how it can be applied to analysing investment problems.

I would like to thank my supervisor at the Department of Industrial Economics and Technology Management, Associate Professor Stein-Erik Fleten, for valuable and encouraging comments throughout the process.

Stavanger, 9th of July 2003

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Summary

The main objective of this thesis was to investigate an investment opportunity in a natural gas storage facility. The valuation method used was a real options approach, which incorporates the flexibility and uncertainty of the storage facility operations into the valuation procedure. The gas storage facility analysed in this thesis was a salt cavern facility, also known as a high-deliverability storage. The main feature of this type of storage facility is its ability to inject and withdraw a large amount of gas in a short period of time. This feature makes it well suited to exploit changing gas prices and serve as an arbitrage mechanism.

The first step in the valuation procedure was to determine a stochastic spot price process to represent the evolution of the spot price. Based on an analysis of the price dynamics for natural gas spot prices, a simple mean-reverting process with constant volatility and mean reversion was chosen. The parameters of the stochastic process were determined using a combination of an ordinary least squares method and a maximum likelihood procedure.

The next step was to value a newly installed storage facility, using the suggested spot price process to simulate the operation of the storage facility for 15 years. In order to simulate the use of the facility, an optimal operation strategy was needed. The strategy should be able to decide whether it is optimal to inject gas into storage, withdraw gas from storage, or do nothing, depending on the spot price and which day of the year it is.

Using a backward recursion scheme, solving for the first order conditions of the value of gas in storage for all storage levels at every time step, the optimal strategy was established. The procedure was based on a technique called stochastic dual dynamic programming, and it solves for the marginal value of gas in storage. Simulated spot price paths were used to create a probability distribution of the spot price every day of the year. The probability distribution together with the operational constraints of the storage facility were fed into the model to produce the optimal decision for all volume levels in storage for all days.

The value of the storage facility was calculated with a constant volatility and mean reversion parameters, which is a major simplification when considering 15 years of operation. However, this simplification could be justified if the constant level reflects the mean value over these 15 years. The long-term level is difficult to predict, but a stochastic process for this value was suggested on the basis of the existing price data. This stochastic process of the long-term level should not be confused with the daily volatility of the spot price. The stochastic process of the long-term level was merely a “tool”

needed in the valuation of the option to invest. It was assumed that long-term volatility and mean-reversion will remain reasonably close to their present value, and the stochastic process was chosen accordingly.

When considering the option to invest in a natural gas storage facility, the optimal investment rule says that if the value of the expected cash flows from the project less the investment costs is larger than the value of the option to invest, the option should be exercised. The value of the option to invest in the storage facility was analysed with changing investment costs and uncertainty related to the long-term value of the volatility and mean reversion.

Different input values were used to analyse the investment opportunity. With a moderate view on the investment costs, the conclusion was generally that the investment should be made right away. With a more pessimistic estimate of the investment costs the option was not exercised, but the option value was significant.

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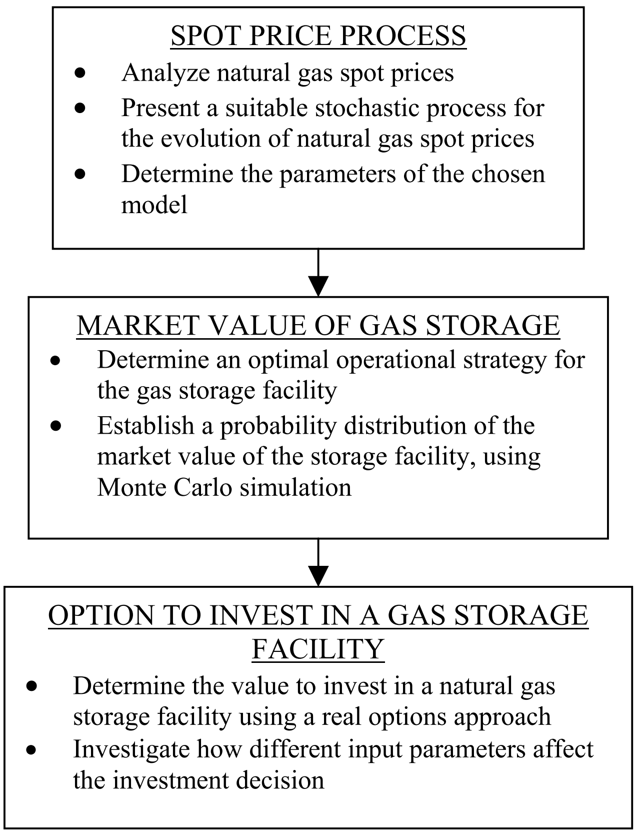
1 Introduction

The European gas market has in the last decade experienced some major changes, and the demand for gas is expected to continue to grow the next decades as well. The International Energy Agency (IEA[1]) predicts that the gas market in Europe will expand from an annual volume of 377 billion m^3 in 2000 to approximately 620 billion m^3 by 2020. Parts of these expanding gas supplies is destined for the residential and industrial sectors, but the power generation sector is predicted to increase the demand for gas the most. In this expanding gas market, gas storage facilities will play a significant role. During periods of high demand the key requirement is high-deliverability of natural gas. This cannot be achieved if the gas is supplied directly from the producing reservoirs, as transportation is limited by pipeline capacity. Storage facilities are the only significant supply regulator and demand buffer, as they are often situated close to the consumers of natural gas.

Gas storage facilities can be used for other purposes than securing supply in periods of high demand. Storage facilities with high deliverability, high injection and withdrawal capacity, can quickly respond to changing gas prices, and be utilized as an arbitrage mechanism. In this thesis, investment in a high-deliverability gas storage facility will be analysed. The analysis procedure can be summarized as follows



2 The financial gas market

In this chapter the general characteristics of the natural gas spot price will be presented. Based on these characteristics, a stochastic process describing the evolution of the spot price will be established later.

2.1 The market place

In this evaluation of an investment opportunity in a natural gas storage facility, market and price data from the International Petroleum Exchange (IPE) in London will be used. The IPE is the second largest energy futures exchange in the world, and has traded Natural Gas Futures since 1997. The traded volumes have increased rapidly, and the Natural Gas Futures are becoming a key indicator for the UK market. The IPE establishes a settlement price for the delivery of gas the following day, or the following days until the next trading day in case of weekends and holydays, for every day of the year. Figure 2-1 shows the daily settlement prices for natural gas deliveries for the period 26. June 1998 until 4. June 2003.

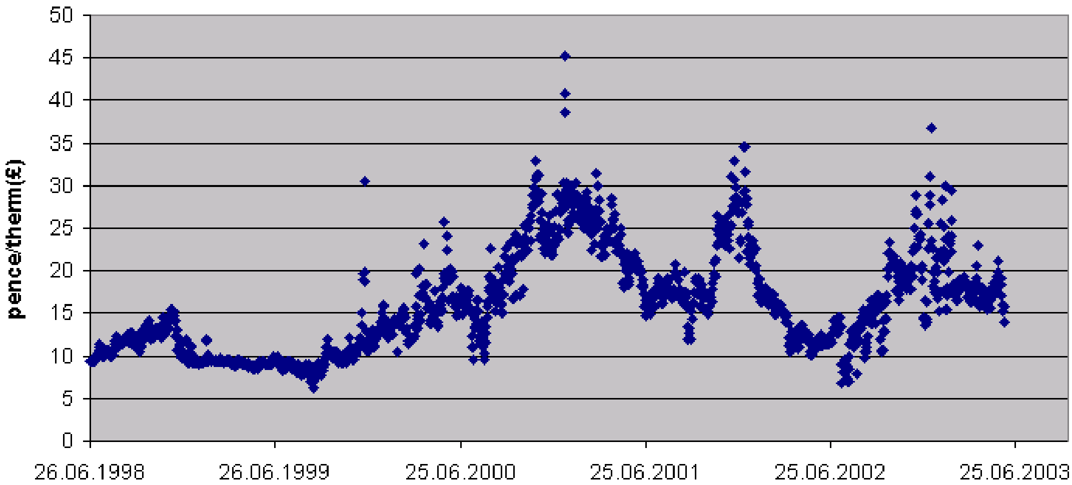


Figure 2-1 Daily settlement prices June 98 –June 2003 (Source: IPE)

The UK market is the most liquid market for natural gas in Europe, and the price data from the IPE will be used throughout this analysis. Although the traded volumes on the UK gas market have increased steadily since 1997 (see figure 2-2), some of the information contained in the historical prices may be considered irrelevant for this analysis. As figure 2-1 shows, the spot prices the two initial years were almost at a constant level, something one would not expect to see if the market was efficient and liquid. This will be further commented when a stochastic spot price process will be established later.

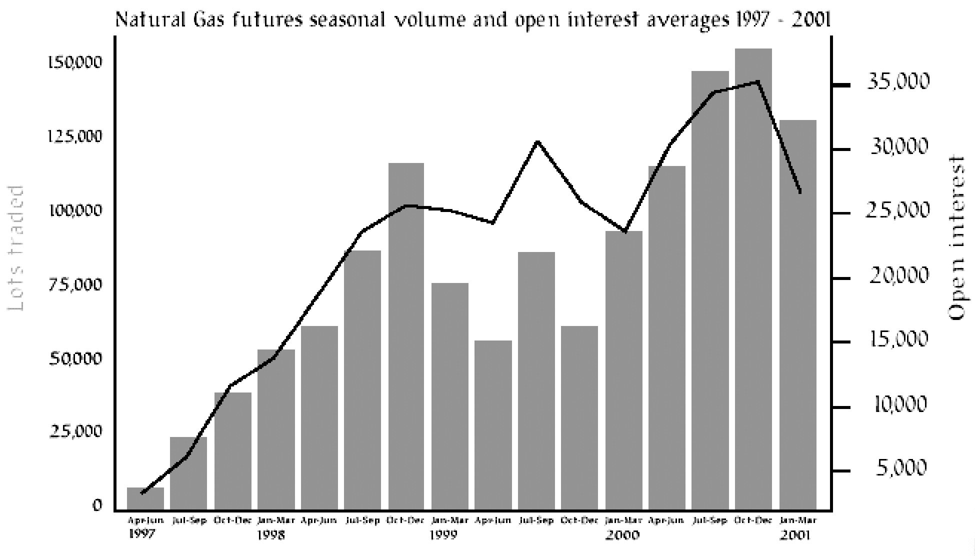


Figure 2-2 Natural gas futures traded volumes and open interest on the IPE (Source: IPE)

2.2 Factors that influence gas markets

There are several factors that may affect the evolution of the spot price, and some of the most significant will be presented in this section.

2.2.1 Demand

Economic growth

From a strong economy follows a growing demand for energy. The use of natural gas in Europe in commercial and industrial sectors has grown substantially in the last decade (figure 2-3). Also, a growing demand for electricity, coupled with a desire for cleaner burning fuel and more stringent environmental standards, has resulted in more natural gas being used to generate power.

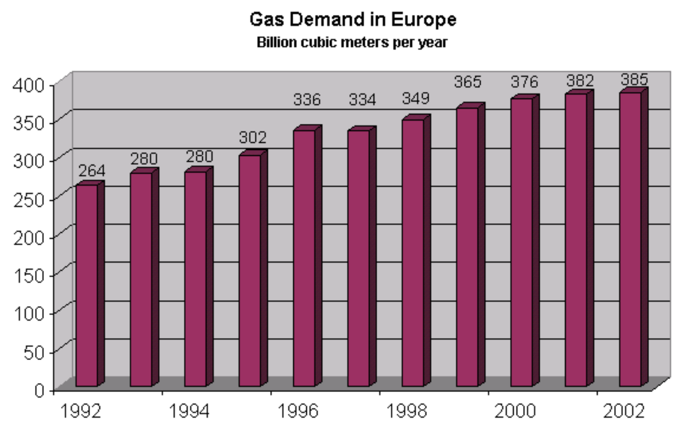


Figure 2-3 Gas Demand in Europe 1992-2002 (Source: BP-Statistical Review of World Energy, 2003)

Weather

The most important factor in determining short-term price movements is the weather (Sailor and Muñoz, 1997). When the temperatures drop, typically during winter, the demand for energy for heating purposes drives the natural gas spot price upwards. The opposite effect occurs during summer. The need for heating during summer months is minimal and the demand for gas is therefore low. The cyclical pattern of the temperature over the course of a year makes the natural gas spot price follow a similar pattern.

2.2.2 Competing fuel prices

Fuel switching is a temporary change from one fuel to another at a particular facility and acts to limit gas price increases. The choice of which energy form to consume is frequently based on relative prices, relative combustion efficiency, availability or security of supply, emissions and other considerations. The ability to switch energy form is of course often limited due to capabilities in the equipment. The most common dual-fuel combinations are natural gas and distillate fuel oils and natural gas and residual fuel oils. Dual-fuel equipment provides flexibility and promotes integrated pricing between fuel markets.

2.2.3 Natural Gas Storage

Underground natural gas storage inventories provide suppliers with means to meet customer requirements during the heating season, especially on peak demand days. The heating season for natural gas markets is considered the five-month period from November through the following March. The other seven months, April through October, become an inventory-building period called either the “non-heating season” or “refill season”. In addition to meeting winter demand loads, storage is also used for load balancing on pipeline systems, short-term “parking” of gas until it is needed, and to provide a physical hedge against price volatility.

2.2.4 Natural Gas Supply

The supply response to increased prices differs in the short run and the long run. As demand increases and causes prices to rise, the immediate response is an attempt to provide a larger volume from the existing wells and facilities. Companies with spare capacity generally respond promptly to opportunities for additional sales. As utilizations rise toward capacity limits, however, further supply increases become more difficult and costly. When utilization rates approach maximum levels, supply cannot adjust to increased demand and market adjustments primarily result in price increases.

Beyond higher utilization from existing wells, further increases in supply require the drilling of new wells. However, a longer term increase in supply requires time for activities such as securing investment capital, acquiring land, planning drilling programs, preparing sites, hiring and training personnel, and developing additional infrastructure. In other words, there is usually a lag in time before production from new wells is brought onstream in response to price signals.

2.2.5 Market Psychology

One of the factors that can influence the price of natural gas either in the short or the long-term is the psychology of the market in reacting to the above noted drivers (factors), including drivers such as availability of pipeline transportation capacity. In examining the price of natural gas and how natural gas markets are working, it is important that market psychology be taken into consideration. Market psychology is very difficult to measure as it is an interpretation by traders and analysts of events that may often lead to a price level that otherwise would not have been expected.

In examining the price movements of natural gas, it is observed that prices may at times appear to overreact to a specific driver both in the short and longer term by moving either higher or lower than would be expected by the specific event (National Energy Board, 2002). For example, markets that anticipate the construction of additional pipeline capacity to remove a capacity bottleneck may see the price of natural gas decline significantly prior to the actual in-service date of the additional pipeline capacity. Similarly, the market may overreact to the announcement of a potential severe weather pattern sending prices higher.

2.3 Gas as a commodity

When analysing the gas spot price characteristics, it is important to understand the underlying concepts of gas as a commodity. Gas is a consumption asset, as opposed to an investment asset such as stocks and other financial instruments (Hull, 2000). In this section the main differences between consumption assets and investment assets will be highlighted.

2.3.1 Delivery

Commodities, or consumption assets, require physical delivery. The delivery process can be both time-consuming and expensive. Consider buying aluminium, sugar or oil. This requires transportation with boats, vehicles or pipelines, which imposes quite substantial costs. Investment assets on the other hand, have practically zero transportation cost because they are mostly paper assets.

Gas is mostly distributed through pipelines, and pipeline capacity may cause delivery problems and hence affect prices. The gas prices reflect the point of delivery, i.e. prices observed in the UK market reflects that physical delivery takes place within the UK natural gas grid at the National Balancing Point (NBP).

2.3.2 Convenience yield and storage cost

Convenience yield defines the benefits that accrue to the holder of the asset (Hull, 2000). The benefits may include the ability to keep production running when there are supply shortages, or just benefit from the high prices when prices rise. However, the benefits of holding the asset are often offset from storage costs. For commodities such as steel, these storage costs are often substantial. Holders of investment assets are neither influenced from convenience yield nor storage costs, and return requirements are of course independent of such.

Natural gas storage

The limited storage possibilities of gas are a significant factor when analysing natural gas spot prices. Gas is consumed at households and other consumers spread over a large area. To prevent the large seasonal changes in prices, the consumer would have to have their own storage unit. This would also erase weekly price fluctuations and price shocks that occur on days of sudden large demand and low supply. However, this is not possible because of the large investment costs in such storage units. Instead, gas must be consumed almost immediately after being produced. "Almost" because of the fact that the pressure in pipelines can be adjusted, and that there are some large central storage facilities. However, these adjustment options serve mostly to secure the distribution, and therefore it can be stated that gas consumption must equal gas production (including shifting in storage inventories) at all times.

2.3.3 General price behaviour

Seasonality

Seasonality in the prices is observed in the spot prices due to the changing demand scenario following the weather climate. During winter, as the temperature drops, the demand for heating of households causes the prices of gas to rise. During summertime, this heating is not necessary, and the prices drop. This general observation is observed in the historical prices, where the winter prices are high during winter and low during summer. In other parts of the world, like in the southern states of USA, the summertime temperatures are so high that energy is required for cooling with air condition. This makes the price pattern exhibit two peaks, one during winter and one during summer, with low prices in the months between.

Weekly pattern

The demand for gas is influenced by the general work activity. Figure 2-4 shows the mean daily prices for gas from 1. January 2000 until 17. March 2003. As the figure shows, during working days (Monday-Friday) the price is significantly greater than during the weekend. The emergence of gas-fired power plants has contributed to increase this effect. Gas-fired power plants are peak-demand units and they absorb the peak electricity demand generated around mid-day during weekdays, hence increasing the gas demand in these periods.

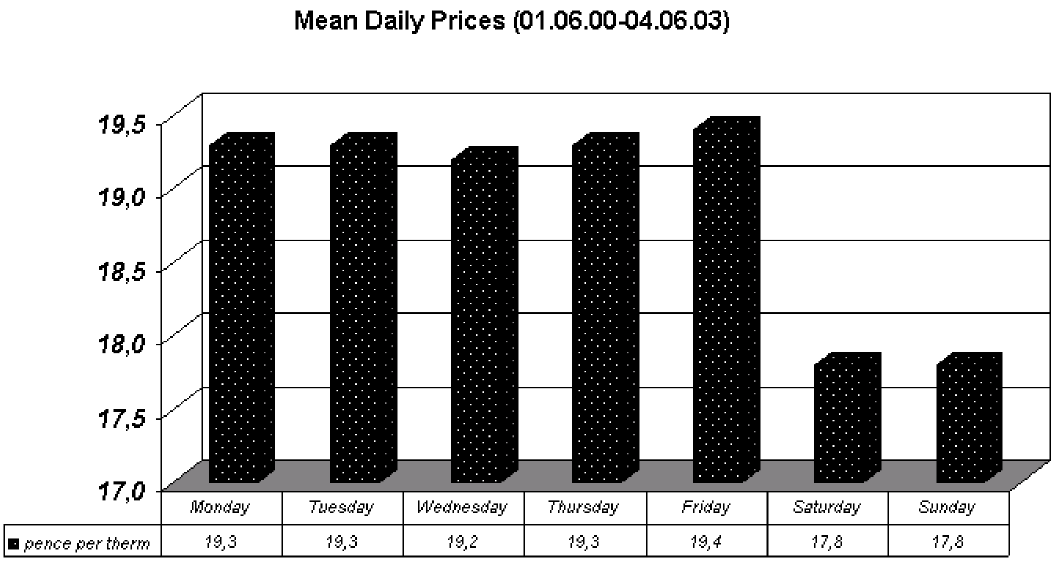


Figure 2-4 Mean daily settlement prices

2.3.4 Equilibrium prices

Natural gas can be considered as a homogenous product transacted in a competitive market. In an efficient market, prices will be driven to marginal cost if demand equals supply (Dixit and Pindyck, 1994). The volatile prices observed in the market are highly correlated with volatility in consumption and production, but with storage available, market-clearing price is determined not only by current production and consumption, but also by changes in the storage level. Prices will then reflect the short-run marginal cost of gas plus the opportunity cost of capacity. For natural gas, as for most commodities, sharp increases in price volatility occur from time to time. These spikes are induced by factors such as extreme weather conditions, distribution problems due to temporary pipeline shutdown or shortage of supply due to temporary production shutdown. These spikes are not sustainable and the demand/supply reverts back to normal levels within a short time.

2.4 Gas Price Volatility

The term price volatility is used to describe rapid fluctuations of a commodity. In analytical terms it is known as numerical variance, or more precisely, standard deviation from a "mean" or average price trend. Natural gas price volatility is measured by the day-to-day percentage difference in its price. The degree of variations defines the volatility of the market.

Volatility of gas, and commodities in general, are quite a bit higher than that for many financial instruments. Among several causes of gas price volatility, the most significant can be attributed to inelasticity in supply and demand. Demand is closely related to weather, and as the weather changes the prices changes to. Production and infrastructure constraints may cause shortage of supply, either locally or generally, and shift the supply/demand balance. Technical trading and market imperfections can also cause volatility.

2.5 Mean Reversion

Mean reversion is a characteristic observed for many energy commodities (Hull, 2000). Mean reversion describes a tendency for the spot price to revert back to a long-term level. This feature is observed because supply characteristic of gas producers is highly price responsive. The supply characteristic is mainly a function of generation technology, fuel costs, availability of generation and the possibility of import/export. For instance, when the gas price is high, gas consumers will try to find a substitute energy source. This will decrease the demand for gas, and the price will start to level off or drop. The opposite will occur if the gas price is low. If this happens, consumers of alternative sources of energy may convert to gas due to favourable prices. This will again result in higher demand and higher prices. The ability to change energy source may not be available to all consumers because this flexibility has its price, but many large energy-consuming companies have this ability .

3 Theoretical framework

In this chapter some of the underlying theory of the valuation procedure used in this thesis will be presented.

3.1 Stochastic processes

In the following analysis some mathematical descriptions of how asset prices evolve through time will be used, and they are introduced here. These mathematical descriptions are known as stochastic processes, which imply that the variable evolves with some sort of randomness.

3.1.1 The Wiener process

A wiener process, also called a Brownian motion, is a continuous-time process with three important features:

- It is a Markov process, which implies that the probability distribution for the next time-step is only dependent on the present state. What has happend in the past is irrelevant.

- It has independent increments, which means that the next change in the process is independent of previous changes.

- The changes are normally distributed over any finite time interval.

A pure Wiener process will have the following mathematical description (Hull, 2000):

$$dz = \varepsilon\sqrt{dt} \qquad (3.1)$$

where dz is the Wiener process, ε is a normally distributed value with mean zero and standard deviation of one and dt is a small time increment.

3.1.2 Geometric Brownian motion

The Wiener process is considered unsuitable for describing the evolution of any security price, because it only has a “noise” term and no drift term. The geometric Brownian motion (GBM) assumes that proportional changes in the asset price, denoted by S, are to have constant instantaneous drift, μ, and volatility, σ. The mathematical description of this property is given by the following stochastic differential equation (Hull, 2000)

$$dS = \mu S dt + \sigma S dz \qquad (3.2)$$

Here dS represents the increment in the asset price during the time interval dt and dz is the Wiener process. The second term of the equation represents the "noise" term, while the first term represents a drift term

3.1.3 A mean-reverting process

It was argued in the previous chapter that commodity prices have a tendency to revert back to a long run equilibrium price. In other words, while in the short run the price of gas might fluctuate randomly up and down, in the long run it ought to be drawn back towards the marginal cost of producing and transporting gas (Dixit & Pindyck, 1994). A geometric Brownian motion have a tendency to wander far from their starting point, which according to the arguments above makes it unable to describe the evolution of gas prices. A mean-reverting process might be better suited for gas prices, and other commodities as well. The simplest mean-reverting process is an Ornstein-Uhlenbeck process

$$dx = \eta(\bar{x} - x)dt + \sigma dz \qquad (3.3)$$

Here, η is the speed of mean reversion, and $\bar{x}$ is the "normal" level of x, the level to which x revert back to. dz is the Wiener process and σ is the standard deviation. In this process the change in x is not independent of its present state. If x is greater than the long run mean, the drift term will pull x down toward the mean value, while if x is less than the long run mean, the drift term will push x up towards the mean value.

3.2 Ito's lemma

The contingent claims analysis requires that an asset in the existing economy can span the stochastic changes in V, the payoff from the investment. Suppose you have an option on a stock. The payoff from the option, G(x), would be dependent of the stock price. Suppose again that you know that the stock price, x, follows a geometric Brownian motion

$$dx = a(x, t)dt + b(x, t)dz \qquad (3.4)$$

When the process of x is known, Ito's lemma tells us the stochastic process followed by some function G(x, t). Here G(x, t) is the value of the option.

Ito' lemma uses the first Taylor series expansions of the function with respect to time, t, and the two first Taylor series expansions of the function with respect to the underlying

$$dG = \frac{\partial G}{\partial t}dt + \frac{\partial G}{\partial x} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}dx^2 \qquad (3.5)$$

If the stock price follows equation 3.4, the option will follow the stochastic process

$$dG = \{\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\}dt + \frac{\partial G}{\partial x}b\,dz \qquad (3.6)$$

With a(x, t) = μx and b(x, t) = σx the result yields

$$dG = (\mu + \frac{1}{2}\sigma^2)dt + \sigma dz \qquad (3.7)$$

3.3 Valuation methods

Real options theory is founded on the same principles as financial options: Real options, like financial options, give an owner the right, but not the obligation, to take action. Real options, unlike financial options, require ownership of real, tangible assets. The traditional discounted cash flow approach underscore the utility of real options analysis(Brealey & Myers, 2000). Single-shot DCF valuations use information that is currently available. Uncertainty, in this modelling framework, translates into a very high discount rate, which yields low or negative net present value. In contrast, real options analysis simplifies the assumptions- probabilities and discount rates are no longer arbitrary. Under an option-based framework, all possible outcomes are assessed and automatically adjusted to reflect the underlying risk profile. This is reflected in the option value, which increases with uncertainty, by quantifying the benefits from managerial/operational flexibility.

How should a firm, facing uncertainty over future market conditions, decide whether to invest in a project or not? The traditional discounted cash flow approach uses the net present value (NPV) to determine whether or not the investment should be undertaken. First, calculate the present value of the expected stream of profits that the project will generate. Second, calculate the present value of the stream of expenditures required to complete the project. Finally, determine whether the difference between the two, the net present value of the investment, is greater than zero. If it is, go ahead and invest. The net present value approach is based on some implicit assumptions that are often overlooked. Most important, it assumes that the investment is either reversible or a now-or-never proposition. If the investment is reversible the expenditures can be recovered should the market conditions turn out to be worse than anticipated. If the investment is a now-or-never proposition the investment must be made immediately or never at all. Some investments meet these conditions, but most do not. Irreversibility and the possibility of delay are very important characteristics of most investments in reality. The ability to delay irreversible investment expenditure can have a significant affect on the decision to invest.

The traditional approach to project evaluation and investment decisions uses discounted present value (DPV) or discounted cash flow (DCF) methods. These methods explicitly assume that the project will meet the expected cash flow with no intervention by management in the process. All the uncertainty is handled in the (risk-adjusted) discount rate. This process is static. At most, the expected value of cash flow is incorporated into the analysis. Management's flexibility to make decisions as states of nature are revealed is assumed away by this methodology. However, management discretion has value, which is not incorporated into the DCF. The real option methodology goes beyond this naive view of valuation and more closely matches the manner in which firms operate. It allows for a firm's flexibility to abandon, contract, expand or otherwise modify its actions after nature has revealed itself.

Real options take these managerial flexibilities in to account when valuing investments(Trigeorgis, 2000). The real options methodology measures the value inherent in the ability to dynamically react to changing market conditions. The methodology is a means of capturing the flexibility of management to address uncertainties as they are revealed. Capital budgeting fails to account for this flexibility and to integrate the flexibility with strategic planning. The flexibility that management have includes: defer, abandon, shut down and restart, expand, contract and switch use.

Real options methodologies can take the best features of DCF and other methodologies like decision-tree analysis (DTA) without their failings. The intuition is simple, but profound – management's decisions skew the distribution of possible outcomes toward the upside. Real option method can make a significant difference in the valuation. It expands the notion of the manager's flexibility and strategic interaction in skewing the results of the traditional DCF analysis that, as with financial options, allows for gains on the upside, and minimizes the downside potential, thus increasing the valuation. Strategic considerations are magnified or made explicit by the analysis. Viewed in light of traditional economic theory, real options methodology suggests that the traditional theory needs re-evaluation.

3.3.1 Contingent claims analysis

Contingent claims analysis is the "new" approach to project valuation. This procedure assumes that any contingent claims on an asset, whether traded or not, can be priced in a world with systematic risk by replacing the expectation of cash flow with a certainty-equivalent rate(by subtracting a risk premium that would be appropriate in market equilibrium) and then behaving as if the world were risk neutral (Trigeorgis, 2000). CCA assumes a risk neutral world, where investor preferences are irrelevant for the solution of the problem. This risk-neutrality facilitates the handy use of the risk free rate of return for discounting, instead of any risk-adjusted discount rate.

Contingent claims analysis builds on ideas from financial economics. Begin by observing that an investment project is defined by a stream of costs and benefits that vary through time and depend on the unfolding of uncertainty events. The firm or the individual of the firm that owns the right to the investment opportunity, or the stream of operating profits from a completed project, owns an asset that has a value. A modern economy has markets for quite a rich menu of assets of all kinds. If our investment project or opportunity happens to be one of these traded assets, it will have a known market price. However, even if it is not directly traded, one can compute an implicit value for it by relating it to other products that are traded. All one needs is some combination or portfolio of traded assets that will exactly replicate the pattern of returns from our investment project, at every future date and in every future uncertainty eventuality. Then the value of the investment project must equal the total value of that portfolio, because any discrepancy would present an arbitrage opportunity: a sure profit by buying the cheaper of the two assets or combinations, and selling the more valuable one. Implicit in this calculation is the requirement that the firm should use its investment opportunity in the most efficient way, again because if it did not, an arbitrager could buy the investment opportunity and make a positive profit. Once we know the value of the investment opportunity, we can find the best form, size, and timing of the investment that achieves this value, and thus determine the optimal investment policy.

Consider an investment opportunity where the problem is when to pay a sunk cost I in return for a project whose value is V. This problem was first discussed by McDonald and Siegel (1986) and later by Dixit and Pindyck (1994).

Suppose that the project value evolves according to the following geometric Brownian motion

$$dV = \alpha V dt + \sigma V dz \qquad (3.8)$$

The decision to invest in this project is equivalent to a perpetual call option. Deciding to invest in the project is therefore equivalent to deciding when to exercise the option.

Dixit & Pindyck obtained the solution of the value of the option to invest, F(V), and this solution will be presented here and later used to value the option to invest in a natural gas storage facility.

Consider the following portfolio: Hold the option to invest, which is worth F(V), and short n = F'(V) units of the project. The value of this portfolio is

$$\Phi = F - F'(V)V \qquad (3.9)$$

The short position in this portfolio will require a payment of $\delta V F'(V)$. The holder of the long position will demand the risk-adjusted return μF, which equals the capital gain αV plus the dividend stream δV. The holder of the short position holds F'(V) unit of the project, and will have to pay out δVF'(V). Now, the payoff from the holding the portfolio over a short period of time dt is

$$dF - F'(V)dV - \delta VF'(V)dt \tag{3.10}$$

Applying Ito's lemma gives

$$dF = F'(V)dV + \frac{1}{2}F''(V)(dV)^2 \tag{3.11}$$

The return from the portfolio now becomes

$$\frac{1}{2}F''(V)(dV)^2 - \delta V F'(V)dt \tag{3.12}$$

Using equation 3.8 for dV ($(dV)^2 = \sigma^2 V^2 dt$). The return of the portfolio now becomes

$$\frac{1}{2}\sigma^2 V^2 F''(V)dt - \delta V F'(V)dt \tag{3.13}$$

This return is risk-free, and to avoid arbitrage opportunities, the return must equal the $r\,\Phi\,dt = r(F - F'(V)V)dt$:

$$\frac{1}{2}\sigma^2 V^2 F''(V)dt - \delta V F'(V)dt = r(F - F'(V)V)dt \tag{3.14}$$

Rearranging and dividing by dt finally gives the differential equation F(V) must satisfy

$$\frac{1}{2}\sigma^2 V^2 F''(V) + (r-\delta)V F'V - rF = 0 \tag{3.15}$$

Applying the appropriate boundary conditions, which will be presented later when the option to invest in a natural gas storage facility is determined, yields the following solution for F(V)

$$F(V) = AV^{\beta_1} \tag{3.16}$$

The optimal investment rule now states that the option should be exercised, and the investment made, if the value of the project less the investment cost (V-I) is larger or equal to the option value, F(V).

$$\text{Invest if:} \qquad V - I \geq F(V)$$

4 Natural Gas storage

Gas storage is set to play an increasingly important role as the European gas markets open up. Storage has traditionally been used to ensure system security and reliability. However, it is now starting to be used for more commercial and trading purposes. In this chapter the different components determining the value of a gas storage facility will be discussed. But first, the different types of storage facilities are presented.

4.1 Natural gas storage types

Gas is most commonly held in inventory underground under pressure in three types of facilities:

- Depleted reservoirs
- Aquifers
- Salt caverns

Each type has its own characteristics, including physical characteristics such as porosity, permeability and retention capability, and economics such as site preparation cost, deliverability rates and cycling capability. Each type has characteristics that govern different application requirements.

Depleted reservoirs
Natural gas or oil fields can be converted to storage duty after the production has stopped. This allows for the continued use of existing wells, gathering systems and pipeline connections, which is a significant advantage when it comes to investment costs. Other advantages are that the reservoirs geology is well known and that it usually has high deliverability. One of the disadvantages is that these facilities often require that as much as 50 % of the total capacity must be kept as base gas. Base gas is the amount of gas that needs to be kept in the reservoir to maintain the pressure support and secure structural integrity, or the gas that cannot be economically removed. These facilities are usually restricted to be cycled once per year, and therefore most commonly offer seasonal service.

Aquifers
Aquifer storages are geological formations that originally contained water, and are converted to gas storage reservoirs. An aquifer is suitable for gas storage if the water-bearing sedimentary rock formation is overlaid with an impermeable cap rock. The rock formation has high deliverability, which allows the working gas to be cycled several times per year. Working gas, or top gas, is the amount of gas that can freely be withdrawn or injected into the reservoir. Unfortunately, aquifers also need a large portion of base gas, often as much as 80%, which generally are impossible to recover. The fact that the geological conditions are untested, as opposed to depleted reservoirs, gives a major risk of

experiencing substantial reservoir leaks. Aquifers are generally the least favourable type of storage facility.

Salt caverns

Salt caverns are constructed within geologic structures called salt domes. To form a salt cavern, one must drill several hundred meters down into a salt formation, and wash the cavern to the appropriate size. The formations get filled with injected gas, and basically act as high-pressure storage tanks. Salt caverns require the least amount of base gas of the different storage types, and also have the highest deliverability. As a result, these facilities can cycle the gas up to four or five times a year. High investment costs and limited capacity are the main disadvantages.

4.2 Need for natural gas storage services

The ability to use storage economically has long been recognized as an important difference between gas and electricity industries (Bates & Fraser, 1974). Storage enables better co-ordination between supply and demand so that a constant supply can be better fitted to a varying demand. Storage permits a higher level of consumer benefit, not only because it permits utilization of off-peak capacity, which might otherwise lie idle, but also because it enables a degree of arbitrage, transferring consumption from the period in which its value is low to where it is higher, to an extent determined by the cost of storage (Rees, 1984).

Traditionally the demand for natural gas has been seasonal, with the demand being higher during the winter months. Corresponding to this highly seasonal pattern, the storage facilities have injected gas during spring and summer, when supply exceeds demand, and withdrawn during winter to meet peak demand. In later years, gas-fired power plants have increased the demand during summer months as well. Such plants are peak-demand units and they absorb the peak demand generated around mid-day during the summer months. According to the fluctuations of electricity load, the demand for gas now swings from day to night and from weekday to weekends. The key requirement during periods of high demand is reliable high-deliverability of natural gas. Gas supplied directly from reservoirs cannot satisfy this requirement. Natural gas storage facilities offer the only significant supply regulator and demand buffer. Reflecting the change in demand patterns during the recent years, the largest growth in daily withdrawal capability has been from high-deliverability storage sites, which are mainly salt cavern storage reservoirs.

4.3 Value components of gas storage

When valuing a natural gas storage facility, its major value determinants must be determined. In short, natural gas storage facilities have two main value components. First, they serve as an arbitrage mechanism to exploit the time spread of gas prices. The simple principal is to sell high and buy low.

Second, they are assets with inherent operational flexibility. This operational flexibility accounts for optionality that enhances the value of the asset.

Along with the trend in natural gas markets towards storage facilities with high deliverability capabilities, such a facility will be evaluated in this analysis. This type of storage facility will mainly have implications of a financial character (optionality), as opposed to a physical character (delivery reliability, etc), and its major value components will be accordingly.

4.3.1 Seasonal price spreads

The natural gas prices exhibit a seasonal pattern due to the changing demand for gas during the winter months and the summer months. Producers and end-users have traditionally used storage to capture this price difference, and the value of injecting gas during summer and withdrawing it during the winter is the price difference less the time value of money and transaction costs

4.3.2 Gas price volatility

Gas price volatility cause prices to fluctuate, and the more they fluctuate the more the facility can exploit the changing prices. Volatility and seasonal price spreads are the two most significant factors when valuing a gas storage facility.

4.3.3 Operational constraints

The operational constraints specify the optionality of the storage facility. The operational constraints are primarily the injection and withdrawal rates, aside from the working volume of the facility. The higher the injection and withdrawal rates are, the better positioned the facility is to exploit changing spot prices.

4.3.4 Operations costs

Operations costs are a major determinant of gas storage value. Storage facilities make money from buying low and selling high. The higher the operations costs are, the bigger difference between the selling price and buying price is needed to secure profit. Higher operations costs reduces the facility's flexibility, and hence the value of the facility.

4.3.5 Market rules

Market rules are similar to operations costs as they reduce the flexibility of the facility. The more rules that need to be followed, the less value the facility got. Market rules could be limitations on the amount of gas that can be withdrawn or injected on a given day, tax rules and so forth.

4.3.6 Other

Other value components can be deliverability, pipeline operation and price management. These will not be analysed in this thesis

5 The spot price process

In order to determine the value of operating a natural gas storage facility, and subsequently the option to invest in such, a model to describe the evolution of the natural gas spot price is needed. In this chapter a stochastic process to describe the spot price is presented and the appropriate parameters are estimated.

5.1 The stochastic process

After analysing the spot prices for natural gas, the following important features are recognized:

- Cyclical patterns over the course of a week and a year
- A slow mean reversion, reverting back to equilibrium prices
- Price spikes or fast mean reversion due to extreme conditions
- Long term factors such as emission costs, climate changes, currency exchange rates and CO_2 taxes may influence the price

A natural approach would be a two- or multifactor model where both the short and long-term uncertainty could be handled. However, in this analysis a single factor model is chosen for the sake of simplicity. The occurrence of price spikes is ignored, as are the long-term factors such as emission costs, climate changes, exchange rates and CO_2 taxes. The complexity of the model increases significantly with the number of factors, and in this analysis a single factor model is considered sufficient.

The natural gas spot price dynamics have a close resemblance to the electricity spot price dynamics. The features presented above; cyclical patterns, equilibrium prices and price spikes due to extreme conditions, are relevant for both commodities. Johnson and Barz (1999) analysed how different stochastic equations managed to model the spot price for different electricity markets. For the Nordic electricity market they suggested a model with mean reversion and jumps to be best suited, followed by a pure mean reverting process. These processes are both capable of describing the natural gas spot price in a sufficient manner. As jumps are ignored in this analysis, the mean reverting process, which is an Ornstein-Uhlenbeck process, is chosen. The suggested model is as follows

Mean reversion, OU: $$dP_t = \kappa\ (\alpha_t - P_t)\ dt + \sigma\ dW_t \qquad (5.1)$$

where P_t is the spot price of gas, α_t is a “long-run mean”, σ is the volatility, κ is the speed of mean reversion and W_t is a Wiener process.

Lund and Ollmar (2002) presented a method for determining the parameters of the model above, and the following calculations is based on this method. To model the weekly and seasonal changes, a time dependent mean is required. Trigonometric functions allow easy incorporation of high prices during winter and low prices during summer. The model is separated into two components, one for the seasonal changes and one for the weekly changes. This is practical when the mean reversion and volatility parameters are estimated. If these components are not separated the slow mean reversion due to seasonal change and the fast mean reversion due to weekly change would get mixed up. This would result in a volatility that is so high that the weekly pattern would vanish and a volatility that is too low to model the large deviation from the long-term mean. The price process will be described by

$$P_t = X_t + D_t \tag{5.2}$$

where X_t represents the seasonal changes and D_t represents the weekly changes. Further the changes in X_t can be specified as

$$dX_t = a_t(b_t + \frac{b'_t}{a_t} - X_t)dt + \sigma_t dW_t \tag{5.3}$$

where a_t is the speed of mean reversion due to long term effects, b_t is the normal seasonal price, $b_t^{'}$ is the derivative and σ_t is the price volatility. The normal seasonal prices, b_t ,and D_t are specified by a sum of trigonometric functions.

$$\begin{aligned} b_t &= b_0 + \sum_{j=1}^{k} R_j^X \cos(w_j^X t + \phi_j^X) \\ &= b_0 + \sum_{j=1}^{k} \{A_j^X \cos(w_j^X t) + B_j^X \sin(w_j^X t)\} \end{aligned} \tag{5.4}$$

$$\begin{aligned} D_t &= d_0 + \sum_{j=1}^{l} R_j^D \cos(w_j^D t + \phi_j^D) \\ &= d_0 + \sum_{j=1}^{l} \{A_j^D \cos(w_j^D t) + B_j^D \sin(w_j^D t)\} \end{aligned} \tag{5.5}$$

where $A_j = R_j \cos(\phi_j)$, $B_j = -R_j \sin(\phi_j)$, w is the frequency, ϕ is the phase, R is the amplitude and b_0 is a constant level. The constant d_0 will be used to ensure that the process D_t starts of at zero every week. By choosing appropriate values for the amplitudes, phases and frequencies, the yearly and weekly price patterns can be modelled.

The explicit solution to the price process, P_t , is given by

$$P_t = (P_s - D_s - b_s)e^{-\int_s^t a_u du} + D_t + b_t + \int_s^t \sigma_u e^{-\int_u^t a_r dr} dW_u \tag{5.6}$$

If $a_t = a$ and $\sigma_t = \sigma$, P_t can be written as

$$P_t = (P_s - D_s - b_s)e^{-a(t-s)} + D_t + b_t + \sigma\left(\frac{1 - e^{-2a(t-s)}}{2a}\right)^{\frac{1}{2}}\varepsilon \qquad (5.7)$$

where ε is a standard normal distributed random variable. From the above equation we see that the process P_t has a conditional mean equal to $(P_s - D_s - b_s)\, e^{-a(t-s)} + D_t + b_t$ and a conditional standard deviation equal to $\sigma\left(\frac{1 - e^{-2a(t-s)}}{2a}\right)^{\frac{1}{2}}$. Since the expected value of P_t when $t \to \infty$ is equal to $b_t + D_t$, $b_t + D_t$ can be interpreted as the long run mean function of the price process.

5.2 Data analysis

The historical daily settlement prices dates back to 1998. For the two initial years, however, the market was in an infantile state. The traded volumes increased steadily, but the market was not mature. This is also seen in the available market data. Figure 2-1 shows a more or less constant price level the initial years, on contrary of what one should expect in a mature market. The main goal when establishing a spot price process is to be able to describe and predict the future spot price as realistically as possible. With this in mind, the market data for the two initial years (June 1998-May 2000) are ignored in this analysis.

5.3 Parameter estimation

To determine the parameters that maximize the sum of log-likelihood regarding the natural gas spot price, the maximum likelihood estimation method is used. By recognizing that the distribution of P_t is known, and letting **P** represent a vector of observations of P_t at $t = t_1, t_2, \ldots, t_n$, the maximum likelihood estimates $\hat{\alpha}$ and $\hat{\beta}$ are the solution to the following maximization problem

$$(\hat{\alpha}, \hat{\beta}) = \arg \max_{\alpha, \beta} \; y(P, \alpha, \beta)$$

$$\text{where } P_t : N(m(p_{t_i} | p_{t_i - 1}; \alpha, \beta), s(\alpha)) \text{ and}$$

$$y(P, \alpha, \beta) = \sum_{i=1}^{n} \log f(p_{t_i} | p_{t_i - 1}; \alpha, \beta),$$

$$f(p_{t_i} | p_{t_i - 1}; \alpha, \beta) = \frac{1}{\sqrt{2\pi}\, s(\alpha)} \exp\left\{ -\frac{(p_{t_i} - (m(p_{t_i} | p_{t_i - 1}; \alpha, \beta))^2}{2 s(\alpha)^2} \right\}$$

$$m(p_{t_i} | p_{t_i - 1}; \alpha, \beta) = (p_{t_{i-1}} + D_{t_{i-1}} + b_{t_{i-1}})e^{-a(t_i - t_{i-1})} + D_{t_i} + b_{t_i}$$

$$s(\alpha) = s\left(\frac{1 - e^{-2a(t_i - t_{i-1})}}{2a}\right)^{1/2}$$

Here the $\hat{\alpha}$ and $\hat{\beta}$ represent the vectors

$$\hat{\alpha} = \{a,\ \sigma,\ w_1^X,...,\ w_k^X,\ w_1^D,...,\ w_l^D\}\ ,\ \hat{\beta} = \{b_0,\ A_1^X,...,\ A_k^X,\ B_1^X,...,\ B_k^X,\ A_1^D,...,\ A_l^D,\ B_1^D,...,\ B_l^D\}$$

With k = 2 and l = 2 this problem has a parameter space of 3*(k + l + 1) = 15. It would be very difficult to solve a numerical maximization problem with such a high degree of freedom. To simplify the problem the frequencies are fixed. With a daily sampling resolution the frequencies can be represented as

$w_1^X = 2\pi/365$	$w_2^X = 4\pi/365$	(year)
$w_1^D = 2\pi/7$	$w_2^D = 4\pi/7$	(weekly)

The frequencies in the left column represents a yearly and a weekly cycle, while the frequencies in the right columns allow the model to incorporate intra –weekly and –yearly changes by doubling the frequencies.

The first step is to take an initial guess for the a and σ parameters. This initial guess allows the $\hat{\alpha}$ vector to be temporarily determined using an ordinary least squares method (OLS) and a weekly sampling rate of the historical spot price. Finally, the daily parameters, $\hat{\beta}$, are established by yet another OLS sequence, this time with a daily sample rate of the historical spot price.

By assuming values for the a and σ parameters, the $\hat{\alpha}$ and $\hat{\beta}$ can be determined, and consequently a value for the log-likelihood maximization problem. This procedure needs to be repeated several times, with different values of the a and σ parameters, to determine the $\hat{\alpha}$ and $\hat{\beta}$ that maximize the log-likelihood function.

5.4 Results

The estimation procedure was implemented and executed in Microsoft Excel using Visual Basic programming. The procedure was performed using different data sets, which will be further discussed in the next chapter. In this chapter an example is shown, using Monday settlement prices from the period June 2000 - June 2003 to estimate the seasonal, mean reversion and volatility parameters. Daily settlement prices were used to estimate the weekly parameters. The procedure includes several iterations, as described in the preceding section, but only the final solution will be presented.

5.4.1 The mean reversion and volatility parameters

In section 5.1 the explicit solution to the spot price process was shown to be

$$P_t = (P_s - D_s - b_s)e^{-a(t-s)} + D_t + b_t + \sigma\left(\frac{1 - e^{-2a(t-s)}}{2a}\right)^{1/2}\varepsilon \qquad (5.8)$$

This process assumes constant values for the mean reversion and volatility parameter. The OLS method suggested the following values of these parameters:

$a = 0{,}0510$ $\qquad$ $\sigma = 1{,}253$ (UK pence/therm)

5.4.2 The yearly changes

The parameters that described the yearly changes were determined to be

$$b_0 = 19{,}30$$

$$A_1^X = 4{,}98 \,,\; A_2^X = 1{,}01$$

$$B_1^X = 0{,}33,\; B_2^X = -0{,}85$$

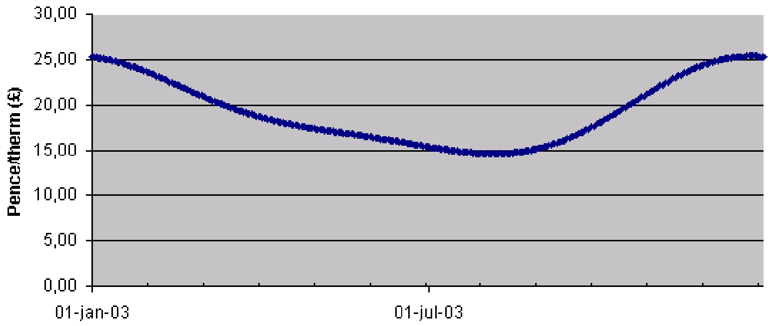


Figure 5-1 Mean seasonal price

Figure 5-1 displays the seasonal changes in the spot price during a year. As one could expect the prices are at their maximum during the winter months with a peak in late December. The lowest prices are experienced late in the summer, with the minimum occurring in August. This path can be viewed as the long run mean price for each day during the year.

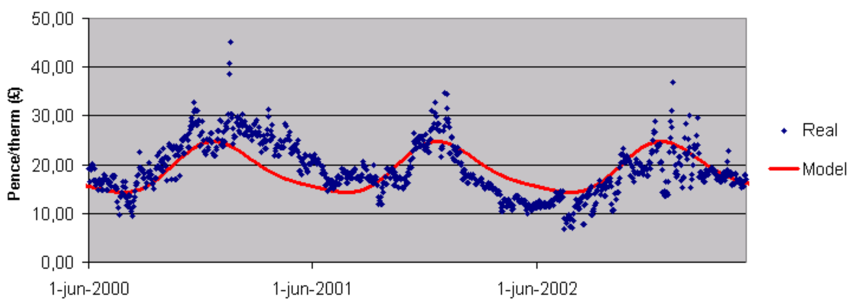


Figure 5-2 Mean seasonal price imposed on observed prices

Figure 5-2 displays the yearly mean price imposed on the daily settlement prices observed in the period June 2000 until June 2003. It is seen that the mean price curve is capable of predicting the general price movements observed over the course of a year.

5.4.3 The weekly changes

Having established the yearly changes using a weekly sampling rate of the historical spot prices, the next step is to determine the intra-week changes. As was shown in chapter 2.3.3, the main feature in the weekly prices is that the weekend prices are significantly lower than midweek prices. The parameters that described the weekly changes were determined to be

$$A_1^D = -0{,}66,\ A_2^D = -0{,}37$$

$$B_1^D = -0{,}32,\ B_2^D = -0{,}46$$

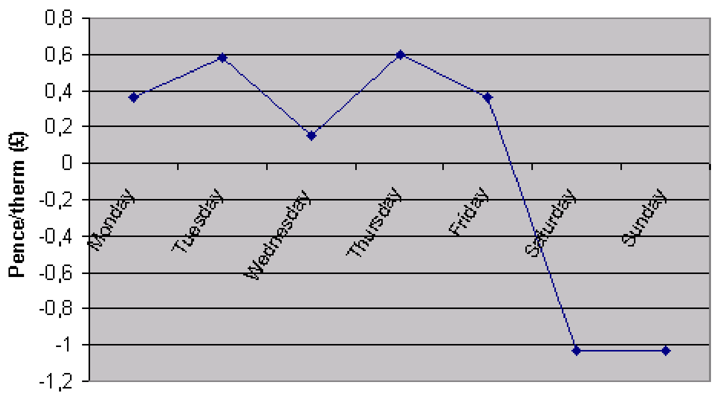


Figure 5-3 Daily changing prices

As figure 5-3 shows, the model is capable of predicting a considerable lower price during the weekend. However, the midweek changes are not equally intuitive. The main cause of the changing prices during midweek is caused by the trigonometric function only having two frequencies. It should not be concluded therefore that Wednesday prices are generally lower than Thursday prices. Still, this model is ok for the purpose at hand.

In the trigonometric function specifying the weekly changes, the parameter d_0 needs to be determined. In this estimation example, Monday settlement prices were used to determine the seasonal changes. The d_0 parameter is merely an adjustment factor, assuring that the weekly changes starts of at zero each week. When using Monday settlement prices, prices on Tuesday should be zero in the weekly cycle. To accomplish this, d_0 is set to be –0,59. If another day was used to determine the seasonal changes, figure 5-3 gives the appropriate adjustment factor, d_0.

5.4.4 The resulting spot price model - example

The final result of this analysis, the explicit solution, is given by

$$P_t = (P_s - D_s - b_s)e^{-0{,}051(t-s)} + D_t + b_t + 1{,}253\left(\frac{1 - e^{-0{,}102(t-s)}}{0{,}102}\right)^{1/2}\varepsilon \qquad (5.9)$$

where

$$b_t = 19,30 + 4,98\cos(\frac{2\pi}{365}t) + 1,01\cos(\frac{4\pi}{365}t) + 0,33\sin(\frac{2\pi}{365}t) - 0,85\sin(\frac{4\pi}{365}t) \quad (5.10)$$

and

$$D_t = -0,59 - 0,66\cos(\frac{2\pi}{365}t) - 0,37\cos(\frac{4\pi}{365}t) - 0,32\sin(\frac{2\pi}{365}t) - 0,46\sin(\frac{4\pi}{365}t) \quad (5.11)$$

where ε is a standard normal distributed random variable. Figure 5-4 shows one simulated price path imposed on the observed price path in the period from June 2000 until June 2003.

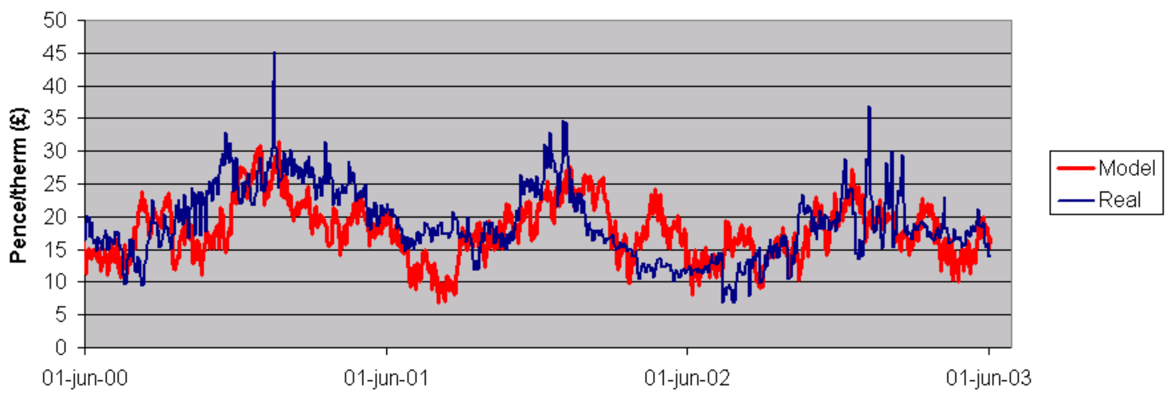


Figure 5-4 A simulated price path vs. real prices

5.5 Investigating the estimated parameters

The procedure presented in the preceding chapters uses a dataset of weekly prices to estimate the seasonal, volatility and mean reversion parameters. The parameter estimates should be reasonably independent of the day chosen for estimation. To examine if this was the case, the estimation procedure was performed using all weekdays as individual sampling sets. The analysis turned out as follows

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
a	0,05	0,07	0,06	0,09	0,05	0,05	0,04
σ	1,25	1,52	1,52	1,92	1,18	1,18	1,03
b	19,30	19,12	19,17	19,31	17,80	17,79	19,31
A_1^X	4,98	5,29	5,16	5,16	4,65	4,63	4,96
A_2^X	1,01	0,74	0,69	0,96	0,82	0,85	0,83
B_1^X	0,33	0,23	0,27	-0,08	0,25	0,22	-0,61
B_2^X	-0,85	-0,58	-0,30	-0,41	-0,76	-0,78	-1,01
d_0	-0,59	-0,15	-0,60	-0,36	1,03	1,03	-0,36
A_1^D	-0,66	-0,66	-0,66	-0,66	-0,66	-0,66	-0,66
A_2^D	-0,37	-0,37	-0,37	-0,37	-0,37	-0,37	-0,37
B_1^D	-0,32	-0,32	-0,32	-0,32	-0,32	-0,32	-0,32
B_2^D	-0,46	-0,46	-0,46	-0,46	-0,46	-0,46	-0,46

Table 5-1Parameters estimated by different datasets

The parameters describing the weekly pattern, A_1^D, A_2^D, B_1^D and B_2^D, are estimated using the complete dataset, and are, as expected, practically identical for all datasets. The seasonal parameters A_1^X, A_2^X, $B_1^X$and B_2^X, on the other hand are estimated using separate datasets. As shown in table 5-1 there are some differences. The d_0 parameter, however, serves as an adjustment factor, and when this is taken into account the parameters are almost equal.

The major differences arise when the volatility and mean reversion parameters are evaluated. A volatility parameter ranging from 1,03 to 1,92, and a mean reversion parameter ranging from 0,04 to 0,09, exhibits quite substantial differences. The weekly datasets from the period June 2000 - June 2003 include 158 weeks (data), which apparently is not sufficient to estimate the parameters with satisfactory precision.

When valuating a natural gas storage facility, and the option to invest in such, the volatility and the mean reversion parameter are expected to have a major impact on the result. The spot price process that was presented earlier, and which will be the underlying process of the following valuation of the option, assumes constant values for both the mean reversion and the volatility. This magnifies the importance of these values for the final result. The assumption that these parameters are constant is a major simplification. One possible solution would be to incorporate stochastic volatility. This would increase the complexity of the model, and is not considered appropriate for this analysis. However, the assumption that the mean reversion and volatility parameters are held constant can be defended if the parameters are close to the mean values of the period.

When valuing investments in assets such as gas storage facilities, several years of operation must be considered. The following analysis will include 15 years of operation. This suggests that the mean reversion and volatility values used in the analysis should represent the expected mean values during this period. However, the long-term values of these parameters are highly uncertain. The question is; which values should be assigned to these parameters? The available historical spot prices give limited information on this matter, as the prices only date back a couple of years. Nevertheless, this information must be used to estimate the values appropriately.

The option valuation method, which will be used later, requires not only the mean value of these parameters, but also a stochastic representation of their uncertainty. This stochastic representation should not reflect the daily variation of these values, but rather their long-term variation. If the daily variation is used, which is significantly larger than the long-term variation, this will result in an unrealistically high option value.

In the following sections appropriate values for the mean reversion and volatility values will be established, as well as a stochastic representation of the long run mean values.

5.5.1 Rolling window estimation

In order to understand the development of the mean reversion and volatility parameters, a time series representation of the parameters was produced. The rolling window includes one year of data, 52 weeks, and was calculated for the period June 2000 - June 2003. In the following figures week 1 refers to the period week 23 2000 till week 23 2001, week 2 refers to week 24 2000 till week 24 2001, and so on. Figure 5-5 shows the rolling window values for the mean reversion factor for each day of the week, and figure 5-6 shows the corresponding volatility.

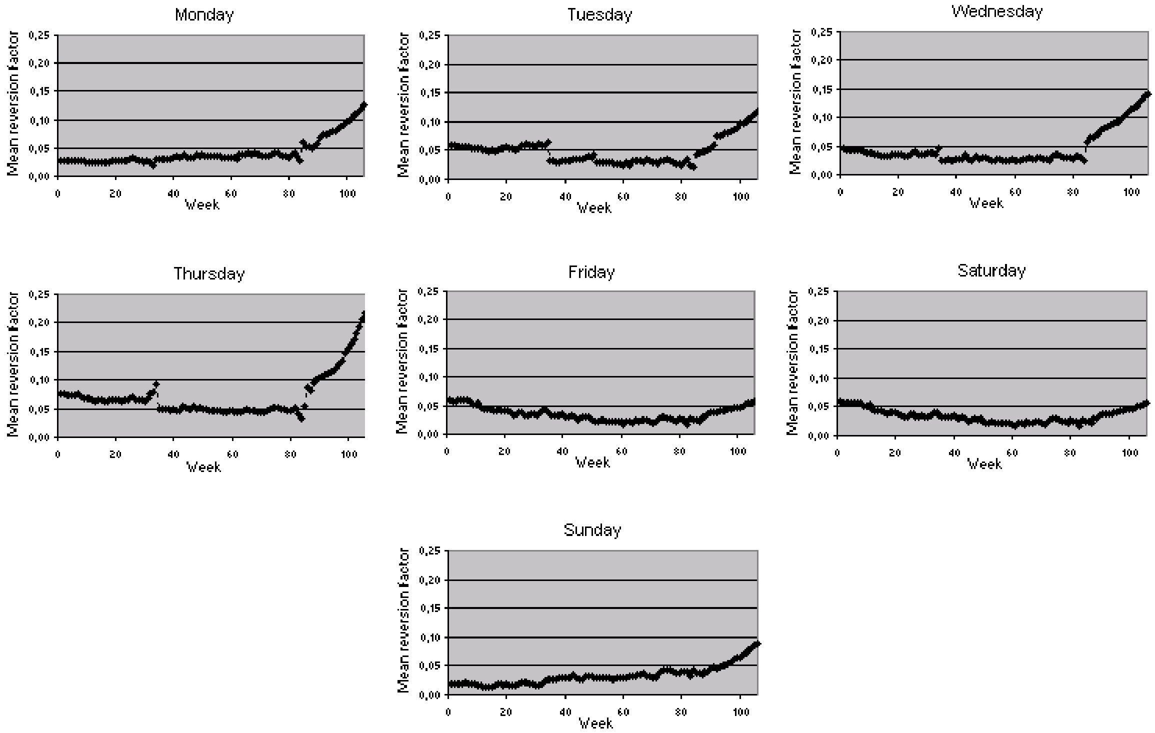


Figure 5-5 Mean reversion value estimated by one year rolling window

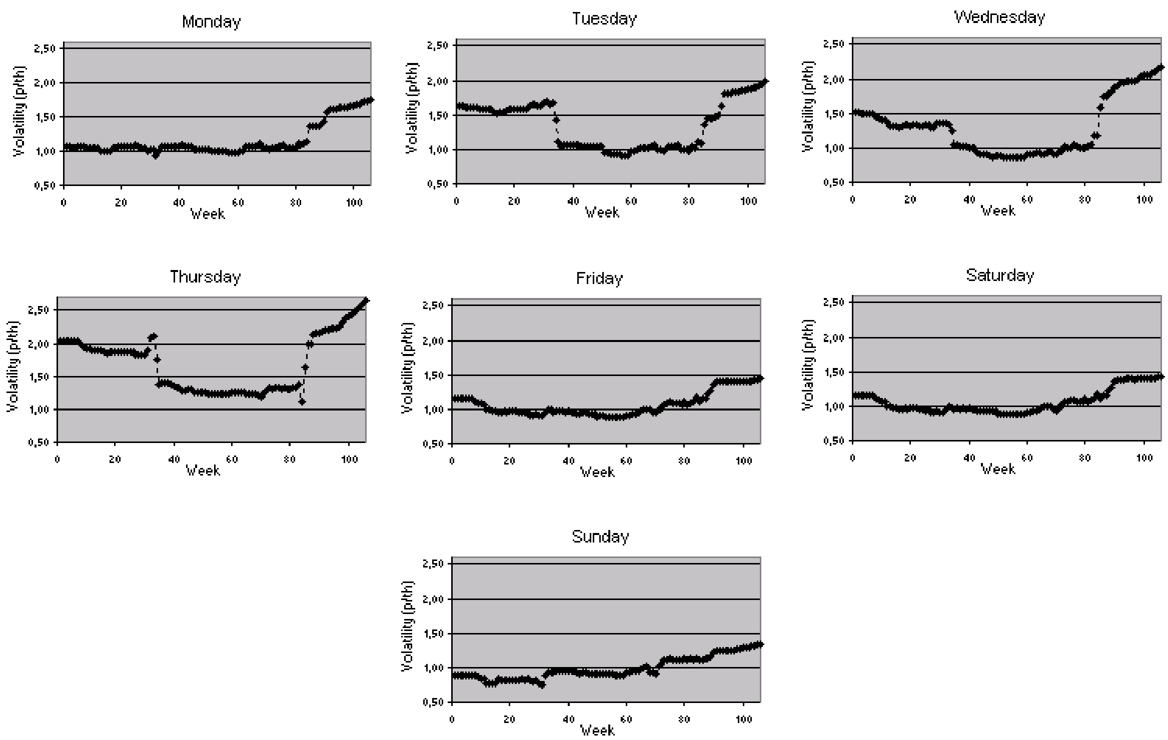


Figure 5-6 Volatility estimated by one year rolling window

A first look at figure 5-5 and 5-6 suggests that there is a possible correlation between the two parameters. To investigate this, a correlation analysis was performed. The analysis included all days and showed a correlation factor of 0,883. Later, when the option to invest in natural gas storage is evaluated, it would be an advantage if these parameters could be described by one single factor. Having established that the correlation is significant, a principal component analysis (PCA) can help determine if it is possible to use one factor to describe the two parameters. The concept of PCA is that although p, in this case 2, components are required to produce the total system variability, often much of this variability can be accounted for by a small number k, 1 in this case, of the principal components. If so, there is (almost) as much information in the k components as there is in the original p variables. The PCA produced the following result

Eigenanalysis of the Correlation Matrix

Eigenvalue	1,8827	0,1173
Proportion	0,941	0,059
Cumulative	0,941	1,000

Variable	**PC1**	**PC2**
Volatility	0,707	-0,707
Mean reversion	0,707	0,707

Table 5-2 Principal component analysis

The PCA showed that one factor is capable of describing 94,1 percent of the total variance. This is considered satisfying, and the following relation is established

$$Y = 0{,}707 \times \text{Mean Reversion Value} + 0{,}707 \times \text{Volatility Value} \qquad (5.12)$$

To further investigate and determine the appropriate value of the volatility and mean reversion factors, the corresponding time series of the new variable Y is shown in figure 5-7.

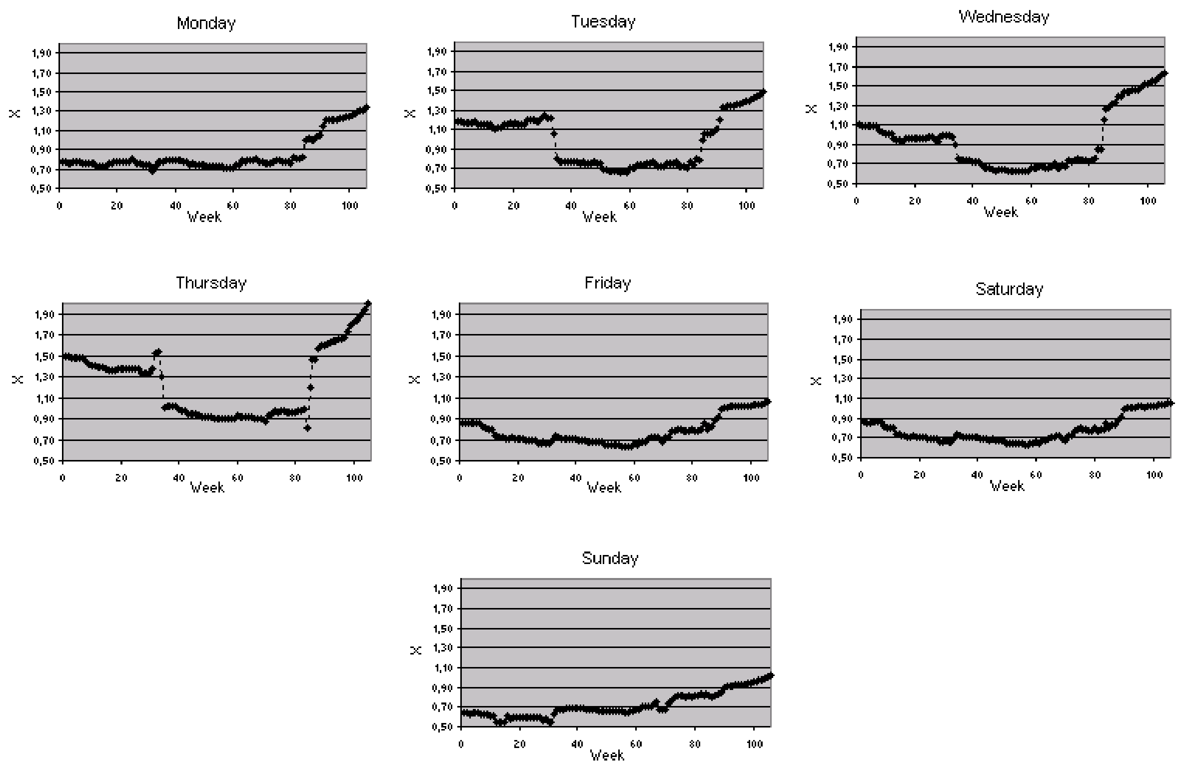


Figure 5-7 Y value corresponding to figure 5-5 and 5-6

Examining figure 5-7 helps establish some important features of the development of the Y parameter. All weekdays exhibit a reasonably constant level during the second year. This constant level is approximately 0,7 for all days except Thursday, which has a constant level at approximately 0,9. This constant level can also be seen as a minimum level for all days, except Sunday. Tuesday, Wednesday and Thursday differ significantly from the other days as they have two major shifts in the parameter value, one after approximately one year and another after approximately two years. In addition, all weekdays have an upward tendency of the parameter value the final year.

Descriptive Statistics: Y

Variable	N	Mean	Median	StDev
Y	742	0,9009	0,78	0,2761

Variable	Minimum	Maximum	Q1	Q2
Y	0,54	2,03	0,70	1,02

Table 5-3 Descriptive statistics of Y

As mentioned earlier, the parameter value should approximate a long-term value. The time series parameter values are one-year averages. As shown in table 5-3, the mean value of all days is 0,9 and the median value is 0,78. The large deviation implies that there are some large values that have a major impact on the mean value and pulls it upwards. This is also seen figure 5-7. The first and third quartiles are correspondingly 0,7 and 1,02. Based on the basic statistics and figure 5-7, a reasonable estimation of the long run mean could be

$$\overline{Y} = 0,85$$

This mean value is considered appropriate, and will be used throughout the valuation analysis.

5.5.2 Stochastic representation of the Y parameter

When estimating the value of the option to invest in a gas storage facility, a stochastic representation of the uncertainty of the storage value is needed. This storage value uncertainty will be represented by uncertainty of the Y parameter, which again represents the long-term mean reversion and volatility. The stochastic process should describe the possible distribution of the long-term Y parameter value, and should not be confused with the daily change of the volatility and mean reversion parameters. A geometric Brownian motion process is chosen, as this process will simplify the option valuation later. This process has the feature that it cannot become negative, which is also true for Y. GBM has the following mathematical description

$$dY = \mu Y\, dt + \sigma Y dz \qquad (5.13)$$

Here dY represents the increment in the asset price process during a small interval of time dt, dz is the underlying uncertainty driving the model and represents an increment in a Wiener process during dt, μ is the constant instantaneous drift and σ the volatility of Y. Difficulty arises when the GBM parameters shall be estimated. The long run mean was established to be 0,85, and it is assumed that this value will remain fairly stable. The drift term is therefore set to zero. The volatility parameter will have a great impact on the option value. Since it is very important to choose this value realistically and also investigate what happens if the value changes, three different values will be used in the analysis.

The three volatility parameters of the long run mean Y parameter value are:

$$\sigma_{\text{small}} = 0,0147$$

$$\sigma_{\text{medium}} = 0,0215$$

$$\sigma_{\text{large}} = 0,0279$$

The volatility parameter reflects the uncertainty of the Y parameter value, and the confidence interval of Y with the different volatility values are shown in figure 5-8.

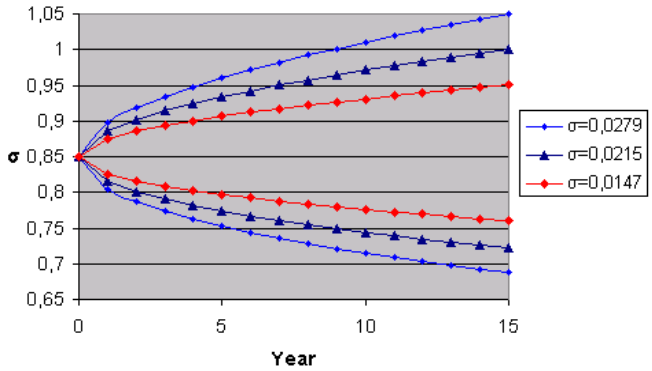


Figure 5-8 Confidence intervals of σ

The strictest volatility parameter, $\sigma = 0{,}0147$, implies that Y has a 95% probability of not exceeding 0,95. The other volatilities imply, with a 95% probability, that Y will not exceed 1,00 and 1,05. As these values are highly uncertain, their values are more of a means to evaluate what happens to the option value as Y's volatility changes.

5.6 Final spot price process

The analysis in the previous sections established a long run mean value of the Y parameter of 0,85. This implies a mean reversion value of 0,047 and a volatility value of 1,155. The choice of the parameters defining the trigonometric functions in the spot price process was fairly independent of the weekday used for estimation. Of the midweek days, the Monday estimation results is closest to the volatility value of 1,155, and the trigonometric values estimated from Monday data are used throughout the analysis. The final spot price process, using the long run mean value of Y, is given by the following explicit solution

$$P_t = (P_s - D_s - b_s)e^{-0{,}047(t-s)} + D_t + b_t + 1{,}155\left(\frac{1 - e^{-0{,}094(t-s)}}{0{,}094}\right)^{\frac{1}{2}}\varepsilon \qquad (5.14)$$

where

$$\begin{aligned} b_t = 19{,}30 + 4{,}98\cos\left(\frac{2\pi}{365}t\right) + 1{,}01\cos\left(\frac{4\pi}{365}t\right) \\ + 0{,}33\sin\left(\frac{2\pi}{365}t\right) - 0{,}85\sin\left(\frac{4\pi}{365}t\right) \end{aligned} \qquad (5.15)$$

and

$$\begin{aligned} D_t = -0{,}59 - 0{,}66\cos\left(\frac{2\pi}{365}t\right) - 0{,}37\cos\left(\frac{4\pi}{365}t\right) \\ - 0{,}32\sin\left(\frac{2\pi}{365}t\right) - 0{,}46\sin\left(\frac{4\pi}{365}t\right) \end{aligned} \qquad (5.16)$$

6 Valuing gas storage - the procedure

In this chapter a procedure that determines the market value of a natural gas storage facility will be presented. In short, the method first establishes an optimal use, or optimal strategy, of the storage facility, and this strategy is then implemented in a Monte Carlo simulation to determine the value of the storage facility. The strategy and the simulation will be based on the spot price process assumptions established in the previous chapter.

6.1 The value of gas storage

The optimal use of a natural gas storage facility is highly dependent on the facility owners' requirements and operational status. The main value components of a gas storage facility were presented earlier, and they could all influence the optimal strategy. Secure deliverability may be of considerable value to a company highly dependent on natural gas for its production. In this analysis, however, the storage facility will be operated to exploit changes in the natural gas spot price; buy low and sell high. The strategy of this facility will not be restricted to keeping a certain volume of gas in storage at any time, which could be the case if the facility should serve to secure delivery. The only restrictions are the specifications of the storage facility; the withdrawal and injection rate, and the maximum and minimum levels of gas in storage. The value of the facility will be decided by its ability to exploit the changing spot prices and earn money. If the facility were to be used for the purpose of securing delivery of natural gas, the restrictions would probably reduce its ability to exploit changing spot prices.

A simple strategy for use of the storage facility would be to buy and store gas during the periods of low demand, typically the summer, and sell during the high demand periods, typically the winter. This strategy is in accordance with the intuitive principal of buying low and selling high. A safe financial strategy would be to optimise and hedge on the forward markets on the first day of the year, and run the facility accordingly. Both strategies would unfortunately under utilize the facility, and fail to exploit the optionality (flexibility) embedded in a storage facility. The faster the gas can move through the storage facility, the better positioned it is to take advantage of price volatility (Scott, Brown & Perry, 2000).

The question of how to operate the storage facility optimal, the optimal strategy, is really a question of what is a good price for gas. One way to determine the right price for gas is to use the mean expected price, sell when the spot price is above and sell when it is below. Another way is to incorporate the fact that it usually takes much longer time to fill up the storage facility than it takes to empty it, and therefore use a weighted average. However, these strategies fail to recognize an important feature when analysing the value of storage; the quantity of gas already in storage.

When buying a unit of gas during the summer months, one would certainly expect that this unit could be sold at a higher price during the winter months. This would probably be the case for the succeeding units as well, but consider what happens when the facility fills up. Due to restrictions on deliverability, there is a limit on the amount of gas that can be sold on the day of the highest price. This implies that subsequent units of gas put in storage are less valuable than their predecessors.

Scott, Brown and Perry (2000) presented a technique, which establishes the value of a unit of gas on a given day, dependent on the amount of gas already in storage. If the storage facility is full, then there will not be bought another unit of gas no matter how cheap it is. If the facility is empty, one will be prepared to pay a relatively high price in the summer, as there is a good chance that during the winter period the price will be higher. In between these two extremes, the price one is willing to pay on a given day will be decreasing as the quantity of gas already in storage increases. The concept that facilitates the incorporation of the amount already in storage, when analysing storage value, is the marginal value of gas.

6.1.1 The marginal value procedure

The marginal value of gas in storage, on any given day, is the additional value that the owner would get if there were on more unit of gas in storage on that day. The additional value will be a result of being in a better position to capture the prices over all future periods.

The procedure used to determine the marginal values is an optimisation routine based on a technique called stochastic dual dynamic programming (SDDP). The technique is directly analogous to stochastic dynamic programming (SDP), but it is significantly more efficient as it solves directly for the first order conditions of the problem at each stage (time step), rather than choosing the best value over a sampled grid. The problem can be expressed mathematically as follows

$$\max_{r_t} Z_0 = \left(c_T - c_0 + \sum_{t=1}^{T} \pi_t(r_t) \right)$$

$$\begin{aligned} \text{s.t.} \quad & s_t = s_{t-1} - r_t \\ & \underline{S}_t \le s_t \le \overline{S}_t \\ & \underline{R}_t \le r_t \le \overline{R}_t \\ & \pi_t = p_t r_t + \alpha_t r_t \\ & \alpha_t = \begin{cases} i_t & \text{if } r_t < 0 \\ -w_t & \text{if } r_t > 0 \\ 0 & \text{otherwise} \end{cases} \\ & \forall\ t = 1, ..., T \end{aligned} \qquad (6.1)$$

where s_t is the amount in storage at the end of period t,
p_t is the (stochastic) spot price
$\underline{S}_t$, $\overline{S}_t$ are the lower and upper storage bounds
$\underline{R}_t$, $\overline{R}_t$ are the lower and upper release bounds
r_t is the release decision of period t
$\pi_t(r_t)$ is the benefit of release decision r_t
c_0, c_T are the (given) starting and ending values of s_0, s_T
i_t is the injection cost per unit
w_t is the withdrawal cost per unit

By recognizing that

$$z_T = c_T \qquad (6.2)$$

and using the relation

$$z_t = \pi(r_t) + z_{t+1} \quad , \forall\ t = 1,....,T \qquad (6.3)$$

the problem can be broken into stages and be solved by adopting a backward recursion scheme. The optimisation problem will maximize the expected present and future return of the units of gas in storage. At each stage and for all storage levels, the optimal withdrawal or injection quantity (release) is determined, under limitations specified by the storage facility. Ultimately, the procedure provides the value of z_0, the expected future value at time 0. However, the important feature is that the first order conditions solved for each stage provides the (continuous) marginal value surface (MVS). Accordingly, the solution provides more than just a single optimal solution at each stage, it provides optimal solution for any allowed storage level at any stage of the time horizon.

The dual dynamic program scheme is based on the observation that the marginal value function (MVF) can be represented by a piecewise linear function (Pereira, 1999). Furthermore, it is shown that the slope of the MVF around a given point can be analytically obtained from a one-stage “release”-problem.

The last stage release problem can be expressed as follows.

$$
\begin{aligned}
z_T &= \text{Max } \pi_T + \omega_{T+1}(s_{T+1}) \\
\text{s.t. } s_T &= s_{T-1} - r_T \qquad \text{simplex multiplier } \beta_{S_{T-1}T} \\
\underline{S}_t &\le s_T \le \overline{S}_t \\
\underline{R}_t &\le r_T \le \overline{R}_t \\
\pi_T &= p_T r_T + \alpha_T r_T \\
\alpha_t &= \begin{cases} i_t & \text{if } r_t < 0 \\ -w_t & \text{if } r_t > 0 \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
\qquad (6.4)
$$

In this problem $\omega_{T+1}(s_{T+1})$ represent the future value function. In the last stage problem the future value function is zero, which implies that any gas left in storage after this period will be worthless. It is well known from linear programming theory that there is a set of simplex multipliers associated with the constraints at the optimal solution. The simplex multiplier, $\beta_{S_{T-1}T}$, represents the derivative of the optimal solution value with respect to the initial (beginning of period) storage volume, or the volume in storage at the end of period t-1. It is equal to the expected increase in the optimal solution value if the right-hand side constraint was eased by a unit; if there was one more unit of gas available for sale. The simplex multiplier is expressed as follows

$$\beta_{S_T\text{-}1T} = \partial z_T / \partial s_{T-1} \qquad (6.5)$$

Figure 6-1 shows the calculation of the marginal value for each storage level. Note that the marginal value is the increase of value in storage if there was one more unit of gas in storage. Calculating the marginal value at each storage level produces a piecewise linear function, where the linear segment with the lowest value is chosen at each storage level. The slope of the marginal value function corresponds to expression 6.3. The piecewise linear function is the marginal value curve.

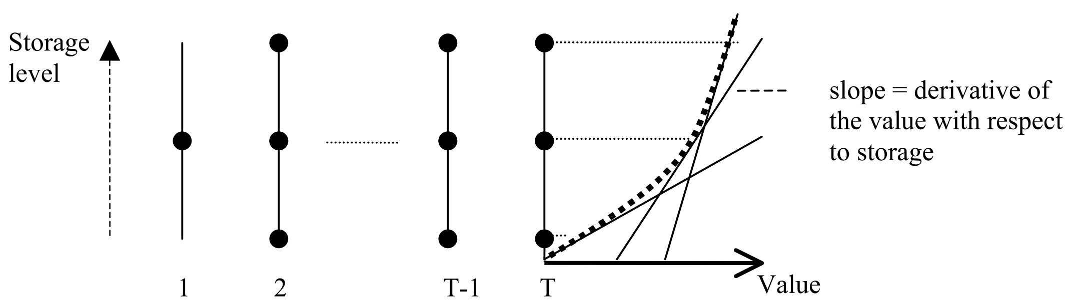


Figure 6-1 Calculation of a piecewise marginal value function

The backward recursion scheme used to determine the marginal value surface, the marginal value functions for all time steps, can be summarized by following algorithm

1. Set the number of linear segments = number of initial storage values N
2. Initialise future value function for the last stage as zero:(φ_t^n -coefficient of the n[th] linear and δ_t^n is the constant term of the n[th] linear segment)

$$\{\varphi_{T+1}^n \text{ and } \delta_{T+1}^n\} = 0 \text{ for n} = 1,...,N$$

3. for t = T, T-1, ..., 1

for each storage value $s = \{s_t^n,\ n = 1, ..., N\}$

for spot price scenario $p_t = \{ p_t^1, ..., p_t^k, ..., p_t^K\}$ with probabilities $e_t = \{e_t^1, ..., e_t^k, ..., e_t^K\}$

solve the one-stage release problem for initial storage s_{t-1}^n and price p_t^k:

$$\begin{aligned}
&\omega_t^k(s_{t-1}^n) = \max \pi_t(r_t^k) + \omega_{t+1} \\
\text{s.t.}\quad & s_t^n = s_{t-1}^n - r_t^k \qquad \text{simplex multiplier } \beta_{s_{t-1}t}^k \\
& \underline{S}_t \le s_t^n \le \overline{S}_t \\
& \underline{R}_t \le r_t^k \le \overline{R}_t \\
& \pi_t^k = p_t^k r_t^k + \alpha_t^k r_t^k \\
& \alpha_t^k = \begin{Bmatrix} i_t & \text{if } r_t^k < 0 \\ -w_t & \text{if } r_t^k > 0 \\ 0 & \text{otherwise} \end{Bmatrix} \\
& \omega_{t+1} = \varphi_{t+1}^n s_t + \delta_{t+1}^n \quad n = 1, ..., N \\
& \forall\ t = 1, ..., T
\end{aligned}$$

next

calculate the coefficient and constant term for the n[th] linear segment of the future value function in the previous stage

$$\varphi_t^n = \sum_{k=1}^{K} e_t^k \times \beta_{s_{t-1}t}^k \quad \text{and } \delta_t^n = \sum_{k=1}^{K} e_t^k(\varphi_t^n \times s_t^n + \alpha_t^k(s_t^n))$$

next

next

Now, the marginal value φ_t^n is dependent on s_{t-1}^n, the storage level at the end of period t-1. This implies that the marginal value φ_t^n defines the optimal strategy at time t-1. For instance, if the value in storage at time t increases by 20 pence if there were 3 units rather than 2 units in storage, φ_t^n =20 pence, this would imply that with 2 units in storage at time t-1, one would be willing to pay up to 20 pence for an additional unit.

In order to determine the marginal value surface, appropriate inputs are needed to define the problem. In the case of a gas storage facility problem, a mean price path, a volatility parameter and a parameter for mean reversion are suitable inputs. These parameters were developed in chapter 5. A probability distribution of the expected spot price on any given day is also needed (p_t-the spot price and e_t-the probability of this spot price), and 1000 Monte Carlo simulated spot price paths is used to determine this distribution. The continuous probability distribution will be represented by 10 spot price scenarios for each day, each scenario with a 10% probability of occurring. The mean value of the 100 lowest spot prices is the first scenario, the mean of the 100 next spot prices is the second and so on. The 10 scenarios are calculated as follows:

N= 1000	- number of simulated prices on any given day
K= 10	- number of price scenarios on any given day
p_t^k	- spot price on day t under scenario k
e_t^k = 10%	- probability of scenario k occurring on day t
$X_t(y)$	- sum of the y smallest lowest spot prices on day t

$$p_t^1 = X(100)/100$$

$$p_t^2 = (X(200)-X(100))/100$$

$$\vdots$$

$$p_t^{10} = (X(1000)-X(900))/100$$

Having established the marginal value surface, the next step is to determine the value of having the gas storage facility; its market value. This procedure is easily conducted using Monte Carlo simulation. Simulating the price path several times allows the market value of the facility to be described by a probability distribution. The stochastic dual dynamic program along with the Monte Carlo simulation has been implemented in Microsoft Excel using Visual Basic programming. This program will be used later to analyse a specific case. But first, a simple example will be demonstrated to describe the meaning of the marginal value.

6.1.2 The marginal value procedure and the optimal strategy

In this section a simple example will be given to illustrate the relationship between the marginal value and the optimal strategy. Consider a storage facility operating in a simple three period timeframe, T =

3. The value of the remaining volume at the end of period three will be worthless. In addition, the facility is capable of either injecting one unit per period or withdrawing two units per period. Maximum capacity is three units, N = 3. Only one price scenario will be considered at each time step, K = 1 and e_t =1. Operation- and transaction costs are ignored in this example.

The mean price of each period, the price scenarios, has been determined to be as follows

Period 1: 23 pence/unit   Period 2: 25 pence/unit   Period 3: 24 pence/unit

Table 6-1 shows the value of the volume of gas in storage on each day of the period

Storage level (start of period)	0 units	1 unit	2 units	3 units
Period 1-Value in storage(pence)	2	27	51	74
Period 2	0	25	50	74
Period 3	0	24	48	48
Period 4	0	0	0	0

Table 6-1 Value of gas in storage

Period 4 is outside the timeframe considered in this example, so any gas left in storage in this period is worthless. If there are 2 units of gas in storage in the beginning of period 3, the gas will have a value of 48 pence because this gas will be sold in period 3 for a unit price of 24. If there are 3 units of gas in the beginning of this period, the value will still be 48 pence. The withdrawal limit allows only two units to be sold, and the last unit will be worthless. Period 2 has the highest price of the three periods, and as much gas as possible will be sold in this period. If there are 3 units of gas in this period, two units will be sold for a value of 50 pence, leaving one unit in storage. The value of having one unit of gas in period 3 was previously determined to be 24, giving a total value of 74 if there are 3 units in storage in the beginning of period 2. Having one unit of gas in the beginning of period 1 will have a value of 2. This value comes from buying one unit of gas in period 1 and selling it the next period giving a profit of 2 pence.

Having established the value of gas in storage on each day, the marginal values can be determined as the first-derivative of the value function. Table 6-2 shows the calculated marginal values.

Storage level (start of period)	0 units	1 unit	2 units	3 units
Period 1-Marginal value(pence)	25	24	23	23
Period 2	25	25	24	24
Period 3	24	24	0	0
Period 4	0	0	0	0

Table 6-2 Marginal value of gas in storage

These marginal values define the optimal strategy of the storage facility operating in this short period of time or more precisely; the marginal value of period 4 determines the optimal strategy of period 3, marginal value of period 3 determines the optimal strategy of period 2, and so on. Table 6-3 shows the optimal strategy values

Storage level (start of period)	0 units	1 unit	2 units	3 units
Period 1-Optimal strategy	25	25	24	24
Period 2	24	24	0	0
Period 3	0	0	0	0
Period 4	-	-	-	-

Table 6-3 Optimal strategy- limits

The optimal strategy defines a limit, at which one would sell if the spot price is higher and buy if it is below. If there is no gas in storage, the sell limit will be adjusted to zero as there is no gas to sell. When the facility is full the buy limit is adjusted to zero, as there is no room for any more gas. The strategy states that with an empty facility in the beginning of period 1, one should buy gas if the spot price is below 25 pence. This limit is set because it is expected that this unit, if bought in period 1, can be sold for 25 pence in one of the following period. If there is one unit of gas in storage in the beginning of period 2, one should sell this unit if the spot price is below or equal to 24 pence and sell if it is higher. As is seen, the strategy does not only determine an optimal storage value at each time period, but rather an optimal action for all storage levels for all time periods.

The backwards recursion scheme presented in the previous section calculated the values, marginal values and the optimal strategy one period at the time, working backwards through the time periods. In this example the value of gas in storage was calculated first for all nodes (time steps and storage levels), then the marginal value and finally the optimal value. The two iterations produce the same results, but the backwards recursion scheme is the one implemented in the developed program.

7 The market value of a gas storage facility

In chapter 5 and 6 a spot price model and a procedure for determining the optimal operational strategy for a natural gas storage facility was presented. In this chapter this will be used to determine the value of having a natural gas storage facility, or the market value of this facility. First the specifications of the chosen facility will be presented along with other factors that will influence the analysis.

7.1 Case specification

The specifications presented in this section will be used to investigate the value of having a gas storage facility, and in the following chapter to investigate the value of the option to invest in such a facility.

7.1.1 The storage facility

As discussed earlier, the specifications of a natural gas storage facility may differ significantly. Depending on the main purpose of the facility, capacity, injection- and withdrawal capabilities are chosen to meet the specific requirements of the facility owner.

In this case, the main interest is to exploit changing spot prices in the market, and a facility capable of withdrawing and injecting large volumes is considered as the best alternative. The following storage facility specifications are chosen:

Space/Working volume:	180 Mill Sm^3
Deliverability:	18 Mill Sm^3/day
Injectability:	3 Mill Sm^3/day

These specifications resemble the specifications of a new storage facility in Aldbrough, UK, which is scheduled to be completed in 2007 (Surface Production Systems). Statoil ASA have bought the rights to operate this underground gas storage facility, which contains three salt caverns with associated wellhead areas, a seawater intake/brine discharge facility and a 8 kilometre gas pipeline connection to the main national transmission system (NTS) pipeline.

7.1.2 Investment costs

The investment costs of the chosen gas storage facility are very uncertain. Attempts have been made to receive some information on this matter from Statoil, but as expected, such information is classified. However, some general guidelines for investment costs are available, and these will be used in this analysis. IEA[2] have collected cost data from different sources in the US and in Europe. The costs are given as estimates of the minimum and maximum costs per thousand cubic feet of working gas. The minimum estimate, converted into £ per Sm^3, is approximately 0,19 £ per Sm^3 working gas, while the maximum is approximately 0,39 £ per Sm^3 working gas. Because the investments costs are so uncertain, three values will be used when the option to build a natural gas storage facility is evaluated in the next chapter.

The facility in this analysis has 180 million Sm^3 of working gas, and an additional 60 million Sm^3 of base gas. The development cost based on the minimum and maximum costs per m^3 will then be

Development costs: Maximum: 70 million £
Minimum: 34 million £

The facility in this analysis contains three caverns with a significant compressor facility, which implies that the development cost would be in the top half of this development cost range. It is assumed that it takes two years to develop the facility, and that the development cost will be incurred at the beginning of this period. The market value of the facility will be discounted back to the start of investments.

The cost of the base gas, the gas that serves as a pressure cushion and cannot be economically removed, will be incurred at start up. Assuming a mean price of 20 pence per therm for the 60 million Sm^3 of base gas, and discounting back two years gives 3,8 million £.

Based on the discussion above, three total investment costs will be used in the option valuation

Total investment costs:	High:	74 million £
	Medium:	62 million £
	Low:	50 million £

7.1.3 Operation costs

Dietert and Pursell (2000) have investigated different aspects of US natural gas storage. They found that average salt cavern storage cost per therm was

Cost Item	1 Cycle	2 Cycles	3 Cycles	4 Cycles	5 Cycles
Demand Charge (cents/th)	6,00	3,00	1,98	1,50	1,20
Injection Fee (cents/th)	0,12	0,12	0,12	0,12	0,12
Withdrawal Fee (cents/th)	0,12	0,12	0,12	0,12	0,12
Fuel (cents/th)	0,24	0,24	0,24	0,24	0,24
Total Costs (cents/th)	6,48	3,48	2,46	1,98	1,68

Table 7-1 Operation costs

It is assumed that these operation costs are relevant also for UK gas storage facilities. The demand charge is a fee paid to the owner of a gas storage facility by an external market participant who wishes to use the storage facility. In this analysis, however, the owner of the facility will not sell capacity, and the demand fee is ignored. The resulting operation cost is therefore 0,36 cents per therm, which is adjusted to 0,5 cents per therm due to costs associated with transportation from the national transportation system to the gas storage facility. The operation fee is set to be constant for both injection and withdrawal.

The cost of keeping inventory is ignored in the analysis.

7.1.4 Market conditions

Spot price

The spot price is assumed to follow the process developed in chapter 5, and the parameters are held constant over the evaluation period.

Risk-free rate of return

The risk-free rate of return was estimated as the historic average of Norwegian long-term bonds. Norges Bank provides data of 10-year bonds, with monthly resolution from the period January 1996 to May 2004. The estimated average was 6,0 %, which will be used throughout the analysis.

Currency

The gas traded on the UK gas market is given in UK sterling (£). Accordingly, the calculated values in this and the following chapters will be given in this currency.

Other

It is assumed that it is possible to buy or sell as much gas as is optimal on any given day. Also, holydays and leap days are ignored.

7.1.5 Valuation method

The valuation method can be summarized as follows:

1. Estimate a probability distribution of the spot price on every day for 15 years, using Monte Carlo simulation.
2. Establish a trading strategy based on the spot price distribution, using the procedure presented in chapter 6.
3. Operate the storage facility according to the suggested strategy using 15 years of Monte Carlo simulated spot prices. This will produce an estimate of the profit from operating the facility for 15 years.
4. Create a probability distribution of the estimated profit by repeating point 3 1000 times.

7.2 The market value results

In this section the market value of storage facility presented in the previous section will be determined.

7.2.1 A good price for gas

The marginal value of gas is the additional value one would get for having an additional unit in storage. As was described in chapter 8, the marginal value on day t determines the price at which gas will be traded, the optimal value, the following day for all possible storage levels. Fig 9-1 shows the optimal value function on day 180 and 320 (day 1 is defined as 1. January). The optimal value curve is adjusted up or down for the costs of injection and withdrawal, which means that if the optimal value is 20 and the operation costs are 0,5, the optimal value for selling will be 20,5 and the optimal value for buying will be 19,5. The optimal value of gas in storage on day 180 with 90 units in storage is 19,61 pence per therm when considering selling and 18,61 for buying.

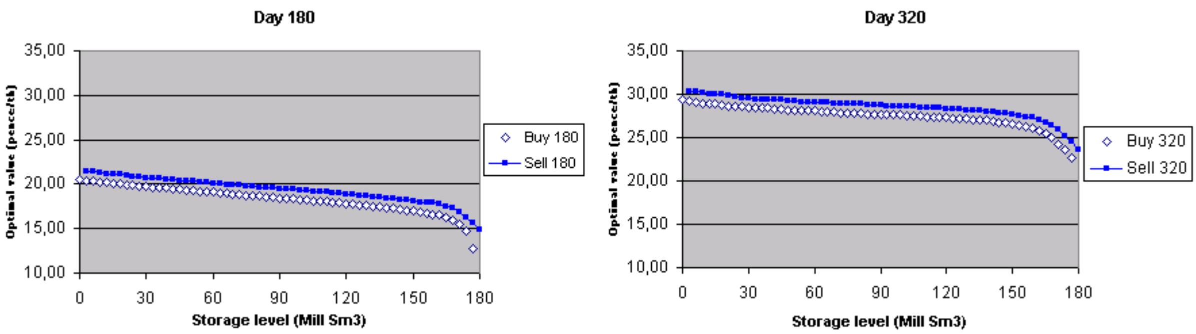
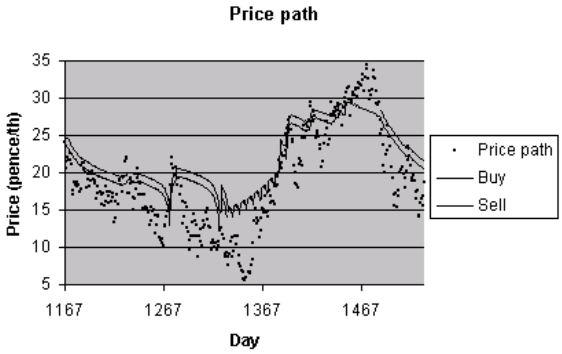


Figure 7-1 Optimal values on day 180 and 320

7.2.2 Storage trajectory

The simulation period is 15 years, and in figure 7-2 one year of simulated spot prices for one of the simulations is shown with the optimal value of natural gas in storage for both buying and selling. Figure 7-2 also shows the corresponding amount of gas in storage on each day of the year.

As the figure shows, the one-year period starts of with lower prices than the buy-limit (the lower line in figure 7-2(left)), and the storage starts to fill up. After approximately 60 days, the prices rise for a short period of time, and the facility release gas for sale. Another short period of low prices fills the storage facility up to maximum, before it again releases gas due to a sudden rise in the prices. It sells off about two thirds of its volume, but still expects to reach maximum before the high winter prices occur. And indeed, a long period of relatively low prices fills the facility back up to maximum. Then the facility waits for quite some time for the prices to increase, before selling of all the gas during the high prices of the winter months. After waiting a while for the prices to decrease, the same cycle starts up all over again.



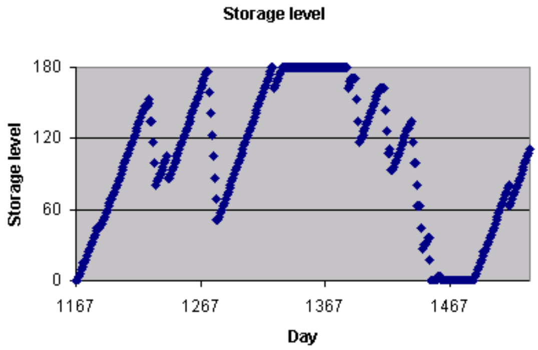


Figure 7-2 Simulated price path and corresponding storage level

7.2.3 The total profit

Simulating 15 years of operation of the natural gas storage facility gave the following probability distribution:

Mean value:	86 323 000 £
Maximum value:	105 428 000 £
Minimum value:	60 369 000 £
Standard deviation:	6 752 000 £

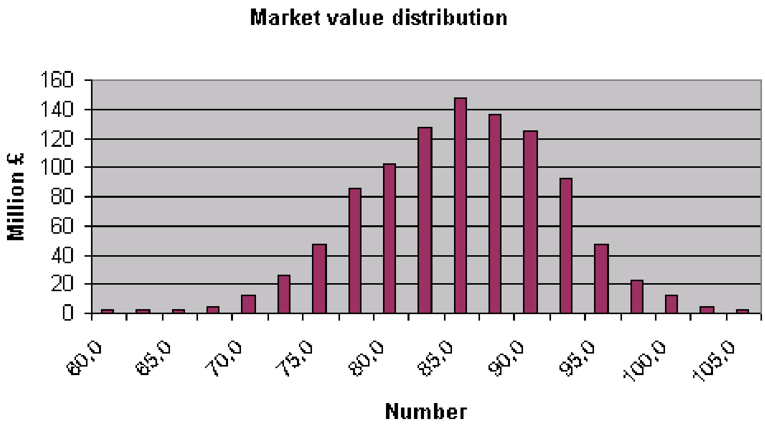


Figure 7-3 Profit distribution – Y = 0,85

The mean expected market value of the facility for 15 years is approximately 86 million £. The market value is however very dependent on the parameter Y, which represents volatility and mean reversion.

7.2.4 Market value related to the Y parameter

The market value is dependent on the Y parameter, which represents both mean reversion and volatility. Figure 7-4 shows the mean expected profit relative to the changing Y parameter, along with the minimum and maximum observed values of 1000 simulations. According to the confidence intervals of Y established in chapter 7, which will be used in this analysis, the possible range of Y will be [0,68-1,05], and the market value distribution will be within this range.

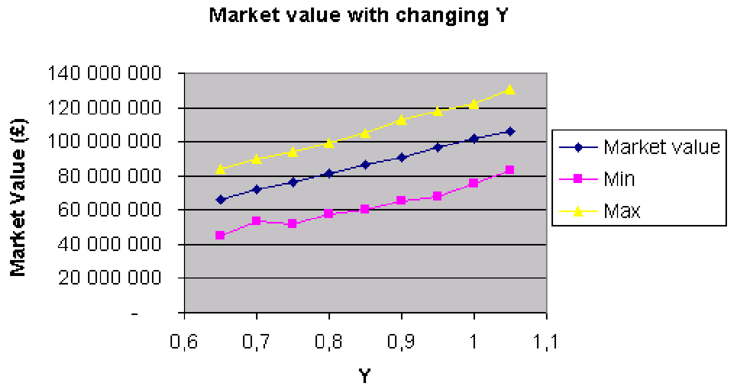


Figure 7-4 Market value with changing Y – value at start-up

As figure 7-4 shows, the mean market value ranges from 66 million £ to 106 million £ depending on the spot price determinant Y.

In figure 7-4 the value corresponds to having a storage facility ready for operation. It was earlier argued that it would take two years to develop the facility, and figure 9-5 corresponds to the expected market value of the facility two years prior to start-up. The values are discounted back two years, and these market values will be used to evaluate the option value.

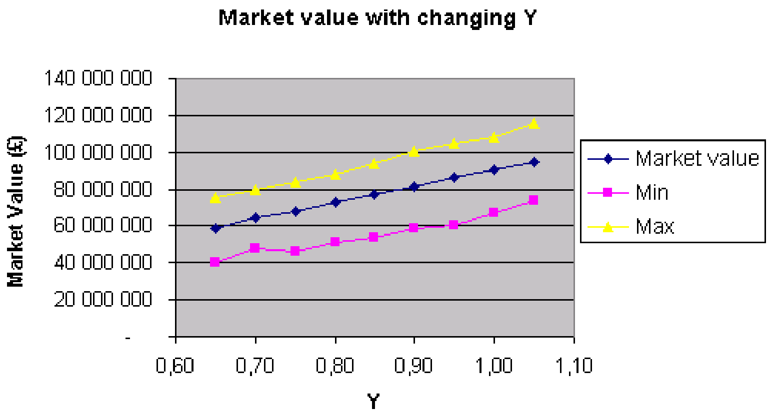


Figure 7-5 Market value with changing Y- Value two years before start-up

In order to evaluate the option in the following chapter, an equation of the market value (discounted back two years) related to Y, V(Y), is needed. To simplify the calculations this equation is preferred to be on the form

$$V(Y) = C \times e^{b \times Y} \qquad (7.1)$$

Microsoft Excel was used to perform the regression analysis and it gave the following results

$C = 27985000$

$b = 1,176$

$R^2 = 0,994$

The R-squared estimate represents the estimated function's ability to describe the observed values, and in this case this ability is 99,4%. The value function related to the Y parameter becomes

$$V(Y) = 27985000 \times e^{1,176 \times Y} \quad (7.2)$$

8 The option to invest in a gas storage facility

In chapter 6, a procedure for determining the market value of a natural gas storage facility was presented, and this procedure was used in chapter 7 on a specific salt dome storage facility. In this chapter the value of the option to invest in this specific facility is examined. The valuation method used in this chapter is a contingent claims analysis. The problem is an optimal stopping problem, where a firm has an option to invest in a gas storage facility. The question is when it is optimal to the investment cost I in return for a project whose value is V. The solution procedure used in this analysis was presented by Dixit and Pindyck (1994) and will establish both the value of the option to invest and the conditions that must be met for the option to be exercised.

8.1 The solution

The basic assumption for the model is that a firm must decide when to invest in a single project. The cost of the investment, I, is known and fixed, but the value of the project, V, follows a geometric Brownian motion. Because future values of V are unknown, there is an opportunity cost to investing today. McDonald and Siegel (1984) demonstrated that the simple net present value, invest when V>I, is wrong under these assumptions. Instead, the optimal investment rule is to invest when V is at least as large as a critical value V* that exceeds I.

The optimal investment rule is as follows

$$V(Y) - I \geq F(Y) \qquad (8.1)$$

where V(Y) is the market value of future cash flows
I is the total investment cost
F(Y) is the value of the option to invest

In equation 8.1 it is assumed that both the market value of the investment, V (Y), and the option value, F (Y), is dependent on the parameter Y. The investment rule states that even if the value of the investment is larger than the investment cost, it may not be optimal to invest. Investment should first take place when the net payoff, V – I, is greater than the value of the option.

The uncertainty of both the market value of the investment and the option to invest is represented by the uncertainty of the parameter Y. This parameter was established in chapter 5, and represents both the mean reversion and the volatility of the underlying spot price process. In chapter 5, three different processes for the long-term Y parameter were suggested. The suggested processes had all a drift rate of zero, but three different volatility parameters:

$$dY_i = \alpha_i Ydt + \sigma_i Ydz \quad (8.2)$$

where $\sigma_1 = 0{,}02790$, $\sigma_2 = 0{,}02151$, $\sigma_3 = 0{,}01467$
$\alpha_i = 0$ for i = 1, 2, 3

The first step is to determine the stochastic process that the value of the option follows. Dixit and Pindyck used Ito's lemma to achieve this, and obtained the following differential equation

$$\frac{1}{2}\sigma^2 Y^2 F'' + \alpha Y F' - rF = 0 \quad (8.3)$$

F'' and F' denotes derivatives. Assuming that the drift term, α, is zero, simplifies the equation

$$\frac{1}{2}\sigma^2 Y^2 F'' - rF = 0 \quad (8.4)$$

In addition, F (V) must satisfy the following boundary conditions

$$F(-M) = 0 \qquad \text{M is a large value} \quad (8.5)$$

$$F(Y^*) = \max\{V(Y^*) - I, 0\} \quad (8.6)$$

$$F'(Y^*) = V'(Y^*) \quad (8.7)$$

Y* is the optimal value of Y, at which the option is exercised. The first boundary condition states that the option has no value if Y goes towards zero. The second and third conditions are the value matching conditions, securing that the two curves are tangent and equal at the optimal value.

Inserting values in equation 8.4 gives the following differential equation

$$3{,}89 \times 10^{-4} F'' + 0{,}0582 \times F = 0 \quad (8.8)$$

In this example $\sigma_1 = 0{,}02790$ is used, and the risk free rate is the continuously compounded interest rate. This process is assumed to neutral and no adjustments are made. Equation 8.3 has a general solution

$$F(V) = A_1 e^{Y\beta_1} + A_2 e^{Y\beta_2} \quad (8.9)$$

where β_1 and β_2 are the roots of the quadric equation

$$3{,}89 \times 10^{-4} \beta^2 - 0{,}0582 = 0$$

$$\beta_1 = 12{,}23 \text{ and } \beta_2 = -12{,}23 \quad (8.10)$$

Boundary condition 8.5 implies that $A_2 = 0$, leaving the solution

$$F(V) = A_1 e^{12,23Y} \qquad (8.11)$$

To be able to use boundary conditions 8.6 and 8.7, the function V(Y) is needed. This was established in chapter 7

$$V(Y) = 27985000e^{1,176Y} \qquad (8.12)$$

Using equation 8.11 and 8.12, the high investment cost of 74 million £, and boundary conditions 8.6 and 8.7, the two unknowns A_1 and Y* can be determined. The solution yields

$$A_1 = 111,6 \qquad \text{and } Y^* = 0,913$$

This solution implies that, under the scenario of an investment cost of 74 million £ and the Y parameter having a volatility of 0,0279, the optimal investment decision would be to wait until the Y parameter reaches 0,913 and then invest. Today the Y parameter is 0,85, and the option to invest has a larger value than the expected market value less the investment cost. Figure 8-1 exhibits this result

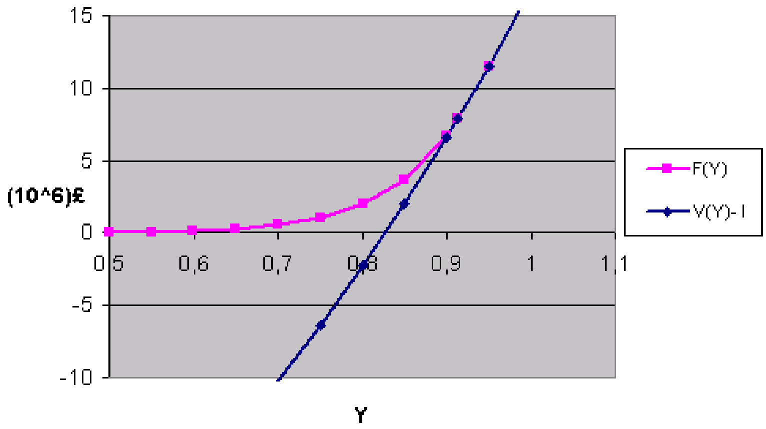


Figure 8-1 Option value: I = 74 million £, σ_Y= 0,02790

With Y=0,85, the option has a value of 3,65 million £. If the simple net present value rule was applied, invest if the present value of the expected cash flows is at least as large as the costs, the decision would be to invest right away, as this yields a profit of 2 million £. Under these specifications, however, the option value is greater then 2 million £, and the optimal decision would be to wait and see if the market conditions improved.

The scenario in the example above used the "high" investment cost of 74 million £. What happens if the investment cost decreases to 62 million £ or 50 million £? The calculations would be equivalent to the example above, and table 8-1 and figure 8-2 exhibit the results

INVESTMENT COSTS	A_1	Y*	F(Y*)	V(0,85) - I	Decision	F(0,85)
74 million £	111,6	0,91	7,6 million £	2 million £	**Wait**	3,7 million £
62 million £	588,8	0,76	6,4 million £	14 million £	**Invest**	-
50 million £	4447,5	0,58	5,3 million £	26 million £	**Invest**	-

Table 8-1 Optimal invest decision with changing investment costs, σ_Y=0,0279

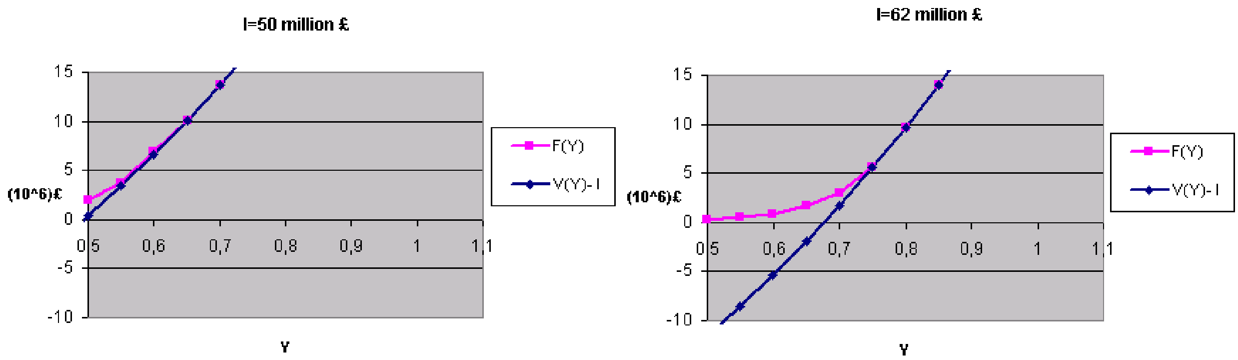


Figure 8-2 Option value--I = 50 million £ and I = 62 million £

The optimal investment limits, when the investment costs are either 50 or 62 million £, are well below today's value of 0,85, and the optimal decision would be to invest right away.

Using the results obtained above, an approximate investment cost limit when σ_Y=0,0279 can be established.

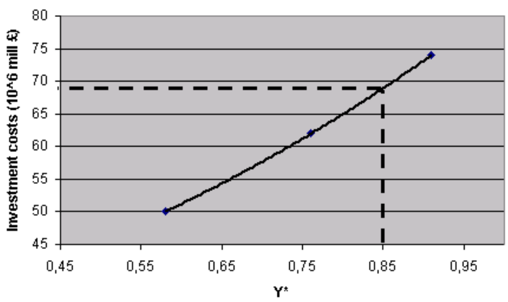


Figure 8-3 Investment cost related to Y*

When σ_Y=0,0279, the value of the option and the future cash flows less the investment is equal if the investment costs are 68,9 million £. The optimal decision rule would then be; invest if the investment costs are below or equal to 68,9 million £, and wait if they are above.

8.2 Results with changing volatility

In the previous section, the result was obtained using a volatility of 0,02790 for the Y parameter. This volatility parameter, however, is very uncertain. In this section the optimal investment decision will be evaluated with both changing volatility and changing investment costs. The calculations are equivalent to the example presented in the previous section, and the results are summarized in table 8-2.

VOLATILITY	INVESTMENT COSTS	A_1	Y*	F(Y*)	V(0,85) - I	Decision	F(0,85)
σ_Y=0,0279	74 million £	111,6	0,91	7,6 million £	2 million £	**Wait**	3,7 million £
	62 million £	588,8	0,76	6,4 million £	14 million £	**Invest**	-
	50 million £	4447,5	0,58	5,3 million £	26 million £	**Invest**	-
σ_Y=0,0215	74 million £	4,2	0,89	5,7 million £	2 million £	**Wait**	3,0 million £
	62 million £	38,2	0,74	4,8 million £	14 million £	**Invest**	-
	50 million £	562,0	0,56	4,0 million £	26 million £	**Invest**	-
σ_Y=0,0147	74 million £	0,007	0,87	4,2 million £	2 million £	**Wait**	2,6 million £
	62 million £	0,18	0,72	3,3 million £	14 million £	**Invest**	-
	50 million £	10,1	0,54	2,8 million £	26 million £	**Invest**	-

Table 8-2 Optimal investment decision

Table 8-2 shows that if the investment costs are 74 million £ the optimal decision would be to wait, and not exercise the option to invest. This decision is based on the fact that the option value is larger than the expected cash flows less the investment costs. The option value with today's Y value of 0,85 will be dependent on the volatility of Y. The results presented in table 8-2 show that the option value increases as the volatility increases, a result well known from traditional option theory. If the investment costs are 62 or 50 million £, the optimal decision would be to invest right away.

The investment cost limit for all the volatilities, the limit at which the option value is equal to the cash flows less the investment cost, is presented in table 8-3.

Volatility	Investment limit
σ_Y=0,0279	68,9 million £
σ_Y=0,0215	70,4 million £
σ_Y=0,0147	72,2 million £

Table 8-3 Investment limits with respect to volatility, Y=0,85

Table 8-3 shows the limit where immediate investment is optimal. Immediate investment is optimal if the investment costs are less or equal to this limit. If the investment costs are higher than this limit the option will not be exercised yet. It is further seen that the limit is less than the "high"-value of the investment cost of 74 million £. This means that with an investment cost of 74 million £ the option should not be exercised for any of the three volatility values. This is in accordance with the results presented in table 8-2.

8.3 Conclusion

Based on the analysis of the market value of a natural gas storage facility, and finally the option to invest in this storage facility, the conclusion is that this investment opportunity is very promising. The three volatility values used in this analysis produced either an incentive to invest in the facility right away, or a significant option value. Further investigations of the investment costs will produce a more conclusive answer. The investment costs used in this analysis were chosen with a "pessimistic" approach, using values in the top range of the costs per Sm^3 storage capacity. It is difficult to predict, but also the spot price parameters were chosen with a moderate outlook for the future. The correct values of the Y parameter and the investment costs are just two of the many uncertainties related to the investment decision. The uncertainty of the valuation method will be further discussed in the following chapter.

9 Discussion

When analysing an investment opportunity in general, and in this case the opportunity to invest in a natural gas storage facility, the results will be affected by the assumptions underlying the analysis. Simplifications of the real problem are inevitable, and sometimes simplifications that are clearly wrong must be made in order to obtain at least an approximate result. This analysis contains simplifications and perhaps erroneous assumptions. In this chapter some of these will be discussed.

9.1 The spot price process

9.1.1 The model

The spot price process is certainly one of the most influential factors in the analysis of the natural gas storage facility. The clearly erroneous assumptions that both volatility and mean reversion is constant over a period of 15 years have already been mentioned. These types of simplifications can easily be improved by incorporating seasonal varying volatility and mean reversion, or even making them stochastic. However, improving the model would also increase the complexity, which was considered unnecessary for this analysis.

Special days, such as holydays, normally imply lower spot prices due to decreased demand. When operating a natural gas storage facility, the low price during holydays will most likely be taken advantage of. In a more advanced model of the spot price, such factors could easily have been incorporated.

9.1.2 The available spot price data

The European gas market has grown rapidly the last decade, but the available market data is still not adequate when attempting to predict future spot prices. In this analysis historical data dating back to June 2000 were used. The storage facility valuation was based on 15 years of simulation, a considerable longer timeframe than the period of observed spot prices. The task of predicting future spot prices will always be difficult, meanwhile better market data would increase reliability of the estimates.

9.2 The valuation method

9.2.1 The optimal strategy

Equally important as the spot price evolution for this analysis, was the operation strategy of the facility. The chosen method for determining this strategy was considered to be quite good. It produced optimal strategies depending on both volume of gas already in storage and the day of the year in an intuitive way. But this method also contains limitations. At first the optimal strategy was determined once, and the facility was operated accordingly for the following 15 years. The operation of a storage facility is in fact much more flexible than that. Decision can, in an ideal world, be made daily. This would allow factors such as the spot price on the day of the decision, weather predictions, and others to be taken into account when making the decision. The increased information would create an upside potential for the value.

The method for determining the optimal strategy only considered the spot price. The storage facility could also use the futures market to hedge its positions, or perhaps sell vacant capacity to other parties willing to pay for the use of the facility. Such considerations are interesting, but would make the problem to large to handle.

Resolution of the input values could also influence the results. For instance, the continuous probability distribution of the daily spot price was approximated by 10 spot prices with equal probability of occurring. This could easily have been extended, but the amount of time the estimation program used increased rapidly with increased resolution and the line had to be drawn somewhere.

9.3 Operation restrictions

The facility in this analysis is only restricted by the injection and withdrawal rates and the maximum and minimum levels of gas in storage. The injection and withdrawal rates are held constant, when they in fact are a function of the volume of gas in storage. As the amount of gas in storage increases the pressure builds up, and the rate of injection decreases. The opposite would be true for withdrawal.

Over a period of 15 years one would expect that some maintenance would be required. It could be argued that maintenance primarily could be scheduled and then performed during periods of unattractive market conditions to reduce the negative impact of such downtime. However, maintenance, scheduled or not, would probably effect the market value negatively.

9.4 Market restrictions

In the analysis it is assumed that the optimal decision given by the strategy can be executed without limitations. The market will buy as much gas as is offered by the facility, and the facility may buy as much gas as it would like. The actions made by the facility have no impact on the prices observed in the market. How much error these assumptions produce is very difficult to predict, but tightening these restrictions would certainly have a negative effect on the market value.

9.5 Costs

9.5.1 Investment costs

The investment costs related to the investment are also highly uncertain in this analysis. However, before investing in a storage facility investment costs are normally well evaluated. As they are relatively certain at the point of investment, they will not cause much uncertainty about the decision to invest. The uncertainty is more likely to be on the income-side.

9.5.2 Operation costs

Operation cost will have a significant impact on the operational strategy. As the operation costs increase, a larger premium is required to make a trade and the difference between the buy limit and the sell limit will increase. In this analysis the operation costs are constant. It is more likely that the operation costs are dependent of how much the facility is used. The more cycles per year, the less operation costs per unit of gas injected or withdrawn from the facility.

9.6 Calculation mistakes

It is possible that calculation mistakes were done when calculating option values and the likes. It is also possible that mistakes were made when writing scripts used in Visual Basic to perform the calculations. The script may be wrong altogether, they may calculate something else than was intended. The scripts have been used on small examples to check for mistakes, but the possibility of mistakes cannot by totally removed.

10 Further work and final remarks

10.1 Further work

The methodology used in this thesis to evaluate an investment opportunity in a natural gas storage facility may be split into three separate sections:

- The spot price process
- The optimal operations strategy for the storage facility
- The market value and the option value of the investment opportunity

The spot price process used in this analysis was a relatively simple stochastic process. There has been done a vast amount of research within the field of energy price modelling, and this theory could be used to improve the spot price process used in this analysis. The problem concerning the amount of available price data will still be present for quite a few years.

The procedure used to determine the optimal strategy for the facility is interesting. Further work should be aimed at improving the restrictions imposed on the facility. In this analysis the facility operated totally free from “outside” interventions. More stringent restrictions are easily added to the stochastic dual dynamic program.

The addition of alternative uses for the facility, i.e. selling spare capacity to other parties interested in storing gas for future use, will improve and increase both the flexibility and the value of the facility. Investigating how alternative use can be exploited, may also increase the value of the option to invest in a storage facility.

10.2 Final remarks

In this thesis an investment opportunity in a natural gas storage facility has been analysed. This has been achieved by analysing the spot price dynamics, establishing an optimal strategy for the facility and finally implementing a real options approach to evaluate the investment decision. Based on the analysis it is difficult to give any final conclusions. The input to the problem is far too uncertain for any conclusions to be made. The calculations showed, however, that storage facilities should be of great interest for gas producers, gas distribution companies and large gas consumers, and that natural gas storage facilities may play an even more significant part in the growing European gas market in the years to come.

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