Electricity Capacity Expansion under Different Market Structures

A Real Options Approach on how to Optimally Expand Non-Renewable Capacity when Power is Treated as a Differentiated Product

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Project Thesis

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Abstract

This paper adopts a real options approach to analyze marginal investments in power markets with heterogeneous technologies and time-varying demand. We present a model for a monopolistic firm controlling the entire power market, and then extend the model to a Cournot duopoly. The main purpose of the paper is to examine the investment behavior of a monopolist, a central planner and two duopolistic firms when two types of power plants are available; base and peak load power plants. We find that in a Cournot duopoly one will install more peak load capacity than base load capacity. A central planner and a monopolist will on the other hand install more base load capacity than peak load capacity. Furthermore, we examine the effect of analyzing power markets without time-varying demand and find that this will lead to shortage of peak load capacity.

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Chapter 1

Introduction

Expansion of capacity in power systems is on the agenda both in developing countries, where demand is growing, and in industrialized countries, where concerns about climate change is a driving force. A non-renewable energy demand is expected to remain despite the government's policies supporting sustainable technologies. Renewable energy sources are non-controllable because they depend heavily on the weather. We thus need a proportion of the total dispatch to be non-renewable. Peak load power plants are well suited to manage energy shortage when the electricity demand exceeds the renewable generation.

Traditional real options capacity expansion theory assumes a constant demand throughout the year. However, this is not the case in the power sector. Thus, power is a differentiated product. Assuming the demand to be fixed over the year may lead to errors when deciding optimal capacity expansions in base and peak load power plants. This can result in excessive investments in base load plants on behalf of peak load plants compared to the requirements in actual power markets, which is undesirable. Hence, we adopt a real options approach for electricity capacity expansion that considers the time-varying demand by dividing the year into load segments, where the power demand is different in each segment. The effects of considering the electricity demand time-varying instead of fixed are discussed in an example with a peak and a base load power plant.

Following the deregulation of European electricity markets in the last decades, the authorities' focus on maximizing social welfare has been replaced by the companies aim to maximize their profits. This change of objective requires decision support that takes the new market structure into account. European electricity markets are considered oligopolies after their deregulation. Several mergers and acquisitions have resulted in markets with few suppliers with significant market shares. Investment decisions in an oligopolistic market depend on actions of the competitors as well as economic variables like demand and marginal cost. Hence, we create a framework that includes market power. We start out by examining a basic monopoly model to discover how market power influences capacity investments. Then we extend the model to a Cournot duopoly setting to better illustrate the dynamics of electricity markets. Hobbs (2007), Jing-Yuan and Smeers (1999), Wogrin et al. (2011) and Gabriel et al. (2012) also view power markets as Cournot equilibriums.

When using a real options approach, the investment is considered irreversible and the investment cost is thus a sunk cost. Although the production assets can be sold from one firm to another, its scrap value is small due to its lack of alternative use. If the electricity demand falls, the value of the electricity producing assets fall as well. The assumptions above suit electricity expansion problems well. Investments in power equipment are capital intensive, and the equipment is difficult to sell once it is installed, particularly when considering the whole industry at the same time.

Capacity expansions are not now-or-never decisions. The investment can be delayed until the company has more information about the uncertainties mentioned above. Treating capacity expansions with a real options approach provides flexibility to the investor because it takes the value of waiting into account. Flexibility with respect to the size of the capacity is also considered. Investments are done marginally so that the companies decide their exact capacity expansion at each time.

Gahungu and Smeers (2012) introduce a real options capacity expansion model for power generation under perfect competition. Their model includes heterogeneous technologies with different investment and variable costs. Power is treated as a differentiated product due to its lack of storability. Thus, the power demand varies between different load segments. The electricity price follows a geometric Brownian motion and an inverse demand function. Instantaneous social welfare is found by an optimization problem that maximizes the social welfare where the annual dispatch is constrained by the installed capacity. To find the value of all capacity expansions, the expected total social welfare from power production and the capacity investment cost are discounted. This represents a stochastic control problem. Then the assumption of myopia is used to convert the stochastic control problem to an optimal stopping problem solved by dynamic programming. To solve the optimal stopping problem analytically proves difficult. Hence, the Lagrange multipliers from the capacity constraints, representing additional instantaneous social welfare from a marginal capacity expansion, is used to compute the marginal value of a capacity expansion though Monte Carlo simulations. Then the discounted marginal values are regressed with the stochastic demand shock as an explanatory variable. The resulting regression coefficients determine the particular solution of the optimal stopping problem.

By taking Gahungu and Smeers (2012) as a starting point, we develop a real options capacity expansion problem under monopoly and duopoly. This makes us able to compute social welfare losses in settings with market power relative to a market governed by a central planner. Combining real options and a social welfare perspective is also done by Huisman and Kort (2016) and Pawlina and Kort (2006) among others. We study capacity expansions for a portfolio of electricity expansion technologies that may differ in both operational and investment costs. In duopolies, both the generation assets and the companies are additively non-separable. Dixit and Pindyck (1994), Madlener et al. (2005), Pindyck (1988), Aguerrevere (2003), Pe (2000) and He and Pindyck (1992) do also model capacity expansions with real options, but they consider one technology or heterogeneous technologies.

We model the electricity price as dependent of a geometric Brownian motion. This is previously proven by Lucia and Schwartz (2002), Schwartz and Smith (2000), Pindyck (2001) and Fleten et al. (2007) to be an appropriate assumption. Basic real options models presented by Dixit and Pindyck (1994) assumes an exogenous price process. However, this is incorrect for electricity markets. An exogenous price process excludes strategic behavior and the feedback effect on the price when new capacity is installed. Thus, we find profits from power production by an optimization problem, like modeled by Madlener et al. (2005) and. Hobbs (2007). Unlike them, we find the operating profits in a continuous time.

The capacity expansion problem is formulated as a stochastic control problem, the aim of which is to optimally expand the capacity of each technology in the presence of demand uncertainty such as to maximize the expected net present value of the portfolio over an infinite time horizon. Hence, capacity is the control variable that must be adapted to the stochastic demand. Stochastic control problems can be converted to optimal stopping problems if the properties of myopia hold, as described by Karatzas and Shreve (1984), Baldursson and Karatzas (1996), Back and Paulsen (2009) and Boetius and Kohlmann (1998) among others. Myopia implies that each investment in incremental capacity is the last one over the time horizon, and it holds for electricity capacity expansions with one technology or several technologies with identical investment cost as stated by Pindyck (1998). As it does not necessarily hold for our capacity expansion problem, myopia is an assumption made to facilitate its solution.

With myopia, we use canonical real options. Such options consider a series of capacity expansions and allow for an endogenous electricity price when the capacity is held constant. Furthermore, the value of the capacity expansion and the optimal expansion path are determined simultaneously. By assuming myopia, also non-additively separable production assets and firms are managed. Canonical real options theory is mainly used in markets within monopoly and perfect competition due to assumptions about homogeneous companies and symmetric technologies. With the assumption of myopia, however, we apply it to a diverse portfolio of technologies.

In a numerical example, we illustrate how investment triggers, social welfare losses and optimal investment paths varies between market structures. Not surprisingly, the welfare losses are largest in a monopoly. The monopolist has no competitors. Thus, the monopolist has the largest incentive to wait. In a duopoly, on the other hand, the companies investment decisions accelerate due to the competition. Hence, the total installed capacity is higher in a duopoly than in a monopoly, and the social welfare exceeds that of the monopoly. We also quantify the effects of considering the electricity demand time-varying instead of constant during the year. With a constant demand, it is optimal to invest only in base load capacity. However, when we model time-varying demand by using six load segments we also invest in peak load capacity. This indicates that capacity expansion models with a fixed demand over the year provide inadequate decision support for power companies having the opportunity to invest in peak load capacity.

The paper is structured as follows. Chapter 2 examines our real options approach. First, we argue why we model the demand shock process as a geometric Brownian motion. Then the capacity expansion approach of a monopolist is presented as a stochastic control problem and an optimal stopping problem with the corresponding investment trigger equation. Furthermore, the duopoly extension of the model is presented. Finally, the concept of social welfare is introduced. In chapter 3 we propose a procedure for solving the models in chapter 2 numerically. We use the shadow costs of the capacity constraints to find the marginal value of investing and hence, the investment triggers. Then we present how to calculate the optimal investment path and the expected discounted value of the social welfare. Chapter 4 presents a numerical example that illustrates our capacity expansion approach and compares decision-making under different market structures. Chapter 5 concludes.

Chapter 2

Capacity expansion under market power

Consider an overall framework based on Dixit and Pindyck (1994) that covers real options theory. We start by arguing why a stochastic shock process and an inverse demand function determine the electricity price. We then derive a real options model for investing marginally under monopoly. The instantaneous profit is found by an optimization problem in section 2.2. To find the value of all capacity expansions, we discount all future cash flows in section 2.3. According to real options theory presented by Dixit and Pindyck (1994), we convert the stochastic control problem of the monopolist to an optimal stopping problem in section 2.4. In section 2.5 our approach is extended to a duopoly to better describe the real world of electricity markets. Section 2.6 describes how our investment approach can be converted to markets with perfect competition. Here, a central planner seeks to maximize the social welfare by taking both the consumer and the producer surplus into account.

2.1 Electricity price and demand shock processes

When a firm decides whether to invest in a capacity expansion, the electricity price is the most uncertain variable. Hence, it is important to consider the fluctuations in the electricity price in the investment analysis. Fleten et al. (2007) argue that electricity prices tend to revert to the long-term generation cost of electricity. The probability of prices reverting to the long-term generation cost is therefore higher than the probability of prices moving away from it. This indicates mean reverting electricity prices. Fleten et al. (2007) concludes that the mean reversion may come as a consequence of reverting fuel prices or varying generation.

Lucia and Schwartz (2002) have studied electricity prices with one and two-factor models. The one-factor models indicate that the electricity price follows a mean reverting process. However, the two factor models explain both short and long-term variations in the electricity prices: Short-term electricity prices follow a mean-reverting process, and the long-term electricity prices follow an arithmetic or geometric Brownian motion. The two-factor models have the best fit with empirical data on the electricity price. However, Pindyck (2001) claims that modelling the long-term electricity price as a geometric Brownian motion results in small errors.

Power producing assets have a long time horizon. Short-term changes in the electricity price have an insignificant influence on the profitability of the capacity investment. Hence, we consider modeling electricity prices by a geometric Brownian motion. A geometric Brownian motion consist of a deterministic growth and a random term determined by the volatility

$$dP_t = \mu_P P_t dt + \sigma_P P_t dz_t. \tag{2.1}$$

In (2.1) μ_P is the drift rate, dz_t is the incremental Wiener process and $\sigma_P > 0$ is the standard deviation. Furthermore, we assume the discount rate ρ to be larger than the drift μ. If this inequality does not hold, the opportunity cost of investing in a unit of capacity always exceeds the benefit of investing. Hence, the firm will never invest in new capacity (Dixit and Pindyck, 1994).

Electricity cannot be stored, and is thus a differentiated product. As a result, the electricity price is time-varying. This is treated by dividing each year into $d(\mathbf{L})$ load segments with different electricity demand. Power generation depends on the load, which is the energy demand per unit of time denominated in MW. The load duration curve in Figure 2.1 illustrates the load during the year, where $\mathbf{L}$ is a set of load segments and τ_l is the duration of load segment $l \in \mathbf{L}$ so that $\sum_{l=1}^{d(\mathbf{L})} \tau_l = 8760$ h.

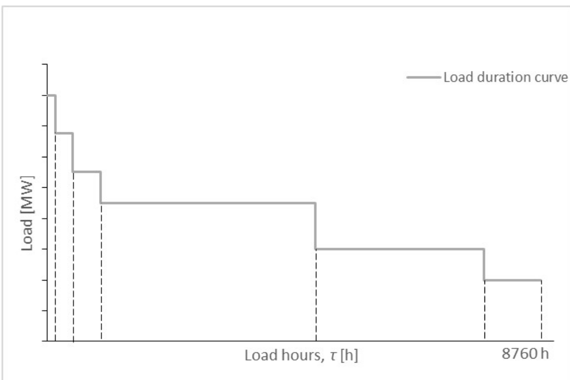


Figure 2.1: Load duration curve for 6 load segments, $d(\mathbf{L}) = 6$.

To find the optimal capacity expansions under different market structures, we also use the inverse demand function $D(Q_l)$ to predict the electricity prices. Q_l is the dispatch in each load segment $l \in \mathbf{L}$. Hence, we let the electricity price depend on both the inverse demand function $D(Q_l)$ and the exogenous shock process Y_t, following the geometric Brownian motion

$$dY_t = \mu Y_t dt + \sigma Y_t dz_t, \tag{2.2}$$

in which μ is the drift rate of the demand and σ is the standard deviation of the demand. The electricity price is then given by

$$P_l = Y_t D(Q_l),\ \forall l \in \mathbf{L}. \tag{2.3}$$

2.2 Instantaneous profit

As advocated in the introduction, capacity systems have heterogeneous generating technologies. Each technology is denoted k and is a part of the set of technologies $\mathbf{K}$. The generators have a capacity K_k for each technology $k \in \mathbf{K}$. $q_{k,l}$ is the produced electricity by technology k in load segment l. Hence, K_k is the maximal value of $q_{k,l}$. The lower limit on the production $q_{k,l}$ of technology $k \in \mathbf{K}$ is 0.

Operational and maintenance costs of each technology $k \in \mathbf{K}$ are given in terms of the installed capacity K_k. Hence, OMC_k is the operational and maintenance cost per unit of installed capacity of technology k and the total capacity related costs are $\sum_k OMC_k K_k$. The unit production cost for each technology k is denoted c_k. This results in the variable costs of $\sum_{k,l} c_k q_{k,l}$. The cost of investing in one additional capacity unit of technology k is denoted I_k.We assume that the cost occurs instantaneously after an investment decision and that the additional capacity is available immediately after the investment. The revenues in load segment l are given as the product of the price function in equation (2.3) and the total quantum sold electricity $Q_l = \sum_{k\in\mathbf{K}} q_{k,l}$ in each load segment $l \in \mathbf{L}$.

As the only player in the market, the monopolist controls electricity production and price. The monopolist seeks to maximize its instantaneous profit, the producer surplus. This is the difference between the instantaneous revenues and costs given a constrained capacity. Pindyck and Rubinfeld (2013) show that the producer surplus is maximized when the marginal production cost equals the marginal revenues of producing. Problem 1 finds the monopolists instantaneous profit by an optimization problem.

Problem 1 Instantaneous profit of the monopolist

$$\pi(Y_t, K_1, \ldots, K_{d(\mathbf{K})}) = \max_q \sum_{l=1}^{d(\mathbf{L})} \tau_l \left\{ P_l(Y_t, Q_l) Q_l - \sum_{k=1}^{d(\mathbf{K})} c_k q_{k,l} \right\} - \sum_{k=1}^{d(\mathbf{K})} OMC_k K_k \tag{2.4}$$

s.t.

$$q_{k,l} \geq 0, \qquad \forall l \in \mathbf{L}, \forall k \in \mathbf{K} \tag{2.5}$$

$$q_{k,l} \leq K_k, \qquad \forall l \in \mathbf{L}, \forall k \in \mathbf{K} \tag{2.6}$$

$$\sum_{k=1}^{d(\mathbf{K})} q_{k,l} = Q_l \qquad \forall l \in \mathbf{L} \tag{2.7}$$

Equation (2.4) and (2.5) constrain the electricity produced $q_{k,l}$ to not exceed its upper limit K_k or fall below its lower limit 0. Equation (2.6) aggregates the the dispatches from each technology $k \in \mathbf{K}$ to simplify the objective function (2.3).

Due to a downward sloping inverse demand curve, the objective in Problem 1 is concave. This combined with linear constraints makes Problem 1 convex. Thus, the problem can easily be solved numerically and has an unique solution. The inverse demand function $D(Q_l)$ proves the profits from each technology k to be additively non-separable. When the dispatch $q_{k,l}$ increases, the inverse demand function $D(Q_l)$ decreases. When holding Y_t fixed, a larger dispatch results in reduced electricity prices. Changes in Y_t have an impact on the optimal dispatch from each technology in each load segment $q_{k,l}$ and the instantaneous profit $\pi(Y_t, K_1, \ldots, K_{d(\mathbf{K})})$. Holding the inverse demand function $D(Q)_l$ fixed, an increase in Y results in a larger instantaneous profit $\pi(Y_t, K_1, \ldots, K_{d(\mathbf{K})})$. Changes in K_k also affect the profit flow $\pi(Y_t, K_1, \ldots, K_{d(\mathbf{K})})$ through setting an upper limit on the electricity generation.

2.3 Value of capacity expansion

Investments are assumed irreversible, and incremental over an infinite time horizon. Future cash flows are discounted with the exogenous annual rate ρ. In year zero, the demand shock is Y_0 and the installed capacity is $K_{k,0},\ k \in \mathbf{K}$. For every demand shock in each time interval Y_t, the monopolist expands its capacity to $K_{k,t},\ k \in \mathbf{K}$ at the per unit investment cost $I_k,\ k \in \mathbf{K}$ that maximizes the profit. Because of the irreversible investments, Gahungu and Smeers (2012) claims the capacity in each time interval $K_{k,t}$, $k \in \mathbf{K}$, should be measurable with respect to the filtration generated by Y_t. $F(Y_0, K_{1,0}, \ldots, K_{d(\mathbf{K}),0})$ represents the value of the expansion path. When the monopolist has no other assets except its generation capacity, $F(Y_0, K_{1,0}, \ldots, K_{d(\mathbf{K}),0})$ is equivalent to the value of the monopolistic firm. Y_t is the stochastic shock process and $K_{k,t}$ is the stochastic capacity level in technology k at time t so that $K_{k,t} \leq K_{k,t+dt}\ \forall k \in \mathbf{K}$. In Problem 2, the value of the capacity expansion is found.

Problem 2 Value function of a monopolistic firm

$$F(Y_0, K_{1,0}, \ldots, K_{d(\mathbf{K}),0}) = \max_{K_{k,t}} \mathbf{E}\Bigg[\int_0^{\infty} \pi(Y_t, K_{1,t}, \ldots, K_{d(\mathbf{K}),t}) e^{-\rho t} dt - \sum_{k=1}^{d(\mathbf{K})} \int_0^{\infty} I_k e^{-\rho t} dK_{k,t} \Bigg]. \quad (2.8)$$

The monopolist invests in new capacity to maximize its expected value over an infinite time horizon. The first term on the right-hand side of the equality represents all future expected discounted profits of the monopolist. The second term on the right-hand side is the total expected discounted investment costs related to capacity investments. Hence, Problem 2 integrates over every point in time t to find the value of installed capacities $F(Y_0, K_{1,0}, \ldots, K_{d(\mathbf{K}),0})$. The problem needs to be transformed to an optimal stopping problem to be solved analytically.

2.4 Optimal stopping problem

Problem 2 is considered a stochastic control problem by Dixit and Pindyck (1994). The complexity of Problem 2 implies that in can not be solved analytical. If the properties of myopia hold, Problem 2 can be converted to an optimal stopping problem. When myopia holds, one assumes the investment to be the last capacity expansion over the time horizon. Then there will not be any further capacity expansions. Hence, we find the optimal timing for investing marginally while holding the capacity fixed.

There are several properties that must hold for myopia to exist, all presented by Gahungu and Smeers (2012). First, the economy is convex. This implies non-increasing returns to scale. Next, investments must be incremental. Market players and the technologies used must also be homogeneous for myopia to hold, such that the profit is additively separable.

The assumption of homogeneous technologies is the only assumption mentioned above that does not hold. Different technologies in an electricity system do not operate independently. Using one technology may be beneficial to other technologies used. Additionally, investing in one technology depreciates the value of investing in other technologies in the power system. This makes the profits and capacities from the different technologies found by the inverse demand curve non-separable.

Because all the other properties of myopia hold, we use myopia as an approximation to solve the optimal stopping problem despite the heterogeneous technologies. One can also argue that myopia is an acceptable approximation because of the monopolists behavior. When the monopolist considers the next investment attractive, this is assumed to be the last one. When time passes and the electricity demand is increased, a new investment might be undertaken as well, despite the earlier belief of the previous investment to be the last one.

Assuming myopia when it is not guaranteed may result in investment triggers which deviate from their optimal values. However, results from the numerical example in section 4, indicate that our capacity expansion approach returns reasonable results. To compensate for non-additively separable profits from different technologies, we propose a regression $\bar{\pi}$ in (2.9) to express the instantaneous profit from Problem 1 analytically as a function of Y and K_k. Then the real options problem can be solved analytically.

$$\begin{aligned}\bar{\pi}(Y, K_1, \ldots, K_{d(\mathbf{K})}) = & \sum_{k=1}^{d(\mathbf{K})} \sum_{i,j=1}^{d(\gamma),d(\alpha)} b_{k,ij} Y^{\gamma_i} K_k^{\alpha_j} + \\ & \sum_{\substack{k,u=1,\\ u\neq k}}^{d(\mathbf{K})} \sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)} c_{uk,ijl} Y^{\gamma_i} K_u^{\lambda_j} K_k^{\lambda_l} - \sum_{k=1}^{d(\mathbf{K})} OMC_k K_k. \quad (2.9)\end{aligned}$$

The first term of the regression shows the profit flow from each technology k. $b_{k,ij}$ are the regression coefficients that describe how changes in the capacity of technology k affect the instantaneous profit flow for a given shock process Y. The second term of the regression corrects for synergies between technologies. For a given Y, the regression coefficients $c_{uk,ijl}$ describe how the different technologies affect each other's profit flows when they are installed.

γ, α and λ are positive base vectors of dimensions $d(\gamma)$, $d(\alpha)$ and $d(\lambda)$ used to describe changes in $\bar{\pi}(Y, K_1, \ldots, K_{d(\mathbf{K})})$ with respect to Y, K_k and K_u. To find a unique root of the trigger equation for each technology (2.21), we constrain the base vectors in the regression like Gahungu and Smeers (2012). We set γ_i by $0 < \gamma_i < \beta_1 \; \forall i$, where β_1 represents the positive solution of the fundamental quadratic equation presented in (2.16). To ensure one unique investment trigger, concavity and non-increasing return to scale, we establish $0 < \alpha_j < 1, \forall j, 0 < \lambda_j < 1 \, \forall j$ and $\lambda_i + \lambda_j \leq 1$ when $i \neq j$. Since new installed capacity has a positive effect on the profits, $b_{k,ij} \geq 0$. Using different technologies causes positive synergies, and thus $c_{uk,ijl} \geq 0$.

We convert the stochastic control problem in Problem 2 to an optimal stopping problem. Myopia implies that the option to invest marginally can be written

$$\frac{\partial F(Y, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k} = V_k(Y, I_k, K_1, \ldots, K_{d(\mathbf{K})}) = \max_{\tau} \mathbf{E}\left[\int_{\tau}^{\infty} \frac{\partial \bar{\pi}((Y, K_1, \ldots, K_{d(\mathbf{K})}))}{\partial K_k} e^{-\rho t} dt - I_k e^{-\rho \tau} \right] \forall k \in \mathbf{K}, \quad (2.10)$$

where $V_k(Y, I_k, K_1, \ldots, K_{d(\mathbf{K})})$ is the option to invest marginally in technology k and τ is the investment timing. Equation (2.10) implies that the option to invest marginally in additional capacity in technology k equals the extra profit gained by a marginal capacity expansion in k less marginal investment cost of expanding when the expansion is timed optimally.

First, we derive the Bellman equation of the optimal stopping problem using dynamic programing. This states that the return from V_k over a time step dt equals the profit of a marginally increased capacity over dt, $\frac{\partial \bar{\pi}}{\partial K_k}$, and the expected change in V_k over dt, $\mathbf{E}[dV_k]$. Thus, the Bellman equation of the optimal capacity expansion problem is

$$\rho V_k dt = \frac{\partial \bar{\pi}}{\partial K_k} dt + \mathbf{E}[dV_k], \; \forall k \in \mathbf{K}. \quad (2.11)$$

We expand the right-hand side of equation (2.11) by Ito's lemma in Appendix A. This gives partial differential equation of the optimal stopping problem stated in Problem 3. We introduce the convenience yield $\delta = \rho - \mu$ to simplify the problem. For sortable commodities, the convenience yield represents the benefit of holding the commodity instead of holding a futures contract on it, according to Bøckman et al. (2008). The convenience yield of electricity is interpreted as the relative benefit of delivering the commodity earlier rather than later.

Problem 3 Optimal stopping problem for the monopolist

$$\frac{1}{2}\sigma^2 Y^2 \frac{\partial^3 F(Y, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k \partial Y^2} + (\rho - \delta) Y \frac{\partial^2 F(Y, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k \partial Y} - \rho \frac{\partial F(Y, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k} + \frac{\partial \bar{\pi}(Y, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k} = 0, \; \forall k \in \mathbf{K}, \quad (2.12)$$

with boundary conditions

$$\frac{\partial F(0, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k} = 0, \; \forall k \in \mathbf{K}, \quad (2.13)$$

$$\frac{\partial F(Y_k^*, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k} = I_k, \ \forall k \in \mathbf{K} \tag{2.14}$$

$$\frac{\partial^2 F(Y_k^*, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k \partial Y} = 0, \ \forall k \in \mathbf{K}. \tag{2.15}$$

Equation 2.13 ensures that the option to invest in new capacity is zero when the value of the capacity expansion is zero. 2.14 and 2.15 are respectively the value matching and the smooth pasting conditions for an incremental investment in new capacity.

The investment problem has the homogeneous solution $F_h(Y, K_1, \ldots, K_{d(\mathbf{K})}) = A(K_1, \ldots, K_{d(\mathbf{K})})Y^{\beta_1}$, with β_1 is given by the positive root of the quadratic equation

$$\beta_1 = (\frac{1}{2} - \frac{\mu}{\sigma^2}) + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2\rho}{\sigma^2}}. \tag{2.16}$$

The particular solution of the Bellman equation 2.12 is the underlying value of the capacity expansion; the monopolists profit flow from investing. This is derived in Appendix B by finding the particular integral of each term in the profit interpolation in equation (2.9).

$$\begin{aligned} F_p(Y, K_1, \ldots, K_{d(\mathbf{K})}) = &\sum_{k=1}^{d(\mathbf{K})} \sum_{i,j=1}^{d(\gamma),d(\alpha)} \bar{b}_{k,ij}(\gamma) Y^{\gamma_i} K_k^{\alpha_j} + \\ &\sum_{\substack{k,u=1, \\ u \neq k}}^{d(\mathbf{K})} \sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)} \bar{c}_{uk,ijl}(\gamma) Y^{\gamma_i} K_u^{\lambda_j} K_k^{\lambda_l} - \sum_{k=1}^{d(\mathbf{K})} \frac{OMC_k K_k}{\rho} \end{aligned} \tag{2.17}$$

where $\bar{b}_{k,ij}$ and $\bar{c}_{uk,ijl}$ are given by

$$\bar{b}_{k,ij}(\gamma) = \frac{b_{k,ij}}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 + \gamma_i(\gamma_i - 1)}, \tag{2.18}$$

$$\bar{c}_{uk,ijl}(\gamma) = \frac{c_{uk,ijl}}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 + \gamma_i(\gamma_i - 1)}. \tag{2.19}$$

The solution of the Bellman equation is the sum of the homogeneous and the particular solution $F(Y, K_1, \ldots, K_{d(\mathbf{K})}) = F_h(Y, K_1, \ldots, K_{d(\mathbf{K})}) + F_p(Y, K_1, \ldots, K_{d(\mathbf{K})})$. This gives the monopolist the following value of investments in technology k.

$$\begin{aligned} F_k(Y, K_1, \ldots, K_{d(\mathbf{K})}) = A_{k1}(K_1, \ldots, K_{d(\mathbf{K})})Y^{\beta_1} + &\sum_{i,j}^{d(\gamma),d(\alpha)} \bar{b}_{k,ij}(\gamma_i) Y^{\gamma_i} K_k^{\alpha j} + \\ &\sum_{u=1, u \neq k}^{d(\mathbf{K})} \sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)} \bar{c}_{kl,ij}(\gamma_i) Y^{\gamma_i} K_u^{\lambda_j} K_k^{\lambda_l - 1} - \frac{OMC_k K_k}{\rho}. \end{aligned} \tag{2.20}$$

By using the value matching and smooth pasting conditions as described in Appendix C, we obtain the equation that when solved returns the investment trigger Y_k^*.

$$\sum_{i=1}^{d(\gamma)}(Y_k^*)^{\gamma_i}\left(\frac{\beta_1-\gamma_i}{\beta_1}\right)\left\{\sum_{j=1}^{d(\alpha)}\alpha_j\bar{b}_{k,ij}(\gamma i)K_k^{\alpha_j-1}+\right.$$
$$\left.\sum_{u=1,u\neq k}^{d(\mathbf{K})}\sum_{j,l=1}^{d(\lambda),d(\lambda)}\bar{c}_{kl,ij}(\gamma_i)K_u^{\lambda_j}K_k^{\lambda_l-1}\right\}=I_k+\frac{OMC_k}{\rho}. \quad (2.21)$$

Y_k^* is the optimal investment trigger of the firm. The investment procedure in each time step is as following. If $Y_t > Y_k^*$, the firm should invest marginally in new capacity K_k with production technology k. Then we repeat the procedure in section 2.2 and 2.4 to find the new investment trigger Y_{new}^*. If $Y_t > Y_{k,new}^*$ still holds, we invest marginally again. This procedure should be repeated until Y_k^* reaches Y_t. Then we move a on to the next time step and repeat the investment procedure. Finally, we calculate the value of all investments, as stated in Problem 2.

2.5 Duopoly expansion of the model

After the restructuring of the electricity market in Europe and the US in the 1990s, the market now consists of several players, each seeking to maximize their own profit under while considering the other producers' dispatch. Thus, Jing-Yuan and Smeers (1999) claims that the electricity markets can be modeled as oligopolies. For illustrative and computational purposes, our model is reduced to a duopoly. However, if desired, one can extend our model to a general oligopoly.

When capacity expansion in a duopoly is considered, one needs to take strategic behavior into account. Consequently, the optimization approach of the monopolist is no longer valid. Jing-Yuan and Smeers (1999) argues that one instead should model a market equilibrium. Huisman and Kort (2016) studies capacity expansion problems in a duopoly without any prior installed capacity using a leader-follower-model. However, in our modeling of the electricity market both players possess an initial capacity. Thus, we do not use the leader-follower-approach.

Jing-Yuan and Smeers (1999) and Wogrin et al. (2011) have previously used Cournot assumptions when modeling electricity capacity expansion. The electricity market consists of firms using heterogeneous technologies to produce a homogeneous product. Although we treat electricity as a differentiated product, it is homogeneous within each load segment. At each point in time, both firms make their production and marginal investment decision simultaneously and independently. This fits well with the Cournot assumptions and we, therefore, choose the Cournot model as a starting point for our approach.

In the rest of this chapter, we assume two players, firm 1 and firm 2. Due to the Cournot model, all equations are symmetric for both firms. Hence, all models and equations are stated only for firm 1. Before applying them on firm 2, the indexes must be switched. The instantaneous profit of a duopolist is stated in Problem 4.

Problem 4 Instantaneous profit of the duopolistic firm

$$\pi_1(Y_{1t}, K_1, K_2) = \max_{q_{1,l}} \sum_{l=1}^{d(\mathbf{L})} \tau_l [P_l(Y_t, q_{1,l} + q_{2,l}) q_{1,l} - c_1 q_{1,l}] - OMC_1 K_1 \tag{2.22}$$

s.t.

$$q_{1,l} \geq 0, \qquad \forall l \in \mathbf{L} \tag{2.23}$$

$$q_{1,l} \leq K_1, \qquad \forall l \in \mathbf{L} \tag{2.24}$$

Appendix E derives the Counot equilibrium.

By using the same assumptions as in the monopoly approach, we obtain the value function of the duopolistic firm, which aims to maximize its own profits over an infinite time horizon. The duopolist's stochastic control problem is stated in Problem 5.

Problem 5 Value function of the duopolistic firm

$$F_1(Y_1, K_1, K_2) = \max_{K_{1,t}} \mathbf{E}\left[\int_0^\infty \pi_1(Y_t, K_{1,t}, K_{2,t}) e^{-\rho t} dt - \int_0^\infty I_1 e^{-\rho t} dK_{1,t} \right]. \tag{2.25}$$

As in a monopoly, firms invest in new capacity to maximize the expected difference between the discounted profits and investment costs over an infinite time horizon. However, the future profits now also depend on the other firm's installed capacity. As well as for the monopolist, the optimal control problem must be transformed to an optimal stopping problem.

In a duopoly, myopia is less realistic than in a monopoly due to heterogeneous players. However, there are two arguments for using myopia as an approximation. First, the players use myopia as a proxy for behavior under uncertainty because it is easy to solve. Second, Gahungu and Smeers (2012) argues that myopia is close to being optimal in symmetric oligopolies. Therefore, we also assume myopia as a simplification in th duopoly.

When assuming myopia, the Bellman equations of the oligopolistic firms are linked through their capacities K_1 and K_2. The partial differential equation for firm 1 is stated in Problem 6.

Problem 6 Optimal stopping problem for firm 1 in a duopoly

$$\begin{aligned}\frac{1}{2}\sigma^2 Y^2 \frac{\partial^3 F_1(Y, K_1, K_2)}{\partial K_1 \partial Y^2} + (\rho - \delta) Y_1 \frac{\partial^2 F_1(Y, K_1, K_2)}{\partial K_1 \partial Y_1} - \\ \rho \frac{\partial F_1(Y, K_1, K_2)}{\partial K_1} + \frac{\partial \bar{\pi}_1(Y, K_1, K_2)}{\partial K_1} = 0,\end{aligned} \tag{2.26}$$

with boundary conditions

$$\frac{\partial F_1(0, K_1, K_2)}{\partial K_1} = 0, \tag{2.27}$$

$$\frac{\partial F_1(Y_1^*, K_1, K_2)}{\partial K_1} = I_1, \tag{2.28}$$

$$\frac{\partial^2 F_1(Y_1^*, K_1, K_2)}{\partial K_1 \partial Y_1} = 0. \tag{2.29}$$

Firm 2 has a corresponding optimal stopping problem. The homogeneous solution of the partial differential equation in Problem 6 is $F_{h1}(Y, K_1, K_2) = A_1(K_1, K_2)Y^{\beta_1}$.

The instantaneous profit regressions in a duopoly are equal to the monopoly regression. In addition to taking the value added from the technologies synergies into account, the impact of the capacity of the other player is considered. Both of these measures are captured in the regression coefficients $c_{12,ijl}$. The coefficients are positive if the technology synergies outweigh the lower price caused by the other players installed capacity, and negative otherwise.

$$\bar{\pi}_1(Y, K_1, K_2) = \sum_{i,j=1}^{d(\gamma),d(\alpha)} b_{1,ij} Y^{\gamma_i} K_1^{\alpha_j} + \sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)} c_{12,ijl}(\gamma_i) Y^{\gamma_i} K_1^{\lambda_j} K_2^{\lambda_l} - OMC_1 K_1 \tag{2.30}$$

By using the procedure described in Appendix A, B and C we derive the solution of Problem 6.

$$F_1(Y, K_1, K_2) = A_1 Y^{\beta_1} + \sum_{i,j=1}^{d(\gamma),d(\alpha)} \bar{b}_{1,ij} Y^{\gamma_i} K_1^{\alpha_j} + \sum_{u=1,u\neq k}^{d(K)} \sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)} \bar{c}_{12,ijl}(\gamma_i) Y^{\gamma_i} K_1^{\lambda_j} K_2^{\lambda_l} - \frac{OMC_1 K_1}{\rho}. \tag{2.31}$$

The corresponding myopic investment trigger is given by

$$\sum_{i=1}^{d(\gamma)} Y_1^{*\gamma_i} \left(\frac{\beta_1 - \gamma i}{\beta_1}\right) \left\{ \sum_{j=1}^{d(\alpha)} \alpha_j \bar{b}_{1,ij}(\gamma_i) K_1^{\alpha_j - 1} + \sum_{u=1,u\neq k}^{d(K)} \sum_{j,k=1}^{d(\lambda),d(\lambda)} \bar{c}_{12,ijl}(\gamma_i) K_2^{\lambda_j} K_1^{\lambda_l - 1} \right\} = I_1 + \frac{OMC_1}{\rho} \tag{2.32}$$

.

When equation 2.32 is solved with respect to Y_1^*, we find the optimal investment trigger for firm 1. It is optimal to invest when $Y_t > Y_1^*$. Then the firm should invest until Y reaches Y_1^*. The identical procedure is completed for firm 2 simultaneously. Firm 2 finds its trigger Y_2^* and invests until Y_2^* reaches Y_t at each point in time.

$\bar{c}_{12,ijl}(\gamma_i)$ is no longer constrained to be non-negative. Thus, we are not guaranteed one unique solution of equation 2.32. This is taken care of as explained in Appendix D.

2.6 Central planner analogy

It is beneficial to compare capacity investment characteristics, like investment triggers and social welfare, under different market structures. This way, we gain information about the changes that have occurred after the deregulation of European electricity markets. This also provides authorities information about how non-renewable electricity markets can be regulated in order to obtain a sufficient social welfare.

As stated in Problem 1 and Problem 4, firms in possession of market power maximize their profits. However, in markets with perfect competition, the producers do not set the price. When each firm seeks to maximize their profits under perfect competition, the result equals the outcome of a central planner maximizing the social welfare. To find welfare losses in monopolies and duopolies, we compare the social welfare under monopoly and duopoly with the social welfare under perfect competition.

Social welfare is the sum of the producer and the consumer surplus: $\psi_l(Y_t, K) = \pi_l(Y_t, K) + cs_l(Y_t, Q_l)$. The producer surplus equals the producers profit, and the consumer surplus cs is given by

$$cs(Y_t, Q_l) = \sum_{l=1}^{d(\mathbf{L})} \tau_l \left\{ \int_0^{Q_l} P(Y_t, x_l) dx_l - P(Y_t, Q_l) Q_l \right\}. \tag{2.33}$$

The social welfare is then found by the equation (2.34).

$$\psi(Y_t, K_1, \ldots, K_{d(\mathbf{K})}) = \max_q \sum_{l \in \mathbf{L}} \tau_l \left\{ \int_0^{Q_l} P_l(Y_t, x_l) dx_l - \sum_{k \in \mathbf{K}} c_k q_k \right\} - \sum_{k \in \mathbf{K}} OMC_k K_k. \tag{2.34}$$

We can derive the discounted social welfare adjusted for investment costs for a central planner under perfect competition as discussed by Gahungu and Smeers (2012). Then the objective of maximizing profits is changed to maximizing the social welfare. Pindyck and Rubinfeld (2013) shows that the market equilibrium is when the marginal cost equals the demand in each load segment.

We use the approach developed in section 2.4 to find the optimal capacity expansion under perfect competition. When substituting the profit $\pi(Y_t, K_1, \ldots, K_{d(\mathbf{K})})$ with the social welfare $\psi(Y_t, K_1, \ldots, K_{d(\mathbf{K})})$, all steps in the calculations are also valid under perfect competition. In section 4 we compare investment characteristics under monopoly, duopoly and perfect competition.

Chapter 3

Numerical procedure

Solving stochastic control problems like (2.8) and (2.25) analytically proves difficult, as pointed out by Pindyck (1988). Consequently, we propose a numerical procedure for solving the models described in the previous section. This implies going from continuous to discrete time. In section 3.1, we propose a procedure for solving the optimal stopping problem and finding the initial investment triggers. In section 3.2, we propose a procedure for solving the corresponding stochastic control problem. This gives us the optimal investment path and the discounted social welfare, producer surplus and consumer surplus. In chapter 4 we solve a numerical example using the procedure described in this section.

3.1 Investment trigger

When assuming a given initial capacity and a set of load segments, the procedure described below allows us to find the optimal investment triggers for each technology. The procedure assumes that there are only two available technologies.

1. Assume that the initial capacities are $K_{1,0}$ and $K_{2,0}$, and the initial shock level is Y_0. Furthermore, assume a set for load segments **L**. Define a time grid $T^{grid} = (0, t_1, t_2, \dots, T)$ and a sett of scenarios $\mathbf{N} = (1, 2, \dots, N)$. Let N and T be sufficiently large. For each scenario in **N**, let Y_t sample from a geometric Brownian motion starting at Y_0. This means that one creates N samples of the stochastic process Y_t over T^{grid}. These paths can be described as $(Y_{0,n}, Y_{t_1,n}, \dots, Y_{T,n}),\ n = 1, \dots, N$.
2. For each point on the T^{grid} in every simulation, solve
   (a) Problem 1 for a monopolist.
   (b) Problem 1 using equation (2.34) as objective function for a social planner.
   (c) Problem 4 for a duopoly.

When solving the problem, compute the shadow costs of the capacity constraint for each technology in each load segment, $\lambda_{k,l}(Y_{t,n}, K_{1,0}, K_{2,0})$. The shadow cost represents the the marginal increase in profit due to a marginal investment in technology k in load segment l.

3. For each scenario in **N**, the marginal value $M_{k,n}(Y_0, K_{1,0}, K_{2,0})$ of additional production capacity in technology k. The marginal value represents the discounted marginal increase in profit due to a marginal capacity investment.

$$M_{k,n}(Y_0, K_{1,0}, K_{2,0}) = \sum_{t=0}^{T} \sum_{l=1}^{d(\mathbf{L})} \lambda_{k,l}(Y_{t,n}, K_{1,0}, K_{2,0}) e^{-\rho t} \\ - \frac{OMC_k}{\rho}, \qquad k = 1, 2. \quad (3.1)$$

By averaging the N scenarios, one determines the expected marginal value of additional capacity when starting at Y_0. This marginal value represents the partial derivative of the particular solution of equation 2.12 with respect to K_k for technology k.

$$\frac{\partial F_{k,p}(Y_0, K_{1,0}, K_{2,0})}{\partial K_k} \approx \mathbf{E}\left[M_{k,n}(Y_0, K_{1,0}, K_{2,0})\right] = \\ \hat{W}_k(Y_0, K_{1,0}, K_{2,0}) - \frac{OMC_k}{\rho}, \qquad k = 1, 2 \quad (3.2)$$

where

$$\hat{W}_k(Y_0, K_{0,1}, K_{0,2}) = \mathbf{E}\left[\sum_{t=0}^{T} \sum_{l=1}^{d(\mathbf{L})} \lambda_{k,l}(Y_{t,n}, K_{1,0}, K_{2,0}) e^{-\rho t}\right] = \\ \frac{1}{N}\left[\sum_{n=1}^{N} \sum_{t=0}^{T} \sum_{l=1}^{d(\mathbf{L})} \lambda_{k,l}(Y_{t,n}, K_{1,0}, K_{2,0}) e^{-\rho t}\right], \qquad k = 1, 2. \quad (3.3)$$

4. The preceding procedure is repeated for every point of a grid of initial values of Y_0 so that $Y_0^{grid} = (Y_0^1, Y_0^2, \ldots, Y_0^G)$. Consequently, we find $\hat{W}_k(Y_0^g, K_{1,0}, K_{2,0})$ for $g = 1, \ldots, G$, where G is a sufficiently large number.

5. For technology 1 and 2, one performs the constrained regressions $\bar{R}_1$ and $\bar{R}_2$ as estimates of $\hat{W}_1$ and $\hat{W}_2$ using a power function of Y_0 as the explanatory variable, where $\gamma_i < \beta_1$. When holding the installed capacity constant, Appendix D proves the regression in equation (3.4) to be equivalent to the regression in equation (2.9) and equation (2.30).

$$\bar{R}_k(Y) = \sum_{i=1}^{d(\gamma)} a_{k,i} Y^{\gamma_i}, \ k = 1, 2, \quad (3.4)$$

where

$$a_{k,i} \geq 0, \ k = 1, 2, \ i = 1, \ldots, d(\gamma). \quad (3.5)$$

6. When assuming myopia and using the regression coefficients from (3.4), one solves the optimal stopping problem initially described by equation (2.21). The investment triggers of the two technologies Y_1^* and Y_2^* are given as the unique solution of equation (3.6) with respect to Y_k^*.

$$\sum_{i=1}^{d(\gamma)} a_{k,i} Y_k^{*\gamma_i} (\frac{\beta_1 - \gamma_i}{\beta_1}) = I_k + \frac{OMC_k}{\rho}, \qquad k = 1, 2. \tag{3.6}$$

3.2 Optimal investment path and social welfare

After solving the optimal stopping problem, we solve the stochastic control problem. This procedure finds the optimal investment path, the discounted social welfare, producer surplus and consumer surplus, the value of the optimal stochastic control problem and the discounted social welfare adjusted for investment costs.

1. Make a marginal investment based on the following investment rules:
   (a) *Monopolist and central planner*: Control both technologies and aim to maximize profit/social welfare.
      i. If $Y_1^* < Y_t$ and $Y_2^* > Y_t$, the monopolist/central planner makes an marginal investment $\Delta\kappa$ in technology 1.
      ii. If $Y_2^* < Y_t$ and $Y_1^* > Y_t$, the monopolist/central planner makes an marginal investment $\Delta\kappa$ in technology 2.
      iii. If $Y_1^* < Y_t$ and $Y_2^* < Y_t$, the monopolist/central planner makes an marginal investment $\Delta\kappa$ in the technology with the investment trigger with the lowest value.
      iv. If $Y_1^* > Y_t$ and $Y_2^* > Y_t$, the monopolist/central planner do not make any investment at the given time.
   (b) *Duopolistic firm*: Each firm controls one technology and aims to maximize its profit. If $Y_k^* < Y_t$, the player controlling technology k makes an marginal investment $\Delta\kappa$ in technology k. This is done independently of the other firm's simultaneous investment decision.

2. After investing marginally, the firm has not necessary found the optimal capacity expansion of the time interval. Consequently, it repeats the procedure in section 3.1 until it is optimal not to invest in any technology.

3. After finding the optimally installed capacity, compute the instantaneous social welfare ψ, the instantaneous consumer surplus cs, the instantaneous profit generated by each technology π_1 and π_2 and the instantaneous producer surplus π corresponding to the optimal solution to the optimization problem

4. Create a second time grid $S^{grid} = (t_i, t_{i+1}, \ldots, T + t_i) = (s_0, s_1, \ldots, s_T)$ so that S^{grid} is dependent on the position on the T^{grid}. For $t_i = 0, 1, \ldots, T$ repeat the procedure described in section 3.1 using S^{grid} as T^{grid}. Invest marginally with the investment rules in step 1 and compute the surpluses as described step 3.

5. Create a second set of scenarios $\Omega = (1, 2, \ldots, d(\Omega))$ in order to compute the expected discounted surpluses. Let $d(\Omega)$ be sufficiently large. Repeat the entire problem for every scenario Ω.

6. Compute the expected discounted social welfare Ψ, the expected discounted producer surplus Π, the expected discounted consumer surplus, Cs and the expected discounted social welfare adjusted for investment costs F_ψ. F_ψ corresponds to the stochastic control problem of the central planner.

$$\Psi = \mathbf{E}\left[\sum_{t=0}^{T} \psi_{t,\omega} e^{-\rho t}\right] = \frac{1}{d(\Omega)}\left[\sum_{\omega=1}^{d(\Omega)} \sum_{t=0}^{T} \psi_{t,\omega} e^{-\rho t}\right] \tag{3.7}$$

$$\Pi = \mathbf{E}\left[\sum_{t=0}^{T} \pi_{t,\omega} e^{-\rho t}\right] = \frac{1}{d(\Omega)}\left[\sum_{\omega=1}^{d(\Omega)} \sum_{t=0}^{T} \pi_{t,\omega} e^{-\rho t}\right] \tag{3.8}$$

$$Cs = \mathbf{E}\left[\sum_{t=0}^{T} cs_{t,\omega} e^{-\rho t}\right] = \frac{1}{d(\Omega)}\left[\sum_{\omega=1}^{d(\Omega)} \sum_{t=0}^{T} cs_{t,\omega} e^{-\rho t}\right] \tag{3.9}$$

$$\begin{aligned} F_\psi = \mathbf{E}\left[\sum_{t=0}^{T}\left(\psi_{t,\omega} e^{-\rho t} - \sum_{k}^{d(\mathbf{K})} \Delta K_{k,t,\omega} I_k e^{-\rho t}\right)\right] = \\ \frac{1}{d(\Omega)}\left[\sum_{t=0}^{T}\left(\psi_{t,\omega} e^{-\rho t} - \sum_{k}^{d(\mathbf{K})} \Delta K_{k,t,\omega} I_k e^{-\rho t}\right)\right]. \end{aligned} \tag{3.10}$$

$\Delta K_{k,t,\omega}$ is the total investment in technology k at time t in scenario ω.

(a) For a monopolist, compute the value of the monopolistic firm corresponding to the stochastic control problem of the monopolist in Problem 2.

$$\begin{aligned} F_\pi = \mathbf{E}\left[\sum_{t=0}^{T}\left(\pi_{t,\omega} e^{-\rho t} - \sum_{k}^{d(\mathbf{K})} \Delta K_{k,t,\omega} I_k e^{-\rho t}\right)\right] = \\ \frac{1}{d(\Omega)}\left[\sum_{\omega=1}^{d(\Omega)} \sum_{t=0}^{T}\left(\pi_{t,\omega} e^{-\rho t} - \sum_{k}^{d(\mathbf{K})} \Delta K_{k,t,\omega} I_k e^{-\rho t}\right)\right]. \end{aligned} \tag{3.11}$$

(b) For the duopolistic firms, compute the value of the firms corresponding to the stochastic control problem in Problem 5.

$$\begin{aligned} F_{\pi_k} = \mathbf{E}\left[\sum_{t=0}^{T}(\pi_{k,t,\omega} - \Delta K_{k,t,\omega} I_k) e^{-\rho t}\right] = \\ \frac{1}{d(\Omega)}\left[\sum_{\omega=1}^{d(\Omega)} \sum_{t=0}^{T}(\pi_{k,t,\omega} - \Delta K_{k,t,\omega} I_k) e^{-\rho t}\right],\ k = 1, 2. \end{aligned} \tag{3.12}$$

Chapter 4

Results

In order to illustrate how capacity optimally is installed under different market structures, we find it beneficial to provide a numerical example. The example includes a market with two types of power plants, base and peak load power plants. The numbers in the example are not taken from a real power market, and are chosen to illustrate our capacity expansion model.

In section 4.1, we present a base case in which we solve the optimal stopping problem and the corresponding stochastic control problem under different market structures. We find the initial investment triggers, the optimal capacity expansion, and the corresponding welfare effects. For the monopoly we also perform a sensitivity analysis in which we examine how the initial investment triggers and the optimal capacity expansion are sensitive to changes in σ and μ. In section 4.2, we illustrate how optimal capacity expansion is affected by fluctuations in the electricity demand. This is done by comparing the monopoly in the base case with a monopoly with one load segment, such that $d(\mathbf{L}) = 1$.

4.1 Capacity expansion under different market structures

There are two different types of power plants available to an investor, a base and a peak load power plant. The demand is spilt into six load segments with different electricity demand, such that $d(\mathbf{L}) = 6$. The inverse demand function described in equation (2.3) is specified to

$$P_l(Y_t, Q_l) = Y_t(A_l - b_l Q_l), \qquad l = 1, 2, \ldots, 6. \tag{4.1}$$

The parameters are defined in the tables below. Table 4.1 describes the different types of power plants, Table 4.2 describes the different load segments and Table 4.3 describes the demand shock process, the discount rate the simulations and the time horizon.

Technology index, k	$Base$	$Peak$
Marginal cost, c [€/MWh]	5	65
O. & M. cost, OMC [€/MWy]	100 000	20 000
Investment cost, I [€/MW]	3 000 000	80 000
Initial capacities $K_{t=0}$ [MW]	15 000	5 000

Table 4.1: Costs and initial capacities of the base and peak load power plants.

Load Segment l	1	2	3	4	5	6
Duration, τ [h]	10	40	310	4400	3000	1000
Maximum demand, A_l	900	180	165	120	90	60
Slope, b_l	0.0070	0.0014	0.0014	0.0014	0.0015	0.0020

Table 4.2: Duration and inverse demand function of the load segments.

Y_0	μ	σ	ρ	β_1	N	$d(\Omega)$	T	Δt	$\Delta\kappa$
1	0.02	0.03	0.1	4.62	50	50	50	$1y$	500

Table 4.3: Modeling parameters.

The problem is solved for a monopolist, a central planner and two firms in a duopoly. In the duopoly, we assume that one firm has access to base load capacity and that the other has access to peak load capacity. We solve the problems using the Y^{grid}

$$Y^{grid} = (Y_t - 0.60, Y_t - 0.55, \dots, Y_t + 0.60). \tag{4.2}$$

The regression described in section 3.1, step 4 is conducted using $\gamma = (0.25, 0.50, \dots, 4.50)$. $\gamma_{d(\gamma)} = 4.5$ is chosen so that $\gamma_i < \beta_1,\ i = 1, \dots, d(\gamma)$. This gives us the optimal initial investment triggers in table 4.4.

	Perfect competition	Monopoly	Duopoly
Y^*_{Base}	0.6276	0.8439	0.8853
Y^*_{Peak}	0.4865	0.9722	0.5688

Table 4.4: The initial investment triggers of base and peak load power plants in perfect competition, monopoly and duopoly.

We observe that the initial demand shock Y_0 exceeds the initial investment triggers Y^*_{Base} and Y^*_{Peak}, and we invest marginally $\Delta\kappa$ according to the investment rules presented in section 3.2, step 1. We repeat the procedure and continue to make marginal investments until it is no longer optimal to invest. We then move one time step Δt ahead and repeat the entire procedure until we reach time T for all scenarios $d(\Omega)$.

The initial investment triggers depend on the already installed capacity. They decrease as the initial installed capacity increases. Since this is an illustrative example, it does not make sense to study the magnitudes of the investment triggers. However, the relationship between the triggers provides information about investments dynamics under different market structures.

In table 4.4, we observe that the central planner has the lowest investment triggers, both for additional base and peak load capacity. Consequently, the central planner has the highest incentive to invest in both base and peak load capacity. This is intuitive, as the central planner aims to maximize the social welfare. With increased installed capacity, the electricity prices drop. Although decreased electricity prices reduce the producer surplus, they also increase the consumer surplus. The monopolist has significantly higher investment triggers for both peak and base load capacity. This is because the central planner aims to avoid very high electricity prices, while this is desirable for a monopolist. Avoiding high electricity prices is particularly important for the central planner in load segment 1 where the electricity demand is at its maximum.

In the duopoly, the market power of the firms also leads to higher investment triggers than under perfect competition. However, the investment trigger of peak load capacity is significantly lower than the investment trigger of peak load capacity in a monopoly. We explain this using the marginal values of additional capacity. The monopolist maximizes the producer surplus by increasing production until its marginal cost equals its marginal revenue. The base load plant has lower marginal costs than the peak load plant and thus, the monopolist employ all base load capacity before utilizing the peak load capacity. In a duopoly on the other hand, each firm maximizes its own profit at the expense of its rival. Each firm will increase its dispatch for as long as a marginal increase in in the dispatch leads to a marginal increase in the firms own profit. The peak load firm is only in possession of peak load capacity. Consequently, it will utilize all of its peak load capacity in load segments where this is not optimal from a monopolist's point of view. Hence, the peak load plant has a higher utilization rate in the duopoly than in the monopoly.

Utilizing all installed capacity implies having a positive marginal value of additional capacity. This contributes to raising the peak load firm's marginal value of additional peak load capacity compared to that of the monopolist. Lower power prices in the duopoly compared to a monopoly implies lower marginal value of additional capacity. In our example, we observe that the effect of a higher utilization rate for peak load capacity outweighs the effect of lower power prices. Thus, the peak load firms marginal value of additional capacity exceeds the marginal value of additional peak load capacity for the monopolist. The investment trigger represents the trade-off between the marginal value of additional capacity, on the one hand, and the sum of the per unit investment cost and the value of postponing the investment, on the other hand. As a result, the peak load firms investment trigger is smaller than the monopolists peak load investment trigger.

The same type of argument is valid for the base load triggers. For the initial capacity in our example, the base load utilization rate is equal in the monopoly and the duopoly. The electricity prices are lower in the duopoly than in the monopoly. Hence the base load firm's marginal value of additional capacity is smaller than the marginal value of additional base load capacity for the monopolist. This leads to higher investment triggers for the base load firm than for a monopolist considering an investment in peak load capacity.

In a duopoly, the initial investment trigger of the peak load capacity exceeds the initial trigger of the base load capacity. This implies that a marginal investment in additional peak load capacity rises the value of the peak load firm more than a marginal investment in additional base load capacity increases the value of the base load firm.

Table 4.5 presents the discounted total surplus, the discounted producer surplus and the discounted consumer surplus. We also examine the value of the firms and how total discounted social welfare adjusted for investment costs is reduced in other market structures than perfect competition.

	Perfect competition	Monopoly	Duopoly
Discounted social welfare [M€]	282 450	186 330	193 410
Discounted producer surplus [M€]	121 940	144 990	128 130
Discounted consumer surplus [M€]	160 500	41 339	65 587
Value of the firm, F [M€]	-	129 090	F_{base} = 99 860 F_{peak} = 21 958
Discounted social welfare adjusted for investment costs [M€]	209 220	170 430	184 930
Percentage loss inn discounted social welfare adjusted for investment costs	-	18.64 %	11.61 %

Table 4.5: Surpluses and welfare losses in perfect competition, monopoly and duopoly.

As expected, the discounted social welfare and consumer surplus of the central planner exceeds the monopolist's and the duopolists'. This is due to central planner's excessive capacity investments compared to the firms' and is a result of his aim to maximize the social welfare. Firms in monopolies and duopolies only consider the producer surplus when deciding to invest and have consequently significantly larger producer surpluses than the central planner. The monopolist controls the entire market, and do thus decide the total dispatch. In the oligopoly, on the other hand, each firm maximizes their own profits when also considering the other firm's dispatch. This leads to a larger dispatch and thus a lower electricity price and producer surplus than in the monopoly.

The value of the firms is found by the stochastic control problems of monopolistic and duopolistic firms in Problem 2 and Problem 5. We observe that the value of the monopolistic firm exceeds the total value of the oligopolistic firms combined. This is a result of differences in both the optimal capacity investments and the optimal dispatch for the monopolist and the duopolistic firms. The monopolist has a flexibility in form of being able to invest in both base and peak load capacity as well as to choose the amount of electricity generated by each technology. The duopolistic firms, on the other hand, have to invest in and generate electricity by the one technology that is available to them. Additionally, both of the duopolistic firms aim to maximize their own profit. This differs from the objective of a monopolist, who maximized its expected discounted producer surplus and thus gains a higher firm value.

As illustrated in Figure 4.3, the duopolistic firm possessing peak load capacity invest in more additional capacity than the firm possessing base load capacity. The value of the firm in the position of base load capacity exceeds the value of the firm in the position of peak load capacity. This is a result of the lower operational costs provided by the base load capacity which leads to a contribution margin that outweighs the lower investment cost provided by the peak load capacity.

When correcting the discounted social surplus for investment costs, the difference between the discounted social welfare in perfect competition and the other market structures is reduced. This indicates high investment costs to be a major investment barrier of the firms operating under market power. The last line in table 4.5 presents the percentage losses in total discounted surplus adjusted for investment costs. The welfare losses are 18.64% in the monopoly and 11.61% in the duopoly. Although the consumer surplus in a monopoly and a duopoly is small compared to the central planners, the total social losses due to market power are modest. This is a result of the investment cost of each unit of capacity installed. The social planner invests in more additional capacity than monopolistic and oligopolistic firms do. This reduces the producer surplus under perfect competition. Huisman and Kort (2016) also examine welfare losses and conclude that the social welfare is reduced by 25% in a monopoly compared to perfect competition. However, they provide a general real options model that is not customized power markets. Their model allows one investment, provides no production flexibility and assumes no installed capacity earlier on.

Figure 4.1, 4.2 and 4.3 illustrates the optimal capacity expansion path of the central planner, the monopolist and the two firms in the duopoly. The charts illustrate total installed base and peak load capacity over a time horizon of 50 years. The dark grey lines represent total base load capacity and the light grey lines represent total peak load capacity installed.

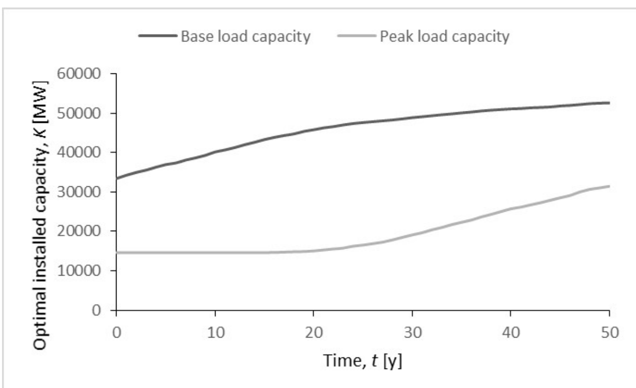


Figure 4.1: The central planner's optimal capacity expansion path.

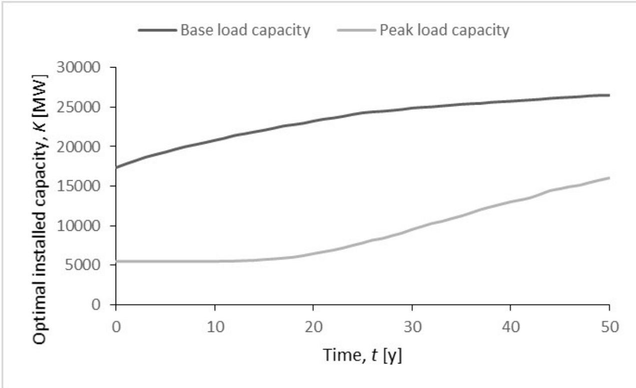


Figure 4.2: The monopolist's optimal capacity expansion path.

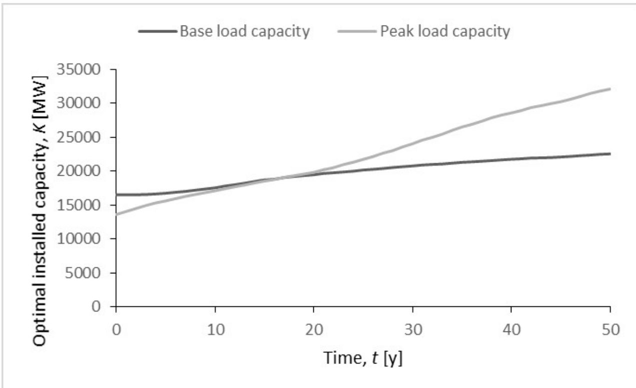


Figure 4.3: The optimal capacity expansion path of the duopolistic firms.

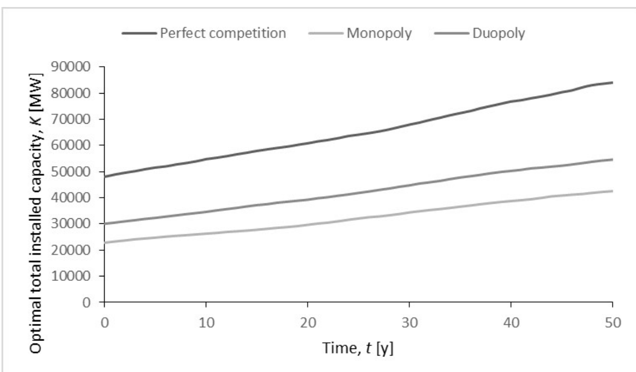


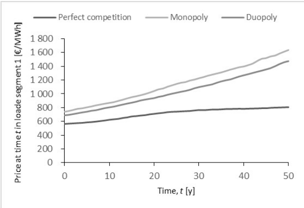
Figure 4.4: Total installed capacity at time t in perfect competition, monopoly and duopoly.

In Figure 4.1 and 4.2, we observe the similarities between the investment patterns of a central planner and a monopolist. At the beginning of the period, they invest clearly most in base load capacity. After approximately 20 years, a higher share of their investments is additional peak load capacity. This is because power prices in the load segment 1, 2, and 3, where the peak load plants run at their maximum capacity, are increasing until year 20. The marginal costs of the different power plants are kept constant. Consequently, the contribution margin of both plants increases. Around year 20, the contribution margins of the two types of plants reaches a trigger in which it becomes optimal to install both additional base and peak load capacity. The marginal values of additional capacity explain the similarities of the investment paths. For both base and peak load capacity, the marginal values of new capacity of the central planner are roughly double of the marginal value of additional capacity of the monopolist. Hence, the central planner installs roughly twice as much capacity as the monopolist.

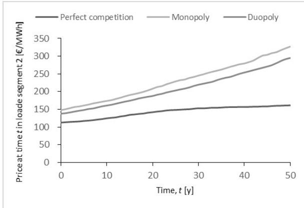
As illustrated in Figure 4.3, the expansion path of the peak load firm in a duopoly differs significantly from the peak load investment path of the monopolist and the central planner. Here, the installed peak load capacity exceeds the installed base load capacity. The expansion path of the base load firm on the other hand, is similar to the monopolists optimal base load expansion plan. The relationship between the marginal value of additional capacity and the marginal investment cost for base load and peak load power plants explains this as well as the investment strategy in a Cournot duopoly. In the duopoly, the firms invests marginally and simultaneously. This means that each firm invest marginally as long as the demand shock at each time step exceeds the optimal investment trigger. This leads to significantly larger peak load installations than in the monopoly and under perfect competition, where the monopolist and the central planner invest relatively more in base load capacity.

Figure 4.4 shows the total capacity expansion under different market structures. We observe that the central planner installs in excessive additional capacity compared to the monopolist and the duopolistic firms. This is a result of the central planners focus on maximizing the social welfare when the aim of the monopolist and the duopolistic firms is to maximize their own producer surplus. The competition in the duopoly results in a larger total installed capacity compared to the monopoly.

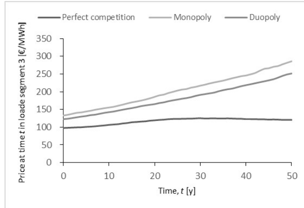
Figure 4.5 illustrates the market price of electricity in load segment l at time t under different market structures. The dark gray, gray and light gray lines represent the electricity price in respectively perfect competition, duopoly and monopoly. Not surprisingly, the electricity price is lowest under perfect competition in all load segments due to the central planners aim to maximize the social welfare. As discussed above, the duopolistic firms generate a larger amount of electricity than the monopolist does. Thus, the electricity price in a monopoly exceeds the electricity price in a duopoly in each load segment. Furthermore, the prices in a monopoly and a duopoly are strictly increasing in all load segments. The demand is expected to rise over time. In order to maximize their profits, the monopolist and the two duopolistic firms install new capacity, but not enough from keeping the electricity price from rising over time.



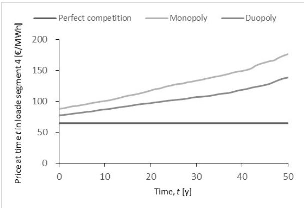
(a) Load segment 1



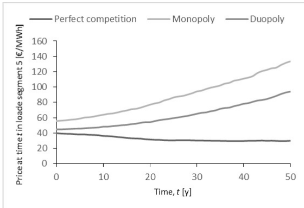
(b) Load segment 2



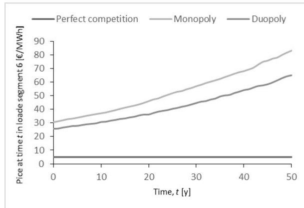
(c) Load segment 3



(d) Load segment 4



(e) Load segment 5



(f) Load segment 6

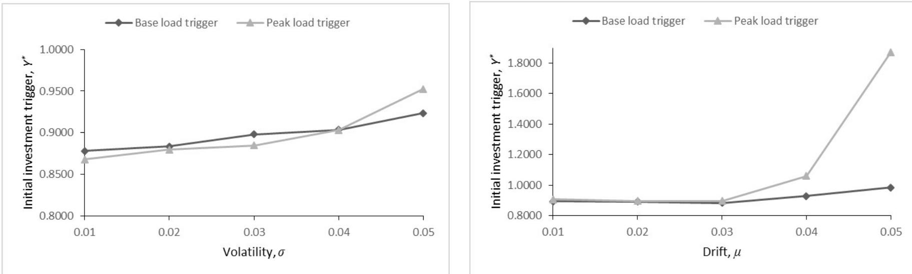
Figure 4.5: The market price of electricity in different load segments in perfect competition, monopoly and duopoly.

In load segment 1, 2 and 3, the prices are strictly increasing for all market structures until year 20. From this point in time, the prices increase less or stop increasing for a market with perfect competition. This corresponds to when the central planner starts to invest in additional peak load capacity. At the same time, the monopolist also invests in additional peak load capacity. However, in the monopoly, the prices continues to increase. This is a consequence of the monopolist making moderate investments compared to the central planner.

In a market with perfect competition, the prices in load segment 4 equal the marginal cost of peak load generation. This is because of the central planner utilizes all the installed base load capacity and some of the installed peak load capacity. In load segment 5, the central planner uses all the installed base load capacity. As the price level is lower than the marginal cost of producing with peak load technology, the central planner does not use the installed peak load capacity. The prices in load segment 6 under perfect competition equal the marginal cost of base load generation. This implies that the central planner has installed enough base load capacity to cover the demand optimally without using any peak load capacity.

Next, we study how the solution of the monopolist in the previous section responds to changes in the the drift rate μ and the volatility σ. Figure 4.6 illustrates how the initial investment triggers vary with the drift rate and the volatility. Figure 4.7 shows how the optimal installed capacity varies with the volatility, and Figure 4.8 illustrates how the optimally installed capacity varies with the drift rate.

According to equation (2.16), changing μ or σ implies changing β_1. As the base vector γ is restricted by β_1, we need to use different γs in the regression in equation (3.4) for each combination of μ and σ in order to find one unique investment trigger. In order to provide comparable results, we continue to use $d(\gamma) = 18$.



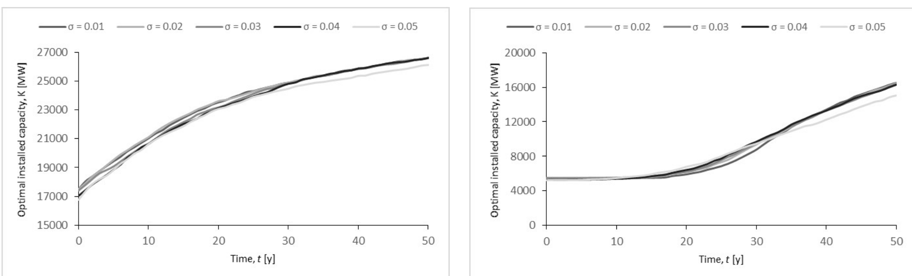
(a) The initial investment trigger for different values of the volatility σ when $\mu = 0.02$.

(b) The initial investment trigger for different values of the drift rate μ when $\sigma = 0.03$.

Figure 4.6: The initial investment triggers for different values of the volatility σ and the drift rate μ.

The initial investment triggers are strictly increasing in σ for both the base load plant and the peak load plant. This corresponds to standard real options theory, in which the value of waiting increase with the uncertainty as advocated by Dixit and Pindyck (1994). The firm invests marginally. For small investments, the value of waiting has a minor impact on the investment decision. Although changes in the volatility affect the value of waiting, changes in the value of waiting have limited impact of the investment decision compared to larger investments. Hence, the investment trigger has a modest response to changes in the volatility.

Increasing the drift rate μ has two contradictory effects, both discussed by Dixit and Pindyck (1994). Both the discounted future profit $\int_0^\infty \pi(Y_t, K_{1t}, \ldots, K_{d(\mathbf{K})t})e^{-\rho t}dt$ and the uncertainty multiplier $\frac{\beta_1}{\beta_1 - \gamma_i}$ increases in μ. From figure 4.6 we observe that for $\mu < 0.03$, the effect of the discounted future profit is dominating, while for $\mu > 0.03$ the effect of the uncertainty multiplier dominates.

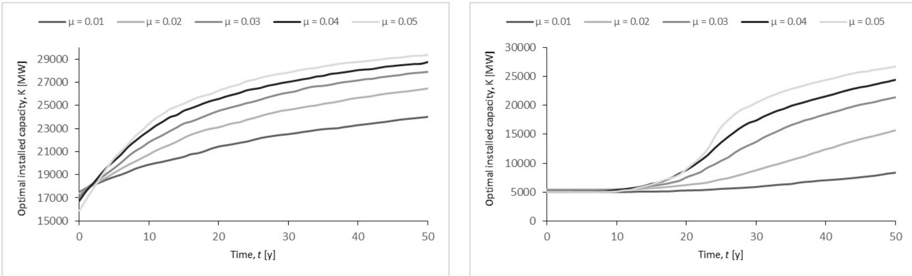


(a) Optimal installed base load capacity.

(b) Optimal installed peak load capacity.

Figure 4.7: Optimal installed capacities for different values of the volatility σ when the drift rate is $\mu = 0.02$.

Figure 4.7 shows that changes in σ has a minor effect on the optimal expansion path. This is also a result of the fact that the value of waiting has limited impact on the investment decision.



(a) Optimal installed base load capacity. **(b)** Optimal installed peak load capacity.

Figure 4.8: Optimal installed capacities for different values of the drift rate μ when the volatility is $\sigma = 0.03$.

As time moves towards $t = 50$, the optimally installed capacity is increasing in μ. An increase in μ implies a higher demand over time. When the monopolist maximizes his expected profits, its dispatch rises with the stochastic demand, Y_t. An increase in the dispatch requires the monopolist to invest in additional capacity. In Figure 4.8 we observe that increases in μ has the same effect on both base and peak load capacity.

4.2 Capacity expansion and time-varying demand

In this section, we compare the investment triggers and the investment paths of the monopolist when the electricity demand is time-varying and fixed throughout the year. We model the fixed demand by reducing the number of load segments to $d(\mathbf{L}) = 1$. The inverse demand curve is thus given by

$$P(Y_t, Q) = Y_t(A - bQ), \tag{4.3}$$

where A and b are calculated as weighted averages of the parameters presented in table 4.2. This corresponds to $A = 105.6$, $b = 0.00153$, $\tau = 8760$ h. Consequently, the fixed demand is the weighted average of the time-varying demand. We solve the capacity expansion problem with fixed electricity demand for a monopolist while using the technologies in table 4.1 and the simulation parameters from table 4.3. The investment triggers are given in table 4.6, and the optimal capacity expansion plan is illustrated in Figure 4.9.

	Fixed demand	Time-varying demand
Y^*_{Base}	0.8669	0.8439
Y^*_{Peak}	2.2871	0.9722

Table 4.6: The initial investment triggers for a monopolist with fixed and time-varying demand.

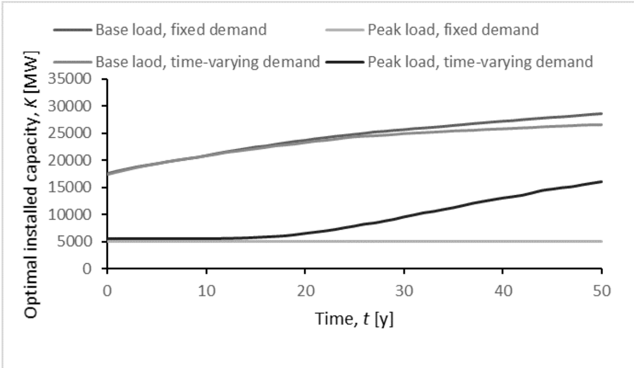


Figure 4.9: The optimal capacity expansion for a monopolist with fixed and time-varying demand.

When modeling the electricity demand as constant throughout each year, no additional peak load capacity is installed. Over time the firm installs additional base load capacity without using all the already installed peak load capacity. This means that the marginal value of additional peak capacity goes towards zeros as t goes towards 50. When the marginal value of additional peak capacity goes towards zero, the investment trigger for peak load, Y^*_{Peak} moves towards infinity. An economical interpretation of the firm not installing additional peak load capacity is as follows. The firm utilizes all base load capacity before using available peak load capacity due to the relatively high operational costs of peak load capacity. Since the electricity demand is constant throughout the year, the firm utilizes installed base load capacity during the whole year. The tradeoff between the investment and the operational costs favors base load capacity. Hence when demand is fixed throughout the year, the monopolist will only invest in base load capacity.

The effect of using several load segments is small for the investments in base load capacity. In Table 4.6, we observe the initial base load capacity triggers to be similar when the number of load segments each year are 1 and 6. Figure 4.9 illustrates that the optimal investment paths of the fixed and the time-varying demand throughout the year are almost identical. The fixed demand over the year equals the weighted average demand when the demand is time-varying during the year. This results in approximately equivalent marginal values of additional base load capacity for the firm, both when demand is time-varying and fixed throughout the year. In both cases, the firm wishes to maximize its producer surplus. Since the marginal values of additional base load capacity are almost equal for a time-varying and a fixed demand, the different base load investment patterns resemble each other. When demand is time-varying throughout the year, the firm invests in additional peak load capacity at approximately $t = 20$ years. Hence, the installed base load capacity is smaller when demand is time-varying than when it is fixed from year 20 and further on.

4.3 Discussion of results

In this chapter, we have studied how different market structures and fluctuations in the electricity demand affect installation of additional base and peak load capacity. As expected, we witness that increased competition leads to more installed capacity and a higher social welfare. Both modelling electricity markets as duopolies and including time-varying demand provides additional incentive to invest in peak load capacity. Power markets are considered oligopolies, and the electricity demand is time-varying. This indicates that installing peak load capacity is more beneficial for power companies when the capacity expansion model reflects properties of real power markets.

Although the results appear reasonable, we address possible limitations of our capacity expansion model. Alexander (2008) argues that when performing Monte Carlo simulations one should use a large number of simulations, e.g. 100 000. To limit the computation time, we use 50 simulations. This might lead to inaccuracies in the results. Our example is not based on empirical data and its motivation is to illustrate our capacity expansion framework. Therefore, minor inaccuracies in the results are of less importance.

Our capacity expansion model includes both a stochastic control problem and an optimal stopping problem, which are equivalent as long as the assumption of myopia holds. Because of non-additively separable technologies and duopolistic companies, we cannot be certain that the properties of myopia hold. This may lead to inaccuracies in the numerical results. However, we argue that myopia is an acceptable approximation because power companies consider the next investment optimal and hence the last investment until the electricity demand has increased sufficiently. Hence, we consider the errors of assuming myopia small.

We assume the electricity demand to have a positive drift. The electricity price is thus expected to rise over time while the marginal cost of generation remains constant. As a result, the marginal contribution margin of the peak load capacity increases over time. Hence, peak load capacity gets more profitable. This results in larger investments in peak load capacity as time passes. One can discuss whether a constant marginal cost of generation is in line with the dynamics of actual power plants. Non-renewable peak and base load power plants consists of mature technologies where the actual costs of running are expected to remain constant. However, power companies might pay a carbon taxes on their emissions that is exposed to policy uncertainty. Changes in the carbon tax impact the marginal cost of generation, and one can thus argue that the marginal cost of generation is stochastic.

Another aspect that effects the capacity investments is the fact that one of the duopolistic firms only has the opportunity to install additional peak load capacity. The base and the peak load capacity are both available to the monopolist and the social planner, who both invest in a highest share of base load capacity. Hence, one can argue that the peak load firm also invests in base load capacity to some extent if this is a possibility. However, we argue that a firm in possession of only initial peak load capacity will face entry barriers when investing in base load capacity. Thus, the peak load firm will not invest in base load capacity. The same argument is valid for a firm only in possession of initial base load capacity. Examples of such entry barriers are political, economic and lack of expertise.

Chapter 5

Concluding remarks

We have adopted a real options approach to analyze marginal investments in peak and base load generation capacity. We study capacity expansion within monopolies, duopolies and markets with perfect competition and compare investment triggers and the optimal capacity installations for peak load and base load power plants. Our approach considers several features of the real world power markets, including heterogeneous technologies, endogenous electricity prices, time-varying electricity demand, and markets with imperfect competition. We find that fluctuations in the electricity demand over the year as well as imperfect competition boost peak load investments. Furthermore, the installed capacity increases with the number of firms in the market.

The renewable energy generation changes the fluctuations in the non-renewable electricity demand. Our capacity expansion framework may provide decision support to both policymakers and private investors. It is important for policymakers to ensure a certain non-renewable dispatch to cover the electricity demand and a certain amount of firms in order to avoid market power. In our example, we find that increased competition leads to a higher installed capacity and lower electricity prices, which result in smaller welfare losses. When observing capacity shortage, policymakers should incentivize new capacity investments. For investors in the power sector, it is important to gain information about how to capitalize on investments in non-renewable peak and base load capacity. By using our investment approach, investors may increase their understanding of the power market they operate in as well as finding their optimal investment strategy.

Extensions of the framework are possible in several directions. One can investigate the mathematical impacts of assuming myopia. Although we argue that the implications of this assumption are limited, it is beneficial to know their magnitudes. Another possible extension is to investigate the synergies introduced by using heterogeneous technologies further. Additionally, our capacity expansion model can be extended to include new features like policy uncertainty, stochastic marginal costs, mothballing of existing power plants and replacement of old power plants to be even more realistic. Finally, one could perform a case study on a power market dominated by non-renewable power generation.

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Appendix

A The option value of the investment

In section 2.4 we convert the stochastic control problem in Problem 2 to the optimal stopping problem in Problem 3. Due to the assumption of myopia, $\frac{\partial F(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k} = V_k,\ k \in \mathbf{K}$. We derive the optimal stopping problem by using dynamic programming and Ito's Lemma. We start out by the Bellman equation

$$\rho V_k dt = \frac{\partial \bar{\pi}}{\partial K_k} dt + \mathbf{E}[dV_k],\ \forall k \in \mathbf{K}. \tag{A.1}$$

Ito's Lemma implies

$$dV_k = \frac{\partial V_k}{\partial t} dt + \frac{\partial V_k}{\partial Y} dY + \frac{\partial^2 V_k}{\partial Y^2} (dY)^2,\ \forall k \in \mathbf{K}. \tag{A.2}$$

As V_k is independent of t, we know that $\frac{\partial V}{\partial t} = 0$. Furthermore we know that Y follows a geometric Brownian motion. This implies that we can re-wright equation (A.2).

$$dV_k = \frac{\partial V_k}{\partial Y} (\mu Y dt + \sigma T dz) + \frac{\partial^2 V_k}{\partial Y^2} (\frac{1}{2} \sigma Y^2 dt),\ \forall k \in \mathbf{K} \tag{A.3}$$

$$\mathbf{E}[dV_k] = \left(\mu Y \frac{\partial V_k}{\partial Y} + \frac{1}{2} \sigma Y^2 \frac{\partial^2 V_k}{\partial Y^2} \right) dt,\ \forall k \in \mathbf{K}. \tag{A.4}$$

When substituting (A.4) into the Bellman equation in equation (A.1), we get

$$\rho V_k dt = \frac{\partial \bar{\pi}}{\partial K_k} dt + \left(\mu Y \frac{\partial V_k}{\partial Y} + \frac{1}{2} \sigma Y^2 \frac{\partial^2 V_k}{\partial Y^2} \right) dt,\ \forall k \in \mathbf{K}. \tag{A.5}$$

$$\frac{1}{2} \sigma Y^2 \frac{\partial^2 V_k}{\partial Y^2} + \mu Y \frac{\partial V_k}{\partial Y} - \rho V_k + \frac{\partial \pi}{\partial K_k} = 0,\ \forall k \in \mathbf{K}. \tag{A.6}$$

We then use the conveniece yield to substitute $\mu = \rho - \delta$ into (A.6). We also substitute $V_k = \frac{\partial F(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k}$ into (A.6). This leaves us with the Bellman equation from the optimal stopping problem in Problem 3.

$$\frac{1}{2} \sigma^2 Y^2 \frac{\partial^3 F(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k \partial Y^2} + (\rho - \delta) Y \frac{\partial^2 F(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k \partial Y} - \rho \frac{\partial F(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k} + \frac{\partial \bar{\pi}(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k} = 0,\ \forall k \in \mathbf{K}, \tag{A.7}$$

Solving equation (A.7) with respect to $\frac{\partial F(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k}$ results in

$$\frac{\partial F(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k} = B_{k1}(K_1,\ldots,K_{d(\mathbf{K})})Y^{\beta_1} + B_{2k}(K_1,\ldots,K_{d(\mathbf{K})})Y^{\beta_2} + \frac{F_p(Y,K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k},\ \forall k \in \mathbf{K}. \quad \text{(A.8)}$$

From the boundary condition in equation (2.13) we know that $B_{k2}(K_1,\ldots,K_{d(\mathbf{K})}) = 0$. When integrating with respect to K_k, we get

$$F(Y,K_1,\ldots,K_{d(\mathbf{K})}) = Y^{\beta_1}\int B_{k1}(K_1,\ldots,K_{d(\mathbf{K})})dK_k + F_p(Y,K_1,\ldots,K_{d(\mathbf{K})}),\ \forall k \in \mathbf{K}. \quad \text{(A.9)}$$

Using $\frac{\partial A_1(K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k} = B_{k1}(K_1,\ldots,K_{d(\mathbf{K})})$, we re-write equation (A.9).

$$F(Y,K_1,\ldots,K_{d(\mathbf{K})}) = A_1(K_1,\ldots,K_{d(\mathbf{K})})Y^{\beta_1} + F_p(Y,K_1,\ldots,K_{d(\mathbf{K})}), \quad \text{(A.10)}$$

where $F_p(Y,K_1,\ldots,K_{d(\mathbf{K})})$ is found in appendix B.

When solving for a duopolistic firm we follow the same procedure. However, as we aim to find the value of each duopolistic firm separately, the Bellman equation for firm k is given by

$$\frac{1}{2}\sigma^2Y^2\frac{\partial^3 F_k(Y,K_1,K_2)}{\partial K_k\partial Y^2} + (\rho-\delta)Y\frac{\partial^2 F_k(Y,K_1,K_2)}{\partial K_k\partial Y} - \rho\frac{\partial F_k(Y,K_1,K_2)}{\partial K_k} + \frac{\partial\bar{\pi}_k(Y,K_1,K_2)}{\partial K_k} = 0,\ k=1,2. \quad \text{(A.11)}$$

Solving equation (A.11) with respect to F_k gives

$$F_k(Y,K_1,K_2) = A_{k1}(K_1,K_2)Y^{\beta_1} + F_{kp}(Y,K_1,K_2)\ k=1,2. \quad \text{(A.12)}$$

B The particular solution

Equation (2.9) describes a regressed estimate of the instantaneous profit $\pi(Y,K_1,\ldots,K_{d(\mathbf{K})})$ for a monopolist. We re-wright (2.9):

$$\bar{\pi}(Y,K_1,\ldots,K_{d(\mathbf{K})}) = \sum_{i=1}^{d(\gamma)} Y^{\gamma_i}\left[\sum_{k=1}^{d(\mathbf{K})}\sum_{j=1}^{d(\alpha)} b_{k,ij}K_k^{\alpha_j} + \sum_{\substack{k,u=1,\\ u\neq k}}^{d(\mathbf{K})}\sum_{j,l=1}^{d(\lambda),d(\lambda)} c_{uk,ijl}K_u^{\lambda_j}K_k^{\lambda_l}\right] - \sum_{k=1}^{d(\mathbf{K})} OMC_kK_k. \quad \text{(B.1)}$$

Y follows a geometric Brownian motion, and the expected profit over an infinite time horizon equals the particular solution $F_p(Y, K_1, \ldots, K_{d(\mathbf{K})})$ of equation 2.12. Thus, we express $F_p(Y, K_1, \ldots, K_{d(\mathbf{K})})$ as

$$F_p(Y, K_1, \ldots, K_{d(\mathbf{K})}) = \sum_{i=1}^{d(\gamma)} \int_0^\infty \mathbf{E}[Y^{\gamma_i}] e^{-\rho t} dt \, [\sum_{k=1}^{d(\mathbf{K})} \sum_{j=1}^{d(\alpha)} b_{k,ij} K_k^{\alpha_j} + \sum_{\substack{k,u=1, \\ u \neq k}}^{d(\mathbf{K})} \sum_{j,l=1}^{d(\lambda), d(\lambda)} c_{uk,ijl} K_u^{\lambda_j} K_k^{\lambda_l}] - \sum_{k=1}^{d(\mathbf{K})} \int_0^\infty OMC_k K_k e^{-\rho t} dt \quad \text{(B.2)}$$

$$F_p(Y, K_1, \ldots, K_{d(\mathbf{K})}) = \sum_{i=1}^{d(\gamma)} \int_0^\infty Y^{\gamma_i} e^{[\gamma_i \mu + \frac{1}{2}\sigma^2 \gamma_i (\gamma_i - 1)] - \rho t} dt \left[\sum_{k=1}^{d(\mathbf{K})} \sum_{j=1}^{d(\alpha)} b_{k,ij} K_k^{\alpha_j} + \sum_{\substack{k,u=1, \\ u \neq k}}^{d(\mathbf{K})} \sum_{j,l=1}^{d(\lambda), d(\lambda)} c_{uk,ijl} K_u^{\lambda_j} K_k^{\lambda_l} \right] - \sum_{k=1}^{d(\mathbf{K})} \frac{OMC_k K_k}{\rho} \quad \text{(B.3)}$$

$$F_p(Y, K_1, \ldots, K_{d(\mathbf{K})}) = \sum_{i=1}^{d(\gamma)} \frac{Y^{\gamma_i}}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 \gamma_i (\gamma_i - 1)} \left[\sum_{k=1}^{d(\mathbf{K})} \sum_{j=1}^{d(\alpha)} b_{k,ij} K_k^{\alpha_j} + \sum_{\substack{k,u=1, \\ u \neq k}}^{d(\mathbf{K})} \sum_{j,l=1}^{d(\lambda), d(\lambda)} c_{uk,ijl} K_u^{\lambda_j} K_k^{\lambda_l} \right] - \sum_{k=1}^{d(\mathbf{K})} \frac{OMC_k K_k}{\rho}. \quad \text{(B.4)}$$

This corresponds to

$$F_p(Y, K_1, \ldots, K_{d(\mathbf{K})}) = \sum_{k=1}^{d(\mathbf{K})} \sum_{i,j=1}^{d(\gamma), d(\alpha)} \bar{b}_{k,ij}(\gamma_i) Y^{\gamma_i} K_k^{\alpha_j} + \sum_{\substack{k,u=1, \\ u \neq k}}^{d(\mathbf{K})} \sum_{i,j,l=1}^{d(\gamma), d(\lambda), d(\lambda)} \bar{c}_{uk,ijl}(\gamma_i) Y^{\gamma_i} K_u^{\lambda_j} K_k^{\lambda_l} - \sum_{k=1}^{d(\mathbf{K})} \frac{OMC_k K_k}{\rho} \quad \text{(B.5)}$$

where $\bar{b}_{k,ij}(\gamma_i)$ and $\bar{c}_{uk,ijl}(\gamma_i)$ are given by

$$\bar{b}_{k,ij}(\gamma_i) = \frac{b_{k,ij}}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 + \gamma_i(\gamma_i - 1)}, \; i = 1, \ldots, d(\gamma) \quad \text{(B.6)}$$

$$\bar{c}_{uk,ijl}(\gamma_i) = \frac{c_{uk,ijl}}{\rho - \mu\gamma_i - \frac{1}{2}\sigma^2 + \gamma_i(\gamma_i - 1)}, \; i = 1, \ldots, d(\gamma). \quad \text{(B.7)}$$

The same solution is valid in the duopoly. We then find one particular solution for the value of each firm $k = 1, 2$. Furthermore, in section 2.5, $c_{uk,ijt}$ is no longer constrained to be non-negative.

C Investment triggers

We have shown that the the value function $F(Y, K_1, \ldots, K_{d(\mathbf{K})})$ can be written as

$$F(Y, K_1, \ldots, K_{d(\mathbf{K})}) = A_1(K_1, \ldots, K_{d(\mathbf{K})})Y^{\beta_1} + \sum_{k=1}^{d(\mathbf{K})} \sum_{i,j=1}^{d(\gamma),d(\alpha)} \bar{b}_{k,ij}(\gamma_i) Y^{\gamma_i} K_k^{\alpha_j} + \sum_{\substack{k,u=1,\\ u\neq k}}^{d(\mathbf{K})} \sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)} \bar{c}_{uk,ijl}(\gamma_i) Y^{\gamma_i} K_u^{\lambda_j} K_k^{\lambda_l} - \sum_{k=1}^{d(\mathbf{K})} \frac{OMC_k K_k}{\rho}. \quad \text{(C.1)}$$

In order to find a unique investment trigger for each technology k, we separate $F(Y, K_1, \ldots, K_{d(\mathbf{K})})$ so that $F(Y, K_1, \ldots, K_{d(\mathbf{K})}) = \sum_{k=1}^{d(\mathbf{K})} F_k(Y, K_1, \ldots, K_{d(\mathbf{K})})$. When finding the investment triggers, we assume each capacity $k \in \mathbf{K}$ to be constant. Consequently, $F_k(Y, K_1, \ldots, K_{d(\mathbf{K})})$ is expressed as

$$F_k(Y, K_1, \ldots, K_{d(\mathbf{K})}) = A_{k1}(K_1, \ldots, K_{d(\mathbf{K})})Y^{\beta_1} + \sum_{i,j}^{d(\gamma),d(\alpha)} \bar{b}_{k,ij}(\gamma_i) Y^{\gamma_i} K_k^{\alpha j} + \sum_{u=1,u\neq k}^{d(\mathbf{K})} \sum_{i,j,l=1}^{d(\lambda),d(\lambda),d(\lambda)} \bar{c}_{kl,ij}(\gamma_i) Y^{\gamma_i} K_u^{\lambda_j} K_k^{\lambda_l} - \frac{OMC_k K_k}{\rho} \; \forall k \in \mathbf{K}. \quad \text{(C.2)}$$

The analytical expressions for value matching and smooth pasting are given by respectively

$$\frac{\partial F_k(Y_k^*, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k} = \frac{\partial A_{k1}(K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k} Y^{\beta_1} + \sum_{i,j}^{d(\gamma),d(\alpha)} \alpha_j \bar{b}_{k,ij}(\gamma_i) Y^{\gamma_i} K_k^{\alpha j-1} + \sum_{u=1,u\neq k}^{d(\mathbf{K})} \sum_{i,j,l=1}^{d(\alpha),d(\lambda),d(\lambda)} \lambda_l \bar{c}_{kl,ij}(\gamma_i) Y^{\gamma_i} K_u^{\lambda_j} K_k^{\lambda_l - 1} - \frac{OMC_k}{\rho} = I_k, \; \forall k \in \mathbf{K}, \quad \text{(C.3)}$$

and

$$\frac{\partial^2 F_k(Y_k^*, K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k \partial Y} = \beta_1 \frac{\partial A_{k1}(K_1, \ldots, K_{d(\mathbf{K})})}{\partial K_k} Y^{\beta_1 - 1} + \sum_{i,j}^{d(\gamma),d(\alpha)} \alpha_j \gamma_i \bar{b}_{k,ij}(\gamma_i) Y^{\gamma_i - 1} K_k^{\alpha j-1} + \sum_{u=1,u\neq k}^{d(\mathbf{K})} \sum_{i,j,l=1}^{d(\lambda),d(\lambda),d(\lambda)} \gamma_i \lambda_l \bar{c}_{kl,ij}(\gamma_i) Y^{\gamma_i} K_u^{\lambda_j} K_k^{\lambda_l - 1} = 0 \; \forall k \in \mathbf{K}. \quad \text{(C.4)}$$

The smooth pasting equation is rephrased

$$\frac{\partial A_{1k}(K_1,\ldots,K_{d(\mathbf{K})})}{\partial K_k} = -\frac{\sum_{i,j}^{d(\gamma),d(\alpha)} \alpha_j\gamma_i\bar{b}_{k,ij}(\gamma_i)Y^{\gamma_i-1}K_k^{\alpha_j-1}}{\beta_1 Y^{\beta_1-1}} - \frac{\sum_{u=1,u\neq k}^{d(\mathbf{K})}\sum_{i,j,l=1}^{d(\lambda),d(\lambda),d(\lambda)} \gamma_i\lambda_l\bar{c}_{kl,ij}(\gamma_i)Y^{\gamma_i}K_u^{\lambda_j}K_k^{\lambda_l-1}}{\beta_1 Y^{\beta_1-1}},\ \forall k\in\mathbf{K}, \quad (C.5)$$

and substituted it into the value matching equation to find the investment trigger

$$\sum_{i=1}^{d(\gamma)} Y^{\gamma_i}(\frac{\beta_1-\gamma_i}{\beta_1})\left\{\sum_{j=1}^{d(\alpha)}\alpha_j\bar{b}_{k,ij}(\gamma i)K_k^{\alpha_j-1}+ \sum_{u=1,u\neq k}^{d(\mathbf{K})}\sum_{j,l=1}^{d(\lambda),d(\lambda)}\bar{c}_{kl,ij}(\gamma_i)K_u^{\lambda_j}K_k^{\lambda_l-1}\right\} = I_k + \frac{OMC_k}{\rho}\ \forall k\in\mathbf{K}. \quad (C.6)$$

D The numerical regression

In section 2.4, equation (2.9) and section 2.5, equation (2.30) we proposed a regressions for handling the fact that the profit is additively none-separable. These regressions specify that the profit is a function of the shock level Y and the installed capacity $K_k,\ k\in\mathbf{K}$. In appendix B we showed the relationship between the the regressions and the particular solution to the optimal stopping problem. In the numerical procedure in section 3.1 we keep the installed capacity constant when solving the optimal stopping problem. Dixit and Pindyck (1994) claims that in order to convert a stochastic control problem to an optimal stopping problem, we need to assume that everything except from the shock process(es) are kept constant. Consequently, we can wright

$$F_{1,p}(Y,K_1,K_2) = \sum_{i,j=1}^{d(\gamma),d(\alpha)}\bar{b}_{1,ij}(\gamma_i)Y^{\gamma_i}K_1^{\alpha_j}+ \sum_{u=1,u\neq k}^{d(K)}\sum_{i,j,l=1}^{d(\gamma),d(\lambda),d(\lambda)}\bar{c}_{12,ij}(\gamma_i)Y^{\gamma_i}K_1^{\lambda_j}K_2^{\lambda_l} - OMC_1K_1 \quad (D.1)$$

$$\frac{\partial F_{1,p}(Y,K_1,K_2)}{\partial K_1} = \sum_{i,j}^{d(\gamma),d(\alpha)}\alpha_j\bar{b}_{1,ij}(\gamma_i)Y^{\gamma_i}K_1^{\alpha_j-1}+ \sum_{i,j,l=1}^{d(\lambda),d(\lambda),d(\lambda)}\lambda_l\bar{c}_{12,ij}(\gamma_i)Y^{\gamma_i}K_2^{\lambda_j}K_1^{\lambda_l-1} - \frac{OMC_1}{\rho} \quad (D.2)$$

$$\frac{\partial F_{1,p}(Y,K_1,K_2)}{\partial K_1} = \sum_{i=1}^{d(\gamma)} Y^{\gamma_i}\Bigg(\sum_{j=1}^{d(\alpha)} \alpha_j \bar{b}_{1,ij}(\gamma_i) K_1^{\alpha_j-1} + \sum_{j,l=1}^{d(\lambda),d(\lambda)} \lambda_l \bar{c}_{12,ij}(\gamma_i) K_2^{\lambda_j} K_1^{\lambda_l-1}\Bigg) - \frac{OMC_1}{\rho}. \quad \text{(D.3)}$$

Since K_1 and K_2 are constant, we can rephrase

$$\frac{\partial F_{1,p}(Y,K_1,K_2)}{\partial K_1} = \sum_{i=1}^{d(\gamma)} a_{1,i} Y^{\gamma_i} - \frac{OMC_1}{\rho} \quad \text{(D.4)}$$

where

$$a_{1,i} = \sum_{j=1}^{d(\alpha)} \alpha_j \bar{b}_{1,ij}(\gamma_i) K_1^{\alpha_j-1} + \sum_{j,l=1}^{d(\lambda),d(\lambda)} \lambda_l \bar{c}_{12,ij}(\gamma_i) K_2^{\lambda_j} K_1^{\lambda_l-1}, \qquad i = 1,\dots,d(\gamma). \quad \text{(D.5)}$$

Due to the symmetry of the problem, substitute the 1-indexes by the 2-indexes and vice versa. In doing so, we do obtain the result for $\frac{\partial F_{2,p}(Y,K_1,K_2)}{\partial K_2}$.

When solving for a duopoly $\bar{c}_{12,ij}(\gamma_i)$ is not constrained to be greater than zero. Consequently, one might find a case in which

$$\sum_{j=1}^{d(\alpha)} \alpha_j \bar{b}_{1,ij}(\gamma_i) K_1^{\alpha_j-1} + \sum_{j,l=1}^{d(\lambda),d(\lambda)} \lambda_l \bar{c}_{12,ij}(\gamma_i) K_2^{\lambda_j} K_1^{\lambda_l-1} < 0,\ i = 1,\dots,d(\gamma). \quad \text{(D.6)}$$

This corresponds to $a_{k,i} < 0$. However, in our model we make the simplifying assumption that $a_{k,i} \geq 0,\ k = 1,2,\ i = 1,\dots,d(\gamma)$. In doing so, we make sure that there are only one optimal investment trigger for each technology. This is a reasonable assumption as the positive effect of firm 1 being able to generate using technology 1 is likely to outweighed the negative effect of the competition from firm 2.

When using the value matching and smooth pasting conditions as in appendix C, one can show that the investment trigger Y_k^* is given as the unique root of

$$\sum_{i=1}^{d(\gamma)} a_{k,i} {Y_k^*}^{\gamma_i} \left(\frac{\beta_1 - \gamma_i}{\beta_1}\right) = I_k + \frac{OMC_k}{\rho}, \qquad k = 1,2. \quad \text{(D.7)}$$

E Cournot Equilibrium

By using standard Cournot assumptions, we create a Cournot equilibrium. This can easily be illustrated assuming d$(\mathbf{L}) = 1$ and

$$P(Y_t,Q) = Y_t D(Q) = Y_t(A - bQ) \quad \text{(E.1)}$$

where $Q = q_1 + q_2$. The profit functions can then be written as

$$\begin{aligned}\pi_1(Y_t, K_1, K_2) = \tau[P(Y_t, Q)q_1 - c_1 q_1] - OMC_1 K_1 = \\ \tau[Y_t(Aq_1 - bq_1^2 - bq_1 q_2) - c_1 q_1] - OMC_1 K_1\end{aligned} \tag{E.2}$$

and

$$\begin{aligned}\pi_2(Y_t, K_2, K_1) = \tau[P(Y_t, Q)q_2 - c_1 q_2] - OMC_2 K_2 = \\ \tau[Y_t(Aq_2 - bq_2^2 - bq_2 q_1) - c_2 q_2] - OMC_2 K_2.\end{aligned} \tag{E.3}$$

We can then use standard Lagrange optimization to solve the problem.

$$L_i = \pi_i - \lambda_i(q_i - K_i) - \mu_i(q_i - 0), \qquad i = 1, 2 \tag{E.4}$$

By solving equation (E.4) with respect to q_1 and q_2 we obtain

$$q_1 = \begin{cases} 0, & q_1 \leq 0 \\ \frac{1}{2b}(A - \frac{c_1}{Y_t}), & 0 < q_1 < K_1 \text{ and } q_2 = 0 \\ \frac{1}{3bY_t}(AY_t + c_2 - 2c_1), & 0 < q_1 < K_1 \text{ and } 0 < q_2 < K_2 \\ \frac{1}{2b}(A - bK_2 - \frac{c_1}{Y_t}), & 0 < q_1 < K_1 \text{ and } q_2 = K_2 \\ K_1, & \text{otherwise} \end{cases} \tag{E.5}$$

$$q_2 = \begin{cases} 0, & q_2 \leq 0 \\ \frac{1}{2b}(A - \frac{c_2}{Y_t}), & 0 < q_2 < K_2 \text{ and } q_1 = 0 \\ \frac{1}{3bY_t}(AY_t + c_1 - 2c_2), & 0 < q_2 < K_2 \text{ and } 0 < q_1 < K_1 \\ \frac{1}{2b}(A - bK_1 - \frac{c_2}{Y_t}), & 0 < q_2 < K_2 \text{ and } q_1 = K_1 \\ K_2, & \text{otherwise} \end{cases} \tag{E.6}$$

with the corresponding shadow costs of the capacity constraints

$$\lambda_1 = \begin{cases} 0, & q_1 < K_1 \\ \tau[Y_t(A - 2bq_1 - bq_2) - c1], & \text{otherwise} \end{cases} \tag{E.7}$$

$$\lambda_2 = \begin{cases} 0, & q_2 < K_2 \\ \tau[Y_t(A - 2bq_2 - bq_1) - c2], & \text{otherwise.} \end{cases} \tag{E.8}$$