Capacity Expansion in the Electricity Market

SIS1101 Accounting and Finance, Specialization

Written by

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Preface

This paper is written in the directed course SIS1101 Accounting and Finance, Specialization, given by the Department of Industrial Economics and Technology Management at The Norwegian University of Science and Technology autumn semester 2002.

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Abstract

This paper opens with a theoretical description of important features necessary for understanding whether to expand the capacity of a hydropower plant or not, and ends with an example of a possible capacity expansion at Svartisen Power Plant.

Introductorily a brief description of the characteristics of the Nordic Power Market is given, and of the properties of hydropower systems in general. Then follows a description of some theoretical frameworks of great importance in understanding capacity expansion problems.

These frameworks are:

- A mathematical description of the generation planning problem, which leads to a description of algorithms for solving such problems. Thus dynamic programming (DP) algorithms are explained.
- Price models that describe the forward power prices are presented. Different one-factor stochastic price models are listed and it is argued for the use of such forward curves instead of the use of expected spot prices.
- A real option approach used for investment problems is described in order to value the option of an investment versus the investment itself. It is argued that such an approach is useful in capacity expansion decisions due to the uncertainty of future prices.

In the end of the paper the theoretical framework is used for a practical purpose. A capacity investment decision at Svartisen Hydropower Plant is valued. The Lucia and Schwartz (2002) one factor model with log prices is chosen as a description of the forward price. This price model is used to modify 70 price scenarios of expected spot price for the area. To be able to do this the price model is adjusted to the behavior of the spot price as a function of the seasonal change. Then these 70 price scenarios are used as an input to the EOPS model from Sintef Energy (EFI's One-Area Power-market Simulator). Input to the model is 70 price scenarios and inflow scenarios. The model uses a stochastic dynamic programming algorithm, and gives NPV of the investment project as output. Finally these numbers are used to value the option of a capacity expansion of 30MW, 150MW and 300MW given different seasonal behavior of the forward price.

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1 Introduction

The background for this paper is the fact that power prices in the Nordic Power Market fluctuates in seasonal, weekly and daily patterns in addition to fluctuations due to stochastic factors as inflow and weather.

This gives a hydropower plant the opportunity to expand their capacity in order to produce more when price is high, and less when price is low.

Some power plants has the option to expand the capacity because they were constructed with penstocks dimensioned for possible future capacity expansion, other plants does not have this opportunity and a capacity expansion will include more expensive construction operations. (Faanes, 2002).

The valuation of such investments has very long time horizon. This emphasizes the necessity of knowledge of prices in the future, as well as an understanding of political, economic, technologic and stochastic factors having influence of this price.

Due to this, it has been chosen to describe the Nordic Power Market as well as the properties of hydropower including algorithms for solving generation planning problems. In addition models for describing forward power prices are presented.

The theories of forward prices, generation planning and option pricing are consecutively used to valuate a capacity expansion at Svartisen Power Plant. Some practical adjustments are done with the theoretical model to accomplish this valuation, and some of the properties of the Plant are simplified. Most of the adjustments to the price model are caused by the decision to use a log price model instead of a price model. The choice of a price model would probably cause fewer adjustments.

Though a practical example of an expansion decision is given in the last chapter, the model is not completely developed. There is still a lot to be done in expressing the forward price better as well as defining the input variables to the option valuation model. However this paper should be a substantial contribution to a complete model.

2 Generation Planning

This chapter presents general theory about the Nordic power market and the properties of hydropower scheduling. A mathematical description of the generation planning problem is presented. In this description, the price of electricity is an input parameter. Different price models that describe the forward prices in the power market will be presented in chapter 3 and different algorithms for solving this planning problem will be presented in chapter 4.

2.1 Introduction

Generation planning at a hydropower plant is a difficult and extensive job. The amount of production at each time step depends on the price in the market and the value of the water in the reservoir. The price depends on the demand and supply in the market. There are uncertainty associated to both future prices and future water values. The producers want to maximize their income in regard to the constraints of the reservoir, watercourses and plants. They have to look at and evaluate tradeoffs among immediate and future uses of water.

2.2 The Market

One special feature in this market is that electricity is not a storable commodity; it has to be used immediately. Because of this, there is low elasticity in the prices, and hence major differences in prices in different time steps. When demand is high, the price is high and when demand is low, the price can be as low as the marginal cost. In a hydro energy system, the capacity is relatively high, but the total amount of energy available is limited. Under normal circumstances the capacity is higher than demand. This causes relatively low peak prices compared to thermal energy systems.

2.2.1 The Deregulation of the Power Market

The deregulation was introduced in Norway with the Energy Act of June 1990. (Fosso et al, 1999) From January 1, 1991, all purchasers were free to choose from whom to buy electricity. Hence, generation and sales activities became exposed to competition, and this affected the generation planning. In the new deregulated market, the producer has in principle no obligation to serve any particular consumer. The main objective is to generate and sell electricity with maximum profit. This is a change from the traditional formulation of the generation planning problem where the main objective was to minimize the costs. This new problem formulation implies that the producer is regarded as a price taker. This means that the producers do not take into account any influence their own production might have on the market price. (Wangensteen [Kraftmarkeder 6], 2001)

2.2.2 The Organisation of the New Market

The Norwegian electricity system has a very decentralised organisational structure. There are about 70 electricity-producing companies, 230 distributors and about two million end users. Large end users trade in the wholesale market on a bilateral or spot basis, while most of the small customers have access to the market either though their local distributor or though an external supplier. (Wangensteen [Kraftmarkeder 4], 2001)

Here is an overview on the main activities and roles of the different entities in the system:

- Market participants
  These are the buyers and sellers in the market place, which can be generating companies, utilities with distribution and more or less generating capacity, or end users.
- Market operator
  The market operator or the exchange, Nord Pool, is responsible for the market clearing process in the spot market and in the futures market. Accounting and invoicing is also a responsibility of the market operator.
- The system operator
  The system operator, which in Norway is the national transmission grid-company Statnett SF, is responsible for system coordination.

(Fosso et al, 1999, Wangensteen [kraftmarkeder 6], 2001)

The Norwegian system does not include a central scheduling or dispatching entity, as the system in England. The generation companies are responsible for production planning, while the power exchange is responsible for market clearing and the system operator is responsible for system coordination. (Fosso et al, 1999)

There are three different organised markets:

- The spot market
  In the spot market the participants submit their bids for buying and selling on an hourly basis. The market is settled every day at noon for delivery for 24 hours following the first midnight. The market operator use the individual bids to aggregate total supply and demand curves. The intersection between those two curves determine the clearing price and the quantity. The spot market is not a spot market in the common meaning of the word; it is actually a 12-36 hours future market.
- The regulating market
  This market is used to adapt generation to the variation in the load. Producers submit their bids to the system operator on how much they are willing to regulate up or down, for different prices and periods of time. In real-time operation, system operator picks the cheapest available regulator from the merit list. Hence, the price in the regulating market is settled ex post, when the price of marginal regulator in each hour is known. All the regulators receive the price of the marginal regulator.
- The future market
  This is purely a financial market. The turnover in the future market is about 9 times bigger than the turnover in the spot market. It is because of the future market that the power market can be viewed as a liquid market. (http://www.nordpool.no)

(Wangensteen [kraftmarkeder 6], 2001)

The market participants, the system operator and the market operator perform the operation planning together in a dynamic interaction.

2.3 Hydropower

More than 99% of the electricity generation in Norway is based on hydro. A hydropower system is characterised by some special features. It has high investment cost and very low variable cost. Consequently, the producers have low cost variability. The variations in prices are not necessarily low because of this. (Wangensteen [Kraftmarkeder 3], 2001) The prices depend on the limitation on reservoir capacities. The storage of water varies between seasons. In the summer, snow melting and rainfall cause high water inflow, while inflow is much less throughout the winter. An important characteristic of a hydropower system is that the market prices varies greatly even in periods of a few months, or weeks, depending on variations in inflow in addition to variations in consumption. (Haugstad et al, [kraftmarkeder 10], 2001).

In Table 2-1 the yearly implied volatility at NordPool from Jan 1. to Des 31. 2001 are calculated with different time resolution. An explanation of the change in volatility as a function of the resolution is that daily resolution hides the predictable daily pattern due to change in demand during the day. Weekly resolution hides the predictable weekly pattern due to different demand in weekends and so on.

	Hour	Day	Week	Month	Year
Volatility (%)	245	120	106	95	55

Table 2-1 Empirical data for the volatility

Hence, there is a considerable difference between generation planning for a thermal power plant and a hydropower plant. The price of fuel is known for a thermal power plant. So the generation planning process is just supposed to determine the number aggregates in operating modus in the next period, the distribution between the aggregates and the amount they want to buy and sell in the market. This generation planning does not need a long analysis period, maximum a week.

A hydropower plant does not have a fuel price, but in the planning process the producer has to consider the value of the water in the reservoir in order to evaluate tradeoffs among immediate and future uses of water. The production planning has to be divided into different horizons of time, short time and long time. In the long horizon, the producer has to determine the disposal of the reservoir. In the short horizon, the producer determines the number of aggregates in operating modus, the distribution between the aggregates and the amount they want to buy in the market. The objective of hydropower scheduling is to determine the sequence of hydro releases that will maximize the operational income. (Wangensteen [kraftmarkeder 4], 2001)

2.3.1 The Value of the Water

The water flows into the reservoir without any cost. That does not mean that the water does not have any value. The producer cannot regulate the inflow of water, and the inflow is not constant. The generation planning problem is then to evaluate tradeoffs among immediate and future use of the water. They can use the water now, produce power and sell it for a known price at the spot market or they can store the water and sell power later for an unknown price. The problem is to evaluate which of the two alternatives that makes the best economic result. (Wangensteen [kraftmarkeder 4], 2001).

The future demand and the future inflow are unknown, but the storage capacity of the reservoir is known. To determine how much water that should be stored, historical demand and inflow or prognosis for demand and inflow can be looked upon. If the inflow becomes large, the risk an overflow of increases. Then the value of the water would be low. If the inflow becomes small, then the value of the water would get high. The production capacity, the storage capacity, the inflow and the power market therefore determine the value of the water.

When the value of the water is determined, the hydropower plant face the same generation planning problem as a thermal power plant. Using the marginal cost principal, the disposal of the production resources can be determined.

2.4 Expansion of the Capacity

Hydropower generators are easy to regulate and have quite low start/stop costs compared to other types of power production. Hydropower producers will offer their surplus power when the price is high and reserve themselves when the price is low. By installing more power, the producer might produce more when the price is high.

The regulation level of the reservoir is also important in a hydro power plant. When the regulation level is high, the plant can choose to produce when the price is high. With lower regulation level or constricted minimal flow the plant has to produce in a more regular schedule. When the regulation level is low a capacity expansion will reduce the risk of an overflow.

2.5 A Mathematical Description

In a deregulated market, a power producer has in principle no other objective than to produce electricity and sell with maximum profit. The generation-planning problem can be formulated like this:

> *Given a forecast of future market price: Establish a generation schedule that maximizes expected profit over the planning period, all relevant constraints taken into account.* (Fosso et al, 1999, Wangensteen [kraftmarkeder 6], 2001)

This formulation does not consider any penalty for possible curtailment of contractual obligations. It is assumed that if a generating company does not produce enough power to cover its contractual obligations, this can be bought on the spot market. Because the objective is to maximize expected profits, it can be assumed that all produced power is sold on the spot market. This planning model is as previously mentioned based on a price taking assumption. The market price is not influenced by short-term variations in the owner’s generation. The assumption that all firms are price takers is a necessary condition for a free market to be economically efficient. The cost for starting and shutting down an aggregate is relatively low for a hydropower plant, so these cost are not taken into the problem formulation.

In order to find an optimum operational strategy, a forecast of the future market price and the future inflow are needed. Different price models will be described in chapter 3.

2.5.1 The One-reservoir Model

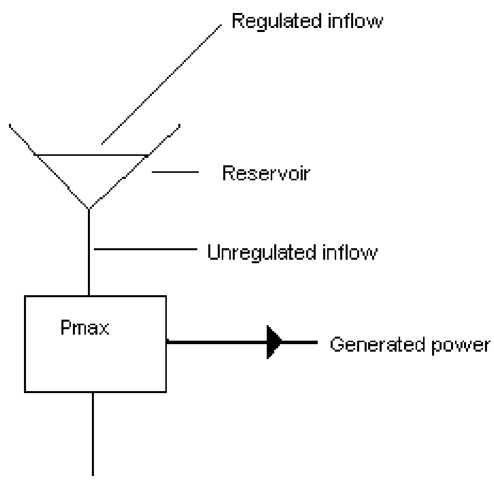

Figure 2-1 The one-reservoir model

The objective function for the one-reservoir model can be written like this:

$$\text{Maximize } E_{vp}\{\sum_{i=1}^{N} q(i)p(i) + S(x(N),p(N))\} \quad (2.1)$$

Constraints are the reservoir equation and variable bounds:

$$x(i+1) = x(i) + v(i) - s(i) - q(i) \quad (2.2)$$

$$x_L(i) \le x(i) \le x_U(i) \quad (2.3)$$

$$q_L(i) \le q(i) \le q_U(i) \quad (2.4)$$

Definition of variables:
E_{vp} - The expected value of the production and the value of the water
q(i) - Generation in time step i
p(i) - The price of electricity in time step i
S(*,*) - The value of storage at the of planning period
i -Time index
N - Time index for the end of the planning period
x(i) - The reservoir level at the end if time step i
$x_L(i)$ - The minimum allowed reservoir level in time step i
$x_U(i)$ - The maximum allowed reservoir level in time step i
s(i) - The spillage in time step i
$q_L(i)$ -The minimum constraint on generation in time step i

$q_U(i)$ - The maximum constraint on generation in time step i

Constraint (2.2) represents the coupling between successive stages. The reservoir storage at the end of stage i, which is the beginning of stage i + 1, is equal to the initial storage plus the inflow minus the outflow.

Constraint (2.3) limits the reservoir storage. The reservoir level at the end of time step i must be above a certain minimum allowed level and under a certain max level. This minimum level is set by the government to assure that natural environment is not damaged. The maximum allowed level is the max storage level.

Constraint (2.4) limits the generation in time step i. The minimum constraint on generation is dependent on the minimum stream flow in the waterfall, which is set by the environmental department. The maximum constraint on generation is the max capacity of the aggregates on the power plant. It is this upper level that will be relaxed in an expansion of capacity of a hydropower plant.

3 Price Models

This chapter presents some price models that can be used in describing the Nordic power market. The price is an important variable in the generation planning problem described in chapter 2. Initially, an empirical description of the Nordic power prices and a macro economic view are presented. Then some mathematical tools for modeling forward prices are presented and consecutively different price models are described.

3.1 Empirical Description of Nordic Power Prices

Nordic electricity prices follow a periodic seasonal behaviour. The prices are at the highest level in cold winter days due to extensive use of electricity for heating purposes, especially in Norway. During the summer months there are lower demand due to lower heating demand and lower demand for indoor lighting. In addition the prices have a characteristic weekly structure with a peak in the morning and in the afternoon, and low prices during the night. Weekends and holidays have lower prices due to less demand in commercial and public utilities.

In addition, several factors can influence the price for longer or shorter time periods. Examples are maintenance of nuclear power plants in Sweden, periods of extreme cold weather or more predictable cases as when everybody turn on the stove at Christmas evening. Prices follow demand in general, but available production will be somewhat lower in low demand periods due to planned revisions.

3.2 A Macro Economic View

There will be no analysis of macro economic factors in this paper, but a brief description some important factors will be given. The uncertainty of these factors contributes to the uncertainty of forward prices.

An important aspect is the exchange capacity through new undersea cables to Mainland Europe. There is a lot of political and economic uncertainty in these projects. Increased exchange capacity will probably lead to higher daily fluctuations within day and night, but will probably also decrease the extreme high peaks in cold winter days.

Another important factor is political decisions about Swedish nuclear plants. A further decrease in this capacity will probably cause higher prices in general.

Trading of green certificates will probably also rise the price level in the Nordic area. How such certificates in great scale will affect European power prices is not known, but the idea is based on the assumption that consumers are willing to pay a price premium for green energy.

Other factors such as technological development, public attitudes to hydropower and economic progress in general will also make contributions to the forward prices.

3.2.1 Forward Prices vs. Spot Prices

Forward prices are often used when a certainty equivalent is needed for the future cash flow. There is uncertainty attached to the future income from trading in the spot market. By trading in the forward market this uncertainty is eliminated and the cash flow can be looked upon as certain. Hence to maintain consistence with the market, the decision maker should set the spot price for a given period equal to the forward price of delivery in that period. (Fleten et al, 2002)

3.3 Mathematical Tools

In this chapter some mathematical tools are listed. These are often used in price models.

3.3.1 Brownian Motions

A Brownian motion is a process of the form

$$\delta x = a\delta t + \sigma\delta z \qquad (3.1)$$

where δz is a standard Wiener process of the form

$$\delta z = \varepsilon * \delta\sqrt{(t)} \qquad (3.2)$$

where ε is a norm inverted random number.

The Brownian motion with a random walk and an expected drift can be used to model for example the spot price or the temperature where the future value is uncertain.

3.3.2 Mean Reverting Processes

A mean reverting process can be used to model for example the spot price. The process has the form:

$$dS = \kappa(\mu(t) - S)dt \qquad (3.3)$$

Definition of the variables:

- dS - The change in the spot price
- κ - This is the mean reverting factor that describes the speed of mean reversion
- $\mu(t)$ - The mean reverting value which the spot price reverts to
- dt - The increment of time

This function can be used to describe non-permanent changes such as high prices due to extreme cold weather. This function will wipe out the high prices in a speed chosen by changing κ

3.3.3 Jump-processes

A jump process can be of the form:

$$dS = \kappa S dq \tag{3.4}$$

Definition of variables:

dS -The change in the spot price

dq -A binary variable that is 1 at a given probability and 0 else

κ -A stochastic variable that describes the jump size

Jump-processes are processes that describe sudden changes in prices, such as peak prices due to extreme cold weather. When dq is 1, the price make a jump. Jumps can be stationary or temporary. Jump processes will not be used or described further in this paper, but should not be ignored in describing special events as political decisions or extreme weather. (Clewlow and Strickland, 2000)

3.4 One-factor Models

In this chapter the Schwartz (1997) one factor model and the Lucia and Schwartz (2002) one factor model will be presented with prices and log prices. The different models are compared. Finally two stochastic factors are briefly mentioned.

3.4.1 One factor models in general

One-factor models describe pricing models with one stochastic factor. This stochastic factor can be the price or the log price.

An important property of the one-factor model mentioned here is the following equation:

$$dX = \kappa(\mu(t) - X)dt + \sigma dz \tag{3.5}$$

Definition of variables:

X - The price/log price

dX - The increment of price/ log price

$\kappa(\mu(t) - X)dt$ - The mean reverting process

$\mu(t)$ - The long run mean of the price/log price

σ - The price/log price volatility

dz - An increment to a standard Brownian motion

This is also known as an Ohrnstein Uhlenbeck mean reverting process (Schwartz, 1997).

3.4.2 The Schwartz (1997) Model

In the article of Schwartz (1997) a one factor model of the forward prices is presented. This model is on the form:

$$F(S,T) = \exp\left[e^{-\kappa T}\ln S + \left(1 - e^{-\kappa T}\right)\alpha^* + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa T}\right)\right] \tag{3.6}$$

Or in log form:

$$\ln F(S,T) = e^{-\kappa T} \ln S + \left(1 - e^{-\kappa T}\right)\alpha^* + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa T}\right) \tag{3.7}$$

The expression for the forward price is the solution of the following differential equation:

$$\frac{1}{2}\sigma^2 S^2 F_{SS} + \kappa(\mu - \lambda - \ln S)SF_S - F_T = 0 \tag{3.8}$$

With terminal boundary condition:

$$F(S,0) = S \tag{3.9}$$

The differential equation (3.8) is derived in Appendix A-1 using Ito's Lemma and a portfolio approach.

Definition of variables:

σ^2 -The volatility of the spot price
μ -The mean reverting level
λ -The market price of risk (assumed constant)
κ -The speed of adjustment
S -The spot price
F_{SS} - $\frac{\partial^2 F}{\partial S}$
F_S - $\frac{\partial F}{\partial S}$
F_T - $\frac{\partial F}{\partial t}$
T -The time at maturity
α -The mean reverting level of log prices

$$\alpha = \mu - \frac{\sigma^2}{2\kappa} \tag{3.10}$$

$$\alpha^* = \alpha - \lambda \tag{3.11}$$

The Schwartz (1997) model can be used to model forward prices in the power market. In this model the spot price is modelled as a log price to avoid negative prices. The model is developed for commodities in general, but is not specially fitted for electric energy.

3.4.3 The Lucia and Schwartz (2002) Model

In the article of Lucia and Schwartz (2002) it is shown how an Ohrnstein Uhlenbeck process leads to forward prices based on spot prices on the following forms:

$$F_0(P_0,T) = E_0^*(P_T) = f(T) + (P_0 - f(0))e^{-\kappa T} + \alpha^*(1 - e^{-\kappa T}) \qquad (3.12)$$

Definition of variables:

$F_0(P_0,T)$	-Forward price as a function of time and starting price
$f(T)$	-A deterministic function
$f(0)$	-The deterministic function at Time 0
P_0	-The starting spot price
P_T	-Spot price at time T
T	-Time
κ	-Speed of adjustment
α^*	$-\frac{\lambda\sigma}{\kappa}$

Based on log prices:

$$F_0(P_0,T) = E_0^*(P_T) = \exp\left[f(T) + (\ln P_0 - f(0))e^{-\kappa T} + \alpha^*(1 - e^{-\kappa T}) + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa T})\right] \qquad (3.13)$$

The Lucia and Schwartz (2002) model is in contradiction to the Schwartz (97) model specially suited for electric energy. The deterministic element given by f(t) makes a description of the seasonal behaviour of power prices possible.

This seasonal behaviour is described by the following equation in Lucia and Schwartz (2002).

$$f(t) = \alpha + \beta D_t + \gamma \cos((t + \tau)\frac{2\pi}{360}) \qquad (3.14)$$

Definition of variables:

α	-The basic level of the sinusoidal oscillation
β	-A constant that describe the consumption on Holidays
D_t	-A binary variable that equals 1 on Holidays
γ	-The level of the amplitude
τ	-The time delay from January 1. to the date of the highest price

This is a sinusoidal curve around a basic level, α, which oscillates at a level given by γ. The maximum of the function during a year can be regulated by τ. The binary variable D_t makes it possible to adjust the prices to special days, such as public holidays.

3.4.4 Comparing the two Models

In Schwartz (1997) the long time equilibrium based on log prizes will be

$$\exp\left[\mu - \frac{\sigma^2}{4\kappa} - \lambda\right] \tag{3.15}$$

In the Lucia and Schwarz 2002 Model with log prices the long-term equilibrium will be

$$\exp\left[f(t) + \frac{\sigma^2}{4\kappa} - \frac{\lambda\sigma}{\kappa}\right]. \tag{3.16}$$

Both equation (3.15) and equation (3.16) leads to the conclusion that higher market price of risk, given by λ, will cause lower forward prices.

The Lucia and Schwarz (2002) model can describe seasonal behaviour; the Schwartz (1997) model cannot do that. The Lucia and Schwartz (2002) model will therefore be a better chose in cases where seasonal behaviour is important, such as in expansion decisions.

From equation (3.15) and equation (3.16) it can be seen that f(t) and μ are not at the same level since both (3.15) and (3.16) describes the same forward price and the equations are different. The two models are different because Lucia and Schwartz (2002) use a slightly different presentation of the mean reverting level than Schwartz (1997). A proper regression of both models with constant f(t) in the Lucia and Schwartz (2002) model will show the same forward curve, but the mean reverting levels will be different.

3.5 Two and three factor models

In the article of Schwartz (1997) it is described models with up to three stochastic variables. In addition to prices, convenience yield and risk free rate are stochastic in these models. In the article of Lucia and Schwartz (2002) it is argued that a two-factor model describes forward prices in Nordpool better than one-factor models. Two-factor models are however more complex and will not be discussed further in this paper.

It is claimed (Schwartz, 1998) that a two factor stochastic forward model can be described almost as a one factor stochastic model in the long run. This is because much of the difference in one and two stochastic factors will appear in the first three years. If the time to enter the market is long or the horizon of the investment is long enough, two factor models can be treated as one-factor models.

4 Algorithms for Solving the Generation Planning Problem

This chapter describes different algorithms for solving the generation planning problem presented in chapter 2. The planning problem for a hydropower producer is very difficult to solve because of the great amount of calculations. Dynamic programming methods are common methods for solving these kinds of problems.

4.1 Dynamic Programming Algorithms

Dynamic programming (DP) is a useful technique for analysing a sequential decision process. It breaks the sequence of decisions into just two components, the immediate decision and a valuation function that captures the consequences of all subsequent decisions, starting with the position that results from the immediate decision. (Dixit and Pindyck, 1994) When DP is used for optimising reservoir operations, the modelling horizon is generally divided into different decision-making stages. For each stage, typically a week or month, a set of system states indexed by m = 1,…,M are defined. The state of the system describes different levels of storage in the reservoir, for example 100%, 90% etc. Figure 4-1 illustrates the system state definition for a single reservoir. (Faber and Stedinger, 2001, Pereira et al, 1999)

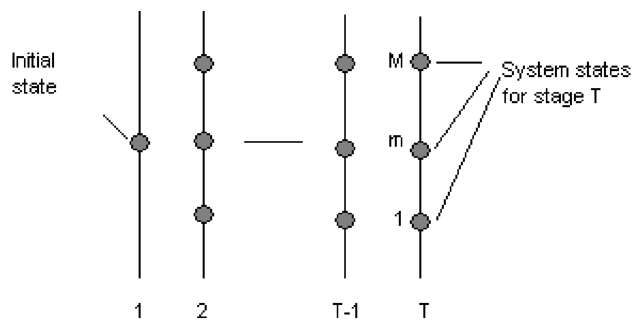

Figure 4-1 Description of system states

A release of water is chosen for each stage in order to maximize the sum of the current benefits of that release and the future benefits. The future benefits are dependent on the resultant storage in the following period. The model is solved with a backward recursive procedure. (Faber and Stedinger, 2001)

The reservoir-storage mass balance can be written:

$$f_t(S_t) = \max_{R_t} \left\{ B_t(S_t, Q_t, R_t) + \alpha \cdot f_{t+1}\left(S_{t+1} \right) \right\} \tag{4.1}$$

$$\forall S_t \vee t \in \{1,....,T\}$$

$$S_{t+1} = S_t + Q_t - R \ - e_t(S_t, S_{t+1}) \tag{4.2}$$

Definition of variables:

t	- The time period (stage)
T	- The final period in the model
S_t	- The reservoir storage vector for period t
S_{t+1}	- The reservoir storage vector for period t+1
Q_t	- The inflow vector for period t
R_t	- The water release in period t
$B_t(*,*,*)$	- The benefit function for period t
α	- The discount factor
$e_t(S_t, S_{t+1})$	- The evaporation loss in period t
$f_t(S_t)$	- The benefits of the reservoir at stage t
$f_{t+1}(S_{t+1})$	- The future benefits

The calculations starts in the final stage, T, calculating $f_t(S_t)$ for every state m. This requires that the current benefits $B_T(*)$ and the ending value for the storage to be known. The set of $f_T(S_T)$ values is then used to calculate $f_{T-1}(S_{tT-1})$ and so on, until $f_1(S_1)$ is found for each state m in stage 1. The final stage, T, is often chosen to be at spring when the likelihood for overflow is greatest. Then the storage is set to be 100%.

4.1.1 Stochastic Dynamic Programming (SDP)

The inflow to a reservoir in a time period is uncertain. The reservoir inflows can therefore be characterised as random variables described with probability distributions. With a stochastic description of the inflow, SDP can be used to compute the expected benefits from each release decision. If the current periods inflow are considered to be unknown, the reservoir mass balance equation can be written:

$$f_t(S_t) = \max_{R_t} \underset{Q_t}{E} \{B_t(S_t, Q_t, R_t) + \alpha \cdot f_{t+1}(S_{t+1})\} \tag{4.3}$$

$$\forall S_t \vee t \in \{1,....,T\}$$

$$R_t = \max\{\min\{R_t^*, S_t + Q_t\}, (S_t + Q_t - S_{\max} - e_t(S_t, S_{t+1}))\} \tag{4.4}$$

Definition of variables:

R_t^*	-The optimal target release vector for period t
R_t	-The optimal release vector modified to honour available storage, space and inflow
$S_t + Q_t$	-The total amount of water available
$S_t + Q_t - S_{\max} - e_t(S_t, S_{r+1})$	-The water that cannot be stored

The maximization in equation (4.3) occurs outside the expected value operator. Because of this a single release target R_t^* is chosen for the entire distribution of unknown inflows. The inflow varies between different seasons of the year. It is therefore possible that the release is not feasible, so R_t must be adjusted to reflect the flow and storage space available. This is done in (4.4). This equation avoid overflow by choosing the highest value of wanted release and the amount of water not storable. Simultaneously, it is not possible to release more water than is available.

To solve the SDP problem, the calculations are computed first in the final stage similar to the DP algorithm. For every state m, there are different inflow scenarios with different probabilities as described in figure 4-2. The expected value of the benefit is calculated for every state m at the last stage T, and these calculations are used to find $f_{T-1}(S_{tT-1})$. This algorithm is follow until $f_1(S_1)$ is found for each state in stage 1. (Pereira et al, 1999)

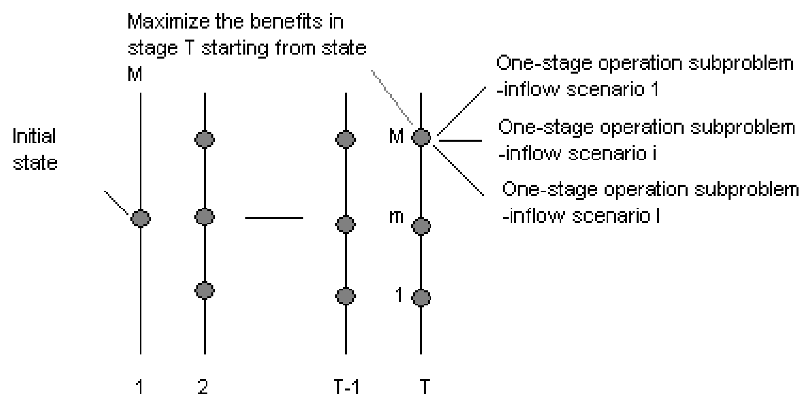

Figure 4-2 Description of stochastic inflow at each state

The SDP formulations do not capture stream flow persistence from one period to the next. By using the SDP formulation, the value of the stream flow at stage t + 1 is independent of the value at stage t. In reality, there is often a strong serial correlation between stream flows in consecutive periods. (Faber and Stedinger, 2001)

SDP models usually describes stream flow with several carefully chosen discrete points, yielding a Markov Chain stream flow model. (Faber and Stedinger, 2001)

4.1.2 Sampling Stochastic Dynamic Programming (SSDP)

Stochastic Sampling Dynamic Programming models (SSDP) use a sample of stream flow scenarios to describe future flows instead of the Markov Chain description in the SDP models. The background for such an extension of the SDP framework is that SDP models often overestimate the benefits with particular release decisions (Faber and Sterdinger)

The model can be written:

$$\max_{R_t}\left\{B_t\left(S_t, Q_t(i), R_t\right) + \alpha \cdot \underset{jIi}{E}\left[f_{t+1}\left(S_{t+1} \right)\right]\right\} \quad (4.5)$$

$\forall$ S_t, i and t$\in${1,….,T}

$$f_t\left(S_{t,i}\right) = B_t\left(S_t, Q_t(i), R_t\right) + \alpha \cdot f_{t+1}\left(S_{t+1}\right) \quad (4.6)$$

$\forall$ S_t, i and t$\in${1,….,T}

Description of variables:
i - The stream flow scenario
$Q_t(i)$ - The stream flow in period t, scenario I

The main difference from SDP given by equation (4.3) is that f $_{t+1}$(S_{t+1}) now is given by a probability of scenario j followed by scenario i. This implies the construction of transition probabilities. This will not be discussed further in this paper.

4.1.3 Stochastic Dual Dynamic Programming (SDDP)

This approach is based on an analytical representation of the future benefits of resources. It does not require discrete states. In SDDP, the benefits at each stage are represented as a piecewise linear function which correspond to the value of benefit in each state as shown in figure 4-3. (Pereira et al, 1999)

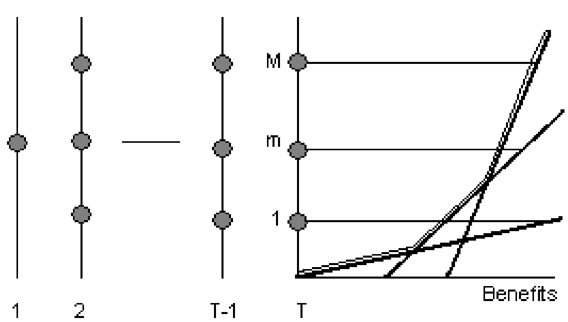

Figure 4-3 Calculation of piecewise future benefits for stage T-1

The model can be written as a LP problem:

$$f_t(S_t) = \max\left\{B_t\left(S_t, Q_t(i), R_t\right) + \alpha \cdot f_{t+1}\left(S_{t+1}\right)\right\} \quad (4.7)$$

subject to simplex multiplier:

$$S_{t+1} = S_{t,m} + Q_t(i) - R_t - e_t(S_t, S_{t+1}) \qquad \pi_{h,t,i} \tag{4.8}$$

$$S_{t+1} \leq S_{\max} \tag{4.9}$$

$$R_t \leq R_{\max} \tag{4.10}$$

$$f_{t+1} \geq \varphi_{t+1,n} S_{t+1} + \delta_{t+1,n} \tag{4.11}$$

$$\varphi_{t,m} = \sum_{i=1}^{I} p_i \cdot \pi_{h,t,i} \tag{4.12}$$

$$\delta_{t,m} = \sum_{i=1}^{I} p_i \cdot f_{t,i}(S_{t,m}) - \varphi_{t,m} \cdot S_{t,m} \tag{4.13}$$

Definition of variables:

$\pi_{h,t,i}$	- The marginal value of the water
$\varphi_{t,m}$	- The coefficient for the mth linear segment of the future benefit function
$\delta_{t+1,n}$	- The constant term for the mth linear segment of the future benefit function
p_i	- The probability of inflow scenario i
i	- The number of inflow scenarios. i = 1…I
m	- The number of states. m = 1…M
n	- The number of linear segments. n = 1…N

It is known from linear programming that there is a set of simplex multipliers associated to the constraints of a problem at the optimal solution. These multipliers represent the value of one additional unit on the constraint right-hand side.

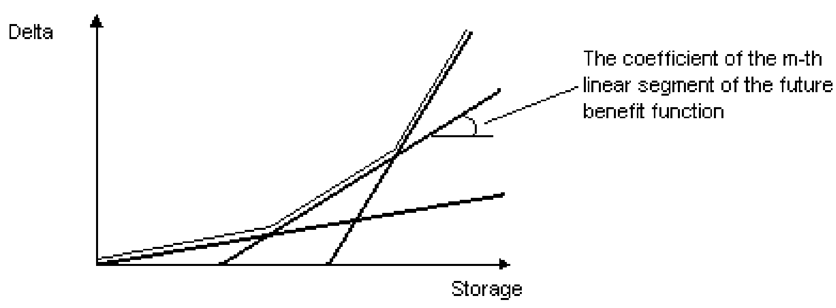

Figure 4-4 Piecewise linear future benefit function

It is shown in figure 4-4 that the benefits of the resources are highest when the storage level is high. At a high storage level the value of the water is low, which can be compared to low cost of the water.

The number of linear segments is equal to the number of initial storage values M. The current benefits and some ending value for storage are known at the last stage.

First the computations are done in the final stage T. Starting in one of the states, $f_t(S_t)$ is calculated for all the inflow scenarios and the maximum benefit of those scenarios are found. Then, the coefficient and the constant term of the mth linear segment of the future benefit function are calculated. When all the storage levels are calculated for the last stage, the future benefits function for stage T-1 is found. This function will be used in the calculations of the previous stages. This algorithm is followed until $f_1(S_1)$ is calculated for all the inflow scenarios at all the states.

By using the stochastic dual dynamic programming method, the output reservoir level can be regarded as a continuous variable at each stage. It is not necessary to calculate as many states for each stage as in common SDP, because the output of the method does not need to be one of the calculated states, but a point on the piecewise linear future benefit function.

4.2 Comparing the different algorithms

In the following chapter the different algorithms described above, will be compared, by looking at advantages and disadvantages of the different methods.

4.2.1 Advantages and Disadvantages

The use of regular dynamic programming is limited to control problems with a small number of state variables, because of the great amount of calculations. Thus, new dynamic programming methods have been developed to solve problems with greater complexity.

In regular DP there is no uncertainty attached to the different states. In real life, the forecast of the future are often uncertain. To solve control problems where the future is uncertain, the state variables have to be stochastic variables. Hence, SDP is a more realistic method for solving real life problems. In common SDP, there are probabilities attaches to different expected future inflows. Thus, in this approach the expected future inflow are used as input variable in every state. The problem with this approach is that there is not modelled any serial correlation between the stream flow in consecutive periods of time, which is often the case in real life.

The SSDP algorithm uses samples of inflow as input variable in every state. Thus, this method takes full advantage of the description of stream flow variability, and temporal and spatial correlations captured within the traces. One disadvantage of the SSDP approach is that the optimal solution is based on one inflow scenario, and it is not possible to move between different scenarios. The SSDP algorithm compared to SDP algorithm will anyway contribute only with marginal improvements.

All the dynamic programming methods described so far is limited to a small number of state variables to keep the number of calculations within a reasonable limit. By using the SDDP method, there is no need to calculate a great number of states to get a detailed result. In this approach the output reservoir storage level can be treated as a continuous variable.

5 A Real Options Evaluation Approach

In this chapter a real option framework will be described. This framework will be used in chapter 6 where a capacity expansion of Svartisen is evaluated. Initially the real option approach will be compared to the traditional approach. Then the contingent claims analyses will be presented. This method is used in finding the value of the option to invest in extra capacity.

5.1 The Traditional Approach vs. the Real Option Approach

The traditional way of determining the investment decision is the net present value approach. This common rule says that an investment should be undertaken when the present value of the projects expected cash flows is at least as large as its cost. This approach ignores managerial flexibility like the option to postpone an investment. (Dixit and Pindyck, 1994, Hull, 2002)

The application of option concepts to value real assets has become an important area in the theory and practice of finance. (Schwartz, 1998) Real options are an important tool in the financial valuation of generation, especially hydroelectric generation. (Frayer and Uludere, 2001) The real option approach has major advantages compared to the traditional NPV approach. The real option approach avoids the need to make assumptions about trajectory of spot prices in the future since this method uses the information contained in futures prices. Another advantage with the real option approach is that it does not require the estimation of risk-adjusted discount rate, since it uses the risk free rate of interest. (Schwartz, 1998)

5.2 Contingent Claims Analysis

Contingent claims analysis is derived from financial economics. An investment project can be defined by a stream of costs and benefits that vary through time and depend on the unfolding of uncertainty of the future. Owning an investment opportunity can be compared to owning an asset that has a value. In the modern economy, there are markets for all kinds of assets. To find the market price of an investment project is thus easy if the asset is traded in the market. If it is not directly traded in the market, it is possible to compute an implicit value for it by relating it to other assets that are traded. (Dixit and Pindyck, 1994)

To find the value of an investment project, it is necessary to look at some combination or portfolio of traded assets that replicate the patterns of returns from the investment project, at every future date and in every future uncertain eventuality. Once the value of the investment is known, it is possible to find the best form, size, and timing of investment that achieves this value, and thus an optimal investment policy can be determined. (Dixit and Pindyck, 1994)

5.3 Option to Invest Model

McDonald and Siegel developed a model to decide when to invest in a single project. In this model, the cost of the investment, I, is known and fixed, while the value of the project, V, follows a geometric Brownian motion:

$$dV = \alpha V dt + \sigma V dz \qquad (5.1)$$

Definition of variables:
V - The value of the project
α - The expected percentage rate of change of V
σ - The proportional variance parameter
dz - The increment of a Wiener process

The future value of the project is always uncertain, which means that there is an opportunity cost of investing today. Hence the optimal investment rule in this model is to invest when V is at least as large as a critical value V* that exceeds I. (Dixit and Pindyck, 1994)

The firm's investment opportunity can be looked upon as a call option. The decision to invest is therefore equivalent to deciding when to exercise such an option. The investment decision problem can now be viewed as a problem of option valuation. (Dixit and Pindyck, 1994)

It is desired to maximize the value of the investment opportunity, F(V).

$$F(V) = \max E\left[(V_T - I)e^{-\rho T}\right] \qquad (5.2)$$

Definition of variables:
F(V) - The value of the investment opportunity
E[*] - The expected value
T - The unknown future time that the investment is made
V_T - The value of the project at the time the investment is made
I - The investment cost
ρ - The discount rate

It is assumed that $\alpha<\rho$, because otherwise waiting would always be the best alternative, and no optimum would exist. From now on, δ, will denote the difference α-ρ.

To solve the investment problem, is equal to determine the point when it is optimal to invest I in return for an asset worth V. V evolves stochastically when $\sigma > 0$. Then it will not be able to determine the time T for the investment. Instead a critical value V* will be found such that it is optimal to invest once V>=V*.

Dixit and Pindyck derive the optimal investment rule by using two different methods; dynamic programming and contingent claims method. In this paper the contingent claims analyses will be used.

5.3.1 The Contingent Claims Method

When using the contingent claims method, it has to assumed that existing assets in the economy span the stochastic changes in V. This means in principle that existing assets can replicate the uncertainty over future values of V. With this assumption, it is possible to solve the investment problem without making any assumptions about risk preferences or discount rate.

Let x be the price of an asset or a dynamic portfolio of assets that are perfectly correlated with V. ρ_{xm} denotes the correlation of x with the market portfolio. Because x and V are perfectly correlated, $\rho_{xm} = \rho_{Vm}$. The assumption is made that x pays no dividends, so it's entire return is from capital gains. Then x evolves according to

$$dx = \mu \cdot x \cdot dt + \sigma \cdot x \cdot dz \tag{5.3}$$

Definition of variables:
μ - The drift rate or the expected rate of return
σ - The volatility of the asset

According to the Capital Asset Pricing Model (CAPM), μ should reflect the asset's systematic risk.

$$\mu = r + \Phi \cdot \rho_{xm} \cdot \sigma \tag{5.4}$$

Definition of variables:
r - The risk-free interest rate
Φ - The market price of risk

μ is according to this, the risk-adjusted expected rate of return that investors would require if they are to own the project. The firm would never invest if α, the expected percentage rate of change of V, was less than μ. Thus, δ denotes the difference between μ and α, and it is assumed that δ > 0. Then the expected rate of capital gain on the project is less than μ. *Hence δ is an opportunity cost of delaying construction of the project, and instead keeping the option to invest alive.* (Dixit and Pindyck, 1994) It is assumed that δ is constant, this means that future cash flows will be a constant proportion of the project's market value.

The market price of risk measures the tradeoffs between risk and return that are made for securities dependent on a specific variable. At any given time, the market price of risk must be the same for all derivatives that are dependent only on this variable and time t. The expression for the market price of risk is described in Hull (2002) chapter 21.

5.3.2 Obtaining a Solution

To find a solution a portfolio consisting of holding an option to invest, which is worth F(V), and going short an amount n= F´(V) units of the project is considered. This gives the following value of the portfolio:

$$\Phi = F(V) - F'(V) \cdot V \tag{5.5}$$

A short position in such a portfolio will require a payment of δV F´(V) dollars per period. This is the amount a rational investor that has a long position on F`(V) units of the project would require to take that position. δ is as previously mentioned the difference between the growth rate of the project, α, and the risk-adjusted rate μ. Thus δ can be compared with the dividend rate. An investor holding a long position in the investment project would require a payment equal the risk-adjusted return.

The return from holding the portfolio over a short time interval, dt, can then be written:

$$dF(V) - F'(V)dV - \delta V \cdot F'(V)dt \tag{5.6}$$

Ito's lemma is used to find an expression for dF. This is explained in Appendix A-2

$$dF(V) = F'(V)dV + \frac{1}{2}F''(V)(dV)^2 \tag{5.7}$$

By inserting equation (5.7) into equation (5.6), the following expression for the total return on the portfolio occurs:

$$\frac{1}{2}F''(V)(dV)^2 - \delta V \cdot F'(V)dt \tag{5.8}$$

From equation (5.1) it is known that $(dV)^2$ can be written as $\sigma^2 V^2 dt$. This is because the standard Wiener process $dz = \varepsilon\sqrt{dt}$, where ε is a random drawing from a standardized normal distribution, N(0,1). By replacing $(dV)^2$ in equation (5.8) the return on the portfolio becomes:

$$\frac{1}{2}\sigma^2 V^2 F''(V)dt - \delta V \cdot F'(V)dt \tag{5.9}$$

This evaluation approach is an extension of the risk-neutral valuation framework. (Hull,2002, chapter 28 and 21) This return is therefore risk-free. Thus to avoid arbitrage possibilities, equation (5.9) must be equal $r \cdot \Phi \cdot dt = r[F - F'(V)V]dt$. Then this expression is derived:

$$\frac{1}{2}\sigma^2 V^2 F''(V)dt - \delta VF'(V)dt = r[F - F'(V)V]dt \tag{5.10}$$

Rearranging equation (5.10) gives the following differential equation that F(V) must satisfy:

$$\frac{1}{2}\sigma^2 V^2 F''(V) + (r-\delta)VF'(V) - rF(V) = 0 \tag{5.11}$$

In addition, F(V) must satisfy the following boundary conditions:

$$F(0) = 0 \tag{5.12}$$

$$F(V^*) = V^* - I \tag{5.13}$$

$$F'(V^*) = 1 \tag{5.14}$$

The first condition (5.12) arises from the observation that if V goes to zero, it will stay at zero. Then the option to invest will have no value. V* is the critical value of the project at which it is optimal to invest. The second condition (5.13) is a value-matching condition. It just says that the value of the option to invest when the value of the project is equal to the critical value, is equal to the payoff the firm will receive upon investing. The final condition (5.14) is a "Smooth-pasting" condition. F(V) has to be continuous and smooth at the critical exercise point V*, or else it could have been done better by exercising at a different point.

The solution for F(V) has the form:

$$F(V) = AV^{\beta_1} \tag{5.15}$$

$$\beta_1 = \frac{1}{2} - (r-\delta)/\sigma^2 + \sqrt{\left[(r-\delta)/\sigma^2 - \frac{1}{2}\right]^2 + 2r/\sigma^2} \tag{5.16}$$

$$A = (V^* - I)/(V^*)^{\beta_1} = (\beta_1 - 1)^{\beta_1 - 1}/\left[(\beta_1)^{\beta_1} I^{\beta_1 - 1}\right] \tag{5.17}$$

$$V^* = \frac{\beta_1}{\beta_1 - 1} I \tag{5.18}$$

Since β_1>1, β_1/(β_1-1)>1 which makes V*>I. *Thus the simple NPV rule is incorrect; uncertainty and irreversibility drive a wedge between the critical value V* and I.* (Dixit and Pindyck, 1994)

6 Capacity Expansion in Svartisen

In this chapter, the theoretical framework presented earlier in this paper will be used in a test case. The EOPS model ("EFI's One-area Power-market Simulator") from Sintef Energy is used to calculate the NPV of an expansion project at the Svartisen plant. As input to the EOPS model a modified version of the Lucia and Schwartz (2002) model, which was presented in chapter 3, is used. Further his chapter describes the assumptions and modifications that have been made in order to use the model in a practical case. The outputs from the EOPS model were used to value the option to expand using the real option framework presented in chapter 5.

The test case is an illustration of how an analysis can be done, and the inputs are not realistic enough for a decision to be taken.

6.1 Methodology

The methodology is as follows:

1. The EOPS model from Sintef Energy was chosen as solution method.
2. The Lucia and Schwartz (2002) model with log prices was chosen. With this price model an analysis of changes in seasonal and weekly fluctuations and mean reverting level was done. These analyses were done with the EOPS model and with analyses of data from Nordpool using Excel.
3. A modified version of the Lucia and Schwartz model was constructed by using the results from 1. This model was fitted to use as input to the EOPS model, requiring 70 price scenarios.
4. NPV were calculated for three different capacity expansion projects using serial simulations in the EOPS model.
5. The value of the option to invest was calculated for the different expansion scenarios using the framework presented in chapter 5.

6.2 The Solution Method

This chapter describes the algorithms used in the EOPS model and the simplifications that were used in the capacity expansion of the Svartisen case.

6.2.1 Algorithms and Methods used in the EOPS Model

In the EOPS model, the inflow is a stochastic variable. There are probabilities associated to different options of inflows for the next period, and the expected value of these inflows is used to calculate the value of the water. Thus, there is not modeled any serial correlation between the inflow in consecutive periods. This has been tried in a previous version of the EOPS model, but it did not make any trivial influence on the calculations. Accumulated probabilities have been used in given time periods. Then the probabilities become accumulated in consecutive periods. However the calculations has not been significantly changed by this method. (Mo, 2002).

The price is modeled as a Markov chain. Hence, the prices in one week influence the prices in the following week. Theoretically, it is possible to have a high price and a high inflow in one period of the analysis, but in practice this is not a problem.

The EOPS model models the hydro system as an equivalent one-reservoir model. The value of the water is calculated with expected inflow and price forecasts as input. Then the value of the water is used in the simulations where relevant constraints are taken into the model. In the simulations, historical inflows are used as input.

Start and stop costs are not modeled in the EOPS model. At a hydropower plant, the cost associated with starts and stops are not considerable high. Thus, these costs do not make a huge influence on the production planning.

6.2.2 Simplifications used in the EOPS Model

In the EOPS model, it is possible to model weekly fluctuations in the prices. Then the highest prices are placed after each other and not in a sequential order. It is not possible to model weekly fluctuations in sequential order. It is neither possible to model daily changes. This can be modelled in the ORP method (Optimal regional planning), another simulation tool made by Sintef Energy.

In addition the ORP method can model inflow at hourly resolution. Our pricing model will not support this function because the prices will not correspond to the inflow-hour. This is of great importance in river power plants with low regulation level, but will be of minor importance in the Svartisen case because the system is regulated well.(Mo,2002)

6.2.3 Settings in the Simulations of the Test Case

An expansion of the capacity can be regarded as an investment. In investment analysis, the inflows are simulated in series instead of in parallel. In a serial simulation, all the historical inflows are places after each other, the inflow for one year starts at the point where the previous year stops. In a parallel simulation, the different inflow scenarios start on the same storage level. Hence, the simulations were run with serial inflow.

In the EOPS model, there is an option to choose calibration or not in the simulations. In the Svartisen case the simulations were run without calibration. If the description of the hydro system does not differ considerable from the equivalent one-reservoir model and the producer is risk neutral, calibration has negligible influence on the calculations. If the hydropower system cannot be described realistic by an equivalent one-reservoir model, or the producer is risk averse, the model can be adjusted with calibrations that take this into account. Calibration can also be used to connect the power production to a local market. Hence, the production is adjusted to serve this market.

6.3 Description of the Power Plant, Svartisen

Some facts about the Svartisen hydropower plant:

- Built in mountain
- 1 generator, 350 MW (Norway's biggest)
- The mean annual production: 2,17 TWh

- The capacity of the reservoir: 3,5 billion m^3
- Tunnels: ca. 100 km

(http://www.statkraft.no/wbch3.exe?p=1890)

The plant is modified to have a linear power curve from origin to the point where the power output is 350 MW and the stream flow is 69,81m3/s.

Other simplifying assumptions made:

- All energy were sold on the spot market
- No limitations on the grid capacity
- No fixed contracts
- Price taking assumptions
- No constraints on water courses and reservoir level

6.4 The Choice of Price Model

The Lucia and Schwartz (2002) one factor model is chosen as basic price model. This model is initially adjusted to describe weekly fluctuations and further modified in order to explain what happens to prices when the amplitude of the seasonal behavior changes. This is of great importance since the expected spot prices from Statkraft were modified to the forward price model. Limitations of the Lucia and Schwartz (2002) log price model and the modified Lucia and Schwartz (2002) log price model are discussed.

6.4.1 The Chosen Price Model

Due to the complexity of two factor-price models, a one-factor model was applied. The Lucia and Schwartz (2002) one-factor model was chosen because it is possible to implement a seasonal behavior in that model. Further the log price model was chosen because that model always has positive prices. Because of the importance of changes in seasonal behavior of the power prices it was necessary to model the amplitude of the deterministic part as a Brownian motion. To model the weekly price changes a technique similar to what SEFAS applies in their EMPS ("EFI's Multi-area Power-market Simulator") and EOPS models was used. The 168 hourly prices in a week were divided in four different categories, each with 42 hourly prices. The lowest prices were in the first category and the highest prices were placed in category 4.By this representation it was possible to construct a price model with the weekly and seasonal changes in prices, which are of utmost importance in planning of power expansion.

The Lucia and Schwartz (2002) model with 4 price categories:

$$F_0(P_0,T_0) = C(T)\exp\left[f(T) + \ln(P_0 - f(0))e^{-\kappa T} + \frac{\lambda\sigma}{\kappa}\left(1 - e^{-\kappa T}\right) + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa T}\right)\right] \qquad (6.1)$$

Definition of the variables:

C(T) - price category factor : c_1 for T=1:42 +n*168,
c_2 for T=43:84 +n*168,
c_3 for T=85:126 +n*168,
c_4 for T=127:168 +n*168

Because $f(T)$ is a deterministic part, it is referred to as the mean reverting level and is given by:

$$\alpha + \gamma \cos\left((T + \tau) \frac{2\pi}{8760} \right)$$

- α - The mean reverting level
- β - A constant
- γ - The level of the amplitude
- τ - The time delay in hours from January 1. to the date of the highest prices
- λ - The market price of risk
- σ - The volatility of the Spot price
- κ - The mean reverting factor
- P_0 - The spot price at t=0

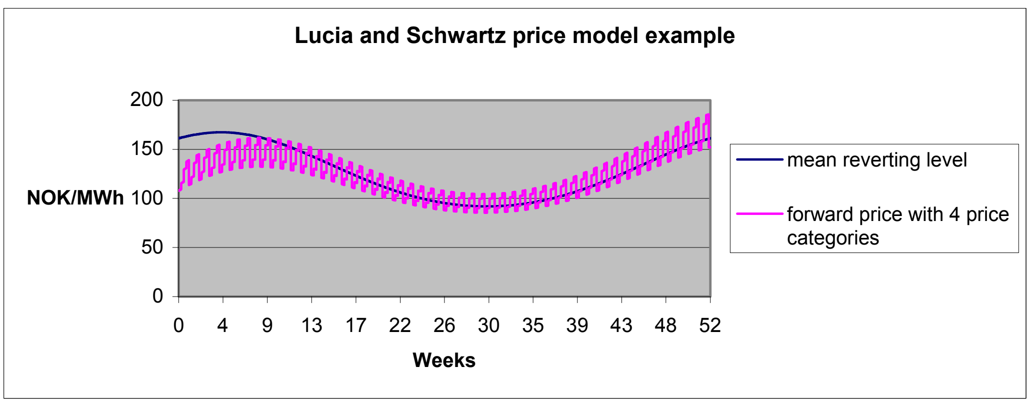

Figure 6-1 The Lucia and Schwartz (2002) model with 4 price categories. P_0 is lower than the long time equilibrium. After a while the forward price approaches long time equilibrium above the mean reverting level

6.4.2 Limitations of the Price Model

The model cannot be used to make production plans in power plants where the aggregates have large start up costs. It is because all the highest prices during a week will be placed after each other in time. With the chosen model the aggregate are just started one time during the week if it is desired to produce at the 42 hours with highest prices. In the real world the aggregates would be turned on several times inflicting start up costs. To solve this “start up cost- problem” a Mixed Integer Programming tool is needed. However there are no long time scheduling programs available today with Mixed Integer Programming algorithms for modeling start up costs. Such algorithms are available in short time scheduling programs such as SHOP from SEFAS, but the calculations will be to heavy for long or medium time scheduling. In other words our pricing model should be appropriate for most of the available long- and medium time- scheduling programs.

Another weakness in the modified price model is that an arithmetic mean price is calculated in each price category and further used as price for the whole bulk. In the real world greater volumes of higher prices and lower volumes of lower prices would have been sold. Due to this a lower economic results will be received by using the chosen price model than by using a price model with real hour values. To avoid this the numbers of categories can be increased.

6.4.3 The Influence of the Mean Reverting Level

An interesting question is what happens if the mean reverting level changes. An increase in the mean reverting level will make a great difference on the total profit, and will have a smaller influence on the expansion decision. In the Lucia and Schwartz (2002) model the mean reverting level is constant and the amplitude, γ, is given by a Brownian motion independent of the mean reverting level. Since the price model has an exponential expression the difference between summer and winter prices will be larger with larger values of γ. In figure 6-2 there are two different forward curves with different mean reverting levels, but with equal γ=0,3.

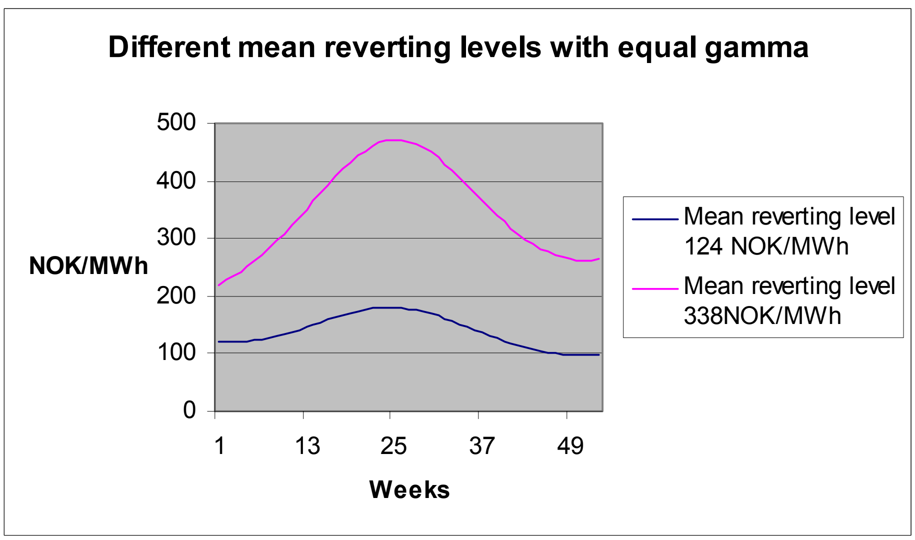

Figure 6-2 Different mean reverting levels with equal γ

It was tested how different levels in the mean reverting level would affect a decision to expand the Svartisen plant using the Lucia and Schwartz (2002) pricing model with 4 price categories. For weekly fluctuations the simple formula below was used:

Price category 1= base price *0,90,
Price category 2 =base price *0,95,
Price category 3 =base price *1,05
Price category 4 =base price *1,10 (6.2)

These last numbers are reasonable assumptions and were not tested empirically.

Mean reverting level (NOK/MWh)	104	114	124	134	144	154	164
NPV of 35MW expansion (mill NOK)	1,94	1,86	1,95	2,19	2,41	2,57	2,82

Table 6-1 The mean reverting level's influence on the NPV

From table 6-1 it can be seen that the mean reverting level will have influence on the expansion decision.

Another aspect is that when the prices are higher the economic losses due to overflow will increase. This will of course favor expansion to minimize the overflow. This phenomenon will probably not influence the decisions made and will not be discussed further.

6.4.4 Modifications of the Price Model

This chapter discusses how the value of the amplitude, γ, a given year will have an influence on the weekly price fluctuations. The Lucia and Schwartz (2002) model is modified, to make a better fit of necessary needs, to a model with precise weekly fluctuations.

An important question for us is if the amplitude, γ, changes with the level of the mean reverting level, and if the weekly price changes as a function of γ. To test this, spot prices from Nordpool in the period 1993 to 2001 were used. As a rough estimator of γ the third lowest and the third highest spot price that year were used in the following formula:

$$\gamma = \frac{(high - low)}{2} \tag{6.3}$$

Figure 6-3 shows that the estimate is not far from the empirical values.

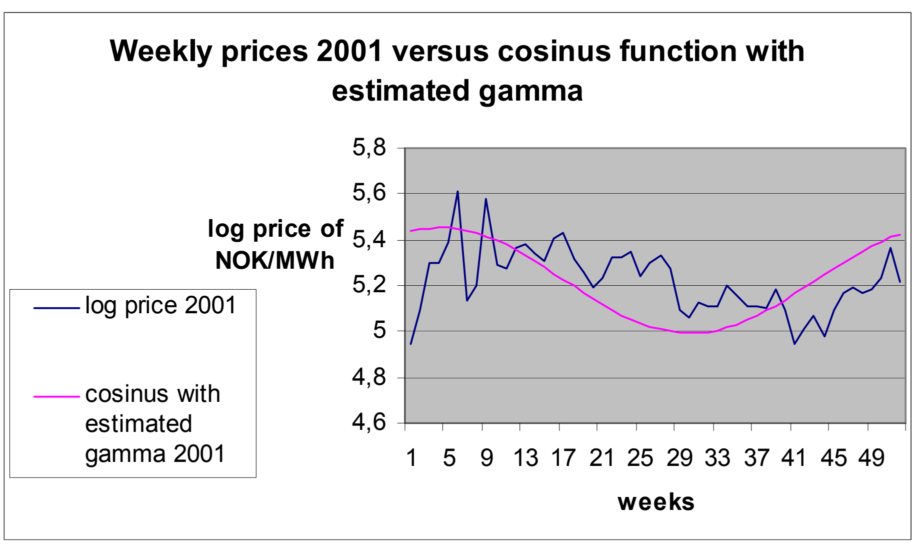

Figure 6-3 The value of γ for 2001 was estimated to 0,23 using equation (6.3). This figure shows that the estimate is acceptable.

Further a regression of γ versus ln(spot price) was run, and this resulted in the following formula:

$$\gamma = 2{,}85 - 0{,}480 \cdot \ln(spot) \qquad (6.4)$$

To test the hypothesis γ = 0 versus γ ≠ 0 we used a statistical analyses used on normal distributed variables with unknown variance; the T-test. A T-test versus a hypothesis of γ = 0 of gives the P-values, of 0 and 1.3 for the parameters in equation (6.4) (Appendix A-3). This implies that the hypothesis that the value of γ is independent of the spot price can be rejected. Appendix A-3 also shows a residual plot for the regression. There are no systematic patterns in the plot. However, this regression line cannot be adequate for all values of the spot. Equation (6.4) implies that for spot prices over 379NOK/MWh, the value of γ will be below zero and prices will be highest in the summer.

Empirical it can be seen that in periods of high spot prices the difference in NOK between summer and winter are highest, but the ratio between winter prices and summer prices are highest in low price years. This can also be deduced from what is known about the prices. The price elasticity is low, and this will result in extreme low prices in wet summers, which give us great values of γ. From equation (6,3) it can be seen that γ will go to 2,85 as prices approaches zero. The log price model prevents the price from crossing zero, but the high γ value will cause great fluctuations using this model.

Further the weekly fluctuations were regressed versus the gamma values and the following results were obtained:

$$\begin{array}{ll} 0{,}982-0{,}034\,\gamma & \text{for } T=1:42 + n*168, \\ 0{,}9973-0{,}016\,\gamma & \text{for } T=43:84 + n*168, \\ 1{,}0021+0{,}0106\,\gamma & \text{for } T=85:126 + n*168, \\ 1{,}0175+0{,}0235\,\gamma & \text{for } T=127:168 + n*168 \end{array} \qquad (6.5)$$

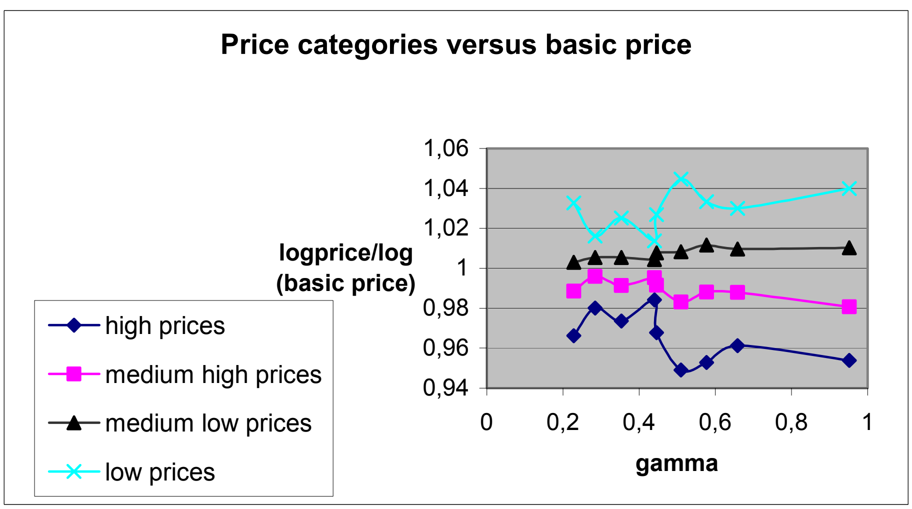

Figure 6-5 The price categories versus basic price

The P-values are between 3 and 15 percent, and a residual plot (Appendix A-3) show that these numbers cannot be looked upon as reliable. High P-values makes it impossible to say with certainty that prices fluctuate during the week as a function of γ. However the values are consistent with each other and are therefore used anyway. If the numbers in figure 6-5 are correct, then lower prices (with their high γ values) should have the greatest fluctuations compared to the log price. Figure 6-6 confirms that this is the truth. However figure 6-6 also shows that there are a lot of uncertainty and that the linear estimate in equations (6.5) probably not is good enough.

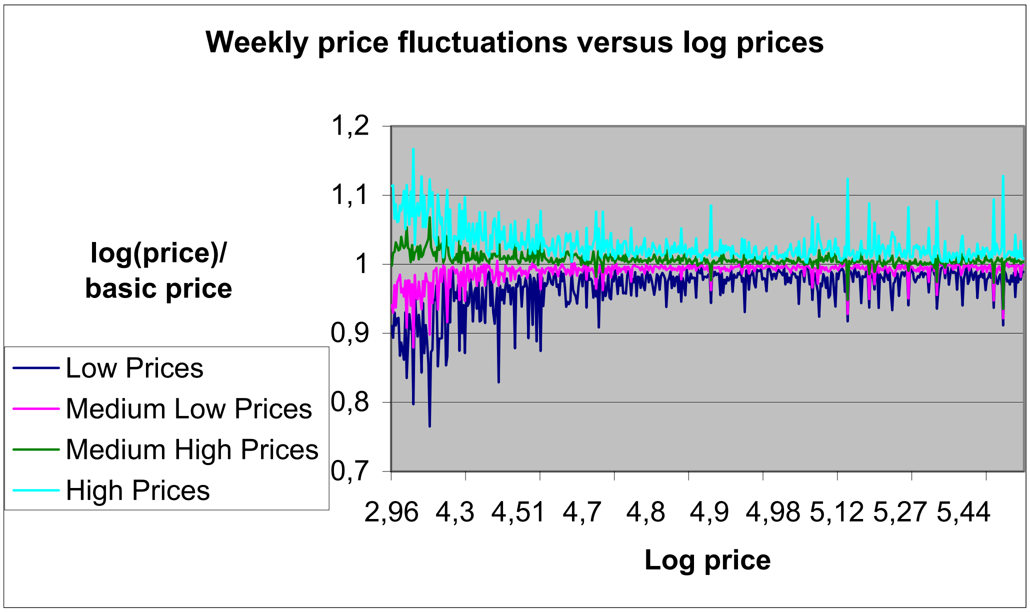

Figure 6-6 Weekly price fluctuations versus log prices

By using a revised model with variable γ it was possible to describe more realistic price fluctuations through a week. When the model was available equation (6.4) were also used to describe the mean reverting level. In this way, it was managed to get a function with lower valleys and lower peaks than an ordinary sinusoidal function. In addition a more realistic weekly fluctuation is achieved.

The revised Lucia and Schwartz (2002) model with 4 price categories

$$F_0(P_0,T_0)=\exp\left[C(T,\gamma)f(T)+\ln(P_0-f(0))e^{-\kappa T}+\frac{\lambda\sigma}{\kappa}\left(1-e^{-\kappa T}\right)+\frac{\sigma^2}{4\kappa}\left(1-e^{-2\kappa T}\right)\right] \tag{6.6}$$

Definition of the variables:

C(T) = price category factor : 0,982-0,034 γ for T=1:42 +n*168,
0,9973-0,016 γ for T=43:84 +n*168,
1,0021+0,0106 γ for T=85:126 +n*168,
1,0175+0,0235 γ for T=127:168 +n*168

Because $f(T)$ is a deterministic part, it is referred to as the mean reverting level and is given by:

$$\alpha + \gamma \cos\left((T + \tau) \frac{2\pi}{8760} \right)$$

α	- The mean reverting level
β	- A constant
γ	- 2,48-0,48 $f(t-1)$
τ	- The time delay in hours from January 1. to the date of the highest prices

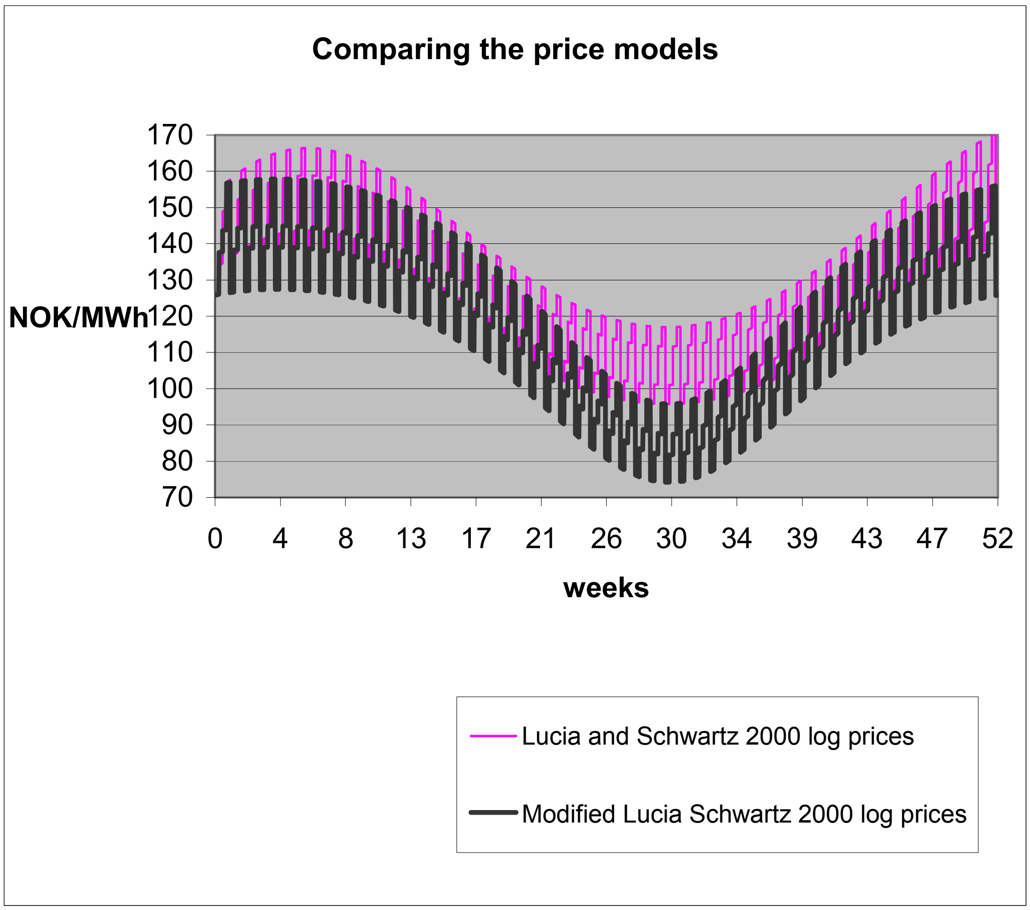

Figure 6-7 Comparison between different versions of Lucia and Schwartz (2002) log price models. The modified version has deeper valleys and rounder peaks. The weekly fluctuations are fitted to empirical data in the modified model.

6.4.5 Limitations of the Modified Price Model

An interesting question is if the modified Lucia and Schwartz (2002) model is better in describing changes in the mean reverting level. This was tested on the Svartisen case with γ described by equation (6.6)

Mean reverting level (NOK/MWh)	94	104	124	134	144	154
NPV of 35MW expansion (mill NOK)	1,92	1,65	0,85	0,84	0,71	0,98

Table 6-2 The mean reverting level's influence on the NPV

From table 6-2 it can be seen that the revised model gives an opposite picture of the influence of a change in the mean reverting level. It is not possible to say how a change in this level will have any influence on the expansion decision. Neither of the models can be used to describe changes in the mean reverting level, but the revised model will better explain the prices given a certain mean reverting level.

6.5 The Simulations

This is a description of how the Lucia and Schwartz (2002) model with 4 price categories was used simulate the NPV of an expansion of 30MW, 150MW and 300MW.

6.5.1 Adjustment to the input price model

A price model with 70 price scenarios was received from Statkraft. It is not known if this model represent an up to date and correct spot forecast, but it was treated as it should have been. To adjust the model to represent forward prices instead of expected spot prices the following algorithm was used

1. Calculate the mean expected spot price for a given week
2. Calculate forward-mean expected spot price for that week
3. Calculate the difference between the numbers in 2 and 1
4. Add the result in 3 to each of the price scenarios for the given week.

In addition 4 different price categories was calculated as explained in the modified Lucia and Schwartz (2002) model. No particular inputs to the forward model were used, instead adjusted the parameters were used. Resonable imputs were used and adjusted so that the forward price was slightly above the mean expected spot price. The price model received from Statkraft had less seasonal fluctuations for the mean expected spot price, than the price series observed at Nord Pool. The value of γ was adjusted with –0,43 compared to equation (6.4) to fit the seasonal behavior of the mean expected spot price.

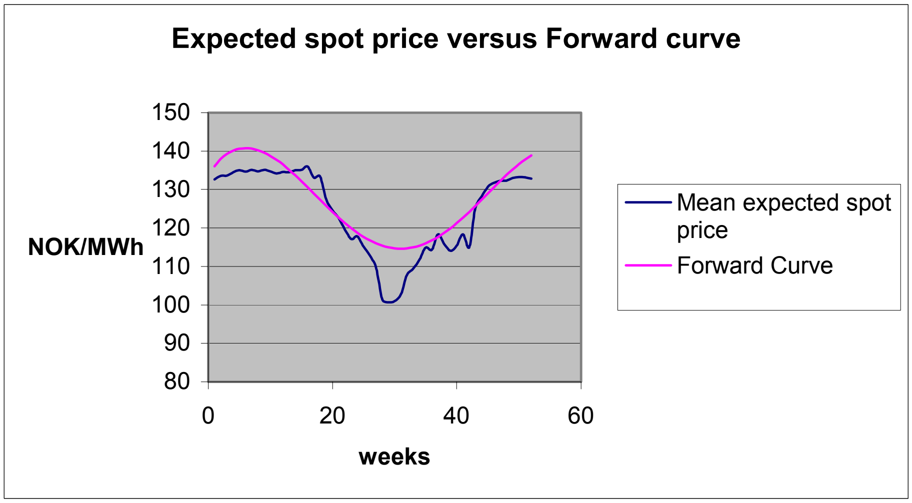

Figure 6-8 Expected spot price versus Forward curve

6.5.2 The simulations

The original plan was to calculate the net present value of building several modules of 30MW with different values of γ. However the EOPS model gave unrealistic values for some input data. Instead a simulation for one module of 30MW was run with different values between 0 and 30 MW and the NPV was calculated as a regression. In addition an expansion of 150MW and 300MW was simulated.

Using 11 simulations for 7 different gamma values made the results that are shown in figure 6-9.

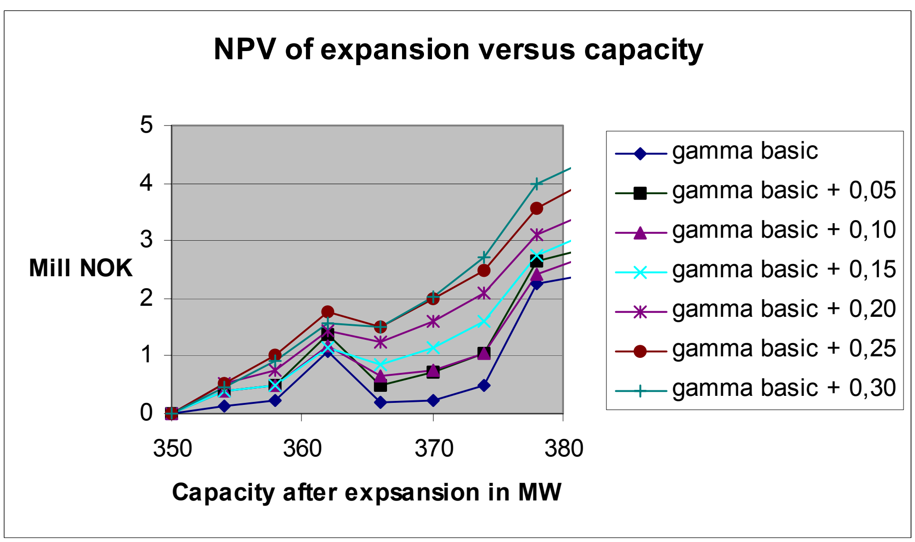

Figure 6-9 The NPV of an expansion versus capacity

As can be seen in figure 6-9 some of the numbers are not optimal solutions. 362MW and 366MW are the most extreme cases. After the regression, the numbers of “gamma basic +0,05” and “gamma basic + 0,10” looks strange, but the other numbers are according to what is expected: Higher γ gives higher NPV.

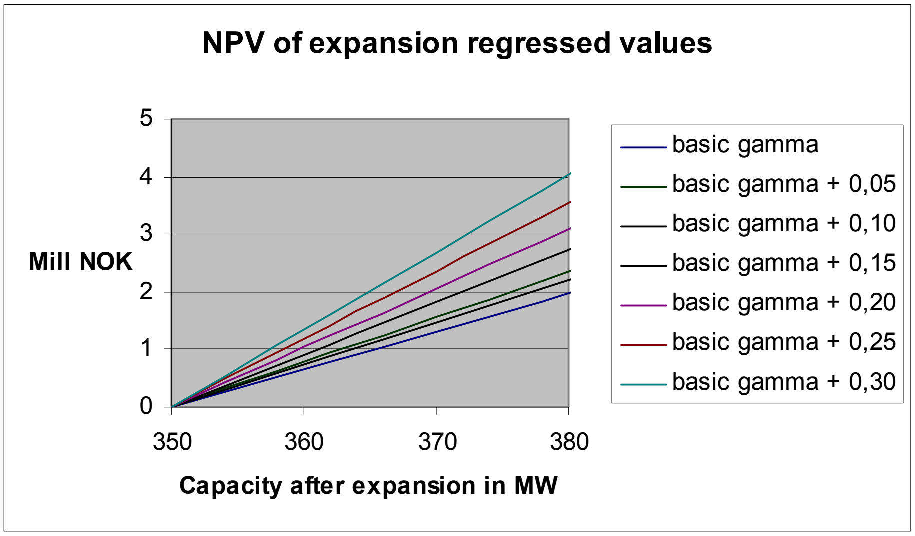

Figure 6-10 The NPV of an expansion with regressed values

gamma	*NPV of 30MW expansion(mill NOK)*	*NPV of 150MW expansion(mill NOK)*	*NPV of 300MW expansion(mill NOK)*
Basic	*1,98*	*5,79*	*11,31*
Basic + 0,05	*2,35*	*7,22*	*13,41*
Basic + 0,10	*2,22*	*8,18*	*14,98*
Basic + 0,15	*2,73*	*9,54*	*17,24*
Basic + 0,20	*3,09*	*11,32*	*19,40*
Basic + 0,25	*3,54*	*12,98*	*21,61*
Basic + 0,30	*4,02*	*14,71*	*23,69*

Table 6-3 The NPV of an expansion of 30MW, 150MW and 300MW after regression

6.6 An Evaluation off the Option to Expand

This chapter describes how to calculate the option to expand given NPVs of an expansion. Further the inputs to the option model and the limitations of the model are described.

6.6.1 The Value of the Option to Expand

To calculate the option to expand the framework of Dixit and Pindyck (1994) described in Chapter 5 was used. The only stochastic factor of importance in the model is σ_γ , which describes the Brownian motion of the amplitude. The spot price itself is also stochastic, but the mean reverting level forces this stochastic process to become unimportant. This is because prices approach long time equilibrium during the first year. After that the forward price will be given from this long time equilibrium. This is also according to the theories published by Schwartz in 1998, as is mentioned in chapter 3. In that article it is claimed that a two factor stochastic forward model can be described almost as a one factor stochastic model in the long run.

The following inputs were used:

R= 7%	Based on the interest rate today. (Forward rates for very long time should be used)
I = 30 mill NOK	Based on the assumption that expansion are possible without further construction in watercourses and penstocks (Faanes, 2002)
V= I/r	Assumption: Constant cash flow
δ=r	Based on the assumption that the value of the generator is the same as the investment costs for all time periods. This means a continuous depreciation at a level of r.
σ_γ =5 %	This is a input to the model, and is not calculated exactly. However by using this number after 10 years a 30MW expansion is trigged in 14.1% of the cases (Appendix A-5). No 150MW or 300MW expansion is trigged in this period even if these projects were looked upon as isolated investment projects.

The results achieved in the tests were regressed linearly to get a smooth line for the net present value of the project. These curves are given in Appendix A-6. By applying equation (5.15-5.18), which is presented in chapter 5, the results given in figure (6-11)-(6-13) were given. In this example 150MW and 300MW will never be built because an option to expand 30MW always is more worth up to delta γ = 0,09 where the option is realized.

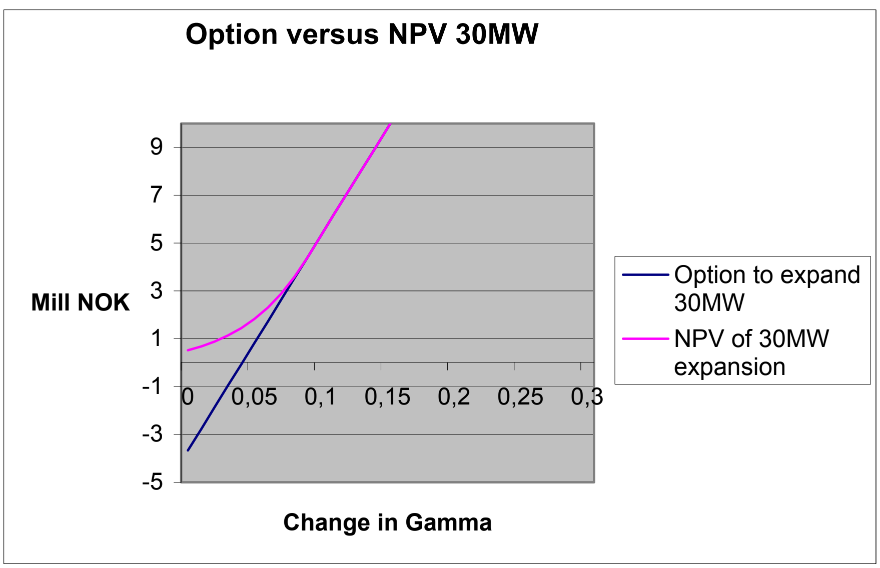

Figure 6-11 The Option to expand 30MW versus NPV

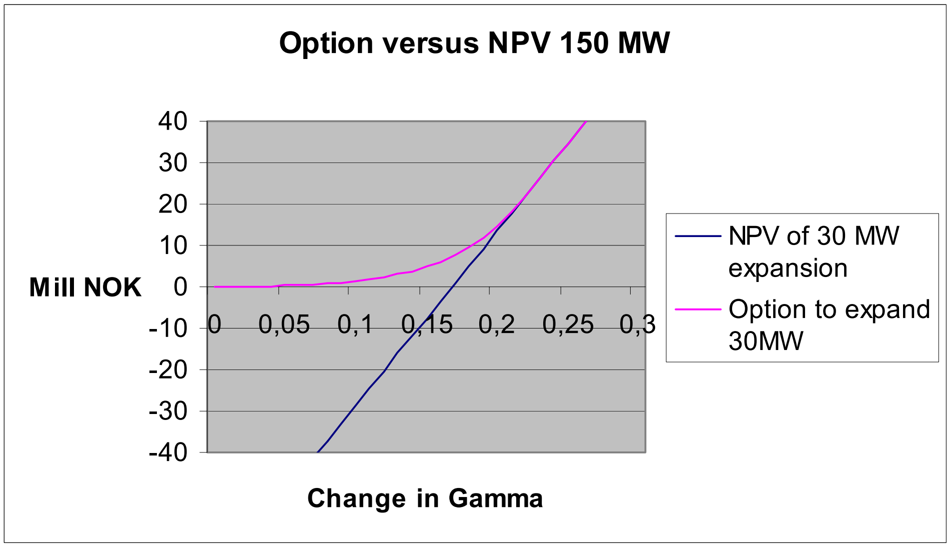

Figure (6-12) The Option to expand 150MW versus NPV

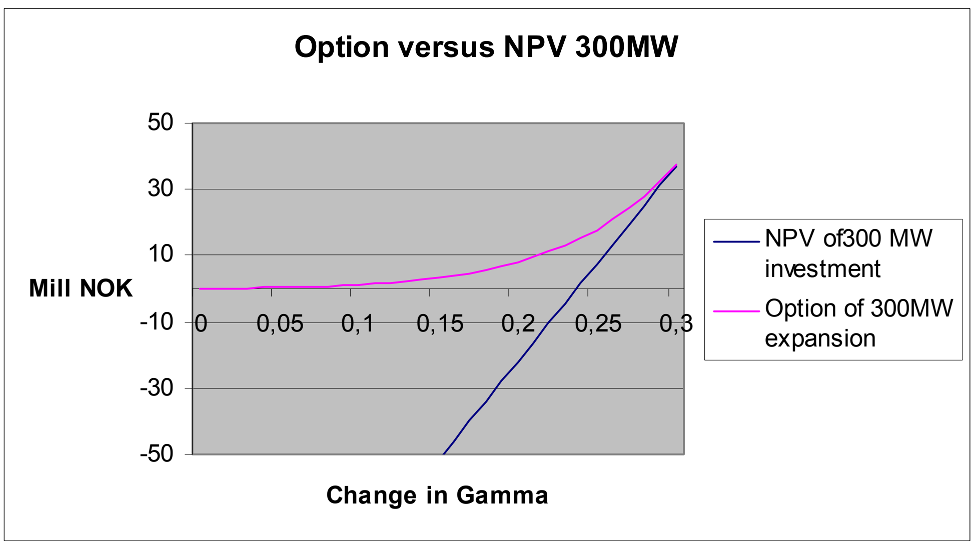

Figure (6-13) The Option to expand 300MW versus NPV

Changes in the value of γ have a great influence on the expansion decision. For a model with a time horizon of 10-20 years it can be dangerous to base the decision on the level of a single factor. A change in the volatility of σ_γ will completely change the decision. There is therefore a lot of uncertainty to the model, and it is important for a potential user to constantly supervise the power market. The empirical data behind figure (6-11)-(6-13) are presented in Appendix A7.

7 Summary

This paper consists of two parts:

- A theoretical part, which describes different theories of importance in understanding generation planning and valuing options to invest.
- A practical part that uses the theoretical framework to value an option to expand Svartisen power plant with 30MW, 150MW and 300MW.

The theoretical part describes The Nordic Power Market and the properties of Hydro Power Scheduling. Further the theoretical fundament of solving the generation planning problem is described, and some algorithms for solving this problem based on Dynamic programming. Different forward pricing models with one stochastic factor are discussed. In addition the framework by Dixit and Pindyck (1994) for valuating an investment option with one stochastic factor is presented.

In the practical part the Lucia and Schwartz (2002) forward pricing model with log prices was chosen as pricing model. Further this model was adjusted to describe the weekly fluctuations of power prices. In next step the EOPS model from Sintef Energy was chosen as dynamic programming tool. Simulations with this price model showed that an increase in the mean reverting level of the spot price increases the NPV of an capacity expansion.

A study of spot prices at Nordpool from 1993 to 2001 showed a correlation between weekly and seasonal fluctuation and between seasonal fluctuations and mean price. In order to be able to use the price model as input to the EOPS model the Lucia and Schwartz model were modified to incorporate this connection between weekly and seasonal fluctuations and mean price.

By this modification of the Lucia and Schwartz (2002) model it was possible to construct 70 forward price scenarios from 70 expected spot price scenarios. By simulating with these 70 price scenarios at the Svartisen Power Plant it was possible to obtain the NPV of investments of 30MW, 150MW and 300MW with different as a function of values of the amplitude of the seasonal price fluctuation.

The value of the option to invest 30MW, 150MW and 300MW was found by using the framework of Dixit and Pindyck (1994). As input to this model some reasonable numbers were used and thereby limiting this paper from deducing correct inputs to the option valuation model. With these inputs, an investment of 30MW was initiated within 10 years in 14% of the cases. An increase in gamma of 0,09 initiated the 30MW expansion. The other options will always be eliminated by an investment in the 30MW project

8 Recommended future work

This chapter describes future recommended work that is necessary before the model can be used in real life projects.

The price model for input in the EOPS model needs to be improved. In this model the Lucia and Schwartz (2002) model with log prices is used. Log prices was chosen because that model always has positive prices. After a while it was recognized that, the log price model was not good enough in describing seasonal and weekly variations, so the model had to be improved. It was rather complicated to modify a log price model, so it was not managed to construct a price model with appropriate properties for modelling power prices.

Our recommendations for future development of this model are to study more thoroughly the relations between high and low prices versus high and low seasonal and weekly prices. It is recommended to use a price model instead of a log price model, since the amplitude anyway has to be adjusted.

Next step in developing the model is to use more appropriate inputs in the price model used in the EOPS model. This implies a study on the general behaviour of forward prices compared to expected spot prices. A result of such a study will be more accurate values of input parameters in equation (6.1) such as market price of risk.

Last step in developing this model will be to estimate the correct values of the parameters in equation (5.15)-(5.18) in order to compute the correct option value. In this paper this issue is not discussed. Some of these parameters is connected to a given investment project, such as the Investment cost. The most important parameter for the result is the volatility of the amplitude, σ_{γ}. This variable should be found by studying the properties of the Nordic Power Market. The value of the option is extremely volatile to this parameter. The option value will therefore not be accurate before such a study is done.

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Index of Appendix

Appendix 1 Schwartz (1997) Differential Equation

In this appendix, equation (3.6) will be derived by using Ito's lemma and by using a portfolio approach.

Ito's lemma

An Ito process is a generalized Wiener process where the parameters a and b are functions of the value of the underlying variable x and time t. Equation (A1.1) is an Ito process, where the variable x has a drift rate of a and a variance rate of b^2. Ito's lemma shows that a function G of x and t follows the process in equation (A1.2)

Ito's lemma:

$$dx = a(x,t)dt + b(x,t)dz \tag{A1.1}$$

$$dG = \left(\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 \right) dt + \frac{\partial F}{\partial x} bdz \tag{A1.2}$$

Deriving the differential equation

In chapter 3, the expression for the differential equation of the one-factor model of Schwartz (1997) is presented. S is the spot price and X is a log price as expressed in equation (A1.3).

$$S = e^x \tag{A1.3}$$

In the article of Schwartz (1997) dX is presentet to avolve according to equation (A1.4).

$$dX = \kappa \left(\mu - \frac{\sigma^2}{2\kappa} - \lambda - X \right) dt + \sigma dz \tag{A1.4}$$

To find an expression for dS, equation (A1.5) from Sydsæter et al (2000) is used.

$$de^x = \left(e^x a(x,t) + \frac{1}{2} e^x b(x,t)^2 \right) dt + e^x b(x,t) dz \tag{A1.5}$$

$$dS = \kappa(\mu - \lambda - x)e^x dt + \sigma e^x dz \tag{A1.6}$$

By inserting equation (A1.7) in equation (A1.6), it is shown that S evolves according to equation (A1.8). S is an underlying variable for the forward curve F(S,T). Hence, Ito's lemma can be used to derive an expression of dF, as shown in equation (A1.9).

$$X = \ln S \tag{A1.7}$$

$$dS = \kappa(\mu - \lambda - \ln S)Sdt + \sigma Sdz \tag{A1.8}$$

$$dF = \left(\frac{\partial F}{\partial S}(\kappa(\mu - \lambda - \ln S)S) + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2 \right)dt + \frac{\partial F}{\partial S}\sigma Sdz \tag{A1.9}$$

The discrete version of equations (A1.8) and (A1.9):

$$\Delta S = \kappa(\mu - \lambda - \ln S)S\Delta t + \sigma S\Delta z \tag{A1.10}$$

$$\Delta F = \left(\frac{\partial F}{\partial S}(\kappa(\mu - \lambda - \ln S)S) + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2 \right)\Delta t + \frac{\partial F}{\partial S}\sigma S\Delta z \tag{A1.11}$$

ΔS and ΔF are the changes in F and S in a small interval Δt. The Wiener process underlying F and S are the same. By choosing a portfolio consisting of holding a forward contract and going short an amount $n = \frac{\partial F}{\partial S}$ units of the underlying commodity, the Wiener process can be eliminated. The value of the portfolio is expressed in equation (A1.12).

$$\pi = 0 \cdot F - \frac{\partial F}{\partial S}S \tag{A1.12}$$

Holding a forward contract where the price is equal to the forward price today has no value to the portfolio.

A short position in such a portfolio will require a payment of δS $\frac{\partial F}{\partial S}$ dollars per period. This is the amount a rational investor that has a long position on $\frac{\partial F}{\partial S}$ units of the derivative would require to take that position. δ is the difference between the growth rate, α, and the risk-adjusted rate μ. δ can be compared with the dividend rate.

The return from holding the portfolio over a short time interval, dt, can then be written:

$$d\pi = dF - \frac{\partial F}{\partial S}dS - \delta S\frac{\partial F}{\partial S}dt \tag{A1.13}$$

The discrete version of equation (A1.13):

$$\Delta\pi = \Delta F - \frac{\partial F}{\partial S}\Delta S - \delta S\frac{\partial F}{\partial S}\Delta t \tag{A1.14}$$

By inserting equation (A1.10) and (A1.11) to equation (A1.14) the following expression for the value of the portfolio is derived:

$$\Delta\pi = \left[\frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2\right]\Delta t - \delta S\frac{\partial F}{\partial S}\Delta t \quad \text{(A1.15)}$$

This return is risk-free. Thus to avoid arbitrage possibilities, equation (A1.15) must be equal $r \cdot \pi \cdot \Delta t = r\left[0 \cdot F - \frac{\partial F}{\partial S}S\right]\Delta t$. Then the following expression is valid:

$$\left[\frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2\right]\Delta t - \delta S\frac{\partial F}{\partial S}\Delta t = -r\frac{\partial F}{\partial S}S\Delta t \quad \text{(A1.16)}$$

By rearranging equation (A1.16), equation (A1.17) is obtained:

$$\frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2 + (r-\delta)S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} = 0 \quad \text{(A1.17)}$$

It is known know that the return on this portfolio is equal to the price change and the dividend, δ. The price change is equal to $\kappa(\mu - \ln S)$. In a risk neutral evaluation, the return is equal to the risk free interest rate and the price change is equal to $\kappa(\mu - \lambda - \ln S)$. An expression for (r-δ) can now be obtained.

$$(r-\delta) = \kappa(\mu - \lambda - \ln S) \quad \text{(A1.18)}$$

By inserting equation (A1.18) in equation (A1.17), the same differential equation as in Schwartz (1997) is obtained, except from the sign in front of $\frac{\partial F}{\partial t}$.

According to Schwartz (2002) the negative sign in front of $\frac{\partial F}{\partial t}$ is because the time scale is Time to Maturity, which is the opposite of calendar time. The expression then becomes:

$$\frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2 + \kappa(\mu - \lambda - \ln S)S\frac{\partial F}{\partial S} - \frac{\partial F}{\partial t} = 0 \quad \text{(A1.19)}$$

Hence expression (3.6) is proven.

Appendix 2 Ito's Lemma

In this appendix, Ito's lemma will be used to find an expression for dF in equation (5.7).

Deriving dF

Ito's lemma has previously been described in Appendix 1. Here follows the expression for an Ito process.

Ito's lemma:

$$dx = a(x,t)dt + b(x,t)dz \tag{A2.1}$$

$$dG = \left(\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 \right) dt + \frac{\partial F}{\partial x} bdz \tag{A2.2}$$

In Chapter 5 a solution of the option to invest is obtained by using the contingent claims method. The value of the project, V, evolves according to equation (A2.3). V is known to be an underlying variable of the function of the value of the option to invest F(V). Ito's lemma can therefore be used to derive an expression of dF, as shown in equation 4.

$$dV = \alpha V dt + \sigma V dt \tag{A2.3}$$

$$dF = \left(\frac{\partial F}{\partial V} \alpha V + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial V^2} \sigma^2 V^2 \right) dt + \frac{\partial F}{\partial V} \sigma V dz \tag{A2.4}$$

By rearranging equation (4), the following expression is derived:

$$dF = \frac{\partial F}{\partial V} (\alpha V dt + \sigma V dz) + \frac{1}{2} \frac{\partial^2 F}{\partial V^2} \left(\sigma^2 V^2 dt \right) + \frac{\partial F}{\partial t} dt \tag{A2.5}$$

From equation (A2.3) dV^2 can be written as $\sigma^2 V^2 dt$. This is because the standard Wiener process $dz = \varepsilon \sqrt{dt}$, where ε is a random drawing from a standardized normal distribution, N(0,1).

Since V evolves stochastically, it is not possible to determine a time, T, when it is optimal to invest. The investment rule take the form of a critical value V* such that it is optimal to invest once V>= V*. dF is therefore not dependent of t, thus the last part of equation (A2.5), $\frac{\partial F}{\partial t} dt$, is equal to zero.

Equation (A2.5) can now be rewritten:

$$dF = \frac{\partial F}{\partial V} dV + \frac{1}{2} \frac{\partial^2 F}{\partial V^2} (dV)^2 \qquad \text{(A2.6)}$$

This is the same expression as equation (5.7) in chapter 5, and hereby it is proved how the equation is derived.

Appendix 3 Gamma Regression

This appendix is a printout from Minitab of the gamma-regression described in Chapter 6.4.4

—————— 22.10.2002 15:45:42 ——————————————————

Regression Analysis: low versus gamma

```
The regression equation is
low = 0,982 - 0,0335 gamma

Predictor        Coef     SE Coef          T        P
Constant     0,981979    0,009166     107,13    0,000
gamma        -0,03353     0,01711      -1,96    0,091

S = 0,01059     R-Sq = 35,4%     R-Sq(adj) = 26,2%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1   0,0004306   0,0004306      3,84    0,091
Residual Error     7   0,0007849   0,0001121
Total              8   0,0012155
```

Plot resmlow * predmlow

```
MTB > Plot 'resmhigh'*'predmhigh';
SUBC>   Symbol;
SUBC>   ScFrame;
SUBC>   ScAnnotation.
```

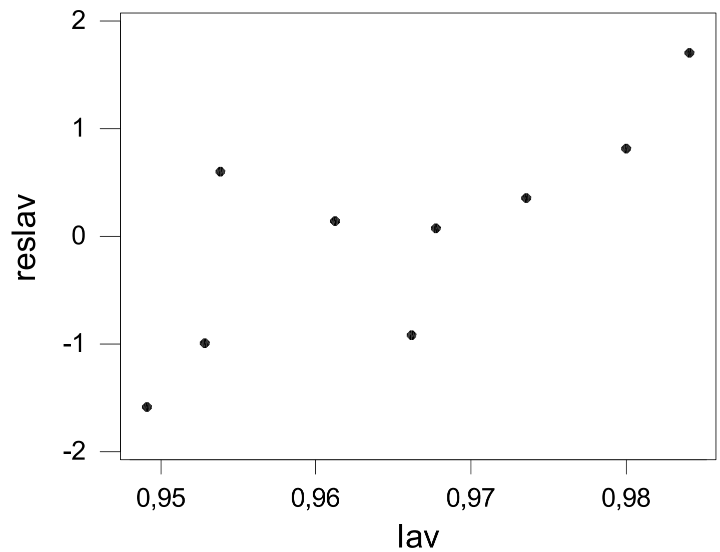


Figure A-3.1Residual plot of low versus gamma

Figure A(3.1) show as residual plot of the regression of the average 42 lowest weekly log prices during a year versus gamma that given year. The plot has a diagonal pattern and shows that the regression is not accurate. Corresponding Residual Plot shows a similar pattern.

```
MTB > regress c2 1 c5 c9 c10
```

Regression Analysis: mlow versus gamma

```
The regression equation is
mlow = 0,997 - 0,0163 gamma

Predictor        Coef     SE Coef          T        P
Constant     0,997279    0,003337     298,89    0,000
gamma       -0,016347    0,006228      -2,62    0,034

S = 0,003854    R-Sq = 49,6%     R-Sq(adj) = 42,4%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1  0,00010234  0,00010234      6,89    0,034
Residual Error     7  0,00010399  0,00001486
Total              8  0,00020633


MTB > regress c3 1 c5 c11 c12
```

Regression Analysis: mhigh versus gamma

```
The regression equation is
mhigh = 1,00 + 0,0106 gamma

Predictor        Coef     SE Coef          T        P
Constant      1,00205     0,00164     612,86    0,000
gamma        0,010588    0,003052       3,47    0,010

S = 0,001889    R-Sq = 63,2%     R-Sq(adj) = 58,0%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1 0,000042935 0,000042935     12,04    0,010
Residual Error     7 0,000024972 0,000003567
Total              8 0,000067907


MTB > regress c4 1 c5 c13 c14
```

Regression Analysis: high versus gamma

```
The regression equation is
high = 1,02 + 0,0235 gamma

Predictor        Coef     SE Coef          T        P
Constant      1,01754     0,00805     126,33    0,000
gamma         0,02353     0,01503       1,56    0,162

S = 0,009304    R-Sq = 25,9%     R-Sq(adj) = 15,3%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1  0,00021197  0,00021197      2,45    0,162
Residual Error     7  0,00060601  0,00008657
Total              8  0,00081797


MTB > regress c6 1 c5 c15 c16
```

Regression Analysis: spotyear versus gamma

```
The regression equation is
spotyear = 231 - 177 gamma

Predictor        Coef     SE Coef          T        P
Constant       230,71       35,16       6,56    0,000
gamma         -177,27       65,63      -2,70    0,031

S = 40,62       R-Sq = 51,0%     R-Sq(adj) = 44,0%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1       12035       12035      7,29    0,031
Residual Error     7       11548        1650
Total              8       23583

Unusual Observations
Obs      gamma     årspot          Fit       SE Fit    Residual    St Resid
  2      0,284      253,6        180,3         19,3        73,3        2,05R

R denotes an observation with a large standardized residual
```

```
MTB > Plot 'resmlav'*'mlav';
SUBC>   Symbol;
SUBC>   ScFrame;
SUBC>   ScAnnotation.
```

Regression Analysis: gamma versus ln(spotyear)

```
The regression equation is
gamma = 2,85 - 0,480 ln(spotyear)

Predictor        Coef     SE Coef          T        P
Constant       2,8500      0,7156       3,98    0,005
lnspotyear     -0,4802      0,1456      -3,30    0,013

S = 0,1463      R-Sq = 60,9%     R-Sq(adj) = 55,3%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1     0,23310     0,23310     10,89    0,013
Residual Error     7     0,14989     0,02141
Total              8     0,38299


MTB > Plot 'resgamma'*'predgamma';
SUBC>   Symbol;
SUBC>   ScFrame;
SUBC>   ScAnnotation.
```

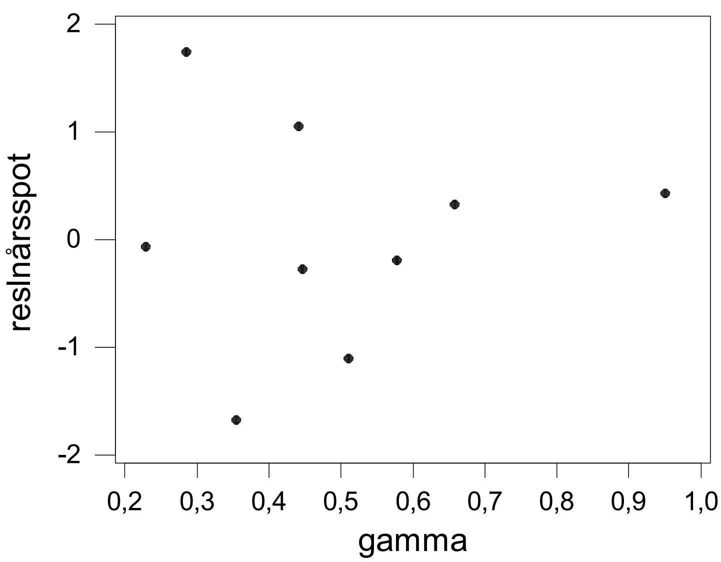


Figure A-3.2 Residual plot of log price versus gamma

In Figure A-3.2 The spot price is ln-transformed and regressed versus gamma. The figure does not have a pattern, but do not have enough to plots to make any conclusion

Appendix 4 Regression of NPV

This appendix is a printout of the NPV regression described in Chapter 6.5.2

06.11.2002 17:28:36

Regression Analysis: NPVgamma0,0 versus Capacity

```
The regression equation is
NPVgamma0,0 = 163 + 0,0659 Capacity

Predictor        Coef     SE Coef          T        P
Constant      163,224       7,590      21,50    0,000
Capacity      0,06588     0,02073       3,18    0,016

S = 0,6423      R-Sq = 59,1%     R-Sq(adj) = 53,2%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1      4,1665      4,1665     10,10    0,016
Residual Error     7      2,8880      0,4126
Total              8      7,0545

MTB > regress c3 1 c1 c11 c12
```

Regression Analysis: NPVgamma0,05 versus Capacity

```
The regression equation is
NPVgamma0,05 = 160 + 0,0782 Capacity

Predictor        Coef     SE Coef          T        P
Constant      160,347       6,824      23,50    0,000
Capacity      0,07817     0,01864       4,19    0,004

S = 0,5775      R-Sq = 71,5%     R-Sq(adj) = 67,5%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1      5,8663      5,8663     17,59    0,004
Residual Error     7      2,3344      0,3335
Total              8      8,2006


MTB > regress c4 1 c1 c13 c14
```

Regression Analysis: NPVgamma0,1 versus Capacity

```
The regression equation is
NPVgamma0,1 = 165 + 0,0740 Capacity

Predictor        Coef     SE Coef          T        P
Constant      164,548       5,657      29,09    0,000
Capacity      0,07403     0,01545       4,79    0,002

S = 0,4787      R-Sq = 76,6%     R-Sq(adj) = 73,3%

Analysis of Variance
```

```
Source           DF         SS         MS        F        P
Regression        1     5,2605     5,2605    22,96    0,002
Residual Error    7     1,6041     0,2292
Total             8     6,8646


MTB > regress c5 1 c1 c15 c16
```

Regression Analysis: NPVgamma0,15 versus Capacity

```
The regression equation is
0,15 = 161 + 0,0910 Capacity

Predictor        Coef     SE Coef          T        P
Constant      161,454       4,529      35,65    0,000
Capacity      0,09105     0,01237       7,36    0,000

S = 0,3833      R-Sq = 88,6%     R-Sq(adj) = 86,9%

Analysis of Variance

Source           DF         SS         MS        F        P
Regression        1     7,9578     7,9578    54,18    0,000
Residual Error    7     1,0282     0,1469
Total             8     8,9860


MTB > regress c6 1 c1 c17 c18
```

Regression Analysis: NPVgamma0,2 versus Capacity

```
The regression equation is
0,2 = 160 + 0,103 Capacity

Predictor        Coef     SE Coef          T        P
Constant      160,481       3,526      45,52    0,000
Power        0,102850    0,009629      10,68    0,000

S = 0,2983      R-Sq = 94,2%     R-Sq(adj) = 93,4%

Analysis of Variance

Source           DF         SS         MS        F        P
Regression        1     10,155     10,155   114,09    0,000
Residual Error    7      0,623      0,089
Total             8     10,778


MTB > regress c7 1 c1 c19 c20
```

Regression Analysis: NPVgamma0,25 versus Capacity

```
The regression equation is
NPVgamma0,25 = 158 + 0,118 Capacity

Predictor        Coef     SE Coef          T        P
Constant      158,372       3,724      42,53    0,000
Capacity         0,11842     0,01017      11,64    0,000

S = 0,3151      R-Sq = 95,1%     R-Sq(adj) = 94,4%

Analysis of Variance

Source           DF         SS         MS        F        P
```

```
Regression         1      13,462      13,462    135,58    0,000
Residual Error     7       0,695       0,099
Total              8      14,157


MTB > regress c8 1 c1 c21 c22
```

Regression Analysis: NPVgamma0,3 versus Capacity

```
The regression equation is
0,3 = 156 + 0,134 Capacity

Predictor        Coef     SE Coef          T        P
Constant      156,284       4,233      36,92    0,000
Capacity      0,13447     0,01156      11,63    0,000

S = 0,3582      R-Sq = 95,1%     R-Sq(adj) = 94,4%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1      17,360      17,360    135,28    0,000
Residual Error     7       0,898       0,128
Total              8      18,258
```

Appendix 5 Probability of an Expansion

Given the inputs in Chapter 6 the development in gamma for 10 years is simulated1000 times. The maximum value in each simulation is noted. In 14.1 % of the simulations the maximum change was over 0,09, hence initiating a 30MW expansion. This is the sum of all columns left of 0,09 in figure (A5-1).

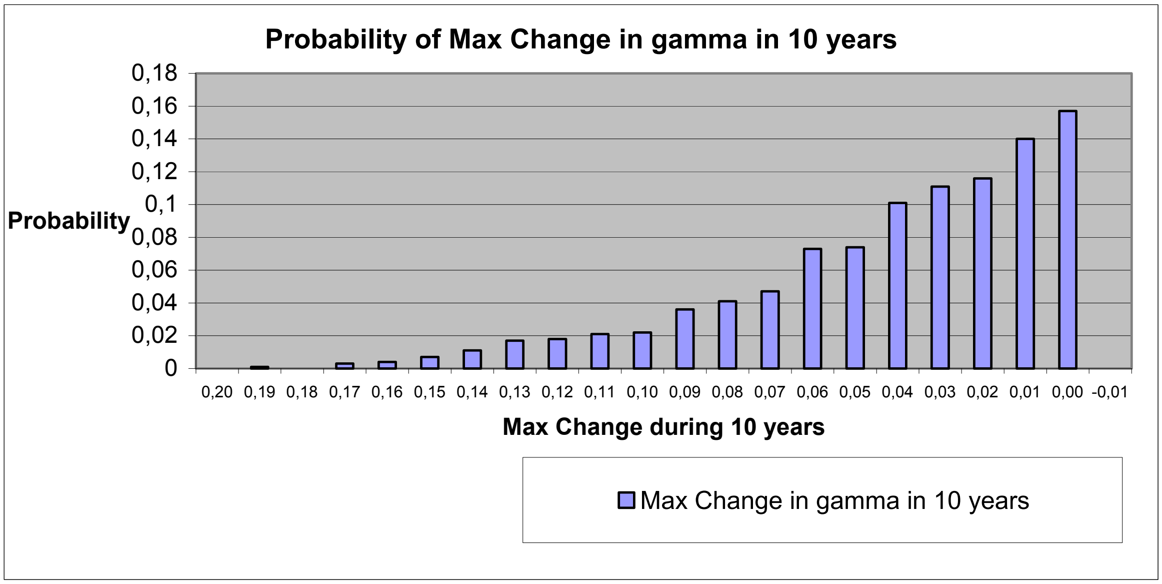


Figure A5-1 The probability for max change in Gamma

To illustrate how the change in gamma have influence on the forward price a simulation of the forward price is given in figureA5-2.

This is an expected forward curve for different forward contracts with infinite long maturity which means that the price today makes no influence.

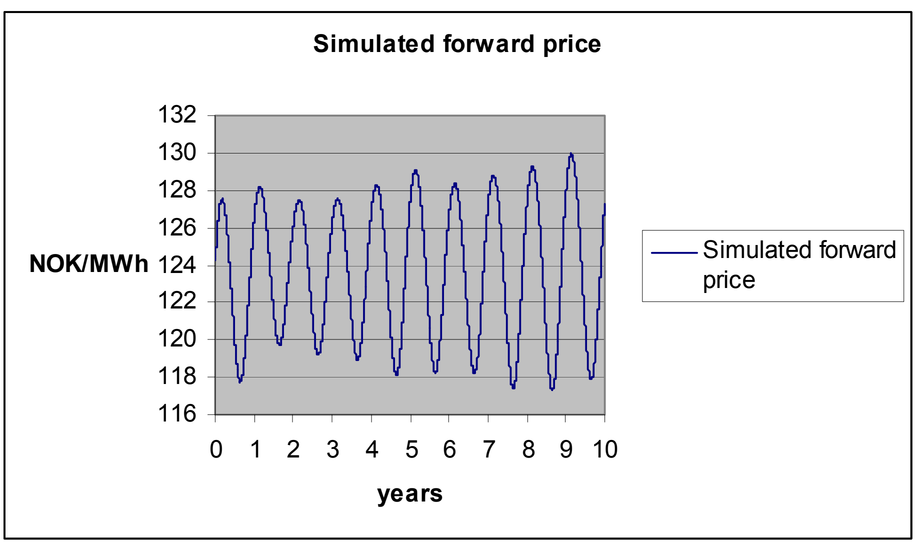
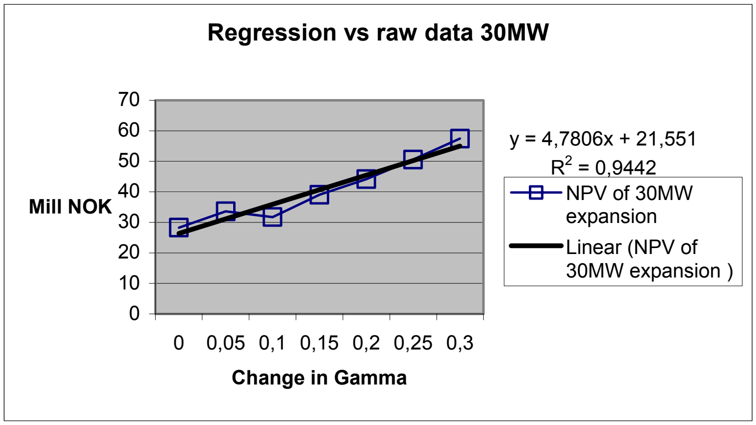
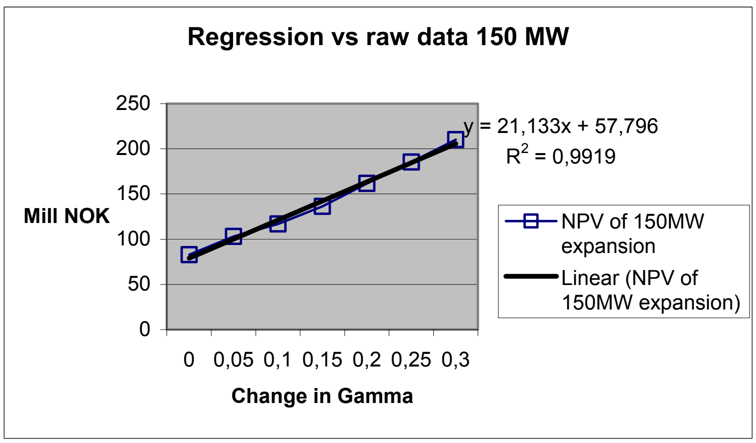


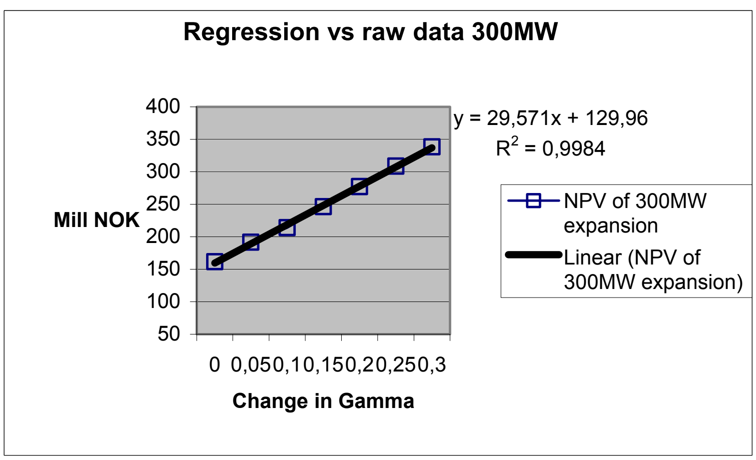
Figure A5-2 Simulation of forward prices

Appendix 6 NPV regression

In this appendix the net present value of investments of 30MW, 150MW and 300MW is regressed versus gamma







Appendix 7 Values for Figures 6.11-6.13

This appendix lists the different values for the expansion options for different gamma values. All numbers are in Mill NOK

	30MW		150MW		300MW	
Change in Gamma	NPV-investment	Option	NPV-investment	Option	NPV-investment	Option
0,00	-3,67	0,52	-71,07	0,04	-140,47	0,09
0,01	-2,77	0,68	-66,84	0,07	-134,56	0,13
0,02	-1,87	0,88	-62,62	0,10	-128,64	0,17
0,03	-0,97	1,13	-58,39	0,14	-122,73	0,22
0,04	-0,07	1,44	-54,17	0,20	-116,81	0,28
0,05	0,83	1,83	-49,94	0,29	-110,90	0,37
0,06	1,73	2,30	-45,71	0,40	-104,99	0,47
0,07	2,62	2,88	-41,49	0,55	-99,07	0,60
0,08	3,52	3,58	-37,26	0,75	-93,16	0,75
0,09	4,42	4,42	-33,03	1,01	-87,24	0,94
0,10	5,32	5,32	-28,81	1,34	-81,33	1,17
0,11	6,22	6,22	-24,58	1,76	-75,41	1,45
0,12	7,12	7,12	-20,35	2,29	-69,50	1,79
0,13	8,02	8,02	-16,13	2,96	-63,59	2,19
0,14	8,92	8,92	-11,90	3,80	-57,67	2,67
0,15	9,82	9,82	-7,67	4,84	-51,76	3,24
0,16	10,72	10,72	-3,45	6,11	-45,84	3,91
0,17	11,61	11,61	0,78	7,68	-39,93	4,70
0,18	12,51	12,51	5,01	9,57	-34,01	5,62
0,19	13,41	13,41	9,23	11,87	-28,10	6,70
0,20	14,31	14,31	13,46	14,64	-22,19	7,96
0,21	15,21	15,21	17,69	17,96	-16,27	9,43
0,22	16,11	16,11	21,91	21,91	-10,36	11,12
0,23	17,01	17,01	26,14	26,14	-4,44	13,07
0,24	17,91	17,91	30,37	30,37	1,47	15,31
0,25	18,81	18,81	34,59	34,59	7,39	17,89
0,26	19,71	19,71	38,82	38,82	13,30	20,84
0,27	20,60	20,60	43,05	43,05	19,21	24,20
0,28	21,50	21,50	47,27	47,27	25,13	28,03
0,29	22,40	22,40	51,50	51,50	31,04	32,37
0,30	23,30	23,30	55,73	55,73	36,96	37,30

Table A7-1 Values for different expansion options for different changes in gamma