A step towards modelling the day-ahead and intraday market as an equilibrium model

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Preface

This report is written at the Department of Industrial Economics and Technology Management (IØT) at the Norwegian University of Science and Technology (NTNU) during the fall semester 2018, as a part of the course TIØ4505 - Managerial Economics and Operations Research.

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Abstract

In the last years there have been a rapid expansion in electricity produced by renewable means in Europe. The intermittent nature of solar and wind production introduces new challenges for the electricity market as it is designed today. Forecast errors are likely to occur between the commitments in the day-ahead market and the actual delivery of electricity, as the output of the renewable electricity production is highly weather dependent. Several authors examine how this may effect the stability of the power system, and if the balancing market is able to provide the sufficient flexibility needed to handle these large imbalances. It is argued that the intraday market can facilitate this integration, as it gives the market participants the opportunity to trade closer to actual delivery when more accurate forecasts are available. This report provides a first attempt to model the day-ahead market as an equilibrium model, facing uncertainty in the realized bid and ask quantities of the market participants in the intraday market.

The report provides two models, each representing a deterministic equivalent of a stochastic two-stage equilibrium model of the day-ahead and intraday market. Equilibrium modelling has not yet been used in the context of modelling the day-ahead and intraday market together, and can provide valuable insight in the market interactions different from the existing multi-stage stochastic optimization models. Four different power producers are introduced in the model. First, a wind power producer with intermittent power production and zero marginal cost of production. Further, a hydro producer and two thermal producers, which have dispatchable power plants and are able to provide flexibility in the power system. Moreover, it is looked into three different cases of future increase in wind power production. This will in turn increase need to correct imbalances in the intraday market, due to forecast errors in the production of the wind power producer.

To solve for market equilibrium, the Karush-Kuhn-Tucker (KKT) optimality conditions are derived for all participants and combined with the market-clearing conditions, which leads to an instance of a mixed complementarity problem (MCP). The first MCP assumes perfect competition, that is all the producers are price-takers. It solves the day-ahead market where the producers face uncertainty in the future deviations. The deviation appears due to forecast error from the wind producers and will occur closer to delivery time. In the model, this deviation has to be settled by the other dispatchable producers, which have the flexibility to change their production plan on short notice. The second MCP assumes Cournot competition, that is, the market participants compete on quantities and can anticipate how their actions affect the market price.

The report provides a starting point in modelling the day-ahead and intraday market as an equilibrium model. It is found that under perfect competition the total surplus in the day-ahead market increases as the wind share increases. Contrarily, under Cournot competition, the total surplus decreases. Thus, the effect of market power eliminates the potential extra

total surplus from increase wind share in the power system. By modelling the day-ahead and intraday market as an equilibrium model some limitations using equilibrium modelling on this application area became apparent. One finding is that by using the market-clearing price as the price of the intraday market, the resulting prices are not realistic compared to the observed intraday prices in the Nord Pool market. Thus, the model was limited to only consider some aspects of the day-ahead and intraday market. The limitations are discussed in further detail in the report.

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Chapter 1

Introduction

In the recent years, there have been a great growth of renewable power production in Europe. In 2016, the electricity production from renewable sources contributed almost 30% of the gross electricity consumption in the EU. The growth has been greatest for wind and solar power, with respectively 3.7 and 44.1 times greater electricity production in 2016 than in 2006 (*Database - Eurostat* 2018). This trend is expected to continue in the coming years, and according to *World Energy Outlook 2018* (2018), it is expected that wind and solar would account for 39.5% and 27.5% of the total gross capacity addition in Europe respectively in the period from 2018 to 2040. As electricity production from burning fossil fuels is one of the main contributors to carbon pollution worldwide, this development is an important part of the necessary measures to meet the EU climate targets. However, the shift of the power production from fossil fuels to wind and solar creates some important challenges for the electricity market the way the market is designed today.

The challenges are mainly caused by the intermittent nature of the wind and solar power plants. As their output is highly weather dependent, it is not possible to forecast the production output without error, and their production output does not necessarily match the consumers demand. This is problematic as it is highly important to maintain balance of demand and supply at all times for the transmission network to remain stable. Historically, the balancing market has been the response to deviations between the day-ahead dispatch and physical delivery of electricity. Although, as a large amount of intermittent electricity production is introduced and imbalances hence are expected to increase, handling all the imbalances in real-time may lead to the use of costly and polluting power plants to provide the flexibility needed. Thus, it is not market efficient to have the balancing market handle the increasing amount of imbalances alone (Weber 2010). In order to ensure a successful integration of the intermittent electricity sources to the power system, it is therefore important to provide good tools to meet the occurring challenges (Selasinsky 2016). In several papers written about the issue, (Weber (2010), Borggrefe and Neuhoff (2011)), the *intraday market* is considered to be such a tool. The intraday market gives the market participants the opportunity to trade electricity closer to physical delivery, when more accurate information is available. Obtaining a well functioning intraday market gives the market participants incentive to balance the deviations from forecasted production and demand themselves.

In this report, it is focused mainly on wind power production. Wind constitutes the largest share of intermittent renewables in the Nordic countries, and is less predictable and more variable than solar power.

Much of the relevant research looks into the state of the electricity market design when a great amount of renewables are introduced. Several authors argue that the intraday market should be used to facilitate the integration of renewable electricity production (Weber (2010), Vandezande et al. (2009), Garnier and Madlener (2014)). The main arguments are that this will give the market participants an incentive to settle imbalances themselves, that the system costs can be kept low and that it is inefficient if the balance market is used to handle significantly larger amount of imbalances. Moreover, the main concern among several of the authors is the low liquidity in the intraday market, and much research has been done with the main focus on this issue (Weber (2010), Henriot (2014)). Skajaa, Edlund, and Morales (2015) argue that the low liquidity causes the economic benefit from trading wind electricity in the intraday market to remain unexploited, and Weber (2010) claims that higher liquidity would improve the overall market efficiency.

In the literature, most of the optimization models that have been made to describe the day-ahead and intraday market have been stochastic multi-stage models, and several of these used a rolling planning procedure. Barth et al. (2006) present a stochastic unit-commitment model to analyze the market impact of the stochastic forecast errors caused by wind power producers and Abrell and Kunz (2012) use stochastic programming techniques to include the uncertain wind generation. Other analytical approaches is used as well, such as simulation algorithms (Skajaa, Edlund, and Morales 2015), dynamic programming with option value techniques (Garnier and Madlener 2014) as well as analytical models to analyze different trading strategies (Henriot 2014).

This report formulates a deterministic equivalent of a stochastic two-stage equilibrium model of the day-ahead and intraday market. Previously, equilibrium modelling has not been used in the context of modelling the day-ahead and intraday market together and it could thus provide different insight in the market interaction than the already existing multi-stage stochastic optimization models. Furthermore, the problem of solving a stochastic MCP is a field mostly unstudied until recently (Gabriel et al. 2013). This report provides a starting point in modelling the day-ahead and intraday market as an equilibrium model. Such a model could give insight to actors in the electricity sector on how the intraday market may affect actions of market participants trading electricity in both the day-ahead and intraday market. The intention is to explore tradings between agents in a situation where the intraday market is becoming more developed, and there is a greater share of renewable electricity. As already mentioned, the electricity production by renewable means is increasing in the coming years, and the intraday market is considered a tool to handle this integration efficiently. The report asks questions on how this integration may affect the system, both in a situation assuming perfect competition as well as Cournot competition.

Chapter 2 is a background chapter that gives the reader basic insight to the electricity market, especially the day-ahead and intraday market. Chapter 3 provides an introduction to the theory relevant to describe the mathematical model in this report. The relevant literature for the research is presented in Chapter 4. Chapter 5 presents two models, the first assuming perfect competition and the second assuming Cournot competition. To solve for market equilibrium, the KKT optimality conditions are derived which leads to a mixed complementarity problem. Following, the results from solving the MCPs are presented and discussed in Chapter 6. The decisions and objective values are analyzed, and the reliability of the results are examined. Chapter 7 provides some concluding remarks, summarizes and highlights the main findings in the report. Finally, suggestions for future research on the area are provided in Chapter 8.

Chapter 2

Background

The background chapter provides a description of the fundamental aspects of electricity systems and markets, focusing on the current structure of the Nordic electricity market.

In the following, Section 2.1 briefly describes the characteristics of the electricity markets and discusses how these characteristics differs from the markets where other goods are traded. Section 2.2, 2.3 and 2.4 describes respectively the day-ahead market, the intraday market and the balancing market trying to provide a basic understanding of the three markets for electricity. Finally, Section 2.5 discuss characteristics of renewable electricity sources and how integration of these may affect power systems.

2.1 The nature of electricity markets

Electricity markets are associated with a physical delivery system, which operates much faster than any commodity market. At all times, production and consumption of electricity must be balanced, implying that all electricity supplied must be the same as the electricity consumed at that time. Maintaining balance is of high importance in power markets because of the inability to store electricity and that there are high costs associated to supply failure. The consequences of a system collapse are massive for the society, and therefore equilibrium is maintained at almost any cost.

The electricity produced by the different suppliers in the power system is aggregated in a pool before it is supplied to the end-consumer. Therefore, the consumer is not able to determine from which power generator the electricity comes from and likewise the supplier cannot direct its production to some consumer. This characteristic is termed pool trading. The producers and the consumers submit their bids to a centralized market operator which determines a price that clears the market. In many cases a forecast of demand is used because the consumers tend to be passive market participants.

Traditionally, electricity demand has been very inelastic, meaning that a change in electricity price do not cause any significant change in electricity consumption (Saez-Gallego 2016). This is mainly due to the fact that electricity do not have a direct substitute product that consumers can use if it becomes too expensive. Another reason is that small consumers are generally not affected by changes in price.

The Nordic region is covered by Nord Pool Spot AS, which manages the largest electricity market in Europe, trading for 512 TWh in 2017 (Nord Pool 2017). Nord Pool offers day-ahead trading for the Nordic, Baltic and UK day-ahead markets as well as intraday trading for in total 12 countries, including Germany and France.

2.2 Day-ahead market

The day-ahead market constitutes of suppliers and consumers trading power for physical delivery of electricity for the hours of the following day. In the Nord Pool day-ahead market, also known as the *Elspot market*, the market participants are able to submit bids and offers for next-day delivery until the market closes at 12:00 Central European Time (CET). Typically, hourly prices are announced to the market at 12:24 CET or later. Once the system prices have been calculated, trades are completed. The trades are physically delivered to the consumers from 00:00 CET the following day.

The day-ahead market for electricity is organized as a periodic double auction. Market participants submit volume bids to the market in MWh, which are collected and aggregated into a supply and a demand curve for the day-ahead market for each hour in the following day. The power exchange then determines which offers will be accepted and the system price which clears the market. Normally, the prices are conducted using a uniform pricing rule. The system price then becomes the intersection of the supply and demand curve, and equals the offer price of the producer that produces the last quantity necessary to satisfy demand, as illustrated in Figure 2.1. The most important role of the system price is to establish equilibrium between supply and demand to avoid the consequences associated with supply failure.

2.3 Intraday market

The intraday market is a continuous market where trading takes place closer in time to the physical delivery. New information emerging after the closure of the day-ahead market, such as improved forecasts and unexpected production output, can be taken into account by the power producers in the intraday market. Thus, the intraday market supplements the day-ahead market, providing an opportunity to use more accurate information about the actual production and balance forecast errors to bring the power system back in balance. It

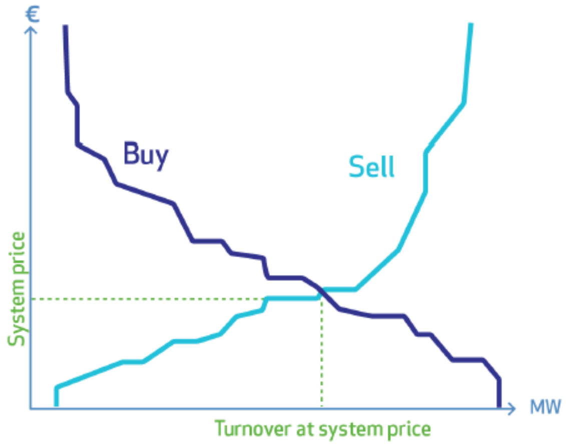

Figure 2.1: Determination of system price, Nord Pool (2018)

is expected that the intraday market will play an important role in the integration of the increasing share of renewable power producers in Europe.

Nord Pool's intraday market, *Elbas*, is organized as a continuous double auction and trading takes place every day until one hour before physical delivery, based on a "first-come-first-served" trade. Hence, in contrast to in the day-ahead market, the market participants are themselves responsible for the clearing process and for deciding at what price the different quantities will be sold. This is done in the framework of a "open order book", where the market participants submit offers by defining a specific quantity at a certain price. On the left-hand side of the order book bids are collected and sorted in a descending order, and on the right-hand side asks are collected in an increasing order. In this way, the top offers of the order book are the current best ones.

One major challenge with the intraday market is that the trading volumes are relatively low and the trading activity is sporadic. Nord Pool (2017) reports a turnover of 505TWh in the day-ahead market compared to 6.7TWh intraday turnover. Moreover, low liquidity is a major concern for the efficiency of the intraday market and is the main focus in several papers, which is discussed to more extent in the Literature review. However, as the amount of solar and wind power production grows, and it thus becomes more challenging for the market participants to stay in a balanced position, the interest for participating in the intraday market increases (NordPool 2018). Also, the Cross-Border Project (XBID), which has created a joint intraday market between several European countries, is expected to help increase both the liquidity and the efficiency of the market as a larger integrated market will increase the probability to get a match for a bid.

2.4 Balancing market

The balancing market, also known as the regulating or real-time market, is a tool used by the transmission system operators (TSOs) to ensure real-time balance between power production and consumption. It allows for the producers to make changes in their production plan on a very short notice as the market is cleared up to 45 minutes before actual delivery (Saez-Gallego 2016). The prices in the balancing market is determined based on the day-ahead price such that the tradings are less favourable in the regulating market. That is, it is more expensive to buy and less profitable to sell in the balancing market than in the day-ahead market (Saez-Gallego 2016). This gives the market participants incentive to schedule their production as precisely as possible in advance.

Historically, the balancing market has been a sufficient solution for ensuring real-time balance in the power system. However, as mentioned, this is not considered an efficient solution as imbalances increases in conjunction with an increasing share of intermittent renewable power (Weber 2010). Figure 2.2 provides a scheme of how the electricity is traded in the different markets.

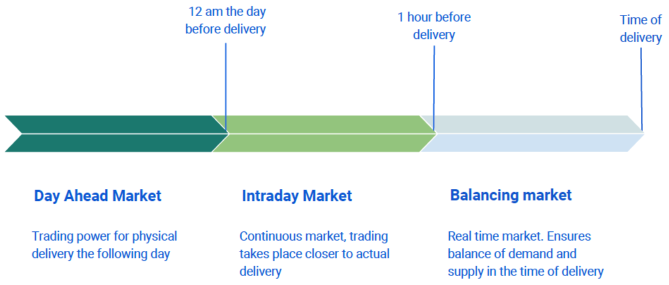

Figure 2.2: Illustration of the different electricity markets

2.5 Renewable energy sources

The renewable energy sources described are wind and solar. These renewable production plants require high initial investments, the electricity produced from renewables has close to zero marginal costs and the production process does not emit any greenhouse gases. This implies that a cost-efficient renewable power producer will try to maximize the feed-in of the installed capacity and that their output should be used whenever possible. Selasinsky (2016) argue that as a consequence the role of the dispatchable power generators will be reduced to cover the remaining difference between electricity demand and the actual production from renewable electricity producers.

Another important characteristic of the renewable energy sources is that the production output is variable and may vary significantly within few hours. Furthermore, the output from renewable energy production is uncertain, meaning that it is difficult to know if the expected electricity production matches the actual real-time production. The uncertainty is mainly due to the fact that the production depends on the environment and cannot be forecasted without some errors. This makes it particularly relevant for these producers to be able to trade electricity closer to actual delivery. The variable and uncertain nature of the renewables production is the main reason why their increasing share raises some challenges in the power system as it is designed today. The main concern is stability in the power system, which depends on an equilibrium of supply and demand.

Figure 2.3 shows a simplified illustration of the effect on the market price when there is a high, medium and low wind share amount. As the wind share increases, the merit order curves shifts towards right, and thus the market price decreases. Selasinsky (2016) argue that from a system perspective it would be preferable if renewables would predominantly replace more inflexible and more carbon-intensive production options, due to the merit order effect.

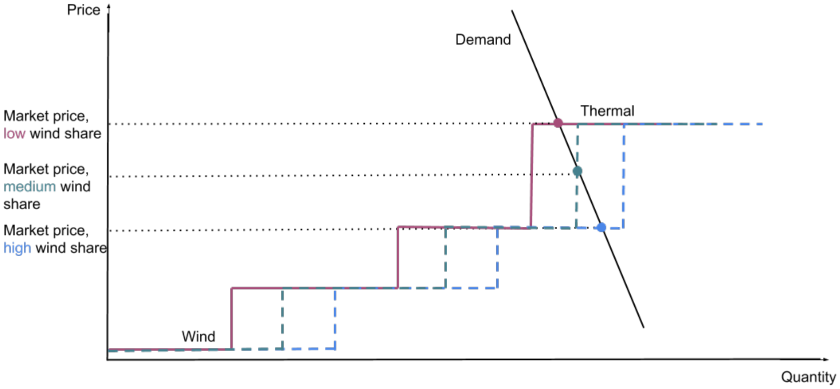

Figure 2.3: Simplified illustration of the merit order effect induced by wind electricity sources

Chapter 3

Theory

This chapter gives a brief introduction to the theory used in this report, and is based on the book by Gabriel et al. (2013) *Complementarity Modeling in Energy Markets.* In Section 3.1 the Karush-Kuhn-Tucker (KKT) conditions are explained for a minimization problem. The KKT conditions can be used to in to model complementarity problems and find equilibrium. Moreover, Section 3.2 describe the mixed complementarity problem (MCP), which is a generalization of the nonlinear complementarity problem (NCP) to include equality constraints and free variables.

3.1 Karush-Kuhn-Tucker conditions

The Karush-Kunh-Tucker (KKT) conditions are conditions that can characterize optimal solutions for a range of optimization problems. For the KKT conditions to guarantee an optimal solution they have to be both *necessary* and *sufficient*. If necessity of the KKT conditions holds, optimal solutions need to meet them. Moreover, if the KKT conditions are sufficient, an optimal solution can be guaranteed. Whether the KKT conditions are necessary and sufficient, depends upon the formulation of the problem which has to satisfy some *constraint qualifications* (CQ's). For linear problems the *linearity constraint qualifications* (LCQ's) need to be satisfied to guarantee an optimal solution. The LCQ's state that the functions defining the feasible region have to be affine functions.

To formulate the KKT conditions we consider a constrained and continuous general minimization problem (3.1). Functions $f(x)$, $g_i(x)$ and $h_j(x)$ are assumed to be continuously differentiable in the feasible region. The dual variables u_i and v_j are also known as the Lagrangian multipliers.

$$min\, f(x) \tag{3.1a}$$

subject to

$$g_i(x) \leq 0 \qquad (u_i) \quad for\, i = 1, ..., m \tag{3.1b}$$

$$h_j(x) = 0 \qquad (v_j) \quad for\, j = 1, ..., n \tag{3.1c}$$

$$x \in R^n \tag{3.1d}$$

Given the formulation of a general minimization problem (3.1), a formulation of the KKT condtitions are presented. To formulate the KKT conditions the gradient of the Lagrangian function with respect to x is defined in constraint (3.2a), which states that at an optimal solution this should be equal to zero. Constraints (3.2b) and (3.2c) state the inequality and equality restrictions from the primal problem, which still are required to ensure feasibility. Constraint (3.2d) represents the complementary slackness conditions, which state that if the dual variable is greater than zero then the corresponding primal constraint must be an equality, otherwise the opposite holds. Lastly, (3.2e) and (3.2f) define the Lagrangian multipliers.

$$\bigtriangledown f(x) + \sum_{i=1}^{m} u_i \bigtriangledown g_i(x) + \sum_{i=1}^{n} v_j \bigtriangledown h_j(x) = 0 \tag{3.2a}$$

$$h_j(x) = 0 \qquad for\, j = 1, ..., n \tag{3.2b}$$

$$g_i(x) \leq 0 \qquad for\, i = 1, ..., m \tag{3.2c}$$

$$u_i \cdot g_i(x) = 0 \quad for\, i = 1, ..., m \tag{3.2d}$$

$$u_i \geq 0 \qquad for\, i = 1, ..., m \tag{3.2e}$$

$$v_j,\, free \qquad for\, j = 1, ..., n \tag{3.2f}$$

Constraints (3.2c), (3.2d) and (3.2e) are also known as the complementarity conditions and can be equivalently written the following way

$$0 \leq u_i \perp g_i(x) \leq 0 \tag{3.2g}$$

The concept of Nash equilibrium is often discussed in the setting of modeling different markets. In particular, Nash equilibrium is reached if all suppliers produce at an output level that leads to a market price where none of the suppliers have the incentive to change their output. If the behaviour of each supplier is modeled by an optimization problem the Nash equilibrium is equivalent to a vector of outputs fulfilling the KKT conditions solving all optimization problems simultaneously. This type of problems are referred to as equilibrium problems (Gabriel et al. 2013). Equilibrium models can represent varying market structures, including perfect competition and oligopoly.

3.2 Mixed Complementarity Problems

Mixed complementarity problems (MCP's) are a generalization of nonlinear complementarity problems (NCP's), i.e. the MCP's are different in that they contain equality constraints with corresponding free variables. Every program of the form (3.1) gives rise to a mixed complementarity problem through its KKT conditions, which are to find x, u and v such that

$$0 = \bigtriangledown f(x) + \sum_{i=1}^{m} u_i \bigtriangledown g_i(x) + \sum_{i=1}^{n} v_j \bigtriangledown h_j(x) \tag{3.3a}$$

$$0 \leq u_i \perp g_i(x) \geq 0 \qquad for\, i = 1, ..., m \tag{3.3b}$$

$$0 = h_j(x), v_j\, free \qquad for\, j = 1, ..., n \tag{3.3c}$$

Then the MCP function is

$$F(x, u_i, v_j) = \begin{cases} \bigtriangledown f(x) + \sum_i \bigtriangledown g_i(x) u_i + \sum_j \bigtriangledown h_j(x) v_j \\ -g_i(x),\ i = 1, ..., m \\ h_j(x),\ j = 1, ..., n \end{cases} \tag{3.4}$$

A Nash-Cournot model can be formulated as a mixed complementarity model by writing out all the agents KKT conditions for optimality. In a Nash-Cournot model it is assumed that the agents choose a quantity that maximizes their own profit using their knowledge about the inverse demand curve, i.e. they recognize how their decisions on quantities supplied effect the market price. Moreover, the Nash-Cournot model do not assume that the decisions of an agent can affect the decisions made by other agents in the market, this is beyond the agent's control.

Chapter 4

Literature Review

This chapter discusses literature on market design, liquidity and modelling of intraday markets. In Section 4.1 research on market design is described, Section 4.2 discusses the research on modelling of intraday markets and Section 4.3 present literature on liquidity. Finally, Section 4.4 describe the contributions made to the literature in this report.

4.1 Market design

The research on market design for intraday markets is often studied in the context of the general market design for electricity markets facing an increasing share of renewable energy sources (Newbery (2010), Smeers (2008), Borggrefe and Neuhoff (2011)). In this context, Smeers (2008) argues that the day-ahead, intraday and real-time markets should be different steps of a single trading process. Hence, the updating process of intraday market should be organized compatible to the day-ahead market, otherwise it prevents temporal arbitrage. Smeers (2008) considers the real-time market as an effective trading period in favor of the intraday market and argues that there are compelling physical and economic reasons for this.

Henriot (2014) argues in his analysis, which is discussed briefly in Section 4.2, that whether a wind power producer should participate in the intraday market would depend on the flexibility of the power plant portfolio in real-time as well as on the forecast errors. He found that situations exists where it would be more costly to correct imbalances in the intraday market than in the balancing market.

Similarly, Weber (2010) analyses the European electricity markets with respect to their ability to absorb great amounts of wind energy. The market designs of several major European power markets are examined with special focus on the liquidity in the day-ahead and intraday markets. In contrast to Smeers (2008), Weber (2010) argues that an efficient market design should favor intraday over real-time balancing to correct for imbalances and highlights that:

> *"In an efficient market design, as much as possible of these adjustments would however be done in the intraday market to avoid the use of more expensive flexible resources in real-time balancing."*

In addition to Weber (2010), several authors argues that the intraday market should be used to facilitate the integration of renewable energy production.Vandezande et al. (2009) discuss interactions between intraday and balancing markets. Their research supports the perception that the intraday market should be used and argue that it could encourage the power producers to provide more accurate production forecasts in order to reduce their imbalances. Borggrefe and Neuhoff (2011) also argue that intraday market may induce market participants to settle imbalances themselves and present intraday markets as a tool to keep system costs low in power systems featuring an increasing amount of intermittent renewable energy sources. Moreover, Garnier and Madlener (2014) point out that the only purpose of the balancing market is to ensure system stability and should not be considered a traditional market place, thus they support the view of Weber (2010). Nevertheless, Weber (2010) specify that a well-designed balancing market is of high importance in order to improve the intraday market. Vandezande et al. (2009) discuss the market design for balancing markets in further detail , and argue that with too low imbalance prices market participants would have less incentive to trade in the intraday market. Lastly, Zipf and Most (2013) support Weber (2010) and Borggrefe and Neuhoff (2011) in their description of the intraday market. They describe the intraday market as a way to compensate the recognizable forecast errors based on the day-ahead forecast errors. Whereas the balancing markets are used to "match real-time electricity demand and supply and guarantee a constant network frequency."

4.2 Modelling intraday markets

The existing models of intraday markets for electricity are mainly stochastic multi-stage optimization models. This is often considered the most appropriate way to model the short-term electricity market because of the uncertainty and continuity in day-ahead and intraday trading. More specifically, the day-ahead market allocates electricity for every hour of the following day. Then, some new information regarding actual production becomes available and the intraday market allows for adjustments of the day-ahead commitments closer to the physical delivery of electricity. Faria and Fleten (2009) develop a stochastic two-stage model for a hydro-power producer trading electricity in the day-ahead market having the opportunity to trade electricity in the intraday market. Their research shows that the impact in profit considering intraday trading for a producer already trading in the day-ahead market is insignificant.

Barth et al. (2006) present a stochastic unit-commitment model to analyze the market impact of the stochastic forecast errors caused by wind power producers. To do so, they use a rolling planning procedure to model the day-ahead and intraday markets in three stages, where the up and down adjustments in the intraday market consequently depends on the different scenarios. Likewise, Abrell and Kunz (2012) develop a stochastic electricity market model and implement a rolling planning procedure in order to incorporate the consecutive clearing of the day-ahead and intraday markets. Both Barth et al. (2006) and Abrell and Kunz (2012) uses case studies to examine the impact of an increasing amount of uncertain wind generation in the electricity market. More recently, Zipf and Most (2013) implement a rolling horizon approach to link the day-ahead market model with the intraday model, and thus introduce a stepwise clearing of the markets similar to the one of Abrell and Kunz (2012). The day-ahead market is modeled to optimize the dispatch of the conventional power plants one day before physical delivery, using a "best guess" of renewable energy production. Afterwards, the intraday model is solved for each hour trying to re-optimize the dispatch from the day-ahead market considering changes in production. Zipf and Most (2013) use the model to calculate the indirect system cost from the uncertain and volatile renewable power plants and conclude that these costs would increase significantly with the share of renewable electricity in the power system.

Even though stochastic multi-stage programming is the main focus among the works on intraday, other approaches has been used in order to model the intraday market. Skajaa, Edlund, and Morales (2015) investigate a simulation algorithm for trading wind energy in a continuous intraday market. Moreover, Garnier and Madlener (2014) combine dynamic programming and option value techniques to model balancing decisions of a renewable power agent who corrects forecast errors in a continuous intraday market. Garnier and Madlener (2014) conclude that in a setting with high price volatility, early trading is preferable due to higher market liquidity, while high forecast volatility should be handled by balancing these errors closer to real-time.

Henriot (2014) applies an analytical model to analyze different trading strategies of a wind power producer in a context of a large increase in intermittent renewable energy sources. The producer is given the opportunity to either adapt to an active or a passive strategy. The active strategy implies that the producer have the possibility to trade in the intraday market and correct imbalances in real-time, whereas a passive strategy implies that the producer is not allowed to participate in the intraday market and have to pay a costly fee to compensate for the forecast errors in real-time.

4.3 Research on liquidity

Low liquidity in the intraday market is discussed by several authors (Henriot (2014), Weber (2010), Hagemann and Weber (2013)), and is one of the major concerns regarding efficient intraday activity. Henriot (2014) argues that the low liquidity in the intraday market can be a result from rational behaviour among the market participant and that it will be unavoidable for some technical parameters. Market participants may see it too costly or unattractive to adjust their positions in the intraday markets due to low participation and volatile markets. Furthermore, Weber (2010) claims that higher liquidity in the intraday market would improve the overall efficiency of the market design and lower the system cost of wind power integration. Moreover, Skajaa, Edlund, and Morales (2015) argue that the economic benefit from trading wind energy in the intraday market remains unexploited due to low liquidity.

Hagemann and Weber (2013) present an empirical analysis of the liquidity in the German intraday market. They suggest two different models to explain the trading in the intraday market and hence liquidity in the market. The first model assumes a perfect competition framework, whereas the second model assumes that all agents aims at profit maximization. They argue that the liquidity in the intraday market is most accurately explained by the model where all market participants aims at maximizing profit. Moreover, they argue that the market participants opportunity to correct forecast errors within their own portfolio will influence the liquidity in the intraday market.

4.4 Contribution to the literature

Several of the existing models that try to model aspects of the intraday market use a stochastic multi-stage approach to capture the inter-temporal character of the intraday market (Barth et al. (2006), Abrell and Kunz (2012), Zipf and Most (2013)). This report provides a deterministic equivalent of a stochastic two-stage equilibrium model of the day-ahead and intraday market. The model describes a situation where the market participants can make decisions in the day-ahead market while facing uncertainty about the future development of the intraday market. By using equilibrium modelling it is possible examine the market from a system perspective, rather than from a single profit maximizing agent's point of view. Equilibrium modeling can possibly give valuable insights to the actions and decisions of market participants that have the opportunity to participate in the intraday market in addition to the regular day-ahead market.

Most of the models that exists in the literature assume perfect competition and minimize system cost, that is they implicitly assume a central planner. Therefore, the market participants are assumed to be price-takers, and thus cannot influence the market price. This may not be a suitable assumption because it does not reflect the aspects of strategic behaviour among the multiple agents in the intraday market (Selasinsky 2016). Moreover, Lundin and Tangerås (2017) argue that the strategic behavior observed in the Nordic day-ahead market is consistent with Cournot competition. In this report, two models are formulated. The first assuming perfect competition and the second assuming Cournot competition, in the day-ahead market. In this way it is possible to examine how the behaviour differs when the market participants can exert market power. Lastly, the report examines how quantities are allocated between the different power producers when there is an increase in the share of renewable electricity production in the power system.

Chapter 5

Mathematical Model

This chapter presents two models, the first assuming perfect competition and the second assuming Cournot competition. In both models, each power producer solve a cost minimization problem in the day-ahead market, subject to capacity and trading constraints. As bid and ask quantities appearing in the intraday market are uncertain at the time of the day-ahead decisions, the nature of the problem is stochastic. These deviations are represented by different scenario realizations with associated probabilities. The problem becomes a deterministic equivalent to the stochastic problem. Under perfect competition, consumers are represented by maximizing consumer surplus. To solve for market equilibrium, the KKT optimality conditions are derived for all market participants. The interactions between the market participants are handled by market-clearing conditions. Combining these conditions leads to an instance of an MCP. Resulting, each MCP represent a deterministic equivalent to a stochastic equilibrium model for a day-ahead and intraday market.

Section 5.1 presents the main assumptions taken. Section 5.2 defines the notation used in the two models, whereas Section 5.3 describes the objective function and constraints of the model. Lastly, Section 5.4 and Section 5.5, formulate the models and derive the MCPs.

5.1 Assumptions

Throughout this section the main assumptions are presented.

It is assumed that the power producers can be aggregated into single players, based on the type of power plants, each selling electricity in the day-ahead and intraday market. Each player thus represent different production characteristics and production costs related to the type of production. It is also assumed that the consumers can be aggregated into one single player.

The production output from the wind producer is characterized by large variations. The intermittency of the power generation, as well as the difficulties in forecasting the production one day ahead, can cause large deviations between the quantities committed in the day-ahead market and the actual production at time of physical delivery. Bidding strategy for the wind power producer in the day-ahead market is an issue in several papers. Among others, Willumsen (2018) argues that for the wind power producer it might be sub-optimal to commit expected forecast in the day-ahead market, as the difference between the price in the day-ahead market and upward and downward balancing price is different. However, for simplicity, it is assumed that the wind power producer trade a quantity equal to the estimated production output at the time of the day-ahead market closure. The wind producers cannot determine their production output themselves, as this is dependent of external factors. Moreover, they produce independently of price as they have zero marginal cost of production. Thus, it is assumed that they become price-takers in the market. Based on these characteristics, the wind producer is not included as a decision maker in the model, but their contribution is rather represented through input parameters.

The forecasts of the wind production output are never perfect, and the resulting production mismatch implies that there is a need to correct for imbalances in the intraday market. Hence, it is assumed that bid and ask quantities from the wind producer in the intraday market is represented by this deviation. It is assumed that the forecast error from the wind power producer is the only factor that constitutes the imbalances that must be settled in the intraday market. This is because other deviations, such as errors in demand forecasts, are expected to be very small compared to these errors. The deviation is an uncertain parameter at the time of the day-ahead market decision as it is dependent on the actual production output of the wind power producer, which first becomes fully aware of the deviation at the time of actual delivery. It is assumed that this uncertain outcome can be represented as a discrete and finite set of scenarios, each with an associated probability of occurring. Thus, the stochastic two-stage model reduces to a deterministic equivalent.

Another assumption made is that there is a market operator that enforces all imbalances to be settled in the intraday market. This obligation is represented by market-clearing conditions. In fact, by making this assumption, the intraday market becomes a balancing market in practice.

The complexity increases rapidly when introducing inter-temporal decisions, especially because a new set of scenarios must be solved for each time step. For simplicity the model is limited to trade electricity in the day-ahead market for only one hour of the following day. Moreover, another assumption made is that the intraday market is limited to only consider tradings taking place the last hour before actual delivery. This assumption is motivated by the findings of Garnier and Madlener (2014), who argue that high forecast volatility should be counteracted by balancing forecast errors closer to real-time. Also, Scharff and Amelin (2015), who analyze intraday trades on Elbas, conclude that most trades occur in the last hours before the intraday market closure. This is illustrated in Figure 5.1.

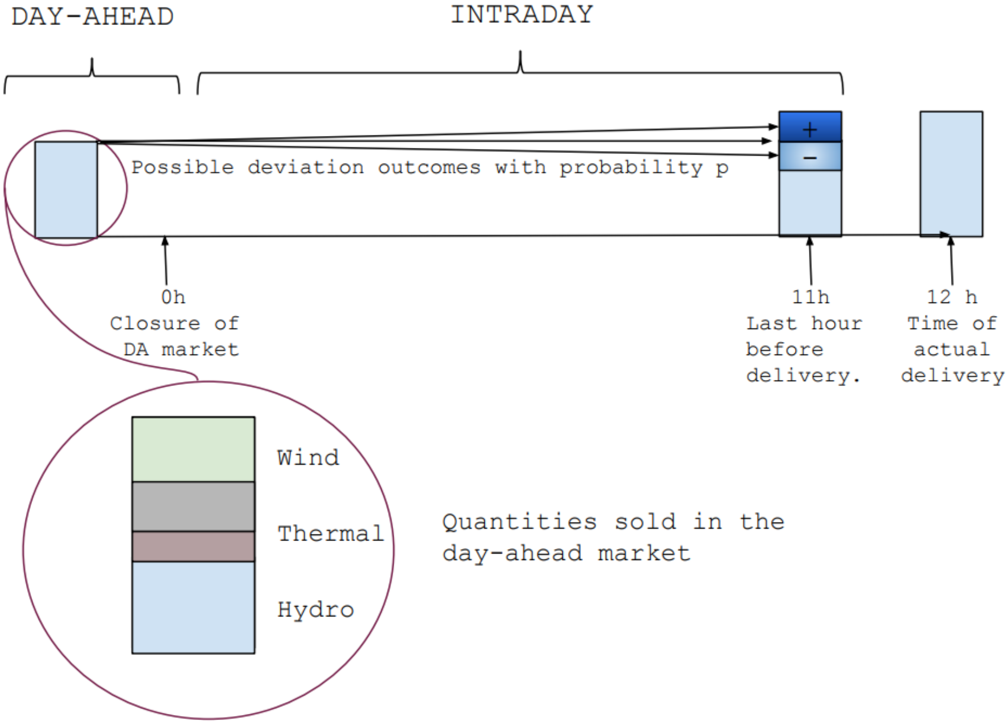

Figure 5.1: Simplified illustration of how the day-ahead and intraday market is modeled

Weber (2010), Henriot (2014) and Hagemann and Weber (2013) all stress the importance of liquidity in the intraday market in order for the market to be efficient. As it is today, the liquidity in the intraday market is in fact low, causing a risk for the market participants that their bid might not be met. In the model however, it is assumed that the intraday market is liquid. That is, the model represents the intraday market as it would function if it was a liquid market.

The first model assumes a perfect competition setting, as has been done in most of the existing models that describe the intraday market (Barth et al. (2006), Borggrefe and Neuhoff (2011) among others). That is, all the producers act as price-takers. In the second model, it is assumed that the market participants compete on quantities and can anticipate how their actions affect the market price. This strategic behaviour among the participants has been observed in the Nordic day-ahead market Elspot by Lundin and Tangerås (2017), who argue that the observed horizontal shifts in bid curves are consistent with Cournot competition. In the intraday market the market participants are assumed to be price-takers. This is due to the assumption that demand is fixed in each scenario, and equal to the deviations of the wind power producer, and must be cleared by the other power producers.

5.2 Notation

Sets:
S: set of scenarios
$S^+ \subseteq S$, scenarios where the wind producer has deficit production and needs to buy quantities in the intraday market
$S^- \subseteq S$, scenarios where the wind producer has excess production and needs to sell quantities in the intraday market
I: set of electricity producers , I = {1, ..., 3}

Indices:
i: electricity producer, $i \in I$
s: scenarios, $s \in S$

Parameters:
q^{max}: max capacity for producer i
$C_i(q_i)$: linear cost function of producer i, given as $C_i(q_i) = d_i q_i$
Q^R: quantity produced by the wind producer in the day-ahead market
D_s^{ID}: deviation in the intraday market in scenario s

a, b: parameters of the inverse demand curve $p(Q) = a - b \cdot (\sum_i q_i + Q^R)$
p_s: probability of scenario s

Variables:
q_i: quantity committed by producer i in the day-ahead market
d: demand of consumers
Δ_{is}^+: quantity sold in the intraday market by producer i in scenario s
Δ_{is}^-: quantity bought in the intraday market by producer i in scenario s
λ_s^+: dual variable to capacity constraint 5.5b in scenario s
λ_s^-: dual variable to constraint 5.5c in scenario s
θ: dual variable to market-clearing constraint of the day-ahead market
θ_s^+: dual variable to market-clearing constraint of the intraday-market in scenario $s \in S^+$
θ_s^-: dual variable to market-clearing constraint of the intraday-market in scenario $s \in S^-$

5.3 Model description

This section provides a description of the models formulated in Section 5.4 and 5.5.

5.3.1 Producers problem

Each power producer i can choose to sell a quantity of electricity in the day-ahead market, q_i, which costs $C_i(q_i) = d_i q_i$ to produce and from which they will gain a revenue $\theta \cdot q_i$. For the decision in the day-ahead market, the objective function (5.5a) aims to minimize production cost minus revenues from selling units in the day-ahead market.

For the wind producer, the deviation between the quantities committed in the day-ahead market and the actual production quantities is dependent on the different scenario realizations. These deviation quantities will constitute the bid and ask quantities in the intraday market. At the time the day-ahead market decisions are made these deviations are unknown. In the model, the scenarios $s \in S$ with probability p_s are used to represent possible outcomes of the stochastic deviation in the intraday market D_s^{ID}.

The quantities traded in the intraday market, Δ_{is}, are split in a positive part Δ_{is}^+ and a negative part Δ_{is}^-. Δ_{is}^+ represent a deficit in the wind production compared to the expected production. Thus the wind producer needs to buy units from the dispatchable power producers to cover the deficit. For Δ_{is}^- the wind producer experience an excess of production, and thus need to sell additional units to the other producers to remain balance. The separation is done in order to distinguish between the two situations that can occur in the intraday market. For dispatchable producer i, selling a unit Δ_{is}^+ in the intraday market implies an extra production cost $C_i(\Delta_{is}^+)$, whereas revenue increases due to the extra amount sold, $\theta_s^+ \cdot \Delta_{is}^+$. Buying a unit in the intraday market implies an additional cost $\theta_s^- \cdot \Delta_{is}^-$, however the producer will cover some of its committed day-ahead amount and will thus get lower production cost. For the decision in the intraday market, the objective function (5.5a) aims at minimize cost minus revenue.

Using quadratic cost functions to represent the production costs require an integral of the marginal production cost in the objective function. This is to find the cost of providing the extra units Δ_{is}^+ and the production cost saved by buying the extra units Δ_{is}^- in the intraday market for producer i. The integrals necessary are displayed in term (5.1) and (5.2) respectively.

$$\int_{q_i}^{q_i+\Delta_{is}^+} MC_i(x)dx \tag{5.1}$$

$$\int_{q_i-\Delta_{is}^-}^{q_i} MC_i(x)dx \tag{5.2}$$

Solving these integrals results in a multiplication of the day-ahead decision variable q and the intraday decision variable Δ in the objective function, as presented below in (5.3) and (5.4). This leads to an instance of a non-convex objective function, and consequently the KKT-conditions cannot guarantee to give a global optimum. Thus, the solution will not be valid as it only provides one out of many, possibly infinite, equilibria. To ensure a global optimal solution when solving the MCP, the model is solved with linear cost curves. The consequences of this will be further discussed in Chapter 6.

$$\int_{q_i}^{q_i+\Delta_{is}^{+}} (d_i + 2e_i x)dx = d_i\Delta_{is}^{+} + 2e_i(\Delta_{is}^{+})^2 + 2e_i q_i \Delta_{is}^{+} \tag{5.3}$$

$$\int_{q_i-\Delta_{is}^{-}}^{q_i} (d_i + 2e_i x)dx = d_i\Delta_{is}^{-} + 2e_i(\Delta_{is}^{-})^2 - 2e_i q_i \Delta_{is}^{-} \tag{5.4}$$

5.3.2 Constraints

Constraint (5.5b) ensures that the sum of the quantity produced in the day-ahead market and the intraday market does not exceed maximum production capacity for each producer i and all scenarios $s \in S^{+}$. Constraint (5.5c) ensures that the quantity bought by producer i in the intraday market does not exceed the quantity the producer has committed the in the day-ahead market for all scenarios $s \in S^{-}$.

5.3.3 Consumers problem

The objective of the consumers is to maximize consumer surplus, i.e. the integral of the demand curve minus expenditures on electricity. In the formulation below, θ^* is the optimal market price and D is the quantity maximized by the consumer.

$$CS = \max \int_{0}^{D^*} (a - b \cdot D)dD - D^* \cdot \theta^*$$

The consumers problem will not be included when assuming Cournot competition as consumer surplus is not maximized when the producers exert market power. There are no constraints in the consumers problem.

5.3.4 Market clearing

The market-clearing constraints (5.7) impose the supply to equal demand in both the day-ahead market and the intraday market. These constraints applies for all players universally.

The prices comes from the dual variables θ, θ_s^+ and θ_s^- which represents the the equilibrium price in the day-ahead market and the equilibrium prices for selling and buying quantities in the intraday market, respectively. This implies that all the market participants are price-takers, i.e. they cannot affect the electricity price through strategic behaviour.

5.4 Model Perfect Competition

For perfect competition, the problem could be solved simpler as a regular optimization problem solving for a market operator that maximize total day-ahead surplus subject to market-clearing. This would give the same results as the MCP, and would be preferable as it is easier to solve. However, the Cournot competition problem cannot be expressed as just one optimization model because the model have multiple players optimizing their own payoffs (Gabriel et al. 2013). Thus, as it is desirable to make the two models comparable, it is chosen to model the perfect competition problem as an MCP.

Producer i

$$\min_{q_i,\Delta_{is}^+,\Delta_{is}^-} \pi = C_i(q_i) - \theta \cdot q_i + \sum_{s\in S^+} p_s \left\{C_i(\Delta_{is}^+) - \theta_s^+ \cdot \Delta_{is}^+\right\} + \sum_{s\in S^-} p_s \left\{\theta_s^- \cdot \Delta_{is}^- - C_i(\Delta_{is}^-)\right\} \tag{5.5a}$$

subject to

$$q_i + \Delta_{is}^+ \leq q_i^{max} \qquad \left(\lambda_{is}^+\right) \qquad \forall i \in I, \forall s \in S^+ \tag{5.5b}$$

$$q_i - \Delta_{is}^- \geq 0 \qquad \left(\lambda_{is}^-\right) \qquad \forall i \in I, \forall s \in S^- \tag{5.5c}$$

Consumer problem

$$\max_d \quad CS = a \cdot d - \frac{b}{2} \cdot d^2 - d \cdot \theta \tag{5.6}$$

Market-clearing

$$d = \sum_{i\in I} q_i \qquad (\theta) \tag{5.7a}$$

$$D_s^{ID} = \sum_{i\in I} \Delta_{is}^+ \qquad \left(\theta_s^+\right) \qquad \forall s \in S^+ \tag{5.7b}$$

$$D_s^{ID} = \sum_{i\in I} \Delta_{is}^- \qquad \left(\theta_s^-\right) \qquad \forall s \in S^- \tag{5.7c}$$

5.4.1 Karush-Kuhn-Tucker conditions

The model above can be solved as an MCP by replacing the optimization problem by its KKT optimality conditions. In the following, the KKT-conditions of all the producers are derived.

$$0 \leq q_i \perp d_i - \theta + \sum_{s \in S^+} \lambda_{is}^+ - \sum_{s \in S^-} \lambda_{is}^- \geq 0 \qquad \forall i \in I \tag{5.8a}$$

$$0 \leq \Delta_{is}^+ \perp p_s(d_i - \theta_s^+) + \lambda_{is}^+ \geq 0 \qquad \forall i \in I, \forall s \in S^+ \tag{5.8b}$$

$$0 \leq \Delta_{is}^- \perp p_s(\theta_s^- - d_i) + \lambda_{is}^- \geq 0 \qquad \forall i \in I, \forall s \in S^- \tag{5.8c}$$

$$0 \leq \lambda_{is}^+ \perp q_i^{max} - q_i - \Delta_{is}^+ \geq 0 \qquad \forall i \in I, \forall s \in S^+ \tag{5.8d}$$

$$0 \leq \lambda_{is}^- \perp q_i - \Delta_{is}^- \geq 0 \qquad \forall i \in I, \forall s \in S^- \tag{5.8e}$$

$$0 \leq d \perp -a + b \cdot d + \theta \geq 0 \tag{5.8f}$$

$$d - \sum_{i \in I} q_i = 0 \qquad \theta\, free \tag{5.8g}$$

$$D_s^{ID} - \sum_{i \in I} \Delta_{is}^+ = 0 \qquad \theta_s^+\, free \qquad \forall s \in S^+ \tag{5.8h}$$

$$D_s^{ID} - \sum_{i \in I} \Delta_{is}^- = 0 \qquad \theta_s^-\, free \qquad \forall s \in S^- \tag{5.8i}$$

In the following, KKT-constraints 5.8a, 5.8b and 5.8c are examined in further detail.

Assuming a positive quantity committed by producer i in the day-ahead market ($q_i > 0$), the right-hand side of constraint (5.8a) equals zero. In addition, if the constraints (5.5b) and (5.5c) are not binding ($\lambda_{is}^+ = \lambda_{is}^- = 0$), the market clearing price equals the marginal cost of producer i, i.e. $\theta = d_i$. Similarly, if producer i either sells or buys an amount in the intraday market ($\Delta_{is} > 0$) and the same constraints are not binding, the intraday price equals marginal cost of supply of producer i for all scenarios.

Contrarily, if the capacity is fully utilized, i.e. $\lambda_{is}^{+} > 0$, the value of having more capacity is expressed as λ_{is}^{+}. Moreover, the market price in the day-ahead market increases in this situation, $\theta = d_i + \sum_{s \in S^{+}} \lambda_{is}^{+}$. In the intraday market where the wind power producer has deficit production $\theta_s^{+} = d_i + \frac{\lambda_{is}^{+}}{p_s}$. In the situation where the wind producer has excess production, and if $\lambda_{is}^{-} > 0$, the market price equals $\theta_s^{-} = d_i - \frac{\lambda_{is}^{-}}{p_s}$.

5.5 Model Cournot competition in the day-ahead market

In the following, a model assuming Cournot competition is formulated. It differs from the model in Section 5.4 as the producers now can exert market power to affect the prices in the day-ahead market, i.e. they are no longer price-takers. In the objective function, the market-clearing price is thus replaced with the inverse demand function $p(Q) = a - b \cdot (\sum_i q_i)$, and the market-clearing constraints in the day-ahead market are removed, as the accepted demand is determined by the producers. The market-clearing conditions for the intraday market (5.10) remains due to the assumption in Chapter 5.1. The model provides a Nash-Cournot equilibrium.

$$\min_{q_i, \Delta_{is}^{+}, \Delta_{is}^{-}} \pi = C_i(q_i) - (a - b \sum_{i \in I} q_i) q_i$$

$$+ \sum_{s \in S^{+}} p_s \left\{ C_i(\Delta_{is}^{+}) - \theta_s^{+} \cdot \Delta_{is}^{+} \right\} + \sum_{s \in S^{-}} p_s \left\{ \theta_s^{-} \cdot \Delta_{is}^{-} - C_i(\Delta_{is}^{-}) \right\} \tag{5.9a}$$

subject to

$$q_i + \Delta_{is}^{+} \leq q_i^{max} \qquad (\lambda_{is}^{+}) \qquad \forall i \in I, \forall s \in S^{+} \tag{5.9b}$$

$$q_i - \Delta_{is}^{-} \geq 0 \qquad (\lambda_{is}^{-}) \qquad \forall i \in I, \forall s \in S^{-} \tag{5.9c}$$

Market-clearing

$$D_s^{ID} = \sum_{i \in I} \Delta_{is}^{+} \qquad (\theta_s^{+}) \qquad \forall s \in S^{+} \tag{5.10a}$$

$$D_s^{ID} = \sum_{i \in I} \Delta_{is}^{-} \qquad (\theta_s^{-}) \qquad \forall s \in S^{-} \tag{5.10b}$$

5.5.1 Karush-Kuhn-Tucker conditions

The KKT-conditions of the model above is formulated to transform the optimization problem to an MCP.

$$0 \leq q_i \perp d_i - a + b(\sum_{j \in I} q_j + q_i) + \sum_{s \in S^+} \lambda_{is}^+ - \sum_{s \in S^-} \lambda_{is}^- \geq 0 \quad \forall i \in I \tag{5.11a}$$

$$0 \leq \Delta_{is}^+ \perp p_s(d_i - \theta_s^+) + \lambda_{is}^+ \geq 0 \qquad \forall i \in I, \forall s \in S^+ \tag{5.11b}$$

$$0 \leq \Delta_{is}^- \perp p_s(\theta_s^- - d_i) + \lambda_{is}^- \geq 0 \qquad \forall i \in I, \forall s \in S^- \tag{5.11c}$$

$$0 \leq \lambda_{is}^+ \perp q_i^{max} - q_i - \Delta_{is}^+ \geq 0 \qquad \forall i \in I, \forall s \in S^+ \tag{5.11d}$$

$$0 \leq \lambda_{is}^- \perp q_i - \Delta_{is}^- \geq 0 \qquad \forall i \in I, \forall s \in S^- \tag{5.11e}$$

$$D_s^{ID} - \sum_{i \in I} \Delta_{is}^+ = 0 \qquad \theta_s^+ \ free \qquad \forall s \in S^+ \tag{5.12a}$$

$$D_s^{ID} - \sum_{i \in I} \Delta_{is}^- = 0 \qquad \theta_s^- \ free \qquad \forall s \in S^- \tag{5.12b}$$

In the following, the KKT-condition (5.11a) is studied. First, if it is assumed that a positive quantity is committed in the day-ahead market ($q_i > 0$), and that constraint (5.9b) and (5.9c) is not binding ($\lambda_{is}^+ = \lambda_{is}^+ = 0$). This implies that marginal revenue equals marginal cost. Moreover, if the shadow price on maximum capacity is positive ($\lambda_{is}^+ > 0$), the constraint becomes $a - b \cdot (\sum_j q_j + q_i) = d_i + \sum_{s \in S^+} \lambda_{is}^+$. Thus, the marginal revenue increases compared to the unbounded case. The value of having more capacity and hence be able to produce additional units is given by λ_{is}^+. If constraint (5.9c) is binding, implying that $\lambda_{is}^- > 0$, the marginal revenue becomes $a - b \cdot (\sum_j q_j + q_i) = d_i - \sum_{s \in S^-} \lambda_{is}^-$. That is, the marginal revenue decreases.

Chapter 6

Computational Study

This chapter examines the two models formulated in Section 5.4 and Section 5.5. Three cases are studied, each representing a possible future situation in terms of increased wind power share in the Nordic power system. The main purpose of the study is to gain insight into the power producers behaviour and the economic consequences as the wind share of the total power production increases. It will also be examined how behaviour under perfect competition differs from Cournot competition, and the socioeconomic consequences of the producers exerting market power.

First, Section 6.1 presents the most relevant input parameters used in the computational study. Moreover it introduces the different cases of increased wind production. The results from running the models are presented in Section 6.2, and relevant observations are stated. Finally, the main findings and results are discussed in Section 6.3.

6.1 Input parameters

The input parameters used in this study are not real data, but based on realistic values for electricity demand, production costs and production capacities in the Nordic countries.

Table 6.1 presents the parameters used to describe the cost functions and the production capacities of producer i. It is assumed that the cost function is linear, $C(q_i) = d_i q_i$.

Table 6.1: Cost function and production capacities

	Hydro	Thermal 1	Thermal 2
d_i (€/MWh)	11	23	37
$q^{max}(MWh)$	20860	12250	27000

In both models the inverse demand curve in the day-ahead market is assumed to be linear. The inverse demand function is given by

$$p(Q) = 104 - 0.0013(\sum_i q_i + Q^R)$$

The function gives the price in the day-ahead market in €/MWh. Further, as the main purpose of the study is to look into the effects of an increased wind share, the same inverse demand curve is used in all cases.

For the wind power producer, the size of the deviation from the day-ahead quantities sold is assumed to be 14% at the lowest, 26% at the highest and 20% on average. The scenarios of realized deviations are based on the root-mean-squared-error of day-ahead wind forecast for the 50Hertz control area in Germany over the period of January 1st 2015 through December 31st 2015, which was 21.67% (Forbes and Zampelli 2017). The deviation of the wind producer will constitute the bid and ask quantities in the intraday market. In this report, D^{ID} describes this deviation for a given scenario s. The set of scenarios for different possible outcomes of the uncertain parameter with their associated probabilities are represented in Appendix A.

It is looked into three cases, each representing different future states in terms of power production by the wind power producer. The base case for the study is based on the current situation in the Nordic countries where wind generators is assumed to account for 4300MWh/h in an average hour of the day. For case 1, 2 and 3, it is assumed that the average wind production will increase to 8600MWh/h, 12900MWh/h and 17200MWh/h, respectively.

6.2 Results

This section presents the results from running the model. First, it is looked at the different producers profits from the day-ahead market, both under perfect competition and Cournot competition. The consumers' and producers' surplus are found based on these results. Second, some of the scenario solutions of the intraday market are provided, which shows the behaviour in the intraday market for the market participants when different outcomes of the uncertain deviation is realized. Last, the values of the objective functions of MCP (5.8) and (5.11) are compared with the objective function values when the models is solved without taking the intraday market into account, i.e. uncertain deviation is not included as the stochastic part is removed.

6.2.1 Day-ahead results

Table 6.2 contains the results of the quantities sold in the day-ahead market, the day-ahead market price θ and the revenue of the different producers in the base case for both MCP

(5.8) and (5.11). It is observed that the total amount of electricity sold by the producers is lower under Cournot competition. This step-back is mainly caused by a decrease in Hydro's contribution, as the producer Thermal 2 has an significant increase in its contribution and Thermal 1 does not change its quantities noteworthy. The resulting market-clearing prices becomes significantly higher in the case of Cournot competition with 46.7€/MWh compared to 37€/MWh under perfect competition. The producers earn higher profits in the Cournot problem.

Table 6.2: Results of quantities committed in the day-ahead market in base case, the market price θ and the day-ahead profit

	Perfect competition		Cournot competition	
	Quantity (MWh)	Profit (1000 €)	Quantity (MWh)	Profit (1000 €)
Hydro	20860	542	20086	717
Thermal 1	12250	172	12250	290
Thermal 2	14128	0	7451	72
Total	47238	714	39787	1079
Price (€/MWh)	37		46.7	

The day-ahead results for all three cases of increasing wind share are given in Table 6.3. The same trends that is observed in the base case applies for these cases as well. That is, the producers sell less quantities in total under Cournot competition compared to perfect competition, and thus yield higher profits. Hydro and Thermal 2 sell less in the day-ahead market under Cournot competition, whereas Thermal 1 has close to constant sales. There is a significant difference in the market price under the two assumptions.

As wind share increases, the market-clearing price under perfect competition is constant in the first two cases at 37.0 €/MWh, and decreases to 34.1 €/MWh in the last case. Likewise, under Cournot competition it is observed that the day-ahead market price decreases in all cases from 44.5 €/MWh to 39.0 €/MWh in the last case. Simultaneously, the profits of all the producers decreases as wind share increases.

Table 6.3: Results MWh sold in the day-ahead market, market prices and day-ahead profits

		Perfect competition		Cournot competition	
		Quantity (MWh)	Profit (1000 €)	Quantity (MWh)	Profit (1000 €)
Case 1	Hydro	20860	542	19140	641
	Thermal 1	12250	172	12250	263
	Thermal 2	9828	0	5774	43
	Total	42938	717	37164	948
	Price (€/MWh)	37		44.5	
Case 2	Hydro	20860	542	18280	572
	Thermal 1	12250	172	12250	236
	Thermal 2	5528	0	4054	21
	Total	38638	714	34584	830
	Price (€/MWh)	37		42.3	
Case 3	Hydro	20860	482	17663	495
	Thermal 1	12250	136	11663	187
	Thermal 2	3440	-10	3440	7
	Total	36550	608	32766	690
	Price (€/MWh)	34.1		39.0	

The producers' and consumers' surpluses from the day-ahead results are displayed in Table 6.4 and 6.5, under perfect competition and Cournot competition respectively. As wind share increases, producers' surplus decreases whereas the consumers' surplus increases. Under perfect competition there is an increase in total surplus for case 3 relative to all the other cases. Contrarily, under Cournot competition an decrease in total surplus is observed from the base case until case 2, then in case 3 the total surplus increases.

Table 6.4: Surplus in the day-ahead market, Perfect competition

	Producers' surplus	Consumers' surplus	Total surplus
Base case	713860	1726538	2440398
Case 1	713860	1726538	2440398
Case 2	713860	1726538	2440398
Case 3	607865	1879234	2487099

Table 6.5: Surpluses and total surplus in the day-ahead market under Cournot competition

	Producers' surplus	Consumers' surplus	Total surplus
Base case	1079149	1263381	2342530
Case 1	948123	1361323	2309446
Case 2	829065	1464188	2293253
Case 3	689500	1622791	2312291

6.2.2 Intraday results

Considering the model under perfect competition, the results for all the scenarios where the wind producer has overestimated his or hers production outcome, i.e in scenarios $s \in S^+$, Thermal 2 covers the entire demand. That is, Thermal 2 sells a quantity in the intraday market equal to the deviation of the wind power producer. The equilibrium selling price becomes the marginal cost of the last unit produced by Thermal 2, i.e. $\theta = d_i \cdot \Delta_{is}^+$. For the scenarios where the wind power producer has excess production, and thus needs to sell this production deviation in the intraday market, Thermal 2 buys these units. In case 3, also Thermal 1 buys some units, and due to a lower marginal cost of Thermal 1 the market price in the intraday market decreases.

Under Cournot competition, the situation is different. For the scenarios where the wind producer experience deficit production Hydro supplies the quantities. The market price in the intraday market becomes the marginal cost of Hydro. In some scenarios, also Thermal 2 supplies some quantities in the intraday market. The resulting market price in the intraday market in this situation increases to the marginal cost of Thermal 2. For almost all the scenarios where the wind producer experience excess production, Thermal 2 buys the entire amount in the intraday market. In these scenarios, the market price becomes the marginal cost of Thermal 2. Moreover, in some scenarios of case 3 Thermal 1 also buys some quantities in the intraday market. Consequently, the market price decreases.

In Table 6.6 the objective function values of Hydro and Thermal 2 is calculated. Moreover, it is compared to the model solved only for the day-ahead market, the model is found in Appendix A. The table shows that all producers gains a higher value under Cournot competition compared to under perfect competition. Regarding perfect competition, the value of the objective function are higher for Hydro when the intraday market is taken into account. The opposite happens for Thermal 2, that experiences a lower value. Likewise, under Cournot competition Hydro experience higher value and Thermal 2 lower value.

Table 6.6: Comparison of objective function values with and without clearing in the intraday market for Hydro and Thermal 2. Values are given in 1000 €.

		Perfect competition		Cournot competition	
		With intraday	Without intraday	With intraday	Without intraday
Hydro	Base case	542	542	724	734
	Case 1	542	542	648	676
	Case 2	542	542	592	617
	Case 3	482	542	512	567
Thermal 2	Base case	0	0	72	65
	Case 1	0	0	43	31
	Case 2	0	0	21	10
	Case 3	0	0	17	3

Table 6.7: Calculated differences of objective function values presented in Table 6.6. Values are given in 1000 €.

		Value of intraday tradings	
		Perfect	Cournot
Hydro	Base	0	-10
	1	0	-28
	2	0	-25
	3	-60	-55
Thermal 2	Base	0	7
	1	0	12
	2	0	11
	3	0	14

6.3 Discussion

In this section, the results in Section 6.2 are discussed. The decisions and objective values are analyzed, and the reliability of the results are examined.

6.3.1 Day-ahead

A trend in the results is that the total committed quantity by the producers is lower under Cournot competition compared to perfect competition, and this applies for all the cases. The producers choose their quantities sold to obtain the market price that yields the highest profit for all the producers and. This result is a Nash-Cournot equilibrium. At the most, Hydro reduces its quantity contribution with 15%, compared to perfect competition and in total a lower demand is satisfied.

Considering the results in Table 6.2 and 6.3, the market price in the day-ahead market decreases as the wind share increases under perfect competition as well as under Cournot competition. This is consistent with the effect of the shift of the merit order curves described in Figure 2.3 of Chapter 2.

It can be observed for all the cases under perfect competition that Hydro and Thermal 1 supplies their full capacity in the day-ahead market. As the cost curves for production cost are linear, and the marginal cost constant, the quantities will be allocated to the producer with the lowest marginal cost until it has reached its maximum capacity and so on until demand is met. It can be discussed whether the linear cost curves gives a realistic representation of the production cost of the producers. This applies especially for Hydro as it ignores the option value of water. Quadratic cost curves might reflect this better, however, it makes the solution of the MCP invalid, as discussed in Chapter 5. This was not further studied in this report.

Since the increased production by the wind producer is produced at zero marginal costs, offering an increased amount in the day-ahead market shifts the supply curve of the producers to the right, as illustrated in Figure 2.3. The effect of this shift is two-folded (Selasinsky 2016). First, some power plants are shifted behind the intersection of the demand and supply curve, and thus are no longer dispatched. This is especially relevant for the producer Thermal 2 that has the highest marginal cost of production. Often these producers are positioned around the intersection of the two market curves. Second, the right shift of the market curves lowers the market clearing price in the day-ahead market. This means that consumers of electricity have to pay less for their electricity, so there is a gain in consumers' surplus, as observed in the results. Regarding the supply side, the producers' surplus is observed to be decreasing, as they earn less profit due to the lowered market price. Selasinsky (2016) argues that this situation may mean a loss in flexibility because there are fewer producers with controllable power plants. This consequence is relevant for ensuring system stability.

Under perfect competition, consumers' surplus equals in the base case and for Case 1 and 2, whereas there is an increase in the surplus found in Case 3. Opposite, the producers' surplus is lower in Case 3 compared to the other cases. Overall, the total surplus increases in Case 3, this is due to consumers' surplus increase more than the decrease in producers' surplus.

In the situation of Cournot competition, the results presented in Table 6.5 indicates that as more wind share is introduced into the power system the total surplus decreases with respect to the base case. That is, the decrease in the producers' surplus is higher than the increase in the consumers' surplus. Thus the positive effect of introducing a larger wind share in terms of increased social welfare, as found under perfect competition, is not apparent under Cournot competition. This observation illustrates the consequences of market participants exerting market power, causing the social welfare to be worse off. A small increase is observed from Case 2 to Case 3, which could indicate that an even higher wind share could result in an increase in total surplus with respect to the base case.

Comparing the two competition situations, Cournot competition yields an overall higher producers' surplus, while perfect competition yields the highest consumers' surplus. However, the overall surplus is found to be higher under perfect competition than Cournot competition, and at most, the surplus is reduced by 6% when producers exert market power. This fall is due to the increased day-ahead market price that has been observed under Cournot competition. This is also termed as *the dead-weight loss*, i.e. loss in economic efficiency that can occur when market equilibrium is not reached. Thus, under Cournot competition the society is worse off considering social welfare. Such a result could give incentive for market regulators to use resources to uncover market participants exerting market power.

6.3.2 Intraday

First, it is looked into the results under perfect competition. For all of the scenario realization of the uncertain deviation in Case 1 and 2, Thermal 2 settles all the imbalances by supplying and buying the deviating quantities. This is due to two reasons. Hydro and Thermal 1 have already used all their production capacities in the day-ahead market, as previously discussed, and can thus not provide any extra quantities. Also, Thermal 2 has the highest marginal cost of production and thus has the most to earn by buying quantities in the intraday market to offset production cost. This is illustrated in Figure 6.1. The resulting prices in the intraday market are equal in all the scenario solution and also equivalent to the corresponding day-ahead market price of 37€/MWh. This equals the marginal production cost for Thermal 2, and thus zero profits are earned from the intraday tradings. This result indicates that Thermal 2 is indifferent to whether he participates in the intraday market or not. This is especially clear observing the results for Thermal 2 in Table 6.2.2, where the value of the objective function is zero both when the intraday decision is included and not. In Case 3, both Thermal 1 and Thermal 2 buys intraday units for the scenarios with the highest deviation. Also here, the results indicates that the market participants are indifferent to the intraday tradings.

Looking into the results of the intraday market under Cournot competition, Hydro supplies most of the balancing quantities in all the scenarios where the wind producer has deficit production. Compared to the situation under perfect competition, Hydro does not use all its production capacity in the day-ahead market and can thus sell the additional units in the intraday market. For a positive deviation from the wind power producer, Thermal 2 buys the majority of this quantity as under perfect competition.

Table 6.6 presents the value of the intraday tradings in the objective function. For the situation of Cournot competition, Hydro yields a higher value when the model is solved *without* clearing in the intraday market. This could partly be explained by the fact that by demanding clearing in the intraday market, the price in the intraday market is lower than the price obtained in the day-ahead market. Other factors from the equilibrium solution could also contribute to such an result. Contrarily, Thermal 2 receives a higher value when the model is solved with intraday clearing. This could partly be explained by the fact that Thermal 2 can trade units

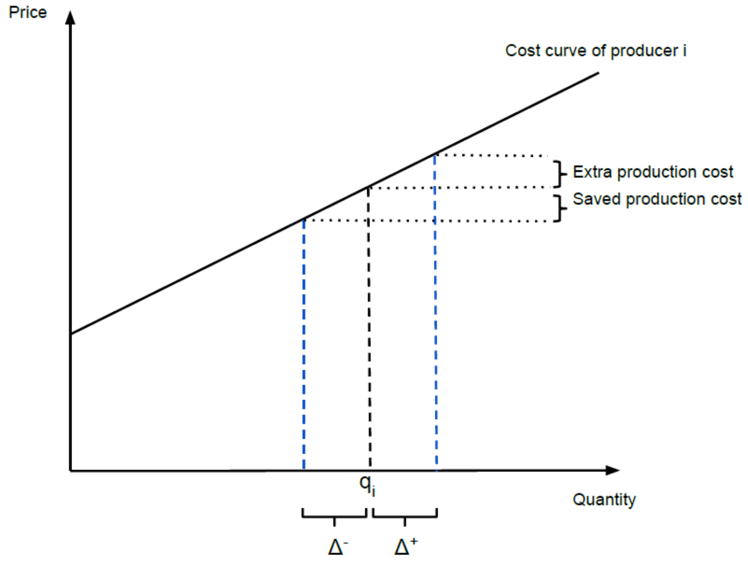

Figure 6.1: Illustration of extra production cost due to selling intraday quantity Δ^+ and saved production cost from buying intraday quantity Δ^-

in the day-ahead market and receive a high price, then buy these units in the intraday market at a price below his marginal cost, and thus earn this difference. Other factors from the equilibrium solution may also play a role.

One can question whether using market clearing constraints is a good representation for the intraday market tradings. In the case of deficit production, balancing the forecast error requires the producers to sell the missing quantities to the wind producer in the intraday market. In a realistic setting, the producers would ask a price above their true valuation for the quantities they sell. Whereas for an underestimation of production from the wind producer, that requires selling the excess production, the producers would bid a price below their true valuations for the quantities they buy (Selasinsky 2016). These price relations are not observed in the model, mainly due to the fact that the price is determined by the dual variable of market-clearing constraint. Also, these constraints make the market more like a balancing market, than a intraday market in practice. The observed prices in the intraday market are caused by shifts in the supply curve due to the different realizations of deviation from actual production by the wind producer in the intraday market. Moreover, the linear cost curves produce a constant marginal cost, thus the prices is constant and equal to the marginal cost of production.

Another important aspect is the state of the liquidity. In the Computational Study, the intraday market is assumed to be a liquid market, whereas in reality the liquidity is significantly low. That is, the risk that the producers are facing by participating in the intraday are not reflected in the model. If the risks are incorporated in the model, it may cause the results to be different.

Chapter 7

Concluding Remarks

In this report, a deterministic equivalent of a stochastic two-stage equilibrium model is formulated for the day-ahead and intraday market. Previously, equilibrium modelling has not been used in the context of modelling these markets together. Modelling the two markets as an equilibrium model allows for solving the market as a whole, which is especially useful when modeling Cournot competition. However, some limitations were discovered in terms of capturing the interactions between the two markets.

The model solves the dispatchable producers problem of selling quantities in the day-ahead market facing an uncertain bid and ask quantity from wind producers in the intraday market. A study of three cases, each representing a different wind share of power production, has been carried out, both under perfect competition and Cournot competition.

Much time has been devoted to adapt the model to the application area, especially due to the stochastic nature of the deviations from the wind producers. The problem of solving a stochastic MCP is a field mostly unstudied until recently (Gabriel et al. 2013). Due to time limit, the models are only solved for one instance of input parameters and it would be hard to draw any conclusions regarding the value of the intraday market. However, some trends are observed.

It is found that the total surplus in the day-ahead market increases as wind share increases under perfect competition. In the Cournot situation, the total day-ahead surplus varies in the different cases. At most the total surplus is reduced by 6% compared to under perfect competition. Thus, under Cournot competition the society is worse off when considering social welfare. This result could give incentive for market regulators to use resources to uncover market participants exerting market power.

Under perfect competition, the results indicate that the producers are indifferent to whether or not they must trade in the intraday market. They most often get an intraday price equal to their marginal cost of production, and thus do not gain any profit from the intraday tradings. Under Cournot competition, Hydro and Thermal 2 gain some additional profit by participating in the intraday market. Based on a comparison of the objective function with and without facing the uncertain deviation, it is found that overall, Hydro would be better off not having to settle the imbalances in the intraday market. Thermal 2 is indifferent under perfect competition, whereas the results indicates that it would be preferable for Thermal 2 to be able to participate in the intraday market when there is Cournot competition in the day-ahead market. It must be emphasized that these results are based on intraday prices that are unrealistic. This issue is discussed in the following paragraph.

It is found that using global intraday prices from the market clearing constraints does not give a good representation of the intraday prices compared to real life observations. By obligating the producers to settle the intraday imbalances, they trade these quantities even if they potentially do not have an incentive to do so in terms of improving their profit. Based on these observations, it is suggested to solve the intraday pricing in another way to make the intraday market representation more realistic. This is further discussed under Future Research in Chapter 8.

Chapter 8

Future Research

In the model provided in this report, the decisions taken in the day-ahead market are limited to only consider trades regarding one hour of the following day. Also, the intraday market is limited to the last hour before actual delivery, i.e. one hour before intraday market closure. The market clearing conditions for the intraday market reduces the market to a balancing market in practice. Altogether, the model in this report is not a realistic representation of the intraday market, but rather a starting point for modeling the effects the uncertainty has on the day-ahead market. For further research, the model could be extended to include time steps and thus allow for tradings taking place at all hours between day-ahead market closure and intraday market closure. Then, further work could examine how the producers alter their production plan in the hours up to actual delivery.

A possible extension of the model in future research is to examine the use of quadratic cost curves in the model instead of linear cost curves as used in this report. The implementation of quadratic cost curves would require additional use of algorithms to solve a non-convex MCP. It could be interesting to examine the effects this could have on the results compared to the results found in this report.

Another possible model extension is to include the flow between regions in the power system. As it is today, the market participants in the electricity market also have to consider flow capacities when trading in different regions. In the intraday market trades between regions will only be possible if there is enough allocated capacity between the regions. One could argue that this could be another area for strategic behaviour among market participants who participate in the day-ahead and intraday market. This extension could make the model more realistic.

Based on the assumptions made in Chapter 5.1, the wind power producer is not included as a decision maker in the model, but their contribution is rather represented through input parameters. It could be interesting to include the wind producer as an agent in the model, and possibly also as a price-setting agent as it in fact the agent with the highest interest of settling imbalances in the intraday market.

In this report, the variable representing the quantity traded in the intraday market (Δ_{is}) is split into a positive variable (Δ_{is}^{+}) and a negative variable (Δ_{is}^{-}). One could question if this is the best approach when it comes to the scalability and this has not been analyzed in this report. It would therefore be interesting to test another approach that is to treat the variable as free, i.e. let Δ_{is} represent both selling and buying quantities in the intraday market. It would be relevant to compare these two formulations with respect to the scalability.

A major challenge in this report has been to estimate the parameters of the cost functions. Therefore, a possible future research could be to develop an inverse equilibrium model. Bertsimas, Gupta, and Paschalidis (2015) define inverse optimization as the problem of finding the cost function or other problem data, given a candidate solution to a optimization problem, that makes that solution (approximately) optimal. The field of inverse equilibrium modelling is to a large extent unstudied.

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Appendix A

Appendix

Table A.1: Input parameters of the scenarios and realized deviations in the intraday-market

Scenario	Probabilities	Base case (MWh)	Case 1 (MWh)	Case 2 (MWh)	Case 3 (MWh)
1	0,05	602	1204	1806	2408
2	0,1	774	1548	2232	3096
3	0,2	860	1720	2580	3440
4	0,1	946	1892	2838	3784
5	0,05	1118	2236	3354	4472
6	0,05	602	1204	1806	2408
7	0,1	774	1548	2232	3096
8	0,2	860	1720	2580	3440
9	0,1	946	1892	2838	3784
10	0,05	1118	2236	3354	4472

```
Model perfect competition
sets
i suppliers /1*3/
s_pos scenarios /1*5/
s_neg scenarios /1*5/;
alias (i,j);

parameters
d(i)             cost function                  /1 11, 2 23, 3 37/

a                inverse demand DA              /104/
b                inverse demand DA              /0.0013/

q_max(i)         max capacity                   /1 20860, 2 12250, 3
27000/

p_pos(s_pos)     probability overestimation     /1 0.05,2 0.1, 3 0.2, 4
0.1, 5 0.05/
D_ID_pos(s_pos)  demand intraday                /1 2408, 2 3096, 3 3440,
4 3784, 5 4472/

p_neg(s_neg)     probability overestimation     /1 0.05,2 0.1, 3 0.2, 4
0.1, 5 0.05/
D_ID_neg(s_neg)  demand intraday                /1 2408, 2 3096, 3 3440,
4 3784, 5 4472/

Q_R              production renewables          /17200/;


*------------------------------------------------------------------------
--------
*                                  VARIABLES
*------------------------------------------------------------------------
--------
positive variables
q(i)
delta_pos(i,s_pos)
delta_neg(i,s_neg)
lambda_pos(i,s_pos)
lambda_neg(i,s_neg)
d_consumer;

variables
theta_pos(s_pos)
theta_neg(s_neg)
theta;

*------------------------------------------------------------------------
--------
*                                  EQUATIONS
*------------------------------------------------------------------------
--------

equations
production(i), adjust_pos(i,s_pos), adjust_neg(i,s_neg),
capacity_pos(i,s_pos),
capacity_neg(i,s_neg), marketID_pos(s_pos), marketID_neg(s_neg),
marketDA,
consur;
```

```
production(i).. d(i) - theta + sum(s_pos,lambda_pos(i,s_pos)) -
sum(s_neg,lambda_neg(i,s_neg)) =g= 0;

adjust_pos(i,s_pos).. p_pos(s_pos)*(d(i)  - theta_pos(s_pos)) +
lambda_pos(i,s_pos) =g= 0;

adjust_neg(i,s_neg).. p_neg(s_neg)*(theta_neg(s_neg) - d(i)) +
lambda_neg(i,s_neg) =g= 0;

capacity_pos(i,s_pos).. q_max(i) - q(i) - delta_pos(i,s_pos) =g= 0;

capacity_neg(i,s_neg).. q(i) - delta_neg(i,s_neg) =g= 0;

consur.. -a + b*d_consumer + theta =g= 0;

*------------------------------------------------------------------------
--------
*                                  Market clearing
*------------------------------------------------------------------------
--------

marketDA.. d_consumer - Q_R - sum(i,q(i)) =e= 0;

marketID_pos(s_pos).. D_ID_pos(s_pos) - sum(i,delta_pos(i,s_pos)) =e= 0;

marketID_neg(s_neg).. D_ID_neg(s_neg) - sum(i,delta_neg(i,s_neg)) =e= 0;



*------------------------------------------------------------------------
--------
*                  MODEL
*------------------------------------------------------------------------
--------

model stochastic /production.q,
adjust_pos.delta_pos, adjust_neg.delta_neg,
capacity_pos.lambda_pos,
capacity_neg.lambda_neg,
marketID_pos.theta_pos, marketID_neg.theta_neg, marketDA.theta,
consur.d_consumer/;


solve stochastic using mcp;
```

```
Model Cournot competition in the day-ahead market
sets
i suppliers /1*3/
s_pos scenarios /1*5/
s_neg scenarios /1*5/
alias (i,j);

parameters
d(i)             cost function                   /1 11, 2 23, 3 37/

a                inverse demand DA               /104/
b                inverse demand DA               /0.0013/

q_max(i)         max capacity                    /1 20860, 2 12250, 3
27000/

p_pos(s_pos)     probability overestimation      /1 0.05,2 0.1, 3 0.2, 4
0.1, 5 0.05/
D_ID_pos(s_pos)  demand intraday                 /1 2408, 2 3096, 3 3440,
4 3784, 5 4472/

p_neg(s_neg)     probability overestimation      /1 0.05,2 0.1, 3 0.2, 4
0.1, 5 0.05/
D_ID_neg(s_neg)  demand intraday                 /1 2408, 2 3096, 3 3440,
4 3784, 5 4472/

Q_R              production renewables           /17200/
;


*--------------------------------------------------------------------------
--------
*                                   VARIABLES
*--------------------------------------------------------------------------
--------
positive variables
q(i)
delta_pos(i, s_pos)
delta_neg(i, s_neg)
lambda_pos(i, s_pos)
lambda_neg(i, s_neg)
;


variables
theta_pos(s_pos)
theta_neg(s_neg);

*--------------------------------------------------------------------------
--------
*                                   EQUATIONS
*--------------------------------------------------------------------------
--------

equations
production(i), adjust_pos(i, s_pos), adjust_neg(i,s_neg),
capacity_pos(i,s_pos)
capacity_neg(i,s_neg), marketID_pos(s_pos),marketID_neg(s_neg);
```

```
production(i).. d(i) - a + b*(sum(j,q(j)) + q(i) + Q_R) +
sum(s_pos,lambda_pos(i,s_pos)) - sum(s_neg,lambda_neg(i,s_neg)) =g= 0;

adjust_pos(i,s_pos).. p_pos(s_pos)*(d(i) - theta_pos(s_pos)) +
lambda_pos(i,s_pos) =g= 0;

adjust_neg(i,s_neg).. p_neg(s_neg)*(theta_neg(s_neg) - d(i)) +
lambda_neg(i,s_neg) =g= 0;

capacity_pos(i,s_pos).. q_max(i) - q(i) - delta_pos(i,s_pos) =g= 0;

capacity_neg(i,s_neg).. q(i) - delta_neg(i,s_neg) =g= 0;


*-------------------------------------------------------------------------
--------
*                                  Market clearing
*-------------------------------------------------------------------------
--------


marketID_pos(s_pos).. D_ID_pos(s_pos) - sum(i,delta_pos(i,s_pos)) =e= 0;

marketID_neg(s_neg).. D_ID_neg(s_neg) - sum(i,delta_neg(i,s_neg)) =e= 0;


*-------------------------------------------------------------------------
--------
*                  MODEL
*-------------------------------------------------------------------------
--------

model stochastic /production.q,
adjust_pos.delta_pos, adjust_neg.delta_neg,
capacity_pos.lambda_pos,
capacity_neg.lambda_neg,
marketID_pos.theta_pos,
marketID_neg.theta_neg/;

solve stochastic using mcp;
```

```
Model day-ahead only, Cournot competition
sets
i suppliers /1*3/
alias (i,j);

parameters
d(i)             cost function                  /1 11, 2 22,3 37/

a                inverse demand DA              /104/
b                inverse demand DA              /0.0013/

q_max(i)         max capacity                   /1 50860, 2 12250, 3
27000/

Q_R              production renewables          /17200/
;


*------------------------------------------------------------------------
--------
*                                  VARIABLES
*------------------------------------------------------------------------
--------
positive variables
q(i)
lambda(i)
;

*------------------------------------------------------------------------
--------
*                                  EQUATIONS
*------------------------------------------------------------------------
--------

equations
production(i), capacity(i);

production(i).. d(i) - a + b*(sum(j,q(j)) + q(i) + Q_R) + lambda(i) =g=
0;

capacity(i).. q_max(i) - q(i) =g= 0;


*------------------------------------------------------------------------
--------
*                  MODEL
*------------------------------------------------------------------------
--------

model stochastic /production.q,
capacity.lambda/;

solve stochastic using mcp;
```

```
Model day-ahead only, perfect competition
sets
i suppliers /1*3/
s_pos scenarios /1*5/
s_neg scenarios /1*5/;
alias (i,j);

parameters
d(i)             cost function                  /1 11, 2 22,3 37/

a                inverse demand DA              /104/
b                inverse demand DA              /0.0013/

q_max(i)         max capacity                   /1 50860, 2 12250, 3
27000/

Q_R              production renewables          /17200/
;

*------------------------------------------------------------------------
--------
*                                  VARIABLES
*------------------------------------------------------------------------
--------
positive variables
q(i)
lambda(i)
d_consumer;

variables
theta;

*------------------------------------------------------------------------
--------
*                                  EQUATIONS
*------------------------------------------------------------------------
--------

equations
production(i), capacity(i), marketDA,
consur;

production(i).. d(i) - theta + lambda(i) =g= 0;

capacity(i).. q_max(i) - q(i) =g= 0;

consur.. -a + b*d_consumer + theta =g= 0;

*------------------------------------------------------------------------
--------
*                                  Market clearing
*------------------------------------------------------------------------
--------

marketDA.. d_consumer - Q_R - sum(i,q(i)) =e= 0;


*------------------------------------------------------------------------
--------
*                  MODEL
```

```
*-------------------------------------------------------------------------
--------

model stochastic /production.q,
capacity.lambda,
marketDA.theta,
consur.d_consumer/;

solve stochastic using mcp;
```