Evaluation of Hydropower Upgrade Projects

Preface

This master thesis was carried out during the spring of 2008 at the Norwegian University of Science and Technology (NTNU), Department of Industrial Economics and Technology Management. The thesis was conducted on the basis of a case generously provided by Hydro.

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Abstract

The purpose of this paper is to develop an investment decision support framework for hydropower producers with aged production facilities, where one has the option to choose between refurbishing existing plants, and investing in production expansion. Aged production facilities normally need a significant amount of refurbishment in order to both maintain production capacity and comply with government regulations. Due to minimization of outage time, refurbishment needs will be aggregated and carried out at specific planned points in time. Because of the significant cost associated with such maintenance the question that arises is whether refurbishment should be carried out or if investments should be made to increase capacity. An increase in production capacity is interesting both because of higher efficiency, and because it enables the producer to produce more when prices are high. The analyses are done assuming liberalized market conditions where the producer is price taker and can time and make irreversible investments in return for uncertain cash flows.

A real options framework is proposed to support investment decisions, and a simulation model is developed to produce inputs to the analysis. Using a case from Hydro, a Norwegian hydropower producer, we employ the simulation model to estimate the required parameters. The real option analysis produces optimal exercise boundaries together with the option value. As such we analyze the optimal timing of capacity upgrades.

Contents

1 Introduction

The Norwegian Water Resources and Energy Directorate states that the potential in refurbishment and expansion of large existing power plants is estimated to around 18 TWh/year, or about 15 % of the total average Norwegian production (NVE, 2006). They also claim that refurbishing older plants to improve efficiency seems economically unattractive due to profit loss caused by outage times and large investment cost. It is therefore assumed that such refurbishments will be done in conjunction with capacity expansions (NVE, 2005).

Large investment costs, long asset lives and uncertainty in future revenues make investments decisions in the hydropower industry challenging. The future electricity prices are an important factor for the future revenues. The Nordic electricity market has been increasingly integrated with the continental market by means of new cable connections, affecting the Nordic electricity prices. Market integration is driven by price differences between the markets, and it is generally expected that the integration process will continue as long as these differences prevail and that they lie above the cost of new transmission capacity. The effect of the integration process is uncertain, but studies indicate that the Nordic prices will to some extent adapt the continental price pattern.

One important feature of the continental market is the high degree of intraday price variation compared to the Nordic market. To illustrate, a peak load contract for 2009 on the German EEX is traded for €104(European Energy Exchange AG, 2008), while the same contract on Nord Pool is traded for €63(Nord Pool, 2008). The increasing intraday price variations are interesting for hydropower producers in the Nordic market due to the excellent load variation capabilities of hydropower. The fact that many hydropower plants in operation today originally were designed for base load generation, the possibility to exhaust increasing intraday price variation to increase profits will be limited, unless production capacity is increased. After expansions, producers can capture eventual higher peak load prices, avoid production at low prices and thus increase total profits. In addition, new renewable electricity production, such as wind power, is expected to increase the total demand for regulative power.

Even though capacity expansion projects in the hydropower industry appear attractive, it seems that investors hesitate to realize projects, unless profits are significant. In general, there are three main factors affecting the investment decisions. First, there is uncertainty regarding the future cash flows. Second, the investments are (at least partly) irreversible. In addition, the investors have the opportunity to time investments. As a result there is an opportunity cost related to the execution of a project (Dixit & Pindyck, 1994).Thus, the reason for this behavior is the high option value, i.e. due to great uncertainty and high and irreversible investment costs, the value of delaying investments often exceeds value of investing immediately. In addition, construction of large plants often implies long outage time for existing plants, resulting in considerable profit loss.

In this paper we analyze a case provided by the Norwegian hydro power producer Hydro in which an aged production facility is in need of extensive refurbishment. Discharge capacity expansions are considered as an alternative to refurbishment.

We use a real option approach to capture the value of expansion options when large refurbishments on existing plants are to take place. Such real options can be regarded as American style options. Often the producers have several alternative options to choose from which complicates the analyses. Kay, Davison and Rasmussen (2005) investigate the early exercise region for American options on multiple underlying assets. Andricopoulos, Widdicks, Newton, & Duck(2003) propose a numerical approach for option pricing based on the quadrature method. They prove that the quadrature method is advantageous compared to more traditional methods because of greater accuracy and faster convergence. They follow up their paper by extending the quadrature method to value multi-asset and path dependent options (Andricopoulos, Widdicks, Newton, & Duck, 2007). Broadie and Detemple (1997) describe the optimal exercise regions for a number of American options on multiple assets.

To calculate the value of ordinary financial options one needs the market value of the underlying asset and the corresponding future dynamics. Real projects are not traded assets in complete markets, and cannot easily be replicated. Various approaches exist to solve or circumvent this problem. In this paper the marketed asset disclaimer (MAD) assumption is made. The essence of this method is that instead of searching in financial markets for replicating portfolios one can use the present value of the project itself as if it were a marketed security (Copeland & Antikarov, 2003).

Real option valuation techniques require estimation of the static present values of the projects with corresponding future dynamics of the present values. To calculate the present values of the projects one needs to estimate the exceeding cash flow they generate. If a replacement of old plants with new plants are current investment projects, entire production plans need to be worked out and accompanying cash flows estimated. Keppo and Näsäkkälä (2005) have developed a way to determine a production strategy depending on electricity forward prices, water reservoir level, inflow and time of year.

We find that the earning capabilities of the alternatives differ considerably, where high capacity installations have the highest earning capability and has the highest volatility associated. The exercise boundaries are found for all points in time, and the optimal strategy is thus established. We also find that refurbishment dates of older plants are decisive for the optimal strategy. Comprehensive refurbishments tend to trigger investment.

The outline of the paper is as follows: Following this introduction, section 2 briefly discusses the theoretical background for the chosen approaches in this paper. In section 3 a concrete investment case given by the Norwegian hydropower producer Hydro is presented. Then we formulate a production simulation model and an investment model for the case at hand. Section 5 describes the implementation of these models. In section 6 a presentation of the empirical results will follow, with a subsequent thorough analysis of the results, before the paper is brought to conclusion. In section 9 we propose some further work.

2 Theoretical background and current approaches

2.1 Hydropower

Most large hydropower plants use reservoirs to store water which enables them to allocate production over time. When water is releases from the reservoir through turbines potential energy caused by the plant's head is utilized to generate electricity. Hydropower has an excellent capability to cheaply adjust generation compared to thermal generation and is therefore well suited to respond to sudden changes in demand.

2.2 The electricity market

The Norwegian power market was deregulated in 1991. This was later followed up by the other Nordic countries. Today electricity is traded on the Nordic Power Exchange, Nord Pool. The Nordic market has a high share of hydro power generation, and is increasingly integrated with the continental market. The electricity prices on the continent have more price structure (Troland & Elverhøi, 2007) compared to the Nordic market, meaning that the intraday prices on the continental market vary more than those on the Nordic market. This is mainly due to two factors. At first, intraday demand varies significantly, with high demand during daytime and lower demand during nighttime. Second, the supply side is dominated by thermal generation, which is very expensive to regulate. Easily adaptive hydropower generation is sought after on the continent, and several new interconnections are planned (Energipartner AS, 2007).

It is challenging to quantify the effect of more interconnections on the Nordic market. A study conducted by Hydro indicates more price structure in the Nordic market, for a given set of new interconnections (Torgersen, 2008). For hydropower producers the increasing market integration represents possibilities to increase profit, given their excellent regulatory abilities. Several of the older power plants today are constructed to generate base load electricity, and an increase in discharge capacity would provide a greater opportunity to take advantage of the varying prices.

2.3 Production planning

Production planning is a vital discipline, as it forms the basis of the profits of the producer. The general objective of production planning is to satisfy electricity demand and maximize profits. The cost of water is zero. However, since the water is a limited resource, it has a value. Producers therefore calculate a water value, which can be regarded as an opportunity cost, in order to determine a production plan. The aim is, in every period, to maximize the profits from water release during the period plus the expected value of the remaining water in the reservoir at the end of the period (Tipping, Grant Read, & McNickle, 2004). The water value is dependent on several factors, such as transmission capacity, reservoir size and level, cost of thermal generation, expected future prices, expected precipitation and contractual obligations.

Production planning is divided into three categories; long term planning, seasonal planning and short time planning. Long term planning of large reservoirs has a time horizon of 3-5 years. A prerequisite for long term planning is that the reservoir is sufficiently large to be able to store water from one year to another. When the reservoirs are smaller, seasonal planning is the main focus, with the main objective to release water when most profitable and avoid spillage. Short time scheduling has time ranging horizons from a couple of hours up to two weeks, depending on the system and the coupling to the seasonal scheduling. The main objective for the short time scheduling is to determine the physical operation plan of the system(Doorman, 2007). Investment decisions require considerably longer time horizons. For small hydropower plants and thermal generation plants it is common to operate with physical lifetimes of up to 30 years. Large hydropower plants have physical lifetimes of 60 years (NVE, 2003).

Most major producers in Norway employ the EOPS (One-area Power-market Simulator) and the EMPS (Multi-area Power-market Simulator) in their long term scheduling, maintenance planning and investment planning (Sintef, 2008). The generation scheduling is based on calculation of incremental water values using a bottom-up approach and stochastic dynamic programming. However, the use of the models is limited by a time horizon of 10 years (Sintef, 2008). For additional decision support the producers employ long term scenario analysis for future price levels, price structures, climate changes and transmission capacity expansions to capture the uncertainty related to the decision.

An alternative approach for production planning based on intuitive water value calculations has been developed by Keppo and Näsäkkälä(2005). They have developed a production threshold based on information in the electricity derivative markets, reservoir level, inflow and time. The threshold acts as marginal water value (MWV) and can be interpreted as the price the producer has to pay to release water from the reservoir. The producer will only generate when the benefits of releasing water exceeds the MWV. Further, they have constructed a production strategy for a profit maximizing producer using information in the forward market, noting that electricity markets are incomplete due to the fact that electricity prices and inflow are not perfectly correlated. Therefore, the production process cannot be perfectly hedged in the derivative markets. The aim of their paper is to develop a hedging strategy, and use the probability that the future spot price is greater than the threshold function to determine this strategy. The intuitive characteristics of the production threshold can be summarized as follows (Keppo & Näsäkkälä, 2005):

- *As the reservoir at the end of the planning horizon is zero, the production threshold must converge to zero as the time approaches the end of the planning period*
- *The probability of spillage increases as the water level increases, and thus the threshold must decrease as a function of water level*
- *Similarly for the future inflow, if the future inflow estimates increase the threshold decreases*
- *If the forward curve increases, the value of waiting increases. Thus, the production threshold increases as the future electricity prices increase*

They formalize these characteristics by a parameterization of the following form:

$$K\big(t, x(t), v(t), s(t)\big) = \propto_s s(t) e^{\left(-\propto_x x(t) - \propto_v v(t) - \frac{\propto_t}{\tau - t}\right)} \qquad \forall t \in [0, \tau] \tag{1}$$

Here $v(t)$ and $s(t)$ represent the average future inflow estimates and the average future forward curve respectively. They assume that the threshold decreases exponentially at a rate of $\propto_t$ as time approaches zero. τ is the planning horizon. In the same fashion, the parameters $\propto_x$ and $\propto_v$ indicate the rate of decrease in the threshold as functions of water level and future inflow estimates respectively. A linear decrease in the threshold as function of the average future forward curve is given by $\propto_s$. For more details concerning this production threshold we refer to Keppo & Näsäkkälä (2005).

Tipping, Grant Read and McNickle (2004) incorporate the hydro storage level into a spot price model used in MWV calculations for a hydropower dominated market. This is based on the intuition that the observed storage level and the spot price are negatively correlated. They assume that the median historic hydro storage trajectory is a good proxy for the optimal trajectory under normal inflow conditions, and define the MWV as a function of the relative storage level, defined in terms of deviation from the expected trajectory. They point out that if inflow can be more or less anticipated, the MWV will change to compensate for this through water release, and consequently the optimal trajectory will tend to parallel the expected trajectory either at a higher or lower level markets (Tipping, Grant Read, & McNickle, 2004). It is shown that water storage levels improve the accuracy of existing spot price models. Other factors influencing production strategy mentioned, but not considered, are load, contracts and flow requirements.

2.4 Real options

As mentioned earlier, we will employ a real options analysis for decision support. The aim is to find the optimal timing and size of an investment. We omit a thorough discussion of basic real option theory, instead some frequently used expressions and the link to the hydropower industry is established.

2.4.1 Real options in general

The real option approach (ROA) stems from financial options valuation. According to Copeland and Antikarov (2001), a real option is "the right, but not the obligation, to take an action at a predetermined cost called the exercise price, for a predetermined period of time – the life of the option". The traditional net present value (NPV) approach fails to capture irreversibility and value of the possibility to postpone a project, in which the ROA succeeds (Dixit & Pindyck, 1994).

There are many different styles of options. In this paper, we employ European options, American options and Bermudan options. The holder of a European call (put) option has the right to buy (sell) an asset at a predetermined strike price at a predetermined time, the expiry date. If the option is American, the holder can exercise the option at any time until expiration. A Bermudan option gives the holder the right to exercise at a predetermined set of discrete exercise points until the option expires. Options with multiple underlying assets are usually referred to as rainbow options.

The American feature of most real options complicates the valuation procedure and rule out closed form analytical solutions (Sullivan, 2000). Numerical approaches and semi-analytical approaches have been developed. For an American option on a dividend paying underlying asset there exist a boundary for which early exercise is optimal, termed the free boundary or exercise boundary. With options on two or more underlying assets the problem of finding the free boundary is further complicated (Broadie & Detemple, 1997). This is because delaying investment enables the holder to capture the gain if one project dominates the others in the future. This gain may be large enough to offset the benefit of immediate exercise of options deep in the money (Broadie & Detemple, 1997). In general, when adding more investment options, one increases demand for more information, and thus the incentive to wait, even if one option value exceeds the other when valuated separately at a given point in time (Décamps, Mariotti, & Villeneuve, 2006).

2.4.2 Real options in the hydropower industry

In the real world there hardly exist well defined simple options, such as an option to build a hydropower plant with a given technology and a given capacity. There is often a continuum of options with strong interdependencies. For the case of hydropower industry there are a few characteristics that are dominant. Since the variable costs are very low, the option to abandon is usually regarded to be of low value. For the same reason, the option to contract is of negligible value. On the other hand, options to expand are assumed to be valuable. The options inherent in a hydropower project are mainly of American character.

Often options have interdependencies; some options are for instance dependent of exercise of other options. These are commonly termed compound options. There are two types of compound options, both relevant in the hydropower industry, namely sequential and simultaneous compound options. In the case of the former, a prerequisite for one option is the exercise of another option, and the latter is when options are underlying assets for other options.

2.4.3 Real option valuation

Underlying asset value

A strict assumption concerning the theory of real option valuation is the existence of a replicating portfolio (Dixit & Pindyck, 1994). When contingent claims analyses are employed, it is assumed that the underlyings of the options can be spanned by the means of existing assets in a sufficiently complete market. However, unlike the case for financial options this is often not the case for real options. There are several approaches developed to solve this problem. Methods commonly used are the classical approach, the revised classical approach, the integrated approach and the Marketed Asset Disclaimer, MAD. We have chosen to use the MAD assumption. The essence of this method is that instead of searching the financial markets for replicating portfolios, we use an unbiased estimate of the present value (PV) of the project itself without flexibility as if it were a marketed security (Copeland & Antikarov, 2003). Thus the PV estimate acts as the twin security in the classical approach. The assumptions made in the MAD approach are not stricter than those traditionally made when estimating the PV, namely that the market contains assets with the same volatility as the real project without flexibility (Schneider, Tejeda, Dondi, Herzog, Keel, & Geering, 2008). All project uncertainties are aggregated to one single uncertainty parameter, which is the volatility of the one period project return. Further we make use of Samuelson’s theorem, namely that properly anticipated prices fluctuate randomly, and thus the present value of a project will follow a random walk. The intuition behind this is that all relevant information is taken into account when estimating the project PV, and hence, if expectations are met, investing in the project will earn exactly the expected cost of capital. Samuelson’s theorem states that deviations from the expected PV dynamics are caused by random events. Copeland & Antikarov (2003) assume that the dynamics of the PV can be modeled as a geometric Brownian motion (GBM). Thus we have a model for the behavior of the PV of the project as if it were a traded asset. This forms the basis of the real option valuation. For more details on Samuelson’s theorem we refer to Copeland & Antikarov (2003).

Estimating volatility and correlation

One of the most challenging input parameter to estimate in ROA is the volatility of project return (Mun, 2006). Unlike the case with financial options, volatility can most often not be estimated historically or from futures markets by implied volatility approaches (Haahtela, 2007). There exists numerous techniques to estimate volatility, but since our approach already relies on the assumptions made in MAD and Samuelson’s theorem, we employ the consolidated approach of logarithmic present value. In this method we use Monte Carlo simulation on gross PV to obtain mean, volatility and correlation of the one period returns. In each simulation, the PV of the cash flows is estimated at two different points in time, present time and after one period, and the one period return, z, is estimated.

$$z = \ln \frac{PV_1 + FCF_1}{PV_0} \tag{2}$$

$$PV_x = \sum_{t=x+1}^{N} \frac{FCF_t}{(1+\mu)^{t-1}} \tag{3}$$

The volatility is then calculated as the standard deviation of the distribution of z (Haahtela, 2007). Here FCF_t is the free cash flow in period t and μ a constant appropriate discount rate. Estimating volatility in this manner assumes constant volatility. This is not always the case, as volatility can vary as function of time and/or asset value.

The logarithmic present value approach of estimating volatility tends to result in too high estimates. This is caused by uncertainty and ambiguity in the estimate of the underlying asset value in the first place (denominator in the expression of z). Unlike in the case of financial options this value is not known. Ambiguity in the present value estimation therefore causes an upward biased volatility as the uncertainty caused by ambiguity and volatility is pooled into one estimate of the volatility (Haahtela, 2007).

Correlation between one period returns can be estimated using Pearson’s correlation coefficient.

Estimating the dividend

If the underlying assets do not pay dividends, the value of an American option will be the equivalent to a European option, and early exercise is never optimal. The continuous dividend yield is calculated as

$$\delta = \mu - \alpha \tag{4}$$

where μ is the risk-adjusted expected rate of return required by the producer to execute the project, and α is the expected rate of change in the underlying asset value (Dixit & Pindyck, 1994). δ can be interpreted as an opportunity cost of delaying project investment. It can also be interpreted as the relative benefit of owning a project rather than holding the option to invest.

When all required inputs are estimated and a suitable stochastic process is chosen to model the underlying assets, one can commence the option valuation. Several numerical approaches exist for real option valuation. The most common approaches are the binomial/trinomial lattice method, Monte Carlo simulation, finite difference methods and quadrature methods. In this paper we employ the quadrature method as proposed by Andricopoulos, Widdicks, Newton & Duck (2007), based on an iterative integral approach. We give a quick review of this procedure.

2.4.4 The quadrature method

The quadrature method can be seen analogous to a multinomial lattice in that the option pricing is done recursively, working backwards from maturity. It also has the added flexibility that nodes can be placed wherever desired and only one time step is required between observations of exotic features (Andricopoulos, Widdicks, Newton, & Duck, 2007).

In order to use the quadrature method, it is necessary to pose the problem in terms of an integral equation. The Black-Scholes equation for n underlying assets is:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_i\sigma_j\rho_{ij}S_iS_j\frac{\partial^2 V}{\partial S_i \partial S_j} + \sum_{i=1}^{n}(r-\delta_i)\,S_i\frac{\partial V}{\partial S_i} - rV = 0 \tag{5}$$

where S_i is the value of underlying asset i, σ_i and δ_i the corresponding asset volatility and dividend yield, r the risk free rate and ρ_{ij} the correlation coefficient between S_i and S_i (Andricopoulos, Widdicks, Newton, & Duck, 2007).

In order to numerically estimate the value of an American option, we consider a finite number of early exercise opportunities, and are as such approximating the American option as a Bermudan option. We have an option value V with N predetermined exercise opportunities at $t \in 0,1,\dots,N$ on n underlying assets with values $S_1, S_2, \dots, S_n$. The American option is for valuation purposes divided into N separate options. These options runs from t to $t+1$. The values at expiry are simply the payoff of the option. At expiry $t = N$, the option value, V_N, is

$$V_N(S_1,S_2,\dots,S_n)=H(S_1,S_2,\dots,S_n) \tag{6}$$

where H denote the payoff function. The payoff function at maturity is equivalent to the value of exercise or zero. At the last early exercise point $t=N-1$, the value of the option is equivalent to a European option on n underlying assets given by

$$V_{N-1}(S_1,S_2,\dots,S_n)=\int_0^\infty \dots \int_0^\infty H(S'_1,S'_2,\dots,S'_n)\,G(S_1,S_2,\dots,S_n;S'_1,S'_2,\dots,S'_n)dS'_1dS'_2\dots.dS'_n \tag{7}$$

where $G(S_1,\dots,S_n;S'_1,\dots,S'_n)$ is Green's function yielding the solution to the Black-Scholes equation on multiple underlying assets (Kay, Davison, & Rasmussen, 2005). Note also that and S'_1 denotes an outcome for S_1 in $t+1$. The integrals can be evaluated using an appropriate quadrature method as the trapezoidal method, the Simpson's rule or the Gauss-Legendre quadrature. In this paper we have chosen the composite Simpson's rule as proposed by Andricopoulos, Widdicks, Newton & Duck (2007).

For the early exercise points $t\in N-2,N-3,\dots 0$ the European option value V_t is calculated in a similar way. After replacing the payoff function in Equation (7) by $H(S'_1,S'_2,\dots,S'_n,V_{t+1}(S'_1,S'_2,\dots,S'_n))$, the value of the European option becomes effectively

$$\begin{aligned}V_t(S_1,S_2,\dots,S_n)=\int_0^\infty \dots \int_0^\infty &H(S'_1,S'_2,\dots,S'_n,V_{t+1}(S'_1,S'_2,\dots,S'_n))\\ &G(S_1,S_2,\dots,S_n;S'_1,S'_2,\dots,S'_n)dS'_1dS'_2\dots.dS'_n\end{aligned} \tag{8}$$

The payoff function H in Equation (8) is the maximum of the value of exercise for a given outcome $S'_1,S'_2,\dots,S'_n$ at an early exercise point $t+1$, or the value of the European option to postpone the investment decision to $t+2$. The option value and the exercise boundaries are determined by working recursively from expiry to $t=0$.

To sum up and introduce some nomenclature we use the following conventions. If the payoff of immediate exercise exceeds the value of holding on to the option, V_t, the option holder will exercise. The value of exercise will hereby be referred to as the exercise value whereas the value of holding on to the option will be referred to as the holding value.

The exercise boundaries are found where the holding value and the exercise value are equal. At every early exercise point there exist n free boundaries, one for each of the assets. These boundaries divide the exercise region into sub regions in which there is optimal to exercise the option with respect to one of the underlying assets. All asset value combinations where immediate exercise is not optimal define a sub region where the optimal strategy is to wait – or hold the option. The holder of the option then waits until the next exercise possibility (Kay, Davison, & Rasmussen, 2005).

The choice of an iterated integral method using quadrature for option valuation is motivated by the method’s superior convergence and hence, increased accuracy and reduced computation times compared with lattice, grid or Monte Carlo methods. A second reason is that the flexibility removes nonlinearity error (Andricopoulos, Widdicks, Newton, & Duck, 2007). In order to derive advantage from this method, a detailed knowledge of the typology of the option valuation problem is required. As the number of underlying assets increases and/or the complexity of the option increases, the more complex the implementation becomes. As in Andricopoulos, Widdicks, Newton, & Duck (2007) we point out that this should not be seen as a drawback, but rather as a possibility to value complex derivatives to any desired level of accuracy.

3 Case presentation

The techniques and ideas described in the previous section will be used to solve a real investment problem given by the Norwegian hydropower producer Hydro.

Hydro has a sequence of five hydraulically coupled power stations installed in a river system. These are mostly aged plants designed to provide base load power to industry in the surrounding area. The existing configuration suffers from high response time, low controllability and low overall efficiency. Almost all inflow used for electricity production is accumulated and stored in one large reservoir with a 67.5 % degree of regulation, defined as reservoir size relative to mean yearly inflow (Norsk Hydro, 1987). Because of the degree of regulation, the reservoir is regarded as an over-seasonal reservoir, meaning that the reservoir is large enough to move generation between seasons.

The last decades the electricity market has changed considerably. As a result the plants are not properly fitted to today's market. Maintenance lags render the fact that the plants are facing large refurbishment costs (Norsk Hydro, 1987).

Price fluctuations and increased intraday price differences offer opportunities for hydropower producers. It is always desirable to reduce the plant's utilization period, i.e. the relation between annual average inflow and maximum generating capacity. This enables the producers to allocate production to peak load periods and reduce or stop production at low prices, rather than being forced to operate at low prices (Johnsen, Verma, & Wolfram, 1999). It is also a desirable property to shift more generation between seasons, i.e. to produce in wintertime when prices generally are higher. To capture the value accompanying these properties, Hydro considers investing in projects improving the generation capacity of the plants.

Hydro contemplates four new expansion projects in addition to refurbishment of the existing plants. All the expansion projects involve replacing three of the existing stations with one large station, and expanding the discharge capacity on the two remaining plants. They differ in geographical location and capacity, and in order to keep the number of options manageable, we limit our analysis to include two of the projects, both located at the same geographical location but with different installed capacities and investment costs.

The problem is then reduced to analyze whether Hydro should conduct the refurbishment of the existing power stations, or if they should invest in higher discharge capacity and when such a decision can be made optimally.

To sum up and to introduce some nomenclature, we say that Hydro can choose to execute one of two upgrade projects, project A or project B, or none of them. In this setting, project A is to replace three power stations by one of intermediate size, project B is to replace the same three stations by one large station, while not executing any of the projects means keeping the existing power stations. It is important to note that if Hydro chooses not to execute today, they do not loose the expansion option. Thus projects A or B can be realized at a later stage. The projects A and B are considered as mutually exclusive.

4 The models

We have developed two models in order to solve the investment problem, a production simulation model and an investment model. The production simulation model generates inputs used in the real option analysis, and relies on a price model developed by Hydro together with historical inflow series.

4.1 The production model

4.1.1 Production planning – reservoir handling

In the spirit of Keppo and Näsäkkälä(2005), we construct a production threshold to develop a production strategy. However, we emphasize that our main purpose of constructing this strategy is to simulate future cash flows, and not a production hedging policy. Our production threshold dictates a "bang-bang strategy", defining when to produce and when not to.

In the model all reservoirs are aggegated into one large reservoir, and all power stations are aggregated into one large power station. All inflow runs to the aggregated reservoir. The output from the reservoir/station pair is considered to be constant, independent of head variation effects. In general, these are the same assumptions as used in long term generation planning (Fleten, Wallace, & Ziemba, 2002).

We assume that inflow comes as random lumps at the beginning of every planning period, meaning that inflow accumulated throughout the whole period is available from the beginning. This is not a strict assumption for a short time planning horizon.

The water reservoir level is constrained by upper and lower bounds, x_{max} and x_{min}, respectively. Upper and lower bounds for the resevoir level can be functions of time. All inflow will either be used for production or creates spillage. All spillage is considered as lost water and cannot be used for production. As mentioned, the lower bound for the discharge is set to zero, while the upper bound is the plants maximumum discharge capacity, u_{max}. The aim is to find an optimal production strategy $u(\cdot)$ satisfying all constraints, where optimal refers to profit maximizing.

$$\max_u V(t,x,S,F,v,u) = E\left[\eta \int u(t)(S(t)-c)e^{-\mu(\tau-t)}dt | F(t), v(t)\right] \tag{9}$$

subject to

$$u(t) \in [0, u_{max}] \tag{10}$$

$$x(t) \in [x_{min}(t), x_{max}(t)] \tag{11}$$

where

V - profits

t - time

$x(t)$ - observed reservoir level at time t

$S(t)$ - spot price at time t

c - variable costs

$F(t)$ - average future forward curve at time t

$v(t)$ - inflow occuring at the beginning of period t

$u(t)$ - total discharge in period t

η - plant efficiency

μ - discount rate

τ - plant lifetime

u_{max} - maximum discharge capacity

$x_{min}(t)$ - minimum reservoir level

$x_{max}(t)$ - maximum reservoir level

The reservoir handling is given by the following equations

$$x(t+1) = x(t) + v(t) - u(t) - s(t) \tag{12}$$

$$u(t) = \begin{cases} min[max[0, x(t) - x_{min}(t) + v(t)], u_{max}] & if\ S(t) \geq K\big(t, x(t), v(t), F(t)\big) \\ min[x(t) - x_{max}(t) + v(t), u_{max}] \vee 0 & if\ S(t) \leq K\big(t, x(t), v(t), F(t)\big) \end{cases} \tag{13}$$

$$s(t) = max\ [x(t) + v(t) - u(t) - x_{max}(t), 0] \tag{14}$$

$s(t)$ is spillage caused in period t.

$K\big(t, x(t), v(t), F(t)\big)$ is the threshold function acting similarly to a MWV.

Equation (12) is the reservoir balance equation. It states that all inflow to the reservoir is either spilled or used for production. In Equation (13) we differ between top and bottom reservoir handling depending on spot price. The equation states that discharge equals the minimum of u_{max} and the maximum of zero and surplus water (current period inflow included) compared to the minimum resevoir level. The lower part of Equation (13) determines the discharge if spillage is inevitable. Regular generation planning generates a MWV of zero if spillage is iminent, which in turn yields full generation. Our threshold does not ensure this, something Equation (13) seeks to correct. Equation (14) yields water spillage in period t, occurant only when reservoir level plus inflow less the released water exceeds maximum reservoir level at time t.

A summary of the production strategy can be stated as follows

$$\begin{aligned} & u\Big(K\big(t, x(t), v(t), F(t)\big)\Big) \\ & \qquad = I\big\{S(t) \geq K\big(t, x(t), v(t), F(t)\big)\big\}(max[0, x(t) - x_{min}(t) + v(t)] \vee u_{max}]) \\ & \qquad + I\big\{S(t) \leq K\big(t, x(t), v(t), F(t)\big), x(t) + v(t) \geq x_{max}\big\}(x(t) + v(t) - x_{max} \vee u_{max}) \end{aligned} \tag{15}$$

4.1.2 Production planning – the threshold function

The threshold function is based on the intuitive assumptions listed below.

- We assume that the outside world can be represented with a price forecast, the same assumption made in general production planning with EOPS. That is, instead of simulating realistic handling of all water reservoirs, thermal generation, changes in demand, import and export and building of interconnections and new generation capacity in a global system, the producer focuses on his own local system, and describes the rest of the global system with price forecasts. This assumption is made by many minor producers in the Nordic system (Doorman, 2007).
- Further we assume that future spot prices can be represented by an average future forward curve. When averaging the forward curves, some information from the market is lost. Averaging a forward curve means that some of the dynamics caused by forward contracts with different maturities is lost.
- We assume that the MWV is a function of the reservoir level. This intuition was supported by Tipping et al for hydropower dominated electricity markets (Tipping, Grant Read, & McNickle, 2004). Similar to Tipping et al we base our MWV as a function of the storage level, in terms of deviation from a median reservoir level. More precisely, if the current storage level is higher than usual for the time of the year, the MWV will decrease and lead to more aggressive water release, and vice versa. As we have seen, this intuition is also supported in Keppo and Näsäkkäläs work, but then in terms of a probability of spillage.
- We assume that the MWV and then the threshold is a function of anticipated inflow in terms of deviation from an inflow median. Again, if anticipated inflow is greater than usual for the time of the year, the MWV is supposed to decrease to stimulate more production.
- Unlike Keppo and Näsäkkälä our planning horizon is extensive, i. e. equal to the lifetime of large hydropower plants. They assume that the value at the end of the planning horizon is zero, and consequently the MWV will also approach zero at the given horizon. In our case, the value of the final non-zero reservoir value will not be estimated. This is not critical since the present value of the final reservoir will, because of discounting, be a small value only. The salvage value of the plant is also not considered.

- We assume that the strategy can be executed as a “bang-bang strategy”. That is, the power plant can shift instantaneously between zero and full capacity. Consequently, we do not take minimum discharge and ramping constraints into account.

The parameterized mathematical form of the threshold function is as follows:

$$K\left(t, x(t), v(t), F(t)\right) = \propto_F F(t) e^{\left(\propto_x \frac{\left(\bar{x}(t) - x(t)\right)}{\bar{x}(t)} + \propto_v \frac{\left(\bar{v}(t) - v(t)\right)}{\bar{v}(t)}\right)} \tag{16}$$

where

$\bar{x}(t)$ - median reservoir level at time t

$\bar{v}(t)$ - mean inflow for current period at time t

The parameter $\propto_x$ gives the rate of change in the threshold function in terms of relative deviation from the median reservoir level. It determines how much deviations from the median affect the threshold. Large values of $\propto_x$ imply that simulated trajectories will be close to the median trajectory, but also less operational flexibility. Similarly, the parameter $\propto_v$ gives the rate of change in the threshold as a function of relative deviation from mean inflow at a given time of the year. Less inflow than normal causes the threshold function to increase and vice versa, where larger values of $\propto_v$ imply greater increase in the threshold function. As in Keppo and Näsäkkälä $\propto_F$ gives a linear change in the threshold as a function of the average forward curve.

An example on how the threshold function behaves is presented in section 7.2.1.

4.2 The price model

The price model employed in our simulations is developed by the Norwegian Computer Center for Hydro. The model consists of several terms, each trying to capture specific characteristics of the spot price observed on Nord Pool. The model was developed to meet three main requirements (Norwegian Computing Center, 2000). It must exhibit realistic long term and short term behavior with a time scale of as little as one day, and coincide with historical data on all levels of time resolutions. The model should also coincide with available market information. The resolution of the price model is daily, with a deterministic price profile over the day.

In order to capture the prerequisites described above, the model is made up of several different terms. Among others there is a term accounting for correlation between reservoir levels and spot price. Due to the high share of hydropower in the Nordic electricity market, this correlation is found to be significant. The price model also stipulates a futures curve simulations scheme, which simulates the futures curve using the spot price as a basis. Knowing the state of all components in the price model, we are able to construct a forward curve, extending to an arbitrary point in time.

4.3 The investment model

The gain from executing one of the projects is the expected excess cash flow generated, compared to the existing plants, while the investment cost is the price Hydro has to pay to receive this excess cash flow. The cash flows from the difference projects are simulated in the production model.

4.3.1 Formulation of the option payoff structures

We model the investment opportunity as a call option on the maximum of two assets or nothing, where the underlying assets are modeled as difference projects. The assumption of mutual exclusivity made in section 3 is thereby preserved. The option valuation framework used in this paper does not allow more than one stochastic variable pro underlying asset. We model the evolution of the present value of each of the difference projects as a random walk.

Refurbishment is required in order to keep the existing plants operative. These needs arise at specific, known, points in time. When the refurbishment need arises, it is assumed that they will be carried out, unless an expansion project is carried out. The relatively low refurbishment costs compared to the present value of the existing plant stipulates that this is not a strict assumption.

To sum up, we formalize the options payoff structure as an option on a maximum of two underlying assets or nothing:

$$Payoff(t) = max\ (Project\ A(t) - I_A(t), Project\ B(t) - I_B(t), 0) \tag{17}$$

$Project\ A(t)$ and $Project\ B(t)$ denote the difference project present values while $I_A(t)$ and $I_B(t)$ are the investment costs less refurbishment costs at time t. The option is an American style option.

4.3.2 Present value dynamics

To determine the option value we need suitable stochastic processes to model the underlying assets. These are formulated as follows

$$dA = A(\mu_A - \delta_A)dt + \sigma_A A dz_A \tag{18}$$

$$dB = B(\mu_B - \delta_B)dt + \sigma_B B dz_B \tag{19}$$

$$Cov(dz_A, dz_B) = \rho dt \tag{20}$$

A - PV of project A

B - PV of project B

μ_A - Expected rate of return of project A

μ_B - Expected rate of return of project B

δ_A - Continuous dividend yield of project A

δ_B - Continuous dividend yield of project B

σ_A - Standard deviation of the one period returns of project A

σ_B - Standard deviation of the one period returns of project B

ρ - Correlation coefficient between one period returns of the projects

dt - Time increment

dz_A and dz_B are correlated Wiener processes

Thus the asset price dynamics are modeled as two correlated geometric Brownian motions.

5 Implementation

Matlab is chosen as the programming language. Most of the analyses, simulations and numerical computations are implemented in Matlab.

5.1 Identification of relevant uncertainty factors

We employ a tornado diagram to illustrate how sensitive the project value is with respect to specific inputs and to find the relative importance of these. Using a base case we determine the static NPV of project A with a mean of MNOK 234. Next we change the inputs one at a time and note the changes in the NPV. Figure 1 shows the sample space of the net present value (NPV) of project A with respect to the inputs we believe to have greatest influence on the project value. A similar diagram can be constructed for project B. Note that the figure is not perfectly scaled and is for illustrative purpose only. Using this information we decide on which inputs to use when estimating the dynamics of the NPV. The price and inflow effect is calculated by using the highest and the lowest simulated NPV. We can see that this greatly influence the project value, and thus contributing to great extent to the uncertainty. Favorable price and inflow scenarios produce high project values, while the opposite causes negative values. To capture the effect of the discount rate we have employed two scenarios, high and low, and we observe deviations of approximately MNOK 500 from the mean. The investment costs are assumed to have only up-side potential. For more details on how the investment costs are treated, we refer to the appendix. As the figure shows, increased intraday price structure does not influence the project value as much as the other factors.

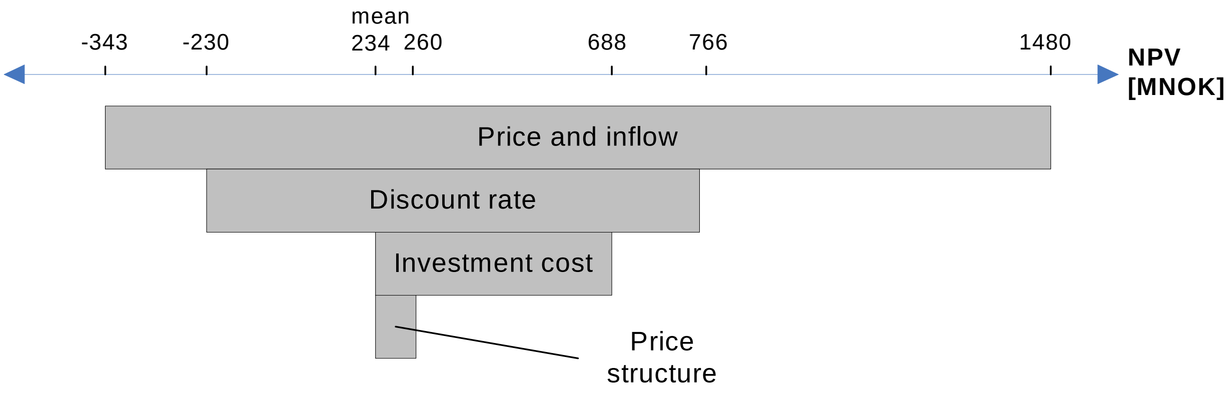

Figure 1 Tornado diagram

We note that there are additional uncertainty factors regarding the NPV which are not quantified here, and that (due to correlation) the total sample space cannot be determined by a simple addition of the uncertainty caused by the factors displayed in Figure 1. We will later come back to a qualitative discussion of the uncertainties that we have not been able to quantify appropriately. The most critical uncertainty factors are modeled as stochastic in Monte Carlo simulations to produce an expected PV.

5.1.1 Stochastic variables

Electricity spot price and inflow are considered to have great influence on project profitability and are both modeled as stochastic. The price is modeled using the price model as described in section 4.2, and inflow series are constructed by random draws from a database containing historically yearly inflow series with all series equally likely to be drawn.

Our framework assumes a competitive market where investors can be regarded as price takers, i.e. their investment decisions do not influence prices. Then price uncertainties can be represented as exogenous stochastic processes (Botterud & Korpås, 2006). Our investment model is based on these assumptions, although we note that these assumptions are not always met. This is further elaborated in the second part of section 7.1.5.

5.1.2 Deterministic variables

Discount factor

The discount factor used in the analyses is similar to the standard discount rate used in the hydropower industry. The Norwegian Water Resources and Energy Directorate propose a guiding discount factor of 6.5 %, and point out that this should be adjusted according to the degree the project is exposed to risk (NVE, 2007). With this in mind, and having consulted with Hydro, we set the discount factor to 7 %. This discount factor is assumed constant and is used in both projects. Thus $\mu_A = \mu_B = \mu = 7\ \%$. A profound study to find the correct discount factor is outside the scope of this paper.

Investment costs

Since both projects can be deferred, investment costs are in reality unknown. Future investment costs can be estimated by using construction cost indexes.

To limit the scope of this text, the investment costs are considered to be deterministic and modeled as constant, but keeping in mind that changing investment costs can be decisive for optimal trigger strategies. For some background information, a short analysis on the evolution of the investment cost is performed in chapter 10.4.

Degree of market integration

Since the degree of integration of the Nordic and the continental market is relevant for the potential profit a plant is able to generate. We incorporate this in the analyses by including two different scenarios. Our hypothesis is that plants with higher capacity profits more when more price structure is introduced compared to low capacity installations. To capture this difference, the degree of market integration is represented by different future intraday price structure scenarios. The scenarios are generated using the ECON BID model in a study conducted by Hydro (Torgersen, 2008). We use a base scenario with expected construction of new transmission capacity and one scenario where some additional transmission capacity is built.

5.2 Implementing the production model

A large hydropower plant usually has a very long lifetime. We have assumed a lifetime of a new plant to be 50 years. A full refurbishment of the existing plants is assumed to bring the plants back to its new condition. It is further assumed that the refurbishment can be split into several minor projects, thus delaying the costs incurred.

To ease production planning, all power stations are aggregated into one large station with a given discharge capacity and efficiency. All aggregated power plants release water from the same reservoir, and consequently they receive the same inflow. A simplification has been made regarding the response time, which is set to zero for all (aggregated) plants.

We have used an estimate of the total costs associated with operation, maintenance, overhead and breakdowns, divided by the average production. After having consulted with Hydro these costs are estimated to 50 NOK/MWh for the existing plant and 30 NOK/MWh for both the new plants. Included in these estimates are breakdown costs, operating costs, fixed yearly costs and regular maintenance upkeep.

To be able to calculate difference project cash flows, the parameters α_F, α_x and α_v in the threshold function must be estimated. These are plant-specific parameters that need to be estimated separately for all three plants.

The time increment in the threshold function is chosen to be one day. However, inflow is anticipated at a weekly basis and compared to the weekly median. The average forward curve is the average of daily forwards for the consecutive 365 days. 1200 price and inflow scenarios are run and production is simulated using the threshold function. All generation is sold at spot price, and a future cash flow series for every price/inflow scenario is calculated and discounted back to current point in time to find the present value of the income. This cash flow simulation is done a number of times, each time adjusting one parameter at a time by a given rate and comparing the mean present value given the set of price/inflow scenarios. The parameter causing the highest improvement in mean present value will be shifted, before a new local search is performed. In this way the parameters are adjusted until the mean present value cannot be further improved. Knowing that this might be a local optimum, the process is repeated with a different set of starting parameters. The parameter optimization heuristic is run for the three power plants, resulting in the parameter sets, as shown in Table 1.

Parameter	Existing plant	Project A	Project B
α_F	0,5647	0,4731	0,3898
α_v	0,1868	0,1962	0,1646
α_x	0,6590	0,7315	0,8103

Table 1 Estimated parameters for the threshold function

Given these three sets of parameters produced by the optimization heuristic, the threshold functions are ready to be used for production planning and cash flow simulation for their respective power plant projects.

5.3 Implementing the investment model

When conducting the real option analysis the time horizon is set to 19 years. This is done to limit computational effort. In addition, experiments show expanding the time horizon has negligible impact on the exercise boundaries and the option value. If the option expires without being exercised it is considered lost, so the investment opportunity at expiry date is worth the exercise value or zero.

We assume refurbishment on existing installations will be done at three predetermined dates, as shown in Table 2. The refurbishment costs at these dates are shown in the second column. At other dates there are no refurbishment costs.

Year	Refurbishment cost [MNOK]
2008	250
2016	300
2026	300

Table 2 Refurbishment on existing plants

We assume further that if an expansion project is conducted all stations are in new condition.

We estimate dividends as described in section 2.4.3 above. Simplifications have been made regarding the input to the analysis. Only electricity spot price and inflow are expected to be time varying processes, neither of them with long term drift. We therefore assume the growth rate α of both project present values to be zero. Using Equation (4) we calculate the continuous dividend yield to $\delta_A = \delta_B = \delta = \mu = 7$ %. The risk free rate is set to 5 %.

The stochastic dynamics of the present values is then reduced to

$$dA = \sigma_A A dz_A \tag{21}$$

$$dB = \sigma_B B dz_B \tag{22}$$

$$Cov(dz_A, dz_B) = \rho dt \tag{23}$$

The parameters needed for the option valuation program are presented in Table 3.

Parameter	Estimated value
Expected present value project A	1738 $MNOK$
Expected present value project B	2000 $MNOK$
σ_A	0,043
σ_B	0,069
ρ	0,94
δ	0,07
r	0,05

Table 3 Estimated parameters for the option valuation program

5.3.1 Implementing the quadrature method

First, we make the logarithmic transformations

$$a = \ln A \tag{24}$$

$$b = \ln B \tag{25}$$

At expiry the option value is

$$V_N(a,b) = H(a,b,) \tag{26}$$

$$H(a,b) = \max\,(a - i_A, b - i_B, 0) \tag{27}$$

where i_A and i_B are the logarithmic transformations of the investment costs I_A and I_B respectively. At early exercise points Equation (8) becomes

$$V_t(a,b) = \int_0^\infty \dots \int_0^\infty H\big(a', b', V_{t+1}(a',b')\big)\, G(a,b;a',b') da' db' \tag{28}$$

Where the payoff function H is given by

$$H\big(a', b', V_{t+1}(a',b')\big) = \begin{cases} ma\,x(a' - i_A, b' - i_B, 0)\,, & t = N-1 \\ ma\,x(a' - i_A, b' - i_B, V_{t+1}(a',b'))\,, & t \in N-2, \dots, 0 \end{cases} \tag{29}$$

We recall that a' denote an outcome for a in $t+1$.

Green's function for a two asset European option following GBM is given by

$$G(a,b;a',b') = \frac{e^{-rdt}}{2\pi dt(1-\rho^2)^{\frac{1}{2}}\sigma_A\sigma_B} e^{\left(\frac{-(\beta_A^2 - 2\rho\beta_A\beta_B + \beta_B^2)}{2(1-\rho^2)}\right)} \tag{30}$$

where

$$\beta_A = \frac{1}{\sigma_A\sqrt{dt}}\left(a - a' + (r - \delta_A - \frac{\sigma_A^2}{2})\, dt\right) \tag{31}$$

$$\beta_B = \frac{1}{\sigma_B\sqrt{dt}}\left(b - b' + (r - \delta_B - \frac{\sigma_B^2}{2})\, dt\right) \tag{32}$$

The quadrature method requires finite upper and lower bounds for integration. At each time step, the probability of movements more than a certain number of standard deviations is negligible (causing the integrand to be small). The integration ranges are therefore truncated to

$$a'_{min} = a - \xi \sigma_A \sqrt{dt} \tag{33}$$

$$a'_{max} = a + \xi \sigma_A \sqrt{dt} \tag{34}$$

The corresponding is true for asset b, where ξ is the number of standard deviation movements per time step. We set $\xi = 7.5$ as this has been proven to be sufficiently accurate (Andricopoulos, Widdicks, Newton, & Duck, 2007). We have determined the distance between two ordinary grid nodes as

$$\Delta a = \frac{\sqrt{\Delta t}}{K} \tag{35}$$

K is an accuracy parameter. After some experimentation we have found that a K of 150 is sufficient.

The advantage of using quadrature is obtained through constructing the grid such that the nodes coincide with discontinuities, which in our case is represented by the strike prices and the exercise boundaries. Working recursively backwards, and knowing the exercise boundaries at the last time step, we iteratively use the Newton-Raphson method at each time step to compute the asset values at the two free boundaries and place nodes exactly on the boundaries.

Knowing the exercise boundaries, we divide each integral into three sub regions accordingly. When the integration range spans all regions, Equation (28) is calculated as a sum of three integrals using separate quadrature calculations for each. The three integrals differ in the manner of the option value depending on the node pair a', b'. If $I1$ equals the integral in the early exercise region for a, $I2$ correspondingly for b and $I3$ the integral in the holding region, we have

$$V_t(a, b) = I1 + I2 + I3 \tag{36}$$

In order to fit the nodes with the free boundaries, we have used the adaptive quadrature. Therefore, around discontinuities the distance between the nodes are different than in the rest of the grid. Because of the two dimension feature and the fact that Simpson's method requires an odd number of equally spaced nodes, additional nodes are placed on all required spots, to allow for integration of the entire range. For more details on adaptive quadrature we refer to Mathews & Fink (2004) or the appendix in Andricopoulos, Widdicks, Newton, & Duck (2007).

6 Results

The model indicates an option value of MNOK 77. The exercise value of project A is MNOK 234 while project B has an exercise value of MNOK 135. The maximum of the exercise values are thus considerably higher than the holding value, and project A should therefore be executed immediately.

	Project A	Project B
Present Value [MNOK]	1535	2000
Exercise Value [MNOK]	234	135
Volatility	0.043	0.069
Cost of electricity [NOK/MWh]	71	79

Table 4 Results

Should this solution not be implemented the model indicates an optimal strategy for future decisions. Figure 2 shows a plot of the holding value less exercise value for points in time where a refurbishment is imminent. It can be seen that the holding value is highest where the exercise values of project A and B are close to each other. This can be seen in Figure 2 as a ridge extending along the line where the exercise values are equal. The difference between holding value and exercise value is indicated by different colors in the figure. The light blue, square region is where both projects are far out of the money. The dark blue region is where the exercise value exceeds the holding value, i.e. the region where it is optimal to exercise immediately.

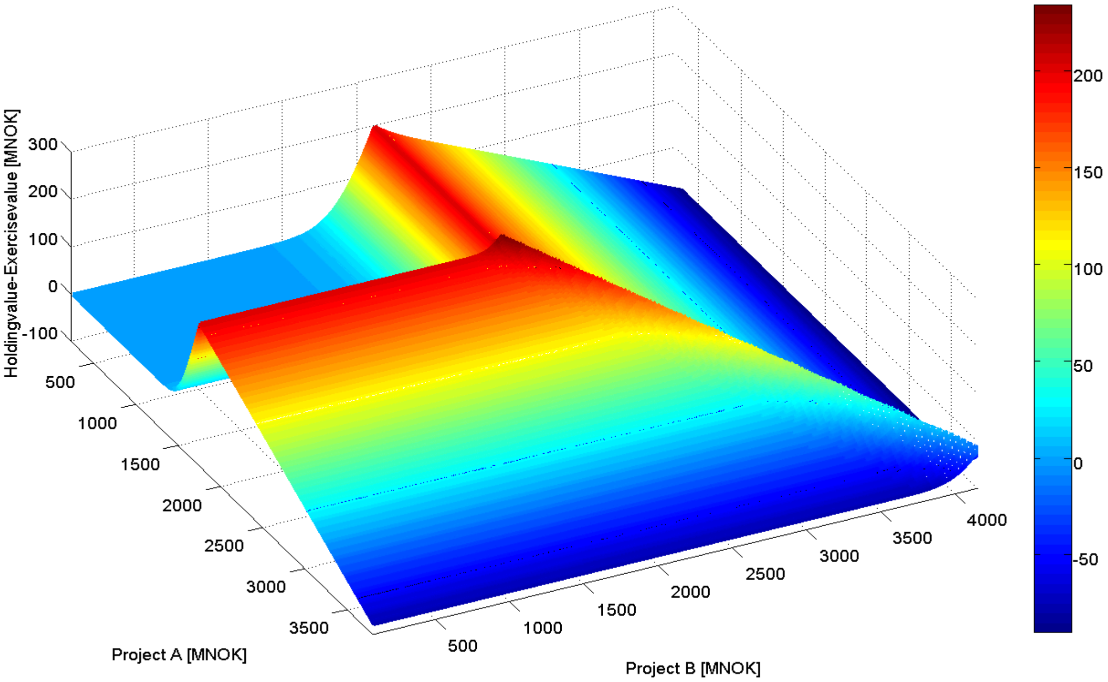

Figure 2 Holding value less exercise value in 2014

The holding region is increasing up to the point in time where the next refurbishment is scheduled. At this point the holding region is diminished. The exercise regions coincide with the points where the project values less the investment costs are positive. Figure 3 shows the situation at the second refurbishment point. It is evident that the holding values are considerably lower at this point.

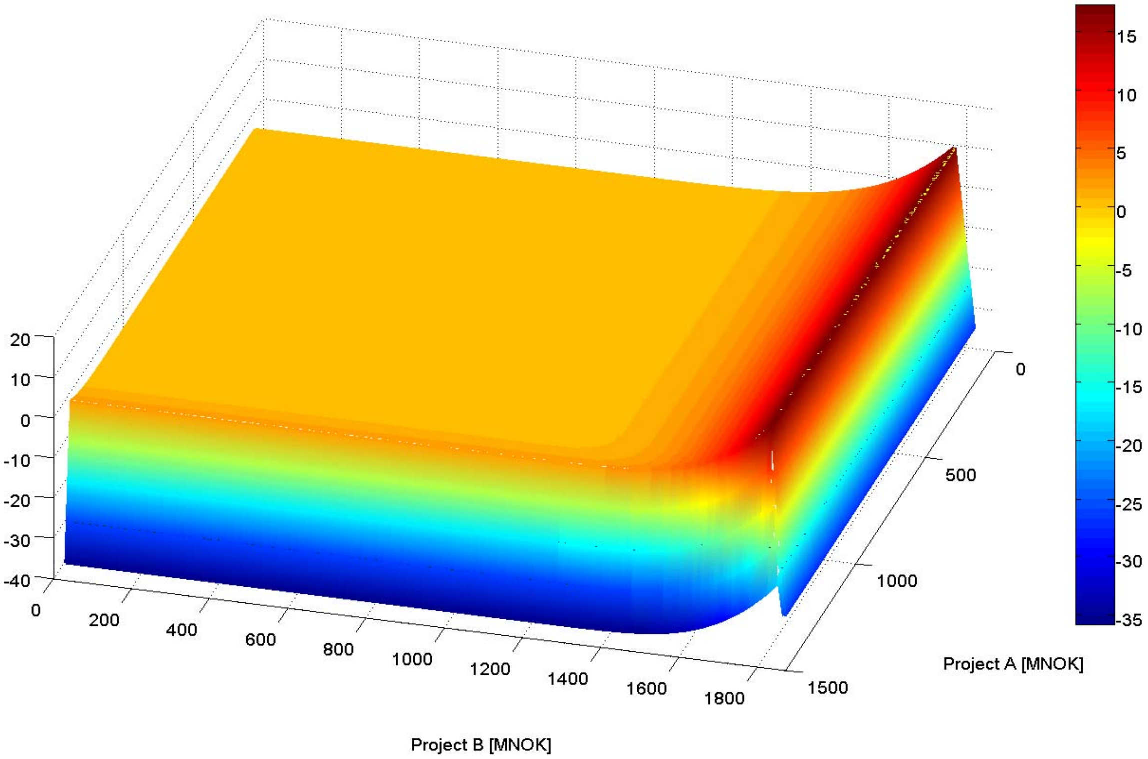

Figure 3 Holding value less exercise value in t=9

The result when running the option value program with the cable scenario is similar, but the holding value of the option is now worth MNOK 93, while the exercise values for project A and project B has increased to MNOK 260 and MNOK 198 respectively.

7 Analysis

7.1 Real option analysis

For cases where both projects are deep in the money, our calculations indicate that it is not necessarily optimal to exercise the option. The model shows a relatively high holding value when the exercise values of project A and project B are approximately equal. This can be observed in the upper right corner of Figure 4. The intuition behind this is that when the exercise values of the projects are equal or approximately equal, it is advantageous to wait to see which one dominating the other in the future. Note that the colors indicate the holding value less exercise value. The holding region can be seen where this value is positive, and the exercise region where it is negative, thus indicating the exercise boundaries. This result is further elaborated in Broadie & Detemple (1997).

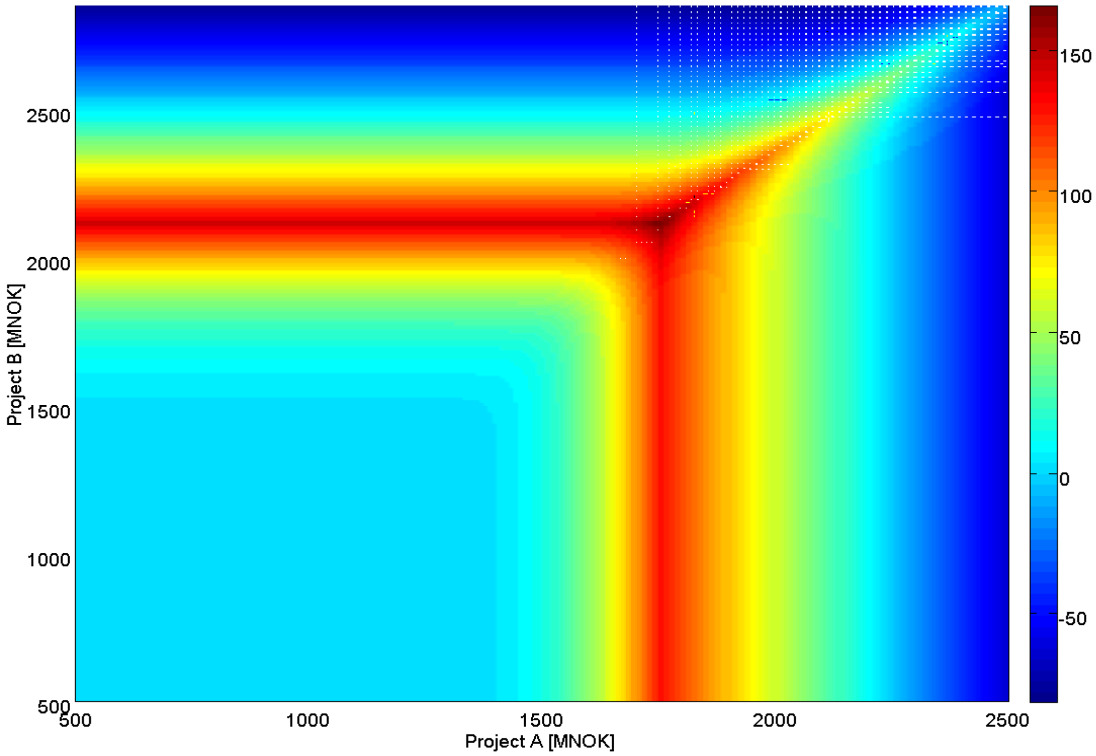

Figure 4 Option topology in 2012

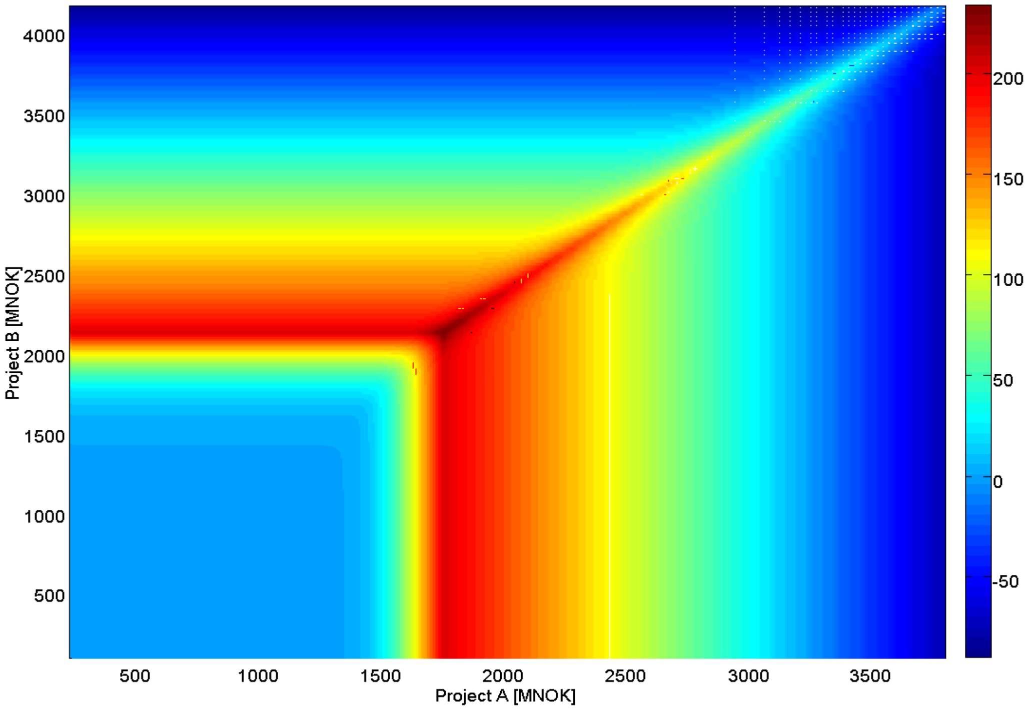

Figure 5 Option topology in 2014

Figure 5 indicates the exercise boundaries in 2014, analogous to Figure 4. Note that the scaling differs. It is evident that the holding region has increased significantly compared to 2012. The intuition behind this is that the strike price decreases when refurbishment needs to be carried out. The next refurbishment is scheduled in 2016, and the projects need to be highly profitable to outweigh the fact that the projects will be relatively cheap in two years time.

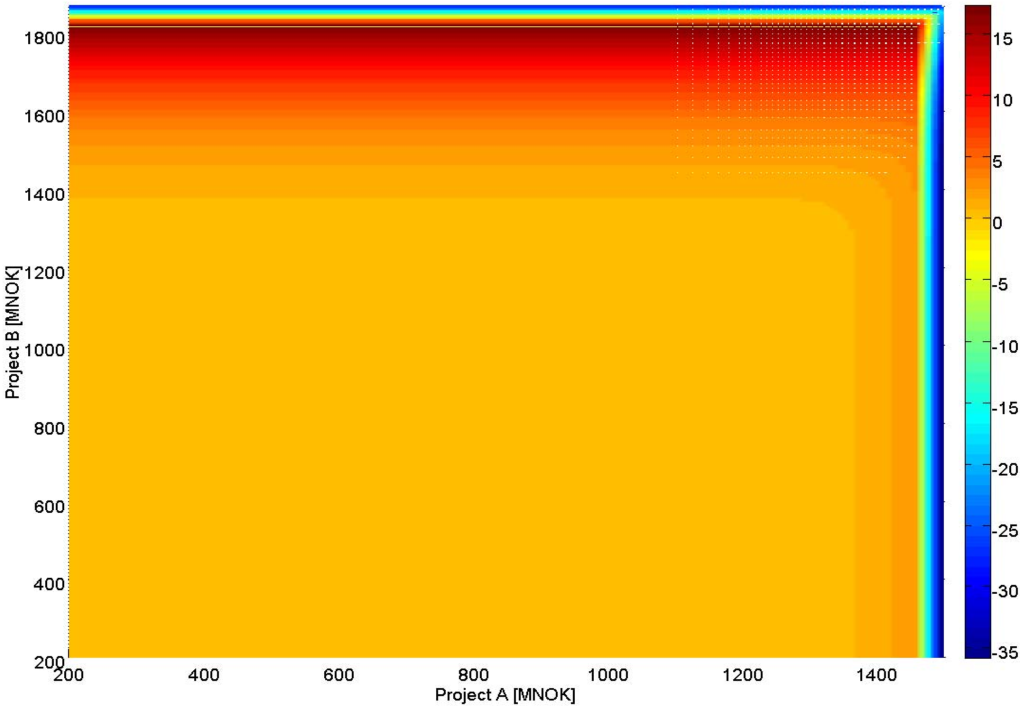

Figure 6 Option topology in 2016

Figure 6 shows the situation in 2016. At this point refurbishment needs to be carried out, and it can be seen that it is optimal to exercise as long as one of the projects are in the money. Again the intuition is that it is relatively cheap to exercise one of the projects at this time, and by waiting one period, the strike price increases significantly.

7.1.1 The effect of the refurbishment costs

The variability of the strike price causes the holding regions to vary considerably. At the stages where the refurbishment costs arise, the holding region is greatly reduced. It is always optimal to exercise as long as at least one asset is in the money. The intuition is fairly simple; the refurbishment costs are considerable, and the possibility that the assets move in such a manner that it is profitable to wait one period is negligible. It is therefore no reason to postpone the decision, given that at least one of the projects is in the money. At the points in time where a refurbishment is imminent, the holding region becomes vast, and the holding value increases considerably. This is also intuitive, as one can run the existing plant for free until refurbishment needs to be carried out, and the upgrade project has to be highly profitable in order to outweigh this. It is therefore likely that one of the projects will be conducted in conjunction with a refurbishment need.

Second refurbishment date	Holding value 2008 [MNOK]
2014	96
2016	77
2018	73

Table 5 Moving the second refurbishment date

Table 5 shows how sensitive the holding value in 2008 is to when the second refurbishment is to be carried out. One can see that the holding value increases as the time to the second refurbishment decreases. The exercise boundaries and hence the strategy is also strongly affected. It will therefore be important to accurately establish when such refurbishments are needed. The strategy up to 2016 is not affected significantly by the last refurbishment.

7.1.2 The Bermudan approximation

The quad method relies on the iterated integral method, and considers the option to be a series of European options. As a result we are actually pricing a Bermudan style option, with exercise points in each time step. Given the fact that a Bermudan style option will always have a lower value than an American style, given the same underlying assets, the option value and the free boundaries will be underestimated. As the time resolution is set sufficiently high, a Bermudan style option will converge towards an American option. By testing the model with different time resolution we have found that a resolution of half a year produces a sufficiently accurate result with an acceptable computational time.

7.1.3 The price profile

We have implemented an intraday price profile that varies throughout the week as well as throughout the seasons. This profile is repeating over the years. However studies indicate that increasing market integration between the Nordic and the continental marked will affect the price profiles. Prices are expected to increase during daytime, and decrease during nighttime as the integration progresses. A study conducted by Hydro (Torgersen, 2008), indicates a price profile for a given degree of integration. Using this price profile in our simulations indicates that increasing market integration will increase the profitability of the plants with high capacity more than the ones with lower. This indicates that the net present value of both project A and project B could be underestimated compared to the existing plants. As a result the option value will be underestimated. However it is beyond the scope of this paper to identify a process to model the degree of market integration, and how this affects the price profile in the Nordic market.

7.1.4 The log normal assumption on the option underlying

The option pricing model is based on the Black-Scholes framework when valuating European options between two consecutive exercise dates. This framework assumes a normal distribution of the underlying assets’ one period returns. Our simulations indicate that this might not be the case for neither project A nor B. The distribution appears to have an excess kurtosis, with a higher peak around the mean and fatter tails. The option pricing model requires the underlying assets to be log normally distributed, and we make the assumption that this is the case, using parameters found in the simulation model. It is reasonable that negative project present values do not occur, as one would assume that in the long run, a large plant can perform at least as well as a small one, even if there is always a risk of making mistakes in the generation scheduling. The assumption of log normality is common in option pricing models. For more details regarding the log normal assumption, we refer to section 10.3.

7.1.5 Other assumptions made in the investment model

Volatility

The volatility is one of the key input parameters in the option valuation and the free boundary calculation. The results are highly sensitive to this parameter. The volatility estimation method employed in this paper is known to overestimate the true volatility. The model will therefore overestimate the option value and the free boundaries. On the other hand, only uncertainty regarding electricity price and inflow is taken into account. By including more stochastic uncertainties in the production simulation model we would increase realism and most likely increase the project volatilities.

Uncertainty caused by varying investment costs can be captured by modeling this by a suitable stochastic process. Incorporating this process will increase the projects' total volatility.

The breakdown probability is assumed to be higher for the existing plant than for the new ones, especially prior to refurbishing. The costs in relation to such breakdowns can be considerable. In this paper, these costs are incorporated as an expected yearly breakdown cost divided by a yearly average production. Inclusion of a stochastic process modeling the breakdown occurrence would also increase the total volatility, and hence the option value.

Construction time and investment cost

Simplifications have been made regarding the construction time. We have not taken into account that there will be a construction period of three to four years, which also results in an overestimation of the present value of both project A and B, and hence the option value. A simplification regarding the timing of the investment is made. Normally, investment costs are spread over the entire construction period, with the largest payments at the last two years. For simplicity, in this paper, the entire investment is assumed to take place when the decision is made. The effect of this assumption will to some extent offset the overestimation made when neglecting construction time.

Competitors' response

It is important to note that other producers face similar investment problems with similar uncertainties. Therefore, competing firms' strategies affect the future profits of the producer. If we consider all firms as price takers, high future prices and more price structure offer profit opportunities also for competing firms. The question is if a response from competitors contributes to dampen the changes in price structure. If it does, the upside will be lower than it first assumed, and since the investments are regarded as irreversible, there can be a considerable downside related to the investment (Dixit & Pindyck, 1994). Relevant in this setting is also the fact that competing firms may upgrade to higher discharge capacities at a lower cost than in our case. It is therefore likely that these firms will exercise their expansion options first. Another question is how much additional capacity is needed to level out the effects caused by more interconnections to other electricity markets. Since cable connections have carrying capacity limitations, the amount of profitable peak load effects is also limited. However, this is a study outside the scope of this paper. In the analysis we have assumed that the producer can take full advantage of both the price and the price structure scenarios proposed in the paper, and thus we disregard the uncertainty represented by competitors' strategies in its entirety.

7.2 The production simulation model

We once again point out that the main purpose of developing the production simulation model is to be able to calculate the input parameters required by the option valuation model. The model is by no means a perfect planning tool to employ in generation scheduling. However, we consider it as a fairly accurate tool by means to construct realistic cash flows for the plant lifetimes, in a setting where there is always a trade off between time consumption and realism.

7.2.1 The threshold function

Figure 7 illustrates the production strategy. A random sample of a period of four years of the threshold function for the intermediate project A is provided for the case of illustration. For a given spot price and forward curve forecast, and with a given reservoir level and inflow, the threshold function is calculated. When the spot price exceeds the threshold function, water is released as long as the reservoir constraints are met. One can see that the threshold function to some extent shaves off bottom parts of the spot price series, although there are some periods where the threshold suggests production on relatively low price levels and vice versa.

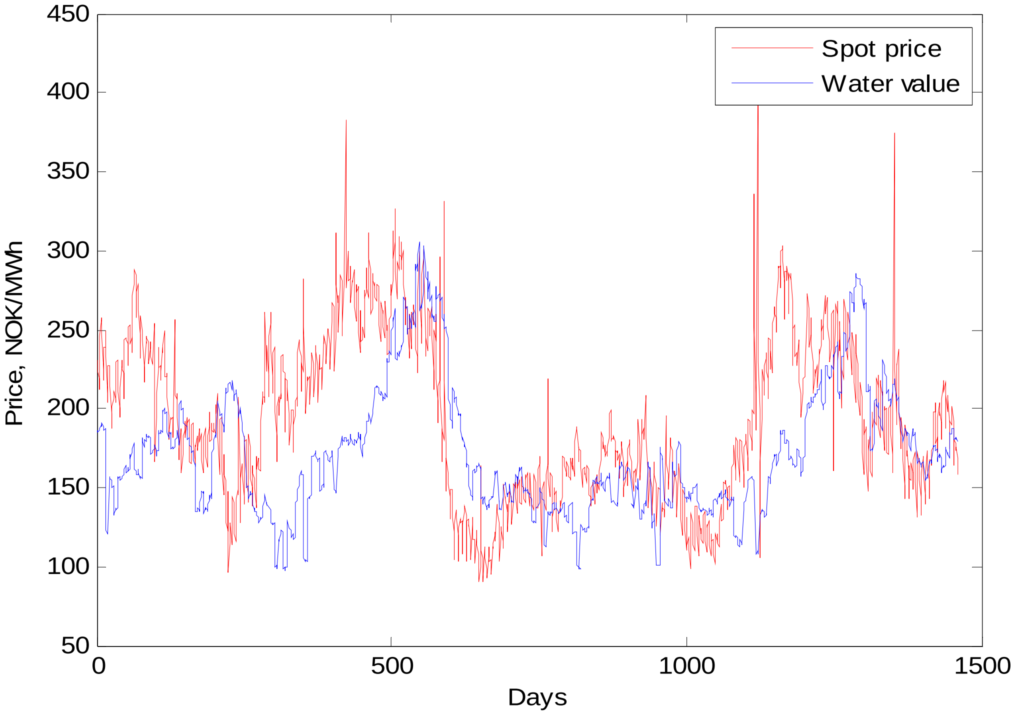

Figure 7 Comparison of spot price forecast and threshold function

The mathematical form of the threshold function implies that deviations in inflow and storage level are penalized asymmetrically, depending on whether deviations are of positive or negative magnitude. For instance, a deviation in storage level of a certain amount below the median will, due to the exponential form, increase the water value relatively more than would be the case if a decrease were to take place because of the same deviation of storage levels above the median. This asymmetry does not apply for the forward curve component.

Knowing that the production threshold defines a heuristic, we tested several versions of the function differing in mathematical form and/or number of parameters included. The simulations showed that inclusion of more parameters did not capture significant additional value.

Response time

In section 5.2 we assumed response times equal to zero for all plants. The cascaded existing plant has considerable longer response times than is the case if either of the projects is completed. By assuming zero response time for all plants, we underestimate the value of both project A and project B. Short response time is also attractive if the regulating power market is taken into account.

7.2.2 Reservoir handling

We have simulated the total amount of spillage for each plant design throughout their lifetimes and come up with approximately the same levels. Table 6 shows the mean yearly spillage levels and spillage in percentage of mean yearly inflow.

Power plant	Mean yearly spillage $[Mm^3]$	Percent of mean yearly inflow
Existing	26,9	0,0170
Intermediate	27,6	0,0175
Large	26,5	0,0168

Table 6 Mean yearly spillage

The similar mean spillage across the designs may seem surprising as one would expect that large power stations easily avoid more spillage because of the increased capability to release water at upper reservoir levels when inflow exceeds anticipated levels. Therefore one can assume that the spillage risk is kept approximately at the same level and that the large plants generally speculate more at higher reservoir levels, taking into account the greater discharge capacity. This fact is also shown in Figure 8. The larger the discharge capacity, the stronger the tendency to operate at high storage levels before the winters. Figure 8 shows simulation medians for the three plants for three consecutive years. From the figure we can see that the larger the plant, the more water is released during wintertime. Compared with the historic median, Figure 8 shows that the production strategy operates at more extreme reservoir levels at both ends of the reservoir.

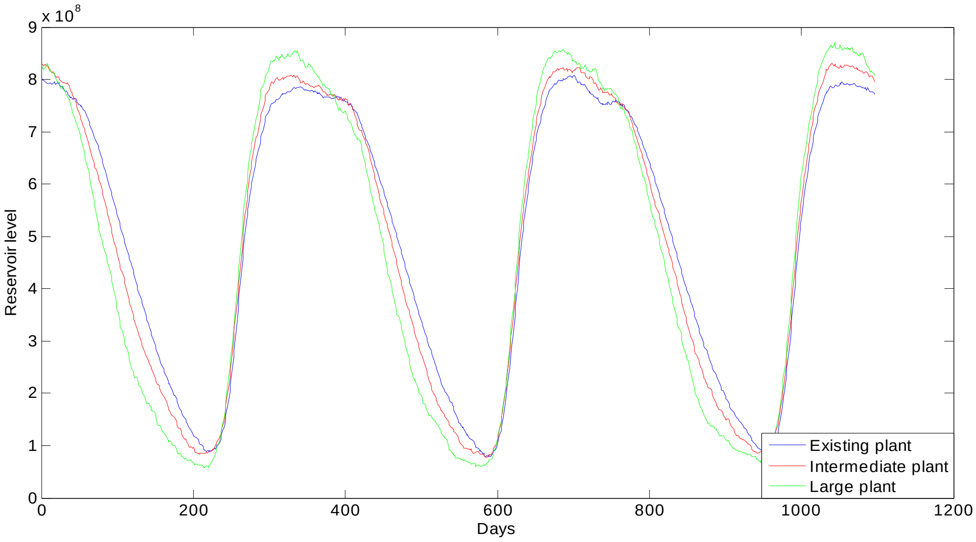

Figure 8 Simulated reservoir medians

In general, the large plants deviate more from their medians than the smaller. This can also be seen in Figure 9 in the appendix. In reality one would not assume that a large reservoir such as the one in this case will be entirely emptied, as this could contribute to situations where rationing must be adopted. Therefore, we suggest that a natural extension of the threshold function is inclusion of a parameter that additionally penalizes the discharge likelihood when the reservoir storage level approaches zero.

7.3 Price model

Since the price model is in commercial use for generation scheduling and investment decision support. It is assumed to be of substantial quality. Given the sensitivity of the model we will not elaborate further on the price model.

7.4 Overall evaluation of the decision support tool

The weakest parts of the model are considered to rest on the uncertainty regarding the estimated parameters from the production simulation. We recommend performance testing of the model, by back testing with historical data. As we have pointed out, reservoir handling can be more realistic if one includes additional parameters to manage handling at extreme storage levels. If computationally possible one should also increase the number of simulations thus reducing the variance of the estimates.

Given the simplifications made in the production simulation and the simplifications made with regards to the discount rate, the present values estimates might be biased and hence the assumptions made in Samuelson's theorem will thus be violated. Although it is difficult to quantify the possible errors caused by this violation, we assume that our results are reasonable enough to be implemented. However, one should be aware of the simplifications, and use the results with care.

8 Concluding remarks

The ambition of this paper was to develop a model to support investment decisions in the hydropower industry. The model is based on valuation of an expansion option, where an investor has the opportunity to choose from a discrete range of plant sizes. We have developed a tool to find the optimal strategy, that is, to determine both the optimal timing as well as the correct capacity, using a real option approach. More specifically we have employed an investment case from Hydro, with an existing production facility in need of refurbishment, and with an opportunity to expand by executing one of two projects; an intermediate expansion project or a large expansion project.

Our model indicates that the optimal strategy would be to immediately invest in the intermediate expansion project. This strategy has a value of MNOK 234, compared to the value of MNOK 135 when realizing the large expansion project. The value of postponing the project is MNOK 77. The profits gained from higher efficiency, lower variable costs and higher mean prices therefore justify the investment cost and exceed the opportunity cost of exercising.

The inputs needed by the ROA are found using a production simulation model. There is some degree of uncertainty regarding these figures, as the production simulation model is simplified. As a consequence our present values might be biased, and thus violating the assumptions of Samuelson's theorem, which forms a basis for the real option analysis.

The refurbishment of the existing plant is assumed to be divided into three planned points in time. The timing aspect is highly sensitive to when refurbishment is required. The analysis shows that the result is rather insensitive to what happens after the second refurbishment. This is mainly due to the fact that the value of holding the option at this point is diminished, and that exercise is always optimal as long as one of the projects is in the money.

When simulating production, our model indicates that larger plants profit from increasing integration of the Nordic and the continental market. Simulations of a scenario with high degree of market integration indicates that the present value of the intermediate sized plant increases by MNOK 25 whereas the present value of the large plant increases by MNOK 63. It is therefore important to note that if more interconnectors than expected are to be constructed the largest project may be preferred. However the effects of market integration are uncertain and further studies are recommended. Participating in the regulating power market is also considered as an up-side potential for large capacity installations, but have not been included our models.

As our analysis show there are several factors influencing the strategy. When evaluating the factors separately it is often possible to assess how these affect the strategy. However the total effect is difficult to predict. Decision makers should be aware of this and use the results with care. Given correct input parameters, our model is a powerful investment support tool, being able to indicate the optimal strategy when there are two expansion projects at hand.

9 Further work

A natural way to improve the overall decision support tool is to improve the quality of the input parameters to the real option valuation. This can be done by including more intuitive explanation factors in the threshold function, or propose another mathematical form of the threshold.

The analysis shows that increased price structure has impact on the project values. It would have been preferable to perform further analyses on the impact of additional future price structure scenarios. An analysis concerning competitors' response to the changing market conditions and how this affects both price level and price structure is also important. We believe that qualitative analyses of these factors will play an important role in the decision process.

The market for regulating power has not been included in this paper. We assume that is possible to increase profits by participating in this market. We assume further that the increased flexibility that project A and B represents would contribute to a rise in the project value when including the regulating power marked. Increasing market integration combined with building of large scale wind power facilities in Norway could require an increased demand for regulating power.

The investment cost is uncertain and can vary considerably. Due to the activity level in the Norwegian economy as well as the global economy, the investment cost has risen considerably higher than the inflation rate in the last decade. To include the investment cost as an uncertainty factor would be possible within our framework.

A natural extension of the investment model is to include more projects. This means letting the producer choose from more than two discharge capacities, and then evaluate options on more underlying assets. Like other lattice or grid methods the quadrature method suffers from what is called the curse of dimensionality. As more underlying assets are implemented, the computation time increases exponentially. In addition, the implementation will be more complicated, as one has to keep track on and place nodes on the discontinuities like the free boundaries and exercise prices.

10 Appendix

10.1 Project screening

At present time, Hydro has four different expansion projects under consideration. There are two discharge capacities considered. Our framework is not able to differentiate between projects with equal discharge capacities, thus favoring the projects with the lowest investment cost. However the more expensive projects are able to capture additional precipitation, and can thus increase generation by around 45 GWh/year. Our simulations indicate that this extra production is not sufficient to justify the additional investment costs. Therefore we limit the analysis to include the two projects with the lowest investment costs. The excluded projects may still be preferable because of other aspects such as environmental issues, governmental requirements or the need to preserve local acceptance.

10.2 Reservoir handling

Figure 9 shows how the reservoir is handled compared to the median level. The red curves represent the median levels while the blue curves denote simulated reservoir level. The black curves dictates upper and lower bounds for the reservoir level. For the case of illustration, the first five years of a random simulation are shown. First of all, one can see that the median differs with different capacity installations, and that it tends to be generally increasing with higher capacity installations. This means that the producer with increasing capacity has greater opportunity of operating with relatively high reservoir levels, with equivalent spillage risk, and thus has greater possibility of speculating in higher future prices. Simulations show that large installations manage the reservoir at significantly higher levels than the existing plant. The simulations also confirm our hypothesis that large plants operate more independently of their median levels. Further this once again indicates more operational flexibility as is expected by, and wanted, from large installations.

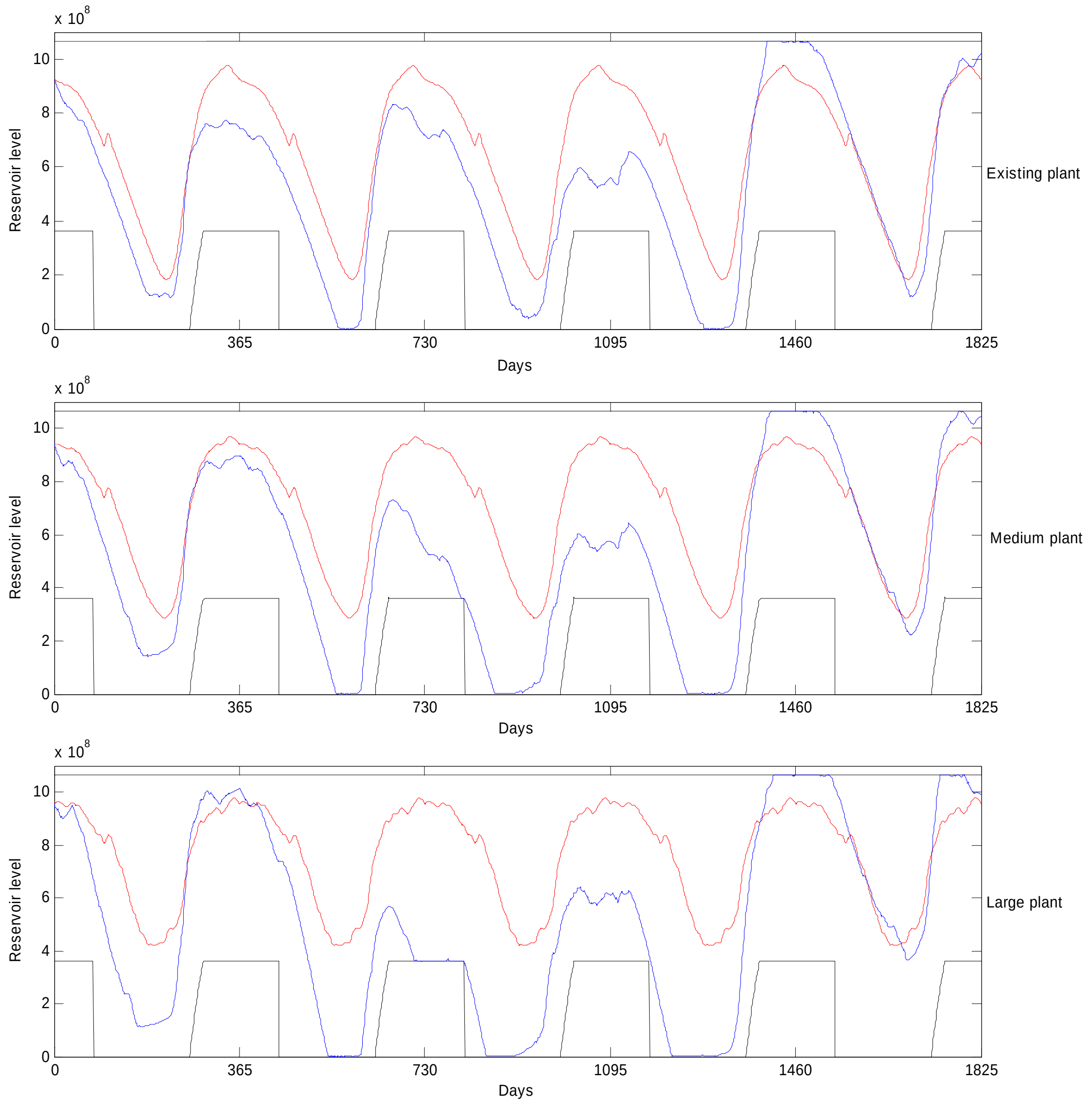

Figure 9 Reservoir handling

10.3 Present value distributions

Figure 10 shows how the present value of difference projects A and B are distributed in histograms. We assumed in the option valuation model that these present values are all non-negative and further log normally distributed. From the histograms, this seems by eye like an acceptable assumption.

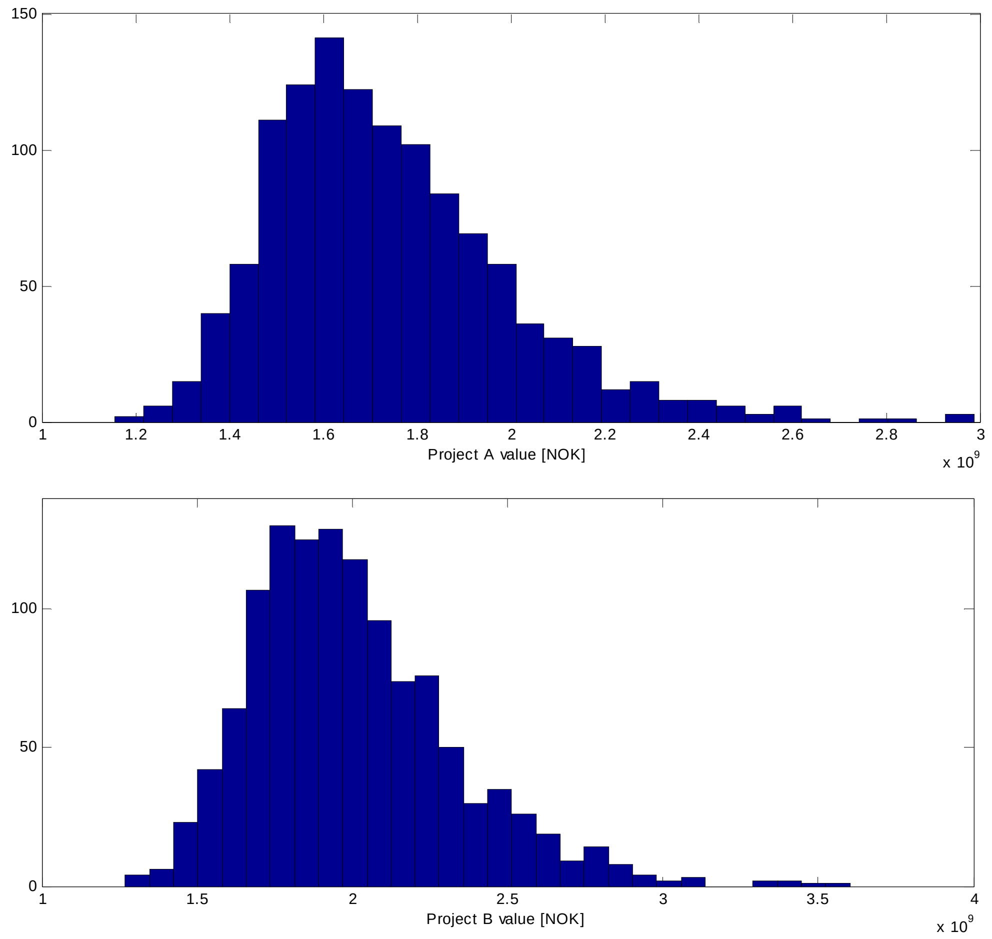

Figure 10 Present value of the projects.

To further test the assumption, the present values are log-transformed and tested for normality using a Ryan-Joiner test. The test for project B is shown in Figure 11. The Ryan-Joiner correlation coefficient close to one indicates normality, but the test rejects the null hypothesis that the distribution of present value of project B is log normal. Because of the need for applicability, we still make the assumption that the underlying assets can be modeled as geometric Brownian motions. According to Samuelson's proof, as long as the project value is properly anticipated (meaning that all investors have complete information about the project), mean reverting and correlated input uncertainties can be combined into one uncertainty, the return of the project, which follows a random walk (Copeland & Antikarov, 2003). This assumption is common in many real option analyses, even if the distribution is not log normal.

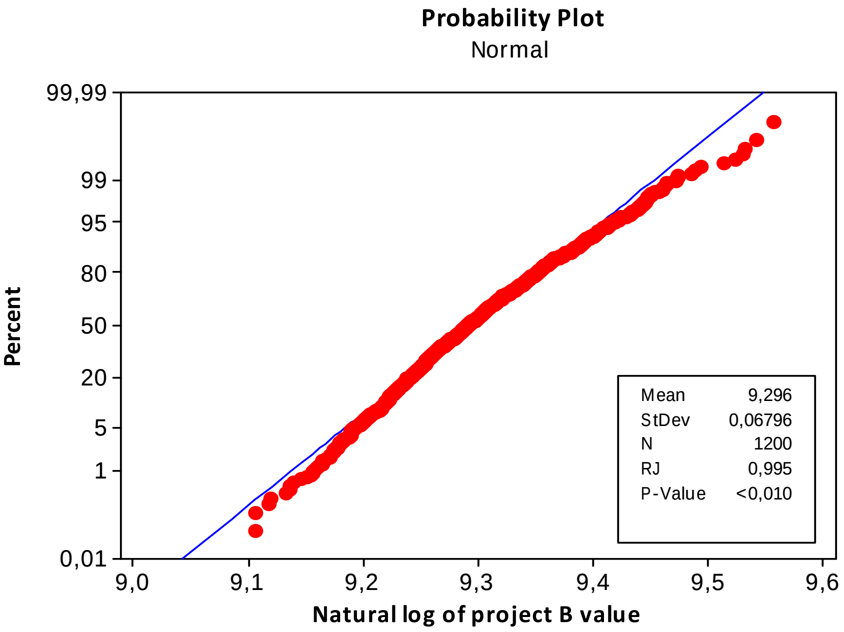

Figure 11 Probability plot of project B

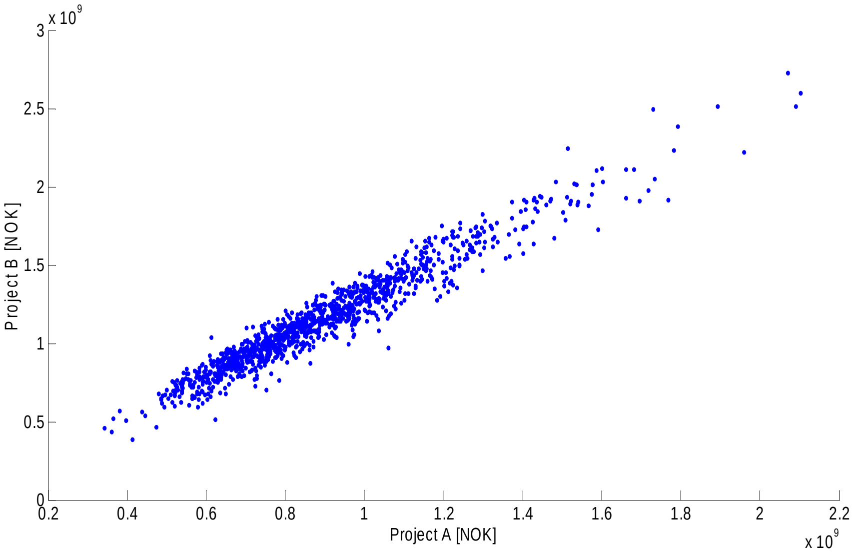

Figure 12 Correlation between the present values of project A and B

Figure 12 shows the correlation between the present values of project A and B. The correlation is significant, with a correlation factor of 0.94. Given that the underlying uncertainties for both projects are the same, price and inflow, this result is expected.

10.4 Uncertainty in investment costs

Figure 13 illustrates how the level of the construction costs has developed in the last 23 years compared to the consumer price index (Statistisk Sentralbyrå, 2008). Both are indexed to a start level of 100 in 1985. The green curve shows the absolute difference between the indexes. We see that the construction costs and the consumer price index are strongly correlated but also that the growth rate of construction costs has considerably exceeded the consumer price the last seven years. However, the extraordinary state of the Norwegian economy has provided a generally high investment level in both industry and power supply sector. Therefore we expect a normalization of the investment costs as the economy cools down. Although difficult to quantify, in our tornado diagram we have roughly estimated the down-side to be a reduction of 25 % and the up-side to 0 %. We point out that these are rough estimates and that a more profound study should be performed to establish good estimates.

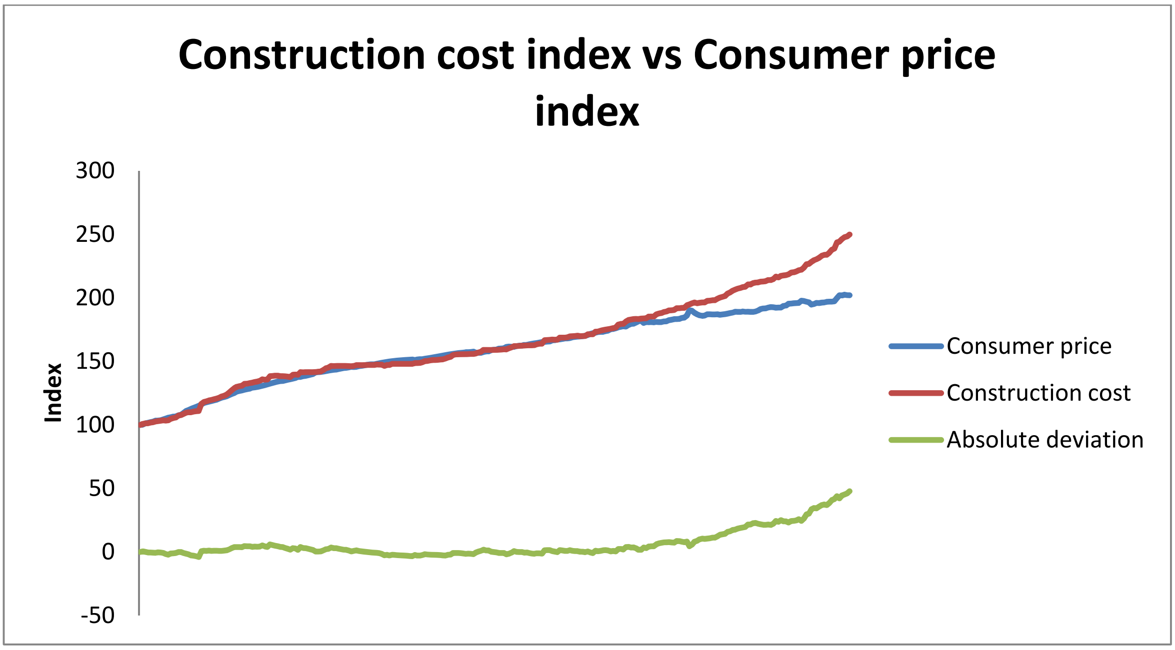

Figure 13 Historic evolution of construction costs

10.5 Calculating the cost of electricity

The cost of electricity (COE) is the total annual cost associated with producing electricity. In the present this constitutes of two components, the annual cost of capital (ACC) and the operation and maintenance costs (OMC):

$$Total\ COE = ACC + OMC$$

The OMC are for both projects estimated to 30 NOK/MWh. The annual capital cost differs between the projects since the largest one is relatively more expensive. By using the discount factor from section 5.1.2 and mean yearly inflow and the efficiency of the plants we calculate the ACC to 41 NOK/MWh and

49 NOK/MWh for project A and B respectively. The COE is then 71 NOK/MWh and 79 NOK/MWh for project A and B.

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