Preface

This report is prepared as our Project Thesis at the Norwegian University of Science and Technology (NTNU) by the Department of Industrial Economics and Technology Management. The preparation of the report has been done during the autumn 2005 as part of the course TIØ 4700 Finance and Accounting, Specialization, and the scope of the project thesis equals 15 ECTS credits. Our work falls within the Group of Investment and Finance Management.

We would first of all like to thank our teaching supervisor, associate professor Stein-Erik Fleten (NTNU), for good support, quick and constructive feedback and interesting discussions during our work on this thesis. We would also like to thank professor Olav Bolland (NTNU) for providing technical guidance and valuable information during the working process.

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Executive summary

In this report a combined heat and power (CHP) plant is analyzed in a real options framework. We consider a CHP-plant that is designated to serve a specified heat load, and the decision to build the plant has already been made. As the investment option is of no interest in this context, we focus on two real options: The option to abandon the plant and the option to operate the plant at part load.

The main purpose of the CHP-plant is to provide heat for an oil refinery, and throughout this report we assume that a fixed heat load at 350 MW must be served for 8100 hours annually. When operating at full load, the plant is able to deliver 280 MW of electricity to the wholesale market in addition to serving the heat demand. There is uncertainty in the prices of electricity, natural gas and CO_2 emission allowances, and this makes it interesting to investigate the CHP-plant by real option analysis. The option to abandon implies that the operator of the CHP-plant can choose to shut down the plant *permanently* and replace it with a set of gas fired boilers. If this option is exercised, the operator will still be able to serve the heat demand, but no electricity output will be available for sale at the power market. A large sunk cost will also have to be accepted. The option to operate at part load implies that the operator can choose to reduce the power output to a specified level during a fixed time period. When the time period ends, the operator can choose to go back to operating at full load or continue operating at part load. The part load option can be exercised several times during the lifetime of the CHP-plant, and no sunk costs are incurred by exercising it.

We use Monte Carlo simulation to estimate the expected cash flows generated by the CHP-plant. Based on these cash flows we calculate the present value of the CHP-plant and evaluate the two real options. The implied value of heat is estimated by calculating the price of heat that is required for the CHP-plant to have a present value of zero. The implied value of heat is calculated both in a traditional net present value perspective and in a real options perspective. The parameter is used to illustrate the importance of the two real options and to compare the cost of producing heat in the CHP-plant with an alternative heat providing technology.

We estimate the implied value of heat to 98 NOK/MWh when no real options are included and to 64 NOK/MWh when the real options are taken into consideration. If the alternative

heat providing technology had been used, the implied value of heat would have equalled 142 NOK/MWh. We also find that the implied value of heat is very sensitive to CO_2 emission costs, and that a strict CO_2 regulation scheme decreases the economic difference between the CHP-plant and the alternative heat providing technology.

Table of contents

List of figures

List of tables

1 Introduction

In this report we perform some analyses on the profitability and risks of operating a combined heat and power (CHP) plant which is designed to provide heat for an industrial process. The analyzes are performed within a real option framework, implying that uncertainty regarding future movements of electricity and natural gas prices as well as prices of CO_2 emission allowances are taken into consideration. The studies are based on Mongstad energy project, which includes the construction of a CHP-plant connected to an existing oil refinery.

The main purpose of the CHP-plant is to provide heat for an industrial process, and this has several economic consequences. The most important consequence is that the CHP-plant is required to run continuously in order to maintain demanded heat deliveries to the oil refinery. This means that one does not have the option to shut down the plant temporarily if market conditions become unfavourable, and one does not have the option to postpone the investment as heat deliveries are required from a certain point in time. One has however two main real options which may affect the profitability of the plant. The first option to be considered is the option to abandon the investment. This means that the CHP-plant is scrapped and replaced with a different heat producing unit. As there will be a certain cost of abandoning the investment, such an option will only be exercised if conditions turn out to be sufficiently unfavourable. In this report we evaluate the option to abandon the investment and also estimate the real economic lifetime of the CHP-plant. The second option to be considered is the option to temporarily operate the CHP-plant at part load. This means that heat is produced partially by the CHP-plant and partially by supplementary duct burners. This option is also likely to be exercised only if market conditions become adequately negative, but as opposed to the option to abandon, this option can be exercised temporarily. In this report the option to operate at part load is evaluated, and its impact on the profitability and real economic lifetime of the CHP-plant is considered. We find that the abandonment option and the part load option both have substantial values, and that they offer downside protection for the owners of the CHP-plant. The real economic lifetime of the CHP-plant equals 32 years when the option to abandon is available. When also the part load option is introduced, the real economic lifetime is increased to 35 years.

In addition to evaluating the two real options mentioned above and analysing the real economic lifetime of the CHP-plant, we also carry out some more general economic analyses. Firstly we evaluate the implied value of the heat that is produced by the CHP-plant. This is done by estimating the economic value of the produced heat that is required in order to break even. This is done both in a conventional net present value perspective and in a real options perspective. We find that the implied value of heat equals 98 NOK/MWh when no real options are considered. Evaluating the implied value of heat illustrates the value of the real options in a very intuitive way, and it also reveals economic information about producing heat with a CHP-plant. We estimate an implied value of heat equal to 64 NOK/MWh when the option to abandon and the option to operate at part load both are taken into consideration. We also estimate the implied value of heat produced in conventional gas fired boilers, and this value equals 142 NOK/MWh.

One of the main risks that come with an investment in gas fired power production is the cost of CO_2 emissions. In this report the cost of CO_2 emission allowances is analysed, and sensitivity analyses is performed regarding the long-term price level of such allowances and the share of allowances required to be bought by the operator of the CHP-plant. We investigate how sensitive the profitability of the CHP-plant, the real economic lifetime and the implied value of heat are to changes in these factors. Both the real economic lifetime of the CHP-plant and the implied value of heat appear to be very sensitive to the cost of CO_2 emissions. We demonstrate that a strict CO_2 regulation scheme, where quota prices are high and a large share of quotas need to be bought, implies that the CHP-plant is more likely to be abandoned or operated at part load.

This report is divided into 14 chapters, which can be separated into five main parts. The first part provides information about the CHP-plant and about Mongstad energy project in general. In the second part the characteristics of the markets for electricity, natural gas and CO_2 emission allowances are given. The third part provides information about the mathematics used for analyzing the profitability of the CHP-plant including spot price models and parameter estimation. The fourth part contains a long-term view on the economics of natural gas fired power production and the carbon market. The last part contains the results of the computations as well as relevant discussions and conclusions.

2 The Mongstad Energy project

The main purpose of this report is to use real option analysis to evaluate and analyze an investment in a CHP-plant located at Mongstad. A more detailed description of the concept of combined heat and power generation is provided in appendix 1, and readers who are unfamiliar with this concept are advised to read appendix 1 before reading this section.

The largest oil refinery in Norway is located at Mongstad, and the main objective of the CHP-plant is to provide heat for the oil refinery. Oil refining is a process that requires a large supply of heat, and the heat demand can be covered in several ways. Today heat is provided by a large network of furnaces and boilers which are used to heat crude oil and to produce steam. The construction of the CHP-plant is part of a larger project named Mongstad energy project, and the CHP-plant will replace several of the boilers and furnaces that are in use today. In addition to this the CHP-plant permits an upgrading of the oil refinery that increases the output amount of marketable products. Mongstad energy project includes the construction of a natural gas pipeline to Mongstad and the tie-in procedure which is necessary in order to transfer heat from the CHP-plant to the oil refinery. Although supplying heat to Mongstad oil refinery is the main purpose of the CHP-plant, the CHP-plant will also produce electricity. The electricity will be supplied to the local electricity grid as well as to the oil refinery, the Troll A platform and to Kollsnes gas processing plant.

2.1 Project description

The CHP-plant will generate electricity with gas turbines, and the hot exhaust gas that exits the gas turbines will be used for crude oil heating and for steam production. According to Statoil this solution has a longer time-frame than today's solution with boilers and furnaces. The company also believes that the CHP-plant will be a vital part of the development of business activity in the Mongstad area.

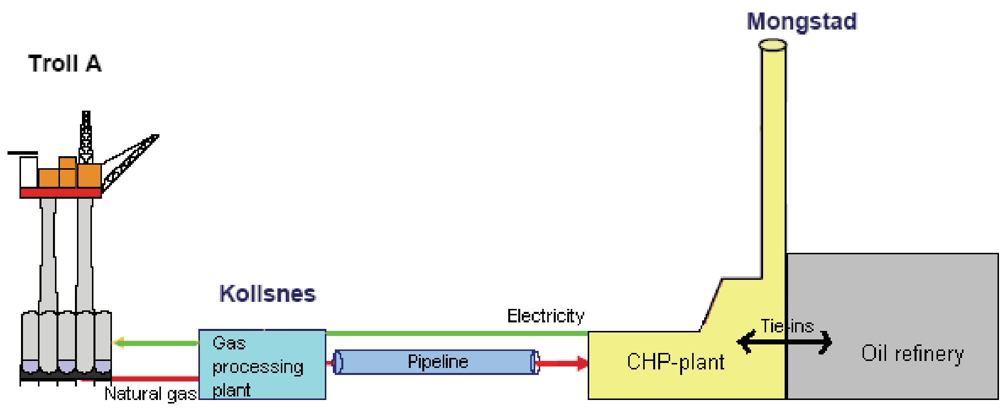
Statoil ASA is the operator of Mongstad oil refinery, Kollsnes gas processing plant and of the Troll A platform. The CHP-plant at Mongstad will be integrated with the oil refinery, and the heat supplied by the CHP-plant will be necessary for the oil refinery to function normally. Statoil has decided that the Danish company Elsam AS is to be responsible for developing and operating the CHP-plant. The reason for this is that such operations are Elsam AS' core competence.

Mongstad energy project includes three sub-projects, which are illustrated in Figure 1.

- A new gas pipeline from Kollsnes gas processing plant to the Mongstad area
- A combined heat and power (CHP) plant located at the Mongstad area with a capacity to produce 280 MW of electricity and 350 MW of heat
- Necessary tie-ins and upgrading of the existing oil refinery at Mongstad

The total investment cost for the project is estimated to be about NOK 3,75 billion[1].

Figure 1: Project overview



Schematic overview of the Mongstad Energy project.

As the objective of this report is to provide economic analyzes regarding the CHP-plant, we consider the CHP-plant to be a separate investment. It is in this context important to mention that the construction of the CHP-plant assumes that the other investments included in the Mongstad Energy Project also are conducted. The natural gas pipeline between Kollsnes and Mongstad is necessary to provide natural gas for the CHP-plant, while the tie-in procedure between the CHP-plant and the oil refinery is necessary to provide heat for the refining process. Despite this the normal solution in the natural gas business is that the owners of Kollsnes gas processing plant will be responsible for constructing the pipeline, while the owners of the oil refinery will be responsible for constructing the tie-ins. The owner structure of the CHP-plant is unknown to us, but it has been stated that Statoil ASA will be the main

[1] Statoil ASA

owner, and that Elsam AS also will be part of the constellation. This means that the owner structure of the CHP-plant will be different than the owner structure of Mongstad oil refinery. Based on this it seems natural to consider the construction of the CHP-plant as a separate investment.

2.2 Project objectives

According to Statoil the main driving force behind Mongstad energy project is to improve the competitive position of Mongstad oil refinery by increasing the plant's energy utilization, as well as securing the energy supply and distribution system in the Mongstad and Kollsnes area. Mongstad energy project will attach the oil refinery to Kollsnes gas processing plant. This means that natural gas form the Troll field will be available at Mongstad, while electricity produced with the same natural gas as input will be available at Kollsnes gas processing plant and at the Troll A platform.

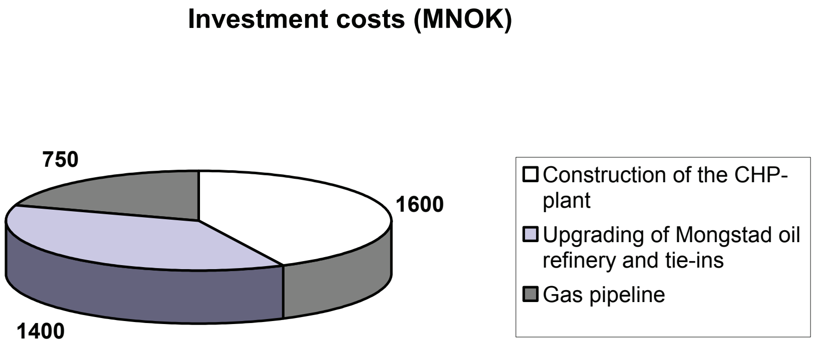
The availability of natural gas at the Mongstad area permits the construction of the CHP-plant which is integrated with the oil refinery. This plant will act as a common energy central for the Kollsnes and Mongstad area. Statoil claims that the CHP-plant is a necessary prerequisite in order to be able to utilize energy at Mongstad oil refinery in an effective and environmentally friendly way. A total energy efficiency of about 70 % is expected for the CHP-plant.

The main energy carrier of the Troll A platform and of Kollsnes gas processing plant is electricity. The electricity is mainly used to drive both onshore and offshore compressors. These compressors are used to increase the pressure of the natural gas which is exported through pipelines from Kollsnes gas processing plant and to increase the downhole pressure in the Troll reservoir in order to maintain the required production rate. The electricity demand at both the Troll A platform and at Kollsnes gas processing plant is expected to increase significantly in the coming years due to reservoir drainage and increased natural gas export from Kollsnes. The CHP-plant at Mongstad will cover the electricity needs at Kollsnes and at the Troll A platform. In 2004 the electricity consumption of the Troll A platform and of Kollsnes gas processing plant was about 1,4 TWh. This number is expected to increase to about 3 TWh within 2020. The electricity supply to the Bergen region may occasionally be problematic, and increased electricity demand at the Troll A platform and at Kollsnes will make this situation even worse. The execution of Mongstad energy project will improve this situation as the electricity production in the Bergen region is increased. According to Statoil this has an economic value because it leads to less power outages in the region meaning that the Statoil operated facilities can improve their availability.

2.3 Economic considerations

The total investment costs of Mongstad energy project can be divided into three main components. The allocation of the investment costs are shown in Figure 2.

Figure 2: Allocation of investment costs



Constructing the CHP-plant requires the construction of the gas pipeline as well as the tie-in procedures with the oil refinery. It is assumed that these investments will be conducted by the operators of the Troll license and Mongstad oil refinery respectively. As the Troll license, the CHP-plant and Mongstad oil refinery all have different owner constellations; they should be treated as separate investments, although the construction of the CHP-plant depends on the other investments being conducted.

In the next chapter it is described how the heat produced by the CHP-plant replaces heat production in furnaces and boilers which use refinery fuel gas as fuel. This product is a by-product of the refinery process, and no alternative market for this product exists. It is assumed that the same amount of fuel gas is available after the construction of the CHP-plant, and that fuel gas is used together with natural gas as gas turbine fuel.

An important parameter in this report is the implied value of heat. This parameter is calculated by determining the value of heat that is required for the investment in the CHP-plant to break even. The implied value of heat indicates the cost of producing heat in the CHP-plant, and the calculation of this parameter enables the possibility of comparing the CHP-plant with conventional gas fired boilers. The value of refinery fuel gas is set to zero in all calculations in this report. The reason for this is that no alternative market for refinery fuel gas exists, so there are no available market prices. The text box shows that in such a setting the price of fuel gas should equal zero. When the CHP-plant is compared with gas fired boilers, it is assumed that the input amount of fuel gas is the same in both cases, and that the remaining fuel requirement is covered by purchasing natural gas.

Price of fuel gas > 0

- The oil refinery receives positive cash flows from selling fuel gas to the CHP-plant
- Cash flows from the CHP-plant decreases
- Implied value of heat increases
- The price of heat increases
- The increased cash flows received from selling fuel gas is exactly offset by the increased payments for heat
- Conclusion: The oil refinery has no reason to charge for the fuel gas

When economic analyzes are performed on a CHP-plant like this, it is important to consider the fact that heat is the main product and electricity is a by-product. The reason for making this assumption is the plant configuration that has been chosen, which results in low efficiency. If electricity was considered to be the main product, a different plant configuration, which resulted in higher efficiency, would have been chosen. Some possible plant configurations are described in appendix 1. It is assumed throughout this report that the price of electricity equals the area price N 01 at Nordpool. This means that it has no economic consequences whether electricity is sold to Kollsnes, the Troll A platform or delivered to the local electricity grid.

2.4 Technical description of the combined heat and power plant

A CHP-plant utilizes a combustion process in order to produce heat and power simultaneously. The main principle of combined heat and power production is that the energy content of any fuel can be separated into high-grade energy and low-grade energy. High-grade energy is also referred to as exergy, and it includes the energy that can be transformed entirely into mechanical work. The remaining energy, which can only partially be transformed into mechanical work, is referred to as low-grade energy. Electricity is an example of high-grade energy, while heat is an example of low-grade energy. A conventional combined cycle gas turbine (CCGT) power plant is designed to maximize the electricity output, and in such a plant no usable heat is produced. A CHP-plant, on the other hand, is designed to produce both electricity and usable heat. This is obtained by sacrificing some of the potential electricity output in order to produce applicable heat.

The main parts of the CHP-plant at Mongstad are two 130 MW gas turbines, each connected to a heat recovery steam generator (HRSG). Inside the HRSG-units both crude oil heating and steam production take place. In addition to this a furnace is also used for crude oil heating. The furnace produces 21 MW of heat, while the remaining heat is provided by the CHP-plant. The CHP-plant provides approximately 76 MW of thermal energy for crude oil heating, while it provides approximately 270 MW of thermal energy for steam production. The total heat production then equals about 350 MW. Some of the steam that is produced is expanded through a 20 MW condensing steam turbine, and the condensed water is injected into the feed water stream before it enters the HRSG-units. This means that the total power production capacity for the CHP-plant equals 280 MW. A gas turbine process connected to a HRSG-unit is shown in Figure 3, while the entire process of the CHP-plant is shown in figure 4.

Figure 3: Gas turbine including HRSG

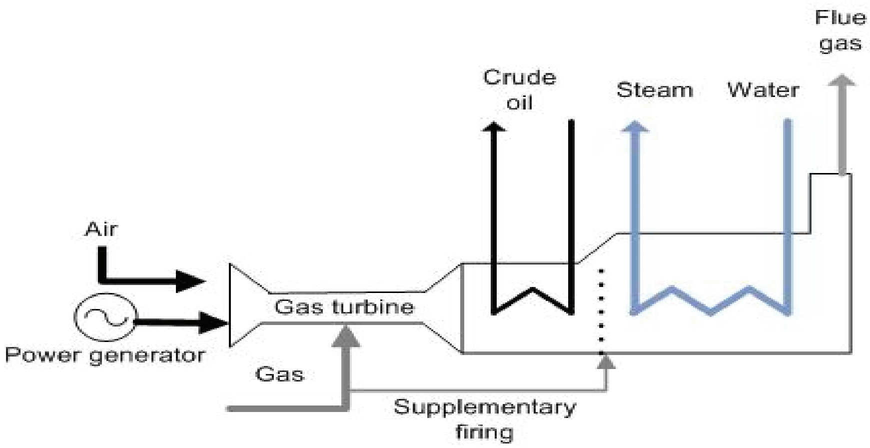
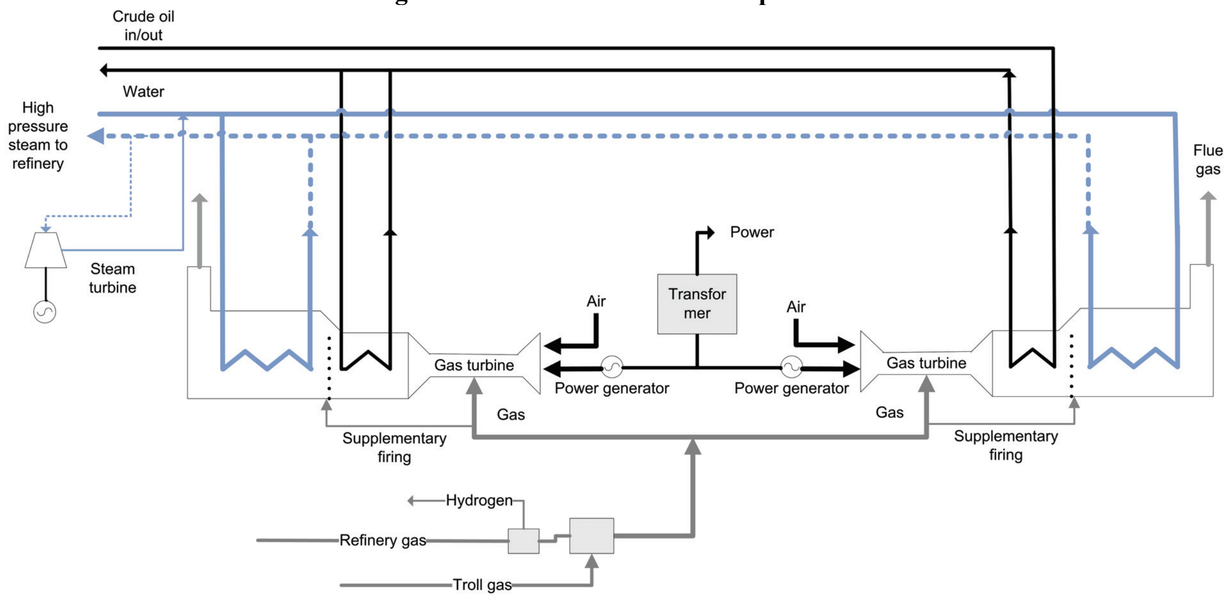


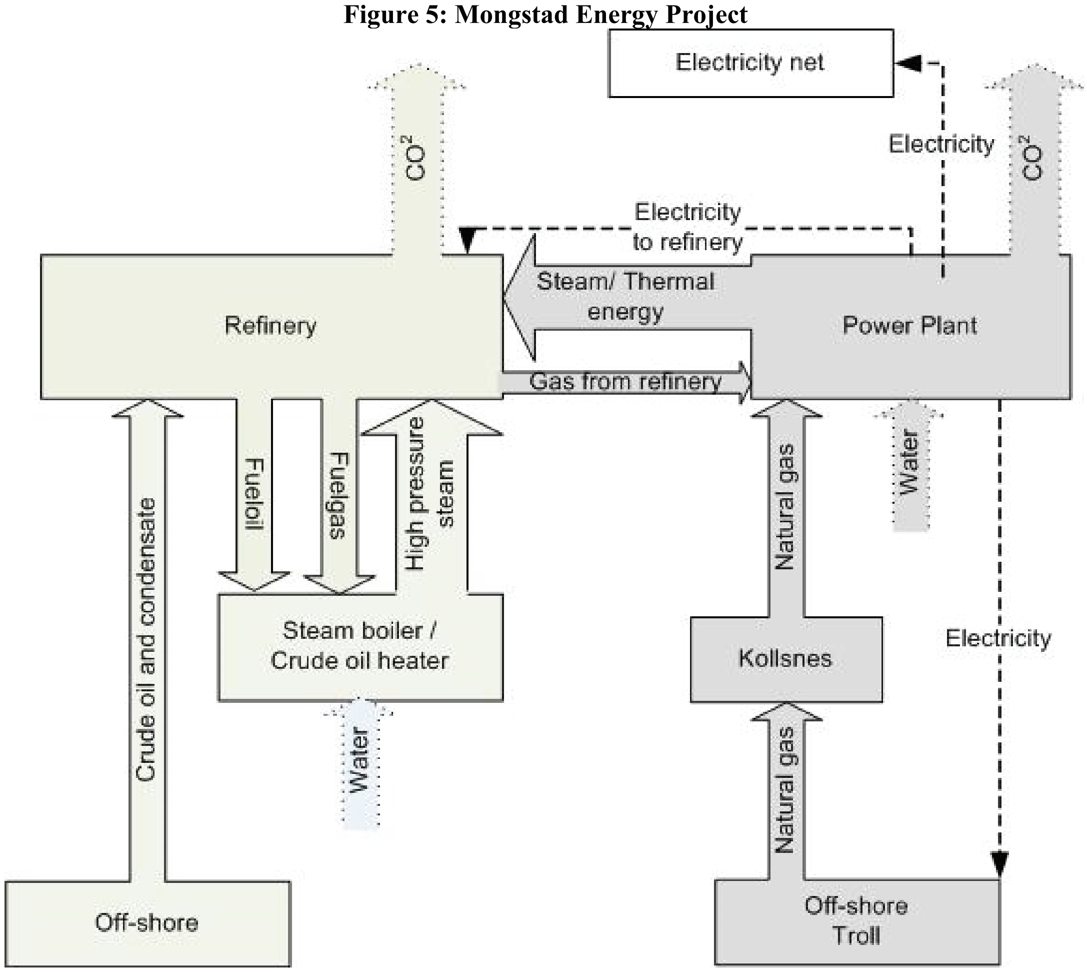
Figure 4: Flow chart for the CHP plant



It is shown in Figure 3 and figure 4 how the hot flue gases from the gas turbines first are used for crude oil heating and then used for steam production. It is assumed that 260 tons of steam at 330°C and 30 bar is required per hour.

Figure 5 provides a schematic representation of the streams that flow in and out of the CHP-plant. Here it can be seen how natural gas, electricity and heat flows between the CHP-plant, the oil refinery and the other installations.

Figure 5: Mongstad Energy Project



Flowchart for Mongstad Energy project

The CHP-plant is expected to operate 8100 hours per year, which corresponds to 92 percent availability of the gas turbines. An overview of the characteristics of the CHP-plant given this availability is shown in Table 1. Efficiency calculations are shown in appendix 3.

Table 1: Characteristics of the CHP-plant

Cost	Value	Unit
Electricity production capacity	280	MW
Expected yearly delivery	2,3	TWh
Heat production capacity	350	MW
Fuel gas consumption	0,2	GSm3/year
Natural gas consumption	0,5	GSm3/year
Efficiency	34	%
Total efficiency	70	%
CO_2 emissions	1 300 000	tons/year

2.4.1 Working conditions and flexibility

The HRSG-units will be equipped with duct burners in order to be able to maintain sufficient heat production if one of the gas turbines is taken out of production. In addition to this the duct burners will also be able to provide further heat production if the heat demand is increased. This provides flexibility and it enables the planning of future projects based on heat deliveries from the CHP-plant. The CHP-plant will be an integrated part of the oil refinery, and under normal conditions the electricity production will depend on the heat requirement of the oil refinery. In this report it is assumed that the heat requirement of the oil refinery constantly equals 350 MW. In reality the heat requirement will not be constant, but the variations are assumed to be so small that they would not affect our results significantly. In situations where the local power grid falls out, it will be possible to take the CHP-plant into island mode. This means that the plant can be disconnected from the electricity grid, but still produce sufficient electricity and heat for the oil refinery to function. This means that the risk of having to reduce production due to electricity outfalls is more or less eliminated, which of course is favourable.

The configuration with two gas turbines and duct burners allows planned maintenance to be performed on one turbine at a time while still being able to deliver the heat that is required by the oil refinery. This configuration also has an economic value as the operator of the CHP-plant has the option to take one of the gas turbines out of production if market conditions become sufficiently unfavourable. Evaluating this real option is one of the topics of this report. The reason for this option having an economic value is that there is uncertainty about the electricity and natural gas prices. If operating conditions become sufficiently negative, meaning that natural gas prices approaches the electricity prices or even become higher than the electricity prices, operating only one of the gas turbines will be more profitable than operating both of the gas turbines.

2.4.2 Variable operating and maintenance costs

The operation and maintenance costs of the CHP-plant are assumed to be 4 percent of the investment costs annually[2]. In addition to this annual insurance costs equal to 0,5 percent of the total investment costs are assumed[2]. As mentioned earlier in the report, 92 percent

[2] Bolland (2005)

availability is expected, and this includes both planned and unplanned outages. This number is based on the operation and maintenance program of the gas turbine vendor General Electric, which the operator of the CHP-plant is expected to follow. This program includes inspection and overhaul of the hot parts of the gas turbines every third year. The hot parts of the gas turbines include the turbine blades, which are the most expensive gas turbine components. Typical O & M costs for a 350 MW CCGT plant using a GE STAG109FA are given in appendix 4. According to the values in appendix 4, the maintenance of the hot parts of the gas turbine constitutes approximately 85 percent of the total maintenance costs. In our valuation model this amount has been assigned to every third operating year according to the GE maintenance program. This could have some implications for the valuation of the abandonment option, as it is possible that the CHP-plant will be abandoned in one of these years.

2.5 Input parameters used for analysis and evaluation

When considering the economics of the CHP-plant, several input parameters are needed. In order to estimate the cash flows from the investment, the most important parameters are the availability of the gas turbines, power output, heat rate, consumption of natural gas, CO_2 emissions, insurance cost and variable O & M costs. The value of these parameters is summarized in Table 2.

Table 2: Operation and maintenance data

Cost	Value	Unit
Gas turbine availability	92	%
Annual operating hours	8100	hours
Power output	280	MW
Heat output	350	MW
Heat rate	2,96	MWh_{fuel}/MWh_{el}
Natural gas consumption	2,48	$MWh_{natural\ gas}/MWh_{el}$
CO_2 emissions	0,565	$Tons/MWh_{el}$
Insurance costs	7	MNOK annually
Regular O & M costs	30,1	MNOK annually
O & M costs every third year	51,8	MNOK annually

The insurance cost is assumed to be 0,5 percent of the total investment cost of the CHP-plant annually. Operational costs are assumed to be 2 percent of the total investment cost of the CHP-plant annually. Maintenance costs are assumed to be 0,15 % of the total investment cost of the CHP-plant annually in regular years and 1,7 % every third year when gas turbine maintenance is performed.

When real option evaluation and economic analysis are performed, some additional input parameters are required. These parameters are given in Table 3. The maximum expected lifetime of the CHP-plant is set to 50 years in this report. This means that we assume that the oil refinery at Mongstad will be operated for the next 50 years, and that heat deliveries are required during the entire lifetime of the refinery. The reason for choosing this timeframe is that there is uncertainty regarding how long crude oil will be available for refining at Mongstad. Only crude oil from fields on the Norwegian continental shelf is refined at Mongstad, and as oil production from these fields has a limited timeframe, 50 years seem like a reasonable estimate.

Table 3: Project data

Cost	Value	Unit
Investment cost	1600	Nok
Construction period	2	Years
Start up time	01.01.2009	
Maximum expected lifetime	50	Years
Risk free interest rate	5	% annually

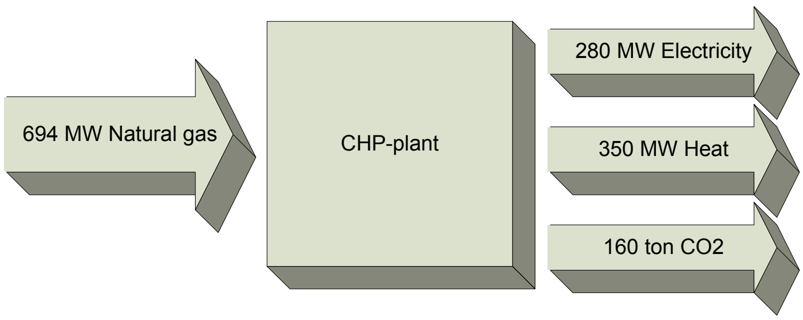
3 Operating scenarios

In this chapter the different operating scenarios for the CHP-plant, which are the foundation for the real option analyses, are presented. We only provide qualitative descriptions of the scenarios here, while the valuation algorithms are given in chapter 7.

3.1 Normal operation scenario

In the normal operation scenario the CHP-plant is run continuously at full load for 8100 hours annually. It is assumed that the operator of the CHP-plant is obliged to deliver heat to the oil refinery for 50 years. This scenario provides the foundation for calculating the NPV of the CHP-plant and the implied value of heat when no real options are taken into consideration. The scenario is illustrated in Figure 6.

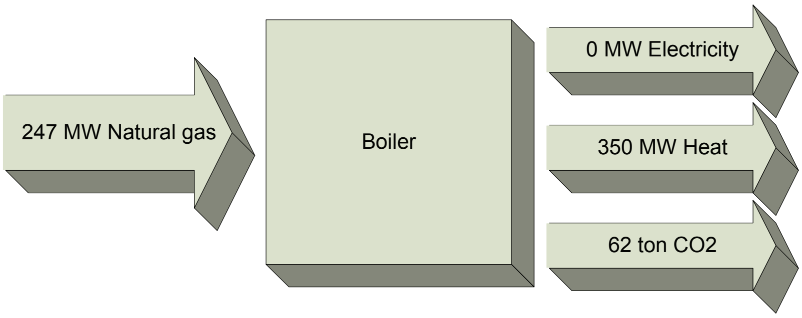
Figure 6: Input/output per operating hour normal scenario



3.2 Abandonment scenario

In the abandonment scenario the operator of the CHP-plant has the opportunity to abandon the plant and replace it with a different heat producing unit which delivers the same amount of heat. The operator is still obliged to cover the heat demand for 8100 hours annually for 50 years. Abandoning the CHP-plant implies that the gas turbines are *permanently* decommissioned. This means that the operator can not restart power production in any period after the abandonment option has been exercised. When the abandonment option is exercised, it is assumed that the operator purchases a boiler with a capacity of 350 MW. It is further assumed that this boiler is operated at full capacity for 8100 hours per year for the remaining expected lifetime of the refinery (50 years minus the time of exercise). The price of the boiler is set to 100 MNOK[3], and the salvage value of the gas turbines is set to zero[3]. This means that a large sunk cost is accepted if the abandonment option is exercised. The value of the abandonment option depends on entire price scenarios[4], and the valuation procedure can be found in chapter 7.2.2. The abandonment scenario is illustrated in Figure 7.

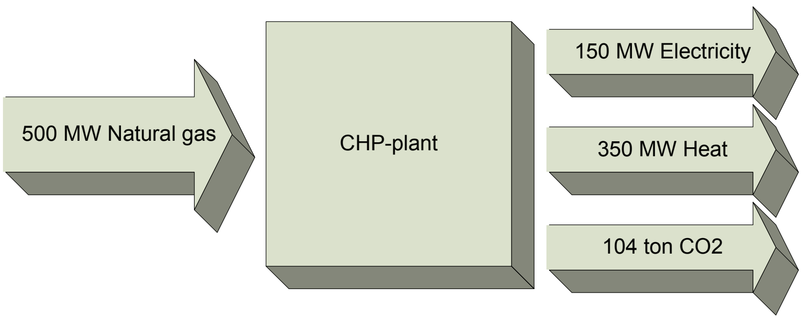
Figure 7: Input/output per operating hour after abandonment



3.3 Part load scenario

In the part load scenario the operator of the CHP-plant has the opportunity to operate only one of the gas turbines. If the operator chooses this solution, it is assumed that one turbine is operated during an entire period. When the next period starts, the operator can choose to continue operating one turbine or to operate both turbines. The part load scenario provides the foundation for evaluating the part load option, which, unlike the abandonment option, can be exercised temporarily. When operating at part load, it is assumed that supplementary duct burners are used to produce heat so that the heat demand is covered. The part load scenario is illustrated in Figure 8.

Figure 8: Input/output per operating hour with part load



[3] Bolland (2005)
[4] A scenario is one set of prices from t_0 to T.

4 Carbon Markets

This chapter is a general description of European carbon markets. Climate change because of the accumulation of so-called greenhouse gases (GHGs) is believed to be our most important environmental issue. Concrete targets for curbing these emissions were established in the Kyoto Protocol in 1997, and the aim is to stabilize the concentration of GHGs in the atmosphere at a level that prevents dangerous human impact on the climate. All EU countries have ratified the Kyoto Protocol, which imposes developed economies to reduce the emissions of greenhouse gases by at least 5 % from their aggregate 1990 levels in the first Kyoto period from 2008 to 2012.

The European Union and Norway have implemented their own schemes for allocating emission rights in a pilot period from 2005 to 2007. These schemes are less extensive than the schemes planned for the first Kyoto-period, and only 10 % of the total GHG-emissions are included in the Norwegian system. The Norwegian Pollution Control Authority (SFT) distributes 95 % of the emission rights free-of-charge in the pilot period[5]. International trading of quotas will be introduced from 2008, making it possible for Norwegian companies to trade emission rights with actors in other countries. Since the two schemes will be indirectly integrated, this paper will focus on the more liquid market in the Union to describe the future trading of emission rights in Norway.

4.1 The EU ETS

The European Union initiated their trading with emission allowances for CO_2 the 1st of January 2005, and the current trading scheme is valid until the first commitment period of the Kyoto protocol. The European Union's Emission Trading Scheme (EU ETS) for this period covers about 12.000 activities and scarcely half of the Union's CO_2-emissions[6].

In addition to the Kyoto agreement, the EU-parlament has stated its own target: to reduce the emissions to a level 8 % below the levels of 1990.The main activities included are power production, refineries, gas terminals, mineral production (fibre glass etc.), iron, steel and

[5] SFT (2005)
[6] Gateway to the European Union (2005)

wood processing. The scheme for the pilot period does only involve CO_2, not the five other greenhouse gases ratified in the Kyoto agreement.

4.1.1 Description of the EU ETS

The EU Emission Trading Scheme is composed of national schemes interconnected, as quotas are traded across borders. Trading can be done on markets or over-the-counter (OTC). Every country has its own plan for allocation of emission rights, which must reflect the country's incentive to reach the goals stated in the Kyoto Protocol. The plans must be approved by the European Commission.

Member States can distribute up to 5 % of allowances by auctioning in the first phase, and up to 10 % in the second phase of the scheme. The reminder of the allowances will be allocated free-of-charge. The share of allowances to be purchased beyond 2012 is unknown. By March 31st every year, each company that is subject to the trading scheme must provide emission rights corresponding to their actual CO_2-emissions the previous year. If a company can not provide sufficient allowances, it has to pay a penalty of about 50 € per ton of CO_2. For the period 2008-2012, this penalty will be raised to at least 100 € per ton of CO_2. The company must also buy the missing quotas in addition to pay the penalty.

4.1.2 Cap-and-trade

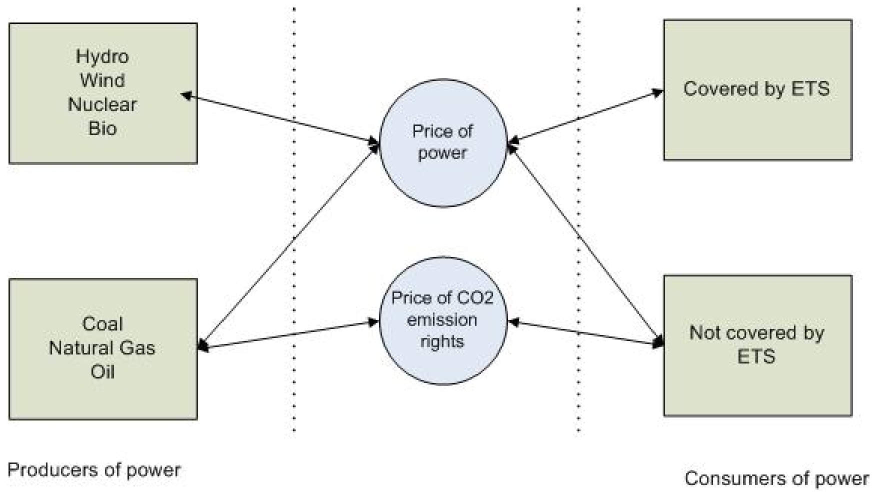
The EU ETS is a so-called cap-and-trade scheme. In addition to be a guarantee for a political given cap to be met, trading of emissions rights is presumable the most cost effective way to meet this cap. An alternative would be to reduce emissions by taxing them. A tax scheme implies exact costs of emission reductions to be known, but there is no guarantee that the cap will be met.

The effectiveness of the cap-and-trade scheme is based upon the companies' incentives to reduce their emissions. If a company can reduce its emissions for a lower cost than the market price of emission rights, it will cut emissions and sell its excess rights at the market. Since other companies can buy them, the market price of the emissions rights will fall to the cheapest cost of reducing emissions.

4.2 The relation between CO_2-allowances trading and the price of power

The total emission limit and the responsibility to own emission rights for its own emissions result in extra constraints in the market equilibrium for power. Since power producers and consumers make up a major part of the allowances market, price of power and emission rights will be set simultaneously. The relation between the markets is shown in Figure 9.

Figure 9: Power market and allowance market



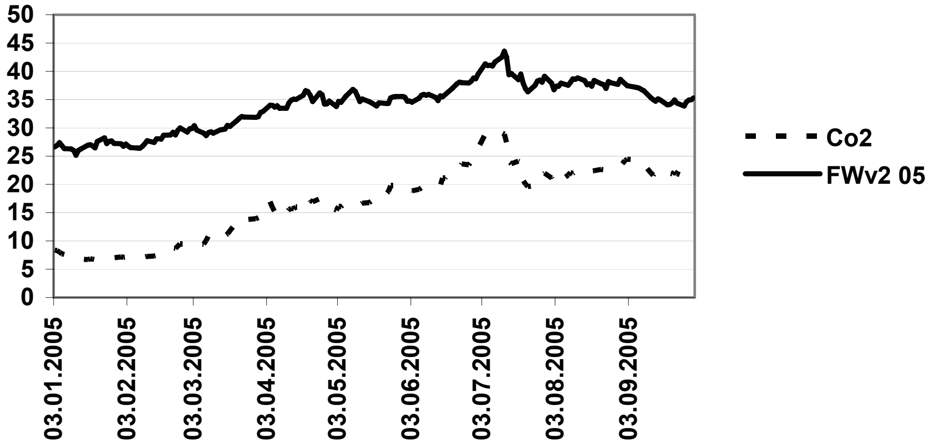
The relation between the market for power and the market for CO_2 allowances indicates a strong connectivity between the markets.

The costs in the power sector are highly dependent on prices of allowances, since this price can influence the merit-order[7] of producers. The price of emission rights results in different marginal costs for producers based on different fuels, and a change in price could lead to a change in the type of power that sets the price in the power market. A high price of CO_2 allowances can result in a substitution from coal plants to natural gas power plants, which might result in higher prices for natural gas.

[7] Producers sorted by marginal costs

Figure 10 shows the relation between the prices of CO_2-emission rights for 2005 and the electricity contract FWV2-05 traded on Nord Pool. The prices have a correlation equal to 0,96, which supports the arguments above regarding the influence of quota prices on power prices.

Figure 10: Correlation between power and allowances



Forward contracts traded on Nord Pool for delivery in December 2005 have a correlation of 0,96 with emission allowances for the same time of delivery. The correlation is estimated from contracts traded in 2005.

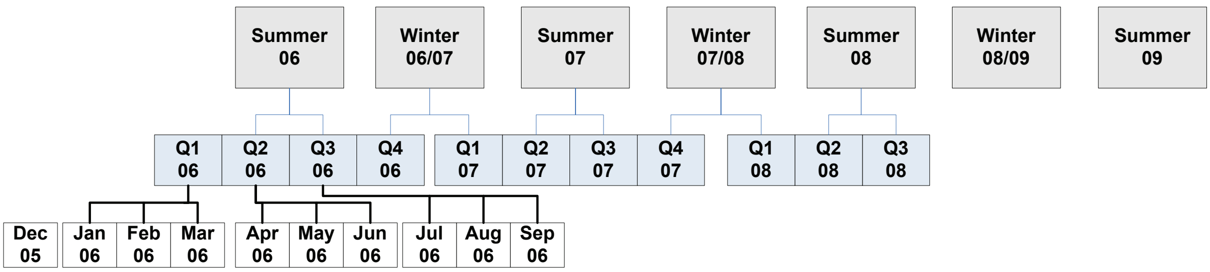
It is worth noticing that this strong correlation does not exist between CO_2 quota prices and spot-price of power in the same period. This is due to restrictions in the transmission system.

5 Markets for electricity, natural gas and CO_2 emission allowances

5.1 Natural gas financial contracts

The International Petroleum Exchange (IPE) in London is the only fully deregulated natural gas spot market in Europe, and because of this price data from the IPE are used in this report. Natural gas futures have been traded at the IPE since January 1997, and three main contracts are traded; Season, Quarter and Month. Season contracts are strips of six individual and consecutive contract months. Season contracts are always an April – September (summer) strip or an October – March (winter) strip. Quarter contracts are strips of three individual and consecutive contract months. Quarter contracts always comprise a strip of January – March, April – June, July – September or October – December. Monthly contracts are contracts for deliveries in each calendar month. The future contracts available in November 2005 are illustrated in Figure 11.

Figure 11: Term structure of IPE-contracts



Contracts traded on the International Petroleum Exchange 4th of November 2005.

5.1.1 TOP contracts

Figure 11 shows that natural gas futures traded at the IPE have relatively short time to maturity compared to the expected lifetime of the project. Based on this it is reasonable to assume that a gas fired power plant in Norway is expected to secure its required supplies of natural gas with a long-term contract. The most common long-term natural gas contracts are Take-or-pay (TOP) contracts. These contracts have two main provisions:

- A positive correlation coefficient is fixed in order to correlate the price of natural gas to the price developments of other energy sources (mostly oil) or of natural gas spot prices, if such exist.
- One party has the obligation of either taking delivery of goods or paying a specified amount.

These provisions implicate that the buyer takes the volume risk and the seller takes the price risk. The reason for this is that the seller needs volume to support large investments in infrastructure, while the buyer needs guarantees of competitive prices to be willing to make a long time commitment. The remaining provisions of TOP contracts mainly cover volume, price and the possibility of renegotiation. The prices of TOP contracts on the European continent are usually indexed against the oil price, but TOP contracts indexed against coal prices also exist. In the UK TOP contracts are usually indexed against the spot price of natural gas. Volume provisions fix the flexibility of the TOP contracts and state to which degree the buyer is allowed to deviate from the predetermined volume. They also state how much the buyer has to pay if the predetermined volume is not taken. Most TOP contracts include the possibility of renegotiation. This means that if market conditions have changed so that the TOP price of natural gas no longer is competitive, the price indexation is renegotiated. This has traditionally benefited the buyer. Further information on TOP contracts can be found in Eydeland & Wolyniek (2003).

Our simulation model is based on a TOP contract indexed against the price levels of natural gas futures quoted at the IPE. It is assumed that the IPE quotations are a good representation for natural gas prices in Norway, and that the prices of deliveries at Mongstad equal the IPE prices less the transportation costs between Norway and the UK. This means that using a forward curve derived from IPE quotations is expected to be a good approximation of a TOP contract.

5.2 Financial electricity contracts at Nord Pool

Nord Pool is the Nordic power exchange, and in addition to the spot market, a wide range of financial contracts are traded. Among the most commonly traded are weekly, monthly and yearly futures and forwards. For any given trading day, eight weekly futures, six monthly forwards and three yearly forwards are available. Before April 7th 2003 a different set of financial contracts were available, but Nord Pool is restructuring its financial contracts to a structure more similar to that of IPE. Since historical data is used in this paper, the old structure makes up a large part of this data set. The main difference is that monthly forwards were not previously traded, but 8-12 block futures were available. Each of the blocks consisted of 4 weekly futures. All financial contracts at Nord Pool are listed in MW for delivery of 1 MW per hour for the duration of the contract period.

5.3 Financial contracts for CO_2 emission allowances

Currently three contracts for CO_2 allowances are traded OTC and on different power exchanges, a yearly forward contract for 2005, 2006 and 2007. New term structures are expected to be introduced when the firs Kyoto-period starts in 2008.

Because allowances in Phase One of the EU ETS are bankable into later years or borrowable into previous years, there is no real difference in the value of the EUAs over time. This is not true for the 2007 contract when phase one ends and the EUAs have no further value. Because of these characteristics, the only proper value of the inter-year spreads has been the cost of carry, i.e. the cost of borrowing to buy EUAs to hold for one or two years.

Due to the similarity of the different year-contracts, the 2005-2006 spread has traded at as much as 25 euro cent contango until September 2005. More recently the price of the 05-06 spread has moved towards balance or even backwardation. One main cause for this change is concerns about the ability of registers in some countries to be operational to the delivery date 1st of December and thus leave the market short. The consequence of these worries has been a move to buy 2005 and sell 2006, which has wiped out the relative values of the two years

6 Price models

The investment valuation of the CHP-plant plant depends on the underlying commodity prices. By modelling prices as stochastic processes, uncertainty in prices is incorporated. This chapter will present the stochastic models used in this paper.

6.1 Model for power and natural gas

The processes for the prices of power and natural gas in this paper are based on the long-term short term model presented by Schwartz and Smith (2000). This model allows mean-reversion in short-term prices and uncertainty in the equilibrium level to which the prices revert.

Let the spot price of a commodity at time *t* be denoted by S_t. This price can be broken down into two stochastic processes X_t and ε_t and a deterministic function $f(t)$ in the following way:

$$\ln(S_t) = X_t + \varepsilon_t + f(t) \qquad (6.1)$$

The factor X_t refers to short-run deviations in the price, i.e. changes in X_t represent changes in price that are not expected to persist. The factor ε_t refers to the equilibrium level and changes in this factor represent fundamental changes in the price. The deterministic function $f(t)$ determines the seasonal characteristics of the price process.

The short-term deviations are assumed to follow an Ornstein-Uhlenbeck process and revert to zero:

$$dX_t = -\kappa X_t dt + \sigma_x dz_x \qquad (6.2)$$

where the mean-reversion coefficient κ is the rate at which the short-term deviation are expected to disappear. The equilibrium level follows a Brownian motion process:

$$d\varepsilon_t = \mu_\varepsilon dt + \sigma_\varepsilon dz_\varepsilon \qquad (6.3)$$

and dz_x and dz_ε are increments of standard Brownian motion processes with correlation

$$dz_x dz_\varepsilon = \rho_{x\varepsilon} dt \tag{6.4}$$

The seasonal component $f(t)$ is given by:

$$f(t) = \gamma cos((t-\eta)2\pi) \tag{6.5}$$

where γ is the strength of the seasonal effect and η is the displacement factor.

Thus the price is given by

$$S_t = \exp(X_t + \varepsilon_t + f(t)) \tag{6.6}$$

6.2 Model for CO_2 emission allowances

The process for CO_2 emission allowances in this paper is modelled as a simple mean reverting process.

$$dS_t = \kappa(\overline{S}_t - S_t)dt + \sigma dS_t \tag{6.7}$$

where κ is the strength of the mean reversion. $\overline{S}_t$ is the level which prices revert to and follows a stochastic process

$$\frac{d\overline{S}_t}{\overline{S}_t} = \rho_{\overline{S},\varepsilon^{el}} \frac{d\varepsilon_t^{el}}{\varepsilon_t^{el}} \tag{6.8}$$

where $\rho_{\overline{S},\varepsilon^{el}}$ is the correlation between the long-term equilibriums for respectively electricity and CO_2 prices. Equation (6.8) describes a correlation between the relative change in the long-term prices of electricity and the relative change in the equilibrium level of CO_2 prices. Combining equation (6.8) with equation (6.3) equation gives the following expression for the equilibrium dynamics:

$$d\overline{S} = \rho_{\overline{S},\varepsilon^{el}} \frac{\overline{S}_t}{\varepsilon_t^{el}} \mu_{\varepsilon^{el}} dt + \rho_{\overline{S},\varepsilon^{el}} \frac{\overline{S}_t}{\varepsilon_t^{el}} \sigma_{\varepsilon^{el}} dz_{\varepsilon^{el}} \tag{6.8}$$

6.3 Risk neutral processes

The models used in this paper are risk-neutral versions of the long-term short term model, where risk-neutral stochastic processes describe the dynamics of the state variables. This introduces the use of the risk-free rate for discounting cash flows.

6.4 Choice of models

In this report the long-term short term model developed by Schwartz and Smith is used for modeling the spot price movements of electricity and natural gas. The reason for this is that it incorporates the effects long-term uncertainty, mean reversion and seasonality.

As mentioned in the previous section, the long-term price level is assumed to follow a random walk, while the short term deviations are assumed to mean revert to zero. From a theoretical point of view these assumptions seem fair. The price level of electricity and natural gas should in the longer term be in equilibrium with the marginal cost of producing and marketing the commodities. Assuming that this equilibrium level follows a random walk is based on the fact that there is uncertainty regarding the future marginal cost of producing and marketing electricity and natural gas. Assuming that the short term deviations from the long-term equilibrium mean revert to zero is based on the expectation that such deviations are caused by temporary effects. These effects could be weather forecasts, storage levels, outages and other temporary effects that cause short term price deviations that are expected to diminish in the longer term. Incorporating seasonality in the models is based on a simple empirical investigation, which indicates that both electricity and natural gas process show strong seasonality.

The spot price movements of CO_2 emission allowances are assumed to follow a simple mean reversion process. This is based on the fact that a long-term equilibrium price level of CO_2 emission allowances is expected to establish as the Kyoto mechanisms are executed in 2008. It is also obvious that a more sophisticated model would be nearly impossible to implement as the amount of price data available is very scarce. Based on this it seems reasonable to assume that future prices of CO_2 emission allowances will fluctuate around an equilibrium level.

As described earlier, the price of power and CO_2 emission allowances is set simultaneously and the price of power is dependent on the price of allowances. This coherence is also evident in the high correlation of 0,96 between price of power and allowances in the limited trading period for allowances. This relationship should therefore be reflected in the price models.

Because of the scarce historic data on prices of allowances and the uncertainty of the future form of the allowance market, our CO_2 quota price model has obvious limitations. Thus making the electricity price process dependent on this limited model has its weaknesses. Since it is desirable to find a model where the level of electricity prices correspond to the level of emission allowances, we have chosen to make the equilibrium level of CO_2 quota prices dependent on the long-term price level of electricity.

Calibrating the price process of CO_2 allowances with the electricity price implies a reversion of the causality from the physical world, where prices of allowances affect the price of power. Our model will still reflect the desired relationship, where high quota prices correspond to high electricity prices and vice versa. Hence, we have chosen to correlate the relative change in equilibrium prices with the relative change in long-term prices of power.

7 Valuation of the plant

7.1 Model parameters

r – Risk free interest rate

T – Lifetime of CHP-plant

t – Time of calculation

t_0 - Time to start operating plant

I - NPV of the investment cost

E – Availability parameter

$Cf(t)$ - Cash flow at time t

H - Implied value of heat

$VC(t)$ – Periodic O & M costs, including insurance

MW_{el} – MW power production

MW_{gas} – MW gas consumption

MW_{heat} – MW heat production

$S_{el}(t)$ – Price of power at time t

$S_{gas}(t)$ – Price of natural gas delivered at Mongstad at time t

$S_{co2}(t)$ – Price of CO_2 emission allowances at time t

ζ – Tons CO_2 emissions per hour

q – Percentage of quotas to be bought on the market

7.2 Valuation models

7.2.1 NPV of the CHP-plant with no real options

The cash flow from the plant at time t is

$$Cf^2(t) = ((MW_{el} \cdot S_{el}(t) - MW_{gas} \cdot S_{gas}(t)) - \zeta \cdot q \cdot S_{co2}(t)) \cdot E - VC(t) \tag{7.1}$$

where the index ^{2} indicates that both turbines operate continuously. Hence the NPV of the project is

$$NPV = -I + e^{-rt_0} \int_{t=t_0}^{t=T} Cf^2(t) e^{-rt} dt \qquad (7.2)$$

7.2.2 NPV of the CHP-plant including the abandonment option

A dynamic algorithm is used to compute the NPV with the option to abandon. The cash flow from operating the power plant is given by equation (7.1), and the cash flow from producing heat in a boiler is

$$Cf^B(t) = (-MW_{gas}^B \cdot E - \zeta^B \cdot q \cdot S_{co2}(t)) \cdot E - VC^B(t) \qquad (7.3)$$

Where

MW_{gas}^B – MW natural gas needed in the boiler to produce steam

ζ^B – Tons CO_2 emissions per hour

$VC^B(t)$ – Operation and maintenance costs, including insurance, for the boiler

I^B – Investment cost for the boiler

Thus the net present value of future cash flows at time t is

$$value(t) = \max \begin{Bmatrix} cf^2(t) + value(t+\Delta t) \\ -I^B + \int_{\tau=t}^{t=T} Cf^B(\tau) e^{-r(\tau-t)} d\tau \end{Bmatrix} \qquad (7.4)$$

And the net present value today of the investment is

$$NPV = -I + e^{-rt_0} value(t_0) \qquad (7.5)$$

7.2.3 NPV of the CHP-plant including the part load option

In this scenario it is possible to temporarily decommission one of the gas turbines in periods where market prices are disadvantageous. Operating with only one turbine gives the cash flow

$$Cf^1(t) \; = \; (MW_{el}^1 \cdot S_{el}(t) - MW_{gas}^1 \cdot S_{gas}(t) - MW_{el}^1 \cdot \zeta^1 \cdot q \cdot S_{co2}(t)) \cdot E - VC^1(t) \qquad (7.6)$$

where all input values with index [1] are adjusted for the scenario. The optimal cash flow for the period *t* is

$$Cf(t) \; = \; \max \begin{Bmatrix} Cf^2(t) \\ Cf^1(t) \end{Bmatrix} \qquad (7.7)$$

And the net present value of this scenario is

$$NPV = -I + e^{-rt_0} \int_{t=t_0}^{t=T} Cf(t) e^{-rt} dt \qquad (7.8)$$

7.2.4 NPV of the CHP-plant including both real options

In this scenario the operator of the CHP-plant has the possibility both to abandon the investment and to temporarily operate at part load. The cash flows from operating the power plant is given in equation (7.7) and the cash flows from operating the boiler is given in equation (7.3). Thus the net present value of present and future cash flows at period t is

$$value(t) \quad = \quad \max \begin{Bmatrix} Cf(t) \; + \; value(t + \Delta t) \\ -I^B \; + \; \int_{\tau=t}^{t=T} Cf^B(\tau) e^{-r(\tau - t)} d\tau \end{Bmatrix} \qquad (7.9)$$

and the net present value of the investment is

$$NPV \quad = \quad -I + e^{-rt_0} \, value(t_0) \qquad (7.10)$$

7.3 Implied value of heat

The implied value of heat is defined as the value of heat required for the CHP-plant to be a zero NPV investment. This can be expressed as

$$NPV + e^{-rt_0} \int_{t=t_0}^{t=T} MW_{heat} \cdot H \cdot E \cdot e^{-rt} dt = 0 \qquad (7.11)$$

After rewriting equation (7.11) the following explicit expression for the implied value of heat is obtained

$$H = \frac{NPV}{e^{-rt_0} \int_{t=t_0}^{t=T} MW_{heat} \cdot E \cdot e^{-rt} dt} \qquad (7.12)$$

8 Estimation of variables and parameters

8.1 Simple mean reverting process (CO_2)

The parameters for the CO_2 process had to be estimated from very scarce historical data. The mean reversion coefficient was estimated by minimizing the root mean square error (RMSE). The RMSE is defined as:

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\hat{F}_n - F_n)^2} \tag{8.1}$$

where $\hat{F}_n$ and F_n are the nth calculated and observed forward prices respectively. The forward price process of CO_2 emission allowances can be stated as

$$F_t - F_{t-1} = -\kappa \Delta t F_{t-1} + \zeta_\tau \tag{8.2}$$

where ξ_t is a normally distributed error term with zero mean and standard deviation σ. The volatility of the mean reversion process can thus be estimated by calculating the standard deviation of all the ξ_t .

8.2 Long-term short-term model

The state variables X and ε in the long-term short-term model cannot be observed directly, but must be estimated from spot and futures prices. Changes in long-maturity futures/forwards indicate changes in equilibrium prices, and changes in the difference between spot/near-futures and long-futures give information about short-term deviations. The estimates can be found using Kalman filtering techniques. The Kalman Filter calculates the likelihood of observing a particular data series given a set of model parameters, and estimates of the parameters are found by maximum likelihood techniques.

8.2.1 Kalman filtering

A Kalman filter is a recursive algorithm for estimating non-observable state variables, based on observations depending on these state variables. In this case, the short-term deviations

(X_t) and the equillibrium level (ε_t) are the state variables and the historical log-forward contracts are the observations.

Given an initial state in step 0 and a set of observations in step 1, the filter calculates state variables for step 1 that minimizes RMSE for a given set of parameters. Detailed equations for the Kalman filter will not be derived in this paper, but can be found in Schwartz and Smith (2000).

Figure 12: The principles of the Kalman filter

	State variables	Observations
t=0	Initial variables	
t=1	Updated variables ←	F1, …, Fn
t=2	Updated variables ←	F1, …, Fn

The Kalman filter calculates the state variables that minimizes the RMSE in time t for a given set of parameters, given the initial variables and the observations at time t. As new observations arrive, the state variables are updated..

A likelihood function L(Θ) must be defined in order to find the model parameters, where Θ is a vector of the variables to be estimated. $\Theta = [\mu, \kappa, \sigma_x, \sigma_\varepsilon, \rho, \gamma, \eta, \varepsilon_0]$. Even if the long-term state variable is not really a parameter, it is treated as an unknown since it cannot be directly observed.

The procedure works recursively, starting with an initial Θ. The parameters are updated every time the filter runs, and the filter runs repeatedly until the likelihood function converges to a given level. The filter computes the estimates more efficiently with initial variables not far from the solution.

8.3 Parameter estimates for electricity prices

8.3.1 Input data

The input data for estimating parameters for the electricity price process are the following contracts: the contract for the block or month closest to maturity and the contracts for the next, second and third nearest calendar year. Prices of contracts traded from the 1st of January to the 4th of November 2005 were used.

8.3.2 Parameter estimates

The parameter estimates and state variables for electricity prices obtained from the Kalman filter are given in Table 4. The state variables are estimated for the 4th of November 2005.

Table 4: Parameter estimates for electricity prices

	μ_ε	κ	σ_x	σ_ε	$\rho_{x\varepsilon}$	γ	η	X_0	ε_0
Estimate	1,29	1,08	55,84	10,50	0,01	-8,59	0,43	0,01	5,54
Denominator	%	$year^{-1}$	%	%		%	year		

8.4 Parameter estimates for natural gas prices

8.4.1 Input data

The input data for natural gas are different future prices observed at the IPE from September 2000 until November 2005. The following contracts have been used:

- First nine monthly futures
- Two quarterly futures with 11 and 25 months to maturity
- Seasonal futures with 30 and 36 months to maturity

8.4.2 Parameter estimates

The parameter estimates and state variables for natural gas prices obtained from the Kalman filter are given in Table 5. The state variables are estimated for the 4th of November 2005.

Table 5: Parameter estimates for natural gas prices

	μ_ε	κ	σ_x	σ_ε	$\rho_{x\varepsilon}$	γ	η	X_0	ε_0
Estimate	-9,79	3,58	61,90	25,01	-0,51	-25,69	0,43	-0,01	5,3
Denominator	%	$year^{-1}$	%	%		%	year		

8.5 Parameter estimates for prices of CO_2 emission allowances

8.5.1 Input data

A very limited amount of historical data exists on prices for CO_2 emission allowances. Since price differentials for inter-year spreads are almost non-existing, using more than one contract is not sensible. The volatility is estimated from all available historic data, i.e. contracts traded from 01.12.04 to 30.09.05. The mean reversion coefficient is estimated from 20.05.05 to 30.09.05, in a period where prices were more stable than the preceding trading period. Correlation between prices for the CO_2 allowances and forward contracts traded at Nord Pool was estimated from contracts traded between 01.01.05 and 20.09.05. CO_2 allowances due to delivery 1^{st} of December 2005 and December forwards for power were used as input.

8.5.2 Parameter estimates

The parameters obtained from the RMSE estimation are given in Table 6.

Table 6: Estimates for the CO_2 price process

	κ	σ	S_0	ρ_{el}
Estimate	0,023	53,00	166,5	0,96
Denominator	year^{-1}	%	NOK/ton	

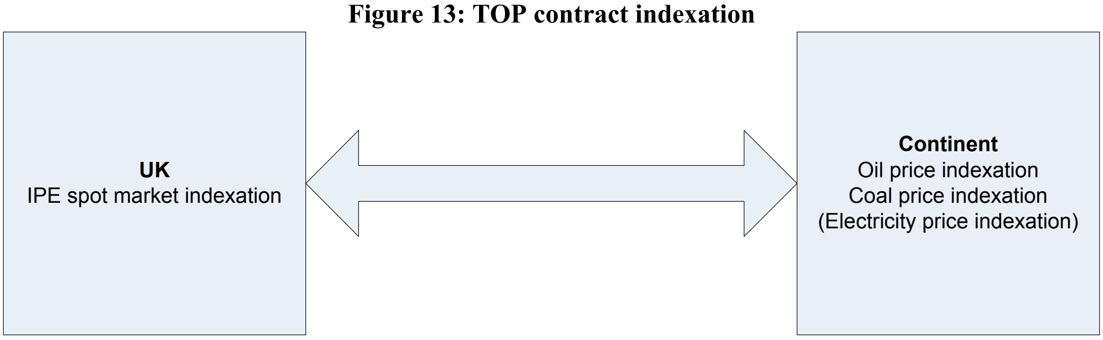
9 A long-term perspective on natural gas fired power production

This section has been included in the report to act as a background for the price simulation models and to illustrate some wider economic aspects of natural gas fired power production. In this section markets for natural gas, electricity and CO_2 emission allowances are discussed, and some trends and expectations regarding these markets are presented.

9.1 Trends within European natural gas markets

In this report natural gas price data observed at the IPE in London have been used for modelling a TOP contract for natural gas delivered at Mongstad. The UK natural gas market has existed since 1997, and the prices observed at this market have historically been somewhat lower than prices observed at the European continent. This price difference is however expected to diminish within a reasonably short time, as large pipelines now exist between the UK and the European continent. The flow direction in these pipelines can be switched in both directions, so any price differences that occur should relatively quickly be levelled out. TOP contracts are the standard tools used by most natural gas purchasers to ensure their supplies of natural gas, but currently such contracts are prices differently. As described in chapter 5.1, the prices of natural gas in TOP contracts are based on indexation, but, as shown in Figure 13, contracts in the UK are generally indexed differently than contracts on the continent.

Figure 13: TOP contract indexation



Typical indexations of natural gas TOP contracts in the UK and on the European continent.

The reason for the different indexation is that on the European continent no deregulated spot market for natural gas exists, and the prices must be indexed against alternative commodities.

Oil price indexation is most common on the continent, but coal price indexation is becoming more popular[8]. Electricity price indexation is wanted by power producers, but presently no such contracts have been signed. From a power producer's point of view the linkage between the global oil price and the local natural gas price is considered to be unfavourable. The reason for this is that the price risks from the oil market are considered to be rather high, while only limited hedging options exist[9]. Moving natural gas price indexation away from the oil price and towards spot prices, coal prices or electricity prices is likely to improve the market transparency[10], and to improve hedging options for power producers. An extensive argumentation on why such a pricing regime is likely to arise on the European continent can be found in Tönjes (2005). It is also stated in Tönjes (2005) that such a pricing regime is likely to ensure long-term natural gas prices that facilitate investments in natural gas fired power production in northern Europe.

As natural gas prices are expected to become less dependent on global oil prices, and as the main factors behind the high natural gas prices that are observed during November 2005 are temporary[11], the natural gas prices in Europe and in the UK are expected to come down from their current levels. From a theoretical point of view long-term commodity prices are expected to converge towards an equilibrium level reflecting the marginal costs of producing the commodities and bring them to the market[12]. In appendix 6 it is shown how most new sources of natural gas can reach the markets in the UK and in Europe at costs of less than $4/MMBtu, which equals approximately 92 NOK/MWh. This suggests that the natural gas price is expected to come down from its current level at more than 200 NOK/MWh, and to remain at a level that makes natural gas fired power production competitive.

9.2 Price of carbon under the Kyoto commitment

Between start of trading allowances in the EU Emissions Trading Scheme (ETS) in December 2004 and October 2005, prices have varied between €6,68 and €29,80 per ton of carbon dioxide[13]. This has lead to great uncertainty and speculations about the expected price in the first Kyoto commitment period (2008-12). Therefore, one important challenge for managing

[8] Indrebø (2004)
[9] Tönjes (2005)
[10] A market is transparent if much is known by many about what products are available at what price.
[11] See appendix 5.
[12] This effect is called mean-reversion
[13] Point Carbon (2005)

risk related to CO_2 prices is to understand the likely drivers of the price in this period and even further.

Most forecasts of the expected market price of CO_2 emissions rights in ETS falls between €15 and €30 per ton of CO_2 emitted. Leyva and Lekander (2003) state that the price is likely to settle somewhere around €25 per ton in 2008.

There are some important reasons for assuming the price of allowances to converge. The main issue lies within the nature of the cap-and-trade-scheme. The total demand for CO_2 emission rights is expected to exceed the cap by 14 to 30 percent in 2008, and the cap-and-trade approach seeks to close this gap at the lowest cost. According to Leyva and Lekander, the cheapest action for the EU would be to switch from coal-fired to gas-fired power plants.

Gas-fired plants emit less than half the amount of CO_2 required by the coal-fired plants to produce one MWh of electricity, hence they also need less then half of the emission rights. Studies show that as the price of emission rights approach €25 per ton, it will be more profitable to invest in higher-efficiency power plants compared to continuing running high-emission plants[14]. The excess supply of emission rights caused by the closing of high-emission plants will drive the price back down. In the case of prices below €25, it will be profitable for high-emission plants to operate and hereby reduce the supply of emission rights.

Switching from coal to gas will be the only realistic way to reach the emission targets required. Technological improvements in power generating and CO_2 sequestration (e.g. reinjecting CO_2 into oil reservoirs) will at best contribute to 10% of required reductions[15]. Renewable energy technologies require a much higher price of emission rights to be competitive. For instance will wind generation require price of emission rights to be approximately €50 per ton to be competitive with gas. Hence, renewable energy sources will not be a way to reduce emissions in the shorter perspective.

Another key variable likely to set he price in the first Kyoto period is the possible entry of Russia and Ukraine in the trading scheme. Both countries are committed by the Kyoto protocol to freeze emissions to 1990 levels. Because of economic disorder in the years after the collapse of the Soviet Union, both countries now have considerably lower emissions than

[14] ICF Consulting (2005)
[15] Leyva & Lekander (2003)

the 1990 level. Hence, if included in the trading scheme, these two countries could sell their surplus of emission rights in the market. This will contribute to extra supply in the market, and be a force to keep the price of emission rights down.

The combination of these factors leads to an expectation of prices to converge to a long-term equilibrium, which most experts forecast to be approximately €25.

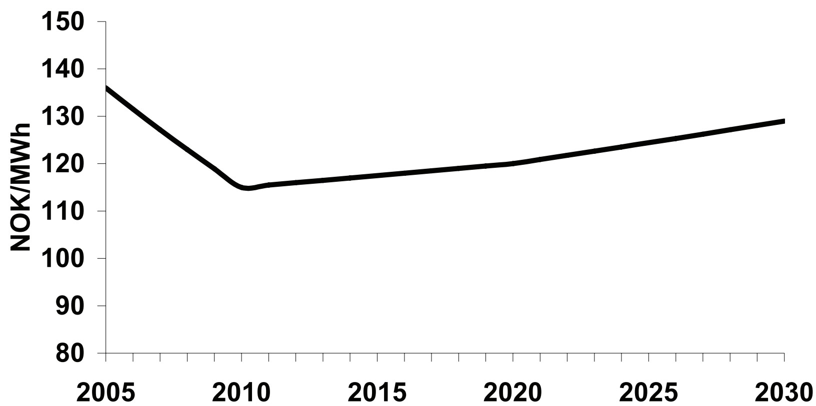
9.3 Expectations and forecasts

This chapter provides some expectations and forecasts regarding future price levels of natural gas, electricity and CO_2 emission allowances. In this context we find it important to mention that *price forecasts are not equal to forward prices*. Two sources of information have been used to obtain data: The International Energy Agency and Markedskraft AS.

9.3.1 International Energy Agency expectations

The annual report produced by the International Energy Agency (IEA), *World energy outlook 2005,* provides several long-term viewpoints on the supply and demand of natural gas, as well as long-term price forecasts. In the base case scenario the IEA expects the global natural gas demand to increase by 2,1 % annually out to 2030. This means that natural gas will grow more than any other energy source in absolute terms during the period. The IEA forecasts three long-term price levels, which are \$5/MMBtu (115 NOK/MWh) in 2010, \$5,20/MMBtu (120 NOK/MWh) in 2020 and \$5,60/MMBtu (129 NOK/MWh) in 2030. These price levels are illustrated in Figure 14.

Figure 14: Forecasted price of natural gas by IEA



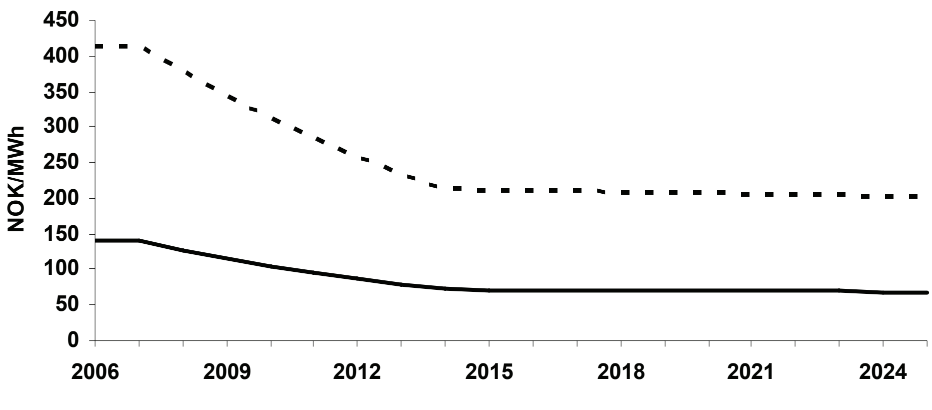
International Energy Agency's forecast for the price of natural gas.

These numbers implicate an annual drift of 0,4 % between 2010 and 2020 and 0,7 % between 2020 and 2030. This equals an average annual drift of 0,6 % during the entire period. The IEA values are $1/MMBtu higher than the values forecasted in 2004, which suggest that the long-term price expectation is higher today than it was a year ago. Still the values are considerably lower than the future prices which are observed today. In this context it is important to remind the reader on the fact that natural gas prices show strong seasonality, and that the price forecasts that are provided by the IEA are average yearly prices. For comparison it can be mentioned that the average price of monthly futures quoted at the IPE from November 2004 to November 2005 is 136 NOK/MWh. This means that the IEA expects natural gas prices to decrease somewhat towards 2010.

9.3.2 Markedskraft AS expectations

Markedskraft AS is an independent company that offers services to the participants in the Nordic and European electricity markets. The services provided are consultancy, financial analysis, risk management and investment management. In the summer of 2005 Markedskraft AS published a report with three main focus areas: Green certificate markets in Norway and Sweden, the CO_2 market in Europe and price forecasts regarding oil, natural gas, coal and electricity. The price forecasts for natural gas delivered at Kollsnes and electricity in the Nordic power market are shown in Figure 15.

Figure 15: Forecasted prices by Markedskraft

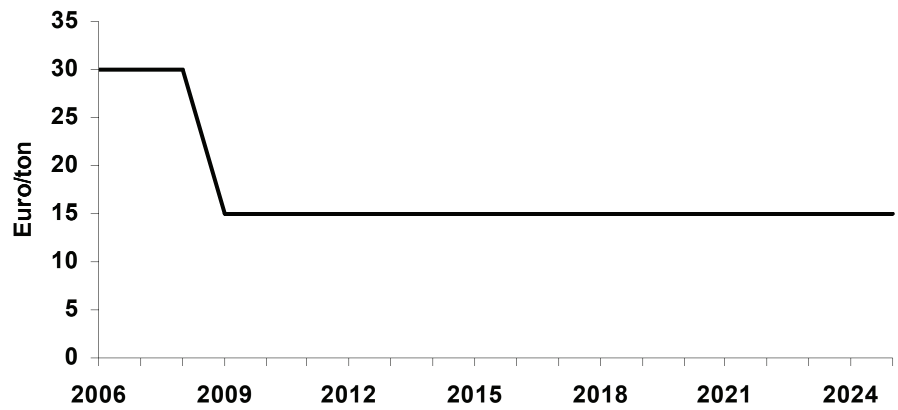


The dotted line represents the electricity price forecast, while the solid-drawn line represents the natural gas price forecast.

These forecasts have been obtained by running a computer based power system operation model. The price forecast for natural gas shows that Markedskraft AS expects the price to decline linearly from today's level at approximately 140 NOK/MWh to about 70 NOK/MWh in 2014. The transportation costs between Norway and the UK are approximately 13 NOK/MWh, which implicates that the forecasted prices at the IPE beyond 2012 equal about 88 NOK/MWh. Figure 15 reveals that Markedskraft AS expects the economic prospects for natural gas fired power generation to be favourable.

Markedskraft AS has also provided a forecast for the long-term price level of CO_2 emission allowances. This is shown in figure Figure 16.

Figure 16: Forecasted price of CO_2 allowances



Forecasted price of CO_2 emission allowances by Markedskraft

The graphs show that the prices for both electricity and CO_2 emission allowances are expected to be high until 2008. After that the forecasted levels are lower, and the price of CO_2 emission allowances is forecasted to stabilize at about 15 Euros per ton. In this context it must be mentioned that the price level of CO_2 emission allowances beyond 2007 is very uncertain because the Kyoto mechanisms come into action in 2008. It is however obvious that Markedskraft AS expects the price of CO_2 emission allowances to decrease when the Kyoto mechanisms are introduced. The main reason for this is probably that a large amount of Russian CO_2 emissions allowances then are likely to become available on the EU market.
The electricity price in the Nordic market is expected to remain at today's high level at approximately 320 NOK/MWh until 2007. As the Kyoto mechanisms are introduced, the

electricity price is anticipated to drop below 260 NOK/MWh in 2012-2013, before it is expected to rise above 280 NOK/MWh beyond 2020.

The long-term price forecasts from Markedskraft AS show that both natural gas prices and electricity prices are expected to decrease towards 2015. The electricity prices are however expected to remain at a high level compared to the natural gas prices. The main argument for this to take place is that the marginal cost of the power produced in the Nordic market is mostly set by coal fired power plants in Denmark and Finland. The marginal cost of coal fired power production is strongly affected by the demand for purchasing CO_2 emission allowances, and this implicates that a long-term high price level for electricity is likely to prevail. The text box shows how the demand for purchasing CO_2 emission allowances increases the marginal cost of coal fired power production. Although the price level of CO_2 emission allowances is expected by Markedskraft AS to decrease, the demand for purchasing such allowances is likely to ensure that electricity prices remain at a high level.

Average marginal cost of coal fired power production in 2004 (No CO_2 emission allowances were required):

Coal purchased:	170 NOK/MWh$_{el}$
+ Variable O & M costs:	40 NOK/MWh$_{el}$
= Marginal production cost:	210 NOK/MWh$_{el}$

Average marginal cost of coal fired power production in July 2005 (CO_2 emission allowances were required):

Coal purchased:	150 NOK/MWh$_{el}$
Variable O & M costs:	40 NOK/MWh$_{el}$
+ CO_2 emission allowances:	200 NOK/MWh$_{el}$
= Marginal production cost:	390 NOK/MWh$_{el}$

9.4 Concluding remarks

- Long-term TOP contracts in Europe are moving away from oil price indexation and towards natural gas spot price- and coal price indexation.
- Long-term natural gas prices are expected to be somewhat lower than the spot prices that are observed today.
- The demand for purchasing CO_2 emission allowances is likely to ensure high long-term electricity prices on the Nordic power market.

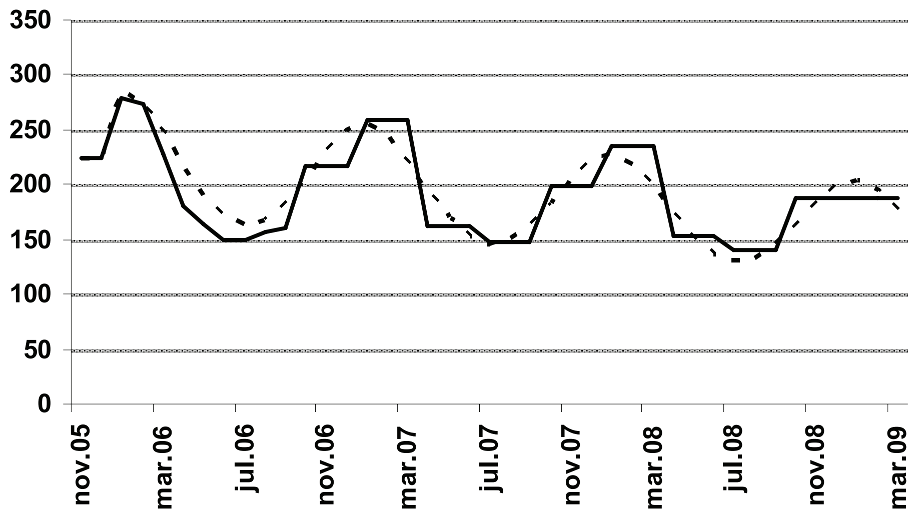
10 Our simulation model

In this section we describe how we combine the historically estimated model parameters with parameters from long-term analyses to construct our simulation models.

10.1 Natural gas

The parameter estimations that were carried out using the Kalman filter suggested a long-term negative drift at approximately 9 percent for natural gas prices. This is consistent with the forward curve observed on the 4th of November 2005 which is shown in Figure 17. This suggests a negative annual drift at about 11 percent.

Figure 17: Natural gas forward curve



The solid-drawn line represents the term structure observed on the 4th of November 2005, while the dotted line represents a smooth approximation of the term structure.

Our simulation model is supposed to simulate the prices of natural gas, electricity and CO_2 emission allowances from 2009. This means that the starting levels in 2009 are required. In order to obtain appropriate starting levels, one can simulate prices based on today's observed values or one can rely on forecasted price levels. In this section we have provided some argumentation that suggests that natural gas prices are expected to come down from their current level. We have also provided some suggestions for the long-term drift of natural gas prices. The negative long-term drift estimate obtained from the Kalman filter is considered not to be realistic on a long-term basis. In order to obtain a simulation model that we consider to be somewhat realistic, we have chosen to perform an ad hoc estimation partly based on parameters obtained from the Kalman filter and partly based on long-term price analysis. Regarding natural gas we have decided to use all the parameters obtained from the Kalman filter except from the long-term drift. For the long-term drift we have decided to use the value forecasted by the IEA, which equals 0,6 percent annually. As a starting value we have decided to use the price forecast from the IEA for 2010, which equals 115 NOK/MWh. We believe that applying these parameters and the forecasted starting value provides us with the most realistic simulation model.

10.2 Electricity

Regarding electricity prices in the Nordic power market there is consistency between parameters obtained form the Kalman filter and price forecasts provided by Markedskraft AS. In November 2005 an average yearly electricity price at approximately 250 NOK/MWh was observed. When applying the long-term drift that is obtained form the Kalman filter, the expected average price of electricity in 2009 is approximately 260 NOK/MWh. This equals the forecasted price of electricity in 2009. Based on these facts we have decided to use a starting value at 260 NOK/MWh, and to apply all parameters provided by the Kalman filter when simulating electricity prices.

10.3 CO_2 emission allowances

Because of limited historical data available, the mean reverting process for CO_2 emission allowances is partly estimated from the scarce market data and partly from expectations of a future equilibrium level. The model parameters are estimated with an equilibrium level of 200 NOK/ton as a moderate forecast. This level is, according to the strong relation to power prices, assumed to be perfectly correlated with the equilibrium of power prices, i.e. $\rho_{\bar{S}_t, \varepsilon_t^{el}} = 1$.

A perfect correlation means that a given change in long-term power prices results in a corresponding relative change in equilibrium of prices for emission allowances. The model assumes that 50 % of the required allowances have to be bought on the market, while the remaining allowances are allocated free-of-charge.

11 Results

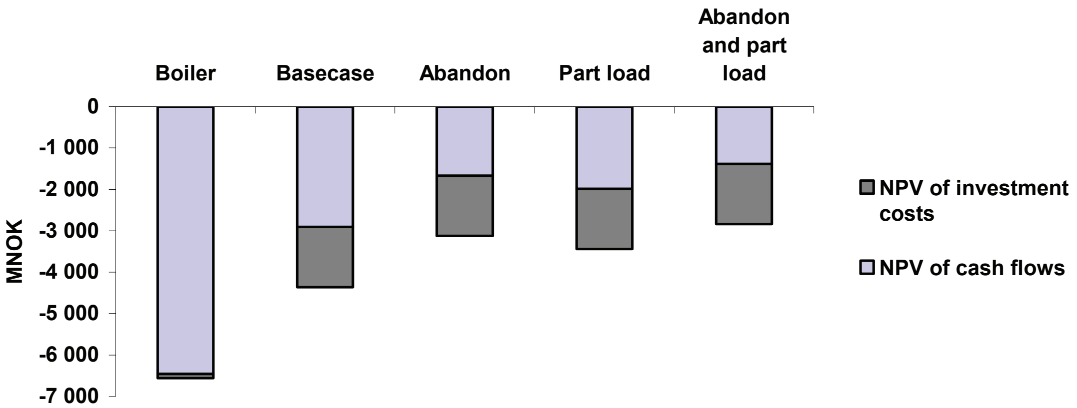
The value of the CHP-plant is analyzed over a finite lifetime of 50-years, based on the expected lifetime of the oil refinery. November 4th 2005 is the reference date of “today” for all net present value calculations. All the inputs for the simulations have been presented in the preceding chapters. The simulation is performed with 6 periods per year, hence the part load option can be exercised every second month. 30 000 simulations are performed.

This chapter investigates the value of the plant and impacts of real options, stated both as net present value (NPV), implied value of heat and expected lifetime of the plant. Valuations are performed with an equilibrium level of 200 Nok/ton for CO_2 if nothing else is explicit stated. Sensitivity analysis of equilibrium level of allowances and percentage of allowances to be bought is performed to examine their impacts on the valuation.

11.1 Net present value of the combined heat and power plant

Figure 18 shows the NPV of the CHP-plant and of a set of boilers with a heat producing capacity equal to the CHP-plant. The NPV of the CHP-plant is also shown when real options are taken into consideration. When performing the calculations, only the ordinary cash flows based on purchasing natural gas and CO_2 emission allowances and selling electricity are considered. In this perspective the NPV is anticipated to be substantially negative because no value is set on the produced heat. This issue is discussed in chapter 11.2.

Figure 18: NPV of different alternatives



NPV of the alternatives is calculated with a zero value for the heat produced.

Even though no value is set on the produced heat, Figure 18 reveals some interesting results. It is shown that the NPV of the CHP-plant is less negative than the NPV of the boiler configuration. This reveals that producing heat in the CHP-plant is a more profitable solution than producing heat in boilers. The difference in NPVs for these solutions equals 2196 MNOK.

The value of the abandonment option and the part load option can be found by comparing the different net present values for each scenario. The option values are summed up in Table 7, and it is shown that the real options have a relatively large value compared to the corresponding net present values.

Table 7: NPV and option values in MNOK, based on Monte Carlo simulation

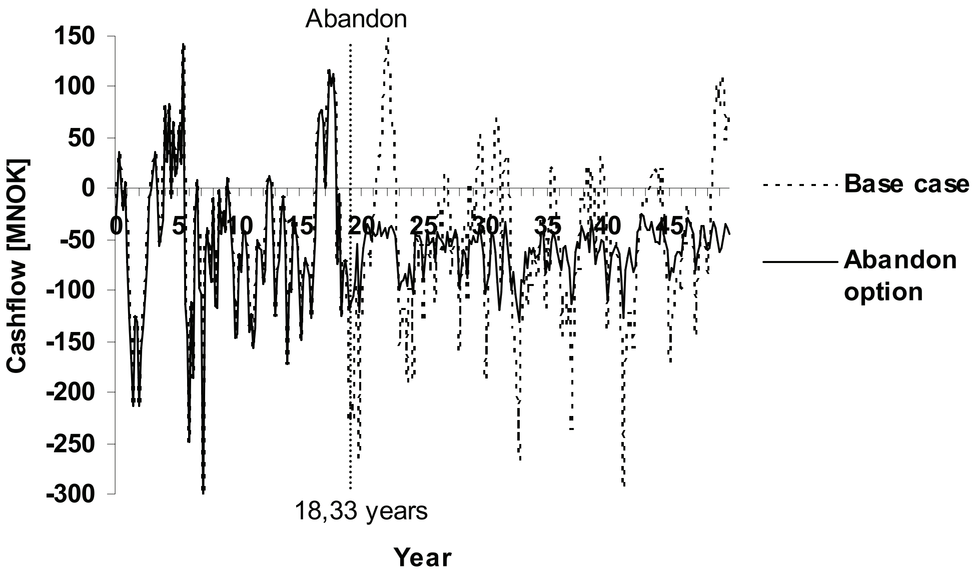
Scenario	NPV	Option value	Unit
Boiler	-6 557		MNOK
Basecase	-4 361		MNOK
Option 1: Abandon	-3 125	1 237	MNOK
Option 2: Part load	-3 440	921	MNOK
Option 1 + 2	-2 839	1 522	MNOK

Net present value of the boiler configuration and the CHP-plant with different real options.
The option values are calculated as the difference between each alternative and the base case.

11.1.1 Investigation of the abandonment option

Table 7 shows that the value of the abandonment option equals 1237 MNOK. This value is relatively high compared to the NPV of the CHP-plant, and thus it indicates that the option to abandon the investment has a substantial value. We repeat from chapter 3.2 that if the abandonment option is exercised, the CHP-plant can never be operated again. This means that the investment in the CHP-plant is deemed a sunk cost. It is described in chapter 7.2.2 how the value of the abandonment option is part-dependent and hence depends on entire price scenarios. This means that the decision to exercise the abandonment option by decommissioning the CHP-plant permanently and investing in boilers is based on observations of sufficiently poor market conditions. This is illustrated in Figure 19, where a scenario of cash flow simulations is shown. The solid-drawn line shows the cash flows when the abandonment option is available, while the dotted line shows what the cash flows would have looked like if the abandonment option was unavailable.

Figure 19: Illustration of the abandonment option

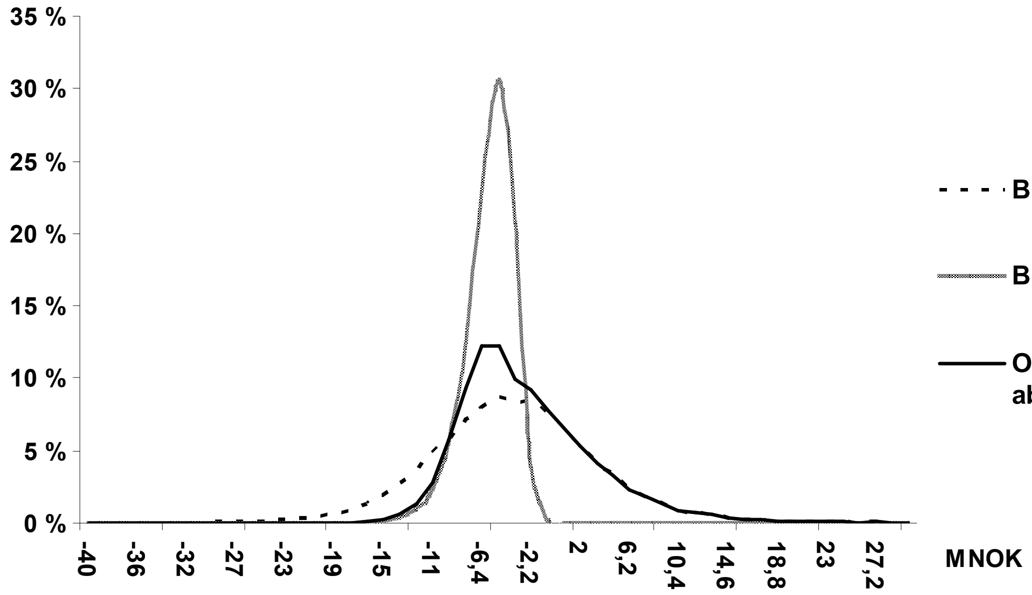


Scenario where the option to abandon is exercised after 18,33 years if this option is available. The figure shows the cash flows with and without this option.

Figure 19 shows that the abandonment option is exercised when it is considerably more profitable to switch from the CHP-plant to a boiler configuration. This is due to the uncertainty regarding future cash flows.

The introduction of the abandonment option results in an average lifetime of 32 years for the CHP-plant. Our simulations demonstrate that in the majority of the price scenarios the abandonment option will not be exercised. Figure 20 shows the NPV distributions for the base case scenario, the abandonment scenario and for a set of conventional gas fired boilers. It is demonstrated that the abandonment option provides downside protection for the owners of the CHP-plant. This is expected, as the option only is exercised when market conditions are sufficiently poor. It can thus be stated that the abandonment option offers a hedging possibility by eliminating the worst case scenarios.

Figure 20: Distributions of NPV



The distribution of net present values for the boiler, the base case CHP and the CHP with the option to abandon.

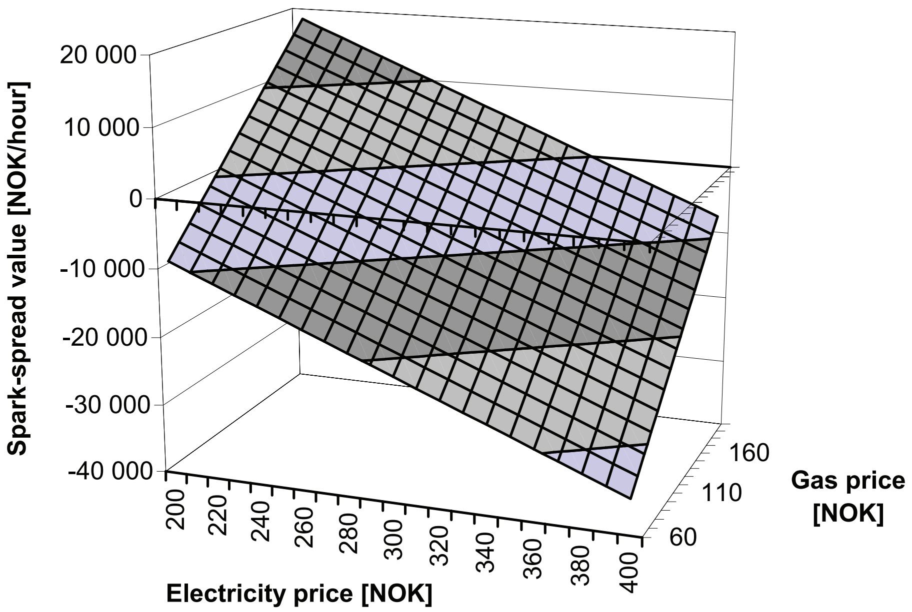
11.1.2 Investigation of the part load option

Figure 21 shows the spark-spread[16] for the part load scenario minus the spark-spread for the base case scenario. The sensitivity interval for electricity prices is 200 – 400 NOK/MWh, and the sensitivity interval for natural gas prices is 60 – 200 NOK/MWh. According to our price simulations, these price intervals capture the majority of the expected observations. The price of CO_2 emission allowances is fixed at 200 NOK/ton, and the share of emission quotas that need to be bought is fixed at 50 %. This equals 648 000 tons annually for the base case scenario and 421 000 tons annually for the part load scenario. The spark-spread is a formula used for estimating cash flows generated by a thermal power plant, and the operator prefers a high spark-spread to a low spark-spread. In operating terms this mean that the operator of the CHP-plant will choose the operating configuration that maximizes the spark-spread during a given period based on observed market prices. This is done in order to maximize the cash flows generated by the CHP-plant during that period. If Figure 21 is considered in such a context, one can observe that the part load option will be exercised when the graph is above zero. The figure reveals that the part load option will only be exercised when electricity prices

[16] Spark-spread = $MW_{el} * S_{el} - MW_{gas} * S_{gas} - \zeta * q * S_{CO2}$. Variable operating costs are not included here since they are assumed to be independent of the operating mode.

are low and natural gas prices are high. In such a setting the CHP-plant is likely to lose money in all operating scenarios, but operating at part load reduces the economic losses.

Figure 21: Exercise of part load option



The part load option is exercised when the graph is above zero.

The fact that the part load option is exercised only when the difference between the electricity price and the natural gas price is small, indicates that the part load option in a financial setting can be compared with a put-option. The reason for this is that both options offer downside protection of an investment, meaning that they reduce the potential economic loss. This fact also emphasizes why the part load option has an economic value, even if the heat rate is increased by more than 30 % when it is exercised[17]. The observations in this chapter correspond with the observations made regarding the abandonment option, and it appears that both real options offer downside protection. The main difference between the real options is that the value of the abandonment option is based on entire price scenarios, while the value of the part load option is based on prices observed within a given period.

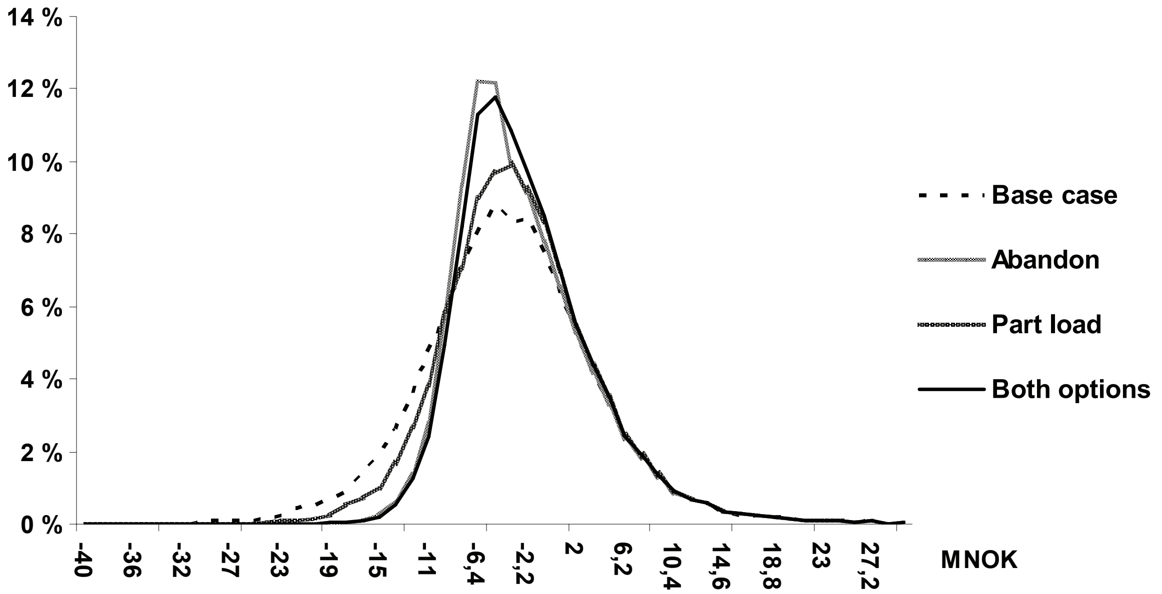
11.1.3 Investigation of both real options considered simultaneously

When the abandonment option and the part load option are considered simultaneously, the total value of both options equals 1522 MNOK. Our simulations reveal that the two options

[17] Heat rate equals natural gas input per MWh electricity output.

interact with each other, and this is demonstrated by the average lifetime of the CHP-plant. The average economic lifetime of the CHP-plant when only the abandonment option is considered equals 32 years, while it equals 35 years when both options are included. This indicates that the possibility of temporarily operating at part load implies that the abandonment option is less likely to be exercised, and the expected lifetime of the CHP-plant is therefore increased. The NPV distributions for the different scenarios are illustrated in Figure 22.

Figure 22: Distribution of net present values



The distribution of net present values for the base case, the base case including the abandonment option, the base case including the part load option and the base case including both real options.

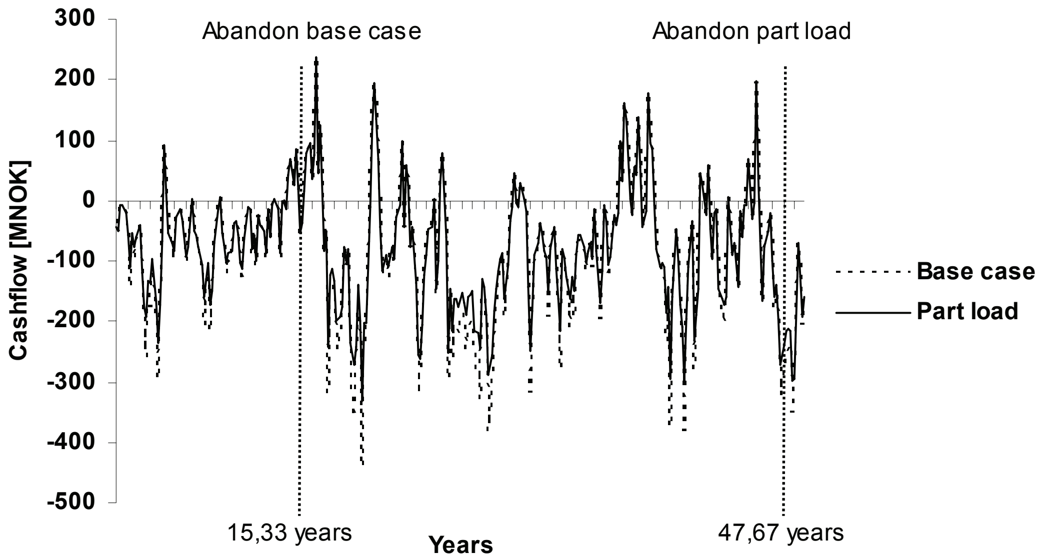
Figure 22 shows how the abandonment option and the part load option both offer downside protection, and the solid-drawn line shows how they interact with each other. This has the following practical interpretations:

- The part load option "replaces" the exercise of the abandonment option in some price scenarios. This means that instead of abandoning the CHP-plant, the plant is operated at part-load in one or, most likely, several time periods. This means that the operator does not have to deem the CHP-plant a sunk cost. The threshold for abandoning the plant is thus increased by the introduction of the part load option.

- In some price scenarios it is still optimal to abandon the CHP-plant at some point, even if the possibility of operating at part load also is available. The introduction of the part load option however delays the exercise of the abandonment option.

These two interpretations are illustrated in Figure 23, where a simulated scenario of cash flows is shown. The solid-drawn line represents the cash flows when only the part load option is available, while the dotted line represents the cash floes when no real options are available. When no real options are available, the CHP-plant is abandoned after 15,33 years, while it is abandoned after 47,67 years when the part load option is available. It can be seen in Figure 23 how the part load option is exercised to avoid large negative cash flows in the periods where market conditions are sufficiently poor.

Figure 23: Illustration of both real options



Scenario where the abandonment option is exercised after 15,33 years if the part load option is not included. When the part load option is included, the abandonment option is exercised after 47,67 years.

The practical interpretations stated above implicate that the conditional value of the abandonment option, given that the part-load option also is available, is lower than the independent value of the option. The independent value of the abandonment option equals 1237 MNOK, while the conditional value equals 601 MNOK.

It has been stated throughout this section that considering the CHP-plant in a real options perspective implies that the downside potential of the investment is substantially lower than if it is considered in a traditional NPV perspective. Table 8 is included to sum up the reason for this.

Table 8: Worst case scenarios [MNOK]

	Base case	Part load option	Abandon option	Both options
1 %	-22 515	-18 022	-13 499	-13 324
5 %	-15 983	-13 160	-10 655	-10 453

The worst 1 and 5 percent scenarios for the plant with and without the real options.

In Table 8 percentiles of the different distributions are given to illustrate how the real options, both individually and together, protect the owners of the CHP-plants from potentially large economic losses.

11.2 Implied value of heat production

The implied value of heat is evaluated by determining the price per MWh of heat that is required for the CHP-plant to be a zero NPV investment. The same principle is used when evaluating the implied value of heat if it was produced in boilers. As heat is considered to be the main product of the CHP-plant, it is very intuitive to use the implied value of heat to illustrate the value of the CHP-plant and, in particular, the real options. The reason for this is that the actual price of heat determines the profitability of the plant, and thus triggers the initial investment decision. This makes it interesting to look at what the implied value of heat actually is, and how it is affected by the presence of real options.

The implied values of heat for the different scenarios are illustrated in
Figure 24. The first column shows the implied value of heat if boilers were used for heat production, while the second column shows the implied value of heat for the base case scenario. The other columns show the implied value of heat in a real options perspective.

Figure 24: Implied value of heat

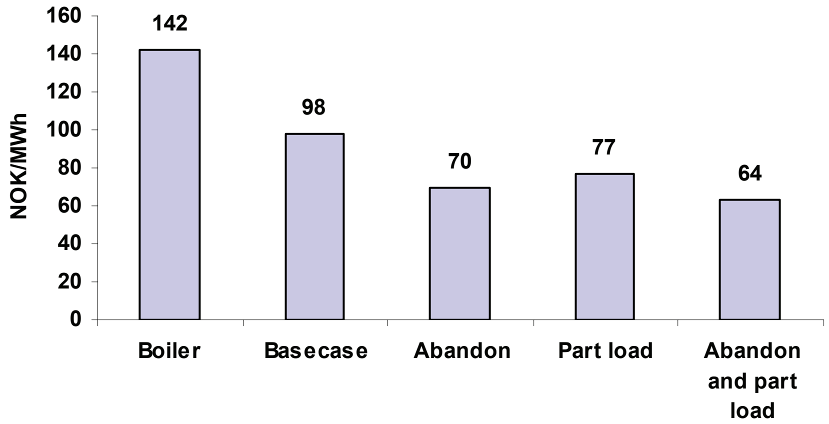
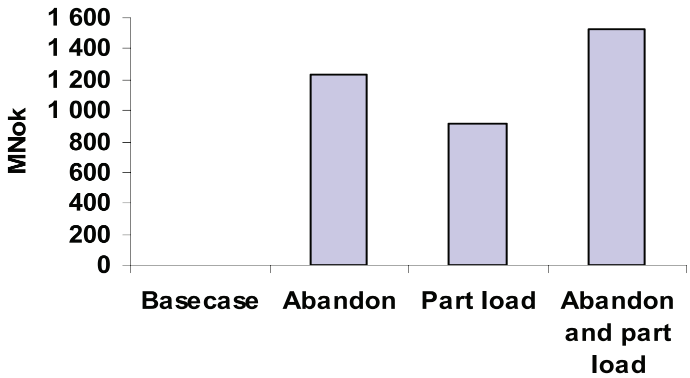


Figure 24 shows that operating the CHP-plant is a more profitable solution than producing heat in boilers. This observation is similar to the observation made in chapter 11.1, and it favours the original investment in the CHP-plant.

Figure 24 also clearly reveals the value of the part load option and the abandonment option, and it shows the effect of both real options considered simultaneously. It is demonstrated that the values of both these real options are substantial, and that they can have a significant impact on the profitability of the plant. In this context it is evident that the real options should be considered when the price of heat is evaluated. If the owners of the oil refinery evaluate the price of heat without taking the real options into consideration, the profitability of the CHP-plant will increase substantiality. This effect is illustrated in Figure 25, where the price of heat is set to 98 NOK/MWh in all scenarios.

Figure 25: NPV with price of heat set to 98 NOK/MWh



The value of heat is set to 98 NOK/MWh, which is the implied value of heat for the base case scenario.

Figure 25 demonstrates how the implied value of heat is affected by taking the real options into consideration. The interesting topic to discuss in this context is what the owners of the oil refinery are likely to pay for the heat deliveries, and how this affects the profitability of the CHP-plant.

In this report it has been suggested that the main purpose of the CHP-plant at Mongstad is to provide heat for the oil refinery. Statoil is the main owner of the oil refinery, and the company will also be the main owner of the CHP-plant. If the CHP-plant is considered to be a utility system for the oil refinery, it seems natural to us that the price of heat will be set so that the CHP-plant is a zero NPV investment. This solution maximizes the profitability of the oil refinery, which, based on Statoil's arguments, is likely to be the company's main objective. It also ensures that the CHP-plant is an acceptable investment on its own.

It appears in Figure 25 that Statoil should take the real options into consideration when the price of heat is set in order to reach this objective. We have demonstrated that the implied value of heat equals 98 NOK/MWh when no real options are taken into consideration. It is interesting to state the fact that if the price of heat is set equal to this value, the CHP-plant, including its real options, can be considered as a *positive* NPV investment.

In this context it is also interesting to state some general considerations regarding the initial option to invest in the CHP-plant. If the investment decision was based on a general NPV-rule, the investment would have been conducted if the price of heat was set to 98 NOK/MWh. If the investment decision was made in a real options perspective, the price of heat would have been set higher than 98 NOK/MWh in order to trigger the investment in the CHP-plant. This is caused by the fact that in a real options perspective the NPV of an investment must exceed the value of keeping the investment option alive in order to trigger the exercise of the option. The price of heat that would have been required if the investment option should have been exercised can be estimated by Least-Squares Monte Carlo simulation[18]. We have not performed such estimation, but it is known that the price of heat would have been higher than 98 NOK/MWh.

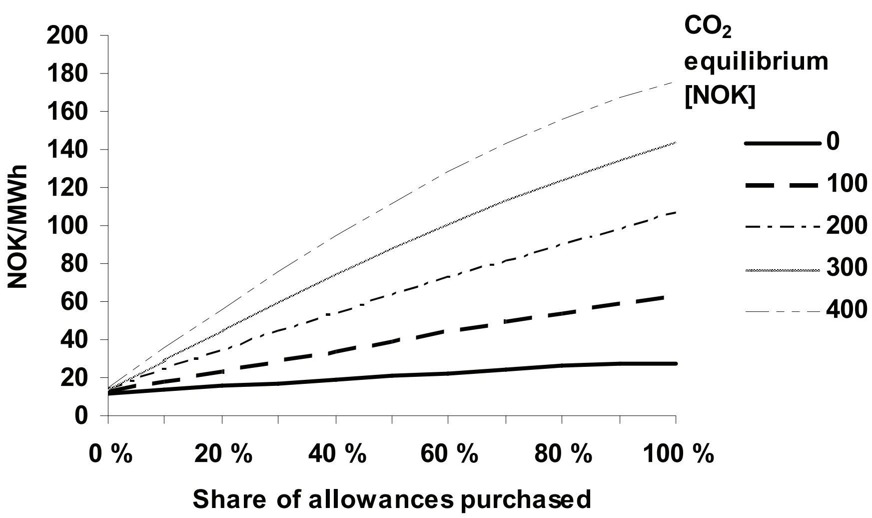
[18] See Longstaff & Schwartz (2001)

11.3 Sensitivity analysis regarding CO_2 emission allowances

As mentioned earlier, large uncertainty is associated with the future costs of CO_2 allowances. The future shape of trading schemes initiated by the Kyoto protocol is unknown and is highly dependent on political decision. The previous valuations are performed with an equilibrium price of 200 NOK/ton in 2009, and 50 % of the allowances are required to be bought on the market.

Because of this uncertainty, it is desirable to investigate how sensitive the value of the plant is to the level of CO_2 emission costs. One approach to study the effect of the emission allowance market is to analyze the impact on implied value of heat. Figure 26 shows the dramatic impact CO_2 allowances have on the implied value of heat, which ranges from 12 to 176 Nok/Mwh.

Figure 26: Implied value of heat for different CO_2 scenarios

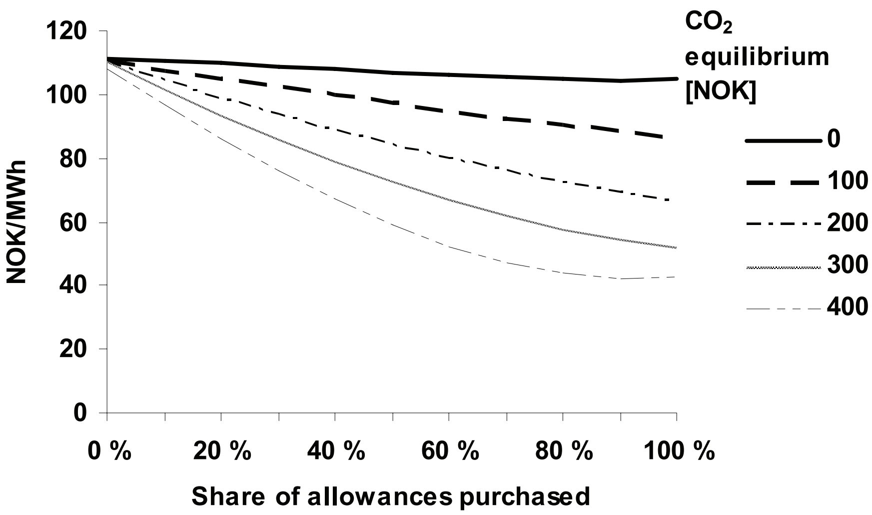


The CO_2 equilibriums above are the initial equilibrium levels for the simulations, i.e. the level at 01.01.2009.

Intuitively, implied value of heat is very sensitive to the level of emissions when prices are high and a large amount of the allowances have to be purchased.

Figure 27 shows the difference between the implied values of heat produced in the CHP-plant compared to heat produced in a conventional boiler. It is clear that the implied value of heat produced in the CHP-plant becomes less superior compared with heat produced in the boiler when CO_2 costs become high.

Figure 27: Difference in implied value of heat between CHP and conventional boiler



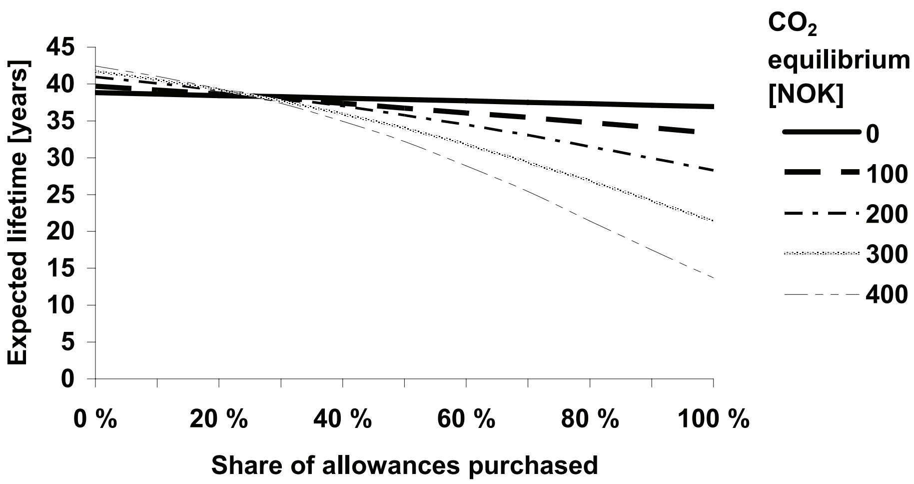
The figure shows the absolute difference between the implied value of heat produced in the CHP-plant and heat produced in conventional boiler, i.e. Heat(CHP) – Heat(boiler). Both real options are included in the CHP valuation.

Figure 27 proves that even if producing heat in the CHP-plant always is preferable, the advantage is reduced as the cost of CO_2 increases. This is due to the lower amount of emissions attached to the boiler.

11.3.1 The abandonment option

Figure 28 shows how the expected lifetime of the CHP-plant decreases as the shares of allowances to be purchased increases.

Figure 28: Expected lifetime of the CHP-plant



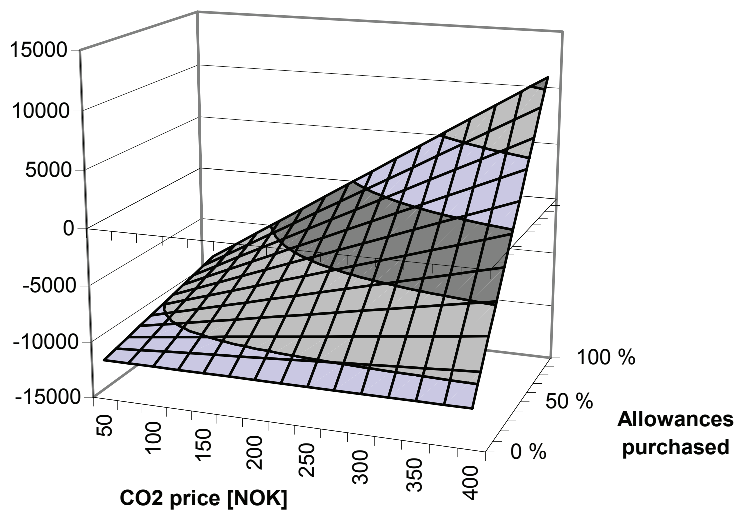
Expected lifetime of the CHP-plant including both real options.

Based on figure 28 it can be stated that the expected lifetime of the plant is very sensitive to the CO_2 regulation scheme, and that a strict scheme makes the exercise of the abandonment option more likely to occur. This is caused by the amount of CO_2 emissions in the two scenarios. When the CHP-plant is operated at full load, the CO_2 emissions equal 1,3 million tons annually. The CO_2 emissions after the abandonment option has been exercised equals 0,5 million tons annually. This clearly means that high CO_2 emission costs favour the exercise of the option.

11.3.2 The part load option

Figure 29 shows the difference in spark-spread between the part load scenario and the base case scenario. In the figure the prices of electricity and natural gas are fixed at 260 and 115 NOK/MWh respectively, and sensitivity analysis is performed regarding the price of CO_2 emission allowances and the share of allowances that needs to be bought. The price is varied from 50 NOK/ton to 400 NOK/ton, while the share is varied from 0 to 100 %. When the spark-spread is above zero, the part load option is exercised.

Figure 29: Sensitivity for the part load option



The part load option is exercised when the graph is above zero.

The interesting aspect of figure 29 is that a strict CO_2 regulation scheme, where quota prices are high and a large number of quotas need to be bought, favours the part load scenario. The reason for this is that when the part load option is exercised, the CO_2 emissions are reduced by 35 %. The marginal emissions[19] are however increased by 21 %, and this indicates that a strict CO_2 regulation scheme may promote non-optimal solutions. From an energy and environmental point of view an optimal energy solution is a way of producing electricity that minimizes the marginal CO_2 emissions. The reason for this is simply that electricity supply is a necessity in out society, and the supply should be obtained with as low CO_2 emissions as possible. This can be stated in an optimization framework as minimizing the total CO_2 emissions from electricity production subject to a given electricity demand. Based on this it is evident that operating the CHP-plant at full load is the optimal solution. This example shows that a strict CO_2 regulation scheme does not always favour such solutions.

[19] Marginal emissions = $\frac{CO_2 emissions}{Electricity\ output}$

12 Discussion and critics

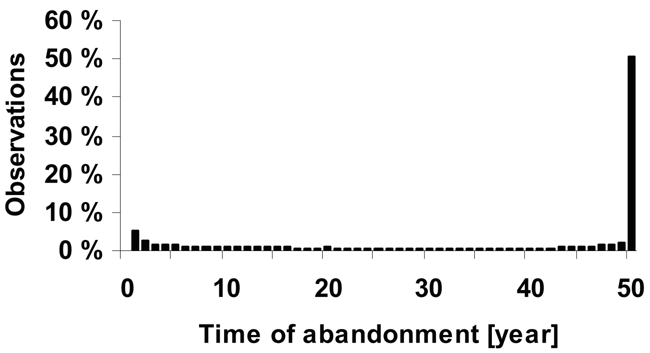
In this section the models, assumptions and results which are presented in this report are discussed and commented. The purpose of this section is to provide some wider interpretations of our results, and to discuss the strengths and weaknesses of our work.

12.1 Analyzing the CHP-plant in a real options framework

The reason for analyzing the CHP-plant at Mongstad in a real options framework is the presence of uncertainty. Our valuation models incorporate uncertainty regarding the prices of natural gas, electricity and CO_2 emission allowances. The presence of uncertainty makes flexibility valuable, and this is demonstrated in this report. It is shown in chapter 11 that the option to abandon the investment and the option to operate at part load both have substantial values. Both real options reduce the downside potential of the investment in the CHP-plant, and thus reduce the financial risk of owning the plant.

The option to abandon the investment is based on the owners of the CHP-plant deciding to permanently decommission the gas turbines and to purchase gas fired boilers to maintain the arranged heat deliveries. In real life it is questionable if such an option would always be exercised, even if exercising it was an economically optimal decision. Exercising such an option includes accepting a large sunk cost, and in some situations this makes little sense. This is illustrated in Figure 30, which shows when the abandonment option is exercised in different price scenarios.

Figure 30: Exercise of the abandonment option



The histogram shows the distribution of the time of abandonment when both real options are included.

Figure 30 shows that the abandonment option in most cases is not exercised during the 50 years of committed heat deliveries. It also shows that when it is exercised, the exercise often takes place very early or very late. We believe that decommissioning the gas turbines only after a few years of operation is unlikely to be done. The reason for this is that by choosing such a solution the owners admit that the CHP-plant should never have been constructed in the first place, and thus they have performed poor judgement when undertaking the investment. We also believe that decommissioning the gas turbines very late is unlikely to happen. By choosing to abandon the investment, the owners commit themselves to purchase gas fired boilers. We believe that making such a commitment is not likely to be done if the remaining timeframe for heat deliveries is short. Based on these arguments, we think it is reasonable to state the fact that the practical value of the abandonment option is likely to be less than the estimated value.

If the results and conclusions of this report are viewed in a critical perspective, it is hard to avoid the fact that a substantial degree of uncertainty is present. It has thus been necessary to make several assumptions in order to produce these results, and the main assumptions are discussed in chapter 12.2. In addition to the assumptions stated in 12.2, we find it appropriate to make a general discussion about making long-term economic calculations in the presence of uncertainty. The CHP-plant is expected to have a maximum lifetime of 50 years, and in this report we have modelled forward prices for this entire lifetime. We have based our models on a relatively small amount of available data constituting of futures and forwards with rather short maturities compared to the relevant time horizon. It can thus be stated that modelling such forward prices is a rather wild guess on future market prices, and that scenario analysis would have been an equally good alternative. It is however clear that as long as liquid future and forward markets exist, and as long as many players take part in these markets, future and forward prices will reflect the weighted average of all players' beliefs. In such a setting the use of current futures and forward data will be the best information available to model forward prices[20].

[20] Alstad & Foss (2004)

12.2 Assumptions

The results that are presented in this report are based on several assumptions. In this chapter the major assumptions are discussed, and possible sources of uncertainty are revealed.

It is assumed throughout the entire report that the available amount of refinery fuel gas is equal in each operating scenario, and that the fuel gas is provided free of charge. These assumptions are based on the application for concession and Sollie (2002). Assuming a constant amount of fuel gas is probably somewhat inaccurate. In reality gas turbine specifications limit the maximum amount of fuel gas allowed to be used in the base case scenario due to restrictions on fuel composition. We use this specified amount of fuel gas in all scenarios, even though a larger amount of fuel gas probably is available and would have been used in boilers and supplementary duct-burners. This assumption favours the base case scenario. The reason for making this assumption is that we do not know the actual amount of fuel gas that is available, and by making this assumption we can evaluate the CHP-plant including its real options in a clear and objective way.

In chapter 5 we assume that natural gas is purchased through a TOP-contract indexed against quoted IPE futures. Exercising any of the real options implies that the purchased volume will be substantially reduced. We assume that the owners of the CHP-plant can reduce the purchased volume without incurring any costs, even though this contradicts one of the main principles of TOP-contracts. The reason for making this assumption is that Statoil ASA is both supplier of natural gas and main owner of the CHP-plant. We believe that this arrangement makes such a solution possible.

In all calculations it is assumed that heat is to be delivered to the oil refinery for 8100 hours annually for 50 years. Assuming 8100 production hours per year is based on the expected availability of the oil refinery, and that heat deliveries only are required when the refinery is in operation. It is thus assumed that the CHP-plant can be operated for 8100 hours per year. In reality this number will not be constant, but the variations are expected to be insignificant. Assuming 50 years of heat deliveries is based on the expected lifetime of the oil refinery, and the reason for choosing this value can be found in chapter 2.5.

12.3 Stochastic price models

We have used stochastic price models to model forward prices for natural gas, electricity and CO_2 emission allowances. The long-term short-term model based on Schwartz & Smith (2000) is used for modelling natural gas and electricity prices, while a simple mean-reversion process is used for modelling the price of CO_2 emission allowances. We believe that the main weakness of this modelling approach is the relatively small amount data available for parameter estimation. When estimating parameters for the long-term short-term models, only quoted futures have been used, and no long-term OTC contracts have been available. This means that the modelled fluctuations of the long-term state variable are based on futures with no more than three years to maturity. Ideally long-term OTC contracts with at least 10 years to maturity should have been used for this purpose, but we did not have access to any price data of such contracts. It is especially the long-term fluctuations of natural gas forward prices that have been problematic to model, and we have not been able to obtain realistic parameter estimates for these fluctuations through historical parameter estimation. This has made it necessary to rely on forecasted values based on fundamental information, and hence an ad hoc procedure was used to model natural gas forward prices.

The parameters in the simple mean-reversion model that was used for modelling forward prices of CO_2 emission allowances were estimated from an extremely small set of data. CO_2 emission allowances have been traded for less than a year when this report is written, and it is thus evident that it is impossible calibrate a price model by only using historical data. We used price forecasts to determine the equilibrium level of the mean-reversion process, and we believe that due to the scarcity of data this was the best solution. In this context we find it appropriate to state the fact that the costs of CO_2 emission used in this report are burdened with uncertainty. This is also the reason for including extensive sensitivity analyses regarding these costs in chapter 11.

We have generally not given priority to perform extensive tests on the model parameters, and thus we cannot make any certain conclusions regarding the robustness of our results. Ideally such tests should have been performed in order to check the stability of the parameter estimates and the out-of-sample performance of the price models. The long-term short-term model was used for modelling natural gas and electricity prices in Alstad & Foss (2004), and in that report a thorough investigation of the model parameters was performed. Alstad & Foss

(2004) states that the long-term short-term model is not able to capture the term structure of natural gas future prices in a satisfactory way, and introducing more complicated models incorporating long-term stochastic volatility is suggested. This has however not been done in this report because calibrating such complicated models using a relatively small set of data is not likely to work well[21]. It is stated in Alstad & Foss (2004) that weakness of the parameters in general is a source of errors when using the long-term short-term model to model forward prices of electricity and natural gas, and this is valid also for our work.

[21] Eydeland & Wolyniek (2003)

13 Concluding remarks

The calculations made in this report suggest that the implied value of the heat produced by the CHP-plant at Mongstad equals 98 NOK/MWh. When the option to abandon the investment and the option to temporarily operate at part load both are included, the implied value of heat equals 64 NOK/MWh. If a set of conventional gas fired boilers were used for heat production instead of using the CHP-plant, the implied value of heat would have been 142 NOK/MWh. This can briefly be summed up with the following conclusions:

- Producing heat in the CHP-plant is a more profitable solution than producing heat in conventional gas fired boilers. This statement is valid both in a real options perspective and in a traditional NPV perspective.
- The abandonment option and the part load option both have substantial values, because they protect the owners of the CHP-plant from large economic losses if market conditions turn out to be poor.

The calculations in this report also suggest that uncertainty regarding the costs of CO_2 emissions have a relatively large impact on the profitability of the CHP-plant. It is demonstrated that the implied value of heat is increased considerably when CO_2 emission costs are high. It is also shown that the real economic lifetime of the CHP-plant is reduced when CO_2 emissions costs are increased. Based on these facts, the following conclusions can be stated:

- High CO_2 emission costs increase the cost of producing heat in the CHP-plant more than it increases the cost of producing heat in a conventional set of boilers.
- It is more likely that the abandonment option and the part load option will be exercised when CO_2 emission costs are high then when they are low.

14 Suggestions for further work

Both the abandonment option and the part load option have been evaluated in this report, and the methodology that has been used has been extensively described. We however believe that the real options could have been evaluated in a somewhat more accurate way. It is stated in chapter 12.1 that the practical value of the abandonment option probably is less than the estimated value. In this context it could have been interesting to incorporate this fact into the valuation procedure, and thus develop a somewhat more sophisticated valuation model. We have not been able to identify such a procedure, but we believe it is possible to do so. The procedure used to evaluate the part load option is based on the assumption that one of the gas turbines can be shut down for two months if it is optimal to do so. Gas turbines are very flexible power generation units, and in reality they can be operated in a more flexible way than what is assumed in this report. It would thus be interesting to evaluate the part load option if the CHP-plant could have been operated at any load between 50 and 100 %, and if the load could have been altered more frequently than every second month. Performing such an evaluation increases the required computational time considerably, and because of this we have not given priority to this task.

In addition to evaluating the abandonment option and the part load option, it would also have been interesting to investigate the original option to invest in the CHP-plant. This is especially interesting in connection with the implied value of heat calculations. Estimating the price of heat that triggers the investment decision would definitively be of interest, and it would be interesting to compare this value to the implied values of heat which have been calculated in this report.

A possible real option which has not been taken into consideration in this report is the option to equip the CHP-plant with a CO_2 removal plant. The profitability of a CO_2 removal plant depends on the costs of CO_2 emissions, and it is possible that in some price scenarios removing the CO_2 emissions would have been more profitable than buying emission allowances. This possibility is however not likely to occur, as CO_2 removal plants are very expensive to install, and by installing a CO_2 removal plant the efficiency of the CHP-plant is reduced. Dobbe, Fleten & Sigmo (2003) states that even under very high prices for CO_2 emissions, the value of a CO_2 removal plant is much less than its investment costs. Based on these facts we have not considered the option to remove CO_2 in this report, but as new information about costs and potential economic gains now are available, such an evaluation could be of interest.

References

Alstad, R. M. & Foss, J. T. (2004) *Investment analysis of CCGT power plants*, Master thesis, Norwegian University of Science and Technology

Bjerksund, P., Rasmussen, H. & Stensland, G. (2000) *Valuation and Risk Management in the Norwegian Electricity Market*, Discussion paper, Norwegian School of Economics and Business Administration

Björk, T. (2004) *Arbitrage Theory in Continous Time*, Oxford University Press Inc., New York, US

Bolland, O. & Sæther, S. (1996) *Notat om samproduksjon av kraft og varme*, Institute for Thermal energy and hydropower, Norwegian University of Science and Technology

Bolland, O. (2005) *Lecture notes in course TEP9 Thermal cycles and cogeneration*, Norwegian University of Science and Technology

Bolland, O., Haglerød, K. K. & Veer, T. (2004) *Measured data correction for improved fouling and degradation analysis of offshore gas turbines*, ASME/IGTI TURBOEXPO Conference 2004, June 14-17, Vienna, Austria

Bolland, O., Jordal, K., Kvamsdal, H. M. & Maurstad, O. (2003), *Benchmarking of gas-turbine cycles with CO_2 capture*, SINTEF Energy Research, Trondheim, Norway

Bossel, U. (2003), *Well-to-Wheel studies, Heating Values, and the Energy Conservation Principle*, European Fuel Cell Forum, Oberrrohrdorf, Switzerland

Botnen, J. B. (2005) *Long Term Price Forecast for Electric Power in the Nordic Countries 2005-2025*, Markedskraft AS, Arendal, Norway

Brealey, R. A. & Myers, S. C. (2003) *Principles of corporate finance, 7^{th} edition*, McGraw-Hill Companies, Inc. New York, US

Brockwell, P. J. & Davis, R. A. (1996) *Introduction to Time Series and Forecasting*, Springer-Verlag, New York, Inc., US

Brooks, C. (2002) *Introductory econometrics for finance*, Cambridge University Press, Cambridge, UK

Chase, D. L. & Kehoe, P. T. (2000) *GE Combined-Cycle Product Line and Performance*, GE Power systems, New York, US

Cortazar, G. & Schwartz, E. S. (1998) *Monte Carlo Evaluation of an Undeveloped Oil Field*, Journal of Energy, Finance &Development, Vol. 3, No. 1, pp. 73-84

Dixit, A. K. & Pindyck R. S. (1994) *Investment under uncertainty*, Princeton University Press, New Jersey, US

Dobbe, T., Fleten, S. E. & Sigmo, S. (2003) *Valuing gas power plants with CO2 capture and tradable quotas*, 16th International Conference on Efficiency, Costs, Optimization, Simulation and Environmental Impact of Energy Systems, Copenhagen, Denmark, June 30 - July 2, 2003

ECON Analysis (2004) *Utsläppsrätter och elhandel*, ECON Analysis AB, Stockholm, Sweden

Energiforum.net (2005), Accessible at http://www.energiforum.net

Ertesvåg, I. S. & Nesheim, S. J. (2005) *Exergetic comparison of energy-based efficiency indicators for combined heat and power (CHP)*, ECOS2005, Trondheim, Norway, 20-22 June 2005

Eydeland, A. & Wolyniec, K. (2003) *Energy and Power Risk Management – New Developments in Modeling, Pricing and Hedging*, John Wiley & Sons, Inc.,Hoboken, New Jersey, US

Fisk, R. W. & VanHousen, R. L. (1996) *Cogeneration Application Considerations*, GE Power Systems, New York, US

Fleten, S. E. & Näsällälä, E. (2003) *Flexibility and Technology Choice in Gas Fired Power Plant Investments*, working paper

Fluor Ltd. (2005) *CO_2 Capture Study at Mongstad*

Gateway to the European Union, Accessible at http://europa.eu.int/

Generating Availability Data System (2005) *2000 – 2004 Generating Availability Report*, North American Electric Reliability Council, Princeton, New Jersey, US

Gibson, R. & Schwartz, E. S. (1990) *Stochastic Convenience Yield and the Pricing of Oil Contigent Claims*, Journal of Finance, Vol. 45, No. 3, July 1990, pp. 959-976

Glasserman, P. (2000) *Monte Carlo Methods in Financial Engineering*, Springer-Verlag New York, Inc., US

Gundersen, T. (1999) *Energibruk i industrien*, Department of Energy and Process Engineering, Norwegian University of Science and Technology

Harvey, A. C. (1989) *Forecasting, Structural time series models and the Kalman filter*, Cambridge University Press, New York, US

Holmelin, E. (2005) *Energiverk Mongstad, Samfunnsmessige konsekvenser*, Agenda Utredning og Utvikling AS, Sandvika, Norway

Holmelin, E. (2005) *Gassrørledning Kollsnes – Mongstad, Samfunnsmessige konsekvenser*, Agenda Utredning og Utvikling AS, Sandvika, Norway
Honore, A. (2005) *Gas Demand in Europe: The role of the power sector*, Energy Risk Management Seminar, CASS Business School, 28th June 2005
ICF Consulting (2005), Accessible at http://www.icfconsulting.com/carbon
IEA (2005) *World Energy Outlook 2005*, International Energy Agency, Paris, France
Indrebø, T. (2004) *Contracts for Pipeline Gas for Power*, Hydro Oil & Energi, Markets
Johnsen, T. A. (2005) *Kvartalsrapport for kraftmarkedet 2. kvartal 2005*, Norges vassdrags- og energidirektorat
Leyva E. & Lekander P.A. (2003), *Climate change for Europe's utilities*, The McKinsey Quarterly, Number 1 2003
Lucia, J. & Schwartz, E. S. (2002) *Electricity Prices and Power Derivatives: Evidence form the Nordic Power Exchange*, Review of Derivatives Research, Vol. 5, No. 1, 2000, pp. 5-50
McDonald, R. L. (2003) *Derivatives Markets*, Pearson Education, Inc., Boston, US
Moran, M. J. & Shapiro, H. N. (2000) *Fundamentals of engineering thermodynamics (3rd edition),* John Wiley & Sons Ltd, Baffins Lane, Chichester, West Sussex, England
Myers, R. H., Myers, S. L., Walpole, R. E., Ye, K. (2002) *Probability and statistics for engineers and scientists, 7th edition*, Pearson Education International, Prentice Hall, New Jersey, US
Northwest Power Planning Council (2002) *Natural gas Combined-cycle Gas Turbine Power Plants*, New Resource Characterization for the Fifth Power Plan, August 8th , 2002
Pettersen, J. (2004) *Industrial Thermal Process Engineering*, Lecture notes, Department of Energy and Process Engineering, Norwegian University of Science and Technology
Point Carbon (2005) Accessible at http://www.pointcarbon.com/
Ronn, E. I. (2002) *Real Options and Energy Management*, Risk books, a division of Risk Waters Group
Rønning, O. (2005) *Kårstøkraft*, Energirike conference 2005, Haugesund, Norway
Sæl, E. (2004) *Utvikling av petroleumsbasert industri i Norge*, Energirike conference 2004, Haugesund, Norway
Schwartz, E. S. & Smith, J. E. (2000) *Short-term Variations and Long-term Dynamics in Commodity Prices*, Management Science, Vol. 46, No. 7, July 2000, pp. 893-911
Schwartz, E. S. (1998) *Valuing Long-Term Commodity Assets*, Financial Management, Vol. 27, No. 1, Spring 1998, pp. 57-66

Schwartz, E. S.(1997) *The Stochastic Behaviour of Commodity Prices: Implications for Valuation and Hedging*, Journal of Finance, Vol. 52, No. 3, July 1997, pp. 923-973

Sollie, O. K. (2002) *Livsløpsanalyse anvendt på energiforsyningssystemet til et raffineri*, Master thesis, Norwegian University of Science and Technology

Statens forurensningstilsyn (2005), Accessible at http://www.sft.no

Statoil ASA (2005) *Energiverk Mongstad - Combined heat and power station, Application for concession*

Statoil ASA (2005) *Energiverk Mongstad – Combined heat and power station with necessary modifications to the refinery, Impact Assessment*

Statoil ASA (2005) *Energiverk Mongstad, Application for revised emission permit*

Tennbakk, B. & Torgersen, L. (2003) *10-årsperspektiv på kraftbalanse og CO_2- utslipp i Nord-Europa*, ECON Analysis, Oslo, Norway

The Intercontinental Exchange (2005) Accessible at https://www.theice.com

The Nordic Power Exchange Nord Pool (2005) Accessible at http://www.nordpool.com

The Norwegian Ministry of Petroleum and Energy (2005), Accessible at http://odin.dep.no/oed/english/bn.html

Tönjes, C. (2005) *Gas to power in Europe*, International Gas Union, 23rd World Gas Conference, Amsterdam, Holland

Trigeorgis, L. (2002) *Real options – Managerial Flexibility and Strategy in Resource Allocation*, The MIT Press, Massachusetts, US

Victor, D.G., Nakicenovic, N., and Victor, N. (1998), *The Kyoto Protocol Carbon Bubble: Implications for Russia, Ukraine, and Emission Trading*, Interim Report 98-094, IIASA, Laxenburg

World Gas Intelligence (2005) *Global perspective on trends and major developments in natural gas markets*, Issues from October and November 2005, Accessible at http://www.energyintel.com/PublicationHomePage.asp

Appendix 1 – Combined heat and power production

Combined heat and power generation is based on the fact that the energy content in any kind of fuel may be utilized for both electricity and heat production at the same time. When applying cogeneration the high grade energy of the fuel is utilized for electricity production, while the low grade energy of the fuel is used for heat production. In this way one can maximize the energy utilization, and construct energy effective plants. This appendix describes some of the technical aspects of cogeneration.

Appendix 1.1 - Concept definitions

Thermal power plant: A facility which applies a combustion process for power generation.
Combined heat and power (CHP) plant: A facility which applies a combustion process for simultaneous power and applicable heat generation.
Efficiency: The percentage share of power generated by a thermal process in relation to the energy content in the fuel.
Total efficiency: The percentage share of power and applicable heat generated by a thermal process in relation to the energy content in the fuel.
Gas turbine: A process which includes a compressor, a combustion chamber and a turbine. The compressor compresses air to high pressure, and the combustion process takes place in the combustion chamber under high pressure. The high pressure exhaust gas is expanded in the turbine.
Heat recovery steam generator (HRSG): A heat exchanger which transfers heat from the exhaust gas after it has been expanded to water or steam.
Combined Cycle Gas Turbine (CCGT): A facility which includes a gas turbine, a heat recovery steam generator and a steam turbine. The steam turbine utilizes heat from the exhaust gas of the gas turbine.
Power generation: Electricity production
Heat to power ratio (HTPR): The relationship between the power and the applicable heat which is produced in a combined heat and power plant.
Saturation temperature: The temperature at which a liquid at a given pressure vaporizes and condenses.
Vaporization: The conversion of liquid water to steam

Superheating: Further heating of steam beyond the saturation temperature. Usually performed by a HRSG in order to maximize the efficiency of a CCGT.

Appendix 1.2 - Description

Combined heat and power production is, from a thermodynamic point of view, a very good way of utilizing the energy content of the fuel. The high grade energy content of the fuel is to a large extent converted to electricity, while the low grade energy content of the fuel is converted to applicable heat. In total this allows for a high total efficiency (up to 90 %), which indicates that only a small fraction of the energy content of the fuel is wasted.

There are several technologies available for combined heat and power production, and the feasible power output ranges from less than 1 MW to about 500 MW. The two major technologies which are available for large CHP-plants (more than 120 MW power output) are:

Condensing steam turbine with steam extraction.

This technology is generally applied when other types of fuel than natural gas is used. The heat to power ratio can be regulated by changing the amount of steam which is extracted. This technology is suitable if steam is available from other processes, and when fuels that are intended for indirect combustion are being used (for example oil or coal).

Combined Cycle with backpressure or condensing extraction steam turbine.

This technology allows for direct combustion of natural gas in a gas turbine, and includes, in the same way as a CCGT, a gas turbine process and a steam turbine process. The heat can be produces either by applying a backpressure steam turbine or by steam extraction. Backpressure steam turbines are used when a high heat to power ratio is wanted, while steam extraction is used for moderate to low heat to power ratios. Steam extraction allows a flexible heat to power ratio, while backpressure steam turbines imply a predetermined heat to power ratio.

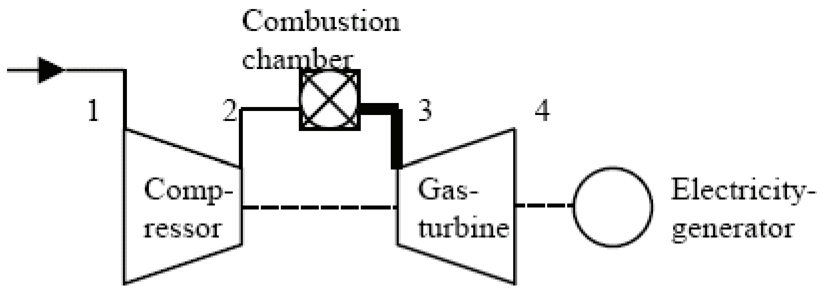
Gas turbine with heat recovery from the exhaust gas

This technology does not include any steam turbines, and the thermal energy which is recovered in the HRSG is entirely used for heating purposes. When this technology is applied, the heat production may be varied independently of the power production. It is also possible not to produce any heat at all.

Appendix 1.3 - Gas turbine systems

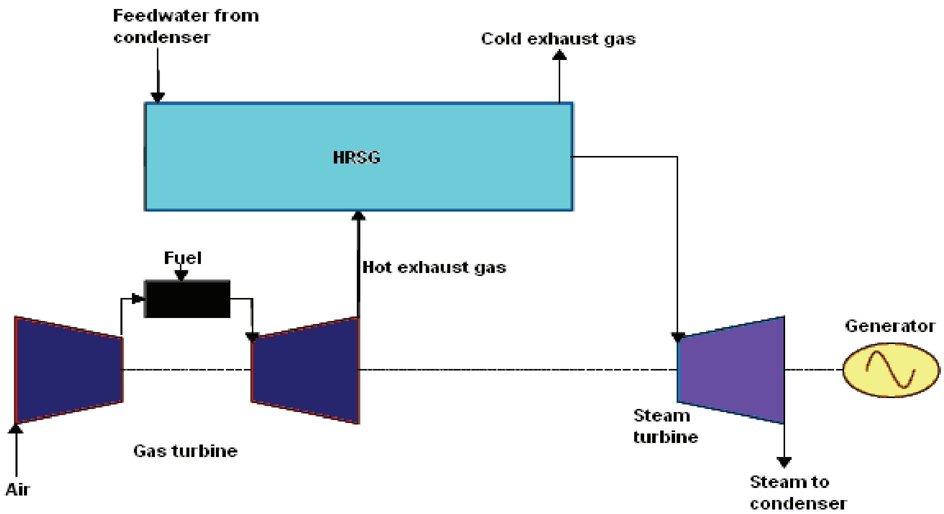
A gas turbine includes a compressor, a combustion chamber and a turbine, and a gas turbine process is generally an open process. This means that the working fluid can not be recycled, and therefore both the compressor inlet pressure and the turbine exit pressure must equal the atmospheric pressure. A schematic gas turbine process is shown in Figure 31.

Figure 31: Gas turbine process



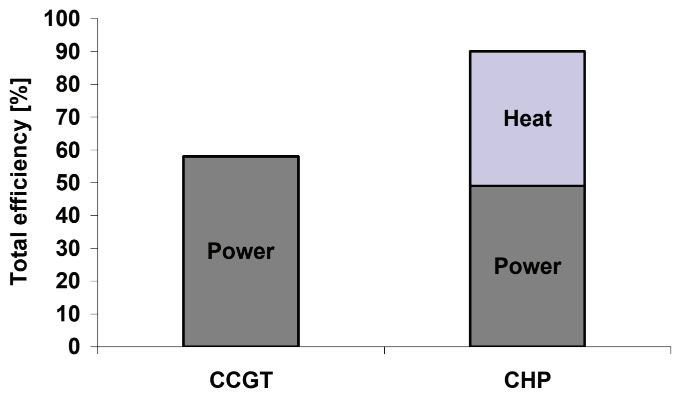
The exhaust gas which exits the turbine is at high temperature (typically 450 – 590°C), and with that still has a high energy content which cannot be utilized if only a gas turbine process is applied. If a HRSG is included in the process, some of the energy content of the exhaust gas may be utilized. The HRSG allows heat to be transferred from the exhaust gas to a feed water stream which is being vaporized and usually superheated. As mentioned earlier, this steam may be used for power production in a steam cycle, heat production or a combination of both. A combined cycle gas turbine process is shown in Figure 32.

Figure 32: CCGT



The efficiency of a gas turbine process is between 20 and 40 %, and the highest efficiency is obtained with the largest and most modern gas turbines. The largest gas turbines available are able to produce more than 250 MW in a single cycle process. For a CCGT it is possible to obtain efficiency as high as 60 %. In a cogeneration system some of the power production is sacrificed in order to produce applicable heat. This means that the efficiency of a cogeneration system is lower than the efficiency of a CCGT, but at the same time the total efficiency is higher. This is illustrated in Figure 33, where typical values for a CCGT and a CHP are shown.

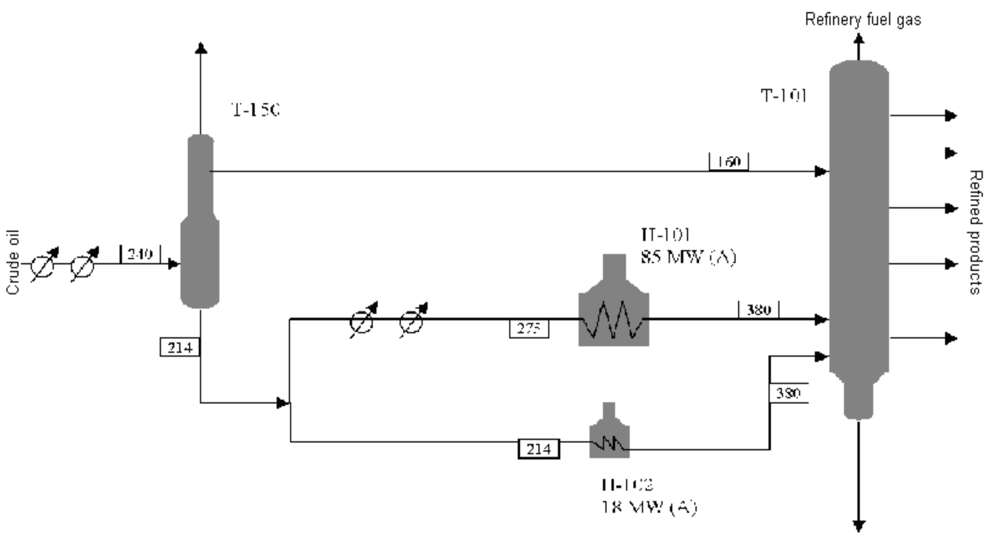
Figure 33: CCGT versus CHP



Appendix 2 – The refining process at Mongstad oil refinery

The purpose of an oil refinery is to convert crude oil into marketable oil products. The oil refinery at Mongstad receives crude oil from the Troll, Statfjord and Åsgård oil fields. In addition to this it receives condensate from the Troll field and from the Oseberg field. The crude oil and condensate are mainly converted into gasoline, diesel, aviation fuel and other light petroleum products. The refining process takes place in a fractional distillation column where the crude oil is heated in a boiler, and the different products are pulled out be their vaporization temperature. This process requires a large amount of heat in the form of crude oil heating and high pressure steam production. Fuel oil and fuel gas are by-products of the refining process, and these components are used in different combustion processes to heat crude oil, to vaporize water and to superheat steam. Today there exist a highly integrated system of steam boilers and steam distribution pipes inside the Mongstad oil refinery, and this system provide the necessary thermal energy for the refining process. Figure 34 shows a schematic drawing of a fractional distillation column including crude oil heating furnaces. The distillation column is denoted with T-101, while the furnaces are denoted with H-101 and H-102.

Figure 34: Oil refining



[Statoil ASA]

Appendix 3 – Efficiency calculations

The efficiency and the total efficiency have been calculated with the input parameters given in Table 9 and

Table 10. These numbers are from Bolland, Jordal, Kvamsdal & Maurstad (2003), and are considered to be typical numbers for North Sea Brent natural gas. The other numbers are publicly available data from Statoil ASA.

Table 9: Natural gas composition

Component	Molar fraction
Methane (CH_4)	82,0 %
Ethane (C_2H_6)	9,4 %
Propane (C_3H_8)	4,7 %
Butane (C_4H_{10})	1,6 %
Pentane (C_5H_{12})	0,7 %
Nitrogen (N_2)	0,9 %
Carbondioxide (CO_2)	0,7 %

Table 10: Natural gas standard conditions

Temperature [C]	15
Pressure [bar]	1,01325
Density [kg/Sm3]	0,851
Lower Heating Value [MJ/kg]	47,6

The numbers given in

Table 10 are called standard conditions for natural gas. It is assumed that both the composition and the properties given above are valid for both fuel gas and for natural gas. The efficiency and the total efficiency are calculated in the following way:

Fuel input:

$0{,}7\ GSm^3/year * 0{,}851\ kg/Sm^3 = \underline{0{,}5957 * 10^9\ kg/year}$

$0{,}5957 * 10^9\ kg/year * 1/(365*24*3600) = \underline{18{,}9\ kg/s}$

Energy input:

18,9 kg/s * 47,6 MJ/kg = 900 MW

Efficiency = 280 MW / 900 MW = 31 %

Total efficiency = (280 MW + 350 MW) / 900 MW = 70 %

The efficiency calculations that have been performed in this section are based on the assumption that the fuel gas provided by the refinery has the same lower heating value (LHV) as North Sea Brent natural gas. This assumption is however not correct as refinery fuel gas has a lower LHV than North Sea Brent natural gas. According to the gas turbine vendor GE the MS9001E gas turbine has a single cycle heat rate equal to 10100 Btu per kWh. This is equivalent to 2,96 MWh natural gas per MWh electricity produced and equals 34 percent efficiency. In order to make our valuation as correct as possible, this value is used in the calculations.

Appendix 4 – Typical operation and maintenance costs

Table 11: O & M costs

Estimated O & M life-cycle costs				
GESTAG 109FA (approximately 350 MW); representative for Norwegian CCGT				
	Average \$/year		\$/kWh/year	
Operations cost				
Direct salary and labour	2 249 520		6,45	31,6 %
Direct materials and subcontracts	745 913		2,14	10,5 %
Total	2 995 433		8,59	42,1 %
Maintenance planned				
Gas turbines	2 949 950		8,46	41,5 %
HRSGs	144 283		0,41	2,0 %
ST-Gs	145 986		0,42	2,1 %
Instruments	37 162		0,11	0,5 %
Systems BOP	154 514		0,44	2,2 %
Total	3 431 895		9,84	48,3 %
Maintenance unplanned	686 379		1,97	9,6 %
Total O & M per year	7 113 707		20,4	100,0 %

[O. Bolland]

The values shown in Table 11 are considered to be realistic for a 350 MW CCGT in Norway using a GE STAG 109FA gas turbine. It is assumed that the allocation of the O & M costs also is applicable for the CHP-plant at Mongstad.

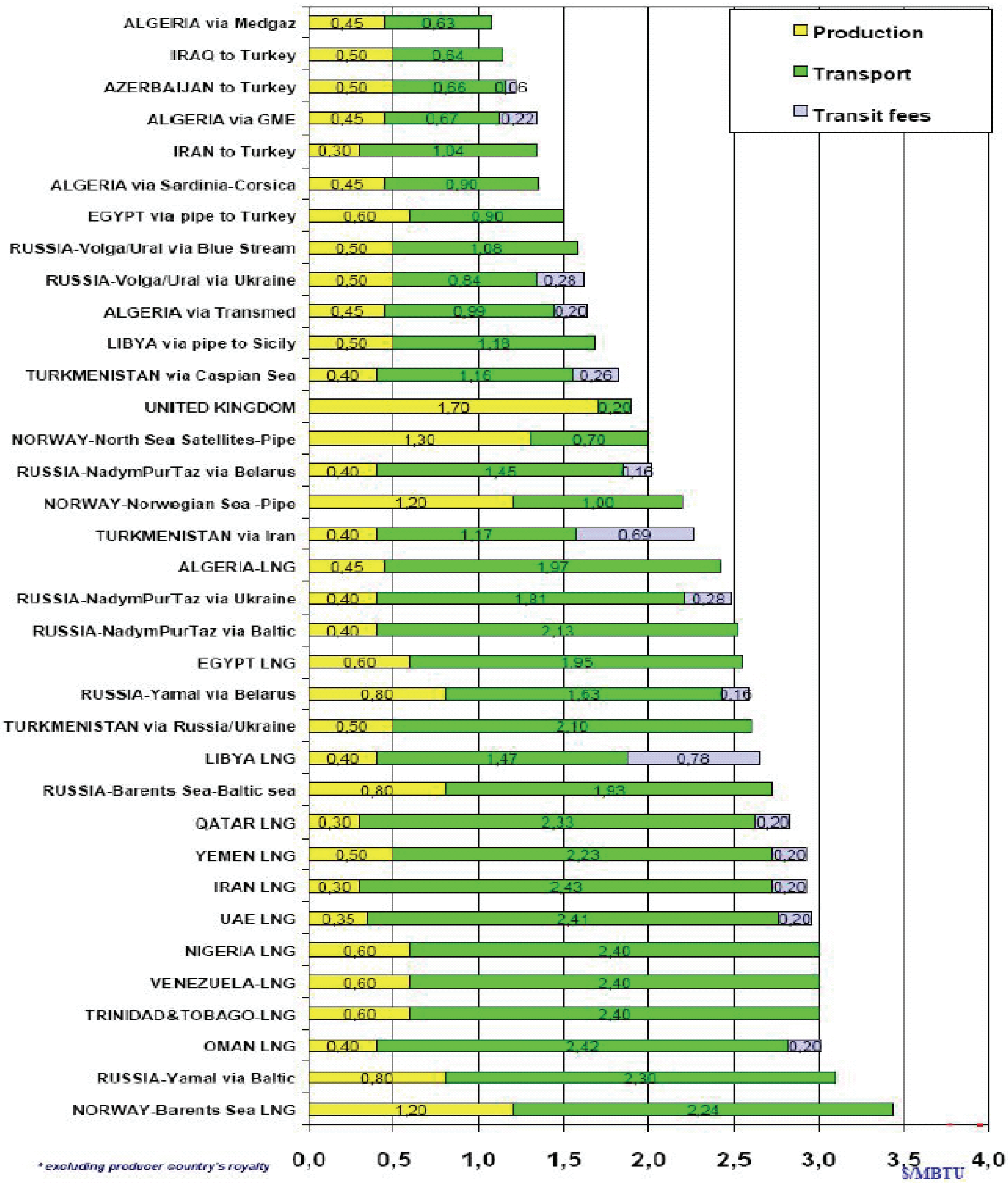
Appendix 5 – UK Natural gas prices observed in November 2005

Recent natural gas prices observed at the IPE have been exceptionally high compared to the average price level that has been observed during the later years. Clearing prices for monthly forward contracts for deliveries during the winter months of 2006 has reached as high as 80 ($14.00/MMBtu) pence per therm. This is an extremely high value compared to the average winter prices observed in November for 2005 and 2004 which were about 60 pence per therm ($11.50/MMBtu) and 30 pence per therm ($5.75/MMBtu) respectively. The high price can to a large degree be explained by the high oil price level, but during 2005 several major outages of the Norwegian natural gas export have occurred. The last major outage occurred at October the 15^{th}, and it involved the Asgard field in the Norwegian Sea. A total of 43,5 million standard cubic metres are usually transported through the Asgaard pipeline, but because of the outage this amount has failed to reach the UK and European markets during October and Novermber 2005. Analysts believe that this outage in combination with the high oil price expectations for the 2006 winter has sent the price of natural gas to an all time high level. The natural gas export from the Asgaard field is however expected to come online again within November 2005, so the high prices that are observed now are not expected to last. A third reason for the extreme natural gas prices expected for the winter months of 2006 are the UK weather forecasts which anticipate a colder-than-average winter. This is however also a temporary factor and it should not affect long-term natural gas prices. During the 3^{rd} and 4^{th} quarter of 2005 UK TOP contracts indexed against the natural gas spot price have been significantly higher priced than contracts indexed against the oil price. One example is British Gas (BG), which revealed that its average third-quarter 2005 realized price on UK gas sales was 20,1 pence per therm ($3.50/MMBtu), 10% up from a year-earlier but less than half spot values. While some of BG's sales probably were on a transfer basis to UK power plants that it partly owns, the rest was presumably oil-indexed or low fixed-price natural gas for the wider market. This example shows that the realized price of natural gas may differ considerably from the observed spot price, especially when large parts of the traded volumes are indexed against other factors than the spot market.

Appendix 6 – Marginal costs of natural gas

Figure 35 illustrates the marginal costs of producing natural gas in different parts of the world and bringing it to the European market. The marginal costs are divided into production costs, transportation costs and transit fees.

Figure 35: Marginal costs of natural gas



Source: OME, 2001

Appendix 7 – Programming code

The programming code can best be studied by viewing the attached files in an appropriate editor. All code is written in C++ by Flåøyen and Kviljo. Code for the Kalmanfilter can be found in Alstad & Foss (2004).

```
/***********************************************
 *  Input data and parameters for the simulation  *
 *                                                *
 *  Note that som parameters might have been      *
 *  changed from the base case simulations        *
 ************************************************/

 // *** Input for the stochastic processes ***
 double rate = 0.05; // risk free annual interest rate

 // *** Input for the electricity price process ***
 double x0_el = 0.0122;            // Short term start parameter
 double epsilon0_el = 5.521;       // Long term start paramater
 double alfas_el=0;                // Value to which short term process revert
 double mu_per_ar_el = 0.0129;     // Annual long term drift
 double kappa_el_pa = 1.08;        // Mean reverting coefficient
 double sigmaShort_el_pa = 0.5584; // Sigma short term
 double sigmaLong_el_pa = 0.1050;  // Sigma long term
 double correl_el=0.0075;          // Correlatrion long and short term
 double season_el_y = -0.0859;     // Strength of seasonal effect
 double season_el_n = 0.4272;      // Seasonal displacement factor


 // *** Input for the natural gas price process ***
 double x0_gas=0;                 // Short term start parameter
 double epsilon0_gas=4.745;       // Long term start paramater
 double alfas_gas=0;              // Value to which short term process revert
 double mu_per_ar_gas=0.0129;     // Annual long term drift
 double kappa_gas_pa = 3.5781;    // Mean reverting coefficient
 double sigmaShort_gas_pa=0.619;  // Sigma short term
 double sigmaLong_gas_pa=0.2501;  // Sigma long term
 double correl_gas=-0.5125;       // Correlatrion between long and short term
 double season_gas_y = -0.2569;   // Strength of seasonal effect
 double season_gas_n = 0.4345;    // Seasonal displacement factor


 // *** Input for the CO2 price process***
 double x0_co2 = 166.5;           // NOK/ton
 double kappa_co2_pa = 0.0022662; // Mean reverting coefficient
 double Seq_co2 = 200;            // Long term equlibrium
 double sigma_co2_pa = 0.53;      // Sigma
 double kvoteandel = 0.5;         // Share of emission rights to be bought
 double correl_el_co2 = 1;        // Correlation with long term el price
 double utslipp = 160;            // Ton CO2-emissions per h basecase


 // *** Input for the simulation ***
 int totaltime = 50;              // Total number of years for project
 int perioder_per_ar = 6;         // Number of periods per year
 int simulations = 50;           // Number of simulations
 double startfraction = 0.85;     // Today (0 = 01.01.05)
 double prodstart = 4;            // Time for start of production

 // *** Input for the project ***
 double investment = 1600000000; // Investment cost
 double insurance = 0.005;       // Annual insurance as percentage of inv.
 double opcost = 0.02;           // Annual operating costs, percentage of inv.
 double vedlikehold1 = 0.003;    // Annual maintenance cost normal year
```

```
  double vedlikehold2 = 0.017;    // Annual maintenance cost "special" year
  double MWEl = 280;              // MW el produced
  double MWGas = 694;             // MW natural gas consumed
  double timer = 8100;            // Annual operating hours
  double transport = 13.34;       // NOK/MWh transport costs from Norway to GB

  // *** Input for the abandon scenario***
  double abandon_invest = 100000000; // Investmentcost boiler
  double abandon_turbineprice = 0;   // Selling price gas turbines
  double abandon_om = 0.03;          // Variable costs as percentage of inv.
  double abandon_gas = 274;          // MW natural gas consumed in boiler
  double abandon_co2 = 62.115;       // ton/h co2 emissions

  // *** Input for part load scenario ***
  double altMWEl = 150;              // Electricity produced with part load
  double altMWGas = 500.3;           // Natural gas consumed with part load
  double altUt = 103.873;            // ton/h co2 emissions

  // *** Calculated parameters
  int periods = totaltime*perioder_per_ar;             // Number of periods
  double periodlength = (totaltime+0.0)/(periods+0.0); // Period (yearfrac)
  double timer_pp = timer*periodlength;                // Operating h per period
  double diskont = log(1+rate);                        // Discount rate
  double preperiod = prodstart-startfraction;          // Time to prod. start


/************************************
 ***** This is the main program *****
 ************************************/

#include <iostream>
#include <fstream>
#include <cstdlib>
#include <string>

#include "time.h"
#include "apmatrix.h"
#include "Simulation.h"
#include "valuation.h"
#include "inputs.h"

using namespace std;

int main ()
{

  // Define variables for finding time of calculations
  int t1; int t2;
  t1 = time(0);

  // ***** El-price process *****
  // Build random matrices for simulation
  apmatrix<double> rxe=FillRandom(simulations, periods); // MR-process
  apmatrix<double> ree=FillRandom(simulations, periods); // ABM-process

  // Calculate x and epsilon
  apmatrix<double> X_el = mcMeanRev(x0_el, alfas_el, kappa_el_pa,
  sigmaShort_el_pa, simulations, periods, rxe, periodlength); //short term state
  apmatrix<double> E_el = ABM(epsilon0_el, mu_per_ar_el, sigmaLong_el_pa,
  simulations, periods, ree, rxe, correl_el, periodlength); //long term state


  // ***** Gas process *****
  // Build random matrices for simulation
  apmatrix<double> rxg=FillRandom(simulations, periods-1);
  apmatrix<double> reg=FillRandom(simulations, periods-1);
```

```
  // Calculate x and epsilon
  apmatrix<double> X_gas = mcMeanRev(x0_gas, alfas_gas, kappa_gas_pa,
  sigmaShort_gas_pa, simulations, periods, rxg, periodlength); //short term
  apmatrix<double> E_gas = ABM(epsilon0_gas, mu_per_ar_gas, sigmaLong_gas_pa,
simulations,
  periods, reg, rxg, correl_gas, periodlength); // simulate long term state


  // ***** Co2 process *****
  // Matrise for korrelasjon med epsilon el
  apmatrix<double> diffEl = diffMatrix(E_el);
  apmatrix<double> random1 = FillRandom(simulations, periods-1);
  apmatrix<double> co2_price = oneFactorMeanRev(x0_co2, kappa_co2_pa, Seq_co2,
sigma_co2_pa,
  periods, simulations, random1, diffEl, correl_el_co2, preperiod, periodlength);

  // ***** Make price *****
  // Combine short-, long- and seasonal term to pricematrix
  apmatrix<double> el_price = makeprice(X_el, E_el, startfraction, periodlength,
  season_el_y, season_el_n);
  apmatrix<double> gas_price = makeprice(X_gas, E_gas, startfraction, periodlength,
  season_gas_y, season_gas_n);

  // Print mean prices to file
  printToFile(arithmeticMean(gas_price), "gasmean.txt");
  printToFile(arithmeticMean(el_price), "elmean.txt");
  printToFile(arithmeticMean(co2_price), "co2mean.txt");


  // Make vector with O & M costs
  apmatrix<double> vedlikehold = vedlikeholdsvektor(investment, vedlikehold1,
vedlikehold2,
  totaltime, perioder_per_ar, opcost, insurance);

  // Cashflow for alternative boiler
  apmatrix<double> abandoncf = abandoncashFlow(gas_price, abandon_gas, abandon_om,
abandon_invest,
  abandon_co2, timer_pp, co2_price, transport, perioder_per_ar,kvoteandel);

  // Cashflow base case
  apmatrix<double> cashflows = cashFlow(el_price, gas_price, co2_price, MWEl,
MWGas, transport,
  kvoteandel, utslipp, timer_pp, investment, vedlikehold, perioder_per_ar);

  // Cashflow for part load
  apmatrix<double> cashflows2 = cashFlow2(el_price, gas_price, co2_price, MWEl,
MWGas, transport,
  kvoteandel, utslipp, timer_pp,  investment, vedlikehold, perioder_per_ar,
altMWEl, altMWGas, altUt);

  // **** Calculate net present value of alternatives and write to file ****

  // NPV alternative boiler
  printToFile2(NPV(abandoncf, diskont, periodlength, preperiod), "boiler.txt");

  // NPV Base case
  printToFile2(NPV(cashflows, diskont, periodlength, preperiod), "basecase.txt");

  // NPV with part load option
  printToFile2(NPV(cashflows2, diskont, periodlength, preperiod),
"base_partload.txt");

  // NPV with option to abandon
  printToFile2(valuation(cashflows, abandoncf, abandon_invest-abandon_turbineprice,
  diskont, periodlength, preperiod), "abandon_base.txt");

  // NPV with option to abandon and part load
```

```
  printToFile2(valuation(cashflows2, abandoncf, abandon_invest-
abandon_turbineprice, diskont, periodlength, preperiod), "abandon_partload.txt");

  t2 = time(0);
  cout<<"\n time computing: "<<t2-t1<<" seconds \n";
  system("pause");

  return 0;
}

/***********************************
 **** Methods for the simulations ****
 **********************************/

#include <iostream>
#include <fstream>
#include "Random.h"
#include "math.h"
#define PI  3.14159265359f

using namespace std;

// Returns [x][y] matrix with random normal(0,1) distributed numbers
apmatrix<double> FillRandom(int x, int y)
{
    apmatrix<double> sim(x,y);
    Random rv;

    for (int i = 0; i<x; i++ )
    {
              for (int j = 0; j<y; j++)
              {
                  double value = rv.normal();
                  sim[i][j] = value;
              }
    }
    return sim;
}


// Prints a matrix to file with name s1[]
void printToFile(apmatrix<double> m, char s1[])
{
  ofstream printfile;
  printfile.open(s1, ios::out);

    for (int i = 0; i<m.numrows(); i++ )
    {
              for (int j = 0; j<m.numcols(); j++)
              {
                 printfile << m[i][j]<<"\t";
              }
              printfile << endl;
    }
    printfile.close();
}


// Calculate arithmetic mean for each column in a matrix
// returns vector of column means
apmatrix<double> arithmeticMean(apmatrix<double> m)
{
    apmatrix<double> temp(1, m.numcols());

    for (int j = 0; j<m.numcols(); j++)
    {
        temp[0][j] = 0.0;
```

```
        for (int i=0; i<m.numrows();i++)
                {
                    temp[0][j] = temp[0][j]+m[i][j];
                }
      }

      apmatrix<double> mean(1, m.numcols());

      for(int j=0; j<m.numcols();j++)
      {
             mean[0][j]=temp[0][j]/m.numrows();
      }

      return mean;
}


// Calculate arithmetic mean for a n x 1 vector
// Returns mean as double
double MeanDouble(apmatrix<double> m)
{
    double temp;

        temp = 0.0;

        for (int i=0; i<m.numrows();i++)
                {
                    temp = temp+m[i][0];
                }
        double mean = temp/m.numrows();

        return mean;
}


// Calculate arithmetic mean for a given row from a given column to end of matrix
// Returns mean as double
double MeanDouble2(apmatrix<double> m, int row, int start)
{
    double temp;

        temp = 0.0;
        for (int i=start; i<m.numcols();i++)
                {
                    temp = temp+m[row][i];
                }
        double mean = temp/(m.numcols()-start-1);
        return mean;
}


// Prints a matrix to file with name s1[],
// including the column means
void printToFile2(apmatrix<double> m, char s1[])
{
  apmatrix<double> mean = arithmeticMean(m);

  ofstream printfile;
  printfile.open(s1, ios::out);

    for (int i = 0; i<m.numrows(); i++ )
    {
              for (int j = 0; j<m.numcols(); j++)
              {
                 printfile << m[i][j]<<"\t";
              }
              printfile << endl;
    }
```

```
    printfile << "*** Mean for "<<m.numrows()<<" simulations ***";
    printfile << endl;

    for (int j = 0; j<mean.numcols(); j++)
    {
        printfile << mean[0][j]<<"\t";
    }

    printfile.close();
}

// Prints matrix means to file with name s1[]
void printMean(apmatrix<double> m, char s1[])
{
  apmatrix<double> mean = arithmeticMean(m);

  ofstream printfile;
  printfile.open(s1, ios::out);

  printfile << "*** Mean for "<<m.numrows()<<" simulations ***";
  printfile << endl;

  for (int j = 0; j<mean.numcols(); j++)
  {
    printfile << mean[0][j]<<"\t";
  }

    printfile.close();
}

// One-factor mean reversion process for CO2
apmatrix<double> oneFactorMeanRev(double X0, double kappa_pa, double Seq,
double sigma_pa, int periods, int simulations, apmatrix<double> m,
apmatrix<double> diff, double correl_el, double preperiod, double length)
{
    // Adjusting factors per period
    double sigma = sqrt(pow(sigma_pa,2.0)*length);
    double kappa = kappa_pa*length;

    // Defining the apmatrix<double> to hold the function value
    apmatrix<double> sim(simulations,periods);

    // The simulation
    for(int i = 0; i<simulations; i++)
    {
        double eq = Seq;
        for (int j = 0; j<periods; j++)
        {
          if (j==0)
          {
            sim[i][0]=X0;
          }
          else
          {
          eq = eq*(1+correl_el*diff[i][j]);
          sim[i][j]=(sim[i][j-1])*exp(kappa*(eq-(sim[i][j-1]))+sigma*(m[i][j-1]));
          }
        }
    }

    return sim;
}


// ABM process (long term variables)
apmatrix<double> ABM(double start, double mu_per_ar, double sigmaLong_pa, int
simulations,
```

```
int periods, apmatrix<double> random1, apmatrix<double> random2, double correl,
double length)
{
  // Adjusting factors per period
  double sigma  = sqrt(pow(sigmaLong_pa,2.0)*length);
  double mu = mu_per_ar*length;

  // Defining the apmatrix<double> to hold the function value
  apmatrix<double> sim(simulations, periods);

  // The simulation
  for(int i = 0; i<simulations; i++)
  {
    for (int j = 0; j<periods; j++)
    {
        if (j==0)
        {
                sim[i][0]=start;
        }

        else
        {
                sim[i][j]=sim[i][j-1] + mu + sigma*(correl*random1[i][j-1]
                +sqrt(1-pow(correl,2))*random2[i][j-1]);
        }
    }
  }

  return sim;
}


// Mean reversion process (short term variables)
apmatrix<double> mcMeanRev(double start, double alfas, double kappa_pa, double
sigma_pa,
int simulations, int periods, apmatrix<double> random, double length)
{
  // Adjusting factors per period
  double sigma = sqrt(pow(sigma_pa,2.0)*length);
  double kappa = kappa_pa*length;

  // Defining the apmatrix<double> to hold the function value
  apmatrix<double> sim(simulations, periods);

  // The simulations
  for(int i = 0; i<simulations; i++)
  {
    for (int j = 0; j<periods; j++)
    {
        if (j==0)
        {
                sim[i][0]=start;
        }
        else
        {
                sim[i][j]=sim[i][j-1] + alfas*(1-exp(-kappa)) + (exp(-kappa)-
1)*sim[i][j-1]
                + random[i][j-1]*sigma*sqrt(1/(2*kappa)*(1-exp(-2*kappa)));
        }

    }
  }

  return sim;
}

// Calculate the seasonal price component
double season(int j, double start, double length, double y, double n)
```

```
{
    double currentfraction = start + j*length;
    double val = y*cos((currentfraction-n)*2*PI);

    return val;
}

// Include short- and long-term variables and seasonal component in one price
apmatrix<double> makeprice(apmatrix<double> x, apmatrix<double> e, double start,
double length, double y, double n)
{
    // Test if the number of short term variables equals the number of long term
variables
    if (x.numrows()!=e.numrows())
        cout << "Ulikt antall rader";
    if (x.numcols()!=e.numcols())
        cout << "Ulikt antall kolonner";

    apmatrix<double> price(x.numrows(),x.numcols());

    for (int i = 0; i<x.numrows(); i++ )
    {
        for (int j = 0; j<x.numcols(); j++)
        {
                price[i][j]=exp(x[i][j]+e[i][j]+season(j, start, length, y, n));
        }
    }

    return price;
}



// Calulate the relative change in a matrix over time
// I.e. delta_x/x
apmatrix<double> diffMatrix(apmatrix<double> m)
{
    apmatrix<double> temp(m.numrows(), m.numcols());

    for (int i = 0; i<m.numrows(); i++)
    {
        for (int j=0; j<m.numcols();j++)
        {
                if ( j==0 )
                   temp[i][j] = 0;
                else
                   temp[i][j] = (exp(m[i][j])-exp(m[i][j-1]))/exp(m[i][j-1]);
        }
    }

    return temp;
}

// Calculate cashflows for base case scenario
apmatrix<double> cashFlow(apmatrix<double> el, apmatrix<double> gass,
apmatrix<double> co2,
double MWel, double MWgas, double trans, double kvote, double ut, double lengde,
double inv, apmatrix<double> v, double periode_pr_ar)
{
   apmatrix<double> cf(el.numrows(), el.numcols());

   for (int i=0; i<el.numrows();i++)
   {
      for (int j=0; j<el.numcols();j++)
      {
            cf[i][j] = (MWel*el[i][j]-MWgas*(gass[i][j]-trans)-
ut*kvote*co2[i][j])*lengde
```

```
            -v[0][j];
      }
   }

   return cf;
}


// Calculate cashflows with part load option
apmatrix<double> cashFlow2(apmatrix<double> el, apmatrix<double> gass,
apmatrix<double> co2,
double MWel, double MWgas, double trans, double kvote, double ut, double lengde,
double inv, apmatrix<double> v, double periode_pr_ar, double altMWEl, double
altMWGas, double altUt)
{

   apmatrix<double> cf(el.numrows(), el.numcols());

   // Matrix with 2 in [i][j] if it operates with two turbines in simulation i,
period j
   // and 1 if it operates with part load
   apmatrix<double> drift(el.numrows(), el.numcols());

   for (int i=0; i<el.numrows();i++)
   {
      for (int j=0; j<el.numcols();j++)
      {
            // Cash flow two turbines
            double cf1 =(MWel*el[i][j]-MWgas*(gass[i][j]-trans)-
ut*kvote*co2[i][j])*lengde
            -v[0][j];

            // Cash flow part load
            double cf2 = (altMWEl*el[i][j]-altMWGas*(gass[i][j]-trans)-
altUt*kvote*co2[i][j])*lengde
            -v[0][j];

            if (cf1>cf2)
            {
               cf[i][j] = cf1;
               drift[i][j] = 2;
            }

            else
            {
               cf[i][j] = cf2;
               drift[i][j] = 1;
            }

       }
   }

   //printToFile(drift, "antallturbiner.txt");
   return cf;
}


// Calculate cashflows for the alternative boiler
apmatrix<double> abandoncashFlow(apmatrix<double> gasprice, double MWGas, double
om, double invest,
double co2emissions, double antalltimer, apmatrix<double> co2, double transport,
double periods_per_year,
double kvote)
{

   apmatrix<double> ab(gasprice.numrows(), gasprice.numcols());

   for (int i=0; i<gasprice.numrows();i++)
```

```
   {
      for (int j=0; j<gasprice.numcols();j++)
      {
            ab[i][j] = (gasprice[i][j]-
transport)*MWGas*antalltimer+om*invest/periods_per_year+co2emissions*antalltimer*co
2[i][j]*kvote;
      }
   }

   return ab;
}


// Calculate net present value of cashflows
apmatrix<double> NPV(apmatrix<double> cf, double diskont, double periodlength,
double preperiod)
{
   apmatrix<double> npv(cf.numrows(), 1);

   for (int i=0; i<cf.numrows();i++)
   {
      npv[i][0]=0;

      for (int j=0; j<cf.numcols();j++)
      {
            npv[i][0] = npv[i][0]+cf[i][j]*exp(-
diskont*(preperiod+periodlength*j));
      }

   }

   return npv;
}


// Calculate periodic operation, maintenance  and insurance costs
apmatrix<double> vedlikeholdsvektor(double invest, double vedl1, double vedl2, int
totaltime,
int perioder_pr_ar, double forsikring, double opcost)
{
    apmatrix<double> vedlikehold(1, totaltime*perioder_pr_ar);
    int temp = 0;

    for (int i = 0; i<totaltime; i++)
    {
        if (temp<2)
        {
                for(int j=0; j<perioder_pr_ar; j++)
                {

vedlikehold[0][i*perioder_pr_ar+j]=(vedl1+opcost+forsikring)*invest/perioder_pr_ar;
                }

                temp=temp+1;
        }

        else
        {
                for(int j=0;j<perioder_pr_ar; j++)
                {

vedlikehold[0][i*perioder_pr_ar+j]=(vedl2+opcost+forsikring)*invest/perioder_pr_ar;
                }
                temp=0;
        }
    }

    return vedlikehold;
```

```
}


// Valuation with option to abandon, returns NPV and time of abandonment for each
scenario
apmatrix<double> valuation(apmatrix<double> cf, apmatrix<double> ab,
 double abandoninvest, double r, double stepsize, double start)
{
  apmatrix<double> simulering(cf.numrows(), 2);
  for (int i=0; i<cf.numrows(); i++)
  {
    // Setter initiell nåverdi lik 0 og abandontime til siste periode
    double nv=0;
    double abtime=cf.numcols()*stepsize;
    double abcashflow = 0;
    double abandonvalue;

    for(int j=cf.numcols()-1; j>=0; j--)
    {
        abcashflow = ab[i][j]+abcashflow*exp(-r*stepsize);
        abandonvalue = -(abandoninvest+abcashflow);

        double v = cf[i][j] + nv*exp(-r*stepsize);

        if (v>abandonvalue)
        {       nv = v;
        }

        else
        {   nv=abandonvalue;
            abtime=stepsize*j;


         }
        if (j==0)
                nv = nv*exp(-r*start);

    }

    simulering[i][0]=nv;
    simulering[i][1]=abtime;

  }
  return simulering;
}
```