Toward a Model for the Swedish-Norwegian Electricity Certificate Market

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Written by:

Magne Hustveit
Jens Sveen Frogner

Supervisor:

Stein-Erik Fleten

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Preface

Several countries have recently been introducing support schemes to favor investments in renewable energy. The intention is to meet the increased demand for electricity with a more sustainable supply. Traditional environmental policies have usually involved providing subsidies, or imposing obligations on market actors. However, over the last decades, market-based energy policies have grown more popular, as governments have sought to maximize the social surplus while addressing the adverse effects of pollution.

The Tradeable Green Certificate (TGC) market is an example of such a market-based energy policy. These markets incentivize investments in renewable energy. Studying the characteristics and performance of these support mechanisms is of great interest as it can provide useful guidance to better design for both new and existing systems.The TGC market introduced in Sweden and Norway is of special interest, as it is a multistate system, causing dynamics worthy of further investigation.

Magne Hustveit   Jens Sveen Frogner

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Abstract

This paper explores the Swedish-Norwegian market for Electricity Certificates, which is a support scheme to establish incentives for investments in renewable electricity production. Producers investing in new renewable capacity receive certificates based on their actual production. Retailers of electricity are required to buy certificates for a proportion of their total sales. If a retailer's obligation is not met, a penalty fee is imposed. The certificates are traded both bilaterally and as a financial instrument on the Nasdaq Commodity Exchange. The design and potential success of this multistate support mechanism will be of great interest to policy makers and green investors. The dynamic equilibrium model of Coulon, Khazaei, Powell (2014) is adapted to the Swedish market. It is found to replicate historical long-term trends and price levels well. Sensitivity analyses show that the key drivers of certificate prices are the penalty levels and the discount rate. Further it is shown that a higher rate of certificate price feedback on the investment rate dampens the price fluctuations around the trend line. The rate of feedback is uncertain, but it is assessed to be larger than zero.

Contents

1 Introduction **2**

2 The Swedish-Norwegian Electricity Certificate Market **3**

3 Methods for Modeling an Electricity Certificate Market **7**
3.1 Fundamental Equilibrium Models 7
3.2 Agent Based Models 8
3.3 Statistical Models 9
3.4 System Dynamics Models 10

4 Price Dynamics **11**
4.1 Price Equilibrium 11
4.2 Technologies for renewable energy in Sweden and Norway 12
4.3 Market Instability 13
4.4 Alternatives to Price Feedback on Investments 14
4.5 Expected Prices in the Market 15

5 Example Model **16**
5.1 Mathematical Formulation 16
5.2 Implementation 17
5.3 Results and Interpretation 23

6 Conclusions and Further Work **27**

1 Introduction

One of history's great works of literature tells the story of Don Quixote, battling the windmills on the plains of La Mancha [1]. One can only imagine Quixote's fear had he seen today's version of these "ferocious giants". However, these giants are not our enemies. Wind turbines, solar cells and hydro power facilities are all feeding clean energy into a power grid struggling to meet the world's ever increasing demand for energy.

"The relation between energy consumption and economic growth is of great interest to the energy economists. It is not possible to achieve high growth in one, without keeping pace with another "[2]. While the economic growth of developing countries is seen as positive, their increased energy consumption brings with it adverse effects [3]. Governments across the globe, concerned with global warming, are currently seeking to establish incentives supporting the further development of clean energy production. The Swedish-Norwegian electricity certificate market is an example of such attempts.

This paper aims to explore the Swedish-Norwegian electricity certificate market and the different methods of valuing these certificates.

2 The Swedish-Norwegian Electricity Certificate Market

This section aims to explain the most important features of a Tradable Green Certificate (TGC) market and specifically the characteristics of the Swedish-Norwegian Market for Electricity Certificates. Traditional environmental policies have usually involved providing subsidies, or imposing obligations on market actors. However, over the last decades, market-based energy policies have grown more popular, as governments have sought to maximize the social surplus while addressing the adverse effects of pollution. The TGC-market is an example of such a market-based energy policy, used by governments to promote the development of increased renewable capacity in the electricity grid. The system is flexible and allows regulators to specify which types of renewables should be favored.

An electricity certificate market must be designed so that there exist both supply and demand, as well as a marketplace in which the certificates can be traded. In a TGC market, supply is established by letting the *regulator* decide which projects fulfill the requirements for receiving certificates. Once qualified, *producers* of eligible electricity are awarded a number of certificates based on their actual monthly production. Demand is then established by requiring *retailers* to buy some of these certificates. The requirement imposed on each individual retailer is based on the amount of electricity he has sold to his customers. If a retailer does not fulfill this requirement by holding enough certificates at some specified compliance date, he is fined a penalty fee. Since the required number of certificates is calculated as a proportion of sold quantity, the total demand for electricity certificates is based on *end users'* consumption of electricity. This demand for electricity is typically found to be fairly inelastic, and the total consumption is predictable within seasonal variance.

One might argue who actually pays for these systems. The legislation states that the costs should be charged consumers over their electricity bill [4]. In his paper from 2000, Morthorst, finds that the costs of such a system is in fact carried by the consumers [5]. Since electricity demand is inelastic, an increased electricity price will not cause significant reductions in the quantity of electricity consumed. Bye [6] disagrees with the view of Morthorst. He argues that the costs of TGC markets are actually paid by the producers. As more projects become profitable, production volumes, and hence supply, rise. According to Bye, this will cause lowered retail prices, even though the certificate costs have been added. Thus, the existing producers carry the costs of the system, and consumers benefit from the lowered prices[1].

The Swedish electricity certificate market was opened in 2003, and is planned to last until 2035. From January 2012, the market was extended to include Norway [7]. By legislating the system, the lawmakers seek to add an additional 26.4 TWh of annual renewable capacity to the Swedish-Norwegian power grid. All sources of renewable electricity production are equally entitled to certificates

[1]Bye notes that the increased consumer surplus is exceeded by the decrease in producer surplus, and hence, the total social surplus experience a net decrease.

and there are no requirements regarding how this new capacity should be located geographically. This ensures that more profitable projects are realized first. Electricity producers receive one certificate per 1 MWh of generation for their eligible power plants. A power plant declared eligible, will generate certificates during its 15 first years of production. While Norwegian power plants must be in operation by the end of 2020 to qualify for certificates, Swedish power plants may wait until later and still receive certificates. The Swedish plants that start operating after 2020 will, however, only receive certificates until 2035.

Electricity production companies are the main sellers of certificates. These companies do not have to meet any obligations. They are allowed to bank their certificates and may time their certificate sales to maximize profits, i.e. either by selling immediately or by waiting for higher expected prices. In the Swedish-Norwegian market, most eligible plants produce power from wind, hydro and bio power. *Run-of-river* hydro power, i.e. with no storage capacity, and wind power are not able to store and time their production. Thus, weather conditions will directly affect the amount of electricity produced, and hence the amount of certificates issued. Weather conditions are therefore of great importance in forecasting certificate prices. *Dammed-hydro* facilities and bio power plants may time their production in order to maximize profits. This implies that the level of electricity production will depend on the price of electricity. Consequently, the price of electricity will affect certificate supply, and thus certificate prices. In the long run, high prices on electricity and certificates will lead to new investments in production capacity and thus, higher supply of both certificates and electricity.

The retailers are the main buyers of certificates. Every year, March 31, they are obliged to prove to the regulators that they are in possession of the required amount. The retailers are free to choose their trading strategy as long as they acquire the amount of certificates necessary to fill their obligations. At April 1, this amount is *cancelled*. If a retailer is not able to fill his obligations, he is charged the penalty fee for the remaining requirement. The penalty fee is calculated as 1.5 times the average price observed over the previous year. The proportion of electricity sales required certified, is increasing until 2020, and decreasing from then towards zero in 2035.

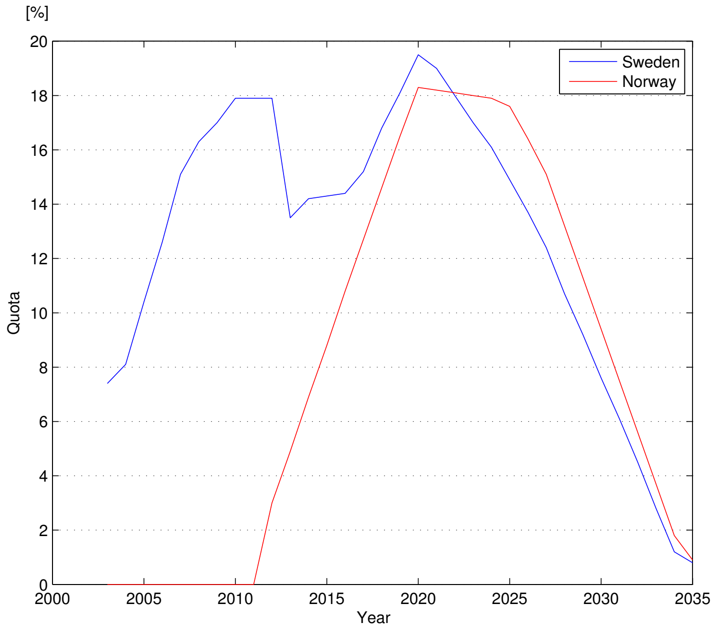

Figure 1: Quotas for Sweden and Norway (2003-2035)

In addition to producers and retailers, there may be other traders that buy and sell certificates, either for hedging purposes or to make profits. From other financial markets it has been observed that such traders can play a role as a stabilizer, making market prices less volatile [8]. How market players form their price expectations and make buying and selling decisions is of great importance when modeling such a market. This will be a central topic in later sections of this text.

The certificates are traded bilaterally, i.e. over the counter, and via brokers as a regular financial instrument on the Nasdaq Commodities Exchange. Most of these trades are done bilaterally between electricity producers and retailers [4]. Traders must have an electricity certificate account with Cesar (The Swedish certificate registry) or NECS (The Norwegian certificate registry). All trades, done bilaterally or on the exchange, are delivered to the designated account in these two registries. On the Nasdaq Commodities Exchange, two types of broker contracts are available for trading, namely the spot price contracts and the March futures. For both these types, delivery takes place against payment, known as "delivery versus payment". The spot contract is traded for five days, and settlement is completed within three bank days from the end of the last trading day. The March futures can have a time horizon of up to five years and are settled in the middle of March of the year of delivery. The contracts can be traded until the specified delivery date and settlement is completed within three bank days after the last trading day. Both contracts are available in EUR and SEK [9], and allow for both short and long positions[2].

[2]Thanks to SKM for provding information.

The Norwegian Water Resources and Energy Directorate and the Swedish Energy Agency are the two market regulators. In addition to the daily operation, the regulators are in charge of ensuring that the market functions properly. Through planned control stations held every fourth year, they evaluate the market and decide whether any adjustments need to be made. As a preparation for the next control station, the two market regulators have been asked to assess the market design and recommend potential changes. The quota curve, the penalty scheme and the accessibility of adequate investment opportunities have been among the areas considered. Their recommendations include minor adjustments to the quota curve and an unchanged penalty scheme. It has also been determined that an adequate amount of investment opportunities exist in the market. [10] The next control station will be held during 2015, and potential changes will be implemented from January 1, 2016.

3 Methods for Modeling an Electricity Certificate Market

Several organizations are currently working on models for the Swedish-Norwegian market for electricity certificates. Most of these are in an early phase of development. Best practices for modeling the market has not yet been established, as can be seen from the several different approaches that are being taken. This chapter aims to give an overview of the various methods and a brief introduction to some of the existing models. Methods for modeling a certificate market can be separated into two types, analytical and numerical. The analytical models can provide a basic understanding of the market and some simple relations and results. An example of an analytical model is described in the paper of Amundsen and Mortensen, on the effects from Green Certificates and CO2-emission permits for the Danish market [11]. The model is used to investigate the effects of varying several factors believed to impact the power grid and its actors. They find that more rigorous CO2 constraints, as well as an increased price for import wholesale electricity, may lead to a reduced capacity for renewable energy production. Further, they are not able to conclude on whether an increase in certificate requirements leads to a change in the capacity of renewable energy, but they find that a higher share of consumption will stem from renewables, as the total electricity consumption is reduced. Another example is the analytical model described in the paper of Morthorst from 2003 where he concludes that the long term certificate price is formed such that the sum of the electricity spot price and the certificate price should equal the long run marginal cost of investing in new renewable capacity [12]. Generally, analytical models can be a good starting point before providing more advanced models. From this point on, this text will focus on numerical models, enabling detailed simulations of the market and its behavior. Numerical models can be divided into four main categories, discussed in the subsequent sections.

3.1 Fundamental Equilibrium Models

Fundamental equilibrium models formulate the market equilibrium models as large optimization problems, minimizing or maximizing an objective function subject to a set of constraints.[3] Estimates for future prices are then obtained by solving the model for market equilibrium. Such models require perfect competition and rational behavior from all players in the market. Modeling supply and demand curves as step-functions yields LP-problems; this subsequently helps lowering complexity and hence lowers the runtime. Equilibrium models can both be static, considering only one time period, and dynamic, with relations between several time periods e.g. inventory equations for storage of certificates and energy over time.

A fundamental equilibrium model can be found in the paper of Unger and Ahlgren from 2005. They apply the Markal model-generator [13] for the energy systems of Denmark, Finland, Sweden and Norway. They maximize the objective function, being the sum of consumer and producer surplus. Among their

[3] An example of such optimization problems could be to minimize the total system costs subject to an inventory constraint and a requirement constraint.

findings is that the introduction of TGC quotas reduces wholesale electricity prices, while retail prices can be higher or lower depending on the TGC quota [14].

Another example of a fundamental partial equilibrium model is the recently developed model by Wolfgang and Jaehnert (2014) of the Swedish-Norwegian Electricity Certificate market [15]. The model is an extension of the EMPS model, a partial model for electricity markets, used by producers, regulators and system operators throughout Scandinavia. The extension introduces electricity certificates to this model. A price of certificates is calculated through stochastic dynamic programming with weekly time resolution for the remainder of the system lifetime, and endogenously determined penalty rates. The strategy calculations are done for several fixed penalty rates. This is done to avoid introducing another state variable in this part of the solution algorithm, for the sake of computational tractability. Further, the expected penalty rate is calculated as an endogenous variable in the formal LP part of the model. The value obtained for the penalty rate is then used to choose the correct strategy, by interpolating between the fixed penalty rates used in the strategy calculation. This seems to be a well-functioning method. A potential consequence of using fixed penalty rates in the strategy calculations, may be the loss of dynamics related to price changes on the penalty rate. By using results from the EMPS model, the dependency between the electricity market and investments in additional capacity and production levels are directly included. They find the results to be consistent with other studies on similar TGC markets, i.e. in particular that "price-scenarios spread out such that the unconditional expected price of certificates is relatively stable in the planning period." [15]

The example model in this text based on the paper of Coulon is also a fundamental equilibrium model, with some aspects from system dynamics modeling. This model will be explained in detail in section 5 [16]. A comparison between this model and the model of Wolfgang and Jaehnert may still prove useful. Both models are fundamental equilibrium models, and prices are calculated in a similar manner, as a function of the expected penalty rate and the probability of this penalty to actually occur. The model of Wolfgang and Jaehnert is linked to the electricity market, has price dependent penalty rates and stochastic issuance of certificates. The model of Coulon et al. has stochastic issuance of certificates and price dependent investments, penalty rates are however assumed fixed and there is no direct link to the electricity market.

3.2 Agent Based Models

Agent based models aim to simulate the behavior and interaction between players in a system. The different players are represented by so-called agents. Every individual agent are acting according to their observed situation based on a set of specified rules. Making these agents act realistically requires detailed information regarding behavioral pattern of each player type. This information is typically gathered by interviewing actual market actors. Once the agents are programmed, the market can be simulated. The simulation continues until it reaches a steady state. Such models allow for analysis of interaction between market actors. From the steady state situation, an equilibrium price can be obtained. Unlike the fundamental models, where rational behavior and perfect competition is assumed, this approach allows for the introduction of irrational behavior. Another positive feature is the possibility of implementing a form of learning process into the agents. Agents that performed well in the first period would want to act similar in the next, while agents that did poorly would try to adjust their behavior, hoping to improve their performance. The core strengths of these models include the ability to handle market power and imperfections more easily. A disadvantage of such an approach is that tracking the dependency between causes and effects could prove difficult.

An example of an agent based model can be found in the paper of Aune et al. from 2012 [17]. They analyze the potential for cost reductions by allowing for trade in green certificates across member states. Their findings indicate that EU-wide trade in green certificates may cut the EU's total cost of fulfilling the renewable target by as much as 70 percent compared to a situation with no trade.[4]

3.3 Statistical Models

Statistical models are purely based on historical price data. They assume that what will happen in the future is reflected in history. From these historical data, parameters can be obtained to calibrate a parametric model. Exogenous variables can still be added to explain expected future changes. These systems has to be static, i.e. there are no dynamic relations between current and future model parameters.

An example of a statistical model can be found in the paper of Fagiani and Hakvoort from 2014 [18]. They use a GARCH model to analyze the causes of volatility and the effects of volatility on certificate prices. From their results it can be seen that regulatory changes strongly affect certificate markets, resulting in periods of higher volatility. During such periods of higher volatility, investors will require a higher rate of return. More specifically, they analyze whether certificate price volatility has changed after the creation of a joint Swedish-Norwegian electricity certificate market. Results indicate that the news about this extended market led to a period of increased price volatility between 2010 and 2011. They also note that this effect is not fully reversed, and that the market is still more volatile than before the change. This effect may be reversed as the market matures and quotas increase. However, a question raised by Fagiani and Hakvoort, is whether a larger market provides more stable prices.

[4] An agent based model for the Swedish-Norwegian Electricity Certificate market is currently under development. The project is led by the Norwegian companies Optimeering and Thema Consulting and is financed by the Norwegian Research Council. They have interviewed several market players, and are modeling the different types of players' behavior to be able to predict prices and identify potential market improvements.

3.4 System Dynamics Models

System dynamics modeling is an approach to understanding the behavior of complex systems over time. Such models are comprised of a coupled set of non-linear differential equations, describing relations between actors, feedback loops and time delays. Further, the set of differential equations is used to simulate the system behavior.

An example of a system dynamics model can be found in the paper by A. Ford et al. from 2007 [19]. They simulate the price dynamics of a TGC market designed to support an aggressive mandate for wind generation in the north-western USA. Their results indicate that the certificate prices climb rapidly to the penalty level in the early years of the market. This is met with increased investments in wind generation capacity, forcing certificate prices back down. Prices then stabilize (though smaller fluctuations still occur). The same results are obtained for simulations with different values for the uncertain parameters, such as price elasticity, penalty level and lead-time on investments. The results are consistent with actual price data from the early years of the Swedish electricity certificate market starting in 2003.

4 Price Dynamics

4.1 Price Equilibrium

The formation of certificate price expectations in the market is of major importance. Two conditions explaining the rational formation of expectations are presented.

1. The certificate price should equal the difference between the levelized cost of energy (LCoE) of the marginal plant and the electricity price.

2. The certificate price should equal the discounted value of the penalty times the probability of having to pay this penalty.

As will be presented in the example in section 5, rational players and perfect competition are assumed. In other words, all players have access to the same market information and act rationally to maximize profits.

The first condition follows from the way electricity producers make investment decisions. In a TGC market, decisions to invest in renewable electricity capacity will be made on the basis of expected electricity- and certificate prices. Assuming a fixed electricity price, investments are made when the certificate price is at a level that makes a new investment profitable. This happens when the price of certificates is higher than the LCoE of the plant less the price of electricity. Consequently, the renewable plants with the lowest LCoE will be built first, since these are profitable at a lower certificate price. As prices increase, so does the number of profitable investments. As more capacity is built, the supply of certificates increases. Subsequently, investments will be made until the price of certificates is equal to the LCoE of the marginal plant less the electricity price. If the price of certificates never rise above the price level necessary to initiate further capacity investments, the 26.4 TWh goal may never be reached. If this situation occurs, the regulators are likely to implement changes in market design, in order to obtain a price level where the target capacity is actually reached. A typical change is an increase in the requirement quota for retailers of electricity. This will yield a positive shift in the demand for certificates, and thus increase prices. On the other hand, in a market without the possibility for interventions the target will not be met if the initial design is such that price equilibrium is reached for a lower capacity. While having the possibility of making regulatory changes seems favorable, regulators should be aware of the consequences of such changes. Namely, increased volatility and thus an increased required rate of return on investments [18].

Furthermore, the second condition follows from the expected payoff of a certificate. When new certificates are issued, electricity producers are faced with the decision of whether to sell their certificates immediately or bank their certificates to wait for higher prices. Producers will sell their certificates at the current price, unless the present value of an expected future price exceeds the current price. When the present value of a future price is higher, producers will bank their certificates until the willingness to pay among retailers increase to a level at which producers are willing to sell. The retailers' willingness to pay is assumed equal to the expected penalty faced if obligations are not met, times the probability of not being able to meet these obligations, i.e. a shortage of certificates in the market. One might think of a scenario where producers bank all their certificates due to an expected discounted future price being higher than the current price. Since demand depends only on the electricity consumption, it can be assumed constant. As more certificates is banked for later years, supply will increase and thus expected prices will decrease. Hence, a player acting to maximize profit is likely to sell/bank his certificates such that he reaches an equilibrium where he is indifferent between selling today and selling tomorrow.

To achieve market equilibrium, both of the above conditions should be met at the same time. If the price of a certificate exceeds the price suggested by condition 1, end-users will be paying too much for the increased share of renewable electricity. If the price is lower than the price suggested by condition 1, the marginal plant will be unprofitable. Similar, if the price of a certificate is not equal to the price suggested by condition 2, a producer can increase profits by banking/selling more certificates. For modeling purposes, an interesting question is whether fulfilling one of the conditions automatically leads to the fulfillment of the other. For a market in equilibrium, one would expect this to be the case, but this question remains to be answered.[5]

4.2 Technologies for renewable energy in Sweden and Norway

The lead time[6] of new power plants will depend greatly on the choice of technology. The Swedish-Norwegian market consists mainly of three competitive technologies; wind, hydro and bio power. Hydro power can be divided into two types; dammed hydro power facilities with reservoirs enabling storage, and run-of-river hydro power, smaller facilities with small or no reservoir capacity.

Technology	Lead Time (years)
Bio	2-5
Dammed Hydro	3-4
Run-of-river Hydro	1-2
Wind	1-2

Table 1: Lead time on investments

As seen in table 1 wind power typically has a lead time of 1-2 years [19]. Run-of-river hydro has a lead time of 1-2 years, while for dammed hydro power, typically 3-4 years.[7] The estimates for bio power varies, but would be in the range 2-5 years depending on the size of the plant. These differences increase the importance of the composition of different technologies in the pool of investment opportunities. In a market where investment opportunities have an on average lower lead time, supply is likely to be more responsive to changes in certificate price, possibly having a stabilizing effect on prices [19]. In scenarios with higher

[5]Thanks to Ove Wolfgang for passing this question.

[6]The lead time is the duration from when an investment decision is taken to the power plant starts operating.

[7]Thanks to Simen Vogt-Svendsen, Statkraft, for sharing this knowledge.

shares of dammed-hydro and bio power, the market will be significantly slower in meeting higher prices with new supply, and hence, in reaching the long run equilibrium. The lower lead time of run-of-river compared to dammed facilities can be explained by the differences of the two building processes. While run-of-river plants consist mainly of prefabricated parts, a dammed facility is a larger structure, requiring a multi-stepped building-process which involves great environmental consequences related to interventions in nature.

Technology	Norway (TWh)	Sweden (TWh)	Percentage of total
Bio	0.00	1.00	15.95%
Hydro	0.74	0.43	18.66%
Wind	0.19	3.90	65.23%
Sun	0.00	0.01	0.16%
Sum	0.93	5.34	100%

Table 2: Expected annual production for eligible plants during 2013

As can be seen from table 2 most of the currently eligible electricity is Swedish wind energy. As more Norwegian facilities are built, the share of hydro power is expected to increase. However, wind power is expected to remain the largest technology. Moreover, the choice of technology and the geographical allocation of new facilities, are also affected by differences in regulation. Swedish taxation rules make investing there favorable, compared to investing in Norway. In a report from 2012, Thema Consulting Group concludes that "up to 5.6 TWh of new renewable electricity in Norway, mainly wind, but also hydro, will be crowded out by more expensive Swedish wind power. The costs of meeting the certificate target of 26.4 TWh will therefore be higher than necessary" [20]. The results from their analysis is presented in table 3.

Technology	Norway (TWh)	Sweden (TWh)	Percentage of total
Bio	0.00	2.90	10.98%
Hydro	5.40	0.50	22.35%
Wind	6.00	11.60	66.67%
Sum	11.40	15.00	100%

Table 3: Expected annual production for eligible plants after the 26.4 TWh target is reached

4.3 Market Instability

Changes in certificate prices cause some interesting effects. The Swedish-Norwegian market operates with a penalty calculated as 1.5 times the average certificate price of the previous year. This leads to a short term mathematical instability. An increase in price will lead to an increase in the expected penalty, which subsequently will cause the certificate prices to rise even further. Given no intervention from market regulators, this spiral could potentially lead to prices climbing infinitely high or collapsing towards zero, depending on the initial price movement.

Dammed-hydro and bio power could to some extent, benefit from higher prices by increasing production. The percentage share of such ”stabilizing renewables” in the mix of certificate eligible electricity, will decide the magnitude of the described instability. In the long run, the market will be able to meet an increase in certificate prices with an increased rate of investment in renewable electricity. This will cause the supply of certificates to increase, which in turns causes prices to decrease. The lead time of new capacity will decide the duration of the time period needed for the system to stabilize at a new equilibrium level.

4.4 Alternatives to Price Feedback on Investments

Both for the discussion in the previous section and for the example model in section 5 it is assumed that the level of investments in new renewable capacity is a positive function of the certificate price level. This assumption is logical, as an increase in prices will directly increase the gross profit of a renewable plant. Two extreme cases are considered. In the case of certificate prices moving towards infinity, all investment opportunities will be taken. In the case of prices collapsing towards zero, only investments made without a TGC support scheme will be taken. However, while prices fluctuate within an expected range, the dependency between prices and investment rate is considered uncertain and may vary for the different technologies mentioned in section 4.2.

Historically, both dammed and run-of-river hydro power have been profitable without subsidies. On the other hand, the profitability of wind and bio power have, with few exceptions, depended largely on subsidies. Assuming that projects waiting to be realized have similar characteristics, one can assess the dependency between investment levels and certificate prices to be less prominent for hydro than for wind and bio. Information from a key producer in the Norwegian electricity market also indicates that their development of new hydro power plants does not depend on prices, but solely on access to capital. Wind and bio power projects will depend on positive certificate prices to be realized. Whether changes in certificate prices affects investment decisions is unclear.

New renewable capacity must be in operation by the end of 2020 to benefit fully from the TGC support scheme. Considering the lead times of the different technologies, investment decisions must soon be done if the new facilities are to be considered certificate eligible. An interesting theory states that the power plants needed to reach 26.4 TWh of new capacity before 2020 have already been planned, since electricity producers have a long planning horizon for future investments. If this is the case, movements in certificate price will be of little to no importance before 2020. Movements in certificate price will thus only affect investments in Sweden after 2020.[8]

[8]Sweden allows plant that start operating after 2020 to receive certificates, while Norway does not.

4.5 Expected Prices in the Market

As described in section 4.1 there are two alternative ways to describe the price equilibrium in a TGC market. However, the market price expected by market players is not necessarily equal to the equilibrium price. Among players, there might be a lack of information and ability to model and forecast prices[9]. Thus actual price expectations may not be consistent with theoretical prices.

For players in the certificate market, price expectation may play a role both in investment decisions and for trading purposes. Today there is a liquid futures market for certificates where contracts can be traded up to five years ahead.

Product	Price (SEK)
Spot	173
March-15	174
March-16	178
March-17	182
March-18	184
March-19	186

Table 4: Spot and futures prices of electricity certificates, December 15 2014 [21]

As can be seen from table 4 the futures market is in contango, with prices increasing with time to maturity. The standard formula for forward prices is defined so that the current price is the futures price discounted at a proper discount rate less the net convenience yield [22]. Since there is no storage cost, and since holding an adequate amount of certificates strengthens the holder's ability to meet his obligations at compliance date, the net convenience yield of certificates is assumed positive. Given a positive convenience yield, the prices in table 4 seems reasonable. Prices are increasing with longer time horizons, however at a rate that is lower than the assumed required rate of return of market players. Thus, prices seem to be in accordance with the martingale condition[10]. Morthorst argues that a liquid futures market for certificates might increase long-term transparency in pricing while stabilizing expectations [5].

[9] Some players in the market may utilize more advanced forecasts for future prices. This can be done by applying models such as those described in section 3. Forecasts can either be bought from market predictors such as SKM, or performed in-house. It is not expected that all players in the market applies such advanced forecasts. Thus, some players are believed to have an advantage.

[10] The price in one time step is the discounted expected price at the next time step.

5 Example Model

This example model is an adaption of Coulon et al.'s modeling of the New Jersey Solar Renewable Electricity Certificate (SREC) market based on the equations described in their paper [16]. The model was chosen due to the promising results it has shown for the New Jersey SREC market. Additonally, to the best of the authors' knowledge, this is the first implementation of such an approach for the Swedish-Norwegian market. The authors are curious of whether Coulon's approach will yield similarly promising results for the Swedish market[11] as for the New Jersey market.

Minor adjustments to the equations have been made to include the possibility of infinite banking of certificates. Parameters have also been updated to reflect historical values from the Swedish market during the period 2004-2011.

5.1 Mathematical Formulation

$$b_t = \begin{cases} max(0, b_{t-1} + \int_{t-1}^{t} g_u du - R_t) & t \in \mathbb{N} \\ b_{[t]-1} + \int_{t-1}^{t} g_u du & t \notin \mathbb{N} \end{cases} \tag{1}$$

Eq. (1) keeps track of the accumulated number of certificates, *banking,* in the market at any given time. At any time step t, the currently banked balance b_t is a function of the previous balance $b_{[t]-1}$ and the accumulated issuance since the previous time step, $\int_{t-1}^{t} g_u du$. If the current time step is part of the set of compliance dates $\mathbb{N}$, eq. (1) accounts for a reduction in the number of certificates in the market, equal to the requirement R_t at the given date. The balance can never be negative, hence the max statement.

$$p_t = \max_{v \in \{[t],[t]+1,\ldots,T\}} e^{-r(v-t)} \pi^v \mathbb{E}_t[1_{\{b_v=0\}}] \tag{2a}$$

$$p_t = e^{-r\Delta t} \mathbb{E}(p_{t+1}) \qquad \text{when } t \notin \mathbb{N} \tag{2b}$$

At a compliance date, the holder of a certificate will avoid the penalty imposed on those who do not comply as he hands in his certificate. Further, if the balance of certificates in the market directly following a compliance date is 0, one can assume that investors would at least have been willing to pay the amount of the penalty fee for one certificate. Eq. (2a) states that at any time t, the value of the certificate p_t is the maximum of the discounted expected future penalty fees it can be used to avoid, discounted at the rate r. Eq. (2b), i.e. the Martingale condition, follows implicitly from (2a) and states that, except at compliance dates, the current price is the discounted expected future price.

$$g_t = \hat{g}_t(p) \exp\left(a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + \varepsilon_t\right) \tag{3}$$

[11] From 2003-2011, the electricity certificate market did only include Sweden. Norway did not enter the market until 2012. Hence, the model has been implemented for the Swedish market, to be able to replicate historical price data.

The seasonality of electricity consumption, and hence certificate generation, is accounted for by a stochastic process on the form shown in eq. (3). $\hat{g}_t$ represents the annualized issuance of certificates and is a function of price, p. This is motivated by the assumption that investors are likely to invest more while certificate prices are high. Seasonal changes are modeled by the sine and cosine functions, while a noise term is added to reflect the uncertainty of generation.

$$p_t = \bar{p}_t \tag{4}$$

Bellman introduced the term "the curse of dimensionality", referring to the exponentially increased execution time associated with the introduction of another state variable [23]. While Coulon's generalized model uses a weighted price average to calculate price feedback, it is here assumed, as stated in eq. (4), that the current average price equals the spot price. This is done to reduce dimensionality, and lower runtime. The result of this adjustment is immediate price feedback on generation.

$$\frac{ln(\hat{g}_{t+\Delta t}) - ln(\hat{g}_t)}{\Delta t} = a_5 + a_6 \bar{p}_t, \text{ for } a_5 \in \mathbb{R}, a_6 > 0 \tag{5}$$

Eq. (5) accounts for increase in generation. a_6 accounts for the logical effect that producers are likely to invest more as prices rise. a_5 represents the growth of generation not related to price increases. It is an independent term describing the drift in investments over time.

5.2 Implementation

The input data for compliance requirements R and penalty fees π are given in table 5 [4]. Further, historical generation data, provided by Svensk Kraftmäkling [21] have been used to calibrate the stochastic generation function. The discount rate has been set to 9.5%. This reflects the required rates of return at which the retail sellers in the Scandinavian electricity market operate.[12] The balance of certificates has been calculated by adding monthly generation data and subtracting cancelled certificates. The monthly balance is shown in figure 2. As can be interpreted from the figure, the balance increases from April until March, followed by an immediate decrease as certificates are cancelled at compliance date. From the figure it is also noted that the accumulated balance is increasing over time.

[12] Courtesy of Bjørn Erik Heiberg, Pareto

Energy Year	R ($\times 10^{-6}$) [TGC]	π [SEK]
2004	7.89	240
2005	10.15	306
2006	12.23	278
2007	14.50	318
2008	15.32	431
2009	15.40	470
2010	17.54	402
2011	16.56	310
2012	16.29	298

Table 5: Historical Requirements and Penalty Fees

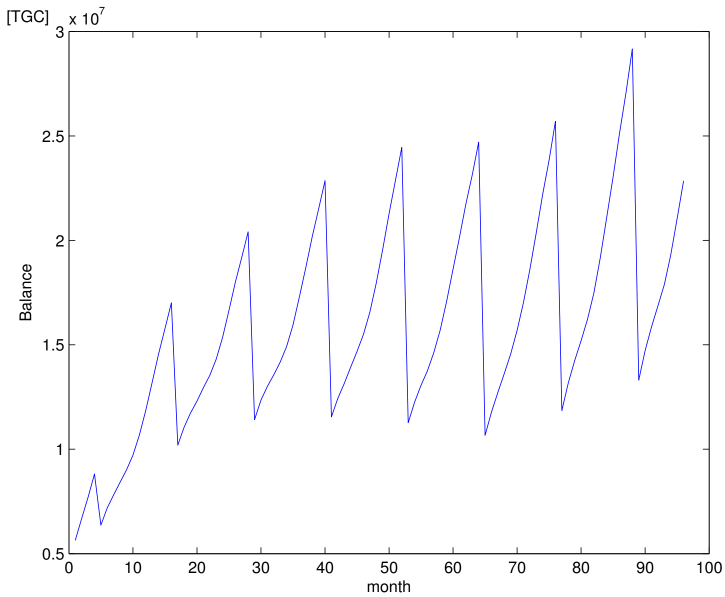

Figure 2: Monthly Accumulated Certificate Balance (2004-2011)

It is assumed that the requirements and the penalties are known and fixed for each year. The authors acknowledge that this is a rather strong assumption. As discussed in section 4, the penalty level is dependent on the previous year's average price level, which in turns depends on other factors, including current investment and trading activity. The assumption is however done for the purpose of computational tractability. Further work will investigate whether it is possible to solve this price model within a reasonable timeframe, without making this assumption.

$$ln(g_t) = a_0+a_1 sin(4\pi t)+a_2 cos(4\pi t)+a_3 sin(2\pi t)+a_4 cos(2\pi t)+a_5 t+a_6 \int_0^t \bar{p}_u du+\varepsilon_t \quad (6)$$

In (6), equations (3), (4) and (5) have been combined to allow for parameter fitting, determining coefficients used in these equations from historical data. The regression, conducted in R [24], yields the regression coefficients shown in table 6. The confidence intervals of these coefficients are shown in table 7.

	Estimate	Std. Error	t-value	Pr(>|t|)
a_0	13.61	0.0180137	755.347	<2,00E-16
a_1	-0.073	0.0112635	-6.454	4.87e-09
a_2	-0.011	0.0113200	-0.990	0.3249
a_3	0.221	0.0112220	19.677	<2,00E-16
a_4	0.308	0.0113873	27.053	<2,00E-16
a_5	0.023	0.0330131	0.698	0.4869
a_6	0.000	0.0001364	1.997	0.0488

Table 6: Regression of Coefficients

	2.5%	97.5%
a_0	13.571	13.642
a_1	-0.095	-0.050
a_2	-0.034	0.011
a_3	0.199	0.243
a_4	0.285	0.331
a_5	-0.043	0.089
a_6	0.000	0.001

Table 7: Confidence Intervals of Regressed Coefficients

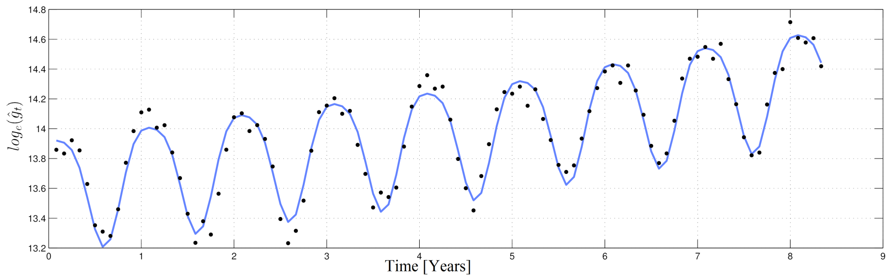

Figure 3: Regression of Issuance

From the regression it is found that there is a relationship between price and generation, that is a_6 exists. Here it has a magnitude in the order of 3×10^{-4}.

Since the regression is nonlinear[13], the goodness of fit is assessed based on the coefficient standard errors, coefficient confidence intervals and a visualization of the regression shown in figure 3. Hence, testing and concluding on whether the coefficient a_6 is significantly different from zero is difficult. It is however unlikely that the coefficient is zero, since some level of price feedback is expected. From this, and the confidence interval of the coefficient, it is concluded that a_6 exists.

The residuals, standardized residuals, the Q-Q-plot and autocorrelation of the residuals are shown in figure 4. The Q-Q plot indicates that the error terms are close to normally distributed [22], though there are some outliers in the lower left corner. It is also noticed that the residuals do not show any patterns of autocorrelation. The residuals seem to be independent and normally distributed. From R the characteristics of the error term, shown in table 8, are obtained. As seen, there is some excess kurtosis, it is however close to zero and hence does not change the assumption that a normal distribution can be used to simulate noise. Neither does the slight negative skewness of the error terms [25]. Based on this discussion, it is decided to use a normal distribution with a mean of 0.077 and a standard deviation of zero for the noise term in eq. (3).

Mean	Std. Dev.	Skew	Excess Kurt.
0.000	0.077	-0.610	0.424

Table 8: Error Term Characteristics

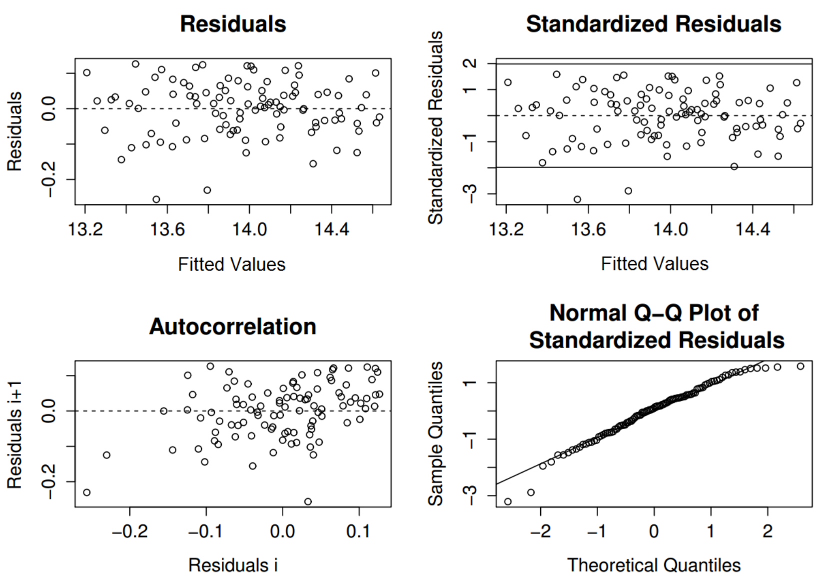

Figure 4: Error Term Plots

The solution algorithm proceeds as follows:

[13] For nonlinear regressions, metrics like R^2 and the p-stat are not suitable for assessing the goodness of fit. These metrics are based on the assumption that the regression is linear.

1. A grid of values for b_t and $\hat{g}_t$ is chosen with lower bounds zero and upper bounds a little above the largest requirement. Time is discretized in monthly steps, matching the frequency of historical generation data.

2. The dynamic program is initialized, evaluating the payoff of the certificate at the end of the market's life $t = T$ for every single gridpoint $(b_t,\ \hat{g}_t)$. At this point, all information is known and hence, the program yields a digital boundary price surface[14]:

   (a) At grid points where there is a shortage of certificates (i.e. the balance is less than the requirement), investors are willing to pay the penalty, $p_T = \pi^T$, for one certificate.

   (b) At grid points where there is a surplus of certificates (i.e. the balance is higher than the requirement), investors are willing to pay $p_T = 0$ for one certificate.

3. From the boundary surface at $t = T$, the dynamic program steps backward to $t = (T - 1)$. Here it solves equations 1-5 at every grid point using price information from the price surface at $t = T$. The same procedure is then followed for every time step; information from price surface $t + 1$ is being used to solve equations 1-5 for every grid point of price surface t with Matlab's $fsolve$ function [26].

4. The expected value statement in eq. (2a) is calculated in the following way:

   (a) For a given time t, every grid point represents a situation $(b_t, \hat{g}_t)$. Using eq. (1), (5) and (6), we follow the evolution from $(b_t, \hat{g}_t)$ one timestep forward to $(b_{t+\Delta t}, \hat{g}_{t+\Delta t})$. Since there is a random variable ε in eq. (6), the obtained $(b_{t+\Delta t})$ is stochastic. (ε is drawn[15] from a normal distribution with the same characteristics as the error term of the regression for generation, shown in table 8)

   (b) Further the procedure determines the future price, i.e. the price at $t+\Delta t$. Since point $(b_{t+\Delta t}, \hat{g}_{t+\Delta t})$ is not a grid point, the future price is obtained by interpolating in the price surface representing timestep $t + \Delta t$.

   (c) To obtain the expected future price, steps (a) and (b) are repeated a sufficient number of times. The obtained future prices are then averaged, yielding an expected future price for timestep t.

The algorithm provides one price surface for every single time step. The price surfaces show what the price would be at this time step, given a situation $(b_t,\ \hat{g}_t)$. An example of the resulting price surfaces is shown in figure 5. In

[14]One might ask whether this boundary condition, with a price equal to zero, is a valid assumption. It is unlikely that anyone would be willing to give up their certificates for nothing. However, it is just as unlikely that anyone would be willing to pay any more for an investment that is destined to yield no payoff. Hence, the digital boundary condition surface is considered the best alternative.

[15]As simulation is not usually conducted within dynamic programs, the authors have investigated whether fetching the noise term from a lookup-table leads to a change in runtime. The two approaches were however found to have the same runtime.

order to compare the modeled prices to historical data, one needs to extract the modeled price for the historical levels of (b_t and $\hat{g}_t$) for every time step. For the given resolution level, the the runtime is approximately 2.5 hours.

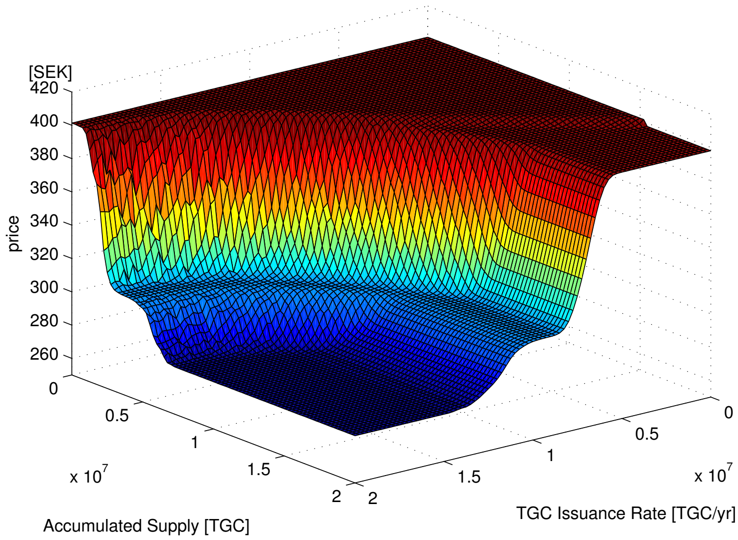

Figure 5: Price Surface for t = 55 (July 2008), $a_6 = 15 \times 10^{-4}$

5.3 Results and Interpretation

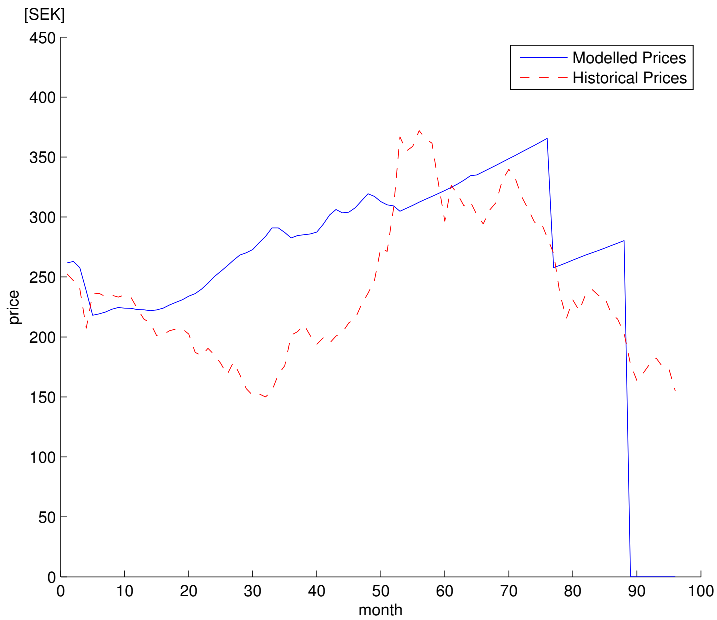

Figure 6: Modelled vs Historical Prices (2004-2011)

Figure 6 shows modeled prices (solid line) reflecting historical $(b_t, \hat{g}_t)$ data from January 2004 to December 2011. It also shows historical price data from the same period (dashed line). Comparing model output to historical prices, we see that historical prices are replicated fairly well with modeled and historical prices fluctuating around the same trend line. While the modeled prices capture long term trends quite well, fluctuations are not captured. The trend is as expected, due to the increasing penalty levels seen until 2009. The drop in 2010 can be explained by the simultaneous drop in penalty fees and the particularly large increase in balance, seen during the same year. The model is requiring prices to equal discounted future prices. These future prices are dependent on penalties occurring once a year, thus short term fluctuations will not be captured. This also follows from the frequency of the input data which never exceeds monthly.

Between compliance dates, graphs are smooth and increasing due to certificate prices satisfying the martingale condition given by eq. (2a) at all time steps. At compliance dates price drops are sometimes observed. These drops stems from foregone possibilities of using certificates for compliance. Another point worth mentioning is that the differences between modeled prices and historical prices have seemed to decrease through the period. Though this might be coincidental, one might speculate that these initial differences might have been due to irrational players in the immature new market. As the market has matured the differences have definitely decreased.

The attentive reader will also observe that the modeled prices go towards zero at the end of the period. This is due solely to an assumption of the system ending at the end of the time frame. This assumption is made to lower the runtime of the model, as it allows for the solving of the dynamic optimization problem with a backwards recursion algorithm. It is worth noting, that the large price drop in t = 88, i.e. April 2011, can be explained by the drop in probability for penalty. At t = 88, a possibility of using the certificate for compliance is foregone, hence the price drop. At t = 100, i.e. April 2012 (the boundary condition), all information is known, and the price is equal to zero since there will without a doubt be a surplus of certificates in the market.

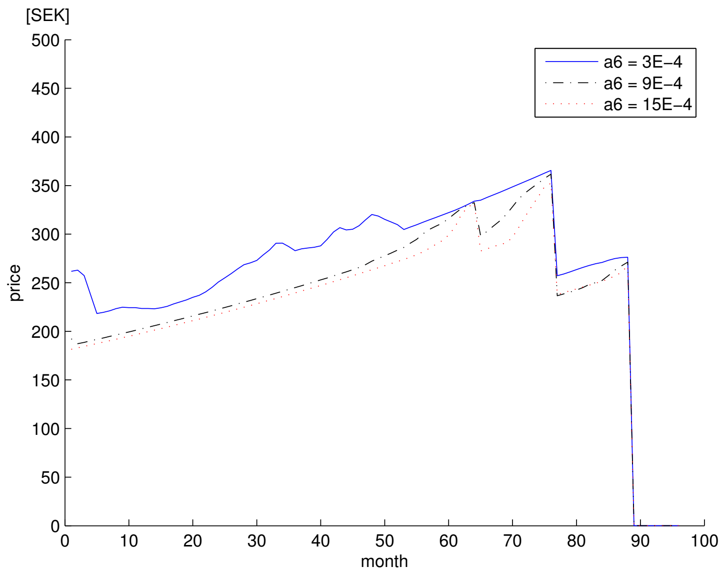

Figure 7: Sensitivity Analysis of the Price Feedback on Investments (2004-2011)

The level of price feedback is determined by regression parameter a_6. A higher a_6 reflects a greater degree of flexibility among producers of electricity. Producers respond more rapidly to price increases, investing in more capacity to overcome a shortage of certificates in the market. The effect of varying a_6 is shown in figure 7. As seen from the figure, prices are slightly lower for a greater level of price feedback. This is due to a potentially faster increase in generation inducing a sustained higher level of supply. From the figure, it is also shown that higher feedback levels dampens fluctuations from the trend line.

From the sensitivity analysis, it is however not obvious which value of a_6 provides the most accurate replication. Recalling that the regression yielded an a_6 of 3×10^{-4}, one might argue to which degree increasing certificate prices are met by increased investments in production capacity. This question was also left unanswered following the discussion in section 4.4.

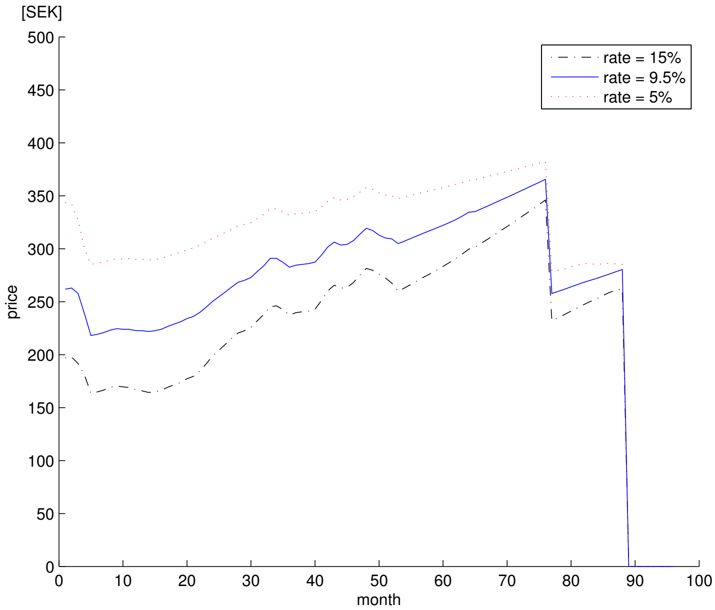

Figure 8: Sensitivity Analysis of the Discount Rate (2004-2011)

As risk increases, so does the required rate of return. Figure 8 shows the effect of increased required rate of return on modeled prices. The higher the required rate of return, the steeper the associated price curves. Similar results are obtained when the model is run for hypothetical situations with a deficit in certificate balance. The sensitivity analysis indicates that a required rate of return of 15% seems to produce the best replication of historical prices. One can argue that this is high. However, not only are investors in the electricity certificate market exposed to price risk. They are also exposed to regulatory risk [18]. This is the risk that changes in regulations will materially impact the certificate price. One reason why such changes might occur is the mathematical instability of certificate prices, mentioned in section 4.3. If the market actually turns out to be unstable, the lawmakers will be required to perform a revision of legislation.

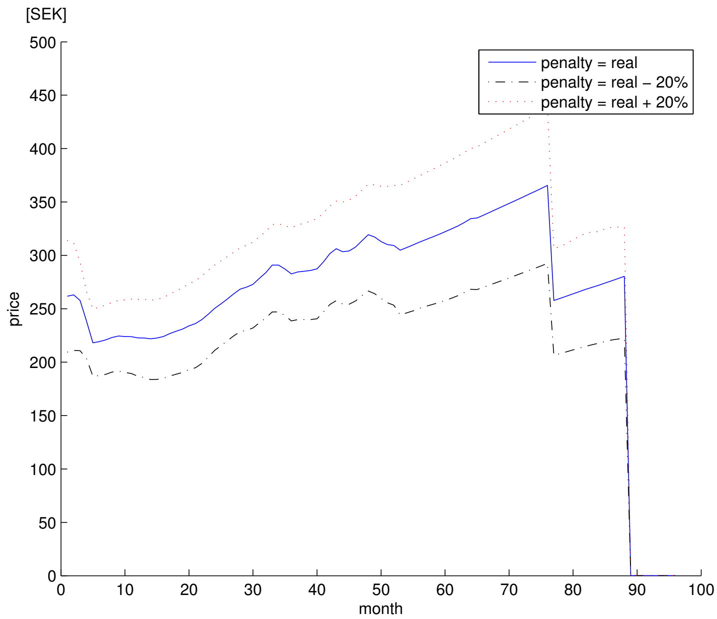

Figure 9: Sensitivity Analysis of the Penalty Level (2004-2011)

From figure 9 it is observed that higher (lower) penalty fees yields higher (lower) prices. This is as expected, as the price of a certificate is a positive function of future penalty fees.

Examining the results from the three sensitivity analyses, it is found that the best replication of historical data are produced using a discount rate = 15% with penalty fees at historical levels. For parameter a_6, the results are inconclusive. Moreover, it is concluded that it is the discount rate and the penalty fees that has the largest impacts on prices, hence these are considered the key drivers for this model.

6 Conclusions and Further Work

Various support schemes have been suggested to promote the development of increased renewable capacity in the electricity grid. The governments of Sweden and Norway have done this by introducing a common TGC market. Throughout this paper, the Swedish-Norwegian electricity certificate market has been explored, in order to investigate its features, assess its performance and explore the dynamics of the system. Additionally, various methods for modeling such a market have been presented and the model by Coulon et al. has been implemented.

The development over the market's first three years indicates that the goal of an additional 26.4 TWh of annual renewable capacity by 2020 is likely to be reached. Due to differences in the tax regimes of Norway and Sweden, some Swedish investment opportunities with higher LCoE are likely to be taken before some of their Norwegian counterparts with a lower LCoE, thus increasing the total social costs of the system. Whether the producers or the consumers are the ones carrying the cost of the system is debatable, however, the system is found to cause a reduction in the total social surplus. From this it is assessed that the system achieves the sought-after effects, but possibly at higher costs than necessary.

The literature suggests two conditions for the rational formation of prices in a TGC market, where both should be met at market equilibrium. Those are:

1. The certificate price should equal the difference between the levelized cost of energy (LCoE) of the marginal plant and the electricity price.

2. The certificate price should equal the discounted value of the penalty times the probability of actually having to pay this penalty.

Whether fulfilling one of the conditions automatically leads to the fulfillment of the other remains an unanswered question, calling for further investigation.

The effects of price changes have several important aspects. Due to the spiral dependence between previous certificate prices and penalty fees, it is pointed out that the system is mathematically unstable. However this instability has yet to result in abnormal changes in the certificate price. Further, the degree to which an increase in prices is met by an increased rate of investments in renewable capacity is unclear. However, from the regression done on historical data, this effect is assessed to exist. Moreover, given an increased investment rate following a price change, the lead times of different technologies will directly affect the time required to reach a new equilibrium. These factors are important for understanding how the price equilibrium is reached.

To the best of the authors' knowledge, this study is the first to implement the approach of Coulon et al. for the Swedish-Norwegian market. The comparison of model output to historical Swedish certificate prices shows that though short-term fluctuations are not captured, long-term trends and price levels are replicated quite well. This indicates that the example model will be a suitable starting point for further work.

From sensitivity analyses done for the penalty fee, discount rate and the feedback effect, results are assessed to be in accordance with the expected behavior. Prices are observed to be positively dependent of the penalty fee, the slope of the price curves increase with the discount rate and an increased feedback effect dampens price fluctuations. Furthermore, the penalty fee and the discount rate seem to be the key drivers of the model.

Some important aspects of the market have yet to be implemented. In contrast to the fixed penalty seen in the New Jersey SREC market described by Coulon et al. the penalties in the Swedish-Norwegian market depend on prices observed over the previous year. To include the penalty fee as an endogenous variable requires the introduction of another state variable, leading to a considerable increase in runtime. Therefore, an implementation must be done in such a manner that the model remains computationally tractable.

Generation of electricity, and thus the issuance of certificates depend on electricity prices and weather conditions. Including electricity price forecasts and weather forecasts into the model, allows for better estimates for the issuance of certificates, consequently increasing the quality of the certificate price forecasts.

Further investigation in the points mentioned above will result in a more sophisticated price model for the Swedish-Norwegian electricity market. This will be a useful tool for investors considering investments in renewable electricity as it can help them predict future prices and hence lower the risks of their projects. Such a model can also provide useful advice to lawmakers and regulators considering whether changes should be made to the market design.

References

[1] M. de Cervantes Saavedra. *The History of the Most Ingenious Knight Don Quixote*. Chiswell, 1700.

[2] P. Shyamal and R. N. Bhattacharya. Causality Between Energy Consumption and Economic Growth in India: a Note on Conflicting Results. *Energy Economics*, 26(6):977–983, 2004.

[3] Intergovernmental Panel on Climate Change. *Fourth Assessment Report: Climate Change 2007: The AR4 Synthesis Report*. Geneva: IPCC.

[4] The Swedish-Norwegian Electricity Certificate Market - Annual Report 2013, 2014.

[5] P. E. Morthorst. The development of a green certificate market. *Energy Economics*, 28(15):1085–1094, 2000.

[6] T. Bye. On the price and volume effects from green certificates in the energy market, 2003.

[7] Lovdata. Lov om elsertifikater, Accessed December 16, 2014. URL `http://lovdata.no/lov/2011-06-24-39`.

[8] Institute for Energy Research. Speculators fixing oil prices? don't bet on it, 2008.

[9] The Nasdaq OMX Group. Electricity Certificates, Accessed December 16, 2014. URL `http://www.nasdaqomx.com/transactions/markets/commodities/markets/electricity-certificates`.

[10] NVE and Energimyndigheten. Oppsummering av energimyndighetens og nves gjennomgang av elsertifikatordningen, February 2014.

[11] E. S. Amundsen and J. B. Mortensen. The Danish Green Certificate System: some simple analytical results. *Energy Economics*, 23(6):489–509, 2001.

[12] P. E. Morthorst. A green certificate market combined with a liberalised power market. *Energy Economics*, 31(13):1393–1402, 2003.

[13] The Energy Technology Systems Analysis Programme. Markal, Accessed December 16, 2014. URL `http://www.iea-etsap.org/web/MARKAL.asp`.

[14] T. Unger and E. O. Ahlgren. Impacts of a common green certificate market on electricity and CO2-emission markets in the Nordic countries. *Energy Economics*, 33(16):2152–2163, 2005.

[15] O. Wolfgang and S. Jaehnert. Methodology for forecasting in the swedish-norwegian market for el-certificates. *Working Paper*, 2014.

[16] M. Coulon, J. Khazaei, and W. B. Powell. Smart-SREC: A Stochastic Model of the New Jersey Solar Renewable Energy Certificate Market. 2013.

[17] F. R. Aune, H. M. Dalen, and Hagem C. Implementing the EU renewable target through green certificate markets. *Energy Economics*, 34(4):992–1000, 2012.

[18] R. Fagiani and R. Hakvoort. The role of regulatory uncertainty in certificate markets: A case study of the Swedish/Norwegian market. *Energy Economics*, 65:608–618, 2014.

[19] Simulating price patterns for tradable green certificates to promote electricity generation from wind. *Energy Policy*, 35(1):91 – 111, 2007.

[20] Thema Consulting Group. Sertifikatkraft og skatt, 2012.

[21] Svensk Kraftmäkling. SKM - Svensk Kraftmäkling El-Certificate Prices (SEK), November 2014.

[22] R.L. McDonald. *Derivatives Markets.*

[23] Richard Bellman. *Dynamic Programming.* Princeton University Press, Princeton, NJ, USA, 1 edition, 1957.

[24] RStudio. *version 0.98.1091.* Foundation for Open Access Statistics, Boston, Massachusetts, 2014.

[25] C. Alexander. *Market Risk Analysis, Quantitative Methods in Finance.*

[26] MATLAB. *version 7.10.0 (R2014a).* The MathWorks Inc., Natick, Massachusetts, 2014.