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An LP-based analysis of the Swedish-Norwegian market for green electricity certificates

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Abstract

Energy certificate policies are rapidly gaining popularity as a governmental tool for promotion of renewable energy technology. This paper aims to contribute to the understanding of the market dynamics as an effect of introducing energy certificates as financial instruments. We evaluate the Swedish-Norwegian joint green certificate system and its characteristics, and review some of the existing academic work on describing the mechanisms of such a market. An initial suggestion for the modeling of the price of green certificates utilizing linear programming with a stochastic generation efficiency factor is proposed. The model presented is intended as an initial step in the direction of using the linear programming approach to illustrate the evolution of green certificate prices. Preliminary results from the adaption are presented and analyzed for viability and realism. In addition the paper presents possible alternative approaches and extensions that can contribute to the improvement of the model's viability and compatibility with the market. Further we consider alternative uncertainty factors and the model's properties are compared to the popular dynamic programming approach.

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1 Introduction

As the world faces increasingly daunting environmental challenges as an effect of human induced climate changes the need for effective actions is severe. The last few decades we have witnessed several large-scale efforts in the attempt of gaining worldwide involvement to overcome this problem. Through the seventh Millennium Development Goal, The United Nations placed an international focus on the importance of environmental sustainability, and the Kyoto Protocol represented the first wide-spanning commitment to a tangible measure[1]. Extensive research on the economic situation of the climate change issues were presented in The Stern Review by Stern

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[1] In this agreement, the signing Annex I parties, industrialized countries with economies in transition, (Barrieu & Fehr (2011)), commit to a collective reduction target on greenhouse gas emissions of 5,2% from a 1990 level during the first commitment period, extending to 2012. As the Protocol presently moves into its second commitment period, lasting until 2020, there is still a high degree of uncertainty concerning which nations will commit for a new period.

(2007), where he describes the climate change situation as ”the greatest market failure the world has ever seen”. In order to act upon these appeals, the topic of efficient environmental policies has been made one of top priority in many countries. The initiatives launched in order to meet the requirements, varies greatly among nations. However, governmental policies are a popular measure and a majority has applied one or several support schemes to induce investment in environmentally friendly technology. Especially technology for production of renewable energy is expected to play an important role in the reduction of greenhouse gas emissions. An often used policy for encouraging expansions of production capacity for renewable energy is tradable green certificates, TGCs. The foundational element of this scheme is the green certificate which trades in the market as a financial security and works to incentivize investment in renewable energy production technology. The demand for such certificates is controlled by the government through a requirement set on the market actors[2] for the acquisition of certificates and a penalty fee collected for the certificates they fail to procure. Among the nations applying such a policy are USA[3], Australia, Holland and England (Morthorst (2003)). In addition, Denmark, Sweden and Norway have applied such certificate schemes.

As the GC system is becoming an increasingly important instrument in energy economics, it is vital to gain insight in its dynamics to be able to arrange the market conditions to be as favorable as possible and maximize the probability of the system achieving its goal. The GC market is however greatly affected by unstable prices which can move from the penalty to close to zero as a result of only minor changes in the underlying market conditions (Morthorst (2000)). This high volatility can scare off potential investors, who wish to finance their investment by the sale of GCs, and base their investment decision on the market’s ability to provide stable income. Distributors of electricity are also depending on stable GC prices, as high volatility will increase their price risk[4]. In order to stabilize the GC price at a sustainable level, it is crucial to have models describing the dynamics and the rules of the market.

On 19 December 2011 the Norwegian directive ”Fornybardirektivet” was incorporated as a part of the EEA[5] agreement, stating the Norwegian government’s commitment to increase the share of the domestic power consumption produced by renewable sources[6] from 65% (2011) to 67,5% within 2020. Representing the highest renewable target in Europe, the directive is a very ambitious commitment, and strengthened the need for governmentally controlled environmental policies (Norwegian Oil and Energy Authorities (2011a)). One of the main tools presented by the government for reaching this target is the Swedish-Norwegian Green Certificate (GC) system, which was launched in Norway 1 January 2012. This system is meant to incentivize investment in renewable energy production capacity by creating an additional source of income for producers in the shape of a certificate awarded for each unit of energy produced by the new capacity. This certificate is a sellable security and is purchased by power distributors to cover the requirement set on them by the government.

Several papers analyzing the European certificate markets have been presented in recent years. However, a majority of these are focused on the choice and design of a policy, comparing the

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[2] Which actors the requirement applies to depends on the detailed features of the specific country’s certificate system.

[3] With their renewable portfolio standards.

[4] The distributor is faced with price risk both when the governmental requirement is set on them, as cost will increase due to the purchase of certificates, and if the producer is charged the penalty for lacking production, as the fees incurred by the producer will be transferred to the distributor through higher electricity prices.

[5] European Economic Area, EØS in Norwegian.

[6] Called ”fornybarbrøken” in Norway.

probability of success of the three main policy alternatives; cap-and-trade, green certificates and feed-in-tariff. An example of such work is Morthorst (2003). The perhaps most discussed such policy is the Emissions Trading System, ETS, also called cap-and-trade. This system has since its origin received great attention for its ability to aid in the implementation of environmental policies. As opposed to the GC system, ETS directly incentivizes cuts in emissions by setting a cap or limit on the allowed amount of emissions and allowing countries which do not use their whole quota to sell their excess emission units to countries who surpass their limit. As presented in Bye (2003), the government increases the cost of producing energy with non-renewable sources by putting a price on emissions, and hence forces a transition from fossil and other non-renewable to renewable energy sources. The author also argues that the ETS system has seen an over-supply of emission credits in recent years, leading to a drastic fall in the price. The low price of CO_2 credits prevents the cap-and-trade system from creating the desired incentive for investment in green technology, as the cost of buying the needed quotas is lower than the cost of new technology.

Another main competitor of the GC policy is the feed-in tariff scheme. The design of this scheme is closer to that of the GC system, as it is applied to directly incentivize investment in renewable technology. By the use of long term contracts and price guarantees, the government reduces the risk held by the producers. Through compensating the producer for the difference between the power price in the spot market and the price guaranteed by the agreement the government transfers some of the price risk to themselves, which according to Huisman *et al.* (2013) makes the system very agreeable for investors. This paper however also discusses the paradox of the policy becoming increasingly expensive for the government as it is more and more successful. The paradox is caused by the fact that the marginal cost of renewable energy, and hydro and wind in particular, is very low. They mention several studies, for example Amundsen & Mortensen (2001), Jensen & Skytte (2002), and Fischer (2006), presenting proof that the power price will go down as a result of increasing market share for renewable energy in a price competitive market due to the low marginal cost. This phenomenon has already been observed in Germany, as shown in Appendix A (web: European Energy Exchange (n.d.)). In turn this means that the overall revenues of the power producer will decrease, leaving the difference to be compensated by the government. Some of the uncertainty that the scheme removed from the investors is by this re-applied in the form of the possibility of the government abandoning the agreement and the losses from the decrease in power price falling on the investors themselves.

The risks of the government abandoning the currently held support scheme is also discussed in Boomsma & Linnerud (unpublished). The authors aim to shed light on how uncertainty connected to the support scheme can affect the rate at which investment is undertaken. When expressing the optimal investment condition, the risk factors are divided into two groups; market uncertainty and policy uncertainty. Market uncertainty will vary among the different schemes according to their specific features[7]. Policy uncertainty affects all schemes and reflects the government's ability to change the market environment by either terminating the support scheme, make revisions in its regulatory conditions or switch among the schemes. In their approach, the authors use Geometric Brownian Motion, GBM, to explain both factors, and the relationships are applied in a dynamic programming framework to express the optimal investment rule.

Another example of a paper doing comparative analysis is Aune *et al.* (2012), which analyzes and compares the cost effectiveness of various designs of GC systems for reaching the EU's

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[7] A fixed feed-in tariff scheme is not connected with any market risk, as the government has committed to account for any losses incurred due to price changes, while a certificate trading scheme implicates risks connected to both electricity and certificate price levels.

renewable target. The authors argue that by introducing trade amongst the EU member states, the overall cost of achieving the renewable target can be cut by almost 70%. They also point to other literature[8] implying that the GC system is well suited for the purpose of securing the share of renewables in final consumption, but not for the purpose of achieving greenhouse-gas emissions reduction. Another perspective covered in the literature is the focus on financial risk, taken by Lemming (2003). This paper analyses the risk concerning the existing producers of renewable electricity as well as that of potential investors in such technology, focusing on the switch from a fixed feed-in tariff system to a TGC system. Other papers combine the task of investigating the success factors for the implementation of a certificate system and modelling the market dynamics. An example is Jensen & Skytte (2002), which presents a deterministic equilibrium model in order to study the price and consumption effects of a demand-driven GC system. They also argue that these effects will be highly influential on the chances of successful implementation.

The feature of banking, that is, storage of certificates from one period for use in future periods, is another issue discussed by the literature. One example of a study with such a focus is Amundsen *et al.* (2006), which investigates to which extent the introduction of banking of GCs will reduce price volatility. The authors also studiy the effect banking would have on surplus and argue that banking indeed will reduce price volatility considerably and have a positive effect on the social surplus, but that the producers' surplus not necessarily will increase. This paper also introduces a basic model for the European GC price using a rational expectations simulation model of competitive storage and speculation. The model however ignores the adaption to the special statutory features of the individual market.

In environmental economics a crucial topic is the price dynamics of financial instruments. The research on this area can be dated back to the early seventies with Montgomery (1972)'s study on the existence of a market equilibrium price in emission licenses as a result from joint cost minimization. Following studies on this topic has turned in two directions; The first direction is in close continuation with Montgomery's work in describing the price formation as an equilibrium formed through supply and demand. Rubin (1996) extends Montgomery's thoughts into a setting which allows banking and borrowing of ETS emissions allowances. He analyzes the inter-temporal effects of these opportunities and shows how they allow the firms to control and adjust their stream of emissions through time. A central assumption stated in this study is that the price of an emission allowance should equal the marginal abatement cost of reducing emissions. A similar stand is taken by Morthorst (2003) with respect to the GC market. His results show that the sum of the electricity spot price and the GC price should equal the long run marginal cost, LRMC, of investing in new renewable capacity. The basic principle from these results is consistent with the belief that a higher price would incentivize companies with lower marginal abatement- or investment cost to exploit the price difference. In general this way of describing the price dynamics simplifies the investment decision as the investment will be favorable only if the marginal investment cost is below the LRMC. However, this approach has been subjected to criticism due to its undermining of the importance of price uncertainty.

The second direction of studies in context with price dynamics on the other hand *does* include this uncertainty by using stochastic modeling of the price. This perspective points out the possession of a martingale property in the price formation, as stated in Seifert *et al.* (2008). Similar results are confirmed by Carmona *et al.* (2010) which argues that the price at compliance must equal either the penalty or 0, and hence that the price at any point in time must equal the discounted expected value of the price at compliance. The statement that the certificate

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[8] See Aune *et al.* (2008); Palmer & Burtraw (2005) and Haas *et al.* (2004).

price depends on the probability of a certain future outcome is also addressed by Chesney & Taschini (2011). They consider the price dynamics of the emission certificates due to asymmetric information where they prove the existence of a martingale condition in the discounted price. Other studies supporting this approach are among others Coulon *et al.* (2013). Another perspective to consider within this group is how to include the effect of price feedback into the optimal abatement or investment decision. One proposed solution to this is presented by Seifert *et al.* (2008), who solves this as an optimal control problem for a central planner considering whether or not to spend money on emission reduction based on total expected emissions.

Bye (2003) and Coulon *et al.* (2013) present holistic approaches to the modelling of GC prices. The first presents a model aiming to maximize producers' profit given by the sum of revenues from sale of "green" and "black"[9] energy less the production cost. The conclusions drawn from this model however rest on the assumption of increasing long run marginal costs of production for green energy. As marginal costs of renewable energy in general can be seen as very low, and for some technologies can be set close to zero, this assumption may lead to faulty conclusions. Coulon *et al.* (2013) presents the SMART-SREC model, calibrated for the New Jersey solar energy certificate market. This model tries to deal with the problem identified in the paper by Amundsen *et al.* (2006), by capturing both the behavior of the market participants and how it is ruled by the unique regulations of this market. The authors propose a stochastic model in continuous time covering both the banking and no-banking cases, and use a backward recursion-like approach to dynamic programming in solving the problem. However, the authors state that the number of states in the state space has reached a limit and that the solving process can take several hours.

This paper tries to find a more efficient solution to a similar problem focused on the Swedish-Norwegian GC market. We present a proposition for the modeling of the GC price through a mixed integer linear programming, LP, approach maximizing the producer's revenue, hereby simply referred to as an LP approach[10]. This may contribute to reducing the size of the state space by simplifying the multi-stage decision process and reducing the number of state variables. The model presented in this paper is not complete, but rather a first step in the direction of introducing linear programming as an approach to gain insight in the evolution of green certificate prices. As it is tailored for monthly time increments within one year, it is not affected by the issues of banking and speculation, but an extension of the time frame will quickly introduce this topic. The main objective is to improve the understanding of the dynamics of green certificates as a policy instrument.

Further the paper is organized as follows; section 2 explains the structure of the Swedish-Norwegian GC market and the different elements affecting supply and demand, section 3 presents the mathematical model simulating the movements in the GC price, and section 4 describes the methodology for the software implementation of the model and the estimation of input parameters. Section 5 presents the input data and output results from the model, and section 6 presents suggestions for possible changes and extensions for future model improvement, as well as discusses alternative uncertainty parameters. Section 7 summarizes and concludes.

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[9] Where green signifies renewable energy and black non-renewable energy.

[10] A linear programming problem where all unknown variables have to be integer, is called an integer programming(IP)problem. In contrast to a LP problem, the IP problem can normally not be solved efficiently in the worst case (there exist some special cases where this is possible). If just some of the variables have to be integers, we have a mixed integer programming problem.

2 The Swedish-Norwegian GC Market

Most large power companies are divided into two distinct parts- a power *producer* and a power *distributor*. The producer owns the individual power plants and feeds the power produced into the grid. The Nordic countries, Norway, Sweden, Denmark and Finland are collected in one area of consumption and electricity trading in these countries is done on the Nordic power exchange Nord Pool Spot. In 2010 Nord Pool Spot traded 307 TWh, representing 74% of the Nordic consumption (web: NVE c (n.d.)).

Almost 100% of the Norwegian electricity production is derived from renewable energy sources, but due to export and import to and from other countries, renewable energy represented only 65% of the Norwegian energy consumption in 2012. As a measure for reaching the renewable target of 67,5% by 2020, the Norwegian government decided in 2011 (Europaportalen-www.regjeringen.no (2011)) to join Sweden's system for GCs which has been operating since 2003. The goal of this cooperative system is to by 2020 have increased the production capacity of renewable energy by 26,4 TWh (web: NVE b (n.d.)), which corresponds to 198 million green certificates from 2002 levels. The goal at the initiation of the Swedish GC system in 2003 was to increase production by 25 TWh by 2020[11], and until Norway joined in 2012, Sweden had seen an increase of 13,3 TWh (web: NVE c (n.d.)). The idea for the joint system is that Norway and Sweden are to contribute half of the target increase, that is 13,2 TWh each. Hence, on completion of this goal, Sweden will surpass their original target by 1,5 TWh.

2.1 Market properties

Producers of renewable energy are through the GC system awarded one certificate for each MWh produced by new[12] capacity. Each GC works as a financial security which can be traded in the marketplace. Within the system, the marketplace is boundless which enables investments at the geographical place where it is most economically beneficial. The demand for GCs is set by each government through the quota they impose on the power distributors. The quota forces these market participants to buy a number of GCs representing a set percentage of their total amount of MWhs distributed. The transaction can happen either directly between the two parties or through a broker. The trades can also be executed in the form of spot contracts or forward contracts[13]. For each GC the distributors fail to acquire within the deadline, they are charged a penalty amounting to 150% of the average GC price calculated over the past year. After eliminating the required amount of certificates or paying the penalty, the distributor's obligations for that year is settled. The distributors' expenses due to the GC system are financed by their customers through an addition in the electricity prices. Swedish and Norwegian customers will however be subject to different additional costs per kWh, as their respective electricity suppliers are subject to the same GC price, but different quotas[14].

Regulatory differences The fact that Norway entered an already established Swedish market, presented the need for differences in the implementation and regulations in the two

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[11] The very first target for which Sweden developed quotas was an increase of 17 TWh within 2016, however this was revised in 2009 to 25 TWh within 2020 (Swedish Energy Agency (2009c)).

[12] Set into production after 1 January 2012 as will be explained below.

[13] In Sweden, the forward contract stretches for four years, with the majority of the trades being executed the first two years.

[14] Norway and Sweden each have their respective level of the requirement quota.

countries. One of the most distinct differences is which facilities are included in the joint system. In Norway, all power producers who have invested in new renewable production capacity with construction starting later than 7 September 2009, is entitled to apply for the rights to receive green certificates for their new production. However, if production started before 1 January 2012, the preceding time will be subtracted from the total allowance time. In Sweden, all facilities which have been set into operation after 1 May 2003 are awarded certificates for the full allowance time. However, only those who started production after 1 January 2012 will be part of the joint system. The remaining facilities entitled to certificates are to be financed by the Swedish state alone. In both systems, the maximal allowance time is set to 15 years, but no longer than until the end of 2035. In general, certificates are awarded only for production by the capacity added after 1 January 2012, but in Sweden extensive improvements or reconstruction can be treated as a new development, and result in the producer being awarded certificates for the whole capacity (Swedish Energy Agency and NVE (2012)). Another regulatory difference between the countries is the existence of a deadline for start of production. In Sweden, all otherwise applicable capacity will be rewarded certificates for either 15 years, if production started before 2020, or until 2035 for production commenced after this time. In Norway, the right to receive certificates for new capacity is revoked if production has not been initiated before 1 January 2020. This legislation has been strongly criticized by Norwegian power producers, as the extensive construction period of such facilities means that the time for making investment decisions at this point in time already is running out.

Another important difference between the two countries is the tax level which applies to the power companies. At this point, tax levels favor Swedish companies, as they are subjected to substantially lower taxes than their Norwegian counterparts. As the agreement between Norway and Sweden states that investments shall take place at the geographical location with the most beneficial conditions, this leads to a majority of the new capacity being built in Sweden (THEMA consulting group for Energi Norge (2012)).

In the joint system's first year of operation the total certificate issuance numbered 21,6 million (Swedish Energy Agency and NVE (2012)). Of this, 21,4 million were issued in Sweden and 0,2 million in Norway. The reason for this difference is the low amount of installed effect in the Norwegian facilities approved for receiving GCs. This is a product of that most of the windpower producers with newly installed capacity in Norway chose to keep the investment support from ENOVA[15] instead of joining the GC system. Sweden was also the largest contributor to the increase in the expected standard yearly production[16], adding 2,8 TWh of a total increase of 3,2 TWh (Swedish Energy Agency and NVE (2012)).

The certificate market is regulated by two authorities which administrates and develops the system. The Norwegian part of the market is regulated by NVE[17] and the Swedish part is regulated by Energimyndigheten[18], these being the energy agencies in the respective countries. The two countries each have an electronic registry where the certificate entitled and –obligated actors have their reporting accounts. The registry in Norway is called Norwegian Energy Certificate System, NECS, and is controlled by Statnett, whereas in Sweden it is called CESAR and is controlled by Svenska Kräftnat, SvK(web: NVE a (2013)).

Setting the quota Figure 1 shows the curves of the Norwegian and Swedish yearly quotas. In both countries the quotas are set by law to see yearly incremental increase until 2020 to

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[15] Organization owned by the oil and energy department of the Norwegian parliament, – created to fuel the change from fossil to environmentally friendly energy sources. Provides financial support for renewable energy

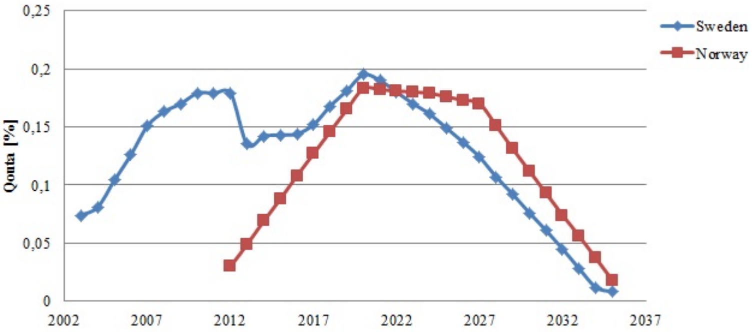

Figure 1: Norwegian and Swedish quotas in percentage of total energy production 2003–2035

ensure an increasing demand for certificates in the market. The quotas are calculated for each country respectively from estimates of the future electricity consumption subjected to certificate obligations and of the expected change in installed capacity. The chance of the actual consumptions not matching the estimates introduces the need for the ability to adjust the quotas, and these adjustments will be performed at so-called control stations, the first of which will be held in 2015[19]. As can be seen from the curves, the Swedish quota is quite high compared to the Norwegian quota at this time. This difference is a result of that the quotas are set to also secure financing for the national certificates not included in the joint system. At present, Sweden has a substantial amount of capacity entitled to certificates which are not included in the joint system. On the other hand, the great majority of actors connected to certificate schemes in Norway are included, hence Sweden is forced to operate with higher quotas to secure sufficient demand.

In Sweden, changes in the quota have already been used to balance the supply and demand of certificates. The quotas which are currently active, were presented in 2009 (Swedish Energy Agency (2009c)). The previous set of quotas were developed for the goal of 17 TWh increase in production capacity within 2016 with a phase-out period stretching to 2030, and can be seen in figure 18 in Appendix B. A control station was initially planned to be executed in 2012, but as a result of the realized consumption of energy subjected to certificate obligation being lower than forecasted in the years 2003–2008, Energimyndigheten was in 2009 ordered to propose adjustments to the quota to be in effect from 2011, where the new goal-date of year 2020 and corresponding target of 25 TWh increase were incorporated. The resulting report (Swedish Energy Agency (2009c)) presents adjusted quotas for the years 2013–2020 and new added quotas for 2021–2035, as can be seen in figure 19 in Appendix B. When determining the new quotas, analyses were based on forecasts of two important parts of data;

1. *The expected level of the energy consumption governed by certificate obligations:*
   The estimation of future energy consumption within the GC system was produced by using the 2009 short-term prognosis presented by Energimyndigheten(Swedish Energy Agency (2009a)) for 2009-2011 and the long term prognosis from the same year(Swedish Energy Agency (2009b)) for the period 2012–2035.

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producers which has to be paid back if the producer joins the GC system.

[16] Expected production during normal operating conditions.

[17] Norges Vassdrags- og energidirektorat, Norwegian Water Resources and Energy Directorate.

[18] Swedish Energy Agency.

[19] Before the Norwegian entrance to the market a control station for the Swedish market was planned to be executed in 2012.

2. *The rate at which new capacity is added to the system:*
   As of year 2008, the total capacity connected to the GC system had increased by 8,5 TWh. When setting the quota, Energimyndigheten estimated the increase between 2009 and 2013 to be 6,3 TWh, which leaves a wanted addition of 10,2 TWh over the period 2014–2020. This remaining increase was linearly distributed over said period, resulting in an estimated yearly increase of 1,46 TWh.

As can be seen from the presented data, the quotas for 2011 and 2012 were kept at their initial levels. This was done deliberately by the government to ensure stability and predictability in the market[20]. As the market already was prepared for a potential change of the quotas at the control station in 2012, changes in the quota beyond this date were considered not to represent noticeable harm to the market actors. On the other hand, changes in the market conditions before the announced control station would change the market premises so close to realization that the actors would not have sufficient time to adapt without being affected negatively. Another argument for the quotas of 2011 and 2012 to remain the same is the nature of the futures contracts on certificates. These are active for four consecutive years, meaning that futures already traded in 2007 and 2008 would experience an unexpected change of market conditions from those which applied at the time of the price-setting. In addition, as of 1 January 2007 the cost of the green certificates is included in the calculated price of electricity in fixed-price agreements that power distributors offer their clients. A sudden change in the quota might harm the distributor, as the price is fixed at initiation and the increased quota would result in a higher certificate price[21].

The initial Norwegian quotas presented in figure 20 Appendix B[22] were made in cooperation between the Norwegian government and NVE with enhanced focus on the expected available resource base in the initial years of operation (Norwegian Oil and Energy Authorities (2011b)). The quotas were produced with special consideration given to the following;

1. *Harmonizing with the existing Swedish market:*
   As the Swedish quotas were already established to balance the Swedish market, the Norwegian quota has to be able to balance the Norwegian market. The presented quotas were set to facilitate a balance between supply and demand, emphasizing the first years of system operation.

2. *Level of ambition:*
   Norway took on the same ambition level as Sweden, aiming for a 13,2 TWh increase in production capacity in the GC system within 2020.

3. *Current and prospective consumption of energy with certificate obligations:*
   Projections of the future consumptions applicable for certificate obligations were made on the basis of a handful of official studies[23] estimating a yearly increase of 0,1–0,8%. These resulted in a final estimate of 0,3% yearly increase in the total energy consumption, from the 73,4 TWh used in 2008, being the basis of the quota calculation.

4. *Prospects of new production capacity:*
   NVE considered hydro- and wind power to be the most significant growth areas for pro-

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[20] Sudden and unexpected changes in the quota might signal increased flexibility in the quotas and hence lessen the appeal to investors.

[21] As a result of the fixed price, the increase in cost of certificates could not be debited the client.

[22] Found in Norwegian Oil and Energy Authorities (2011b) and in *Prop.101 L- Lov om elsertifikater*- Norwegian Oil and Energy Authorities (2011c).

[23] "Finansdepartementets Perspektivmelding", the "Klimakur" study made for ENOVA in 2009, and study by Statnett on future consumption on the central grid. (Norwegian Oil and Energy Authorities (2011b)).

duction capacity. When estimating the short term growth rate, existing lisenses for development had to be considered. Several lisenses had already been given, but been delayed by lack of grid capacity, and the quotas were set with estimates of the probability of short-term realization of these. On a long-term basis, Norway followed Sweden's example and the total desired capacity increase of 13,2 TWh was linearly distributed over the years 2012–2020.

Certificate surplus Each issued certificate is valid until 2036, meaning that both producers and distributors are able to store certificates and trade/eliminate them at a later point in time. Hence the certificate market allows for banking, but it is important to note that there is no borrowing of certificates from future supply. In periods when the issuance of certificates exceeds the number of eliminated certificates, the total reserve or surplus will increase. At the end of 2012 the total reserve amounted to 11,7 million certificates[24], which represented an increase of 3 million (Swedish Energy Agency and NVE (2012)). This reserve has successively increased in the Swedish GC market from 2003, as can be seen in Appendix C, and is now included in the joint system (Swedish Energy Agency and NVE (2012)). In the process of changing the Swedish quotas, the government increased the requirements of 2013 and 2014 more than the successive years in order to decrease the reserve. Despite this increase the Swedish quota level in these years was subject to a significant drop, apparent in figure 1, due to the about 1400 Swedish facilities which at this time were phased out of the GC system (Swedish Energy Agency (2013)).

The ratio of the banked certificates to the number of certificates to be eliminated in a given year can give an indication of the temperature in the certificate market. A certain level of surplus has proven an important instrument in stabilizing demand and supply. However, if the ratio is high, that is if there is a large amount of stored certificates compared to the amount that is to be eliminated, this could contribute negatively to the price of the certificate. On the other hand, if the reserve is low in comparison with the quota, the pressure in the market will rise, leading to an increase in the price. The level of surplus can also greatly affect the rate at which investments are taken on. A large surplus decreases the probability of not reaching the quota, which lessens the pressure on investing and vice versa (Swedish Energy Agency and NVE (2012)).

3 Model

In this section we introduce a stochastic mixed-integer linear programming model which aims to describe the GC price by maximizing the power producer's revenue subject to the amount of generated GCs sold, the price received and an investment cost. The demand for GCs is known and regulated by the government through the required quotas, which places the market uncertainty in the supply variations. We abstract from possible market interactions from competitors and elasticity in demand, and model the market supply of a representative producer. We assume that the marginal cost of energy production is zero, and hence face the following optimization problem;

$$\max g \times p - k$$

Where g is the amount of generated GCs, p is the GC price and k is the investment cost incurred per MW of expansion in the production capacity. In this problem we assume that the whole amount of generated certificates is sold, as long as this stays below the demand set by the quota. As any market actor facing the requirement from the government sees no increase of

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[24] Certificates eliminated for 2012 excluded.

utility for certificates acquired beyond this demand[25], the price will be zero if the amount of generated GCs is above the quota. In the full model presented later in this chapter, this property is handled with the use of binary variables. For simplicity we assume a risk-neutral world as this allows for the focus being set on the appropriate interactions.

3.1 The SMART-SREC model

A model for a similar market as the one treated in this paper has been introduced by Coulon *et al.* (2013). They introduce a stochastic model called the SMART-SREC where a dynamic programming approach is utilized to simulate the Solar Renewable Energy Certificates, SREC, prices. The model is calibrated to the New Jersey market, which has a similar structure as the Swedish-Norwegian market. However, instead of giving the solar power producers a second source of income through the sale of GCs, the system charges them a penalty for each certificate they fail to produce below a set requirement level. The certificates are valid[26] for four years after the corresponding power production year, which presents the need for the model to allow banking of certificates for this limited time. The main underlying stochastic process for the SREC prices is the generation of solar energy, corresponding to the rate of issuance of certificates, implying that the uncertainty lies in the supply of certificates, not in demand. The SREC price is hence assumed to depend on the amount of generated certificates, relative to the requirement level, and the current penalty for not generating sufficient certificates. The price is presented as the discounted value of the product of the penalty and the probability of the considered period's certificate generation being lower than the requirement[27] less the accumulated reserve of valid certificates from previous periods. The rate of growth in solar energy capacity due to new investment is assumed to be directly dependent on the SREC market price.

3.2 The Linear Programming Approach

In the development of our own model, we have based our approach on a similar set of assumptions except the obvious differences between the underlying market structures and the allowance of banking[28]. As mentioned, we have chosen a linear programming approach as an alternative to Coulon *et al.* (2013)s backwards recursion-based dynamic programming solution. This is a first effort to conceive a more efficient way of modeling the certificate price evolution, with the core objective of creating a leaner process as we, opposed to the dynamic programming approach, avoid the need for discretizing the state variables in the state space.

This paper presents a model considering the period of one year. This simplification eliminates the issue of banking as the time frame only includes one annulment of certificates. The incremental time period equals one month, hence we have $t = 1, ..., 12$.

The factor of uncertainty in the market is the level at which the capacity for generation of renewable electricity is utilized at any time t. The utilization of capacity in the Norwegian and Swedish renewable sector is affected by seasonal effects and a stochastic factor accounting for the uncertainty in inflow of hydropower, wind power etc. The combined impact of these factors is expressed in the variable C_{ts} [h], measured in effective output hours. The values of

---

[25] We ignore those rewards given by buying certificates for the sake of "being green".

[26] Can be used to fulfill the quota.

[27] The probability of underproduction.

[28] As the model presented in this paper only considers a one-year period.

C_{ts} at any point in time is characterized by a scenario tree which considers m different outcomes in each time period, resulting in m^T possible end-scenarios, $s_1, ..., s_{m^T}$. The total installed production capacity [MW] in the system at any time t, given scenario s, is given by $\hat{g}_{ts}$. As we have the practical one-for-one relationship between MWh of electricity produced and certificates generated, the generation of certificates g_{ts} [MWh/t] at time t and scenario s is expressed as follows:

$$g_{ts} = \hat{g}_{ts} \times C_{ts}$$

In the Coulon *et al.* (2013) paper it is argued that in the case of modelling a single one-year period with no storage of certificates, the SREC price at the compliance date must be either the penalty, if total generation is below the requirement, or zero, if it is above. This is evident by no arbitrage and based on the results from Carmona *et al.* (2010), who argue that the price at any point in time preceding the compliance date must be the discounted expected value of the final pay-off. This martingale property also states the foundation of the price dynamics in our model. Thus the price of a GC, p_{ts}, is stated as the discounted expected value of the certificate at compliance given the information known until time t;

$$P_{ts} = e^{-r\frac{T-t}{T}} Q \sum_{s' \in \phi_{ts}} \mu_{s'} \pi_{s'}$$

where Q is the fixed penalty, r the yearly discount rate and $\frac{T-t}{T}$ the amount of time for which the value is discounted over[29]. We define an additional set ϕ_{ts} containing scenarios with equal certificate generation until time t, noted s';

$$\phi_{ts} = \{s' \mid C_{ts'} = C_{ts} : t = 1...t\}.$$

This is a method for including information revealed through time according to the principle of non-anticipativity, and hence ensuring consistency in information between the model and an efficient market. With respect to the model this ensures that all scenarios following the same path until time t will make the same decision at this time. These scenarios are included in a subscenario, s', and we state this property by

$$\hat{g}_{ts} = \hat{g}_{t,s'}$$

The binary variable $\mu_{s'}$ is equal to zero if scenario s' results in overproduction, that is, if $\sum\limits_{t \in T} g_{ts} > R$ for the individual scenario, and one in the case of underproduction, $\sum\limits_{t \in T} g_{ts} \leqslant R$, where R is the quota set by the government. The probability of scenario s' occurring is noted $\pi_{s'}$ and takes into account the information known up to t.

If the initial installed capacity, $\hat{g}_{t=1}$, or the prospective efficiency factors are too low, investments to expand the capacity may become relevant. The cost of such investment, K, is counted per MW capacity added, and the amount of additional capacity paid for at time t will be given as the increase in $\hat{g}$ from the previous period to the current[30].This gives the actual cost of the increase in capacity for the period, k_{ts};

$$k_{ts} = K e^{-r\frac{T-t}{T}} \sum_{s' \in \phi_{ts}} (\hat{g}_{ts'} - \hat{g}_{t-1,s'}) \pi_{s'}$$

---

[29] Where t is the number of months spanned and T is the total time period – here 12 months.

[30] For a clearer illustration of the relation between the studied factors, we here assume no lag from the investment until the added production capacity is in operation.

Now we can express the initial optimization problem as the expected value of future profits;

$$\max \sum_{s\in S}\sum_{t\in T}\pi_s\hat{g}_{ts}C_{ts}e^{-r\frac{T-t}{T}}Q\sum_{s'\in\phi_{ts}}\mu_{s'}\pi_{s'}-\sum_{s\in S}\sum_{t\in T|t>1}\pi_sKe^{-r\frac{T-t}{T}}\sum_{s'\in\phi_{ts}}(\hat{g}_{ts'}-\hat{g}_{t-1,s'})\pi_{s'} \quad (1)$$

Binary variables In order to give value to the binary variable, we need constraints for the over- and underproduction for each given scenario s. For the case of overproduction we have[31];

$$R-\sum_t g_{ts}-M\mu_s \leqslant \eta$$

Where η is a infinitly small number allowing us to make the constraint linear while still remaining the property of $\mu_s=1$ when $\sum_{t\in T} g_{ts}=R$. For the underproduction we have;

$$\sum_t g_{ts}-R-M(1-\mu_s)\leqslant 0$$

Where M depicts a number bigger than the largest possible difference between the requirement and the number of generated GCs over the whole time period and μ_s is a binary variable which in the case of overproduction is forced to 0 and in the case of underproduction forced to 1, due to the constraints above.

Further we state that the installed capacity cannot decrease, imposing that the total growth in capacity will be larger than the amount of capacity phased out in the same time period[32]. Hence we have;

$$0\leqslant \hat{g}_{ts}\leqslant \hat{g}_{t+1,s}$$

Dealing with Non-linearity From equation (1) we see that the problem we are facing includes the product of a continuous variable, $\hat{g}_{ts}$, and a binary variable, $\mu_{s'}$. This complicates the model by introducing non-linearity. The required linearization is performed by constructing a new auxiliary variable, $z_{tss'}=\hat{g}_{ts}\mu_{s'}$. To simplify the objective function we include C_{ts} in $z_{tss'}$;

$$z_{tss'}=\hat{g}_{ts}C_{ts}\mu_{s'}=g_{ts}\mu_{s'}$$

From this we see that if

$$\mu_{s'}=0 \text{ then } z_{tss'}=0$$

This relation is expressed by the constraint

$$z_{tss'}\leqslant \mu_{s'}g_{ts}^{max}$$

Here g_{ts}^{max} is the highest possible value that g_{ts} can take. We also see that when $\mu_{s'}=1$;

$$z_{tss'}=g_{ts}$$

And when $\mu_{s'}=0$;

---

[31] We assume the demand for electricity to be constant. Since the required quota changes annually, we also assume constant demand for GCs within the time period of one year.

[32] We do not explicitly consider the phasing out of facilities in this paper, but positive capacity growth is imposed.

$$z_{tss'} \leqslant 0$$

This again implies that both

$$z_{tss'} \leqslant g_{ts} \text{ and } z_{tss'} \geqslant g_{ts}$$

must hold. The first constraint is implied as we see in the equations above, hence both for $\mu_{s'} = 0$ and $\mu_{s'} = 1$, whilst the second constraint is enforced by the following statement;

$$g_{ts} - z_{tss'} + g_{ts}^{max} \mu_{s'} \leqslant g_{ts}^{max}$$

To simplify the expression we introduce a variable $r_{tss'}$, representing the difference between g_{ts} and $z_{tss'}$, hence we write

$$r_{tss'} \leqslant g_{ts}^{max}(1 - \mu_{s'})$$

Summarized we have the linearization constraints

$$r_{tss'} = g_{ts} - z_{tss'}$$

$$z_{tss'} \leqslant \mu_{s'} g_{ts}^{max}$$

$$r_{tss'} \leqslant g_{ts}^{max}(1 - \mu_{s'})$$

Rewriting the objective function with $z_{tss'}$ gives

$$\max \sum_{s \in S} \sum_{t \in T} \pi_s e^{-r\frac{T-t}{T}} Q \sum_{s' \in \phi_{ts}} z_{tss'} \pi_{s'} - \sum_{s \in S} \sum_{t \in T | t > 1} \pi_s K e^{-r\frac{T-t}{T}} \sum_{s' \in \phi_{ts}} (\hat{g}_{ts'} - \hat{g}_{t-1,s'}) \pi_{s'}$$

Full Model This results in the full model;

$$\max \sum_{s \in S} \sum_{t \in T} \pi_s e^{-r\frac{T-t}{T}} Q \sum_{s' \in \phi_{ts}} z_{tss'} \pi_{s'} - \sum_{s \in S} \sum_{t \in T | t > 1} \pi_s K e^{-r\frac{T-t}{T}} \sum_{s' \in \phi_{ts}} (\hat{g}_{ts'} - \hat{g}_{t-1,s'}) \pi_{s'} \tag{2}$$

$$\begin{aligned}
s.t.\quad & g_{ts} = \hat{g}_{ts} C_{ts} && t \in T, s \in S && (3)\\
& P_{ts} = e^{-r\frac{T-t}{T}} Q \sum_{s' \in \phi_{ts}} \mu_{s'} \pi_{s'} && t \in T, s \in S && (4)\\
& R - \sum_t g_{ts} - M\mu_s \leqslant \eta && s \in S && (5)\\
& \sum_t g_{ts} - R - M(1 - \mu_s) \leqslant 0 && s \in S && (6)\\
& 0 \leqslant \hat{g}_{ts} \leqslant \hat{g}_{t+1,s} && t \in T\ |t < T, s \in S && (7)\\
& \hat{g}_{ts} = \hat{g}_{t,s'} && t \in T, s \in S, s' \in \phi_{ts} && (8)\\
& z_{tss'} = g_{ts} - r_{tss'} && t \in T, s \in S, s' \in \phi_{ts} && (9)\\
& 0 \leqslant z_{tss'} \leqslant \mu_{s'} g_{ts}^{max} && t \in T, s \in S, s' \in \phi_{ts} && (10)\\
& 0 \leqslant r_{tss'} \leqslant (1 - \mu_{s'}) g_{ts}^{max} && t \in T, s \in S, s' \in \phi_{ts} && (11)\\
& \hat{g}_{ts}, P_{ts} \geq 0 && t \in T, s \in S && (12)\\
& z_{tss'}, r_{tss'} \geq 0 && t \in T, s \in S, s' \in \phi_{ts} && (13)\\
& \mu_{ts} \in \{0, 1\} && t \in T, s \in S && (14)
\end{aligned}$$

Which is a mixed integer linear programming problem.

4 Methodology

In order to use our model to produce numerical output for the GC price, P_{ts}, and installed capacity, $\hat{g}_{ts}$, it was implemented in the optimization solver Xpress using the Mosel modeling language. The model and the input data file can be seen in Appendix F.1 and F.2 in Appendix F. This implementation naturally called for some minor deviations from the theoretical model, explained in Appendix F.3, but none which will change the fundamental understanding of the approach. The steps needed to implement the model will be explained in this section.

As previously stated, this model's main input is the value of the stochastic inflow variable or production efficiency measure, C_{ts}, which is represented by a scenario tree. In reality, the value of C can take on any value between 0 and an upper bound set by the highest inflow possible for the producer's total capacity with a given probability distribution. A similar view on the concept of uncertainty connected to renewable energy production is held by Wagner (2012)[33]. In a market where energy from non-renewable, conventional generation technologies sets the price at the exchange, it is argued that electricity price models need to account for the risks introduced by the volatility in the inflow to renewable energy production. The explained situation has been created by discretizing time in t discrete time steps. Given the decisions made at previous time steps, each step has been allowed m different outcomes, each with a set probability. Hence, this leads to a scenario tree with t time periods and m nodes springing from each ancestor node, resulting in m^t possible leaf-nodes. In our solution, we have chosen $t = 2$ and $m = 2$, resulting in $2^2 = 4$ leaf-nodes.

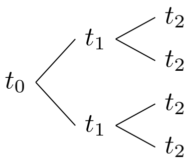

Figure 2: Illustrative scenario tree, T=2, m=2

4.1 Preliminary Implemented Model

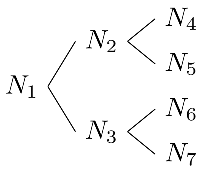

Figure 3: Numeration of the nodes, N=7

For simplicity and efficiency the model has been implemented in Xpress with indices according to nodes and not time periods. This allows us to avoid the coding of non-anticipativity constraints for each decision variable with corresponding auxiliary variables, hence saving computational

---

[33] A stochastically modelled electricity infeed from renewable sources is used in the residual demand model for non-renewable sources. The author however gives separate models for the infeed of wind and solar power.

capacity[34]. The nodes are counted as in figure 3, whereas the number of scenarios equals the amount of leaf nodes.

Sets and Variables In the implementation, some additional parameters and variables have been created[35]. For clarity we start by defining all input parameters and variables:

Sets:
N: Set of nodes.
S: Set of scenarios.

Input Parameters:
Q: Penalty.
π_{ns}: Probability of scenario s when standing in node n.
ρ_{ns}: Probability contribution to objective function from scenario s to node n. [36]
K: Investment cost due to increased capacity.
C_n: Capacity efficiency in node n.
R: Demand according to set quota.
η: Infinitely small number.
G_{max}: Maximum value of g_n.
M: Big number used forcing the binary value.
r: Discount rate.
D: Amount of relevant discounting periods.
T: Total amount of time periods.
Γ: Dynamic array directing to parenting node of node n.
Ω: Dynamic array determining active scenarios in node n.
Ψ: Dynamic array determining the discount period at each node n.

Variables:
g_n: Generated GC in node n.
$\hat{g}_n$: Total installed capacity at node n.
p_n: GC price in node n.
z_{ns}: Equal g_n if underproduction and 0 in overproduction in node n scenario s.
μ_s: Binary variable equal to 1 if underproduction and zero if overproduction.

Here ρ_{sn} is a matrix consisting of probabilities corresponding to the additional GCs produced in the time between node n and its parent node[37]. As the model is now counting through nodes and scenarios, and not time, the array Ψ is needed to give time values for the discounting. This array consists of one value for each node indicating how many time periods, out of the total T, its contribution to the revenue or cost should be discounted for. The η is a small number allowing the constraint to display the "less than or equal"-quality from equation (5).

---

[34] As the model gets expanded for longer time periods the amount of state variables in the state space increases rapidly and the computational capacity will be strained.

[35] Note that these additional parameters and variables are only added for easing the actual computation process, and do not change the essence of the model.

[36] As explained in Appendix F.3.

[37] In the implementation we distinguish between π and ρ to ease the coding process. The two are based on the same set of probabilities, but have different configurations to ease the counting.

Implemented Model Using the notation presented above the actual implemented model is as follows:

$$\max_{\hat{g}_{ts}} \sum_s \sum_n e^{-rD_{(n' \in \Psi)}T^{-1}} \left(z_{ns}\rho_{sn}Q\right) - \sum_s \sum_n e^{-rD_{(n' \in \Psi)}T^{-1}} K\rho_{sn} \left(\hat{g}_n - \hat{g}_{(n' \in \Gamma)}\right)$$

$$\begin{aligned}
&s.t. && && (15)\\
&g_n = \hat{g}_n C_n && n \in N && (16)\\
&p_n = e^{-rD_{(n' \in \Psi)}T^{-1}} Q \sum_s \mu_s \pi_{ns} && n \in N && (17)\\
&R - \sum_{n' \in \Omega} g_n - M\mu_s \leqslant \eta && s \in S && (18)\\
&\sum_{n' \in \Omega} g_n - R - M(1-\mu_s) \leqslant 0 && s \in S && (19)\\
&\hat{g}_{n' \in \Gamma} \leqslant \hat{g}_n && n \in N && (20)\\
&z_{ns} - G_{max}\mu_s \leqslant 0 && n \in N, s \in S | s \in \Omega && (21)\\
&g_n - z_{ns} + G_{max}\mu_s \leqslant 0 && n \in N, s \in S | s \in \Omega && (22)\\
&z_{ns} - g_n \leqslant 0 && n \in N, s \in S | s \in \Omega && (23)\\
&z_{ns} \geq 0 && n \in N, s \in S && (24)\\
&g_n, \hat{g}_n, p_n \geq 0 && n \in N && (25)\\
&\mu_s \in \{0,1\} && s \in S && (26)
\end{aligned}$$

4.2 Modeling the generation of certificates

In order to attain numerical values for the optimal price and installed capacity, we need to approximate an expression for the generation factor, C_t[38], at a given time t. As mentioned, the total amount of issued GCs, g_t, depends on the total installed capacity, $\hat{g}_t$ [MW], and the generation factor. This relationship can be used to produce realistic input values for C_t.

Figure 4 shows the evolution in the issuance of green certificates, g_t, in Sweden in the time between the start in 2003 and August 2013. From this we can see that there is an obvious seasonality factor affecting g_t, and the yearly mean issuance seems to increase approximately linearly with time. Another logical assumption is that the generation at time t depends on the current GC price, p_t, as has been assumed in the model. This relation is however not evident in the graph. There also appears to be a stochastic factor producing random spikes and drops. We suggest a linear relationship to $\hat{g}_t$;

$$g_t = \hat{g}_t(a_1 sin(4\pi t) + a_2 cos(4\pi t) + a_3 sin(2\pi t) + a_4 cos(2\pi t) + \varepsilon_t) \tag{27}$$

Where $\hat{g}_t$ will account for the price dependence of g and the sine and cosine expressions account for the seasonality. The stochastic factor is captured in the error term, ε_t. This results in the following final expression for C_t;

$$C_t = \frac{g_t}{\hat{g}_t} = a_1 sin(4\pi t) + a_2 cos(4\pi t) + a_3 sin(2\pi t) + a_4 cos(2\pi t) + \varepsilon_t \tag{28}$$

---

[38] In this section we note the parameters as dependent on t, not t and s, as earlier. This is because the regressions done here are estimates of real values and not values according to a given scenario. Further the scenario generation will be based on the regressions calculated here.

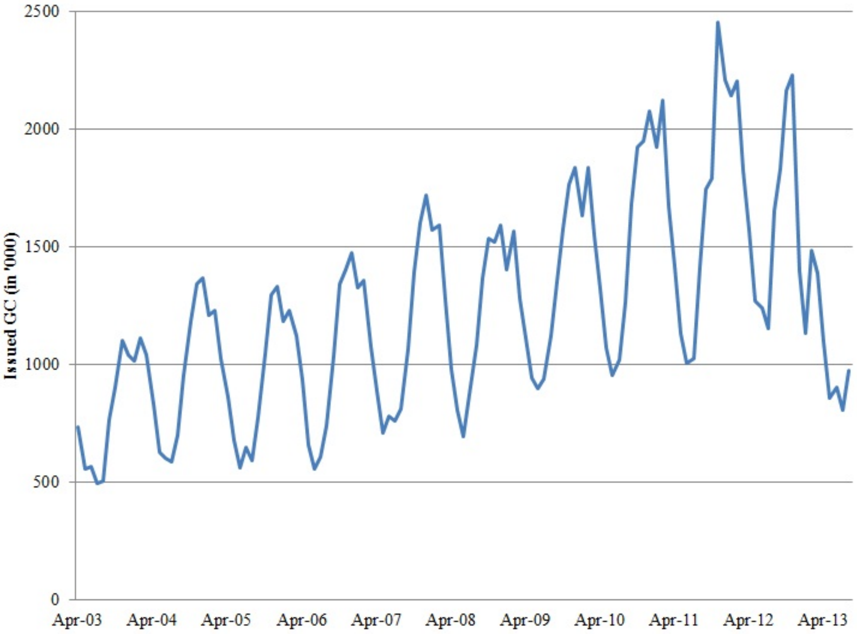

Figure 4: Issued GCs in Sweden 2003-2013 [1.000 MWh]

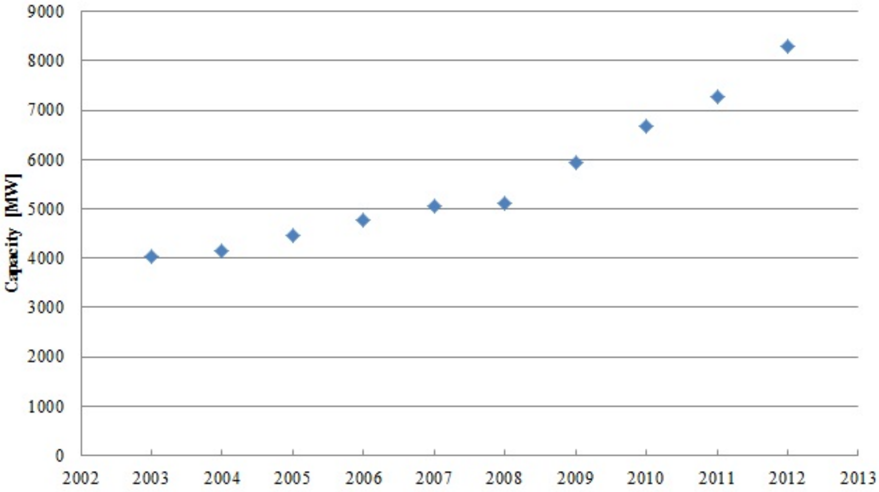

Figure 5: Installed capacity in the GC system 2003-2012 [MW]

Modeling Installed Capacity In order to find an expression for C_t using the regression, we need monthly input data for $\hat{g}_t$. The data accessed are however given in yearly granularity, as is natural for this kind of data. The evolution in installed capacity connected to the GC system can be seen in figure 5 [39]. In line with the assumptions stated above, our hypothesis is that the installed capacity varies linearly with time and the GC price;

---

[39] It is important to note that only capacity enrolled in the GC system is included.

$$\hat{g}_t = a_0 + a_5 t + a_g p_t \tag{29}$$

This relationship is however not totally consistent with the model, as equation (29) displays g_t's dependency on price as linear, which is not the case. It represents a reduced form estimate[40], where the most correct path may have been to perform a structural estimation based on the proposed model. This process would entail assuming the model to correctly describe the historical data and from this produce estimates for the needed parameters(Reiss & Wolak (2007)). However, this lies beyond the scope of this paper and a linear dependence can be seen as a valid, though not optimal, compromise. In section 5 we will also test an exponential relationship solely on time;

$$\hat{g} = ae^{bt} \tag{30}$$

This expression also faces the consistency issue introduced above, as it does not link the value of $\hat{g}_t$ to the price at all, however, this is not considered here. Monthly estimates were made for both methods and compared by studying which results in the best fit for the final model.

5 Results

In this section the data used in the modelling process and the results from running the preliminary implementation will be presented[41]. First, the data used in the regression for finding C_t is discussed. Second, the results from the modelling of $\hat{g}_t$ and the estimation of C_t is presented, followed by a comparison of results from the implemented model to our initial intuitions. Finally the model's sensitivity to selected parameters is analyzed.

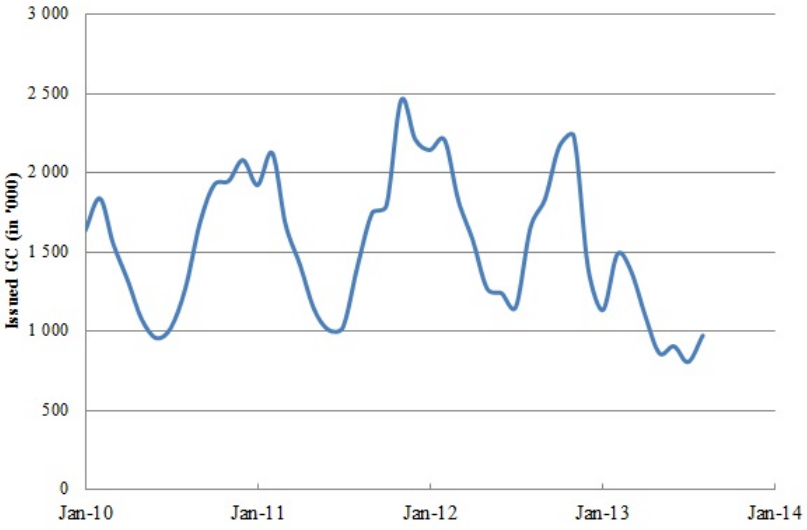

Figure 6: Issued certificates in Sweden 2010-2013 [1.000 MW]

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[40] Statistical estimation without reference to a specific economic model like the one presented in section 3.

[41] Data and explanations of the inputs to the implemented model can be found in Appendix F.

5.1 Data

Regression Data For the regression, data presenting the historical total monthly certificate issuance in Sweden were collected from the Swedish energy authorities, (CESAR), 21 October 2013. The data used shows the period between June 2003, directly following the launch of the GC system in Sweden, and December 2012. This time interval is intentionally set to minimize the "noise" caused by the incorporation of Norway in the system and the large phase–out in 2012, while still obtaining a maximum amount of usable data. Although it had alreasy been launched, the Norwegian system was in a highly initial phase and did not cause the generation data to deviate very much from previous tendencies during the year 2012, as can be seen in figure 4. Over the year 2013, the new market conditions have however caused irregularities in the number of generated certificates as illustrated in figure 6. This can be seen as a result of the drop in the Swedish quota due to the phase–out and the low Norwegian quota that by 2013 the market had managed to adjust to. The fact that there is uncertainty about how the market will develop in the future as a consequence of the new configuration naturally represents a weakness in the adaptability of the regression results. However, we believe that there is reason to assume that the main factors affecting the issuance level will remain the same, and that we will be able to extrapolate the future C_t values of the joint system from the performed regression. The data for the GC price, p_t, were also gathered from (CESAR) and is given as a monthly average price in SEK per certificate. Yearly data for historical installed capacity, $\hat{g}_t$, were found from the report "Et norsk-svensk elsertifikatmarked" Swedish Energy Agency and NVE (2012) and is presented in Appendix E, table 5.

5.2 Regression

Expanding $\hat{g}$ to Monthly Data As explained in section 4.2, we consider two possible expressions for the installed capacity in the process of extrapolating monthly estimations from the yearly collected data, given by equation (29) and (30) respectively. As we will show, they both provide strong fits, both graphically and by the R^2–value, and we will decide which to recommend by performing the regression for equation (27) with both and comparing the final fit.

Table 1 displays the output of the linear regression of equation (29). This expression produces an R^2 of 0,954 and a very good graphical fit as can be seen in figure 7a[42].

a_0	a_5	a_6
4805,08	471,592	-5,98270

Table 1: Output from linear regression on yearly $\hat{g}_t$.

The exponential regression for equation (30) and its results are shown in figure 7b and table 2. As can be seen, this expression also yields a very good fit both by an R^2 of 0,962 and graphically. Figure 8 shows the resulting curves for the estimates of the monthly $\hat{g}_t$–values resulting from the expansion of the yearly data.

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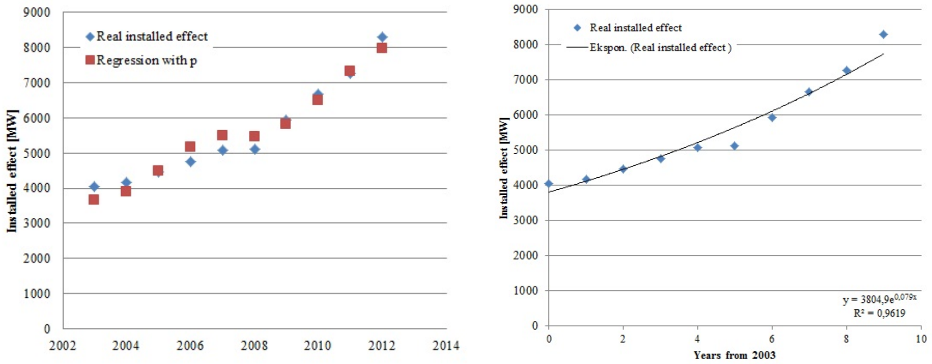
[42] A normal probability plot was produced to prove normal distribution of the error terms for both expressions.



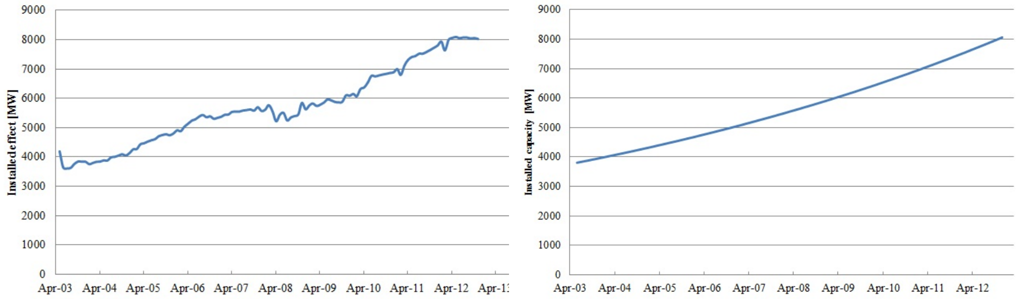
(a) Linear regression of $\hat{g}_t$with price p_t and time t [MW]

(b) Exponential regression of $\hat{g}_t$ with time t [MW]

Figure 7: Results of linear and exponential regressions of installed capacity $\hat{g}_t$

a	a
3804,9	0,079

Table 2: Output from exponential regression on yearly $\hat{g}$.



(a) Linear regression

(b) Exponential regression

Figure 8: Yearly capacity data expanded to monthly granularity using regressions

The linear regression in figure 8a shows a $\hat{g}_t$ which can display negative growth between months due to a negative price change. As it is unlikely that installed capacity will be shut down and restarted based on the GC price on a monthly basis, this expression might lose some of its credibility[43]. However, if this is overlooked, it can be an effective way of incorporating the price effect in the final relationship[44]. As the main purpose of the GC system is to increase the ratio of renewable energy production to non-renewable, it is viable to assume that the growth in renewable energy technology and production facilities will be strong, at least until 2020. This argument may favor the exponential relationship in figure 8b. Hence, both can be seen as viable options and will therefore be inserted in the final expression.

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[43] Although the small spikes and drops might have a valid cause in phase-outs set into action and production on new facilities being initiated.

[44] Also ignoring the consistency issue explained above.

Estimating C_t Table 3 and 4 show the results of the final regression of equation (28) using the estimated monthly data from the two expressions for $\hat{g}_t$. We see that they provide almost equivalent solutions, both with a high fit and with coefficient a_2 statistically insignificant. The normal distribution plots show that normally distributed error terms follow both regressions, with standard deviations of 21,796 and 18,575 respectively. As the exponential expression for $\hat{g}_t$ gives the largest[45] R^2, we choose to utilize this output in our following illustrations. The graphical fit of this final expression is illustrated in figure 9.

a_0	a_1	a_2	a_3	a_4	R^2	σ
218.819	–8,58211	–	–45,6025	–63,8286	0,874	21,796

Table 3: Result of final regression when $\hat{g}_t$ is modeled linearly.

a_0	a_1	a_2	a_3	a_4	R^2	σ
218.604	–8,52914	–	–45,4202	–63,1313	0,903	18,575

Table 4: Result of final regression when $\hat{g}_t$ is modeled exponentially.

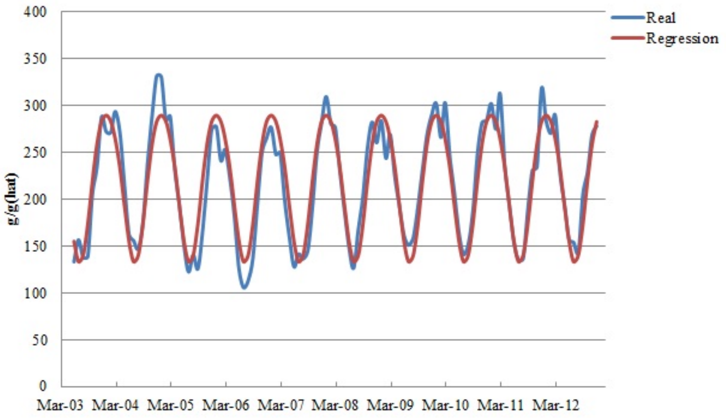

Figure 9: Realized vs. Estimated $\frac{g_t}{\hat{g}_t}$–ratio using data from exponential expansion for $\hat{g}_t$

6 month data As the implemented preliminary model explained in section 4.1 uses two time periods of length 6 months, we need to adapt the estimation of C_t to this granularity. Firstly, we need to study the applicability of the model in this case. Hence, we calculate the realized 6 month certificate generation, the average installed efficiency during the same period, and the average generation efficiency factor over the same time. The product of the estimated $\hat{g}_t$ and C_t will be the estimation of the period generation, which is tested against the historical realized values. This comparison is illustrated in figure 10 and shows an acceptable fit even with this breakdown.

We use the calculated standard deviation σ to set scenario values for C_t as shown in figure 11 and 12.

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[45] R^2 difference of 0,029.

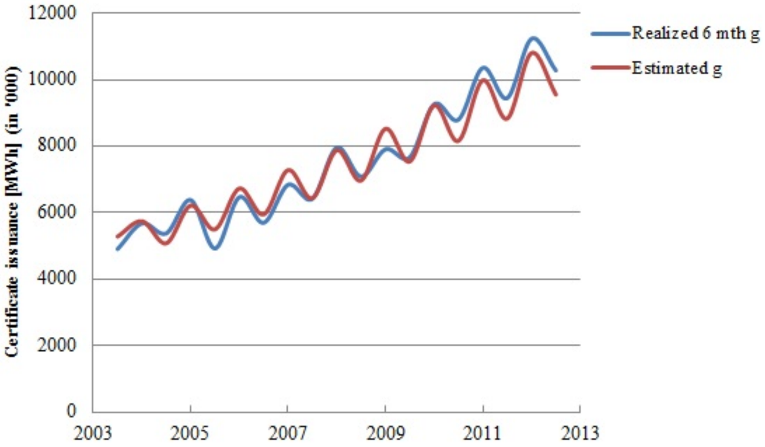


Figure 10: Realized vs. estimated 6 month total certificate issuance g_t

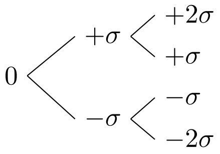


Figure 11: Estimation process of C_t

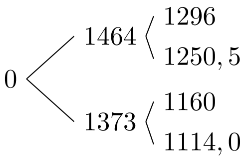


Figure 12: Calculated values of C_t

5.3 Results from preliminary implemented model

Here we present the results from running the model in Xpress. We start by stating our preconceived intuitions concerning the relation between the input factors and the output variables. Next we perform a sensitivity analysis based on possible events and comment on the results.

Intuitions Our initial intuition about the GC price can be stated as follows:

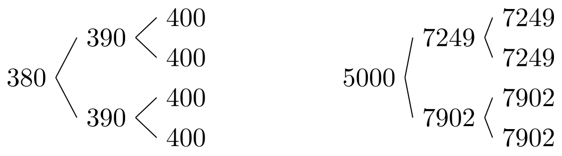
1. When we have overproduction in a scenario, the price at this scenario will equal zero at compliance.
2. When we have underproduction in a scenario, the price at this scenario will equal the penalty at compliance.
3. If there is a possibility of both under– and overproduction according to the given scenarios, the price should be the sum of the discounted expected values of the penalty at the scenarios with underproduction.

4. If the marginal investment cost is higher than the marginal revenue, there will be no investment and hence no expansion in the production capacity.

Case: Realistic input from normal production year We start the first case by giving the inputs reasonable values for the Swedish-Norwegian GC market based on the actual numbers from year 2010. The average GC price in 2010 was 267 SEK/GC (Swedish Energy Agency (2011)), giving a penalty per missing GC equal to 400 SEK/GC[46]. Based on the regression, the total developed capacity in the start of 2010 was approximately 6000 MW. Given the fact that parts of the generated GCs were banked (Swedish Energy Agency and NVE (2012)), we reduce the initial capacity to 5000 MW as this is assumed to give a more reasonable estimate for the case without banking[47]. The Swedish quota in 2010 was calculated to 17,5 million GCs and the Norwegian quota from 2012 was almost 2,5 million giving a total quota of approximately 20 million GCs for the joint system. The investment cost of increased capacity is estimated to about 1 million US dollar per MW, about 6 million SEK/MW[48]. Added capacity can be assumed to contribute to the total profit for several years, implying that the investment cost should not be attributed solely to the first year. In this case we divide the investment cost over 10 years, resulting in a yearly cost of 600 000 SEK/MW. Additionally would an investment also induce increased profit due to higher energy production, but this is disregarded in this estimate. The input for the efficiency of the production capacity, C, is based on an estimate from the regressions above. These are highly correlated with the actual observed values, giving a quite low standard deviation, and hence, by using these values we get reasonable estimates for a "normal" production year with a reasonable level of variance connected to the efficiency factor. Given this normal case of efficiency scenarios, we use the values of C presented in figure 12 above.

Result With these data inputs the model gives the results[49]:

1. The output values for the installed capacity are as displayed in figure 13b.
2. $\mu_s = 1$ for all s, indicating underproduction in every scenario.
3. $p = Q$ for every s. The price equals the discounted penalty at all times and in every scenario as seen in figure 13a



(a) Normal C: Output for P (b) Normal C: Output for $\hat{g}_n$

Figure 13: Output results for $p_t s$ and $\hat{g}_t s$ for the normal operating condition case

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[46] As explained above, the penalty equals 150% of the average price the preceding year.

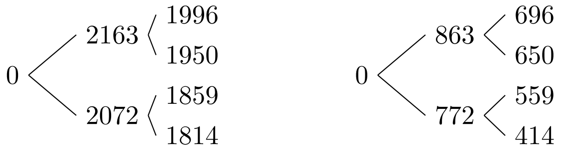
[47] For the input to give a valid estimate not only for the year 2010, but also for the following years, the fact that Norway joins the system has to be considered. However Norway's contribution to issued GCs has been negligible the first years.

[48] Investment cost can vary greatly by type of technology, geography of the plant and other factors. 1 MUSD/MW is an initial approximation.

[49] The actual output value can be found in Appendix F.4

We see that the investment in new capacity is larger for scenario 3 and 4 than for 1 and 2. This is a consequence of scenario 3 and 4 having lower efficiency values and therefore having to compensate by adding more capacity. As can be seen in Appendix F.4 the total amount of GCs generated in scenario 1 and 3 equals the quota, while in scenario 2 and 4 the total generated GC is just below. The reason why some scenarios do not fulfill the quota entirely is that investment cost for additional capacity is higher than the possible income from the same capacity. The appendix also show the price of GCs at time t. The average price in the first half of 2010 was 293,10 SEK/GC and in the second half it was 241,40 SEK/GC. Hence, at the level of accuracy expected at the current state of the model, the price can be seen as being in the right range.

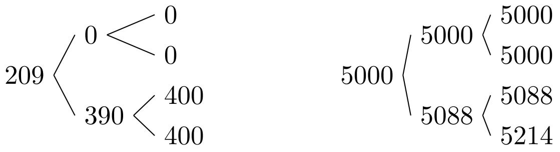
Case: Extreme weather In this case we study what happens if the amount of rain, wind etc are higher or lower than normal. This is done by manipulating C_t. All the other input factors are held constant. The values of C_t is found by increasing the normal values from above with 700 hours in all nodes in the high C_t case[50] and decrease the normal values by 600 hours in the low C_t case[51]. This gives the following scenario trees:



(a) Input value C: High (b) Input value C: Low

Figure 14: C_ts-values for the high and low efficiency cases

Results The price and installation results from the extreme weather simulations can be seen in figures 15 and 16[52]:



(a) High C: Output value for P (b) High C: Output value for $\hat{g}$

Figure 15: Output values for p_{ts} and $\hat{g}_{ts}$ in the high efficiency case

In the high C_t case scenario 1 and 2 results in overproduction, with prices equal to zero, and scenario 3 and 4 in underproduction, where the price equals the penalty. As we can see this leads to a reduced price at $t = 0$ compared to the normal case, due to the possibility of ending in overproduction with a price equal to zero. Further we see that the capacity remains unchanged in the scenarios with overproduction, as these are already forced to produce more GCs than the quota due to the high C_t-values. In scenario 3 and 4 there is a small expansion in order to achieve the required amount of GCs.

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[50] Where increased levels of wind/rain has increased the efficiency factor.

[51] Draught, less wind.

[52] The complete output results can be seen in Appendix F.4 and F.4.

(a) Low C: Output value for P (b) Installed Cap low C

Figure 16: Output values for p_{ts} and $\hat{g}_{ts}$ in the low efficiency case

From the low C_t case we see that the price always equals 400 at compliance, hence we have underproduction in every scenario. More interesting are the features in the installed capacity. Because of the low C_t, scenario 1 and 2 compensates by investing a lot to increase the installed capacity. With this expansion, in scenario 1 we achieve to generate GCs according to the required quota, whereas in scenario 2 the total amount of GCs generated is a bit below[53]. The limitation in the expansion exists because the marginal investment cost per extra generated GC is higher than the corresponding profit gained. As the investment cost is fixed per added MW and the amount of actually obtained GCs are dependent on the level of the C_t, the required investment for generating one GC more causes higher expenses than the profit lost by not expanding.

Through these different cases we have seen that:

1. When a scenario ends in overproduction, the price equals zero.
2. When a scenario ends in underproduction, the price equals the penalty.
3. When there is a possibility of ending in either over– or underproduction, the price is the sum of the expected values of the scenarios ending in underproduction.
4. If the marginal investment cost is higher than the marginal revenue, the producer will not invest.

These results summarize and confirm our initial intuitions. Although the model considers a very simplified version of the market, the essence of the results give a logic understanding of the market's reactions to the considered input factors, which gives the model credibility. It also raises the value of future efforts made to expand and adjust the model to incorporate more market features.

6 Alternative Approaches and Further Work

6.1 Model extensions

As expressed previously, the model presented in this paper is at a highly initial level, and the realism of its features is limited. Some main steps in the work to improve the applicability have been identified;

1. *Expand to include more periods and several years*
   The model is at present calibrated for at most 12 months, with the only change needed to

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[53] The combination of low initial capacity and low possible efficiency factors has "forced" the producer to uncerproduce.

go from the implemented 2 periods to 12 would be expanding the input data in the data-file correspondingly. Expanding it to several years would however need further alterations. The manual calculation and insertion of input-data would become tedious[54] and a scheme for calculating these relationships within the model would be advisable.

2. *Include banking*
   The inclusion of several years in the model would also present the issue of banking of certificates from one year to the next. As has been explained, the size of the accumulated surplus from previous periods can greatly affect both the price of GCs and the decisions of investors and firms considering expanding their production capacity.

3. *Calculate penalty as 150% of average price*
   In the implemented model, the penalty, Q, has been exogenously set in the data-file. In reality, this depends on the average certificate price during the corresponding year. This dynamic would also have to be included in the model for it to correctly capture the market features.

4. *Account for proximity of a control station*
   As a new control station and potential changes in the quotas approaches, the willingness to invest in new production capacity might change. Past market activity and the level of the accumulated reserve might give a clue of whether the quota will be increased or decreased, and this expectation is likely to have an effect on the willingness to invest.

5. *Expand to include the electricity market*
   A possible further expansion of the model could be to incorporate the dynamics of the electricity spot price. This alteration would enable the model to illustrate the relationship between the price of green certificates and the spot and would provide deeper insight to the effect of the policy on the energy market.

6.2 Alternative schemes

The point of view taken in this paper is, as explained above, that of a representative producer of renewable electricity in a market free of interactions and elasticity[55], who because of his position is the only supplier of green certificates in the market. This position gives the producer the ability to control whether the set quota is reached, which of course is a highly unrealistic setting. In this situation, the producer will at all times adjust its production in order to supply a number of certificates just below/approaching the requirement. This section will present some suggestions of changes to the model which will make it more adaptable to reality.

Oligopoly model One way to make the model more realistic is to include additional certificate suppliers[56] who all aim to maximize their total profit of selling certificates. However, this situation produces the need for considering competitors' supply and the risk of the total supply of certificates resulting in over- or underproduction in the total market. The issue of optimizing one market actor's decision subject to the market conditions would in this case bring an uncertainty factor connected to the actions of the other actors. This approach should be handled with care, as it could raise the need of an extensive amount of assumptions and additional parameters.

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[54] Even after the need for set non-anticipativity constraints has been avoided by the node approach discussed in section 4.1.

[55] That is, the producer is the only power producer within the GC system.

[56] Power producers.

Welfare Maximizing Model Another possible approach is to model the market by utilizing a social planner aiming to maximize social welfare by reaching the requirement for written certificates while minimizing cost. However, the intuition is that this approach would present quite similar results as the model already presented for the same combination of possible scenarios, as the social planner would be likely to install the capacity in each period which results in an expected supply of certificates just above the set requirement.

6.3 Other Uncertainty Factors

In the presented approach we have based our arguments on the market uncertainty being portrayed mainly in the efficiency factor C_t. However, in the real world, many other factors can bring uncertainty into the model. Some examples can be;

1. *Cost of capital*
   The interest levels are connected to a certain degree of uncertainty and can greatly affect the willingness to invest.

2. *Future tax changes*
   At this point, tax levels for power companies in the GC system are acceptable, even though they are less favorable for Norwegian companies than for Swedish (THEMA consulting group for Energi Norge (2012)). However, changes in the tax-regime might have tremendous effects on the market dynamics. We can already see that the beneficial tax levels in Sweden push more of the investments within the system to that side of the border (THEMA consulting group for Energi Norge (2012)). Tax levels, and their possible development, are also an important factor investors need to consider when making an investment decision. Investment considerations are made on the basis of price expectancy, and the expected price of both electricity and certificates is affected by the tax level.

3. *Cost of development in new technology*
   The cost incurred by increasing production capacity can vary with seasons, with the price of materials and with the general state of the economy. As the GC scheme is intended to incentivize exactly such investment, the cost of new technology can represent a very important source of risk.

6.4 LP versus SMART-SREC

In the SMART-SREC model presented by Coulon *et al.* (2013) the price formulation is modeled in a backward recursion dynamic programming approach. A problem with this method is, as mentioned, the scale of the state space when facing problems of practical scale as the size of the state space grows exponentially with the number of state variables (de Farias & Van Roy (2003)). Linear programming might provide a more efficient alternative in solving these problems (Büyüktahtakin (2011)). Another strength to the LP-model is its ability to determine the installed capacity according to the investor's optimization problem, as to the SMART-SREC model where the capacity is assumed to follow a given function representing the increase of the capacity with respect to the GC prices.

7 Conclusion

This paper has been created with the intention of contributing to the already extensive literature on the area of green certificate markets and policies. By introducing a new approach to simulate the green certificate market mechanisms and indicate the interdependency of the market factors, we aim to make a foundation for further explorations of the applicability of this method. We want to urge further investigations of the model's ability to provide insight to the properties of the market, and the effect of potential changes in these. In this context an initial model based on the fundamental presumption that the certificate price dynamics contains a martingale property is presented.

The solution is based on the specifications of the newly launched Swedish-Norwegian green certificate market, and hence we also wish to facilitate increased insight in the workings of this market and its distinctive features. To achieve this, the first contribution of the paper is a quite detailed description of the market and its most important properties and the internal dynamics between the two actors in the market.

Our approach uses mixed integer linear programming in the context of replicating the market's evolution. It works from the foundational assumption that the certificate price at the compliance or elimination date has to represent the penalty paid by the distributor, if the producer fails to supply the required amount of certificates, or zero, in the case of the producer flooding the market with more certificates than required. By implementing a highly preliminary representative of this philosophy, we have shown its ability to produce results in line with some fundamental intuitions. By this we have taken the initial step in validating the method as an effective and applicable tool in the task of understanding the dynamics of the green certificate market.

In addition to relevant expansion possibilities, we present alternative and supplementary schemes which could be relevant for improving the applicability of the approach both by replacing characteristics and expanding it to incorporate more of the distinctive features of the market.

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Appendices

A Electricity Spot Prices in Germany

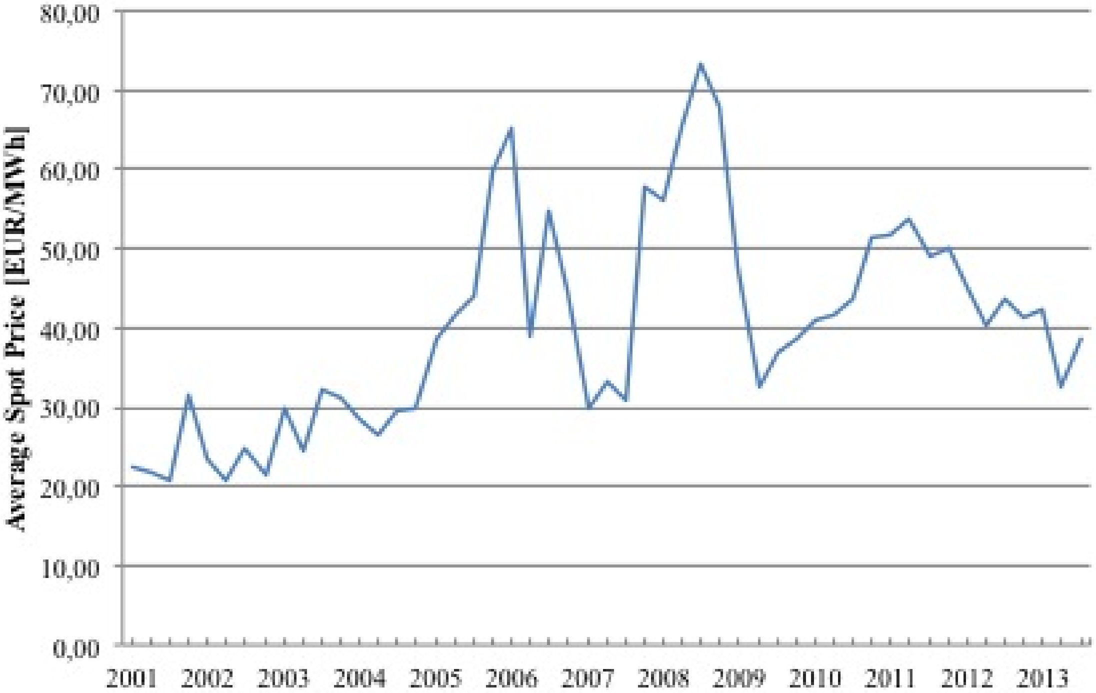
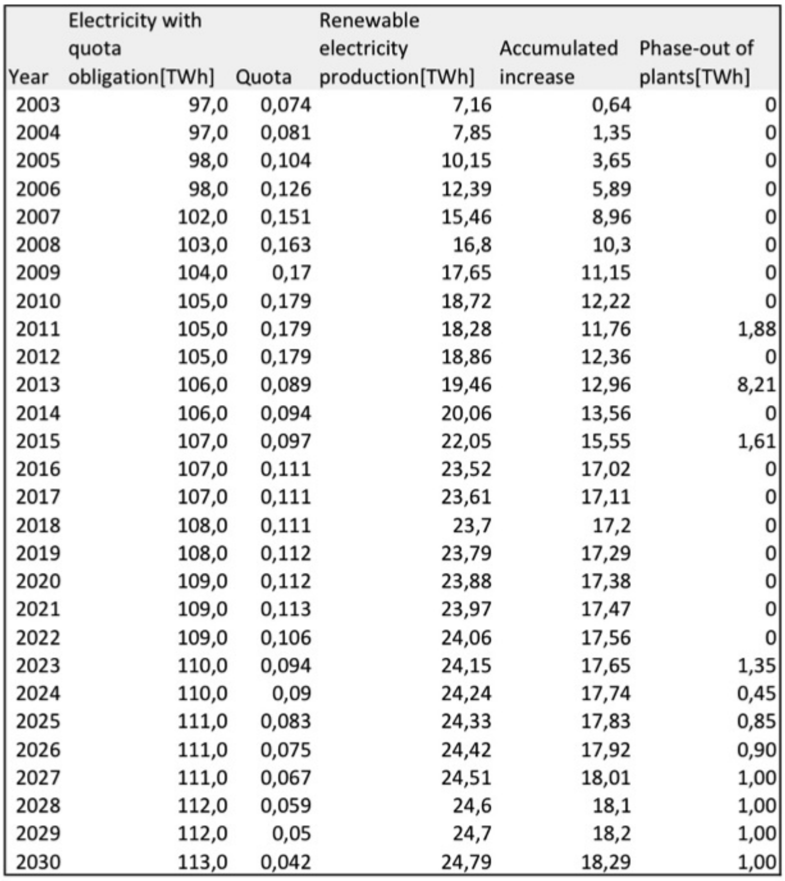


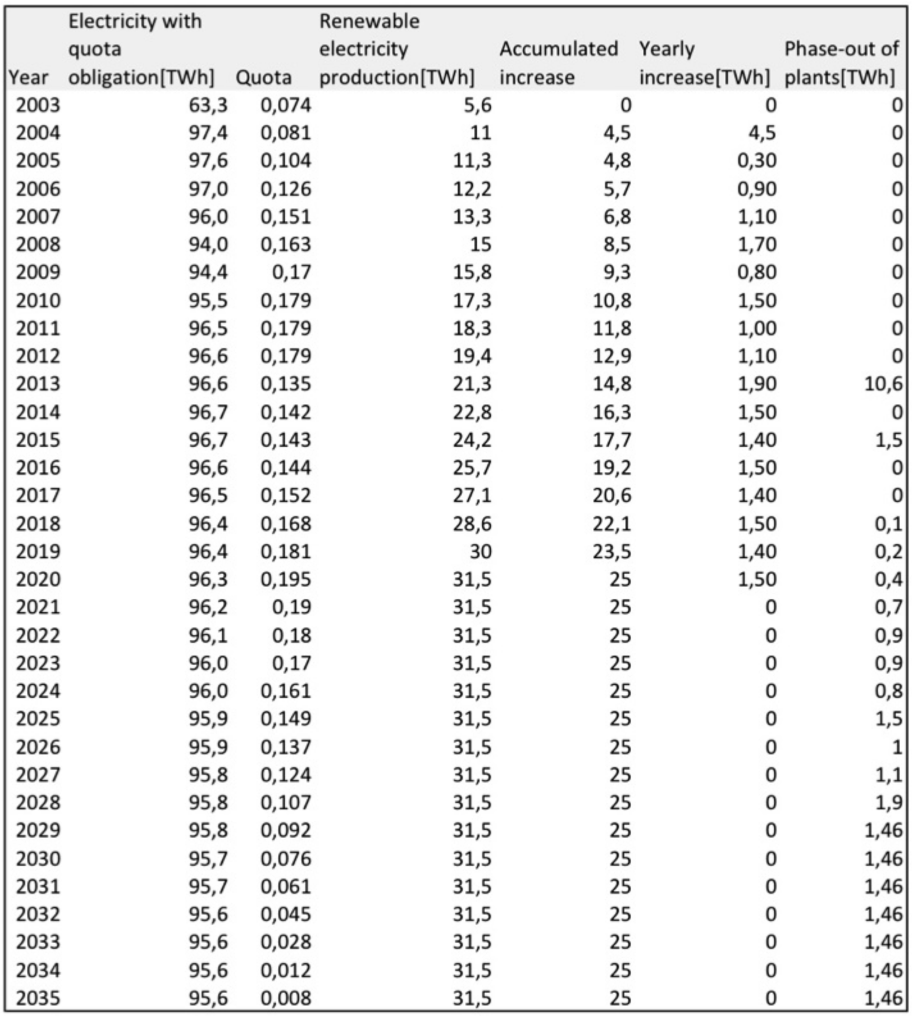
Figure 17: Average quarterly electricity spot prices Germany 2001–2013

B Yearly quotas for Sweden and Norway



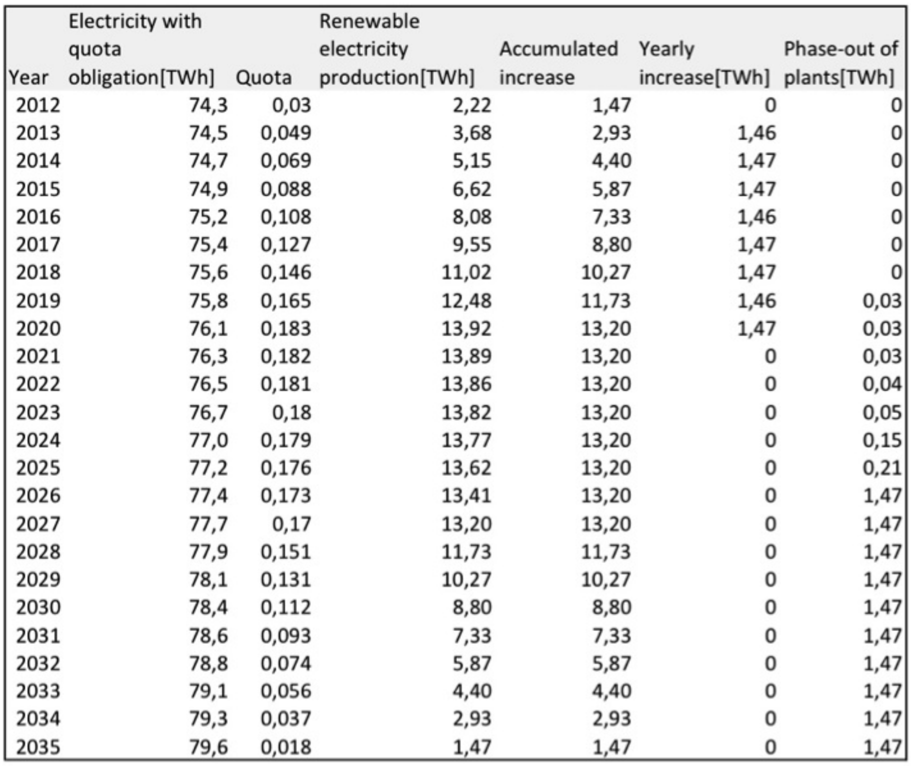
Year	Electricity with quota obligation[TWh]	Quota	Renewable electricity production[TWh]	Accumulated increase	Phase-out of plants[TWh]
2003	97,0	0,074	7,16	0,64	0
2004	97,0	0,081	7,85	1,35	0
2005	98,0	0,104	10,15	3,65	0
2006	98,0	0,126	12,39	5,89	0
2007	102,0	0,151	15,46	8,96	0
2008	103,0	0,163	16,8	10,3	0
2009	104,0	0,17	17,65	11,15	0
2010	105,0	0,179	18,72	12,22	0
2011	105,0	0,179	18,28	11,76	1,88
2012	105,0	0,179	18,86	12,36	0
2013	106,0	0,089	19,46	12,96	8,21
2014	106,0	0,094	20,06	13,56	0
2015	107,0	0,097	22,05	15,55	1,61
2016	107,0	0,111	23,52	17,02	0
2017	107,0	0,111	23,61	17,11	0
2018	108,0	0,111	23,7	17,2	0
2019	108,0	0,112	23,79	17,29	0
2020	109,0	0,112	23,88	17,38	0
2021	109,0	0,113	23,97	17,47	0
2022	109,0	0,106	24,06	17,56	0
2023	110,0	0,094	24,15	17,65	1,35
2024	110,0	0,09	24,24	17,74	0,45
2025	111,0	0,083	24,33	17,83	0,85
2026	111,0	0,075	24,42	17,92	0,90
2027	111,0	0,067	24,51	18,01	1,00
2028	112,0	0,059	24,6	18,1	1,00
2029	112,0	0,05	24,7	18,2	1,00
2030	113,0	0,042	24,79	18,29	1,00

Figure 18: First set of quotas in Sweden



Year	Electricity with quota obligation[TWh]	Quota	Renewable electricity production[TWh]	Accumulated increase	Yearly increase[TWh]	Phase-out of plants[TWh]
2003	63,3	0,074	5,6	0	0	0
2004	97,4	0,081	11	4,5	4,5	0
2005	97,6	0,104	11,3	4,8	0,30	0
2006	97,0	0,126	12,2	5,7	0,90	0
2007	96,0	0,151	13,3	6,8	1,10	0
2008	94,0	0,163	15	8,5	1,70	0
2009	94,4	0,17	15,8	9,3	0,80	0
2010	95,5	0,179	17,3	10,8	1,50	0
2011	96,5	0,179	18,3	11,8	1,00	0
2012	96,6	0,179	19,4	12,9	1,10	0
2013	96,6	0,135	21,3	14,8	1,90	10,6
2014	96,7	0,142	22,8	16,3	1,50	0
2015	96,7	0,143	24,2	17,7	1,40	1,5
2016	96,6	0,144	25,7	19,2	1,50	0
2017	96,5	0,152	27,1	20,6	1,40	0
2018	96,4	0,168	28,6	22,1	1,50	0,1
2019	96,4	0,181	30	23,5	1,40	0,2
2020	96,3	0,195	31,5	25	1,50	0,4
2021	96,2	0,19	31,5	25	0	0,7
2022	96,1	0,18	31,5	25	0	0,9
2023	96,0	0,17	31,5	25	0	0,9
2024	96,0	0,161	31,5	25	0	0,8
2025	95,9	0,149	31,5	25	0	1,5
2026	95,9	0,137	31,5	25	0	1
2027	95,8	0,124	31,5	25	0	1,1
2028	95,8	0,107	31,5	25	0	1,9
2029	95,8	0,092	31,5	25	0	1,46
2030	95,7	0,076	31,5	25	0	1,46
2031	95,7	0,061	31,5	25	0	1,46
2032	95,6	0,045	31,5	25	0	1,46
2033	95,6	0,028	31,5	25	0	1,46
2034	95,6	0,012	31,5	25	0	1,46
2035	95,6	0,008	31,5	25	0	1,46

Figure 19: Second set of quotas in Sweden



Year	Electricity with quota obligation[TWh]	Quota	Renewable electricity production[TWh]	Accumulated increase	Yearly increase[TWh]	Phase-out of plants[TWh]
2012	74,3	0,03	2,22	1,47	0	0
2013	74,5	0,049	3,68	2,93	1,46	0
2014	74,7	0,069	5,15	4,40	1,47	0
2015	74,9	0,088	6,62	5,87	1,47	0
2016	75,2	0,108	8,08	7,33	1,46	0
2017	75,4	0,127	9,55	8,80	1,47	0
2018	75,6	0,146	11,02	10,27	1,47	0
2019	75,8	0,165	12,48	11,73	1,46	0,03
2020	76,1	0,183	13,92	13,20	1,47	0,03
2021	76,3	0,182	13,89	13,20	0	0,03
2022	76,5	0,181	13,86	13,20	0	0,04
2023	76,7	0,18	13,82	13,20	0	0,05
2024	77,0	0,179	13,77	13,20	0	0,15
2025	77,2	0,176	13,62	13,20	0	0,21
2026	77,4	0,173	13,41	13,20	0	1,47
2027	77,7	0,17	13,20	13,20	0	1,47
2028	77,9	0,151	11,73	11,73	0	1,47
2029	78,1	0,131	10,27	10,27	0	1,47
2030	78,4	0,112	8,80	8,80	0	1,47
2031	78,6	0,093	7,33	7,33	0	1,47
2032	78,8	0,074	5,87	5,87	0	1,47
2033	79,1	0,056	4,40	4,40	0	1,47
2034	79,3	0,037	2,93	2,93	0	1,47
2035	79,6	0,018	1,47	1,47	0	1,47

Figure 20: Quotas in Norway

C Certificate issuance, elimination and surplus

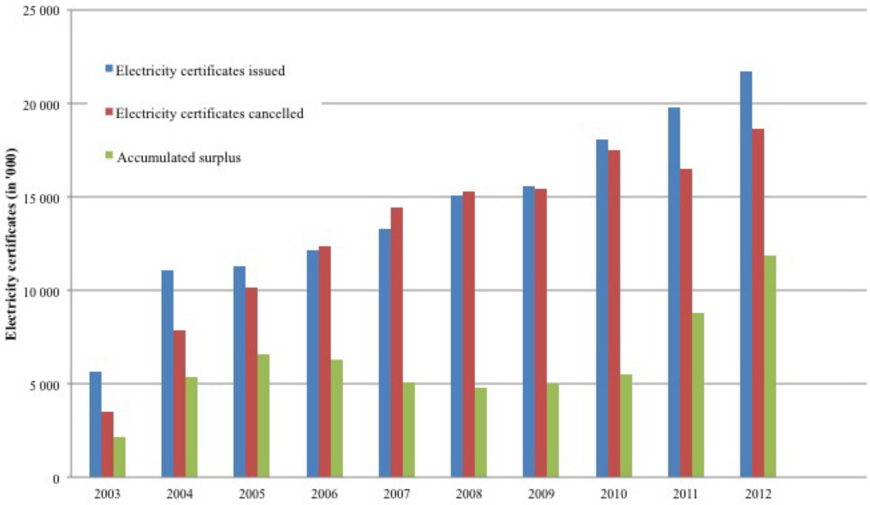


Figure 21: Electricity certificates issued, cancelled and the accumulated surplus during 2003–2012

D Average GC price

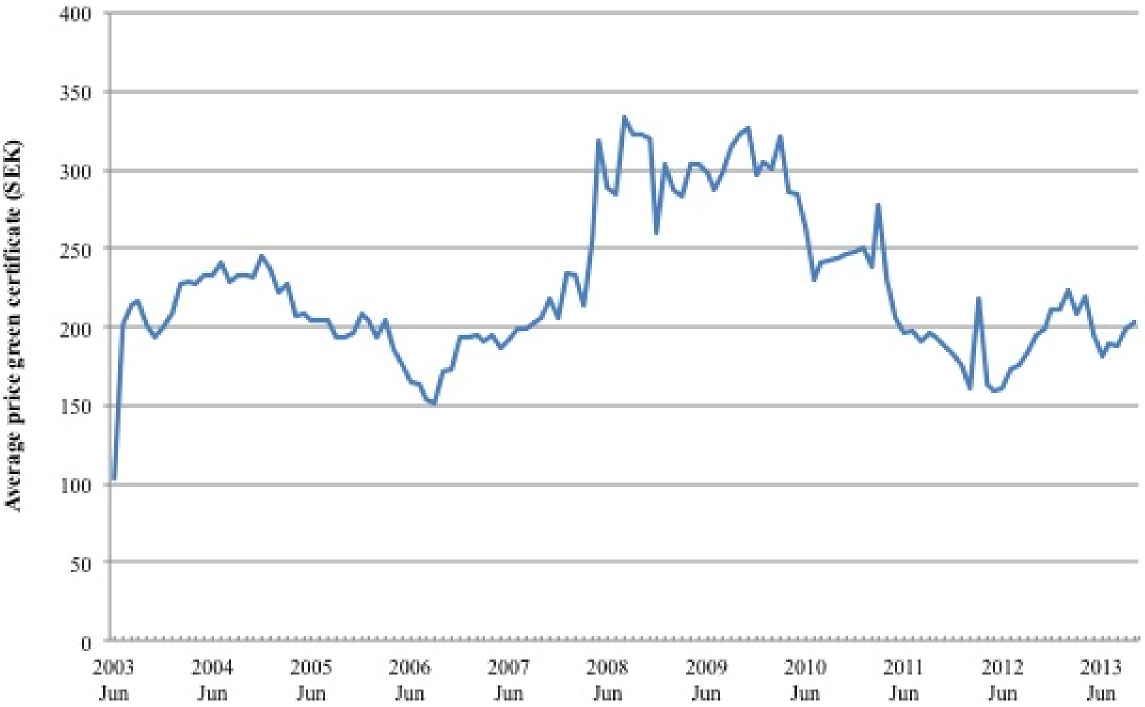


Figure 22: Average certificte price in SEK from 2003–2013

E Installed Capasity

Installed Capacity	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Total	4049	4161	4471	4765	5066	5123	5935	6674	7271	8296

Table 5: Total installed renewable energy capacity [MW] in Sweden from 2003–2012.

F Implemented model and input data

F.1 Mosel code

```
model GCmodel
uses "mmxprs"; !gain access to the Xpress-Optimizer solver

options explterm
    ! This option means that all lines must end with a ;
options noimplicit

uses "mmxprs"; !gain access to the Xpress-Optimizer solver


parameters
Datafile = 'GCmodelDataMWh.txt';
end-parameters

setparam("XPRS_MIPRELSTOP", 0.0002);

!sample declarations section
declarations
  nScenarios:        integer;
  nNodes:            integer;
end-declarations

initializations from Datafile
  nScenarios;
  nNodes;
end-initializations

declarations
  Scenarios:   set of integer;
  Nodes:  set of integer;
end-declarations

Scenarios := 1 .. nScenarios;
Nodes  :=1 .. nNodes;


finalize (Scenarios);
finalize (Nodes);

declarations
  Penalty:              real;
  ProbSubScenario:      array(Nodes, Scenarios)     of real; !pi
  RequiredQuota:        real;
  BigEnoughNumber:      integer;
  Gmax:                 real;
  GenerationFactor:     array(Nodes)  of  real;   !Noted as C_ts in model.
  SubSet:               dynamic array(Nodes)       of set of integer;
  Growth:               dynamic array(Nodes)       of integer;
  SmallNumber:          real;
  Cost:                 integer;
  ObjectiveProb:        dynamic array(Scenarios, Nodes)     of real;
  InitialCapacity:      real;
  Discount:             real;
  T:                    real;
  DiscountStage:        dynamic array(Nodes)       of integer;
end-declarations

initializations from Datafile
  Penalty;
  ProbSubScenario;
  RequiredQuota;
  BigEnoughNumber;
  Gmax;
  GenerationFactor;
  SubSet;
  Growth;
  SmallNumber;
  Cost;
  ObjectiveProb;
  InitialCapacity;
  Discount;
  T;
  DiscountStage;
end-initializations

!Variables
declarations
  installedCap:    dynamic array(Nodes)              of mpvar;  !g_hat_ts
  priceGC:         dynamic array(Nodes)              of mpvar;
  generatedGC:     dynamic array(Nodes)              of mpvar;  !g_ts

  auxiliaryVarZ:   dynamic array(Nodes, Scenarios)   of mpvar;  !z_tss'
```

```
  binaryS:          dynamic array(Scenarios)          of mpvar;
end-declarations

forall (nn in Nodes) do
create(installedCap(nn));
end-do

forall (nn in Nodes | nn=1) do
installedCap(nn)=InitialCapacity;
end-do


forall (nn in Nodes) do
create(priceGC(nn));
end-do

forall (nn in Nodes) do
create(generatedGC(nn));
end-do


forall (nn in Nodes, ss in Scenarios) do
create(auxiliaryVarZ(nn, ss));
end-do


forall (ss in Scenarios) do
create(binaryS(ss));
end-do

forall (ss in Scenarios) do
binaryS(ss) is_binary;
end-do

!Constraints
declarations
  TotalRevenue:             linctr;
  GeneratingCon:            dynamic array(Nodes)                of linctr;
  PriceCon:                 dynamic array(Nodes)                of linctr;
  UnderProductionCon:       dynamic array(Scenarios)            of linctr;
  OverProductionCon:        dynamic array(Scenarios)            of linctr;
  PosGrowthCapCon:          dynamic array(Nodes)                of linctr;
  AuxVarzCon:               dynamic array(Nodes, Scenarios)     of linctr;
  AuxiliaryCon1:            dynamic array(Nodes, Scenarios)     of linctr;
  AuxiliaryCon2:            dynamic array(Nodes, Scenarios)     of linctr;

end-declarations

TotalRevenue :=
sum(ss in Scenarios, nn in Nodes)
      (exp(-Discount*(DiscountStage(nn)/T))*auxiliaryVarZ(nn,ss)
      *ObjectiveProb(ss,nn)*Penalty)
- sum(ss in Scenarios, ii in Nodes)
      exp(-Discount*(DiscountStage(ii)/T))*Cost*ObjectiveProb(ss,ii)
                     *(installedCap(ii)-installedCap(Growth(ii)));

forall(nn in Nodes) do
   GeneratingCon(nn) :=
       generatedGC(nn) = installedCap(nn)*GenerationFactor(nn);
end-do

forall(nn in Nodes) do
   PriceCon(nn) :=
      priceGC(nn) =  exp(-Discount*(DiscountStage(nn)/T))*Penalty
      *sum(ss in Scenarios)(binaryS(ss)*ProbSubScenario(nn,ss));
end-do

forall(ss in Scenarios) do
   UnderProductionCon(ss) :=
      RequiredQuota- sum(nn in Nodes| ss in SubSet(nn))(generatedGC(nn))
                        - BigEnoughNumber*binaryS(ss)<= SmallNumber;
end-do

forall(ss in Scenarios) do
   OverProductionCon(ss) :=
      sum(nn in Nodes | ss in SubSet(nn))(generatedGC(nn)) - RequiredQuota
                                - BigEnoughNumber*(1-binaryS(ss))<= 0;
end-do

forall(nn in Nodes ) do
  PosGrowthCapCon(nn) :=
      installedCap(nn)>= installedCap(Growth(nn));
end-do
```

```
forall(nn in Nodes, ss in Scenarios | ss in SubSet(nn)) do
   AuxVarzCon(nn, ss)  :=
      (auxiliaryVarZ(nn,ss))- Gmax*binaryS(ss) <= 0;
end-do

forall(nn in Nodes, ss in Scenarios| ss in SubSet(nn)) do
   AuxiliaryCon1(nn,ss)  :=
       generatedGC(nn)-auxiliaryVarZ(nn,ss) + Gmax*binaryS(ss) <= Gmax;
end-do

forall(nn in Nodes, ss in Scenarios | ss in SubSet(nn)) do
   AuxiliaryCon2(nn,ss)  :=
      (auxiliaryVarZ(nn,ss)) - generatedGC(nn)  <= 0;
end-do

maximize(TotalRevenue);

fopen("Results.txt",F_OUTPUT);


writeln;
writeln('Optimal objective value: ',getobjval);


forall(nn in Nodes) do

writeln('Node', nn);
    writeln(strfmt(getsol(installedCap(nn)),4),
          ' is the total installed capacity ');

    writeln(strfmt(getsol(generatedGC(nn)),4), ' is the additional
          amount of certificates generated ');

    writeln(strfmt(getsol(priceGC(nn)),4), ' is the price of a GC  ');


    writeln('---------------------------------------');
end-do

forall(ss in Scenarios) do

writeln(' Scenario: ', ss);

writeln(strfmt(getsol(binaryS(ss)),4), ' is the binary variable  ');

writeln(strfmt(getsol(sum(nn in Nodes | ss in SubSet(nn))generatedGC(nn)),4),
       ' is the total generated GC  ');

writeln('----------------------------------------');

end-do


fclose(F_OUTPUT);

end-model
```

F.2 Data file

```
! This is the data file to the GCmodel model with dimentions in MWh

nNodes            :    7
nScenarios        :    4

T: 2



ProbSubScenario: [

    0.09    0.36   0.385   0.165

    0.2     0.8      0      0

    0       0       0.7    0.3

    1      0       0       0

    0      1       0       0

    0     0        1       0

    0     0        0       1
 ]



ObjectiveProb: [

    0   0.45   0    0.09   0     0    0

    0    0     0    0     0.36   0   0

    0    0     0.55  0    0     0.385   0

    0    0      0  0   0     0      0.165

]

Penalty: 400

Discount: 0.05

InitialCapacity: 5000        !Initial capacity 2010 were around 6000,
                             !but this includes GC for banking.

RequiredQuota:  20000000

BigEnoughNumber: 30000000

SmallNumber: 0.01

Gmax        : 50000000

Cost: 600000               !$ 1 000 000 per ectra installed MW. Since we calculate the profit
                           !of only one year, we distribute the investment over 10 year.

!Normal conditions:
GenerationFactor:  [ 0   1463   1372   1296  1250  1159  1114 ]

!High (+ 700 hours in every  scenario at each step)
!GenerationFactor:  [ 0   2163   2072   1996  1950  1859  1814 ]

!Low (-600 hours in every scenario at each step)
!GenerationFactor:  [ 0   863   772   696  650  559  414 ]


SubSet :  [ (1) [1 2 3 4 ]  (2)  [1 2 ]  (3)  [3 4]  (4) 1   (5) 2  (6) 3   (7) 4 ]

Growth:   [ 1  1  1   2   2  3   3 ]

DiscountStage : [ (1) 2 (2) 1 (3) 1  (4) 0  (5) 0  (6)  0   (7)  0 ]
```

F.3 Input Data

In order to run the preliminary model described in section 4.1, some external input is needed to construct the solution. The main inputs to the code are the probability distribution, π_{ns}, the estimated efficiency factor C_n, and the contribution-probabilities ρ_{ns} which are given in the matrices and arrays named ProbSubScenario, GenerationFactor and ObjectiveProb. We have utilized two different matrices for the probability, both based on the same probabilities, but with different configuration in order to facilitate the summations. These probabilities are set based on the discretization of C_{ts} which leads to discretized probabilities corresponding to each outcome[57]. The ProbSubScenario-matrix is a [nxs]=[7x4]-matrix where each row represents a unique node in the tree and each column represents a leaf-node. The number displayed at place $[n, s]$ in the matrix shows the probability of ending up in end-scenario s, given that the present position is node n. These data were set by the assumption that the probability of reaching the mid-leaf nodes, nodes 5 and 6, is a bit larger than reaching the more extreme cases, nodes 4 and 7.

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{14} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{24} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{71} & \pi_{72} & \cdots & \pi_{74} \end{bmatrix}$$

Figure 23: ProbSubScenario matrix

The second important piece of input is the GenerationFactor array illustrated in figure 24. This array has a length=n and gives the C_t at each node, denoted in [h]. The values in this array is simulated using the procedure explained above.

$$\begin{bmatrix} C_1 & C_2 & \ldots & C_7 \end{bmatrix}$$

Figure 24: GenerationFactor

The final input needed in the datafile is the [4x7] ObjectiveProb matrix. This matrix is used in the objective function in the calculation of the total expected revenue. If the period leading *to* node n from its parent node in the path leading to leaf node s represents a potential contribution to the expected income, ρ_{ns} represents the probability with which that contribution will be received, seen from time 0.

$$\begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{17} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{27} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{41} & \rho_{42} & \cdots & \rho_{47} \end{bmatrix}$$

Figure 25: ObjectiveProb matrix

---

[57] In reality each node have an infinite amount of outcomes leading to a continuous probability distribution in each node.

F.4 Output Implemented Model

Normal C Output values in the normal case scenario:

```
Optimal objective value: 6.29623e+009
Node1
5000 is the total installed capacity
   0 is the additional amount of certificates generated
380.492 is the price of a GC
----------------------------------------------------------------------------------
Node2
7249 is the total installed capacity
1.06053e+007 is the additional amount of certificates generated
390.124 is the price of a GC
----------------------------------------------------------------------------------
Node3
7902.02 is the total installed capacity
1.08416e+007 is the additional amount of certificates generated
390.124 is the price of a GC
----------------------------------------------------------------------------------
Node4
7249 is the total installed capacity
9.39471e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
Node5
7249 is the total installed capacity
9.06125e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
Node6
7902.02 is the total installed capacity
9.15844e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
Node7
7902.02 is the total installed capacity
8.80284e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
 Scenario: 1
   1 is the binary variable
2e+007 is the total generated GC
----------------------------------------------------------------------------------
 Scenario: 2
   1 is the binary variable
1.96665e+007 is the total generated GC
----------------------------------------------------------------------------------
 Scenario: 3
   1 is the binary variable
```

```
2e+007 is the total generated GC
----------------------------------------------------------------------------------
 Scenario: 4
   1 is the binary variable
1.96444e+007 is the total generated GC
----------------------------------------------------------------------------------
```

High C Output values in the high case scenario:

```
Optimal objective value: 4.302e+009
Node1
5000 is the total installed capacity
   0 is the additional amount of certificates generated
209.27 is the price of a GC
----------------------------------------------------------------------------------
Node2
5000 is the total installed capacity
1.0815e+007 is the additional amount of certificates generated
   0 is the price of a GC
----------------------------------------------------------------------------------
Node3
5087.76 is the total installed capacity
1.05418e+007 is the additional amount of certificates generated
390.124 is the price of a GC
----------------------------------------------------------------------------------
Node4
5000 is the total installed capacity
9.98e+006 is the additional amount of certificates generated
   0 is the price of a GC
----------------------------------------------------------------------------------
Node5
5000 is the total installed capacity
9.75e+006 is the additional amount of certificates generated
   0 is the price of a GC
----------------------------------------------------------------------------------
Node6
5087.76 is the total installed capacity
9.45815e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
Node7
5213.98 is the total installed capacity
9.45815e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
 Scenario: 1
  -0 is the binary variable
2.0795e+007 is the total generated GC
----------------------------------------------------------------------------------
```

```
 Scenario: 2
  -0 is the binary variable
2.0565e+007 is the total generated GC
----------------------------------------------------------------------------------
 Scenario: 3
   1 is the binary variable
2e+007 is the total generated GC
----------------------------------------------------------------------------------
 Scenario: 4
   1 is the binary variable
2e+007 is the total generated GC
----------------------------------------------------------------------------------
```

Low C Output values in the low c scenario:

```
Optimal objective value: 2.79953e+009
Node1
5000 is the total installed capacity
   0 is the additional amount of certificates generated
380.492 is the price of a GC
----------------------------------------------------------------------------------
Node2
12828.7 is the total installed capacity
1.10712e+007 is the additional amount of certificates generated
390.124 is the price of a GC
----------------------------------------------------------------------------------
Node3
5000 is the total installed capacity
3.86e+006 is the additional amount of certificates generated
390.124 is the price of a GC
----------------------------------------------------------------------------------
Node4
12828.7 is the total installed capacity
8.9288e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
Node5
12828.7 is the total installed capacity
8.33868e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
Node6
5000 is the total installed capacity
2.795e+006 is the additional amount of certificates generated
 400 is the price of a GC
----------------------------------------------------------------------------------
Node7
5000 is the total installed capacity
2.07e+006 is the additional amount of certificates generated
```

```
 400 is the price of a GC
--------------------------------------------------------------------------------
 Scenario: 1
   1 is the binary variable
2e+007 is the total generated GC
--------------------------------------------------------------------------------
 Scenario: 2
   1 is the binary variable
1.94099e+007 is the total generated GC
--------------------------------------------------------------------------------
 Scenario: 3
   1 is the binary variable
6.655e+006 is the total generated GC
--------------------------------------------------------------------------------
 Scenario: 4
   1 is the binary variable
5.93e+006 is the total generated GC
--------------------------------------------------------------------------------
```