Master Thesis

Pricing and Hedging Electricity Swing Contracts

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Preface

This master thesis was performed during the spring of 2007 at the Norwegian University of Science and Technology, NTNU, Department of Industrial Economics and Technology Management, section of Accounting and Finance.

This thesis is written in LaTeX with TeXnicCenter as user interface. The modeling and optimization have been implemented and executed in Xpress-IVE.

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Abstract

In this master thesis, an efficient and flexible model for pricing and hedging swing contracts, in an incomplete market, is presented. The model supports an hourly withdrawal strategy and includes all available future and forward contracts in the market. The model is a two stage stochastic optimization model, constructed to price and hedge simultaneously. It uses exogenous spot prices and swap contracts, and the scope is limited for one kind of swing contracts; flexible load contracts.

The model is a good foundation to develop into a multi stage stochastic model. The models flexibility makes it possible to use a time resolution of blocks of hours, in order to value and hedge other swing contracts, and use different price models for the spot and swap prices. A block resolution of small number of hours is preferred, compared to an hourly resolution, due to a large decrease in computation time at the expense of a minimal loss of value. However, a daily or larger resolution of the withdrawal schedule and spot price will drastically reduce the value of the optimization.

Because of the incompleteness in the electricity market, the market player's risk aversion will influence the hedging and withdrawal strategy, and therefore also the value of the contract. We look at several volume risk functions and a CVaR profit risk function, and we optimize a flexible load contract by creating a static hedge. A utility function minimizing the profit risk, results in a better reduction of downside risk in an optimization compared to a hedge created by a volume risk aversion model.

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Chapter 1

Introduction

Due to the many different contracts existing in the Nordic electricity market, risk management is an important aspect for market players. Since the deregulation of the Nordic power market, the need for methods to minimize risk has increased and therefore several tools and methods have been developed, both for the standardized contracts traded through Nord Pool and the contracts traded over the counter. To value equity derivatives the model presented by Black & Scholes(1973) is used, and Black(1976) developed a valuation formula for commodities. By reason of the unique characteristics of the electricity spot price, there is to date no consensus in a valuation model for electricity derivatives in the literature or in the industry.

An important group of contracts traded over the counter are swing contracts. A swing contract is a contract which gives the buyer the right to purchase electricity at a fixed price within a period of time. The buyer can choose when to purchase the electricity within a set of restrictions written in the contract. The degree of possibility to swing the load during the contract period is known as the flexibility of the contract. By increasing the flexibility, the value of the contract is augmented, but the valuation and exercise schemes are made more difficult. Swing contracts, also known as virtual power plants (VPP), are difficult to manage owing to the multiple exercise decisions. Regarding the valuation, there are thousands of different combinations[1] to exercise these types of contracts; and the most optimal strategy for exercising might change when including risk management. Due to the large number of different swing contracts, we will look at one specific type of contract. The objective of this master thesis is to develop a model for pricing and hedging of a flexible load contract. A flexible load contract is a type of swing contract traded over the counter.

This master thesis is motivated by the work done by Mo & Gjelsvik(2002). They developed a model that optimizes the withdrawal from a flexible load contract and hedged the contract based on profit risk. The aforementioned model is a development of a model

[1]Number of combinations depend on the length and flexibility of the contract

Mo, Gjelsvik & Grunt(2001) proposed for scheduling and contract management in a hydrothermal system.

The model presented in this thesis will make use of flexible load contracts with the duration of one month and six months, with different types of constraints. The contracts will be hedged against all available forwards and futures. Furthermore, the time resolution will be both hourly and in blocks of 6 hours. The reason for partitioning time resolution is the computation time of the model, and we will compare the results from the two resolutions. Moreover our model will hedge against volume risk or conditional value at risk of low profit.

Keppo(2002) claims in his paper that hedging swing contracts is possible. This makes it possible to value a swing option as long as there exists an option market. The main problem with this method is the time resolution. We have to assume that the forward market has an equal or higher resolution than the flexible load contract in order for this to work. Lund & Ollmar(2003) use an hourly resolution, in their complex stochastic optimization problem, to formulate and then manage the flexible load contract numerically. Their results indicate that *their* model performed better than some market players. Bjerksund, Myksvoll & Stensland(2006) on the other hand look at two simple strategies to manage a flexible load contract; the first is a deterministic strategy and the second is a stochastic strategy. They compare their results with the result from a more complex dynamic approach and the comparison shows that their models perform better than the alternatives on average. But their assumption about a frictionless market with no risk premium violates the characteristics of an incomplete market and they use constructed hourly forward prices. Haarbrücker & Kuhn (2006) use a multistage stochastic program to value swing options by aggregation of decision stages, discretization of the probability space, and reparameterization of the decision space.

As we do not assume the market to be complete, we encounter challenges as the non-existent possibility of a perfect hedging portfolio by market traded instruments. Particularly the volume risk is not traded in the electricity markets. Oum, Oren & Deng(2005) look at volumetric hedging in electricity procurement for load serving entities and reach a hedging strategy, with the use of a utility function, that can be implemented through a portfolio of call and put options. On the other hand Keppo, Meng & Sullivan(2006) introduce a fictitious risky asset to the market and by that they can use methods of a complete market to optimize the utility.

Some of the main conclusions from this thesis are the importance of reflecting the hourly variation in spot prices when valuing a flexible load contract. For a monthly flexible load contract with a maximum withdrawal in 400 hours, the loss in value can be as much as 12.63 NOK/MWh for an optimization with daily resolution of spot prices and withdrawal, compared to hourly resolution. We also show the importance of modeling a good utility function, due to the incompleteness of the electricity market. The formulation of the utility function, affects the decision for both withdrawal and hedging, which lead to valuation and risk management of a flexible load contract.

The rest of this thesis is organized as follows: Chapter two describes some basic theory and stylized effects of electricity prices. In chapter three we will describe the model for pricing and hedging. Modeling of uncertainty in the electricity market is then described in chapter four. Data estimations and contract specification are shown in chapter five and chapter six deals with the results from different cases where the model is applied. Concluding remarks and the need for further work are addressed in chapter seven and eight.

Chapter 2

Theory

Valuation of electricity derivatives[1] cannot simply rely on models developed for financial or other commodity markets. That is because of the unique characteristics of the spot price. We will briefly present the stylized effects of electricity prices in this chapter. We will also take a look at the basic theory needed for pricing and hedging a flexible load contract.

2.1 Effects and characteristics of an electricity market

The electricity price is affected by different factors, both directly and indirectly; rain, temperature and fuel prices are important factors since they affect both supply and demand. Other important factors are seasonality, mean reversion, price spikes and extreme volatility (Knittel & Roberts, 2001).

Seasonality is an important aspect in the electricity market. The variations appear over the course of the day, week and year. Accordingly, Lucia & Schwartz(2002) argue that seasonality is one of the most important aspects in the shape of the forward curve. Knittel & Roberts (2001) shows, through empirical studies of the Californian electricity market, that there are strong deterministic cycles within the daily, weekly and yearly effects.

Different from most other commodities markets, the electricity market is extremely volatile. In the period between 1993 and 1999, the volatility of the Nordic spot electricity market was estimated to 189% (Lucia & Schwartz, 2002). It was also proven that the volatility of the spot price was non stationary. Other factors influencing the volatility are seasonality and demand. The volatility is higher during periods of high demand and vice versa (Knittel & Roberts, 2001).

Lund & Ollmar(2003) analyze the Nordic electricity market and find that price spikes or fast mean reversion, due to abnormal load conditions, will have a strong influence

[1] A derivative is a financial instrument that is derived from an underlying asset's value. Instead of trading the specific asset, the market players agree on exchanging money, assets or some other value at a future date, based on the price of the underlying asset

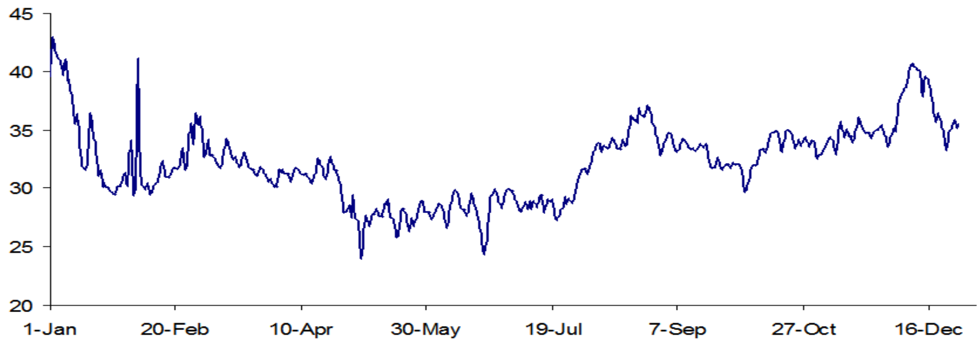

Figure 2.1: Seasonality graph from Nord Pool with average value in EUR for 2001-2006

on the spot price process. Still, the Nordic market usually has a slow mean reversion because of the hydro plant production (Lund & Ollmar, 2003). The non-storability of electricity is one of the most important reasons for the spiky nature of the spot price. The price spikes and extreme volatility in the market also lead to a non-normal and fat tailed distribution of returns for the spot price. Figure 2.2 shows a typical curvature for a fat tailed distribution. Usually, there is a positive skewness in the spot price as well, which means that there is a higher probability of extreme high prices compared to extremely low prices in the electricity market.

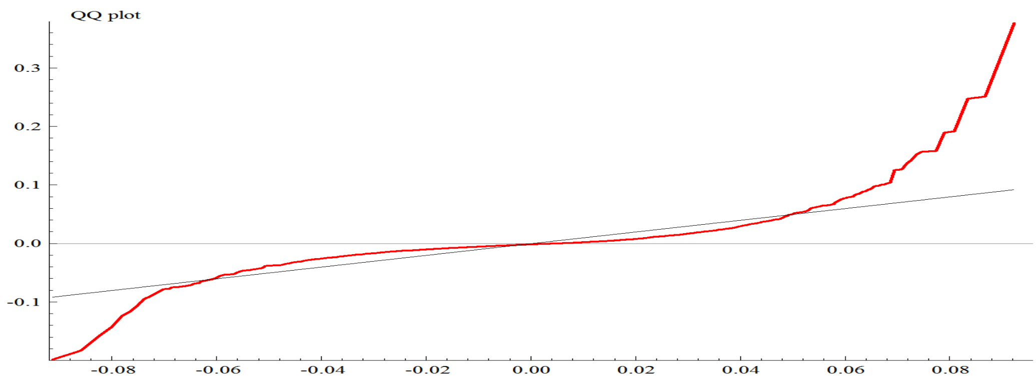

Figure 2.2: Quantile-Quantile-plot of the log-returns of the Nord Pool spot prices from 01.01.2001-31.12.2006 against N(0,1).

2.2 The structure of Nord Pool

The Nordic electricity market is divided into one physical and one financial market. The physical market handles the physical contracts for the next 24 hours and the market delegate present price and supply/demand for each of the hours. The financial market consists of different types of standardized future and forward contracts and some standardized European options. The future contracts are traded for days and weeks, and the forwards contracts are traded for months, quarters and years. The main difference between a future and a forward contract is that the future is handled with mark-to-market[2] and the forward are calculated at the last day of the contract. All of the futures, forwards and European options are standardized contracts(Nord Pool, 2007).

Table 2.1: Future and forward contracts traded at Nord Pool in May 2007

Type	*Duration*	*Contracts available in the market*
Future	day	4 - 9
Future	week	6
Forward	month	6
Forward	quarter	10
Forward	year	5

Benth & Koekebakker(2005) refer to future and forward contracts as swaps. This reference is used because future and forward contracts reflect an exchange between fixed contract price and floating spot prices. Accordingly, we will adopt this concept in this thesis.

There is also an Over-The-Counter(OTC) market in the Nordic electricity exchange. Most OTC contracts are not standardized and can often be of exotic[3] nature. Some of the most common contracts are Contracts for Difference (CfD) and swing contracts[4]. The liquidity in the OTC market is often thin and this might result in higher risks. For standardized contracts in the OTC market, clearing service is provided by Nord Pool. For other contracts the counterparties themselves must take the financial counterparty risk.

One type of a swing contract is called flexible load contract[5]. A flexible load contract can be compared to a hydro power plant without inflow of water, and that must be depleted within a given time frame. This is also known as a Virtual Power Plant (VPP). The buyer of a flexible load contract pays a fixed price for every MWh of withdrawal. The buyer can choose to withdraw power at any hour during the contract period as long as it is within some predefined limits. At contract formation, the exact number of MW available in the lifetime of the contract, and the maximum and minimum withdrawal in one hour, are determined. This arrangement sets the contract limits. The hours chosen for withdrawal must be set at least the day before exercising.

[2] Mark-to-market means recording the spot price on a daily basis, in order to calculate profits/losses against the contract price

[3] An exotic option is a derivative which has features making it more complex than commonly traded products

[4] See Unger(2002) for more details on different types of OTC contracts

[5] These contracts are also referred to as load factor contracts or FLC

2.3 Hedging

Hedging is, in this context, defined as an action which reduces the risk of loss, possibly at the expense of potential profit. There are several ways of hedging; two of the most common are replicating a portfolio and delta hedging. A replicating hedge is to take an identical, but opposite, position and compare it to the position you already have. In delta hedging, the delta for a position with value c is given by:

$$\Delta = \frac{\delta c}{\delta S} \tag{2.1}$$

Delta hedging is a first order Taylor-approximation to the value of the asset. By taking an opposite position in the approximated commodity, it is possible to hedge the commodity. It is also possible to improve this hedge by doing a higher order hedge.[6](McDonald, 2006)

Because of the complexity of most electricity contracts, it is only possible to replicate small parts of the different contracts in the electricity market. This makes a replicating portfolio unsuitable for complex electricity contracts. There are also some problems with using delta hedge in the electricity market. One of them is the non-storability of electricity. You cannot hold an amount of electricity to make the derivatives locally immune against each other. Another problem is to obtain the price of the derivatives. This is because of the unique characteristics of the spot price.(Unger, 2002)

2.4 An incomplete market

Incompleteness is another factor which makes the electricity market even more complex. The reasons for an incompleteness of a market can vary. Stochastic volatility and mixed jump-diffusion price process for an asset (Fedotov & Mihkailov, 1998) are two examples. Another example is the limited available contracts to the possible states for the commodity. This means that it is not possible to hedge a single hour, day or week which is sufficiently far into the future.

A flexible load contract has exercise decisions for each hour during the period of the contract, but hourly forwards or futures for financial trading do not exist. As a result, there are not enough contracts to cover all the hours, and therefore we have an incomplete market for this contract.

A complete market is one requirement that has to be fulfilled in order for a unique equivalent martingale measure to exist. This martingale measure is often referred to as the risk neutral probability measure[7], with which a pricing formula can be derived

[6]For more details about replicating a portfolio, delta hedging and other hedging methods, see McDonald(2006)

[7]Martingale measure or risk neutral probability measure is often referred to as Q in the literature.

without taking the different traders utility functions in to discussion(Constantinides, Jackwerth & Perrakis, 2005).

When the market is incomplete, several different probability measures exist which lead to different prices on a given derivative. The probability measure will depend on the risk attitude of the traders, and this attitude will vary among the different traders. The risk attitude is one factor in the process to derive a risk premium, which also will vary from trader to trader. For each of the risk premiums it will be a risk neutral probability measure. Because of this it will be hard to value something under the risk neutral probability measure. Consequently you have to use the true probability measure[8] to derive a pricing formula in an incomplete market. The true probability measure will often vary because the traders consider the probabilities and consequences of the possible outcomes differently(Constantinides et al, 2005)).

Based on the aforementioned, we know that only one unique risk neutral probability measure does not exist, but several non-unique ones in an incomplete market, and therefore we cannot find an arbitrage-free price for the asset. The non-uniqueness means that it is relevant to use the utility functions of the different traders to derive an acceptable pricing formula.

[8]The true probability measure is often referred to as P in the literature.

Chapter 3

The Model

In this chapter we introduce a two stage stochastic model which is constructed to price and hedge a flexible load contract simultaneously. This model is motivated by Mo & Gjelsvik's paper from 2002, where they propose a stochastic multi stage optimization model for simultaneous withdrawal from a flexible load contract and financial hedging. They find that a simultaneous optimization reduces the uncertainty of the profit of a portfolio(which includes a flexible load contract) and that the risk aversion of the contract holder does not affect the withdrawal strategy measurably, as long as the transaction costs are small.

Mo & Gjelsvik's(2002) proposed model has weekly resolutions for both the spot price and the available swaps in the model. With identical resolution on both spot and swap prices, the optimization is done in a complete market which is not realistic. Another drawback with their model is that with weekly spot prices, the model ignores the daily and hourly variations during the week, and therefore gives an imprecise illustration of the spot market.

We propose a model that has hourly resolution on the spot price and includes all available swap contracts in the market. This result in a more realistic withdrawal schedule and hedging compared to Mo & Gjelsvik(2002). The drawback with hourly resolution on the spot price is computation time due to expansion of dimensions for long lasting flexible load contracts. Our model can be adjusted for blocks of hours in order to reduce the dimension. In order to model hedging decisions, several versions of volume risk aversion models, and a risk aversion model to reduce low profit scenarios, are introduced.

There are some disadvantages to a two stage stochastic model compared to a multi stage stochastic model[1]. In a two stage stochastic model, the first stage is to optimize the problem based on the expectation of the future. The second stage is to optimize when the outcome of the future is know, through different scenarios. In a multi stage stochastic model, new information is gradually available through time. With new information, the

[1]See chapter 8 for one method for deriving spot and swap prices for a multi stage model.

expectations of the future may change creating new decisions(Dyer & Stougie, 2005). On the ground of the previous, our two-stage model will only create a realistic static hedge. The proposed model in this chapter establishes a good foundation to build a multi stage stochastic model, which will lead to realistic discrete dynamic hedging decisions.

3.1 A flexible load contract model

We consider a flexible load contract that lasts for a given number of days or months. To model the maximization of the contract value, we define the following quantities:

s	Index for scenario
i	Hourly index for exercising/withdrawal
I	Last day in the exercising period
Pflc	Price per MW exercised from the FLC (NOK/MWh)
Qmin	Minimum hourly withdrawal from the FLC (MWh)
Qmax	Maximum hourly withdrawal from the FLC (MWh)
Qsum	Total withdrawal from the FLC, in hours (MWh)
$Sp_{(i,s)}$	Spot price for hour i, for scenario s (NOK/MWh)
$q_{(i,s)}$	Amount of MW withdrawn from the FLC at hour i, for scenario s (MWh)

The optimal exercising schedule for a flexible load contract, without using the market, is given by a maximization of profit. The total optimization problem is calculated for each scenario.

$$Max \quad W = E\left[\sum_{i=1}^{I}\left(Sp_{(i,s)} - Pflc\right) q_{(i,s)}\right] \tag{3.1}$$

The flexibility of the flexible load contract is determined through the minimum and maximum withdrawal for each hour, and the total withdrawal in the lifetime of the contract. These properties are modeled with the following restrictions:

$$\begin{aligned} q_{(i,s)} &\geq Qmin \qquad \text{for all } i \text{ and } s \\ q_{(i,s)} &\leq Qmax \qquad \text{for all } i \text{ and } s \\ \sum_{i=1}^{I} q_{(i,s)} &= Qsum \qquad \text{for all } s \end{aligned} \tag{3.2}$$

A flexible load contract is only flexible if the total withdrawal, $Qsum$, is less than $Qmax*I$ and larger than $Qmin*I$.

3.2 Market model

To reduce the uncertainty of the profit of the flexible load contract, we model the swap market in order to set up a simultaneous optimization of the withdrawal schedule, and the financial hedging.[2] The financial swap contracts available in the market are of unequal lengths and are overlapping, which makes a multi stage modeling of the market more complex. Even though our model is only capable of making a realistic static hedge, we model the market with the possibility to make a discrete dynamic hedge. Multiple trading points are modeled by reason of a possible development from a two stage stochastic model into a multi stage stochastic model.

In order to simultaneously optimize the flexible load contract and make a financial hedge with all available contracts in the market, we use these definitions:

t	Index for trading dates
T	Last trading day
k	Index for number of available contracts in the market for the existing period
K	Last contract
l	Index for contracts of different lengths
L	Total number of different contract lengths in the market
Ct	Transaction cost per traded MW (NOK/MWh)
$M_{(k,l)}$	Number of hours in the contract k of length l (h)
$Fs_{(t,k,l,s)}$	Selling price for contract k of length l, at trading time t for scenario s (NOK/MWh)
$Fk_{(t,k,l,s)}$	Buying price for contract k of length l, at trading time t for scenario s (NOK/MWh)
$vs_{(t,k,l,s)}$	Amount of MW sold of contract k of length l, at trading time t for scenario s (MW)
$vk_{(t,k,l,s)}$	Amount of MW bought of contract k of length l, at trading time t for scenario s (MW)

Since the spot price has an hourly resolution, a realistic representation of the market has contracts that are expressed in resolutions as multiples of the hourly spot price. Figure 3.1 illustrates the basic idea of the setup of our model. It displays a simple market with a total of I available hours (i) from which to withdraw, two different swap contract lengths(l), where each contract (k) of length $l = 1$ covers half of the available period, whereas there is one contract of length $l = 2$ that covers the whole period. The setup displays $T = 4$ possible trading points (t) where the model gives the option to buy or sell swap contracts in the market.

[2]By including other derivatives correlated with the electricity price, for instance weather derivatives, one may create an even better hedge(Mount, 2002), but this is beyond the purpose of this thesis.

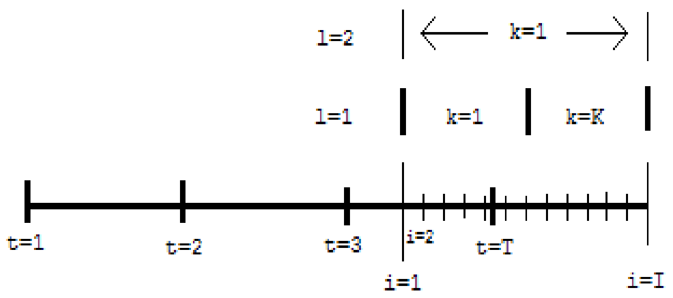

Figure 3.1: Index description for the market modeling

With different resolutions for the expected spot price, $Sp_{(i,s)}$, and the swap contracts in the market, $Fs_{(t,k,l,s)}$ and $Fk_{(t,k,l,s)}$, the respective hourly spot prices have to be matched with each contract of different lengths that include the respective hour. For every swap contract you sell in the market at the price Fs, the profit of the contract is the agreed swap contract price minus the spot price for the contract period. By selling a swap contract for the period the buyer withdraws from a flexible load contract, the buyer reduces the risk of low prices on the hours the buyer expects too withdraw. However the buyer gains a risk of high prices for the hours that are not withdrawn. In order to model the option to buy a swap contract, the price of a swap contract, Fs, is multiplied with the number of hours in the contract $M_{(k,l)}$[3]. The sum is then subtracted with the spot price, Sp, for all hours within the contract. The total value of the contract is then multiplied with the volume sold (vs) of the contract. Equation 3.3 shows the model for selling all possible contracts available at the market:

$$+\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{t=1}^{T}\left(\left(Fs_{(t,k,l,s)} * M_{(k,l)} - \sum_{i=1+\sum_{m=1}^{k-1} M_{(m,l)}}^{\sum_{m=1}^{k} M_{(m,l)}} Sp_{(i,s)}\right) * vs_{(t,k,l,s)}\right) \tag{3.3}$$

With the same method, we model the purchase function for the objective function in our two stage stochastic model:

$$-\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{t=1}^{T}\left(\left(Fk_{(t,k,l,s)} * M_{(k,l)} - \sum_{i=1+\sum_{m=1}^{k-1} M_{(m,l)}}^{\sum_{m=1}^{k} M_{(m,l)}} Sp_{((i,s)}\right) * vk_{(t,k,l,s)}\right) \tag{3.4}$$

Depending on the volume for withdrawal for one hour($Qmax$) in the flexible load contract, the decision variables vs and vk are either integer or continuous. Swap contracts in the

[3]The value of $M_{(k,l)}$ changes for swap contracts with a duration of a month or more. For example is the value of M equal to $31 * 24 = 744$ for a January contract while it is $28 * 24 = 672$ for the February contract

market are only traded in volumes of 1 MW, so a correct modeling of vs and vk is to model them as integer variables. However, if $Qmax$ is significantly large, the integer assumption can be relaxed. With continuous variables, the value of the market model in the optimization will be equal or better than with integer variables. The relaxation will also decrease the computation time of the optimization problem[4].

In order for the signs before both 3.3 and 3.4 to be valid, both vs and vk must be larger or equal to zero. This is modeled:

$$vs_{(t,k,l,s)}, vk_{(t,k,l,s)} \geq 0 \qquad \text{for all } t,\ k,\ l \text{ and } s \tag{3.5}$$

The displayed model for swap contracts in the market, models contracts that may not be tradeable in all the trading points by reason of the market incompleteness. At Nord Pool there are for instance only six weekly and six monthly contracts available for trading at one point. These unavailable contracts are not modeled.

In the electricity market there are transaction costs for every MWh traded. At Nord Pool the trading fee for a swap contract is 0.0035 € /MWh(Nord Pool, 2007). Table 3.1[5] displays the total value of the transaction cost for different contract lengths.

Table 3.1: Transaction cost for 1 MW traded swap contracts at Nord Pool

Contract	*hours*	*Price EUR*	*Price NOK*
Week	168	0.59	4.82
Month	744	2.60	21.35
Quarter	2184	7.64	62.68
Year	8760	30.66	251.41

Compared to the spot price of 1 MWh, the transaction cost for one transaction is very small, however with an active hedging policy the cost may become significant in the optimization. The value of all transactions is modeled by multiplying the total volume of bought and sold swap contracts in the market with the constant transaction cost value Ct. The contribution to the objective function becomes:

$$-Ct * \sum_{l=1}^{L}\sum_{t=1}^{T}\sum_{k=1}^{K}\left(vs_{(t,k,l,s)} + vk_{(t,k,l,s)}\right) * M_{(k,l)} \tag{3.6}$$

3.3 Block profile

The model presented in chapter 3.1 and 3.2 uses hourly resolution on the decision variable $q_{(i,s)}$, and the spot price $Sp_{(i,s)}$. Since all other resolutions in the model are daily or larger,

[4]See Rardin(1998) for theory about integer versus continuous variables

[5]In Table 3.1, the exchange rate between EUR and NOK is 8.2 NOK/€

the model supports block profiles of identical lengths with resolution smaller or equal to a day. The value of the optimization must then be multiplied with the number of hours in a block. The value of the optimization will for instance be multiplied with 6, if the hourly resolution is combined in 4 blocks of 6 hours.

By arranging the daily hours in blocks, the optimization takes less time at the expense of a less exact value. The value of the loss is investigated in 6.2. The reduction of computation time is a result of the reduction of dimensions.

3.4 Modeling risk aversion

As a consequence of the incompleteness of the electricity market, traders' utility function must be modeled to value and hedge a flexible load contract. Modeling the utility function for the whole market will make the hedging decisions the most realistic, but the total markets utility function is unknown. Even the utility function of one player can be impossible to determine.(Varma, 1989). We will in our modeling of risk aversion address the utility functions for one trader, not for the complete market.

The main purpose of our risk modeling is to construct a proper hedge in the market either by reducing the volume risk(quantity risk) or limit the possible profit losses(profit risk). Li & Flynn(2004) show that in most deregulated electricity markets, including Nord Pool, demand is the most important factor affecting the spot price for electricity. The correlation between demand and price for Nord Pool is 0.53, signifying that high demand will increase the spot price and vice versa. For a flexible load contract with physical load, the uncertainty of the demand quantity creates a risk that cannot be completely explained by price risk(Oum et al, 2005). By reducing the volume of the flexible load contract exposed to changes in spot price, the contract holder reduces the effect of low profit scenarios. We will compare the effect of reducing the risk of volume exposed with a model to reduce profit risk.

The swap contracts at Nord Pool range from daily contracts too year long contracts. Each of the different contract lengths lasts an exact number of days. In order to model the total volume traded with swap contracts for each day, we define the following quantities:

d	Index for number of days the flexible load contract lasts
D	Last day of the flexible load contract
Bl	Number of hours the day is divided in (h)
$H_{(l,d)}$	Reference matrix for matching day d with the correct contract of length l
$vsday_{(d,s)}$	Total volume sold of swap contracts for day d (MW)
$vkday_{(d,s)}$	Total volume bought of swap contracts for day d (MW)

Each matrix, $H_{(l,d)}$, shows which contract of length l that corresponds to a specific day. An exemplification of this is the daily and weekly values in $H_{(l,d)}$:

Daily contracts	$H_{(1,d)} =$	[1 2 3 4 5 6 7 8 9 10 ....]
Weekly contracts	$H_{(2,d)} =$	[1 1 1 1 1 1 1 2 2 2 ......]

The total volume that are sold or bought of swap contracts for day d are modeled with the restrictions:

$$\begin{aligned} vkday_{(d,s)} &= \sum_{l=1}^{L}\sum_{t=1}^{T} vs_{(t,H_{(l,d)},l,s)} \qquad \text{for all } d \\ vkday_{(d,s)} &= \sum_{l=1}^{L}\sum_{t=1}^{T} vk_{(t,H_{(l,d)},l,s)} \qquad \text{for all } d \end{aligned} \tag{3.7}$$

To evaluate the hedging, profit, and calculation time of the risk aversion in the model, we propose three functions, case 2 through 4, that model a minimization of the volume risk. In case 5 we model a profit risk with conditional value at risk. The cases we will test are:

Case 1:

As a reference we create a risk neutral case, which means the withdrawal schedule is optimized independently from the swap market. In order for us do this, no extra function is added to the model.

Case 2:

We propose a quadratic utility function to reduce the exposure risk. The model penalizes every MWh that are exposed to changes in the spot price. If either withdrawal from the flexible load contract are not hedged, or the hedging with swap contracts result in a hedge in periods where you do not withdraw, the model penalizes. The risk aversion function becomes:

$$-\lambda \sum_{d=1}^{D} \left(\left(Bl \left(vkday_{(d,s)} - vsday_{(d,s)} \right) + \sum_{i=1+(d-1)*Bl}^{d*Bl} q_{(i,s)} \right)^2 * \sum_{i=1+(d-1)*Bl}^{d*Bl} Sp_{(i,s)} \right) \tag{3.8}$$

By multiplying each contract period with the spot price, you get a more volatility adjusted model. According to Knittel & Roberts(2001) the volatility rises with higher spot prices. Rise in volatility gives a bigger chance for lower prices than expected and a risk averse player will have more reason to hedge. The λ is an adjustment factor to value the risk aversion function. Table 3.4 shows the value of the risk aversion function with $\lambda = 1$ and $\lambda = 0.001$, when the exercised hours from a flexible load contract is not hedged, and the average spot price is 150 NOK. A large value of λ will increase the impact of the risk aversion.

Table 3.2: Value of risk aversion function for non-hedged hours with different values for the adjustment factor λ in a quadratic volume risk aversion function

Hours exercised	1	6	12	18	24
Value; $\lambda = 1$	3 600	129 600	518 400	1 166 400	2 073 600
Value; $\lambda = 0.001$	3.6	129.6	518.4	1166.4	2073.6

Case 3:

This function is a simple on/off hedging function. It states that you will hedge in the market if, and only if, the sum of withdrawal from the flexible load contract within a day is above a certain level or percentage. This is done with the following restrictions:

$$\begin{aligned} \sum_{i=1+(d-1)*Bl}^{d*Bl} q_{(i,s)} - P_{level} * Bl &\leq (vsday_{(d,s)} - vkday_{(d,s)}) * Bl \qquad \text{for all } d \\ \sum_{i=1+(d-1)*Bl}^{d*Bl} q_{(i,s)} + P_{level} * Bl &\geq (vsday_{(d,s)} - vkday_{(d,s)}) * Bl \qquad \text{for all } d \end{aligned} \tag{3.9}$$

where:

P_{level} Percentage level for when to hedge in the market

The linearity of the utility function makes the computation time fall drastically compared to the quadratic function in case 2. With a hedging level(P_{level}) at 50 percent, a withdrawal less than 50 percent has the risk of a low price scenario for the hours exercised, while a withdrawal over the P_{level} has the risk of a high price scenario for hours not exercised.

Case 4:

If, in one hour, the flexible load contract has a maximum withdrawal per hour larger than 1 MW, the function in case 3 may be expanded to include several levels or boundaries in order to get a more realistic hedging sequence. Say you can withdraw 2 MW per hour from the flexible load contract in the model, the model hedges 1 MW when the exercised hours from flexible load contract passes boundary 1(P_{level1}) and 2 MW when it passes boundary 2(P_{level2}). In order to include this flexibility in the model, we need to include some additional quantities:

P_{level1} First hedging barrier
P_{level2} Second hedging barrier
$vs1_{(d,s)}$ First barrier variable, binary variable
$vs2_{(d,s)}$ Second barrier variable, binary variable

Similar to case 3, the hedging decisions are included in the model by adding restrictions. For each level there are both a $\leq$ and a $\geq$ function to model the on/off functionality. The total hedging decision for each day is then matched with the total value of bought and sold swap contracts.

$$\begin{aligned}
\sum_{1+(d-1)*Bl}^{d*Bl} q_{(i,s)} - P_{level1} * Bl \leq vs1_{(d,s)} * Bl \qquad & \text{for all } d \\
\sum_{1+(d-1)*Bl}^{d*Bl} q_{(i,s)} + P_{level1} * Bl \geq vs1_{(d,s)} * Bl \qquad & \text{for all } d \\
\sum_{1+(d-1)*Bl}^{d*Bl} q_{(i,s)} - P_{level2} * Bl \leq vs2_{(d,s)} * Bl \qquad & \text{for all } d \\
\sum_{1+(d-1)*Bl}^{d*Bl} q_{(i,s)} + P_{level2} * Bl \geq vs2_{(d,s)} * Bl \qquad & \text{for all } d \\
vs1_{(d,s)} + vs2_{(d,s)} = vsday_{(d,s)} - vkday_{(d,s)} \qquad & \text{for all } d
\end{aligned} \tag{3.10}$$

Case 5:

In order to compare the volume risk functions, we construct a profit risk function by using the Conditional Value-at-Risk(CVaR) methodology. CVaR is a linear model to minimize the possible loss of a scenario optimization. The proposed model adds the daily losses and test them towards an acceptable level of total loss.[6] In order to model CVaR in our maximization set-up, the objective function stays the same, while restrictions are added. CVaR modeling creates a necessity for the following definitions:

$z_{(d,s)}$	loss value for day d in the contract period
α	Value of lowest optimal Value-at-Risk
β	Confidence level for value at risk
C	Acceptable level of total loss

The restrictions included are:

[6]See Unger(2002) for theory about CVaR and modeling CVaR in electricity markets

$$
\begin{aligned}
z_{(d,s)} \geq -\Bigg[\sum_{i=1+(d-1)Bl}^{d*Bl} (Sp_{(i,s)} - Pflc) * q_{(i,s)} \\
+ \sum_{l=1}^{L} \sum_{t=1}^{T} \Big(vs_{(t,H_{(l,d)},l,s)} * (Fs_{(t,H_{(l,d)},l,s)} * Bl - \sum_{i=1+(d-1)Bl}^{d*Bl} Sp_{(i,s)}) \\
- vk_{(t,H_{(l,d)},l,s)} * (Fk_{(t,H_{(l,d)},l,s)} * Bl - \sum_{i=1+(d-1)Bl}^{d*Bl} Sp_{(i,s)}) \Big) \\
- Ct * Bl(vkday_{(d,s)} + vkday_{(d,s)}) \Bigg] - \alpha \qquad \text{for all } d
\end{aligned}
\tag{3.11}
$$

$$
z_{(d,s)} \geq 0 \qquad \text{for all } d \tag{3.12}
$$

Restriction 3.11 and 3.12 define $z_{(d,s)}$ equal to *MAX[0, Possible loss for day d]* for all days in the flexible load contract period. The total value of the daily losses is then tested versus a total acceptable level of loss, hence giving the minimization of CVaR.

$$
\alpha + \frac{1}{(1-\beta) * D} \sum_{d=1}^{D} z_{(d,s)} \leq C \tag{3.13}
$$

The value of C, α and β are defined by the traders utility function.

3.5 Complete model

Combining the equations shown in chapter 3.1 and 3.2, the complete objective function that are computed for every scenario s, becomes:

$$
\begin{aligned}
Max \quad W = E\Bigg[\sum_{i=1}^{I} \big((Sp_{(i,s)} - Pflc) q_{(i,s)} \big) \\
+ \sum_{l=1}^{L} \sum_{k=1}^{K} \sum_{t=1}^{T} \left(\left(Fs_{(t,k,l,s)} * M_{(k,l)} - \sum_{i=1+\sum_{m=1}^{k-1} M_{(m,l)}}^{\sum_{m=1}^{k} M_{(m,l)}} Sp_{(i,s)} \right) * vs_{(t,k,l,s)} \right) \\
- \sum_{l=1}^{L} \sum_{k=1}^{K} \sum_{t=1}^{T} \left(\left(Fk_{(t,k,l,s)} * M_{(k,l)} - \sum_{i=1+\sum_{m=1}^{k-1} M_{(m,l)}}^{\sum_{m=1}^{k} M_{(m,l)}} Sp_{(i,s)} \right) * vk_{(t,k,l,s)} \right) \\
- Ct * \sum_{l=1}^{L} \sum_{t=1}^{T} \sum_{k=1}^{K} \big(vs_{(t,k,l,s)} + vk_{(t,k,l,s)} \big) * M_{(k,l)} + \phi \Bigg]
\end{aligned}
\tag{3.14}
$$

Here ϕ is zero for case 1, 3, 4 and 5 but equal equation 3.8 for case 2. For all cases the value of the flexible load contract is equal to equation 3.14 with $\phi = 0$. The restrictions in the model are the following:

$$\begin{aligned} q_{(i,s)} &\geq Qmin && \text{for all } i \text{ and } s \\ q_{(i,s)} &\leq Qmax && \text{for all } i \text{ and } s \\ \sum_{i=1}^{I} q_{(i,s)} &= Qsum && \text{for all } s \\ vs_{(t,k,l)}, vk_{(t,k,l,s)} &\geq 0 && \text{for all } t,\ k,\ l \text{ and } s \end{aligned} \tag{3.15}$$

For case 1 and 2, no restrictions are added. For case 3 and 4, equations 3.9 or 3.10 are added respectively as restrictions. For case 5 with CVaR minimization equations 3.11, 3.12, and 3.13 are included.

The model is run for each scenario in the total optimization of the flexible load contract. With the use of block profiles, the number of hours in a block is multiplied with the total value of the objective function. The model assumes that the players are price takers and therefore cannot manipulate the prices in the spot market to the players own advantage[7].

[7] See Overbye, Weber & Patten (2001) for theory about market power in the electricity market

Chapter 4

Modeling the uncertainty in the electricity market

The proposed model for a FLC-optimization shown in chapter 3 uses exogenous spot prices and swap contracts. The complexity of the electricity market, as mentioned in chapter 2, makes the modeling of future uncertainty in the market important to the market players.

Most of the existing literature focuses on developing realistic spot price models, where you can derive swap price dynamics through arbitrage theory based on the time evolution of a stochastic spot price model. Of more recent work, Geman & Roncoroni(2006) suggest a stochastic mean reverting jump process, and successfully fit the model to several markets. Furthermore Schindlmayr(2005) propose a stochastic regime-switching model, and adjust it to the European Energy Exchange. For references to other articles and models on this subject, see Kluge(2006) or the introduction in Benth & Koekebakker(2005).

Even though the predicted spot and swap prices are important to the price and outcome of the FLC-optimization model, the demonstration and evaluation of our proposed model only need data that represent a typical electricity market. Nord Pool, the Nordic power market, is the oldest and most extensively researched power market. Lucia & Schwartz(2002) present a simple one factor model for spot, future and forward prices that incorporate a seasonal pattern, which is adequate for our purpose.

The spot price is estimated with the Lucia & Schwartz(2002) logarithmic one factor model with a simple sinusoidal function to capture the seasonal pattern[1].

$$
\begin{aligned}
Ln(P_t) = \alpha + \beta * D_t + \gamma \cos((t+\tau)\frac{2\pi}{365}) + Y_t \\
\text{where} \\
Y_t = \phi Y_{t-1} + u_t
\end{aligned}
\tag{4.1}
$$

[1]Equation 38 in Lucia & Schwartz, 2002

Here P_t is the daily spot price at time t. α, β, γ, and τ are constant parameters, and the cosine function captures the seasonal pattern in the electricity market. Y_t is a discrete stochastic process, where u_t are i.i.d.[2] normal random variables with mean zero and variance σ^2.

Based on the results from the spot price simulation, the future/forward price for day T at time 0 is estimated with Lucia & Schwartz(2002)[3].

$$F_0(P_0, T) = \exp[f(T) + (lnP_0 - f(0))exp^{-\kappa T} + \alpha(1 - \exp^{-\kappa T} + \frac{\sigma^2}{4\kappa}(1 - \exp^{-2\kappa T})] \quad (4.2)$$

Equation 4.2 is not optimal for representing Nord Pool, but is adequate for our purpose. The future/forward price model estimates the daily swap price. In order to calculate the price of the forward or future contract, Lucia & Schwartz(2002) use the arithmetic average of the daily contract prices, denoted $F_0(P_0 : T_1, T_2)$ where P_0 is the spot price at t=0. T_1 and T_2 represent the number of days till the beginning and the end of the contract period, respectively. It states: [4]

$$F_0(P_0 : T_1, T_2) = \frac{1}{T_2 - T_1} \sum_{T=T_1}^{T_2} F_0(P_0, T) \quad (4.3)$$

Our proposed FLC-optimization model uses hourly spot prices or small blocks of hours to determine which hours to exercise in the contract. Equation 4.3 estimates the daily prices and we therefore need to adjust and expand the data with a profile for the daily variation in spot prices. Using historic spot data from Nord Pool in 2006, we acquire a daily profile by averaging every hour of the day, and show the percentage change that every hour has got from the daily average. For the block hours we define a series of 4 blocks with 6 hours. Figure 4.1 shows the average percentage change from the daily mean for both the hourly profile and the block profile, where daily mean is given by 100%.

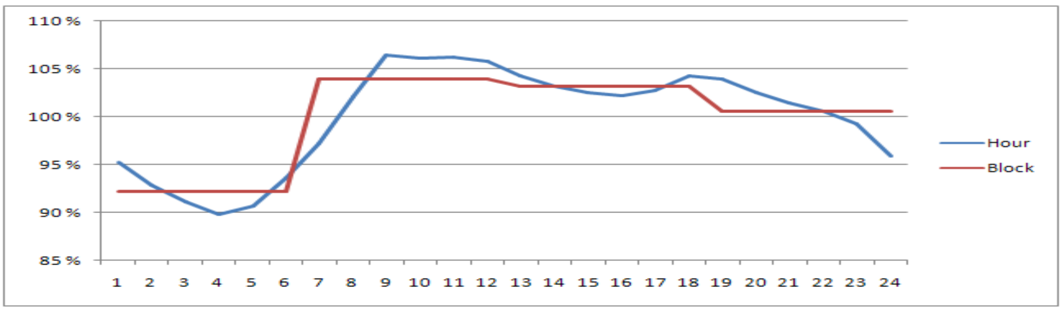

Figure 4.1: Average daily profile from Nord Pool in 2006

[2]i.i.d is short for independent and identically distributed

[3]Equation 23, Lucia & Schwartz, 2002

[4]Equation 41, Lucia & Schwartz, 2002

Chapter 5

Contract specification and data estimations

The proposed cases in chapter 3 will in chapter 6 be analyzed for flexible load contracts of different lengths; a month, and half a year. For simplicity, the contract agreement, scenario simulation, and the creation of a static hedge will be set 8 weeks in advance for the half year contract, and 2 weeks in advance for the month long contract. The monthly contract will last from 1st of January 2007 to the 31th of January 2007, while the half year contract from January 2007 till June 2007.

By reason of the model being a two stage stochastic model, rather than a multi period stochastic model, trading in the market will be modeled only at contract agreement. For the monthly optimization, there are 5 weekly swap contracts along with a swap for January in the model. The half year optimization has 6 monthly contracts. The transaction cost for trading in the market is set equal to Nord Pools trading fees for swap contracts, which are 0.0287 NOK[1].

Applying the simple one factor model(4.1) with parameters Lucia & Schwartz(2002) estimated from Nord Pools system price from 1993 - 1999, one hundred spot price scenarios are generated. Figure 5.1 displays the distribution of 100 scenarios for the period 2 weeks before 1st of January till the beginning February. Similarly, one set of a hundred simulations are generated for half a year. Figure 5.2 shows the spread of the spot price simulations used in the optimization for the 6 month long flexible load contract[2]. Even though 100 scenarios result in a limited preciseness of the valuation, it is still adequate for our test of the model. Chapter 5.1 shows the stability of our model with different number of scenarios.

The modeled spot price ignores holidays, weekends and the transition between summer and winter time. A modeled smoothness between the dates will make the hourly prices

[1] Calculated from the trading fee of 0.0035 Euro (Nord Pool, 2007), with an exchange rate at 8.2 NOK/EUR

[2] Both 5.1 and 5.2 displays the scenarios before they are adjusted with an hourly profile

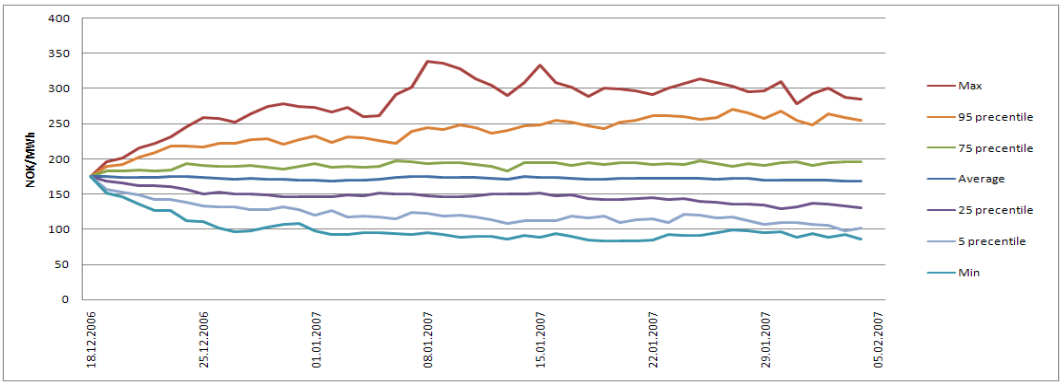

Figure 5.1: Spot price distribution of 100 scenarios for the monthly contract optimization

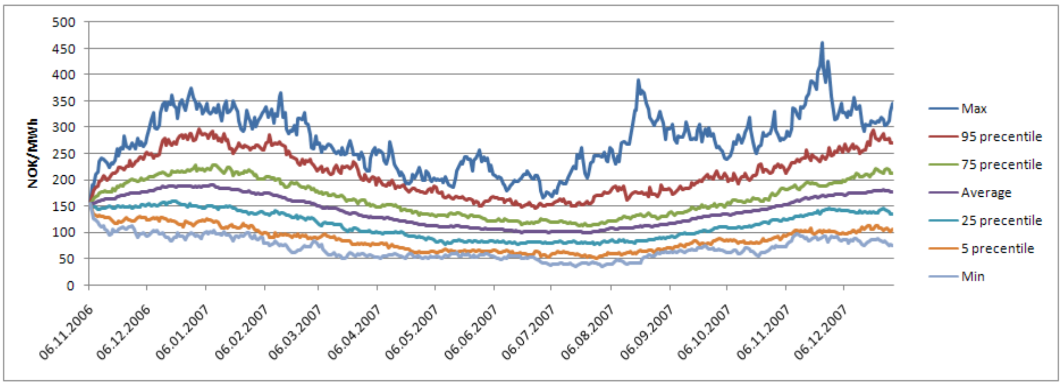

Figure 5.2: Spot price distribution of 100 scenarios for the half year contract optimization

more realistic, but it is not important for the demonstration and evaluation of our FLC-optimization model.

The forward contracts corresponding to each contract are calculated using equation 4.3, which has parameters from the Nord Pools system price from 1993 - 1999. Figure 5.3 displays the available contracts for the one month flexible load contract optimization. In the optimization the difference between buying and selling price is ignored. Nord Pool's weekly and monthly swaps are traded with a bid-ask spread, ranging from 0.05€ up to 2€. Since our model only supports a realistic static hedge, the spread has small effect on the trading decisions.

Both of the two contract lengths are optimized with two different set of flexibilities. Year long flexible load contracts with maximum withdrawal of 3000 or 5000 hours were common in Norway before the deregulation of the power market. These contracts have a flexibility of respectively 65.7 % and 42.9 %. For a similar flexibility of a monthly contract with 31 days, we optimize contracts with 250 and 400 hours. For the 6 months contract we optimize contracts with 1500 and 2500 hours to withdraw. The maximum

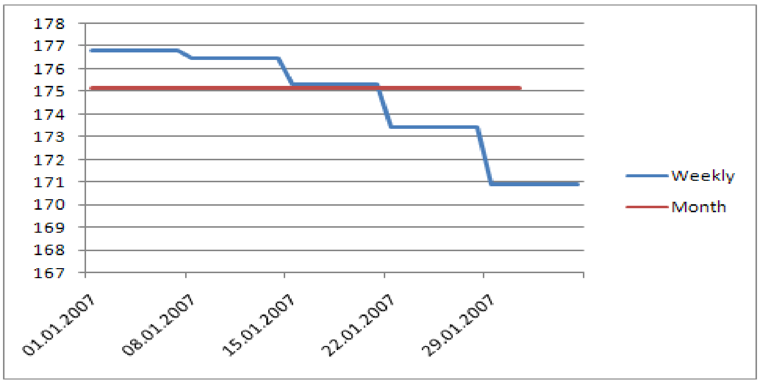

Figure 5.3: Month long and weekly swap contracts available for January

withdrawal per hour is set at 50 MW, whereas the minimum is set equal to 0 MW. Table 5.1 displays the different contract specifications that we analyze.

Table 5.1: Specifications for modeled flexible load contracts

Length	*Total withdrawal*	*Max hourly withdrawal*	*Contract price per MWh*
1 Month	250 hours	50 MWh	184.22 NOK
1 Month	400 hours	50 MWh	182.41 NOK
6 Months	1500 hours	50 MWh	171.89 NOK
6 Months	2500 hours	50 MWh	159.31 NOK

The proposed model supports block profiles of the exercising schedule up to a daily block. In order to test which effect blocks of hours has on the value of the flexible load contract compared to hourly profiles, we analyze four blocks of six hours. The first block starts at the first hour of the day. Figure 4.1 displays the block profile and the hourly profile tested.

For each computation, we are interested in the value of the contract, and the hedging decisions. In order to compare and analyze the accumulated profit for the different cases, the value of risk aversion is not included. We are also interested in the withdrawal schedule for each case, due to the hedging in the market. For practical use of the models, the computation times are important and they are analyzed accordingly.

5.1 Model Validation

The stability of the proposed model is tested by optimizing a 500 MWh flexible load contract with maximum of 2 MW withdrawal per hour with different numbers of non-identical scenarios. Figure 5.4 and 5.5 displays the average value of the flexible load contract with 50, 100, 250, 500, 1000, and 10000 scenarios for an optimization of case 1 and case 3 with a static hedge, respectively. The value for one MWh is set equal to the value for a monthly contract with 500 hours displayed in table 5.1.

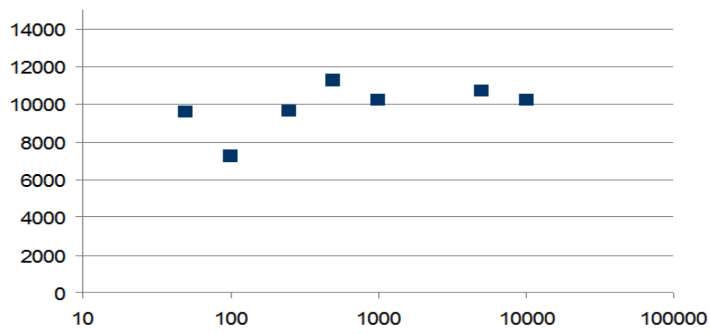

Figure 5.4: Average value of case 1 with multiple scenarios

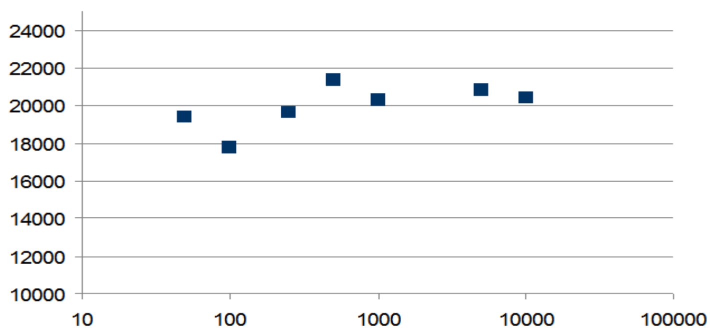

Figure 5.5: Average value of case 3 with multiple scenarios

Figure 5.4 and 5.5 show the average value for each optimization, with the number of scenarios on a logarithmic scale. The value of the simulation converges towards an exact value for each case.

Chapter 6

Results

In this chapter we will look at results from the two stage stochastic optimization model from chapter 3, and analyze how the value of the optimization is altered through the different hedging strategies described by case 2 - 5. For each case the optimal strategy and value have been simulated with the use of 100 different price scenarios. Downside risk and upside potential will be compared for each static hedging strategy. Withdrawal strategy and computation times are tested for each case. All results for each case will be compared to the risk neutral optimization in case 1. Our model supports both hourly and block profiles, and the impact of block profiling will be tested for both value and computation times.

6.1 Monthly contract

A monthly flexible load contract for January is tested with two flexibilities. A 400 hours contract that has a flexibility in the withdrawal of 46.2 percent[1] and a 250 hours contract that has 66.4 percent flexibility. Comments on the withdrawal schedule are made after the profit results from both contracts.

6.1.1 50 MW 400 hours contract

Case 1 is the risk neutral case and is used as a reference case. With a 400 hours flexible load contract with maximum withdrawal of 50 MW per hour[2], the maximum value is almost 2.4 MNOK. The minimum value for case 1 is -1.6 MNOK. The break even value, which describes the percentage of the scenarios that are below 0 profit, is 57%. The average optimization value of all scenarios is 45 700 NOK. Table 6.1 displays the main results for all cases analyzed and Figure 6.1 displays the accumulated profit for each case.

The quadratic risk aversion from case 2 sets up a static hedge where 31 MW are hedged in week one, then 30 MW, 29 MW, 23 MW, and 0 MW are hedged respectively in the

[1] The flexibility is calculated by (1 - 400/744) = 46.2%

[2] Also referred to as a 50 MW 400 hours contract or 400 hours 50 MW contract

following weeks. The January swap is not bought. This static hedge cuts the downside risk with 92.4 percent to -125 012 NOK. The static hedge gives a 36% chance of a non profit scenario. The upside potential is drastically lowered to only 263 647 NOK, but this reduction of 89% is smaller than the reduction in the downside risk. The average value for the optimization is just over 31 000 NOK.

Case 3 is an on/off-hedge. The on/off-hedge demands a full hedge of 50 MW in a swap contract if the withdrawal within a contract period is 50% or higher. The optimization with case 3 results in a static hedge, at contract agreement, of 50 MW for week 1, 2, and 3, and zero for the last two weeks and the January swap. Compared to the risk neutral case, the reduction in downside risk is 81.66%, and profit is generated for 80% of the scenarios. The average value of the optimization is increased to almost 147 000 NOK, which is over three times as much as the risk neutral case. The maximum upside potential is 890 000 NOK.

Case 3 is developed further in case 4 and has two limits of hedging(25% and 60%), which each demands independently a hedge of 25 MW. By optimizing a 400 hours flexible load contract, a static hedge is created, identical to case 3. Since the demand of 60% withdrawal within a contract period is stricter than the demand of 50%(which is the demand of case 3), the contract value of case 4 is smaller than the contract value of case 3. After a static hedge is created, case 3 has more freedom to choose hours than case 4. This explains why case 4 is smaller than case 3 in all scenarios.

Profit risk is in case 5 modeled through CVaR. The CVaR calculates possible loss for each day, and tests this value towards a total acceptable loss. For case 5, 50 MW of swaps are bought for week 2 and 3. 19 MW are bought for week 4, creating the static hedge. The minimum value for a scenario is -89 120 NOK, which is a reduction of 94.5 percent compared to case 1. The break even is reached at 21 percent and the maximum is 863 902 NOK.

Table 6.1: Value in NOK for a 50 MW 400 hours 1 month flexible load contract with a static hedge

	Minimum value	*Maximum value*	*Break even*	*Average value*
Case 1	-1 640 611	2 396 188	57%	45 000
Case 2	-125 012	263 647	36%	31 270
Case 3	-300 851	890 451	20%	146 946
Case 4	-302 751	744 089	20%	127 531
Case 5	-89 120	863 902	21%	148 834

As Figure 6.1 shows, a static hedge of the flexible load contract will reduce the downside risk at the expense of upside potential. Of the three volume risk aversion models, case 2 reduces the downside risk almost 3 times more than case 3 and 4. The reason is that the

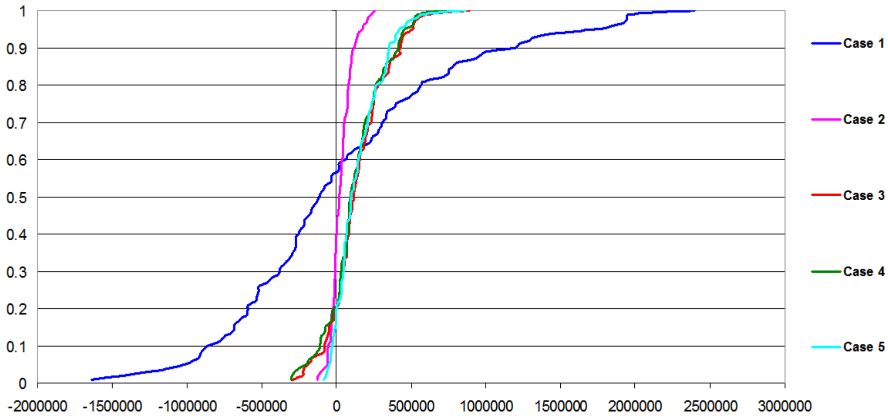

Figure 6.1: Accumulated profit for a 50 MW 400 hours monthly flexible load contract with a static hedge and hourly resolution

static hedge created by case 2, hedges a volume almost exact to the total withdrawal of the flexible load contract, whereas case 3 and 4 for this contract overhedges.

Compared to the volume risk aversion function, case 5 with CVaR reduces the downside risk slightly more than case 2. However, the upside potential is reduced for case 2 with 89% compared to case 1, the profit potential in case 5 is only reduced with 63.95%. The reason for the upside potential being over 3 times higher in case 5 compared to case 2 is the lack of withdrawal flexibility in case 2. Whereas case 5 seeks profit, case 2 penalizes the optimization, but only if the withdrawal is not scheduled where the model has created a static hedge.

6.1.2 50 MW 250 hours contract

A 250 hours monthly flexible load contract has a withdrawal flexibility of 66.4 percent. Table 6.2 and Figure 6.2 illustrate, respectively, the main values from the optimization and the accumulated profit. Since non of the cases in the 400 hours optimization bought the swap for January, the January swap is not modeled for this 250 hours optimization.

Table 6.2: Value in NOK for a 50 MW 250 hours 1 month flexible load contract with a static hedge

	Minimum value	*Maximum value*	*Break even*	*Average value*
Case 1	-989 903	1 601 970	50%	124 089
Case 2	- 79 051	279 372	18%	54 229
Case 3	-382 352	915 433	43%	97 164
Case 4	-199 547	785 016	14%	164 483
Case 5	- 58 979	562 979	14%	160 117

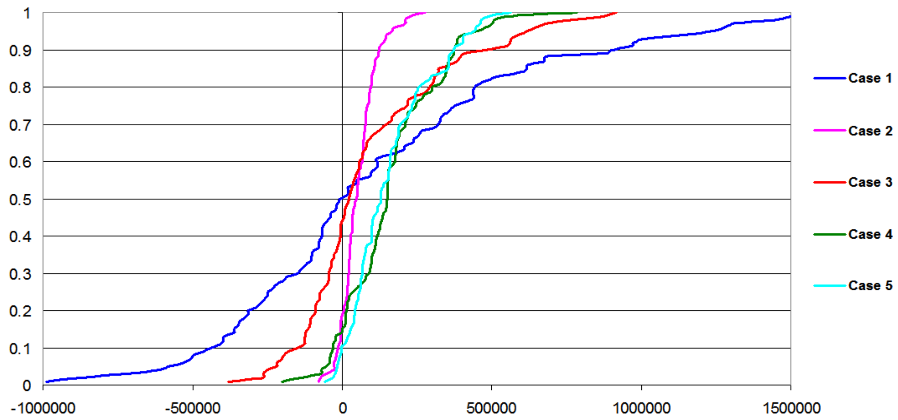

Figure 6.2: Accumulated profit for a 50 MW 250 hours monthly flexible load contract with a static hedge and hourly resolution

The risk neutral case 1 has a possible loss of almost 1 MNOK and a 50 percent chance of loss. For case 2, a static hedge is created by 23 MW for week 1 and 21 MW, 15 MW, 11 MW, and 0 MW for the following 4 weeks. The downside risk is reduced with 92% compared to the risk neutral case, at a cost of an 82.6% loss in possible profit.

The on/off hedging barrier in case 3, displays a lack of flexibility in the hedging decisions, that reduces the optimality of the hedge. The static hedge created by case 3 is 50 MW for week 1; no other swap contracts are bought. Even though the static hedge created by case 3 reduces the downside risk compared to the risk neutral case, the possible loss of case 3 is over four times the loss of case 2. Case 4 has with its two barriers a more attuned hedge compared to case 3. This results in a 47.8 percent reduction of downside risk compared to case 3, but the risk is over 2.5 times the risk of case 2.

The CVaR profit risk optimization in case 5 establishes a static hedge with 50 MW for week 3 and 24 MW for week 4. The downside risk is reduced with 94 percent compared to the risk neutral case 1. The downside risk reduction is slightly better than the volumetric risk function of case 2, however, similar to the 400 hours flexible load contract, the upside potential is significantly better for the optimization of case 5 than for the optimization of case 2.

6.1.3 Withdrawal strategy

Mo & Gjelsvik(2002) state that the withdrawal of the flexible load contract is not affected by the risk aversion. They model a complete market with both weekly resolution on both spot and forward prices. Our model, with higher resolution on the spot prices compared to the available swap contract, shows that the withdrawal is affected by the modeled risk aversion for the monthly contracts. Table 6.3 displays the withdrawal strategy of the

400 hours flexible load contract, and Table 6.4 indicates the results for the 250 hours contract.

Table 6.3: Average withdrawal strategy for a 50 MW 400 hours 1 month flexible load contract with a static hedge

	Week 1	*Week 2*	*Week 3*	*Week 4*	*Week 5*
Case 1	23.09 %	24.56 %	22.84 %	21.24 %	8.27 %
Case 2	27.31 %	26.45 %	25.63 %	20.60 %	0.02 %
Case 3	27.09 %	26.82 %	24.75 %	14.2 %	7.15 %
Case 4	25.79 %	28.15 %	26.10 %	9.76 %	10.21 %
Case 5	24.53 %	25.53 %	20.26 %	20.80 %	8.98 %

Table 6.4: Average withdrawal strategy for a 50 MW 250 hours 1 month flexible load contract with a static hedge

	Week 1	*Week 2*	*Week 3*	*Week 4*	*Week 5*
Case 1	25.79 %	28.15 %	26.10 %	9.76 %	10.21 %
Case 2	32.38 %	29.68 %	21.64 %	16.27 %	0.03 %
Case 3	47.28 %	15.91 %	14.25 %	14.22 %	8.34 %
Case 4	24.53 %	25.43 %	20.26 %	20.80 %	8.77 %
Case 5	27.60 %	27.96 %	24.91 %	10.76 %	8.77 %

For all cases of a 50 MW 400 hours contract, the withdrawal schedule is slightly altered. The volume risk models either penalize(case 2) or limits the withdrawal restricting the exercising flexibility(case 3 and 4). The CVaR profit risk model barely alters the withdrawal schedule to a higher withdrawal in the first two weeks.

The withdrawal strategy for case 2, 3, and 4 of the 50 MW 250 hours flexible load contract is greatly influenced by the volume risk aversion. The extra flexibility in the 250 hours flexible load contract compared to the 400 hours contract, results in a larger difference in withdrawal strategy by reason of the restrictions or penalties in the volumetric hedging strategy. The difference between case 5 and case 1, compared to the difference between the quantity risk cases(2, 3, and 4) and case 1, is smaller, but the profit risk model in case 5 is influenced by the hedging decisions.

The results from the withdrawal strategy of case 5 show that, with a modeling of the market incompleteness and a static hedge, the most profitable strategy might not be identical for profit risk optimization and risk neutral optimization.

6.2 Hourly versus block resolution

By optimizing the 50 MW 400 hours flexible load contract with our simple block profile of six hours displayed in 4.1, the hedging decisions are not changed for any of the 4 risk aversion cases. The average reduction in value is calculated to 0.35 NOK/MWh. The value is identical to the transaction cost at Nord Pool for one MWh. For a 50 MW 400 hours flexible load contract, the total average value reduction is 6 924 NOK.

A block profile with 6 hours in each block, significantly reduces the computation time with at least 75% for the static hedge computation. This reduction in computation time is important for a trader's possibility to utilize an optimization model for valuation of a derivate.

Table 6.5: Computation time and average value for a month long 50 MW 400 hours flexible load contract with hourly and block profile

	Average value			*Computation time*		
	Hour	Block	NOK/MWh	Hour	Block	% change
Case 2	31269	27 687	-0.179	124.6 s	3.4	97.3 %
Case 3	146 945	138 019	-0.446	12.8 s	2.4 s	81.3 %
Case 4	127 531	119 013	-0.426	22.2 s	3.8 s	82.9 %
Case 5	148 834	142 163	-0.333	35.6 s	8.9 s	75.0 %

Even though the value of computation times displayed in Table 6.5[3] are small, a multi period stochastic model have additional decision variables to calculate, resulting in longer computation times. Arranging the hourly profile in small blocks of hours will, as a consequence, drastically reduce the computation time without a significant loss in value.

An optimization where the resolution of the spot price is 1 day(blocks of 24 hours), the loss in value of an optimization for case 1 is 14.44 NOK/MWh for a 250 hour flexible load contract and 12.63 NOK/MWh for a 400 hour contract. By this reason, a daily profile divided into blocks will give a better value of the optimization compared with a daily or larger resolution of the spot price. However, estimating the optimal block resolution based on value and computation time is beyond the purpose of this paper.

6.3 6 months contract

In the valuation of a six month long flexible load contract, ranging from January till June, the block profile of six hours displayed in Figure 4.1 is used in the optimization. The swap contracts in the market are modeled through six monthly contracts. Accumulated value and a static hedge are created for both a 2500 hours(42.4% flexibility) and a 1500 hours(65.5% flexibility) with a maximum hourly withdrawal of 50 MW.

[3] All test are run with a P4 2,4GHz with 4 GB ram computer on a Win-XP platform. The programming code for Xpress IVE might not be the most efficient.

6.3.1 50 MW 2500 hours

The optimization of a 2500 hours flexible load contract with maximum withdrawal of 50 MW, results in a downside risk for the risk neutral case of 8.68 MNOK. Table 6.6 and Figure 6.3 illustrates the optimization for all five cases evaluated.

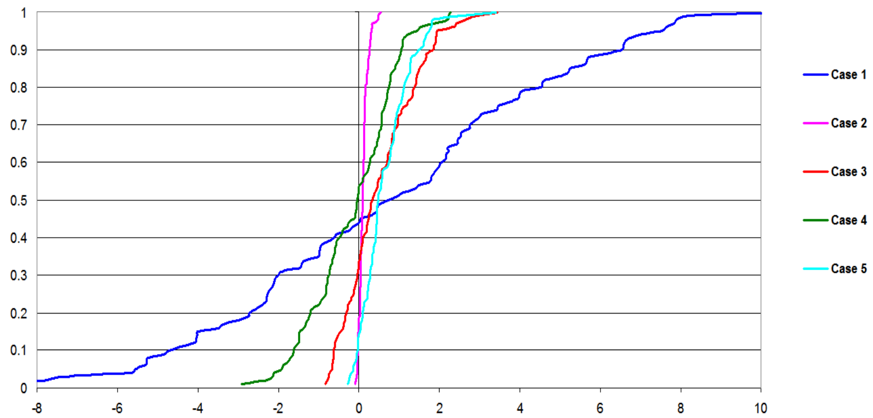

Figure 6.3: Accumulated profit for a 50 MW 2500 hours six month long flexible load contract with a static hedge and block resolution

Table 6.6: Value in NOK for a 50MW 2500 hours 6 month flexible load contract with a static hedge

	Minimum value	*Maximum value*	*Break even*	*Average value*
Case 1	-8 682 765	10 491 977	43%	124 089
Case 2	-92 577	536 125	18%	107 460
Case 3	-840 465	3 444 824	33%	541 630
Case 4	-2 919 807	2 272 278	51%	-135 297
Case 5	-276 701	3 406 661	14%	657 572

The static hedge created by case 2, hedges 50 MW for January and February, 49 MW for March and 24 MW for April. For May and June, no swaps are bought. The static hedge reduces the possible losses with 98.9% compared to case 1, at the cost of a reduced profit potential of 94.5%.

Similar to the monthly contracts of case 3 and 4, the static hedge created by case 3 and 4 for the half year contract, results in an overhedge for months with high expected withdrawal and underhedge for months with little expected withdrawal. This results, compared to case 2, in a hedge that is not optimal. The downside reduction of risk is 90.3% and 66.4% for case 3 and 4, respectively. Compared to the risk reduction of 98.9%

for case 2, the downside risk reduction of case 3 and 4 are low and comes as a result of the poor static hedge.

Case 5, with a CVaR profit risk modeling, creates a static hedge with 50 MW for the first two months, 28 MW for March, then 24 MW, 19 MW and 0 MW for the last three months, respectively. The downside risk is reduced with 96.8% compared to the risk neutral optimization. The upside potential is, similar to the optimization of a monthly contract, far better than the possible gains optimized for case 2.

6.3.2 50 MW 1500 hours

The 50 MW 1500 hours flexible load contract has a possible loss of 5.34 MNOK for case 1. The results from case 1 and the four other cases evaluated, are given in Table 6.7 and Figure 6.4.

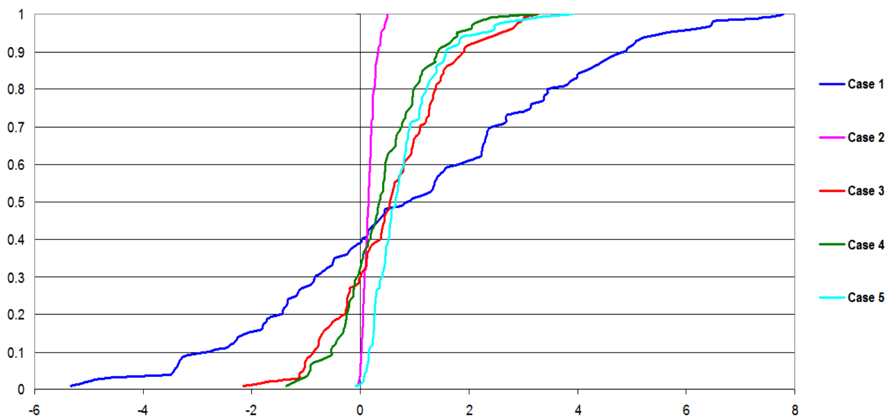

Figure 6.4: Accumulated profit for a 50 MW 2500 hours six month long flexible load contract with a static hedge and block resolution

Table 6.7: Value in NOK for a 50 MW 1500 hours 6 month flexible load contract with a static hedge

	Minimum value	*Maximum value*	*Break even*	*Average value*
Case 1	-5 338 915	7 795 034	38%	1 019 506
Case 2	-47 609	493 974	5%	161 925
Case 3	-2 163 547	3 285 921	29%	580 518
Case 4	-1 363 359	3 268 206	32%	414 920
Case 5	-84 662	3 916 554	1%	822 306

The quadratic volume risk aversion modeled with case 2, reduces the downside risk with 99.1%. This is accomplished through a hedge with swap contracts of 50 MW in January, 43 MW in February and 15 MW in March. For case 3 and 4, a poor static hedge results in a possible downside of -2.16 MNOK and -1.63 MNOK respectively.

Case 5 reduces the possible loss with 98.4% to 84 662 NOK compared to Case 1. However the reduction is slightly less than the reduction of case 2, but the upside potential is, similar to the three previously analyzed contracts for case 5, far better than the potential of case 2. The static hedge created by case 5 is 50 MW, 29 MW and 25 MW, respectively, for the three first months.

6.3.3 Withdrawal strategy

As for the monthly contracts, the withdrawal strategy is altered from the risk neutral case 1 to the volume risk aversion cases of case 2, 3, and 4. Table 6.8 and 6.9 display the withdrawal strategy for the 1500 hours and the 2500 hours contract respectively. The alteration of the withdrawal in the volumetric risk cases is(similar to the monthly contracts), due to the restrictions in withdrawal the static hedge creates.

Table 6.8: Average withdrawal strategy for a 50 MW 2500 hours 6 months flexible load contract with a static hedge

	January	*February*	*March*	*April*	*May*	*June*
Case 1	26.93 %	23.07 %	22.10 %	13.14 %	8.39 %	6.36 %
Case 2	29.76 %	26.88 %	29.22 %	13.91 %	0.12 %	0.11 %
Case 3	27.96 %	24.36 %	24.33 %	10.62 %	7.35 %	5.47 %
Case 4	21.92 %	19.40 %	21.23 %	19.01 %	13.43 %	5.01 %
Case 5	26.93 %	23.07 %	22.10 %	13.14 %	8.39 %	6.36 %

Table 6.9: Average withdrawal strategy for a 50 MW 1500 hours 6 months flexible load contract with a static hedge

	January	*February*	*March*	*April*	*May*	*June*
Case 1	38.60 %	29.54 %	15.91 %	7.49 %	5.17 %	3.30 %
Case 2	49.55 %	38.49 %	11.90 %	0.03 %	0.02 %	0.01 %
Case 3	37.98 %	27.70 %	26.56 %	4.07 %	2.37 %	1.32 %
Case 4	34.19 %	29.34 %	23.42 %	5.61 %	4.43 %	3.01 %
Case 5	38.60 %	29.54 %	15.91 %	7.49 %	5.17 %	3.30 %

The CVaR profit risk model in Case 5, has an exact withdrawal schedule similar to risk neutral optimization. This result is different from the withdrawal strategy of the monthly contract. The monthly contract has different withdrawal strategies, because the static hedge created by the model. Whereas the static hedge for both half year contracts resemble the hedge from case 2, the case 5 hedge for the monthly contracts are more different. This results in the alternation of the withdrawal strategy for monthly case 5 optimizations.

6.4 Discussion

The best withdrawal strategy is reached by modeling the profit risk aversion. From a static hedging perspective, profit risk modeling does not alter the withdrawal strategy compared to a risk neutral strategy, if the static hedge is a good hedge. In volume risk modeling, the restrictions or penalties will limit the model to exercise the most profitable hours.

For a static hedge, the ability to create the best possible hedge, is extremely important. In our modeling, the quadratic volume hedge creates the best hedge, while the CVaR model, of case 5, has more freedom in the exercising strategy, and therefore picks the best hours based on the hedge. From the perspective of a static hedge, a CVaR risk reduction based on the hedge created by the quadratic volume risk of case 2, will in our modeling create the best results.

In the linear integer function of volume risk aversion(based on the method of case 3), additional barriers must be included in order to give a good hedge. The inclusion of more barriers, will however result in a slower model. In the interest of profit and computation times, there are no grounds for a further development of this method of valuating flexible load contracts.

For the half year optimization, the seasonal fluctuations modeled, results in spot and swap prices in the first two months being far higher than the of the last two months. As a result, the reduction in downside risk for the optimization is better than the of the monthly contract, where the weekly differences are less pronounced.

Comparing the flexibility of the contracts, the 400 hours monthly contract and the 2500 hours half year contract have a higher value of break even compared to the 250 hours monthly contract and the 1500 hours half year contract. This indicates that extra flexibility in a flexible load contract results in a higher probability of profit. However, if one compares the flexibility to real option theory, additional flexibility may cost more(Trigiorgis, 1996). Higher flexibility in a flexible load contract might result in a larger risk premium.

Chapter 7

Conclusion

We propose a two stage stochastic optimization model, in order to simultaneously valuate and hedge a flexible load contracts in an incomplete electricity market. The model includes all available swap contracts in the market and supports hourly resolutions for the withdrawal strategy and spot market. The aforementioned, which results in a complete presentation of the market, leads to good and realistic withdrawal and hedging decisions. On the basis of withdrawal schedule and hedging decisions, the value of a flexible load contract can be derived with the traders utility function.

We test our model with hourly resolution and a block resolution of six hours. We show that a block resolution with a small number of multiple hours drastically reduce the computation time, without a significant loss in value of the optimization compared to the hourly resolution. However, a daily or larger resolution of the spot price will result in an extreme reduction of the value of the optimization. The loss in value arises because the daily average spot price ignores the hourly variation in spot prices.

Risk aversion is modeled through utility functions for both volume risk and profit risk. The best static hedge is created with a utility function, which minimize the volume exposed of the expected withdrawal schedule. However, based on a static hedge, a utility function minimizing profit risk creates the best withdrawal schedules. This illustrates the importance of modeling a good risk aversion for reduction of the downside risk in a flexible load contract.

Our results indicate that a flexible load contract with higher flexibility increases the chance for higher profit. This may result in a higher risk premium. However, our results are not conclusive.

Chapter 8

Further work

The model presented in this thesis is only capable of optimizing a static hedge for a flexible load contract. The reason for the inability to create a realistic dynamic hedge, is the limitations of a two stage stochastic optimization model. By evolving the model to a multi stage stochastic optimization model, a discrete dynamic hedge can be created for the contract. This method will probably give more realistic results and the potential profit might increase. With a multi stage stochastic optimization the stochastic movement of the electricity price will be modeled better. The uncertainty modeling is therefore improved with a multi stage stochastic optimization. One way to solve this challenge is by using a discrete Markov chain.

A discrete Markov chain has N number of states for each time step and each of these states has transition probabilities which lead the next state. This method is used by Mo et al.(2001) in the price model they present. Vogstad(2004) also makes use of this method, but instead of grouping the spot price scenarios into N bins, with as equal bin width as possible for each time period, he minimizes the area differences between the bins. This method gives a better representation of the distribution of the data.

Another possible adjustment to the model is the spot and swap price modeling. By changing to a more complex price model, more of the characteristics of the electricity price will be captured, giving more realistic prices for the spot and swap markets. An adjustment in the spot and swap price modeling will have an effect on the hedging sequences.

The hedging strategy of a small maximum withdrawal per hour(10 MW or less), produce the problem of integer variables for the computation. Since swap contracts only are available in an integer multiple of 1 MW, optimizing the best hedge might therfore result in large computation times. Further research may be assigned to the problem of integer variables for small flexible load contracts.

From the 12th of June 2007 Nord Pool includes peak contracts to complement their product range. This inclusion reduces the incompleteness of the market. If peak contracts are included in our modeling, better pricing and hedging decisions may be derived. Another small development of the model presented in this thesis is to include discounted cash flow.

Asian options can also reduce the risk of a flexible load contract, but they are rarely traded in the electricity market and have low liquidity. If the liquidity of asian options rises, the effects of risk management with asian options should be studied on a flexible load contract.

Further, it is important to consider the buyers and sellers perspective, and determine a reasonable and mutual utility function for all the market players. Based on the previous, risk premium can be derived for the contracts in the market and get an approximately correct price for the flexible load contracts.

Chapter 9

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