Norwegian University of Science and Technology

Project Report

# Capacity Allocation of Electricity Generation Between a Primary Reserve Market and a Day-Ahead Market

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# Abstract

With an increasing number of ancillary services and energy markets, the decision making process for a power producer is becoming more complex. In this report a stochastic dynamic model is developed to find the optimal bid in the primary reserve market for a thermal, CCGT, power producer who coordinates bidding in the primary reserve market and day-ahead market. The reserve market in question is modeled with a pay-as-bid auction and limited competition, while the day ahead market is modeled with a uniform price auction and is seen as competitive. The report concludes that it is profitable for the modeled power plant to participate in both markets. A sensitivity analysis with respect to characteristics of the power plant and prices in both markets, indicate that the results are sensitive to changes.

# Preface

This report is written as a specialization project within the field of Applied Economics and Operations Research at the Department of Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU).

We would like to thank our supervisors, professor Stein-Erik Fleten and postdoc Gro Klæboe for their valuable guidance and helpful discussions. Their insight in the problem has been immensely valuable. We would also like to thank Alois Pichler for guidance on lattice generation, provision of source code and helpful inputs. In addition we would like to thank Christian Skar and Lars Olof Nord for valuable insight in cost modeling of thermal power plants.

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# Nomenclature

### Sets

| | |
|---|---|
| $H$ | Set of settlements in the day-ahead market within a day |
| $I$ | Set of generating units within the producers portfolio |
| $T$ | Set of stages, days, in the bidding period |
| $T^D$ | Set of stages with day-ahead bidding , $T^D \subseteq$T |
| $\Pi$ | Set of policies |

### Indices

| | |
|---|---|
| $h$ | Settlement hour |
| $i$ | Generator |
| $t$ | Stage |
| $\pi$ | Policy |

### Parameters

| | |
|---|---|
| $c_i^S$ | Start-up cost for generator $i \in I$ |
| $\hat{p}^R$ | Relevant market price of capacity in the primary reserve market |
| $\hat{p}^E$ | Efficiency market price of capacity in the primary reserve market |
| $\hat{p}^M$ | Market price of capacity in the primary reserve market |
| $\hat{p}^{EM}$ | Difference between efficiency and market price of capacity in the primary reserve market |
| $\hat{p}_{th}^D$ | Price of electricity in hour $h \in H$ of stage stage $t \in T^D$ in the day-ahead market |
| $\hat{p}_t^D$ | Vector with $t \in T^D$ elements in which each element is given by $\hat{p}_{th}^D$ for $h \in H$ |
| $Q_i^{max}$ | Maximum production level of generator $i \in I$ |
| $Q_i^{min}$ | Minimum production level of generator $i \in I$ |
| $Q^{Pmax}$ | Maximum bid in the primary reserve market |
| $Q^{Pmin}$ | Lowest bid in the primary reserve market |
| $R_i^{up}$ | Maximum ramp-up rate of generator $i \in I$ |
| $R_i^{down}$ | Maximum ramp-down rate of generator $i \in I$ |
| $T_i^{on}$ | Minimum number of time periods generator $i \in I$ must remain on |
| $T_i^{off}$ | Minimum number of time periods generator $i \in I$ must remain off |

**Variables**

| | |
|---|---|
| $a_t$ | Decision made in stage $t \in T$ |
| $q_{tot}^P$ | Total capacity bid in the primary reserve market in stage t=1 |
| $q_{thi}^P$ | Primary reserve capacity reserved on unit $i \in I$ for $h \in H$ of stage $t \in T^D$ |
| $q_t^P$ | Vector with $t \in T^D$ elements in which each element is given by $q_{thi}^P$ for $h \in H$ and $i \in I$ |
| $q_{thi}^D$ | Energy bid into the day-ahead market to be delivered by unit $i \in I$ for $h \in H$ of stage $t \in T^D$ |
| $q_t^D$ | Vector with $t \in T^D$ elements in which each element is given by $q_{thi}^D$ for $h \in H$ and $i \in I$ |
| $p^P$ | Price of capacity bid in the primary reserve market for $t = 1$ |
| $u_{thi}$ | 1 if generator $i \in I$ is turned on from an off-state in hour $h \in H$ of stage $t \in T^D$, 0 otherwise |
| $y_{thi}$ | 1 if generator $i \in I$ is on in hour $h \in H$ of stage $t \in T^D$, 0 otherwise |

**Functions**

| | |
|---|---|
| $c_i(\cdot)$ | Cost function of generator $i \in I$ |
| $C_t(\cdot)$ | Contribution function in stage $t \in T$ |
| $f^{\hat{p}^R}(\cdot)$ | Probability density function of the relevant market price in the primary reserve market in stage $t = 1$ |
| $f^{\hat{p}^E}(\cdot)$ | Probability density function of the efficiency price in the primary reserve market in stage $t = 1$ |
| $f^{\hat{p}^{ME}}(\cdot)$ | Probability density function of the difference between the efficiency and the marginal price in the primary reserve market in stage $t = 1$ |
| $P^A(\cdot)$ | Probability of acceptance of a bid in the primary reserve market in stage $t = 1$ |
| $S^M(\cdot)$ | State transition function |
| $V_t(\cdot)$ | Value function in stage $t \in T$ |

# Contents

# 1 Introduction

Availability of electricity is central for modern society and security of supply is dependent on a continuous balance between electricity consumption and generation. In the past years, almost all areas within the European Network of Transmission System Operators for Electricity (ENTSO-E) have experienced an increasing number of frequency deviations, and an increase in the amplitude and duration of the deviations [1]. These patterns are most prominent at the change of hour in the morning and evening due to an increased power supply during these periods of the day. Frequency deviations are also the result of critical stochastic events such as outages of power plants and loads. The increased amount of reserves needed to counteract the deviations at the change of hour necessarily reduces the available reserves in case of stochastic events. The result is a tighter reserve limit and a risk of insufficient availability of reserves in the case of an outage.

Demand for reserves is also expected to grow in the years to come, due to a closer connected European power system and a larger share of non-flexible renewable, intermittent energy. The intermittent, renewable resources are characterized as being non-dispatchable, hence the generating units cannot be regulated in order to match changes in demand. Consequently, the remainder of the generation capacity has to complement the variability of the non-flexible resources [2]. This challenges, and will continue to change, the way power markets are operated.

Several markets have been established in order to ensure system security and keep the system in balance. In European power markets, the different ancillary services and energy markets are cleared in sequence. All markets provide revenue for power producers, but the amount bid in one market reduces the possible amount that can be bid in other markets. Hence, the bidding decisions are interdependent and the decision problem becomes increasingly complex. Coordinating the multimarket bidding decisions can increase the possible revenue of the power producers.

This report considers the unit commitment problem of a thermal power producer in Switzerland whose objective is to maximize profit by bidding into both the primary reserve market and the day-ahead market, taking uncertain prices into account. The problem is solved by using stochastic dynamic programming. The day-ahead market and the primary reserve market are cleared in a uniform price auction and a pay-as-bid auction respectively. The report concludes that there is potential to increase profits by coordinating bidding in the primary reserve market and the day-ahead market.

To our knowledge, no other work has analysed coordinated multimarket bidding within the Swiss primary reserve market and day-ahead market, and no other work uses stochastic dynamic programming to model multimarket bidding in these markets. The report contributes to the literature by assessing the value of optimal multimarket bidding in the primary reserve market and the day-ahead market. Moreover, the model can serve as a decision support tool for producers bidding in the primary reserve market.

In Section 2 relevant literature on bidding in the primary reserve market and day-ahead market is discussed. Section 3 introduces the Swiss power market and the operation of thermal power plants. The stochastic dynamic problem of coordinated bidding in the primary reserve market and the day-ahead market is presented in Section 4. Modeling of the pay-as-bid auction in the primary reserve market as well as an analysis of historical primary reserve prices are presented in Section 5. In Section 6 the methodology of scenario generation for uncertain historical prices in the day-ahead market is presented. In Section 7 the model is tested with anonymized and slightly modified data from a continental European thermal power producer. The case study investigates bidding in the primary reserve market and day-ahead market during a 12 day bidding period from 2015-02-03 to 2015-02-14. The week of delivery is from 2015-02-09 to 2015-02-15. The section further discusses sensitivity of the results. Section 8 presents the conclusion and discusses future work on coordinated bidding in the primary reserve market and the day-ahead market.

# 2 Literature

The changing market environment has lead to extensive research on building models for decision and analysis support that are consistent with the new market context [3]. In [3] three major trends are identified within the area; optimization models, equilibrium models and simulation models. Optimization models are models that consider a single supplier, whose objective is to maximize profit given either exogenous prices, or prices as a function of the producers decisions. Several models and solution procedures within this area have been proposed, some of which will be presented in the following.

Optimal bidding in the day-ahead market is modeled in many papers, and a review can be found in [4]. Bidding within the day-ahead market often consists of integrating optimal bidding with unit commitment. In [5] a model is developed to build optimal bidding strategies for a Nordic hydro producer taking the unit commitment decision and uncertainty into account. Other models discussing unit commitment are [6] and [7]. The unit commitment problem in [6] is formulated in a real time market setting, while [7] takes unit commitment constraints into account when using real options in valuation of power plants. A model for profit maximisation in a reserve market with pay-as-bid pricing rules is described in [8].

Since deregulation, an increasing number of markets have been established and market participants are becoming more aware of the opportunities seen within these markets [9]. In [9] the effects of committing several levels of regulation obligations in different regulation markets are investigated seen from a hydropower producer's perspective. Some of these markets are modeled with pay-as-bid auctions.

In [10] evolutionary programming is used to maximize profit for a supplier with several generators, taking both the deregulated day-ahead market and the reserve market into account. In [11] a genetic algorithm is proposed to be used in order to solve the coordinated bidding problem in situations with different start-stop conditions. Stochastic dynamic programming is used in [12] to schedule production in a hydroelectric system for a price-taking producer bidding energy and capacity in the day ahead market and primary reserve market, respectively. In [8] bidding in a competitive day-ahead market with uniform price auction and a reserve market with a pay-as-bid auction, in which the bidder has the opportunity to behave strategically, is considered.

In contrast to [10] and [11], [8] does not consider the bidding information of rival bidders to be known, leading to an approach in which the probability distribution of the market price is used to model the behaviour of the other bidders. The problem is furthermore solved analytically. Both [9] and [12] consider optimal multimarket bidding for a hydropower producer. The Swiss market is analyzed and prices are treated as deterministic parameters in [9]. In [12] the Norwegian market is analyzed and prices are treated as exogenous, stochastic variables in a stochastic dynamic problem.

# 3 Electricity markets in Switzerland

In this section, the structure of the Swiss power market is presented with a focus on the primary reserve market and the day-ahead market. Characteristics of a gas-fired thermal power plant are also discussed.

## 3.1 The liberalized Swiss power market

The Swiss power market became liberalized in 2008 when the Electricity Supply Act came into force. The act included the establishment of an independent regulator and an independent system operator, the Swiss Federal Office of Energy, SFOE, and Swissgrid respectively. Regulated third-party access to the grid and freedom to choose supplier was included in the liberalization process [13]. In the first phase only participants with production or consumption above 100 MWh were included in the liberalization. However, full market opening was realized in 2014.

To keep a secure supply of electricity in Switzerland, electricity is traded in multiple markets. The goal is that markets further away from operating time settle the majority of demand, while markets closer to operating time take care of the smaller deviations [14]. An overview of the Swiss electricity market is given in Table 3.1.

Table 3.1: Electricity markets in Switzerland [15], [16]

| Market Place | Physical trade | Financial trade | Time frame |
|---|---|---|---|
| EPEX spot | Day-ahead auction | - | 24 hours |
| | Intraday auction | - | 1 hour |
| Regelleistung | Primary reserves | - | 1 week |
| Swissgrid (TSO) | Primary reserves | - | 1 week |
| | Secondary reserves | - | 1 week |
| | Tertiary reserves | - | 4 hours and 1 week |
| EEX | - | Futures | > 6 years |
| | - | Options | > 6 years |

EPEX spot operates both the day-ahead auction and the intraday auction in several European countries, including Switzerland. Most of the electricity on EPEX spot is traded in the day-ahead auction. The intraday market offers an opportunity for the market participants to change their production schedules closer to real time. Both the day-ahead market and intraday market are open to cross-border trading between France, Germany, the Netherlands and Switzerland. A common European electricity market coupled with the PCR algorithm is also in the process of being implemented across Europe. The algorithm will be used to allocate capacity across borders in Europe and by this provide fair and transparent determination of day-ahead prices. This will ensure overall maximization of welfare and an increase in the transparency of computation of prices and flows [17].

In order to keep the system in balance and to ensure security of supply, the TSO has access to ancillary services, including frequency control, reactive power for voltage regulation, and black start capabilities [18]. The required amounts of ancillary services in Switzerland is set by the Union for Co-ordination of Transmission of Electricity (UCTE), and follow ENTSO-E requirements. Frequency control mechanisms can be divided into primary, secondary and tertiary reserves, as presented in Table 3.2. Primary reserves are activated automatically when the system experiences frequency deviations of ±200 mHz from the set standard of 50 Hz. Secondary reserves will be activated if the primary reserves are not able to handle the deviations, or it takes over in order to free primary reserves capacity. If needed, tertiary reserves can be manually activated within 15 minutes [19].

Table 3.2: Control reserves in Switzerland

| Reserve | Activation time | How | Supply |
|---|---|---|---|
| Primary reserves | Immediately | Automatically | +/- 90 MW |
| Secondary reserves | 30s | Automatically | Varies |
| Tertiary reserves | 15 min | Manually | Varies |

The European Power Exchange operates a derivative market where market participants can purchase futures and options in order to hedge against risks of price changes [16]. Maturities of months, quarters and years are offered, and the underlying price of the derivatives is the system price.

Electricity production in Switzerland is dominated by hydro power and nuclear energy. In 2014 hydropower plants contributed about 55% to overall electricity production, nuclear power plants contributed with about 41%, thermal power plants with about 3% and other renewable energy sources contributed with about 1% of overall electricity production [20].

## 3.2 Day-ahead market

Trading in the day-ahead market takes place until gate closure at 12:00 the day before delivery. Electricity is traded in hourly time intervals and the price and volume for each hour are settled by the intersection between the aggregated supply and demand curve. The auction for day-ahead energy is a uniform price auction, meaning that every producer whose bid are accepted receive the same price per MWh accepted. The prices bid on EPEX Spot must lie between -500 €/MWh and 3000 €/MWh. Historical day-ahead prices from 2007 to 2015 are presented in Figure 3.1

Market participants can bid energy in the day-ahead market as standalone bids, where volume and price for each hour are specified independently from hour to hour, or as different types of block bids. There are different block orders ranging from standard block orders to user defined block orders and smart blocks. A standard block order is an order for a set time interval, e.g $Night$ from 1am-6am. The market participants also have the option to bid for a set of linked hours of their own choice. Smart blocks consist of linked block orders or exclusive block orders. A linked block order is an order with a linked execution constraint, meaning that all bids in the block have to be accepted in order for the block to be executed. An exclusive block order is a set of block orders where a maximum of one of the block orders can be executed [19].

## 3.3 Primary reserve market

Imbalance between supply and demand will cause the frequency in the power grid to deviate from its setpoint value of 50 Hz. This will affect the behaviour of electrical equipment and can in the worst case lead to system breakdown [1]. Primary reserves are activated automatically within seconds when a deviation of $\pm 200$ mHz from the setpoint value arises in the system. Primary reserves can be provided by different types of power plants, including thermal and hydroelectric power plants, responsive loads and storage able to change output in a short period.

Primary reserves are purchased by the TSO to ensure secure operation of the power system. In Switzerland the demand for primary reserves is 71MW. Swissgrid procures 25 MW of this demand through the open tender process in the common primary reserve market where German, Austrian, Dutch and Swiss producers participate. Participating countries must provide a minimum of 90

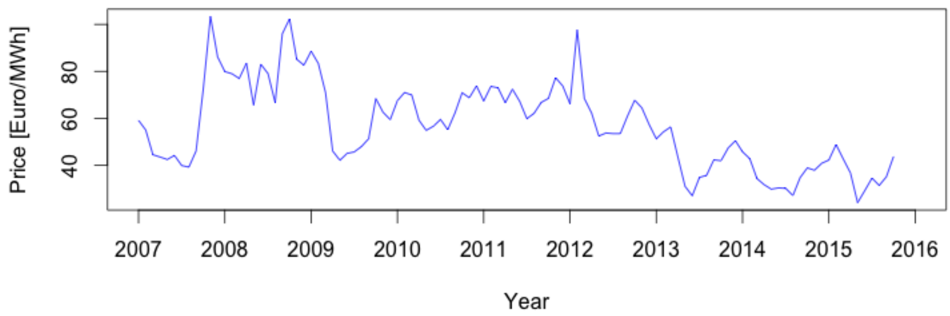


Figure 3.1: Average monthly day-ahead prices in Switzerland EPEX Spot, January 2007-October 2015 [21]

MW, but a maximum of 30% of its own demand for primary reserves. If 30% of a country's demand is less than 90MW, the country's suppliers must in total supply 90MW. For Switzerland this results in a total supply of 90MW. The remaining demand for primary reserves is covered by other requests for tender issued by Swissgrid [15]. In this report, only participation in the common market is considered. There is no obligation for producers to participate in the market.

Producers who wish to participate in the market must enter into a framework agreement with Swissgrid following a prequalification where the technical and operational status of their generators are assessed [15]. The generators are tested with a frequency deviation of ±200 mHz, and the power deviation 30 seconds later is measured. The aims of the tests is to find the deadband and droop of the generating unit. The droop determines how fast a unit can regulate its production up or down under frequency deviations. The droop, denoted $D$, is given as the ratio between the relative frequency change and the relative power change, as stated in Equation (1). The relative frequency change is given by the size of the frequency deviation with full activation of the primary reserves compared to the nominal frequency in the power system. The relative power change is given by the primary reserve capacity the unit has bid into the market. Hence, a higher droop setting means that the unit reacts less strongly to frequency deviations [22].

$$D = \frac{\frac{\Delta f}{f_{nom}}}{\frac{\Delta P}{P_{nom}}} 100\% \tag{1}$$

Primary reserve bidding takes place before 15:00 every Tuesday. The tendering period extends from Monday 00:00 until Sunday 24:00. The total volume of primary control power must be available without interruption in this period. Allowed bids run in increments of 1 MW starting at 1 MW. An upper limit on the tendering is individual for each supplier and is decided in the prequalification described above. The primary reserve capacity is based on a supplier's portfolio of power plants and there is no link between a specific generator and the primary reserve bid. Hence, a supplier can freely distribute the capacity over the portfolio of its generators, allowing the reserve capacity available at a single generator to vary within the delivery week. The TSO must be informed of which generators that are available, so that it is clear that the contracted reserves are available within the portfolio of the supplier [23].

The auction for primary reserves takes the form of a pay-as-bid auction where suppliers whose offers are accepted receive the price quoted. The suppliers only receive a capacity price for reserved capacity, hence producers are not remunerated for energy delivered [24]. The incentive of the supplier will in such an auction be to bid as close to the clearing price, the price of the maximum bid accepted in the auction, as possible [25].

The price of primary reserves vary throughout the year. In Figure 3.2, the average price for primary reserves in Switzerland from July 2012 until November 2015 are illustrated. The efficiency price is the lowest bid in the market and the marginal price is given by the highest accepted bid in the market.

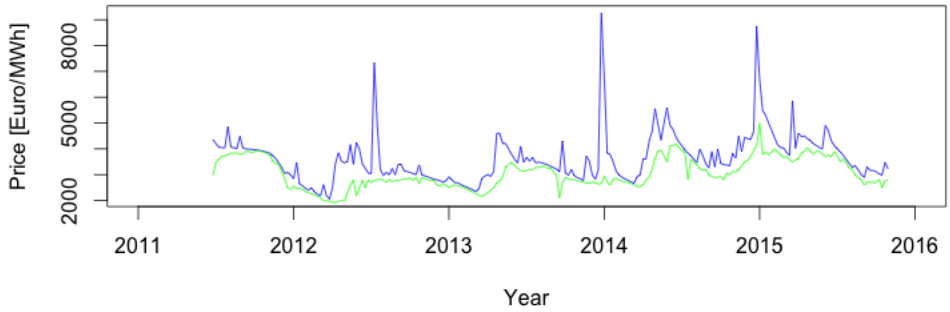


Figure 3.2: Weekly primary reserves prices, marginal price (blue) and efficiency price (green), 2011-06-27 to 2015-11-01 [26]

## 3.4 Thermal generators

In a thermal generator, energy from different sources is converted to electricity in high-pressure turbines. The choice of fuel, which can be either renewable or non-renewable, largely impacts the functioning, as well as the cost, of the thermal power plant. In this report, a thermal CCGT power plant consisting of a gas and steam turbine is taken into account, and is thus the focus of this section.

Gas-fired power plants can be divided into two sub-groups; combustion turbines and steam turbines. The technology of combustion turbines are based on the principle of expanding warm gas, while a steam turbine is based on expansion of steam. A steam turbine is characterized by being more efficient but less flexible than a combustion turbine. A combustion and steam turbine can also be combined in a combined cycle power plant (CCGT). By using waste heat from the gas combustion process to produce steam in a heat recovery steam generator, the two technologies together increase the efficiency of the plant [27]. A combustion turbine can be said to generally have an efficiency of 36-38%, a steam turbine an efficiency of 42-45% and a CCGT plant an efficiency of 55-58% [28]. Moreover, the emissions from a CCGT plant are lower and the plant is more flexible than a conventional thermal plant. An example of the topology of a CCGT plant is given in Figure 3.3.

Thermal power plants are designed and operated differently and the costs of each unit are unique. Characteristics of their operation are however similar. These characteristics restrict the operation of the power plant and affect both the efficiency and costs of generation.

A thermal power plant is characterized by a minimum and maximum rated capacity. Below the minimum level, a plant's operation is characterized by instability due to insufficient temperature and excessive emissions. Hence, optimal production lies above the minimum rated capacity and below the maximum rated capacity. Furthermore, a thermal power plant is characterized by having limited flexibility. This leads to restrictions on dispatch and unit commitment, meaning

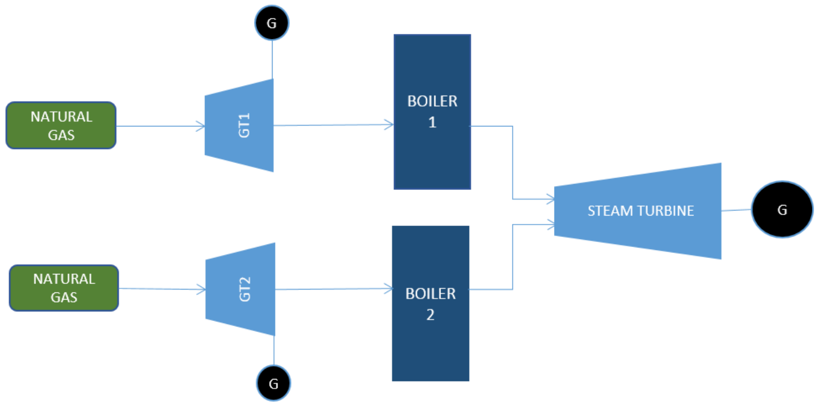


Figure 3.3: Illustration of plant topology, CCGT [29]

that there are restrictions on the possibility of changing the output of the generator and on changing the status of the generator by turning it on or off. The limited flexibility of the power plant is a consequence of thermal stress associated with cycling. During cycling, the boiler, steam lines, turbine, and auxiliary components experience large thermal and pressure stresses, which cause damage to the equipment [30]. Ramping rates, start-up and shut-down time and minimum up- and down-time are often used in order to model this limited flexibility. The ramping rate restricts the possibility to change generation. Start-up time represents the time it takes for a power plant to start-up and reach a stable state of operation. The status of the generator determines the time it takes for the power plant to start-up. Consequently, starting the power plant after the plant has completely cooled down requires more time than a warm or hot start. Minimum up- and down-time are used in order to take the economic limits of operations into account. These constraints should not be viewed as hard physical constraints, but as economic limits [27].

Thermal stress leads to higher operation and maintenance costs. There are costs associated both with ramping and start-up. Start-up costs can be broken down into direct costs, depreciated costs and lost profit [31]. The direct costs include fuel cost and cost of additional manpower due to an increased need to supervise the process. Depreciated costs include costs related to an increased need for maintenance and a shortening of the unit's life due to thermal stresses. Lost profits during start-up should also be taken into account.

Producers participating in the market for primary reserves need to take the above-mentioned characteristics into account when planning their bid. Operation when delivering primary reserves requires continuous changes in output. This increases the thermal stress on the plant and consequently the operation and maintenance costs. Units supplying primary reserves must furthermore be running in order to deliver primary reserves immediately when needed. Running on partial load reduces the efficiency of production and consequently increases the marginal cost [30].

The technical ramping rates mentioned above do not necessarily restrict the amount of primary reserves a thermal power plant can deliver. In the short term, gas-fired power plants are able to provide additional energy very quickly by releasing thermal energy stored in the generation process [27]. The amount of primary reserves a thermal power plant can deliver is given by the droop setting of the plant as discussed in Section 3.3. Primary reserves from thermal power plants can be delivered mainly in two ways; high pressure control valve dethrottling and condensate flow stoppage [32].

# 4 Problem formulation: Stochastic dynamic programming

The objective of the thermal power producer is to maximize profit when bidding in the day-ahead market and the primary reserve market. Bidding in the two markets is done sequentially. The primary reserve market is settled before bidding in the day-ahead market takes place. Hence, the capacity to be delivered in the primary reserve market restricts the bidding decisions that can be made in the day-ahead market.

The problem is a type of unit commitment problem. In a deregulated market the unit commitment problem consists of deciding on the amount to dispatch from every generating unit with the objective to maximise profit. This problem also includes decisions on when to start-up and shut-down the generating unit [6]. Unit commitment depends on the market rules and the physical and economical restrictions on the operation of thermal power plants as discussed in Section 3. Constraints on ramping, production limits and run-times are therefore taken into account. This requires binary variables the use of, which means that the unit commitment problem is a mixed integer problem.

Different solution techniques have historically been used in order to solve the unit commitment problem. A bibliographical survey covering unit commitment can be found in [33]. The survey furthermore gives an overview of solution methods for unit commitment problems. The methods range from theoretically complicated methods to simple rule-of thumb methods. Examples of methods are mixed-integer linear programming, Lagrangean relaxation, heuristics and dynamic programming. In this report, stochastic dynamic programming is used to solve the unit commitment problem taking both the primary reserve and day-ahead market into account. By using dynamic programming the multistage decision problem of the producer is solved by independently solving a sequence of simpler sub-problems. Advantages of dynamic programming are that the method can be used to solve many types of problems of varying sizes, and that the method can be modified in order to take problem-specific characteristics into account. On the contrary, the method makes it difficult to include constraints across stages, an issue dicussed in both [6] and [33]. This is further discussed in Section 7.1.

## 4.1 Assumptions

It is assumed that the producer operates a portfolio of thermal generators which may be of the same type. All generators have an availability of 100%, meaning that there is no probability of failure and there are no risks associated with not being able to deliver contracted capacity or energy.

The producer participates only in the day-ahead market and the primary reserve market. Demand in the primary reserve market is low compared to what a producer might deliver and the market is seen as having limited liquidity. One reason for this is that not all generators are capable of providing regulation reserves due to operational practice or lack of necessary equipment to follow the regulation signals [34]. This reduces the number of participants in the auction for primary reserves and the producer is consequently modeled with the possibility to affect prices in the primary reserve market. The market for day-ahead energy is on the contrary seen as deep, with no possibility to affect marginal prices. Hence, the producer is modeled as a price taker in the day-ahead market. The model takes only hourly bids in the day-ahead market into account, and block bids and linked hourly bids are left out.

It is assumed that the producer is risk neutral and seeks to maximize expected profit. Profit is considered only for the given period, and conditions before and after this period are not taken into account. This means that the optimization problem is not constrained by previous or later bidding decisions. Final electricity prices are exogenous information that first become known after bidding decisions are taken.

## 4.2 Stage, state and decisions

The stochastic dynamic problem is solved sequentially moving through the stages of the problem. A stage is defined as a day when a producer makes a bid for delivery of primary reserve capacity or day-ahead energy. The state is denoted by an index t=1,...,$|T|$, in which $T$ is the set of all stages. The stages in which day-ahead energy is bid are denoted by $T^D \subseteq T$. The reserved capacity resulting from the bidding decision in the primary reserve market in stage $t = 1$ will affect production for a period of one week. According to the market rules, the available primary reserve capacity must be the same in every hour in every stage of the period. In the day-ahead market, the bidding decisions and settlements occur on two different schedules, every day versus every hour respectively. Bidding in the day-ahead market takes place at t=2,...,$|T|$ and the respective settlements are made for each hour the following day. The hourly settlements will be denoted by index h=1,...$|H|$ , in which $H$ is the set of settlements within a day. The bidding process is illustrated in Figure 4.1.

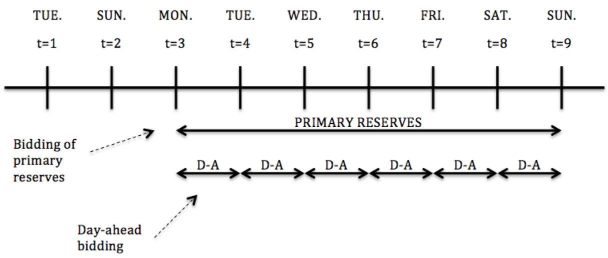


Figure 4.1: Illustration of the bidding decision and stages

In each stage, the state of the system is defined by a state vector $S_t = (\hat{p}^R, \hat{p}_t^D, q_t^P, q_t^D) \in S$ in which S is the state space, the collection of all states. The state is given by the price of capacity, $\hat{p}^R$, and the price in the day-ahead market, $\hat{p}_t^D$. The state variable $\hat{p}_t^D$ is a vector of $|H|$ elements in which each element $h$ corresponds to $\hat{p}_{th}^D$. The state is furthemore given by the states of the thermal generators, including the amount of capacity delivered, $q_t^P$, and the amount of day-ahead energy, $q_t^D$, they supply.The state variables $q_t^P$ and $q_t^D$ are two dimensional vectors of $|I|$ rows and $|H|$ columns in which each element $i, h$ corresponds to $q_{thi}^P$ and $q_{thi}^D$ respectively.

The decision to be made in the first stage is the amount and price of capacity the producer is to bid in the primary reserve market for a given week. The decision variables are denoted by $q_{tot}^P$ and $p^P$ respectively. This is decided by calculating the value of having all generators $i \in I$ reserve capacity $q_{thi}^P$ and deliver electricity $q_{thi}^D$ for all $h \in H$ in all stages $t \in T^D$. The aim is to decide the amount $q^P$ to bid in the primary reserve market in order to maximize expected profit. The values given to $q_{thi}^P$ and $q_{thi}^D$ in the optimal decision are decided under uncertainty. These values are given by the policy of the optimal solution, but should be reevaluated when there is no or less uncertainty. The decisions in every state will, when needed, be denoted by $a_t$ which is defined as given in Equation (2).

$$a_t = \begin{cases} \{p^p, q_{tot}^P\} & t = 1 \\ p_t^D & \forall t \in T^D \end{cases} \tag{2}$$

## 4.3 Objective function

The objective is, as previously mentioned, to maximize total profit for the thermal power producer bidding in the day-ahead market and the primary reserve market assuming random prices.

In the following, the objective function will be presented and its components will be discussed.

The contribution function for the problem is given by

$$C_t(a_t|S_t) = \begin{cases} P^A(\hat{p}^R > p^P; q^P_{tot})p^P q^P_{tot} & t = 1 \\ \\ \sum\limits_{i\in I}\sum\limits_{h\in H}[q^D_{thi}\hat{p}^D_{th} - c_i(q^D_{thi}) - c^S_i u_{thi}] & \forall t \in T^D \end{cases} \tag{3}$$

Giving an objective function

$$\underset{\pi\in\Pi}{max} \quad E[\sum_{t\in T} C_t(a_t|S_t)] \tag{4}$$

For most problems, solving this equation is computationally intractable. The objective function can however be expressed recursively by using the standard form of the Bellmann equation. For stochastic dynamic problems the expectation form of the Bellmann equation is often used [35], and will be used here as well. In this formulation, the sum over probabilities is given by expectation as stated in Equation (5).

$$V_t(S_t) = \begin{cases} \underset{q^P_{tot}}{max}(C_t(p^P, q^P_{tot}|S_t) + E\{V_{t+1}(S_{t+1})|S_t\}) & t = 1 \\ \underset{q^D_t}{max}(C_t(q^D_t|S_t) + E\{V_{t+1}(S_{t+1})|S_t\}) & \forall t \in T^D \end{cases} \tag{5}$$

The formulation often includes a discount factor but due to the short time horizon of the problem it will not be taken into account in this report. The value of being in stage $t$ is given by the value function $V_t(S_t)$. A value must be given to $V_{|T|+1}$ to solve the problem. In this case $V_{|T|+1} = 0$, since it is assumed that the current bidding period does not affect the next one.

The contribution function $C_t(a_t|S_t)$ is state dependent and the expected value of the next stage $E\{V_{t+1}(S_{t+1})|S_t\}$ is dependent on the state transition to $S_{t+1}$. The state transition function, $S^M(S_t, a_t)$, in a stochastic dynamic program is given by the current state, the decision made in the current state, and the outcome of the random parameters of the problem [35] and is described for this problem by Equation (6).

$$S_{t+1} = \begin{cases} S^M(S_t, p^P, q^P_{tot}, \hat{p}^R) & t = 1 \\ S^M(S_t, q^D_t, \hat{p}^D_t) & t \in T^D \end{cases} \tag{6}$$

To apply this model, the uncertain prices must be independent from stages before the immediately preceding stage for all $t$, meaning that $p^D_t$ is independent from previous prices except from the price $p^D_{t-1}$ in the stage before. Thus, the price has to fulfill the Markov Property.

The cost of day-ahead energy is given by a cost function dependent on delivered energy. There are no explicit costs associated with bidding and reserving capacity in the primary reserve market. The cost of bidding in the primary reserve market is the opportunity cost of not being able to bid the reserved capacity in the day-ahead market. Increased level of operation and maintenance (O&M) costs associated with delivering primary reserves, as discussed in Section 3.4, are not taken into account due to the time frame of the model and are thus the same whether or not primary reserves are delivered. Such costs could be taken into account in a study of the long-term costs and benefits of participating in reserve markets.

The generation cost function is often modeled as a quadratic function of the output [7]. It can however be modeled as a piecewise linear function to allow for the use of linear programming methods. There are several different methods proposed in the literature for linearizing functions. Using the most common method, the cost function for unit $i$ can be linearized as described by (7)-(10) [36].

The breakpoints of the cost function $c_i(q^D_{thi})$ are defined as $B_{ik}$ and lie in the interval [0, $Q^{max}_i$]. The set of breakpoints is denoted by $M_i$ for generator $i$, and the weight that is put on breakpoint $B_{ik}$ in hour $h$ of stage $t$ is denoted by $t_{thik}$.

$$l(c_i(q^D_{thi})) = \sum_{k\in M_i} c_i(B_{ik})t_{thik} \qquad \forall t\in T^D, h\in H, i\in I \tag{7}$$

$$q^D_{thi} = \sum_{k\in M_i} B_{ik}t_{thik} \qquad \forall t\in T^D, h\in H, i\in I \tag{8}$$

$$\sum_{k\in M_i} t_{thik} = 1 \qquad \forall t\in T^D, h\in H, i\in I \tag{9}$$

$$t_{thik} \geq 0 \qquad \forall t\in T^D, h\in H, i\in I, k\in M_i \tag{10}$$

Only two adjacent $t_{thik}$ can be larger than 0. This is taken care of by the problem formulation itself, due to the maximization of revenue in the objective function. Revenue is maximized when the cost is minimized, thus two adjacent breakpoints will be the optimal solution since the cost function is convex.

The generator cannot generate electricity immediately after starting up. An idle thermal generator will need a specified time to start-up. The start-up costs are given by the term $C^S_i u_{thi}$, where $u_{thi}$ is equal to 1 when generator $i$ is turned on.

## 4.4 Constraints

A producer whose bid in the primary reserve market has been accepted is obligated to supply $q^P_{tot}$ at all times within the bidding period. This means that the available primary reserves from all units in the portfolio has to be equal to $q^P_{tot}$ for all hours within each stage as described by (11).

$$\sum_{i\in I} q^P_{thi} = q^P_{tot} \qquad \forall t\in T^D, h\in H \tag{11}$$

Each generator has a minimum and maximum rated capacity. This has to be taken into account when bidding into both the primary reserve market and the day-ahead market. A unit providing primary reserves must be on, and it must have the ability to both decrease and increase production by the amount of capacity the generator has reserved in the primary reserve market.

$$q^D_{thi} + q^P_{thi} \leq Q^{max}_i y_{thi} \qquad \forall t\in T^D, h\in H, i\in I \tag{12}$$

$$q^D_{thi} - q^P_{thi} \geq Q^{min}_i y_{thi} \qquad \forall t\in T^D, h\in H, i\in I \tag{13}$$

Constraint (12) states that the maximum energy bid in the day-ahead market and the primary reserve market every hour in every stage has to be less than the maximum capacity of the thermal generator. The same is governed by constraint (13) with respect to the minimum capacity of the generator.

In order to change the generated power of a thermal power plant, thermal and mechanical inertia in the system have to be overcome [37]. As explained, there are consequently ramping constraints associated with the operation of the thermal generator.

$$q^D_{thi} - q^D_{t(h-1)i} \leq R^{up}_i + (Q^{min}_i - R^{up}_i)(1 - y_{t(h-1)i}) \qquad \forall t \in T^D, h \in H \setminus \{1\}, i \in I \tag{14}$$

$$q^D_{t1i} - q^D_{(t-1)24i} \leq R^{up}_i + (Q^{min}_i - R^{up}_i)(1 - y_{(t-1)24i}) \qquad \forall t \in T^D \setminus \{2\}, i \in I \tag{15}$$

$$q^D_{t(h-1)i} - q^D_{thi} \leq R^{down}_i + (Q^{min}_i - R^{down}_i)(1 - y_{thi}) \qquad \forall t \in T^D, h \in H \setminus \{1\}, i \in I \tag{16}$$

$$q^D_{(t-1)24i} - q^D_{t1i} \leq R^{down}_i + (Q^{min}_i - R^{down}_i)(1 - y_{t1i}) \qquad \forall t \in T^D \setminus \{1\}, i \in I \tag{17}$$

Constraints (14) and (16) take care of ramping restrictions within the stage, while (15) and (17) handle these restrictions across stages. The ramping restrictions are only defined for production within the minimum and maximum production levels given by constraints (12) and (13). The reason for this is that when the maximum ramping rates are smaller than minimum production, (12) and (13) restrain the model from ramping production down below $Q^{min}_i$ to turn a unit off, or up to $Q^{min}_i$ when it is turned on. This is solved by constraining a unit $i$ to not produce when generation falls below $Q^{min}_i$. In reality a unit would gradually ramp down to or up from zero production. The costs incurred in this process can be included in the start-up costs.

The dynamic programming algorithm cannot handle ramping restrictions across stages in a straightforward manner. This can be taken care of by discretizing the production variables [7]. Note however that this will increase the state space of the problem and moreover increase the computational burden [37].

Costs arise when units are turned on/off and this is handled by constraints (18)-(20). These costs arise when a unit shifts from an off-state to an on-state, and $u_{thi} = 1$. Constraint (18) models this for units that are turned on within a stage and (19) takes care of the constraints between stages. The generator is assumed to be in the same on/off state in the first hour in the first stage as it was in the last hour the day before, which is given by (20).

$$y_{thi} - y_{t(h-1)i} \leq u_{thi} \qquad \forall t \in T^D, h \in H \setminus \{1\}, i \in I \tag{18}$$

$$y_{t1i} - y_{(t-1)24i} \leq u_{t1i} \qquad \forall t \in T^D \setminus \{2\}, i \in I \tag{19}$$

$$u_{11i} = 0 \qquad \forall i \in I \tag{20}$$

The generating units have to be on/off for a given time when turned on and off. This is modeled with minimum up- and downtime constraints specifying the time the unit has to be on/off, following [6].

$$y_{thi} - y_{t(h-1)i} \leq y_{tki} \qquad \begin{aligned} \forall t \in T^D, i \in I, h \in H \setminus \{1, |H| - T^{on}_i + 2, ..., |H|\}, \\ k = h+1, ..., h + T^{on}_i - 1 \end{aligned} \tag{21}$$

$$y_{t1i} - y_{(t-1)24i} \leq y_{tki} \qquad \forall t \in T^D \setminus \{2\}, i \in I, k = 2, .., T^{on}_i - 1 \tag{22}$$

$$y_{thi} - y_{t(h-1)i} \leq y_{(t+1)ki} \qquad \forall t \in T^D \setminus \{|T|\}, i \in I, h = |H| - T_i^{on} + 2, ..., |H|, \\ k = 1, ..., T_i^{on} - |H| + h - 1 \tag{23}$$

$$y_{t(h-1)i} - y_{thi} \leq 1 - y_{tki} \qquad \forall t \in T^D, i \in I, h \in H \setminus \{1, |H| - T_i^{off} + 2, ..., |H|\}, \\ k = h + 1, ..., h + T_i^{off} - 1 \tag{24}$$

$$y_{(t-1)24i} - y_{t1i} \leq 1 - y_{tki} \qquad \forall t \in T^D \setminus \{2\}, i \in I, k = 2, .., T_i^{off} - 1 \tag{25}$$

$$y_{t(h-1)i} - y_{thi} \leq 1 - y_{(t+1)ki} \qquad \forall t \in T^D \setminus \{|T|\}, i \in I, h = |H| - T_i^{off} + 2, ..., |H|, \\ k = 1, ..., T_i^{off} - |H| + h - 1 \tag{26}$$

Constraints (21)-(23) handle the minimum up-time, and (24)-(26) handle the minimum down-time of the units. Constraint (21) models the minimum up-time within stages, while (22) models the minimum up-time between the first hour in the current stage and the last hour in the stage before. Constraint (23) models the minimum up-time between the current stage and the next stage. (24)-(26) are similar; (24) is the within-stage down-time constraint while (25) and (26) take care of the special conditions between stages.

$$p^P \geq 0 \tag{27}$$
$$q_{tot}^P \geq 0 \tag{28}$$
$$q_{thi}^P \geq 0 \qquad \forall t \in T^D, i \in I, h \in H \tag{29}$$
$$q_{thi}^D \geq 0 \qquad \forall t \in T^D, i \in I, h \in H \tag{30}$$
$$u_{thi} \in \{0, 1\} \qquad \forall t \in T^D, i \in I, h \in H \tag{31}$$
$$y_{thi} \in \{0, 1\} \qquad \forall t \in T^D, i \in I, h \in H \tag{32}$$

Constraints (27)-(32) handle the domain restrictions of the variables.

# 5 Primary reserves pay-as-bid auction

Studies of pay-as-bid auctions in electricity markets have used different approaches. In [38] a method that uses a combination of particle swarm optimization and simulated annealing is used in order to predict bidding strategies of other suppliers in a setting of incomplete information. In [39], [40] and [41] uniform and discriminatory (pay-as-bid) pricing rules are compared. In [40] a model-based approach is used in order to model the pay-as-bid auction and in [41] a combination of game theory and auction theory is used. In this report, the approach in [25] and [8] is used.

In this section, the theory behind modeling the reserve market pay-as-bid auction as described in [25] and [8] is explained, and a Gaussian-mixture model and an Erlang distribution are fitted to the historical primary reserve prices in Switzerland. Distribution fitting and testing is conducted in the statistical software R applying the maximum likelihood method. Historical prices of primary reserve are from [26].

## 5.1 Pay-as-bid modeling

The price that is received for an accepted bid in a pay-as-bid auction is the bid quoted. This means that a bidder should bid as close to the marginal price as possible. The marginal price is the price of the highest accepted bid. To find the optimal bid, the probability of acceptance given by $P^A(\hat{p}^M > p^P)$ needs to be taken into account. This is the probability that the quoted price of the bid $p^P$ is lower than the marginal price $\hat{p}^M$.

In a market with perfect competition the marginal price will not be affected by the size of the capacity, $q^P_{tot}$, bid by a single bidder. However, in a market where bidders possess market power their bid can influence the price. In Switzerland the pay-as-bid market can be said to have limited liquidity. As explained in Section 3.3, a producer must be prequalified before he can participate in the primary reserve market. Not all units have the technical or operational possibility to fullfill these requirements. The cost of prequalification furthermore vary from case to case [24]. These factors create barriers for some producers to enter into the primary reserve market, and can explain the low number of participants in the auction for primary reserves. This indicates that it can be reasonable to take into account bidders possibility to affect the market prices. The marginal price resulting from the price and capacity bid, is denoted the relevant market price and is given by $\hat{p}^R$.

The probability of acceptance, considering that a bid is accepted only, and entirely, if the relevant market price is higher than the bidding price is given by Equation (33), where $f^{\hat{p}^R}(p^P; q^P_{tot})$ is the probability density function of the relevant market price given by (34). The density function, $f^{\hat{p}^R}(p; q^P_{tot})$, of the relevant market price is found by applying a single-sided convolution of the probability density function of the efficiency price, $f^{\hat{p}^E}(p)$, and the probability density function of the difference between marginal and efficiency price ,$f^{\hat{p}^{ME}}(p)$. The efficiency price is the lowest price bid in the auction.

$$P^A(\hat{p}^R > p^P; q^P_{tot}) = 1 - \int_{-\infty}^{p^P} f^{\hat{p}^R}(p; q^P_{tot})\mathrm{d}p \tag{33}$$

$$f^{\hat{p}^R}(p^P; q^P_{tot}) = \int_0^{\infty} f^{\hat{p}^E}(p^P - k(q^P_{tot})u) f^{\hat{p}^{ME}}(u)\,\mathrm{d}u \tag{34}$$

The term $k(q^P_{tot}) \in [0, 1]$ in Equation (33) is the linear index of the merit order given by Equation (35). $Q^{Pmax}$ and $Q^{Pmin}$ represent the size of the primary reserve market, $Q^{Pmax}$ being the maximum amount of MW bid into the market and $Q^{Pmin}$ being the minimum amount of MW in the market. In Switzerland these are 90 MW and 1 MW respectively. Hence, the linear index represent the inverse of the market share of the producer.

$$k(q_{tot}^{P}) = \frac{Q^{Pmax} - q_{tot}^{P}}{Q^{Pmax} - Q^{Pmin}} \tag{35}$$

Using the probability of acceptance the price and capacity that maximizes expected profit of bidding in the primary reserve market as stated in the contribution function, given by Equation (3), can be found. The effect of the suppliers possibility to affect the market price in subsequent periods is not considered.

Using this approach and considering the assumption of risk neutrality of the producer, a single bid is sufficient for optimized bidding. It is not necessary to submit a discrete supply curve as is often proposed. A bid will only be accepted if it is advantageous to the procurer and if it displaces the offers of the competing bidders. Bidding more than one would result in displacing another bid on the bidder's own supply curve.

The derivation of (33)-(35) follows from [25] and [8] and can be found in the appendix.

## 5.2 Probability distributions of primary reserve prices in Switzerland

Figure 5.2 illustrates the historical prices in the primary reserve market in the period 2011-06-27 to 2015-11-01. The marginal and efficiency price are seen to follow each other closely, but the marginal price exhibit more price spikes than the efficiency price. Both the marginal and efficiency prices vary with the season; prices fall during late winter/early spring before they increase towards the summer and autumn.

The case study in Section 7 investigates bidding in the primary reserve market and the efficiency market during a 12 day bidding period from 2015-02-03 to 2015-02-14 Due to the small amount of available data relevant for the bidding week of the second week of February, all data available from the period 2011-06-27 to 2015-11-01 are used in the analysis of primary reserve prices. The component model of time series, states that a time-series of prices can be decomposed into four components; a seasonal component (S), a cyclic component (C), a trend component (T) and an irregular component (I). Hence using all available historical data, mean that the data have to be adjusted. Cycling and trend have not been taken into account, but the data have been seasonally adjusted. As seen in Figure 5.1, the two time series are highly cross-correlated. In order to keep the correlation between the two time series, only the efficiency price has been seasonally adjusted. The difference between the marginal price and efficiency price is therefore assumed to be represented by the original distribution. The seasonal adjustment is done by multiplicative monthly adjustment by dividing the weekly value of the primary reserves time series with the corresponding monthly seasonal index. The monthly seasonal index represents the normal, typically observed values within the given season and is found by dividing the average monthly primary reserve prices on the overall average for the year. The seasonally adjusted time series, adjusted to February price level, are illustrated in Figure 5.3. Price characteristics of both the original and seasonally adjusted time series are given in Table 5.1.

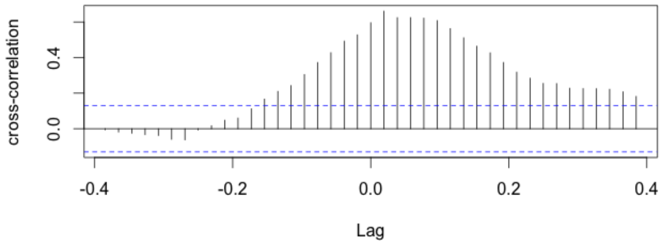


Figure 5.1: Cross correlation between the efficiency and marginal price, 2011-06-27 to 2015-11-01

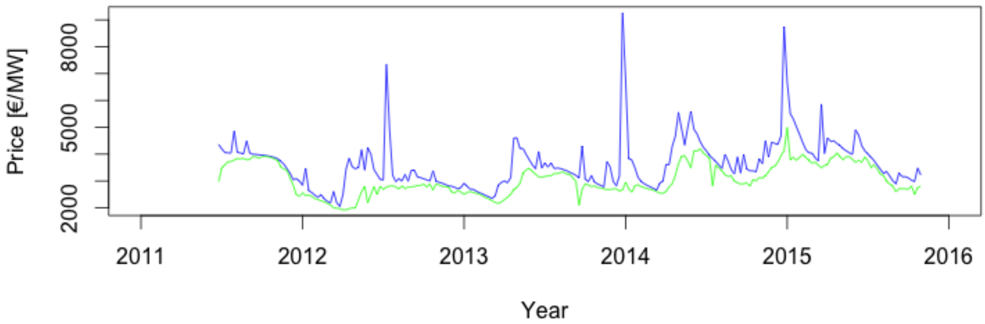


Figure 5.2: Weekly historical primary reserves prices, marginal price (blue) and efficiency price (green), 2011-06-27 to 2015-11-01. [26]

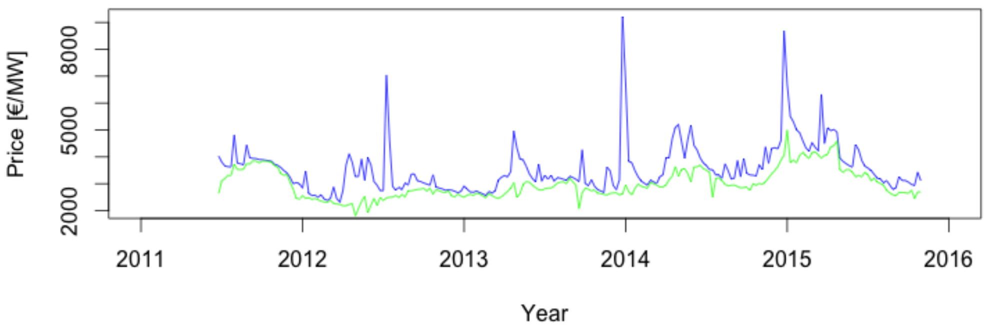


Figure 5.3: Weekly seasonally adjusted primary reserves prices, marginal price (blue) and efficiency price (green), 2011-06-27 to 2015-11-01 [26]

By further analysing the historical price data in the primary reserve market, the distribution of $\hat{p}^E$ and $\hat{p}^{ME}$ that are used in order to find the distribution of the relevant market price are estimated. Histograms of the seasonal adjusted time series are given in Figure 5.4. The difference between the marginal and efficiency price is positively skewed to the right and can be modeled

Table 5.1: Price characteristics of historical and seasonally adjusted data

| **Price** | **Mean** | **Median** | **Min** | **Max** |
|---|---|---|---|---|
| Marginal price | 3651 | 3463 | 2052 | 9253 |
| Efficiency price | 3060 | 2888 | 1925 | 4995 |
| Marginal price* | 3578 | 3328 | 2314 | 9210 |
| Efficiency price* | 2987 | 2859 | 1805 | 4995 |

by a log-normal or gamma distribution. The distribution of the efficiency price is seen to consist of two modes with two distinct spikes. Hence, a multimodal normal probability density functions with two modes can be seen to fit the data. If the distributions are to be integrated analytically, the choice of probability density functions representing the efficiency price and the difference between the marginal and efficiency price is limited. In [8], this is taken care of by modeling efficiency prices with a Gaussian-mixture model with two modes and the difference between the marginal and efficiency price with an Erlang distribution. The Erlang distribution is a variant of the gamma distribution with integer shape. The shape parameter of the Erlang distribution in [8] is two, facilitating analytical integration of the convolution. These distributions are seen fit the data of the Swiss market well, and are consequently used to model the efficiency price and the difference between the efficiency and marginal price in the Swiss market.

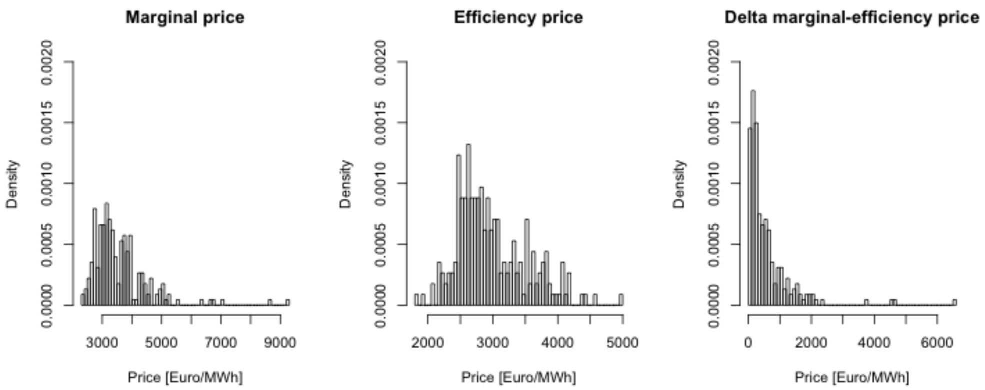


Figure 5.4: Log-transformed, seasonal adjusted primary reserves prices, marginal price (blue) and efficiency price (green), 2011-06-27 to 2015-11-01

The probability density function of the multimodal normal probability distribution is given by Equation (36) and the probability density function of the Erlang distribution is given by Equation (37). The probability distributions are fit to the historical data of the efficiency price and the difference between the marginal and efficiency price respectively. The resulting characteristics of the fitted distributions are given in Table 5.2 and 5.3. Histograms of the historical data with the approximated distributions are illustrated in Figure 5.5.

$$f^E(p) = \sum_{m \in M} \lambda_i \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(p-\mu)^2}{2\sigma^2}} \tag{36}$$

$$f^{\Delta ME}(p) = \frac{p}{b^2} e^{-\frac{p}{b}} \tag{37}$$

Table 5.2: Multimodal normal distribution

| Mode | Mean | SD | Lambda | Log-lik |
|---|---|---|---|---|
| Mode 1 | 2665.75 | 234.43 | 0.48 | -1730.504 |
| Mode 2 | 3286.90 | 577.30 | 0.52 | -1730.504 |

Table 5.3: Erlang distribution

| Mode | Shape | Scale | Log-lik |
|---|---|---|---|
| Erlang distribution | 2 | 295.60 | -1711.825 |

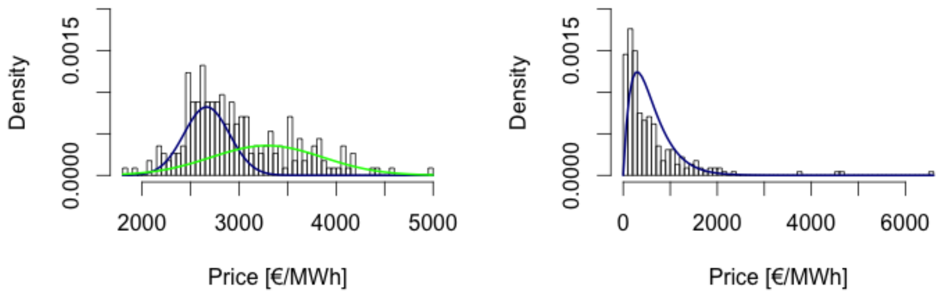


Figure 5.5: Probability density functions, efficiency price (left) and difference between marginal and efficiency price (right)

# 6 Day-ahead price scenario generation

Forecasting electricity prices has become a more prominent task after deregulation due to the risks associated with volatile prices. Volatility, often correlated with price level, is one on of the key features often observed within electricity markets. Other features observed within electricity markets are seasonal patterns, periodicity, price spikes, mean reversion and long-term non-stationarity [42]. Seasonal patterns and periodicity are closely connected to variations in demand. Price spikes with following mean reversion is often observed due to e.g stochastic events such as power plant outages with following recovery. Due to uncertainty in the long term, prices seem to follow a non-stationary model in the long-term, meaning that the statistical properties of the time series are non-constant over time. In the short term prices are however seen to be stationary. Different forecasting methods reflecting these characteristics have been proposed in the literature. The methods can be divided in three main groups; game based models, simulation based models and time series models [43]. Time series models uses historical behaviour of electricity prices in order to forecasts future price development, and is used in order to model day-ahead prices in this report.

Prices in a stochastic dynamic program are required to follow, and hence to be modeled, as Markov processes. This means that $\hat{p}_t^D$ only depends on $\hat{p}_{t-1}^D$. This limits the possible models that can be used in modeling and forecasting day-ahead prices. The first order autogregressive model, AR(1), is consistent with the Markov Property and is used to model and forecast day-ahead prices within the Swiss market. A scenario generating procedure based on optimal discretization by the use of nested distance is thereafter applied in order to discretize the distribution of prices. Price modeling is conducted in the statistical software R applying the maximum likelihood method. Historical day-ahead prices are from [21].

## 6.1 Day-ahead price modeling

The average day-ahead price is given in Figure 6.1. From the figure, it can be seen that day-ahead prices before 2010/2011 were significantly higher than prices following 2011/2012. The reasons for this can be the recent deregulation and the economic recession following the financial crisis. Due to this, only prices from 2011 have been considered relevant in analysing, modeling and forecasting electricity day-ahead prices.

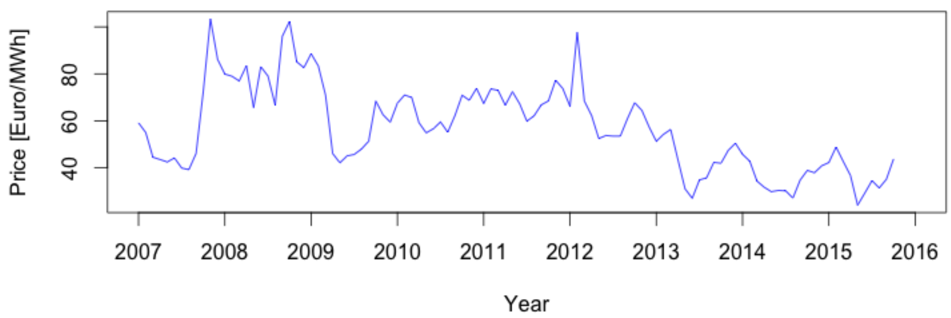


Figure 6.1: Average monthly day ahead prices in Switzerland EPEX Spot, January 2007-October 2015 [21]

Modeling day-ahead prices, historical values during six weeks of January/February in the period 2011-2015 have been taken into account. The historical prices and their characteristics are illustrated in Figure 6.4 and in Table 6.1 respectively. By using historical data for January only, only data most relevant for the chosen period is taken into account and hence no seasonal or cyclic adjustment is needed. Trend adjustment is neither conducted, but could be included in a future study. Taking only separate weeks into account however means that the months have to be treated as separate time series. Since the hourly prices for the next day are revealed at the same time, the hourly day ahead prices are not characterized as a time series. Hourly prices are in fact a panel of 24 cross-sectional hours that vary from day to day [44]. Thus the time series of the day-ahead prices are modeled on a daily basis.

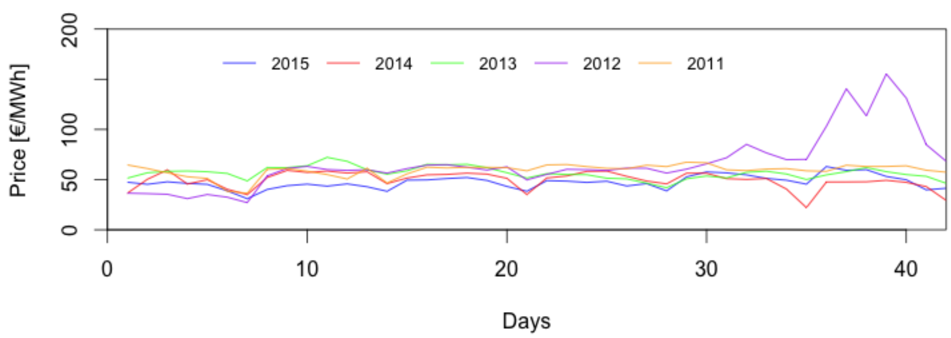


Figure 6.2: Daily prices January/Febuary, 2011-2015

Table 6.1: Price characteristics, January/February 2011-2015

| **Date** | **Mean** | **Median** | **Min** | **Max** |
|---|---|---|---|---|
| 2015-01-05 to 2015-02-15 | 47.39 | 47.29 | 30.98 | 62.99 |
| 2014-01-06 to 2014-02-16 | 49.37 | 51.00 | 22.08 | 59.65 |
| 2013-01-07 to 2013-02-17 | 56.58 | 56.46 | 41.93 | 72.18 |
| 2012-01-01 to 2012-02-12 | 66.24 | 61.09 | 27.04 | 155.30 |
| 2011-01-03 to 2011-02-13 | 59.13 | 60.98 | 36.45 | 67.40 |

The AR(1) model is given by Equation (38), where $\epsilon_t$ is i.i.d white noise with 0 mean, $E(\epsilon_t) = 0$, and a finite variance. $E(\epsilon_t^2) = \sigma^2$. The model assumes no correlation within the time series, $(\epsilon_t.\epsilon_s) = 0$. The time series has to be stationary in order to use this model, and testing the separate time series with the Augmented Dickey-Fuller test indicate stationarity.

$$y_t = c + \phi_1 y_{t-1} + \epsilon_t \tag{38}$$

A common convention in the literature is to model prices in electricity markets as log-prices. A log transformation is applied in order to attain a more stable variance. Moreover, if prices had been negative, it would constrain the price to be positive. Day-ahead prices are consequently transformed to log-prices before the parameters of the AR(1) model are estimated. The parameters of the AR(1) model fitted to historical log-data for six weeks in January/February in 2011-2015 are given in Table 6.2. Note that the model for 2015 is based on historical prices for only the four weeks of January up until the Tuesday the week before delivery. This is due to the analysis focusing on bidding into the Primary reserve market and day-ahead market for the the second week of February, 2015-02-09 to 2015-02-15. The quality of the model is estimated by the log-likelihood values and t-values. Estimation of t-values and p-values indicate that the significance of the parameters are high, rejecting the null-hypothesis that the coefficient have no effect. The plot of the ACF of the residuals in Figure 6.3 furthermore illustrates the randomness of white noise of the fitted distributions. Note that the ACF plot in R always plot the 0-lag value as 1.

Table 6.2: AR(1) model, January/February 2011-20155

| **Date** | **Param.** | **Estimate** | **Std. error** | **Log-lik** | **t-value** | **p-value** |
|---|---|---|---|---|---|---|
| 2015-01-05 to 2015-02-03* | $\phi_1$ | 0.4148 | 0.1764 | 23.54 | 2.35 | 0.0263 |
| | c | 3.8185 | 0.0339 | 23.54 | 112.64 | 0.0001 |
| 2014-01-06 to 2014-02-16 | $\phi_1$ | 0.3474 | 0.1612 | 10.13 | 2.16 | 0.0370 |
| | c | 3.8718 | 0.0448 | 10.13 | 86.42 | 0.0001 |
| 2013-01-07 to 2013-02-17 | $\phi_1$ | 0.6879 | 0.1158 | 46.40 | 5.94 | 0.0001 |
| | c | 4.0165 | 0.0380 | 46.40 | 105.70 | 0.0001 |
| 2012-01-01 to 2012-02-12 | $\phi_1$ | 0.8803 | 0.0684 | 12.14 | 12.90 | 0.0001 |
| | c | 4.0660 | 0.1993 | 12.14 | 20.40 | 0.0001 |
| 2011-01-03 to 2011-02-13 | $\phi_1$ | 0.5517 | 0.1259 | 35.42 | 4.38 | 0.0001 |
| | c | 4.0747 | 0.0347 | 35.42 | 117.4 | 0.0001 |

As seen from table 6.2, the parameters of the price paths vary some from year to year. The mean value of $\phi_i$ is 0.5764 and the mean value of $c$ is 3.9695. The parameters of the AR(1) model for 2015 can be seen to lie not far from the mean values of the parameters. Moreover, they lie in an

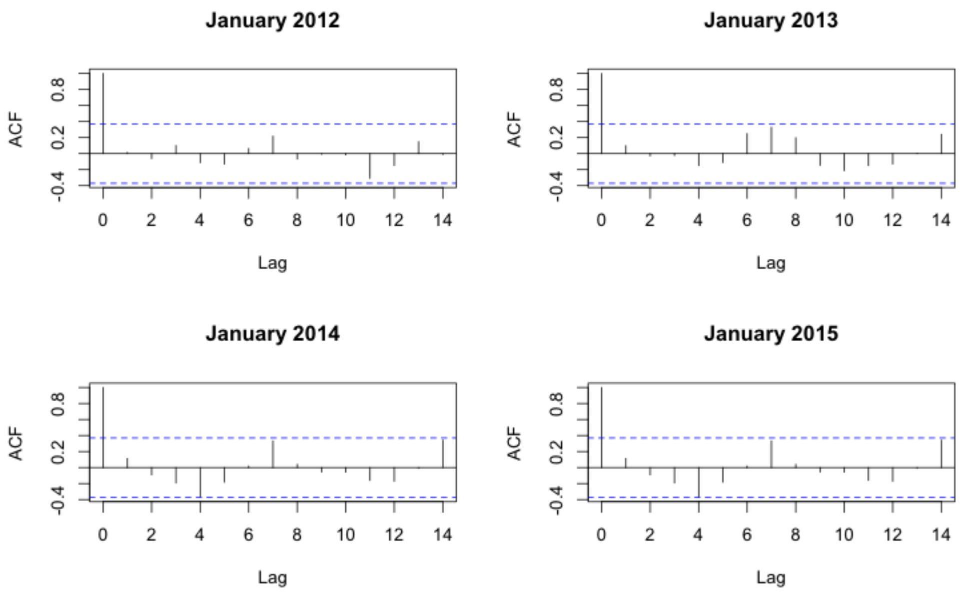


Figure 6.3: ACF plot of residuals from AR(1) model

interval between the values of the parameters from previous years. The price path for 2015 can also be seen from Figure 6.4 to follow the price paths from previous years, except from the last days of February 2012. Due to this and also the fact that the historical prices used to estimate the 2015 AR(1) model are closer in time to the bidding week, the forecast for the bidding week is based on the historical 2015 AR(1) model.

Bidding into the primary reserve market takes place Tuesday the week before the week of delivery, and hence prices for 12 consecutive days following the bidding day are forecasted. 5000 individual price paths are simulated by the AR(1) model for 2015 and Figure 6.4 illustrates 25 of the simulated price paths.

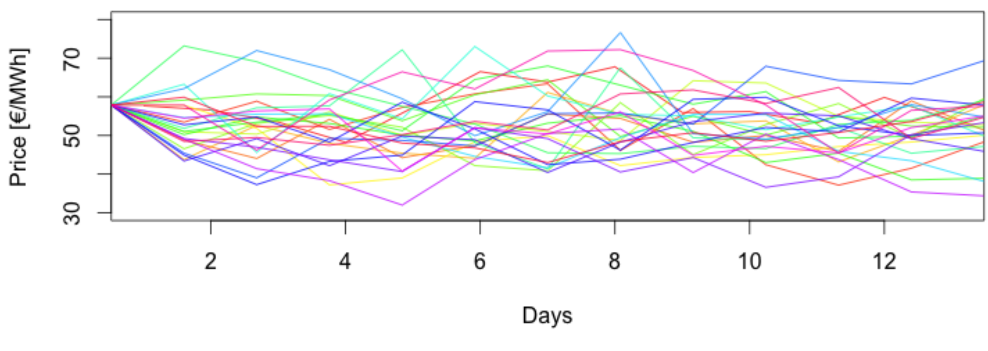


Figure 6.4: Simulated prices, 2015-02-03 to 2015-02-14

## 6.2 Multinomial lattice

A stochastic dynamic program cannot, except in a few cases, be solved with a continuous distribution. The stochastic variables consequently have to be discretized in a discrete distribution with a limited set of outcomes. One of the main challenges of scenario generation, is the trade-off between a good approximation of the continuous distribution, and the dimension and the complexity of the model. The solution of the stochastic dynamic program will be an approximation of the real phenomena and the quality of the scenario generation algorithm largely affects the results of the stochastic problem.

The stochastic dynamic problem in this report is solved on a multinomial lattice. This is a structure that models the stochastic dynamics of the problem and the evolution of information. There is a finite number of outcomes of the price at each stage, all with a given probability of occurrence. A lattice is similar to a scenario tree, but considers identical children for all nodes at the same stage [45]. The result is that the number of variables does not grow exponentially as it does in a scenario tree. The number of possible paths in the lattice is however large. An illustration of the lattice is given in Figure 6.5.

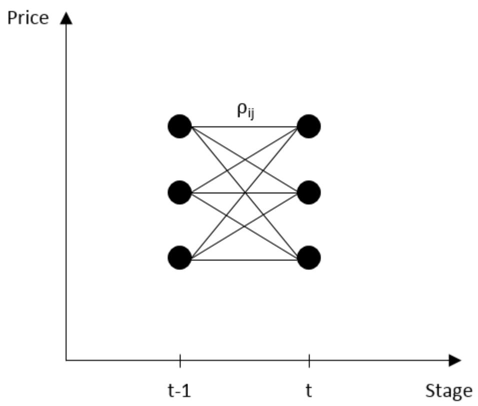


Figure 6.5: Illustration of lattice

There are several methods for generating scenario trees and lattices. An overview can be found in [46]. The best method will secure a low computational complexity and a high approximation quality. The scenario generation method chosen in this paper is based on the concept of nested distance and the work of Pflug and Pichler in [45], [47] and [48]. The method is a non-parametric, optimal discretization method. A high level of approximation quality is secured through the use of nested distances, which is a generalization of the Wasserstein distance for stochastic two-stage problems [47].

The goal of the model is to minimize the distance between the approximated stochastic process and the real process. For a two-stage problem this can be described by Inequality (39) where L is a Lipschitz constant of the cumulative distribution function and $d(\tilde{\xi_T}, \xi_t)$ is the Wasserstein distance. The Wasserstein distance is used for optimal discretization of the stochastic process due to its nice relations to the distance of the objective function [48]. The goal of the scenario generating algorithm is to minimize the Wasserstein distance.

$$e_f(\tilde{\xi_T}, \xi_t) \leq 2\underset{x}{sup}[F(x; \xi_t) - F(x; \tilde{\xi_T}] \leq 2Ld(\tilde{\xi_T}, \xi_t) \tag{39}$$

Complete data series generated from the AR(1) model described in Section 6.1 are used as input to the lattice. A Matlab script developed by Pichler is thereafter used in order to generate the lattice given in Figure 6.7 [49]. The script calculates the resulting distance of the scenario generation procedure to be 6.4157. The aim of the scenario generating procedure is to minimize this distance. The algorithm for nested distance will be optimal given infinitely many input-scenarios. Having only 5000 scenarios as input consequently affect the result of the procedure. A larger stepsize used in finding the optimal clusters, i.e pricespoints, between each scenario reduces the distance, but gives an uneven scenario tree with outliers. Hence, there is a trade-off between minimizing the distance and obtaining an even tree which approximates the real values in a good way.

The number of states per stage is chosen as input to the scenario-generating algorithm. The number of states per stage affects the resulting shape of the lattice and the nested distance. The approximation of the solution space should be small enough to allow for an efficient numerical solution but also secure a small approximation error. Having more states decreases the distance and contributes to creating a more even tree. However, the time taken to solve a problem on a scenario lattice increases with the number of states. Due to each bidding-day in the day-ahead market being similar, the number of states in each stage is chosen to be the same.

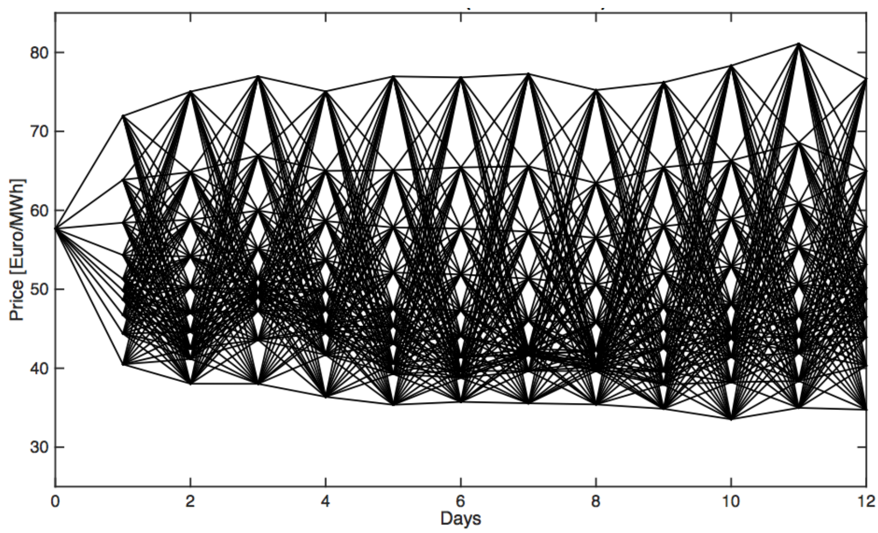


Figure 6.6: Generated lattice with forecasted day-ahead prices to be used in the stochastic dynamic program [49]

There are other methods which can be used in order to generate scenario trees and lattices based on historical data, e.g moment matching or quantile regression. In order to assess the quality of the scenario generating method described above, and used in this report, 5000 random samples from the lattice were sampled with replacement and compared to the data simulated from the historical AR(1) model. The mean value and the 10th, 30th, 60th and 90th percentile of the random sampled values from the lattice and values generated from the AR(1) model are given in Figure 6.7. As can be seen from the two graphs, the lattice provides values giving approximately the same mean value but with a more varying price level in the percentiles between each stage. The histograms in 6.7 illustrate the outcomes in the last stage for both sampled values from the scenario lattice and from values generated by the AR(1) model. The bin size in the histogram is chosen to be 10 €/MWh since the lattice provides 10 possible price outcomes in each stage. Given the results from this analysis, the scenario generating procedure based on nested distance is seen to satisfactorily represent the day-ahead prices modeled by the AR(1) model.

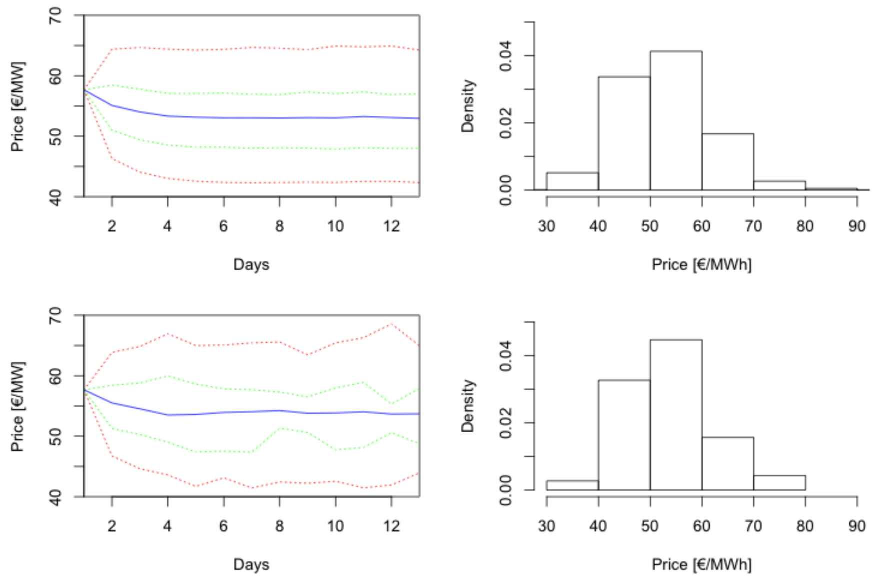


Figure 6.7: Percentiles and histogram of historical values in last stage, AR(1) values (above) and 5000 random samples (below)

The lattice provides daily electricity prices and their corresponding probability. In order to find the contribution in each stage, hourly prices are needed. By using multiplicative adjustment with factors calculated from the historical prices for each day within the analysed period, hourly prices can be found. The hourly factors are found similarly to the monthly adjustment factors in 5.2. However, average hourly prices divided by the overall daily average is used instead of respectively average monthly values and yearly values. Calculated hourly factors for each day within the bidding week are illustrated in Figure 6.8.

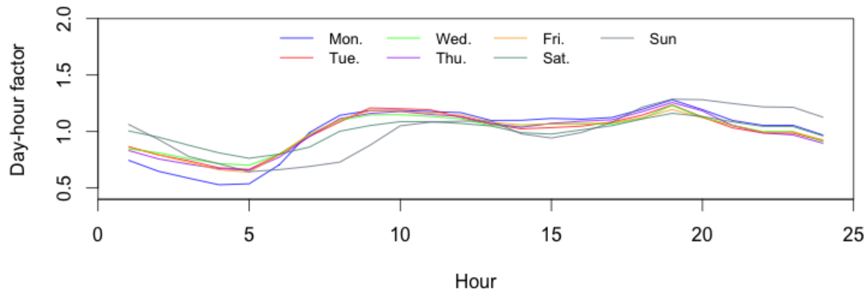


Figure 6.8: Hourly factors, Monday-Sunday

# 7 Case study

A case study has been conducted to evaluate the model presented. Simplifications that have been made are described below. In the study the optimal bid of a power producer bidding into the Swiss primary reserve market and day-ahead market during a 12 day period from 2015-02-03 until 2015-02-14 is found. The week of delivery is from 2015-02-09 to 2015-02-15.

## 7.1 Implementation

In the implementation, a power producer with only one generator is considered. Hence, the subscript $i$, used in the model formulation in Section 4, is not considered in the description of the implementation. Minimum up- and down-time are not included in the implementation due to large increases in the state space if they are included. As discussed in Section 3.4, these constraints are seen as economical constraints and do not represent strict technical restrictions. A high start-up cost is however taken into account in order to reflect the cost of starting up and avoid production with short start-stop periods.

The problem is solved using backward induction, treating each stage as a separate subproblem. As the decision made in the first stage restricts the possible bidding decisions in all the remaining stages, the problem is solved for every possible bidding decision in $t = 1$. The model is run with a scenario lattice generated with 5000 scenarios and 10 states per stage as described in Section 6. The lattice represents the development of day-ahead prices for 12 days following bidding into the primary reserve market. Bidding in the day-ahead market takes place only during the last seven days represented by the lattice, but the lattice needs to represent prices for all 12 days and their corresponding probability of occurrence in order to find the transition probabilities from the first to the second stage.

Given the bidding rules presented in Section 3.3 the possible capacities to bid in the primary reserve market run in increments of 1 MW from 0 MW to 90 MW, and the producer must have the same capacity available for the whole bidding week. The model is therefore implemented with 90 iterations through the scenario lattice with the value of $q^P_{tot}$ running in increments of 1 MW between each iteration. The parametric value of $q^P_{tot}$ is denoted by $Q^P_{tot}$. This is similar to restricting the transition between two states to only be allowed if $q^P_{tot}$ is the same in both states.

Ramping constraints for each hour within each stage are considered by each subproblem as described by Constraints (14) and (16). Handling the ramping restrictions described by Constraints (15) and (17) is however difficult since they run between stages. To incorporate these constraints, $q^D_{th}$ would have to be discretized. This would increase the state space and the computational burden considerably. The proposed approach in this report is to predefine an allowed production interval for $q^D_{(t-1)24}$ and $q^D_{t1}$ in which the ramping restrictions are not violated. The interval is found by solving a small optimization problem with an the objective function as given by (40) for all $t \in T^D \setminus \{2\}$ with respect to Constraints (41) and (42). The objective is to find the production level, $z$, that maximizes the profit given price, $P^{avg}$, and cost of production, $C(z)$. The price, $P^{avg}$, is the average of the expected prices in the first hour of the current day, $t$, and the last hour the previous day, $t - 1$. Constraints (41) and (42) restrict the production level to lie in the interval between $Q^{max} - Q^P_{tot}$ and $Q^{min} + Q^P_{tot}$ when the unit is delivering primary reserves. When the unit is not delivering primary reserves, the constraints restricts the production level to lie between $Q^{max}$ and $Q^{min}$ or to be 0. The parameters of the problem are the same as in the main stochastic dynamic problem. The parameter $V^{cap}$ is specific for this problem, and is 1 if $Q^P_{tot}$ is larger than 0, meaning that the unit has reserved capacity in the primary reserve market, and 0 otherwise. The problem is solved separately for all $t \in T^D \setminus \{2\}$ and the results are stored in an array where each element $z$ in the array corresponds to the day $t$ they were solved for. These parameters are denoted $z_t$ and are used in the subproblem.

$$\max \quad P^{avg}z - C(z) \tag{40}$$

st.

$$z \leq (Q^{max} - Q^{P}_{tot})y \tag{41}$$

$$z \geq (Q^{min} + Q^{P}_{tot})V^{cap} + (Q^{min} + Q^{P}_{tot})(y - V^{cap}) \tag{42}$$

$$z \geq 0 \tag{43}$$

The resulting production restrictions for $q^{D}_{(t-1)24}$ and $q^{D}_{t1}$ in the stochastic dynamic optimization problem is given by Constraints (44) and (45). These are added to the problem described in Section 4. Because the production in the last hour one day is restricted to be in the same interval as the first hour the next day, no start-up costs occur in the first hour of any day.

$$z_t - \frac{R^{down}}{2} \leq q^{D}_{(t-1)24} \leq z_t + \frac{R^{up}}{2} \qquad \forall t \in T^D \setminus \{2\} \tag{44}$$

$$z_t - \frac{R^{down}}{2} \leq q^{D}_{t1} \leq z_t + \frac{R^{up}}{2} \qquad \forall t \in T^D \setminus \{2\} \tag{45}$$

While moving backward through the stages, the value function is iteratively updated by finding the optimal policy and corresponding contribution function. The optimal policy of each state is the optimal bidding decision given the available information in that state. For every value of the bidding capacity, $q^{P}_{tot}$, the expected profit for the period considered is calculated. The contribution in the primary reserve market in the first stage, $t = 1$, is calculated by Equation (3). The optimal policy in the first stage is the decision that maximizes the expected value, as given by Equation (5). A high-level pseudocode of the implementation is given below.

**Algorithm 1:** The implemented problem

**input** : Distribution of historical primary reserve prices, scenario lattice for day-ahead prices
**output**: Optimal policy for t=1

Initialize Policy array
Initialize Contribution array
Initialize Value array
**for** $q^{P}_{tot} \leftarrow 0$ **to** $Q^{Pmax}$ **do**
    Find production intervals for the first and last hour for every day of the week as given by (40)–(45)
    Set $V_{|T|+1}$ to 0
    **for** $t \leftarrow |T|$ **to** 1 **do**
        **for** *state* $\leftarrow 1$ **to** *nStatesPerStage* **do**
            **if** *t=1* **then**
                Find the optimal policy by solving th subproblem given by (3), (5), (33)–(35) and (27)–(28) and store it in the Policy array
            **else**
                Find the optimal policy by solving the subproblem given by (3), (5)–(14), (16),(18)–(20) and (29)–(32) and store it in the Policy array
            Calculate the contribution of the optimal policy using (3) and store it in the Contribution array
            Calculate the expected value of the optimal policy using (5) and store it in the Value array

Find the optimal policy by finding the maximum expected value in the Value array

## 7.2 Technical data of the CCGT power plant

The model has been tested with operational cost data for a CCGT plant received from Powel [29]. The data is anonymized and slightly modified data from a continental European power producer. The implemented model takes only the cost, production limits and ramping rates of the CCGT system into account. As a simplification the size of the plant is measured by the maximum and minimum production levels of the gas-unit. Table 7.1 displays the relevant technical data of the unit.

Table 7.1: Technical data of the generating unit

| $Q^{max}$ [MW] | $Q^{min}$ [MW] | $R^{up}$ [MW/min] | $R^{down}$ [MW/min] |
|---|---|---|---|
| 292.09 | 74 | 24.33 | 11.23 |

The costs of running the CCGT plant can be broken into start-up costs, fuel costs, O&M costs and costs of $CO_2$. Fuel costs for the modeled CCGT plant are adjusted values from the gas turbine fuel consumption received from Powel so that the efficiency of the plant mirrors the efficiency of a CCGT plant. The part-load efficiencies of the CCGT plant are measured at the same levels of production as given in the data received from Powel, and the maximum efficiency of the plant is assumed to be 60%. The energy of natural gas used in calculating the cost curve is $40MJ/Sm^3$ [50].

Table 7.2: Efficiency of the generating unit

| Generation [MW] | 77 | 100 | 160 | 270 |
|---|---|---|---|---|
| Efficiency [%] | 48 | 51 | 55 | 58 |

The cost function is dependent on the consumption and price of gas. The price of gas is a stochastic, varying parameter, but due to the short time frame of the period concidered, uncertainty and variability in the gas price are not taken into account. This can also be justified by the possibility to enter into long-term contracts with gas suppliers. Terms of long-term gas contracts are not found to be publicly available, hence the day-ahead price of gas is taken into account. An average day-ahead market settlement price for natural gas of 23.07 $€/MWh$ for the relevant bidding period starting at 2015-02-03 is used. This value is reported by [51]. A rate of pollution of 0.34 $tCO_2/MWh$ used in order to calculate the cost of $CO_2$ are estimated by [52]. A cost of 10 $€/tCO_2$ is furthermore used as an average price of $CO_2$ estimated from the ETS market. O&M costs are also taken into account

Table 7.3: Costs of the generating unit

| | | | Parameters of the fuel cost function | | |
|---|---|---|---|---|---|
| $c^S$ [€] | $c^{CO_2}$ [€/Mwh] | $c^{O\&M}$ [€/MWh] | $c$ [€/MWh] | $b$ [€/MWh] | $a$ [€/MWh$^2$] |
| 6 000 | 3.4 | 2 | 1 130 | 28 | 0.010 |

## 7.3 Results

The optimization problem has been implemented in MATLAB R2015a (Version 8.5) and Mosel/ Xpress MP. The implementation is written by the authors with inspiration from the MDP toolbox available in Matlab. Matlab handles the recursive value iterations, while each subproblem is solved in Xpress MP. All tests have been run on a 64-bit Windows 7 PCs with 3.40 GHz Intel Core i7-3770 CPUs (4 cores) and 16 GB RAM. The solution time of the problem is 277.93 seconds.

This section presents the results from running the original case as well as results from a sensitivity analysis which has been conducted with the purpose to investigate how the optimal decision in the primary reserve market is affected when chosen parameters of the model are changed. Sensitivity with respect to costs, technical restrictions and prices of primary reserves have been investigated.

The optimal solution of the original case is given in Table 7.4. The deviation given in the table is the percentage difference in expected profit from bidding the optimal amount of primary reserves in the primary reserve market compared to participating only in the day-ahead market. Participation in the day-ahead market only is given by $q^P_{tot} = 0$. The deviation represents the value of bidding in the primary reserve market; a higher percentage difference in expected profit between the two markets indicates a higher percentage change in profit by participating in the primary reserve market. Figure 7.1 shows the expected profit of delivering 0–90 MW in the primary reserve market. Bidding the optimal amount of 67 MW is the maximum value on the curve. The optimal solution indicates that it is beneficiary for the CCGT plant to bid a large amount of capacity in the primary reserve market.

Table 7.4: Optimal solution

| Expected profit [€] | $q^P_{tot}$ $[MW]$ | $p^P$ $[€/MW]$ | Expected Profit, $q^{tot}_P = 0[€]$ | Deviation [%] |
|---|---|---|---|---|
| 653 672 | 67 | 2 514 | 632 005 | 3.4 |

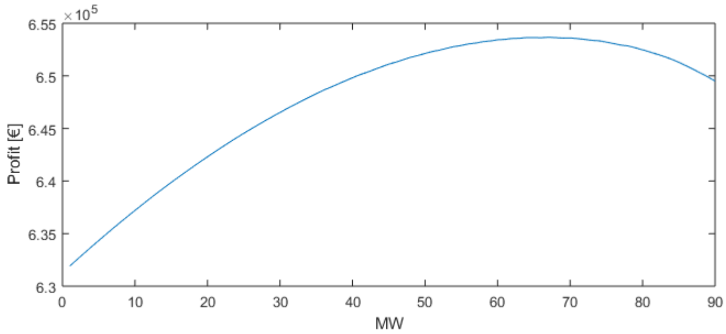


Figure 7.1: Profit from bidding 1–90 MW in the primary reserve market

Table 7.5-7.8 present the results from the sensitivity analysis. In each analysis one parameter is changed, either by a percentage change with respect to the original parameter or by changing the parameter by an absolute value. The results presented in table 7.4 are referred to as the base case in the sensitivity analysis.

The costs of running thermal power plants differ from unit to unit. The cost function used in the base case represents the cost of a CCGT plant with a high efficiency compared to other types of thermal generators. It is therefore interesting to see how the bidding decision changes when the the cost of generation is higher or lower. Such an analysis will hence indicate what type of unit that is profitable to use to deliver primary reserve capacity. The sensitivity with respect to cost is tested by changing only the fixed cost parameter, $c^{on}$, of the quadratic cost function. This is the same as shifting the graph vertically. The results are given in table 7.5.

Table 7.5: Sensitivity with respect to costs, $c^{on}$

| | Percentage change in $c^{on}$ [%] | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | −40 | −30 | −20 | −10 | – | +10 | +20 | +30 | +40 |
| $q^P_{tot}$ [MW] | 45 | 51 | 56 | 62 | - | 70 | 74 | 78 | 81 |
| $p^P$ [€/MW] | 2 612 | 2 586 | 2 564 | 2 537 | - | 2 500 | 2 481 | 2 462 | 2 447 |
| Expected Profit [€] | 714 028 | 697 500 | 682 450 | 667 889 | - | 639 774 | 626 156 | 612 806 | 599 593 |
| Expected Profit, $q^{tot}_P = 0$[€] | 704 142 | 685 871 | 667 864 | 649 920 | - | 614 125 | 596 310 | 578 584 | 560 920 |
| Deviation [%] | 1.4 | 1.7 | 2.2 | 2.8 | - | 4.2 | 5.0 | 5.9 | 6.9 |

One of the technical restrictions governing the operation of thermal power plants are ramping restrictions. The ramping rate is furthermore one of the parameters reflecting the level of flexibility of a thermal power plant. Sensitivity with respect to ramping rates is given in Table 7.6. Maximum ramp-up and ramp-down rates are changed such that they are on the same level, and seven different levels of $R^{up}$ and $R^{down}$ are analyzed.

Table 7.6: Sensitivity with respect to ramping rates

| | Ramping rates, equal values of $R^{up}$ and $R^{down}$ [MW/min] | | | | | | |
|---|---|---|---|---|---|---|---|
| | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| $q^P_{tot}$ [MW] | 70 | 67 | 64 | 60 | 58 | 56 | 51 |
| $P^P$ [€/MW] | 2 500 | 2 514 | 2 528 | 2 546 | 2 555 | 2 564 | 2 586 |
| Expected Profit [€] | 651 633 | 653 860 | 655 692 | 657 093 | 658 222 | 659 114 | 659 747 |
| Expected Profit, $q^{tot\,P} = 0$ [€] | 627 625 | 631 098 | 634 101 | 636 761 | 639 229 | 641 501 | 643 649 |
| Deviation [%] | 3.8 | 3.6 | 3.4 | 3.2 | 3.0 | 2.7 | 2.5 |

The primary reserve market has been analyzed as a market with low liquidity and corresponding possibility for market participants to affect primary reserve prices. If more participants enter the market, prices may drop and profitability in the market decrease. Another scenario is for prices to increase when the need for ancillary services increases due to a larger amount of intermittent, renewable energy in the power system. It is therefore interesting to investigate how the optimal decision is affected when prices in the primary reserve market are changed. The sensitivity with respect to the primary reserve prices is investigated by changing the expected revenue of the optimal bid in the primary reserve market. The results are given in Table 7.7

Table 7.7: Sensitivity with respect to primary reserve prices

| | Change in expected revenue in the primary reserve market [%] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | −20 | −15 | −10 | – | +10 | +15 | +20 | +25 |
| $q^P_{tot}$ [MW] | 0 | 29 | 45 | - | 81 | 85 | 89 | 90 |
| $P^P$ [€/MW] | 0 | 2 276 | 2 350 | - | 2 692 | 2 791 | 2 888 | 3 002 |
| Expected Profit [€] | 632 005 | 634 493 | 639 359 | - | 672 093 | 682 234 | 692 749 | 703550 |
| Expected Profit, $q^{tot}_P = 0$ [€] | 632 005 | 632 005 | 632 005 | - | 632 005 | 632 005 | 632 005 | 632 005 |
| Deviation [%] | 0 | 0.4 | 1.2 | - | 6.3 | 7.9 | 9.6 | 11.3 |

The sensitivity of primary reserve prices illustrates how the bidding decision changes when the difference between day-ahead prices and primary reserve prices change. Sensitivity with respect to day-ahead prices is also interesting to take into account in such an analysis. The sensitivity is analyzed by changing the overall price level in the primary reserve market, i.e changing the level of prices in the scenario lattice. The results are given in table 7.8.

Table 7.8: Sensitivity with respect to day-ahead prices

| | Change in day-ahead prices [%] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | −40 | −35 | −30 | −20 | −10 | – | +5 | +10 |
| $q^P_{tot}$ [MW] | 0 | 90 | 90 | 90 | 90 | - | 25 | 0 |
| $P^P$ [€/MW] | 0 | 2 402 | 2 402 | 2 402 | 2 402 | - | 2 694 | 0 |
| Expected Profit [€] | 16 357 | 62 284 | 141 366 | 305 283 | 475 175 | - | 759 631 | 883 944 |
| Expected Profit, $q^{tot}_P = 0$ [€] | 16 357 | 38 052 | 75 884 | 188 726 | 382 099 | - | 757 154 | 883 944 |
| Deviation [%] | 0 | 63.7 | 86.3 | 61.7 | 24.4 | - | 0.3 | 0 |

## 7.4 Discussion

The results of the coordinated bidding model indicate that the opportunity cost of reserving 67 MW of capacity in the primary reserve market is lower than the expected profit received from selling the capacity in the day-ahead market. Hence, the value of participation in the primary reserve market is high.

In the implementation there are no constraints on the amount of primary reserves a unit can deliver, but a producer owning only one generator would be constrained by droop restrictions in the generator. The optimal solution found can however be viewed as an estimation of a solution for a producer owning several similar generators. For a producer owning generators with different operating constraints, optimizing for each generator separately can be viewed as an approximation of the value of bidding the combined capacity into the market. A producer who owns several generating units can however vary the amount of primary reserve capacity that is provided by each generating unit during the week, as long as the total amount delivered is constant. Hence, the flexibility of having several generating units available has a value which is not included if optimizing for each unit separately. If there are restrictions on the amount of capacity a single unit can reserve, the optimal bid can be found from Figure 7.1. Figure 7.1 shows the expected profit of delivering 1 to 90 MW in the primary reserve market, and it can be seen that for a producer with an upper limit on supply lower than 67 MW it will be optimal to bid the upper limit.

The developed model can be compared with an existing bidding model in order to assess the value of the model developed. Information regarding current bidding practices for a power producer in Switzerland has been received from [29]. The current procedure builds on comparison between three deterministic scenarios, with different commitments in the primary reserve market, and a base case of no commitment. The costs of delivering 1 MW more of primary reserves is equal to the lost profit in the day-ahead market from delivering this incremental amount of primary reserve capacity. This cost is used to build a bidding curve which is manually adjusted in order to get a smooth curve with increasing price for increasing volume. The way producers manually adjust their bids in order to get a smooth curve is information unknown to the authors. The exact value of the model developed in this report compared to the current bidding practice is consequently unknown. However, the procedure described indicates that the practice today is to bid an amount given by the break even price of delivering primary reserves and adjusting the bid with a manually set value. If the manually adjusted value is assumed to be zero, the value of the bidding model in this report is given by the difference in profit between bidding the optimal amount of primary reserve capacity and bidding only in the day-ahead market. This can be seen as an optimistic estimate of the value of the model developed.

The results from the sensitivity analysis show that the bidding decision in the primary reserve market is sensitive to changes in input parameters. The analysis with respect to costs shows, as expected, that the overall profit is higher when costs are lower. The amount bid in the primary reserve market is however lower with lower costs, indicating that the increase in profit stems mainly from bidding in the day-ahead market. With lower production costs, the value of delivering energy in the day-ahead market is higher and the opportunity cost of production is therefore also higher. This explains why it is optimal to deliver less capacity in the primary reserve market. The opposite is true when costs are higher; the optimal amount of capacity bid in the primary reserve market increases when costs increase. When $c^{on}$ is increased, the amount bid in the primary reserve market moves towards 90 MW. However, when costs get sufficiently high, the costs of being on become higher than the expected profit of delivering primary reserve capacity and day-ahead energy. The optimal bid in the primary reserve market will then be 0 and the unit will be turned off when prices are low. This analysis indicates that the optimal decision largely depends on the operating costs of the generator and that the benefits from bidding in the primary reserve market differ between units.

The ramping rates of the CCGT unit are parameters representing the level of flexibility of the power plant. The results from the sensitivity analysis in which the ramping rates vary, indicate that higher ramping rates give an increase in the overall expected profit. The optimal capacity to bid in the primary reserve market and the deviation between the expected profit in the coordinated bidding model and in the solution taking only the day-ahead market into account, are however lower. This indicates that the value of flexibility is higher in the day-ahead market than in the primary reserve market. Units with higher ramping rates can vary their production more, and the production can consequently be better adjusted to changing prices in the day-ahead market. The profitability in the day-ahead market is therefore higher, and the alternative cost of primary reserve capacity, which is the only cost of delivering primary reserves in this model, is correspondingly higher. Due to an increased level of intermittent renewable energy, there is a large focus on developing more flexible thermal power plants with shorter start-up time and ramping rates.

There are factors, that the model does not take into account, which could further affect the value of delivering primary reserve capacity. The model developed assumes 100 % availability and does not take into account the possibility of not being able to deliver primary reserve capacity. If such risks and associated non-delivery costs were taken into account, the value of supplying primary reserves would be smaller. Such effects in combination with an increased ramping rate would mean that the value of participating in the primary reserve market would be lower than given in the sensitivity analysis.

If more producers enter into the primary reserve market, the market will become more liquid and prices can be expected to fall to a competitive level. This effect is investigated in sensitivity of primary reserve prices in Table 7.7. The profit in the day-ahead market without bidding capacity in the primary reserve market is naturally the same. The amount bid in the primary reserve market is however lower when prices decrease. At a level of 20 % lower costs, profit in the primary reserve market is not high enough to cover the opportunity cost of the reserved capacity in the day-ahead market.

The AR(1) model used to estimate day-ahead prices takes only historical day-ahead prices into account. The lattice consequently does not consider any correlation between primary reserve prices and day-ahead prices. Figure 7.2 illustrates the cross-correlation between day-ahead prices and marginal and efficiency price in the primary reserve market. The figure shows negative correlation between the time series, indicating that prices in the day-ahead market and primary reserve market tend to move in opposite directions. The figure furthermore indicates that the correlation is strongest for no displacement of the curves. When prices are low, fewer generators will be spinning and fewer generators are consequently able to supply primary reserve capacity and prices in the primary reserve market are higher. Because negative correlation between prices is not taken into account, the results from the sensitivity analysis with respect to day-ahead prices show that more primary reserve capacity should be bid at a lower price when prices in the day-ahead market fall. This is contrary to the price development seen in the market and is hence a weakness of the model developed.

It can be seen that it is no longer profitable to deliver primary reserves when day-ahead prices fall by 40%. The model will only find it optimal to bid capacity in the primary reserve market if the opportunity cost of delivering energy in the day-ahead market is lower than the *expected* value of delivering primary reserves. However, when the optimal solution given by the model is zero, capacity can be bid at a price level covering the costs associated with keeping the generator running. In this case, other methods, such as the one discussed earlier in this section, can be used in order to decide the bid.

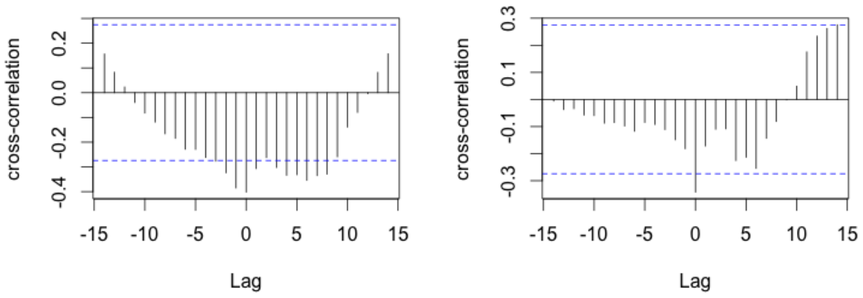


Figure 7.2: Correlation between efficiency prices and weekly average day-ahead prices (left) and marginal prices and weekly average day-ahead prices (right), 06.01.2014 - 28.12.2014

# 8 Conclusion and future work

This report investigates the use of stochastic dynamic programming on a sequential bidding problem for a thermal power producer bidding capacity in the primary reserve market and energy in the day-ahead market. The stochastic problem is solved on a multinomial lattice developed with prices forecasts for day-ahead prices in Switzerland. The pay-as-bid auction is modeled based on historical primary reserve prices. Uncertainty in the primary reserve prices affect the optimal bid of the thermal power producer in the pay-as-bid auction.

A case study is conducted in order to assess the value of coordinated bidding in the two sequential markets. It has been found that there are large potential profits of coordinating bidding in the primary reserve market and the day-ahead market. The potential profits however depend on the characteristics of the generator, and it has been found that lower costs and higher ramping rates imply a lower amount bid in the primary reserve market. Technological advances will hence affect the optimal bid in the primary reserve market. The results are sensitive to input parameters, indicating that it is important for a producer to estimate the parameters well in order to bid optimally in the pay-as-bid auction. The analysis of sensitivity with respect to prices, indicate that possible changes in prices will affect the bidding decision.

The bidding problem has been analyzed by using stochastic dynamic programming. The method is a well known method used for solving unit commitment problems, but has not, to the authors' knowledge, been used when taking coordinated bidding in the primary reserve market and the day-ahead market into account. Multi-market optimization problems have historically often been modelled as stochastic programming problems, with the problem that an increasing number of stages leads to an exponential increase in problem size. Using stochastic dynamic programming and representing the prices by a scenario lattice, have contributed to limit the problem size and solution-time of the problem.

A challenge with using stochastic dynamic programming is to implement constraints across stages. The proposed method in this report, which involves finding an interval of optimal production for the last and first hour of two consecutive days in which ramping restrictions will not be violated, facilitate the use of dynamic programming without increasing the state space. This method however restricts the production to lie within an interval and hence reduces the solution space for the first and last hour each day. The implication is lower flexibility of production during these hours. Another drawback of the stochastic dynamic solution method is that prices must fulfill the Markov property. There is extensive research on price modeling of electricity prices, and other price models could be beneficial to take into account if the method had not been restricted to fulfill the Markov property.

## 8.1 Future work

The implemented model does not take the possibility of owning several different generating units, with different technical restrictions and costs, into account. Owning different generating units is a source of flexibility when delivering ancillary services. In order to determine the value of this flexibility, a more comprehensive model taking several generating units into account should be implemented and tested.

In the implementation of the model, constraints regarding ramping were simplified. An approach using stochastic programming would allow these constraints to be more easily implemented. However, a problem with stochastic programming is that solution times often are high due to exponential growth of scenario trees. Methods to reduce solution times of stochastic programs exist and it is suggested to explore such methods for this problem in future research.

Analysis has showed that the model can be improved by taking correlation of prices in the primary reserve market and the day-ahead market into account. Such an analysis should be combined with an analysis of parity between primary reserve prices and day-ahead prices. Parity between prices in the day-ahead market and the primary reserve market describe the level of equality between the prices, and may affect the results of the model.

Only coordinated bidding in the primary reserve market and day-ahead market is taken into account. However, it is believed that there are synergy effects of delivering capacity in several reserve markets. In order to assess the value of coordinated bidding in more markets, suggested future work is to extend the model developed to include other markets in addition to the two markets considered.

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# Appendix A. Pay-as-bid auction

The derivation of the expected profit from bidding in the primary reserve market presented here is taken from [8] and [25].

The market price is assumed to be a stochastic variable $\chi$ following a density function $f^{\chi}(p)$ : $\mathbb{R} \mapsto \mathbb{R}_+$ with the probability distribution $F^{\chi}(p) : \mathbb{R} \mapsto [0,1]$. For a bidding capacity $q^P_{tot}$ the probability of acceptance is:

$$P^A(\chi > p^P) = 1 - F^{\chi}(p^P) = 1 - \int_{-\infty}^{p^P} f^{\chi}(p)\,\mathrm{d}p \tag{46}$$

It is assumed that only one bidder behaves strategically, and that the behaviour of other bidders can be summarized in a probability distribution of the market price. There will not be a uniform price and therefore no single probability distribution of the market price is readily available. However, in a multi-unit pay-as-bid auction the market price can be between the efficiency and the marginal offer. The efficiency offer and the marginal offer is the less and most expensive offer accepted, respectively. Their values are unknown before the auction is held. The respective prices are defined to be the efficiency price $\hat{p}^E$, with density function $f^{\hat{p}^E}(p)$, and the marginal price $\hat{p}^M$, with density function $f^{\hat{p}^M}(p)$.

The density function of the distribution of the efficiency price can for instance be found through historic time series. Because of the always holding inequality $\hat{p}^E \leq \hat{p}^M$, the density function of the marginal price cannot be estimated independently of the efficiency price. Therefore the density function of the difference between the marginal price and the efficiency price $f^{\hat{p}^{ME}}(p)$ must be calculated. Now, a convolution of these two distributions can be applied to find the distribution of the marginal price. Because the distribution of the difference between the marginal and the efficiency price is defined for positive values only, a single sided convolution can be applied:

$$f^{\hat{p}^M}(p) = \int_0^{\infty} f^{\hat{p}^E}(p-u) f^{\hat{p}^{ME}}(u)\,\mathrm{d}u \tag{47}$$

Due to the assumption of a non-competitive primary reserve market, the bidding capacity must be taken into account to find the relevant market price $\hat{p}^R \in [\hat{p}^E, \hat{p}^M]$. Assuming a linear approximation between the efficiency and marginal price, the relevant market price is as follows:

$$\hat{p}^R = (\hat{p}^P - \hat{p}^E) k(q^P_{tot}) + \hat{p}^E \tag{48}$$

With $k(q^P_{tot}) \in [0,1]$ as the index of the merit order:

$$k(q^P_{tot}) = \frac{Q^{Pmax} - q^P_{tot}}{Q^{Pmax} - Q^{Pmin}} \tag{49}$$

Extending (47) by taking the bidding capacity into account results in the density function of the relevant market price that depends both on the price and capacity bid:

$$f^{\hat{p}^R}(p^P; q^P_{tot}) = \int_0^{\infty} f^{\hat{p}^E}(p^P - k(q^P_{tot})u) f^{\hat{p}^{ME}}(u)\,\mathrm{d}u \tag{50}$$

Following Equation (46), the probability of acceptance, considering that a bid is accepted only and entirely if the relevant market price is higher than the bidding price, may generally be calculated using the primitive of the distribution of the relevant market price in Equation (50) by:

$$P^A(\hat{p}^R > p^P; q_{tot}^P) = 1 - \int_{-\infty}^{p^P} f^{\hat{p}^R}(p; q_{tot}^P)\,\mathrm{d}p \tag{51}$$

The expected profit $Pi^R$ is given as:

$$\max_{p^P} \quad E[\Pi] = P^A(\hat{p}^R > p^P; q_{tot}^P) q_{tot}^P (p^P - c^P) \tag{52}$$

In which $c^P$ is the cost of delivering the primary reserves.

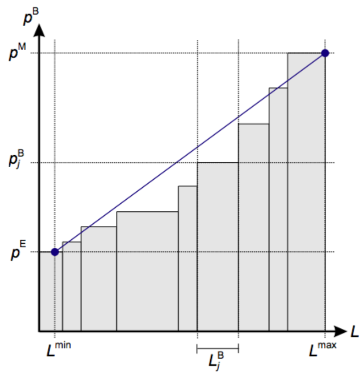


Figure 8.1: Pay-as-bid modeling [25]