Short-term Hydropower Scheduling - A Stochastic Approach

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Preface

This paper is prepared as our Project Thesis at the Norwegian University of Science and Technology (NTNU) at the Department of Industrial Economics and Technology Management. The work has been accomplished during the autumn semester 2006 and the paper is a part of the course TIØ 4700 Finance and Accounting, Specialization.

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Abstract

The common practice among Norwegian hydropower producers is to use a deterministic approach in the bidding into the day-ahead market. However, day-ahead market prices and water inflow to the reservoirs are uncertain also within a short time frame. Based on this fact we propose a stochastic short-term bidding and scheduling model for a price-taking hydropower producer who participates in the Nordic electricity day-ahead market, Elspot. Bidding to the spot market and a simplified version of the unit commitment problem is modeled within the framework of a two-stage linear stochastic model and solved as a deterministic equivalent. The objective is to maximize the revenues from sales in the market. To substantiate the model, relevant aspects of the Nordic day-ahead market, hydropower scheduling, bidding and stochastic programming are illustrated.

A demonstration of the model is presented using data from a Norwegian hydropower producer. To represent the uncertainty, scenarios for price and inflow are generated using a moment-matching scenario generation method. The model is run with sets of scenarios consisting of respectively 1, 10, 100 and 250 scenarios. The preliminary results show a slight improvement in the objective value when the model is run with increasing number of scenarios.

Contents

Chapter 1

Introduction

1.1 Background and Motivation

The Nordic power producers are exposed to a volatile and competitive market when they are to schedule production. In 2005 about 40% of the total concumption in the Nordic market were traded at the Nordic day-ahead market and the share is increasing (NVE 2006). Hence, the sales of power into the day-ahead market constitutes a substantial part of the revenues for the producers. This makes the bidding into the day-ahead market one of the most important tasks the power producers are faced with.

In the process of planning hydropower production, problems are usually categorized according to their time horizon. The focus in this paper is on short-term hydropower scheduling. The most important activities within the short-term scheduling include the bidding of the production into the electricity spot market a day in advance and the establishment of a production plan which complies with the day-ahead commitments from the bidding. In these activities the future price and inflow are important factors.

Future price and inflow are stochastic variables in the short-term perspective. Nevertheless, hydropower producers today use deterministic models in the short-term scheduling. Based on this we present a stochastic optimization model for short-term scheduling for a hydropower producer who only participates in the Nordic electricity spot market. The objective is to maximize profit from sales of power. We use the model presented in (Fleten & Kristoffersen 2006) as a starting point.

We have accomplished a case study based on one of Statkraft AS´s hydropower plants. Our motivation behind the case study is to see if a stochastic model performs better, i.e. have a higher objective value than the deterministic one. If so, one can argue that the stochastic model provides better support in the day-ahead bidding. The topology of the plant is rather complex and simplifications is necessary to construct a linear model. First a deterministic linear model is formulated where we regard the bidding into the

day-ahead market and a simplified version of the unit commitment suited for the case. The model is then extended into a two-stage stochastic linear model and solved as a deterministic equivalent. To account for uncertainty, scenarios of price and inflow are constructed applying a moment-matching scenario generation method from (Høyland, Kaut & Wallace 2003). The models were programmed in Xpress Version 1.6.2. and some results from the demonstration of the model are presented in the paper.

1.2   Structure of the Paper

The structure of the paper is as follows; Chapter 2 is an introduction to the Nordic power market. Further in chapter 3 the concept of hydropower scheduling is presented. In chapter 4 we discuss approaches to bidding. Chapter 5 deals with stochastic optimization and has the purpose of being a study of the topic of stochastic optimization. Finally in chapter 6 we introduce the case study which is based on one of the hydropower plants belonging to Statkraft AS. Chapter 7 states a conclusion and suggests some improvements left for future work.

Chapter 2

The Nordic Electricity Market

2.1 Nord Pool

Nord Pool ASA is the Nordic power exchange. It has developed from being solely a Norwegian power exchange to be a multinational exchange for electrical power. Nord Pool provides the population of Norway, Sweden, Denmark and Finland with supply of electrical power and optimal use of total system resources.

With the liberalization of the Norwegian power market in 1991, the power sector changed from having monopoly areas under governmental regulation to be a competitive and market oriented sector. This process later proceeded in the rest of the Nordic region. As a cause of the liberalization the producers had to change their focus from reliable and cost-efficient energy supply to more profit oriented and competitive objectives (Fleten, Wallace & Ziemba 2002). The liberalization of the market led to the need of a market place where a price could be set. Nord Pool offers a market for physical contracts and a market for financial contracts. The market for physical contract is provided by Nord Pool Spot AS. Nord Pool also offers clearing services.

The market for physical contracts, Elspot, is an auction-based day-ahead market where electrical power contracts are traded for each hour the following day. Elspot provides an effective system for letting supply and demand set the market price, and it gives the participants the possibility to balance their portfolios of power contracts close to real-time load. Nevertheless, the time span between the day's Elspot auction and the actual delivery hour of the concluded contracts is 36 hours at the most. As consumption and production situations change, a market player may find a need for trading during these 36 hours. The Elbas market provides continuous power trading 24 hours a day, up to one hour prior to delivery. The Elbas market is only available in Eastern Denmark, Finland and Sweden. Since the purpose of this paper is to present a short-term scheduling model for a Norwegian hydropower producer, the Elbas market would not be treated further.

There are four types of bids available at Elspot; hourly bids, block bids, flexible hourly bids and linked block bids. In an hourly bid the participants submit how much they are willing to buy or sell at a given price for every 24 hour starting at 00.00 the following day. Flexible hourly bids are only sales bids for one single hour with a fixed price and volume. When bidding, a lower price limit and a volume is stated. The flexible bid will be accepted in the hour with the highest price. Block bid is an aggregated bid for minimum four consecutive hours with a fixed price and volume. Linked block bids consist of a main block bid and a dependent block bid. If the main block is accepted, then the dependent block bid is also considered (NordPool 2004).

2.1.1   Bidding Process at Nord Pool

The participants at Elspot submit sales- and purchase bids for every hour of the following day of operation. The bidding is done under uncertainty since the system price is not yet known. Because of the uncertainty, the bidding process is a difficult task. We will later discuss this topic thoroughly.

All the participants deliver their bids at the latest at 12.00. After the bidding Elspot calculates the prices by aggregating the sales- and purchase-curves for every hour the following day from the hourly bid curves. The spot prices are determined by the intersection-point between the resultant sales- and purchase-curves for every 24 hours the following day (See figur 2.1). An hourly sales bid is accepted as long as the bid price is equal or lower than the spot price. The opposite is true for the purchase bids. Sales block bids are accepted when the average spot price for the block period is equal or lower than the given block bid price. Again the opposite is true for purchase bids. Dependent linked block bids are accepted under the same rules as regular block bids, but since linked block bids are dependent of each other, the main block bid has to be accepted before the dependent one is considered. Flexibly hourly bids are not given for a specific hour, but are accepted in the hour with the highest spot price given that the bid price is lower. The system price for a given day is an average over the 24 spot prices within that day.

If there are congestions in the grid, separate area prices will be established. In the following we will assume that there are no different area prices, hence the spot prices are the only prices in the whole Nordic area. The calculation of the spot prices are completed at the latest at 13.30. The spot prices and the belonging volume are then published. All parties are notified how much volume they are obligated to dispatch. All transactions are handled at the spot price, and the accepted contracted volume, not the metered volume, decides the financial settlements. If the metered volume differs from the contracted volume, imbalances arise. This will be handled by the short-term balancing market (NordPool 2006).
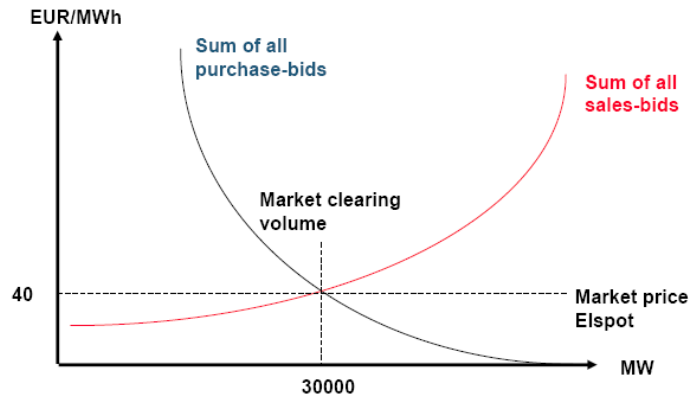
Figure 2.1: Calculation of the spot price

2.2   The Short-term Balancing Market

The special feature of electricity is that it can not be stored in substantial part, and hence it has to be generated and consumed at the same time. We need a system that handles the imbalances between load and generation, such that the generation always equals the load. This is the objective of the short-term balancing market. The transmission system operators (TSOs) are responsible for this balancing within each country. The TSOs in the Nordic countries cooperate to focus on the real time balance of the overall Nordic grid. Statnett is the Norwegian system operator, and is responsible for the regulating power in Norway.

The participants in the balancing market submit their bids to the transmission system operator after the Elspot marked has closed, at the latest at 19.30. Balancing bids are divided in two; bids for upward- and downward regulation. Bids for upward regulation are bids for increased generation or decreased consumption, where the participants submit how much they require for increasing generation or decreasing consumption for a specified volume. Bids for downward regulation signal how much they are willing to pay to decrease generation or increase consumption.

The TSO sorts each bid according to price for every hour during the day. In case of upward regulation, there is a power deficit in the market. The TSO therefore has to activate the power reserves by informing the participants with the accepted bids from the balancing market. The accepted bids are chosen according to the ascending price, and the last call-upon unit sets the price for upward regulation. For downward regulation the bids are accepted according to descending prices. In addition, spatial aspects are taken into account. For example, in the case of upward regulation within a area the TSO calls a local generator to increase generation.

There are different practices within the Nordic countries regarding how the imbalances are priced. In Norway there is only one price for every hour in every price area. The financial settlements take place after real time regulation. The participants receive payment for positive imbalances, and are charged for negative imbalances. Positive imbalance for a producer is when he has generated more than he was obligated to in contracts at Elspot or in other physical contracts. A consumer will have positive imbalances when he has consumed less than the volume in the contracts. The volumes traded in the balancing market are often of insignificant quantity compared to the spot market. Hence, we will only focus on the bidding in the spot market and not include the balancing market in the short-term stochastic model presented later.

Chapter 3

Hydropower Scheduling

3.1 The Concept of Water Value

Water is the" fuel" in a hydropower plant. To be able to optimize the production it is necessary to set a value on the water stored in the reservoirs. Even though water is for free, it has a value given that it is a scarce resource and one is free to decide whether to produce today or to store it for later production.

The water value is often referred to as the marginal operating costs of the power plant. In reality, the water value is a function of the future development of the power market and the inflow. It should be noted that it is a complicated connection between the parameters that affect the water value. Load and price are closely linked to one another. A high load necessarily results in a high price in a competitive market. This again gives an incentive to discharge a considerable amount of water given that the market price is higher than the present water value of the reservoir. The discharge level in combination with the inflow decides the reservoir level, which again influences the water value. For instance if there is a small amount of water left in the reservoirs, the water will have a high value since the "fuel" is a scarce resource. In the opposite case, lets say the reservoir is nearly full, then there is a chance that the water flows over and becomes worthless since it will never contribute to any production. In addition to reservoir level, the overall market situation affects the water value. Since all these variables are stochastic, we express the water value by the expected value of the marginal kWh that is stored in the reservoirs (Fosso & Gjengedal 2006).

If the purpose is to maximize profit, one should produce until the short-term marginal cost equals the price. This is illustrated in section 4.1. Thus, a producer who participates in the day-ahead market would like to bid equal to his marginal cost curve. Hydropower plants in general have a low operating cost, and the only considerably cost is linked to the discharge of water. Therefore, the marginal cost of operations equals the water value. This makes the water value an important aspect in the bidding process. In what follows,

we will show that the water value equals marginal operational cost when minimizing total costs over a long time horizon. It should be noted that one may derive the same expression when maximizing revenues. This is a more suitable approach in a deregulated market, but since the notion marginal cost is important in short-term production scheduling, cost minimization will be used to derive the water value.

Let $J(l, t)$ denote the expected total cost from period t until period T, where T is some remote future date and l is the reservoir level in period t. The costs of changing the water level l is denoted $\Lambda(l, T)$ and is equal to the value of the water in the reservoir at period t, minus the value of the water in the reservoir at period T. In addition to this cost, the expected total cost also consists of the sum of all operating cost, $L(l, w, i)$ within a period i from t to T. The operating cost within a period i is dependent on the reservoir level l and the discharge of energy, w.

$$J(l, t) = \Lambda(l, T) + \sum_{i=t}^{T} L(l, w, i) = L(l, w, t) + J(l, t+1) \tag{3.1}$$

Equation (3.1) allude that the expected total costs from period t until T equal the sum of the operating cost in period t and the expected total costs from period $t + 1$ until T. Optimal disposal of the water in period t is achieved when the expected total costs, $J(l, t)$ are minimized in consideration to the energy disposal, w.

$$\min_{w} J = \min_{w} \{L(l, w, t) + J(l, t+1)\} \tag{3.2}$$

$$\Rightarrow \frac{dJ}{dw} = 0 \tag{3.3}$$

$$\frac{\delta J}{\delta w} = \frac{\delta L}{\delta w_t} + \frac{\delta J}{\delta l_{t+1}} * \frac{\delta l_{t+1}}{\delta w_t} = \frac{\delta L}{\delta w_t} + \frac{\delta J}{\delta l_{t+1}} * (-1) = 0 \tag{3.4}$$

It should be noted from equation (3.4) that the marginal change in the reservoir level caused by a marginal change in the energy discharge in the anterior period is equal to -1.

Optimal strategy for period t can now be derived as

$$\frac{\delta L}{\delta w_t} = \frac{\delta J}{\delta l_{t+1}} \tag{3.5}$$

The left side of the equation (3.5) equals the marginal operating costs. The right side equals the expected total costs derived regarding to the reservoir level, which per definition is the water value at time $t + 1$. Thus, the optimal strategy is to produce when the price is higher than the water value.

The water value in period t is equal to the water value in period $t + 1$, given that the optimal production strategy described above is applied in period t. When calculating the water value in period t one therefore needs the water value in the subsequent period. The calculation of water values is done using long-term models. One way to calculate the water values is to choose the end of the planning horizon T such that it coincides with a point in time where the water value is known. For instance when the snow melt is at its highest and flooding appear, one knows that the water value equals zero. This result can be used to calculate backward until present period t is reached, for example using a stochastic dynamic programming model.

3.2 The Hydropower Scheduling Problem

3.2.1 Medium- and Long-term Scheduling

The hydropower production scheduling is because of its complexity, decomposed into a long-, medium- and short-term problem, each being solved by suitable models and solution techniques (Flatabø, Fosso, Haugstad & Mo 2002).

The goal of the long-term production planning is to maximize the market value of the water resources. The long-term production planning seeks to analyse the long-term fluctuations in price and inflow and by this find an optimal strategy for the hydropower operations in the long run perspective. The modeling of the production is often simplified by aggregating the reservoirs into one equivalent reservoir. Stochastic dynamic programming can be applied to predict the water values from present time up to a point in time in the future (Fleten et al. 2002). In this calculation price and inflow prognoses provide important input. Output from the long-term planning which among other factors are the water values derived in equation (3.5), is further used as boundary condition in the medium-term production planning. The medium-term model has an increasing detail level and serve as a link between the long-term model and the short-term model (Fosso, Haugstad & Mo 2006).

SINTEF Energy Research has developed a model for long-term scheduling purpose, the EMPS model. This model is widely used in the Nordic areas. Within this model the whole market including all the producers are considered. The EMPS model describes production, consumption and transmission within the Nordic and adjacent areas. For each sub area in the market the model gives an indication of the long-term situation of the water values, reservoir level, generation, sales and purchase of spot power. Important input in the model are load, thermal generation costs, and initial reservoir level (Flatabø et al. 2002).

3.2.2   Short-term Scheduling

Short-term hydropower scheduling primarily deals with the physical operations of the power plant within a time horizon of a day up to a week, depending on the coupling to the medium-term model, and with a time resolution of up to one hour (Fleten & Kristoffersen 2006). The main activities in the short-term scheduling are to make decisions that build up under the planning of the physical production and can be listed as follows

- The bidding of production into a power exchange one day before actual time, often referred to as the day-ahead commitments.

- Set up a detailed production plan which meets the terms from the day-ahead bidding.

- The real-time balancing, i.e. the establishment of the bids for the short-term balancing markets.
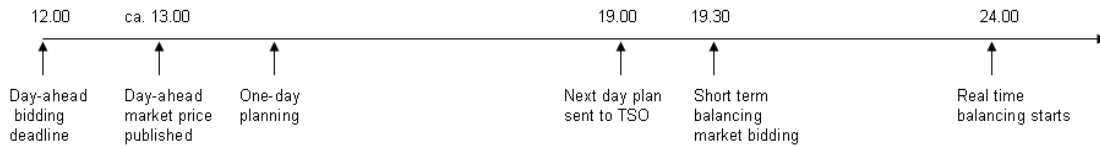
Figure 3.1: Time schedule

We present how the procedure of the short-term scheduling can be accomplished according to the deadlines at the Nordic power exchange. The first task in the short-term scheduling is to prepare the submitting of bids to the power exchange for the day-ahead commitments. The bids have to be submitted to the power exchange at 12.00 at the latest. Next, after the spot price is published around 13.00, one has to establish a production plan that complies with the day-ahead commitments. Not all, but some of the power producers participate in the short-term balancing market. The final deadline for submitting bids to this market is at 19.30.

The day-ahead bidding in the spot market is completed before the bidding in the balancing market has been accomplished (Fleten & Kristoffersen 2006). From section 3.1 we know that it is optimal to bid to the marginal costs of your production for every hour. As stressed earlier, this is the water value of the reservoirs. In the modeling of the short-term scheduling it is complicated to derive the water value from equation (3.5), thus alternative approaches to express the water value may be used. Later in section 6.2.4 an alternative way to derive an expression of the water value is applied. This approach is based on a method used in (Fleten & Kristoffersen 2006).

To sum up, the main tasks in the short-term scheduling can be separated in two; First is the submitting of bids for the day-ahead production, second is the unit commitment of production which meet the terms of the day-ahead bidding, i.e. the decision of how to

distribute the production between the different aggregates. The bidding to the balancing market will be handled in chapter 4. It should be noted that bidding and unit commitment are closely related to one another, mainly because of the water value. As stressed earlier, the bids are set according to the water values. The production plan of how much to produce from each reservoir is determined according to each reservoirs water value. For example, one will obviously discharge from the reservoirs with the lowest water value first.

Decisions of how to utilize the water resources are based on profit maximization of the expected revenues from power sales. The short-term scheduling problem is rather complex, thus to be able to carry out the bidding and the unit commitment in light of profit maximization, a detailed modeling of the physical system is essential. Since the bidding and the unit commitment are dependent on another, an optimal model would include both operations. In chapter 6 we present an optimization model for short-term scheduling which include both the bidding and a simplified version of the unit commitment. Further in this chapter we will concentrate on only the unit commitment problem. In chapter 4 the bidding issue will be carefully discussed.

How to Model the Unit Commitment

At this stage in the short-term scheduling the spot price is known and the remaining task is to make a production plan for the next day which is consistent with the accepted bids. This task is often referred to as the hydropower unit commitment problem. More precisely it is that of determining which turbines should be on and the levels at which to generate in each turbine so as to meet the commitment made in the spot market (Philpott, Craddock & Waterer 2000).

As stressed in the previous section the short-term scheduling is a rather complex task. The unit commitment problem requires that the detailing level of the physical system is high. In a model of the system, all the relevant details which affect the production must be taken into consideration. For example, hydropower plants may have quite complex topologies with several cascaded reservoirs or power plants in the same river system. The different reservoirs may have different storage capacity and significant water travel time. This has the effect that the decision in one time interval have strong impact on what is possible to do in later time steps (Belsnes, Honve & Fosso 2005). In cases where the reservoirs are linked in series, water release from an upper reservoir leads to water inflow in a lower reservoir. See figure 3.2. In addition, the start and stop costs makes the decisions of production in one time step dependent on the decisions of the adjacent hours. The modeling of start and stop costs often requires binary variables expressing whether each turbine is on or off. To avoid the computational effort by introducing binary variables one can model the start and stop in an alternative way which is introduced in section 6.2.5.

Each power plant may include several turbines which has a minimum and maximum
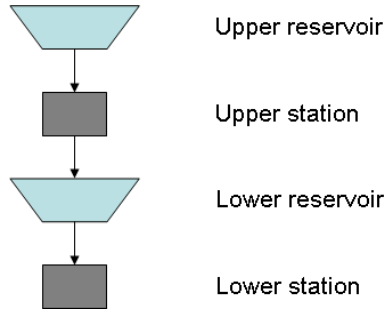
Figure 3.2: Two reservoirs in a cascade

operating level. The electricity is generated by letting water flow through a turbine. Potential energy from the water is changed into electrical energy. The power generated is a nonlinear function of the flow rate x and its net head, that is the difference between the headwater elevation e_h and the tail water elevation e_t. See figure 3.3. The flow rate is again a function of the volume of the reservoir y so that the net head can be represented by some function $h(x, y)$. There is a loss in power in the transfer of water. An efficiency function $\eta(h, x)$ represents the loss of power in the transfer of water flow to electricity. In summary, the power generated by a turbine with flow rate x and headwater volume y is

$$g(x, y) = xh(x, y)\eta(h, x) \tag{3.6}$$

It is reasonable to make the assumption that $h(x, y)$ does not vary much with y over the course of a short-term planning horizon, especially in the case where the reservoir is large. If we let y be a constant then the generation function is only dependent on the flow rate x and becomes $g(x)$. The function $g(x)$ is typically a concave function or it can be approximated by a concave function (Philpott et al. 2000).
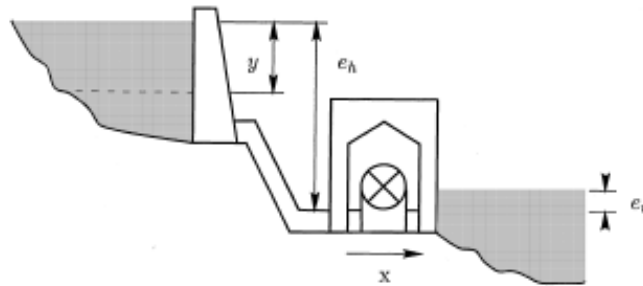
Figure 3.3: A typical hydropower station: $e_h - e_t$ is the net head

It is difficult to consider all aspects of the physical system. For instance, in the case when a power plant has several owners the modeling is complicated. In addition there are often legal requirements that have to be considered in the model. Hence, it is a challenge to develop a model with a high enough detailing level.

Simulation and Optimization - Methods for solving the unit commitment problem

Providing the utilities with optimal scheduling plans for each generator in the system is a difficult task. Existing approaches to unit commitment include both simulation and optimization. A simulation is based on adjusting manual suggestions until a convincing plan is found. This approach is very user dependent and hence does not guarantee an optimal plan. On the contrary, optimization represents a relatively impartial way of making an optimal unit commitment (Fleten & Kristoffersen 2006).

Further we will concentrate on the important aspects of an optimization model that satisfy the need for a high detailing level. Clearly, the objective in such a model should be either profit maximization or cost minimization. In a deregulated market where the price is set in a market clearing process at a power exchange, profit maximization is reasonable. Moreover, the cost in the objective function is related to the use of water and the start and stop costs. The constraints constitute the modeling of the physical system, as described in the previous section.

While price and inflow are treated as stochastic variables in the long- and medium-term model, they are often treated as deterministic variables in the short-term scheduling. This is the praxis in spite of the fact that price and inflow are subject to uncertainty also in the short-term perspective, at least in the case where both the bidding and the unit commitment are included in the same optimization model. If the model only concentrate on the unit commitment, one assumes that the market price is known in advance i.e. the bidding has already been done and the market price is set. Nevertheless, the inflow is still an uncertain variable in the unit commitment and should be modeled in a stochastic approach. The reason for the deterministic praxis is that a stochastic approach is deemed to be computationally demanding because of the required detailing level. This is especially the case for cascaded reservoir systems (Flatabø et al. 2002).

Examples of Theory and Methodology of Short-term Scheduling in the Literature

In this section we will look at some approaches of how to model the short-term scheduling in the literature. A general model formulation of the short-term hydro scheduling is presented by George, Read and Kerr (George, Read & Kerr 1995). This is a deterministic modeling approach where integer variables are used to represent the number of turbines operating at each station along with piecewise linearization of the unit efficiency curves.

The objective is to maximize profit accrued from generation and the value of end of period storage of water in the reservoirs, less the costs of failing to meet generation targets and the start and stop costs. No heuristic is applied to obtain faster solution time and the model is solved with a standard IP solver.

Hreinsson (Hreinsson 1988) has made a deterministic optimization model with the purpose of finding the optimal short-term production scheduling of a hydropower system. More specifically, the model optimizes the hourly power production by minimizing losses in turbines and waterways, while maintaining production to meet load. The optimization problem is inherently formulated as a nonlinear mixed integer problem, but an algorithm has been applied to solve the problem in two stages by linear programming. The model is further simplified by letting the power production and the water resources be treated separately. In praxis, this means that the production is modeled without considering any of the variables associated with water or reservoir content. With this simplification the number of variables is kept to a minimum and the problem can be solved with less computational effort.

As mentioned before, the short-term modeling is often subject to a deterministic treatment in spite of its stochastic parameters. Philpott, Craddock and Waterer (Philpott et al. 2000) have regarded the uncertainty in the scheduling of daily hydro-electric generation. With appropriate approximations the problem of determining what turbine units to commit in each half hour of the day can be formulated as a large mixed-integer linear programming problem. To be able to solve this stochastic problem they suggest using an optimization-based heuristic.

SHOP (Short-term Hydro Operation Planning) is an example of a commercial modeling tool which is developed by SINTEF Energy Research. This is a deterministic linear programming model adjusted for solving complex hydropower scheduling problems. As in the modeling approaches introduced above the power plant is modeled at unit level. Unlike the model of Philpott, Craddock and Waterer SHOP does not regard uncertainty in the modeling formulation (Flatabø et al. 2002).

Chapter 4

Approaches to Bidding

4.1 Bidding in Practice

At the time of bidding the price is not yet known, thus the bidding is done under uncertainty. To reduce the uncertainty the planning of the bids will normally take place close to the deadline because then the latest information can be used. For a profit maximizing producer the philosophy behind the bidding should always be to maximize the expected revenues, i.e. to sell when the prices are high, and to buy when the prices are low. Thus the important and difficult task is to find out what a "high" and a "low" price is.

From microeconomic theory one knows that a producer who acts as price taker should to maximize profit, produce until his short time marginal cost equals the price (Wangensteen 2005). To see this, let $C(w)$ be the producer's cost function, w the volume produced and ρ the price set by the market. Then his profit π may be formulated as

$$\pi = w \times \rho - C(w) \tag{4.1}$$

Profit maximizing behavior implies

$$\frac{d\pi}{dw} = \rho - \frac{dC(w)}{dw} = 0 \tag{4.2}$$

which gives

$$\rho = \frac{dC(w)}{dw} \tag{4.3}$$

Hence, to maximize profit the producers would bid equal to their marginal cost curve. From section 3.1 we know that the marginal costs equal the water values.

When constructing the bids there are several important issues the producer has to consider. For some power producers locked production is an issue, that is power they have

to produce regardless of the price. Examples of this could be wind power or hydropower from a river plant with no store possibilities. And even if there are store possibilities in the water chain, there are often legal requirements that state that the water flow has to be at least at a given minimum level at all times because of ecological or esthetical considerations. Since they have to produce this power no matter what, the producers are willing to sell this power at any price. Thus the operator bids this locked volume to a price as low as zero, so that he is ensured a knockdown on this volume.

Then one has to consider how to bid the rest of the production. One way of constructing these bids, which is used in practice, is to bid the water value at the best point of production and at the maximum point of production for every generator. By doing this, the producer gets two price-volume points for every generator. The water value used in the bidding process is usually found from long-term models, but some adjustments may be done. For example the EMPS model, see section 3.2.1, could be run a few times per week to get the water values given a long- and medium-term strategy. These weekly water values would then be used as indicators of how the water value will be within that given week. Since the market situation and inflow vary from day to day, the weekly water values may be modified on daily basis. The calculation of the daily water values are usually based on experience and analysis. Factors which indirectly influence the decisions are for example the expected weather forecast and the hydrological situation and the expected gas- and coal-price. The latter is important in the Nordic market since it consists mainly of hydropower and thermal production.

The decision of how much to bid for every hour is dependent from hour to hour. As already mentioned in section 3.2.2, the topology of the hydro system and the start and stop costs cause the production plan to be dependent on the consecutive hours. At the time when the bidding schemes are constructed, there already exists a scheduling plan for the remaining hours of the day. This is based on the commitment made in the spot market the day before. Since this scheduling plan effects the system state at the beginning of the next bidding period, this too as to be considered when constructing the bids.

From the above discussion we see that in practice the process of making the bids is often based on experience and the skills of the operator. In the literature we find models which have a more theoretical approach and in the reminder of this chapter we will discuss two articles which both deal with bidding strategies. We illustrate two different approaches to the bidding problem. The first one (Fleten & Pettersen 2005) addresses the possibility of willingly bid too high or too low volumes in the day-ahead market to earn a profit in the balancing market. The second one (Wen & David 2001) concerns the start and stop costs issue.

4.2 Day-ahead Bidding in View of the Balancing Market

Fleten and Pettersen propose in (Fleten & Pettersen 2005) a stochastic linear programming model for constructing piecewise-linear bidding curves for a price-taking retailer. In their model they consider both the day-ahead energy market and the balancing market, and their objective is to minimize the total cost for the retailer from both these markets. In the following we will apply their model seen from the producer's point of view. To do this one first has to see if all the presumptions made in (Fleten & Pettersen 2005) also hold for the producer. One important assumption that Fleten and Pettersen do is that they assume the retailers to be price takers. They argue for this by saying that the Nordic market consists of many small retailers which none of them hold a substantial share of market power. Although most of the producers in the Nordic market are considerably larger, we feel that this is an assumption one can accept at least for most of the producers in the market.

In the balancing market the producers behave differently from the retailers. All participants that have the ability to alter production or consumption significantly on 15 minutes notice are allowed to place bids in the balancing market. Although this also includes the retailers, Fleten and Pettersen do exclude this from the model with the argument that the demand side bidding of the balancing market is still immature. It is common for the producers to place bids in the balancing market, nevertheless we will as a simplification disregard this fact.

4.2.1 Day-ahead Bidding in View of the Balancing Market: Producer's Perspective

As explained earlier the balancing market balances the production and consumption close to real time. When there is deficit or surplus of power in a price area, respectively up-regulation or down-regulation will be done. This is controlled by the system operator. Because of the way the balancing market is run, up-regulating power are offered at a higher price than the price in the day-ahead market. The opposite is true in hours of down-regulation. In the following discussion remember that the financial settlement in the day-ahead market is handled according to the contracted volume, not the metered volume. That is, the producers receive the day-ahead market price for the contracted volume independent of the actual amount of energy they produce.

In the case of up-regulation consider a producer who generates less than he has committed himself to in the day-ahead market. The reason for this can for instance be unavailability in production. Because of his negative imbalance the producer has to pay the balancing price for the deficit. Since the market is up-regulated the producer will have to pay a higher price in the balancing market than he received in the day-ahead market. At the same time there might be some producers who generate more than committed. The cause for this may be locked production, i.e. production that has to be produced regardless.

Examples of this are wind power generation or hydropower generation from a river power plant. These producers will receive the balancing market price for this extra energy. As mentioned this price will be higher than the spot price and thus the producer would be better off selling power in the balancing market than in the spot market.

Let us now consider the case of down-regulation. A producer who generates less than he obligated himself to in the day-ahead market will have to compensate for the deviation in production and pay the balancing market price for this. Nevertheless, this price is lower than the price he received in the spot market. Hence, the producer in this situation would have gained. The opposite is true for a producer who generates more than he is obligated to. He will for his surplus power receive the balancing market price which is lower than the spot price that he otherwise could have received.

From the above discussion one sees that the producer has to be regulated in the same direction as the rest of the market to make some extra gain in the balancing market. That is when the market is up-regulated it is beneficial for the producer to produce more than his day-ahead market contract states. The opposite is true in the case of down-regulation, then it is favorable for the producer to produce less than committed. Thus, the producer has the possibility to speculate by bidding too low volumes in the spot market if he expects the market to be up-regulated and to bid too high volumes if he believes that the market will be down-regulated. Such speculations may be profitable for one producer. But since the day-ahead market should reflect physical supply and load conditions, this kind of speculation would be unfortunate for the market. Therefore the TSO monitors such practice and can impose penalties for it.

Let β be the balancing market price and ρ the spot price. From the above discussion we see that $\beta - \rho > 0$ in hours of up-regulation, $\beta - \rho < 0$ in hours of down-regulation and that $\beta - \rho = 0$ when no regulation is needed. Then let I_d and I_b be the income from the sales of energy from the day-ahead market and the balancing market, respectively. Thus the total income from the two markets is

$$I = I_d + I_b \tag{4.4}$$

Further, let I_d and I_b in addition to prices, be expressed by the volume dispatched in the day-ahead market, y, and the real physical production, ξ.

$$I_d = y\rho \tag{4.5}$$

$$I_b = (\xi - y)\beta \tag{4.6}$$

Combining equation (4.4), (4.5) and (4.6) gives

$$I = y\rho + (\xi - y)\beta = \xi\beta + y(\rho - \beta) = \xi\beta + y\delta \tag{4.7}$$

where $\delta = \rho - \beta$ is the difference between the spot price and the balancing market price.

The spot price and the balancing market price are exogenous. As mentioned previously, ξ constitutes the real physical production. The real physical production can be seen as the sum of the planned production before the bidding to the day-ahead market takes place and the alteration made in the production plans. This alteration takes place after the spot price is known, but before the balancing market price in known. In addition the real physical production includes alteration caused by errors or locked production. Because of these unexpected incidents the real physical production ξ, is uncertain and should be treated as a stochastic variable. Since the producer would like to maximize his revenues, his objective should be to maximize the expected revenues from both markets;

$$\max E[\widetilde{\xi}\beta + y\delta] \tag{4.8}$$

So far we have seen that there is a possibility for the producer to speculate if the market will be up- or down-regulated, and thus bid accordingly too low or too high volumes in the day-ahead market on purpose. Hence, the real physical production may deviate from the dispatched production from the spot market either because of speculation or because of unexpected incidents. This kind of speculation will increase the risk of the producer considerably. This is also a very unfavorable situation for the system operator, and in the Norwegian market the system operator Statnett will penalize participants who are detected in showing this kind of behavior. To include this in the model Fleten and Pettersen include shortfall costs. Define the variables $w_{1s}^+, w_{2s}^+, ..., w_{ms}^+$ and $w_{1s}^-, w_{2s}^-, ..., w_{ms}^-$. If the producer is up-regulated, i.e. he produces more than his obligation from the day-ahead market, $\sum_{m \in M} w_{ms}^+ = \widetilde{\xi}_s - y_s > 0$. In the opposite case, if the producer is down-regulated $\sum_{m \in M} w_{ms}^- = y_s - \widetilde{\xi}_s > 0$. Let $T_m^+ > 0$ and $T_m^- > 0$ represent the marginal cost of piece m on the volume deviation risk function for positive and negative deviations, respectively. Then, by introducing the term

$$-V \sum_{m \in M} (T_m^+ w_{ms}^+ + T_m^- w_{ms}^-) \tag{4.9}$$

in the objective function one may penalize volume deviations. In equation (4.9) V measures the producer's aversion to volume deviation. Since the risk will increase with higher volume deviations it is naturally to let the marginal penalty increase with increasing deviations. This would also require the following constraints

$$\sum_{m \in M} w_{ms}^+ + y_s \geq \widetilde{\xi}_s, \quad \forall s \tag{4.10}$$

$$\sum_{m \in M} w_{ms}^- - y_s \geq -\widetilde{\xi}_s, \quad \forall s \tag{4.11}$$

$$0 \leq w_{ms}^{\pm} \leq W_m, \quad \forall m, s \tag{4.12}$$

To model the bid curve Fleten and Pettersen suggest in (Fleten & Pettersen 2005) an approximation of the bid curve with a linear model where price points $P_0, ..., P_n$ are fixed in advance. They argue that for each scenario, the spot price ρ_s will lie between two certain price points and that this eases the formulation of the relationship between the bid volume and the dispatched volume. Let $i(s)$ denote the largest line segment i between P_i and P_{i+1} for which $P_{i+1} > \rho_s$. This means that the volume dispatched y_s will lie on the line segment described by linear interpolation between the price-volume pairs $(P_i(s), x_i(s))$ and $(P_{i(s)+1}, x_{i(s)+1})$. Thus the relationship between the dispatched volume y_s and the bid volume x_i can be written as

$$
\begin{aligned}
y_s &= \left(1 - \frac{\rho_s}{P_{i(s)+1} - P_i(s)} + \frac{P_i(s)}{P_{i(s)+1} - P_i(s)}\right) x_i(s) \\
&+ \left(\frac{\rho_s}{P_{i(s)+1} - P_i(s)} + \frac{P_i(s)}{P_{i(s)+1} - P_i(s)}\right) x_{i(s)+1}
\end{aligned}
\tag{4.13}
$$

To summarize, Fleten and Pettersen propose a model where they discuss the possibility of constructing bids to the day-ahead marked to maximize the expected profit from the balancing market. The model applied from a producers perspective is as follows

$$\max \sum_{s \in S} p_s \left(\widetilde{\xi}_s \beta_s + y_s \delta_s - V \sum_{m \in M} (T_m^+ w_{ms}^+ + T_m^- w_{ms}^-) \right) \tag{4.14}$$

subject to

$$
\begin{aligned}
y_s &= \left(1 - \frac{\rho_s}{P_{i(s)+1} - P_i(s)} + \frac{P_i(s)}{P_{i(s)+1} - P_i(s)}\right) x_i(s) \\
&+ \left(\frac{\rho_s}{P_{i(s)+1} - P_i(s)} + \frac{P_i(s)}{P_{i(s)+1} - P_i(s)}\right) x_{i(s)+1}
\end{aligned}
\tag{4.15}
$$

$$\sum_{m \in M} w_{ms}^+ + y_s \geq \widetilde{\xi}_s, \quad \forall s \tag{4.16}$$

$$\sum_{m \in M} w_{ms}^- - y_s \geq -\widetilde{\xi}_s, \quad \forall s \tag{4.17}$$

$$0 \leq w_{ms}^{\pm} \leq W_m, \quad \forall m, s \tag{4.18}$$

$$x_i \leq x_{i+1}, \quad \forall i \in I \tag{4.19}$$

$$x_i \geq 0, \quad \forall i \in I \tag{4.20}$$

For further information we refer the reader to (Fleten & Pettersen 2005).

The disadvantage with this approach is that the volumes in the balancing market are relatively small compared to the volumes in the day-ahead marked. Hence, for most producers are the figures they may earn or possible lose in the balancing market accordingly small. Because of this a model which consider the possibility of speculation in the two markets, may have insignificant value for producers. Since participation in the balancing market provides little profit the focus should be on markets which gives greater benefit, for instance the spot market.

4.3 Strategic Bidding

Wen and David present in (Wen & David 2001) two different bidding schemes, and based on this an overall bidding strategy is developed. Their starting point is that they consider a day-ahead market in which the participants trade and schedule for next day's delivery. This market is operated by a power exchange (PX), which conducts a series of 24 auctions simultaneously and separately, one for each hour. This formulation of the market is consistent with the Nordic power market.

Since the PX evaluates the hours independently, a dispatch in one hour do not guarantee for a dispatch in the adjacent hours. Therefore the producers have to internalize all involved cost and physical constraints in preparing their bids since the bidding structure do not take this into account. An example of cost they have to include is the start and stop costs. For producers with low generation costs, it is not difficult to build bids to make sure that their units can be dispatched at each hour. The opposite is true for producers with relatively high generation costs. It is likely that some of his units will not be dispatched in one or more hours, and hence it is difficult to construct bids which guarantee acceptance.

Wen and David propose two different bidding schemes. The first called "maximum hourly-benefit bidding strategy" is to bid such as to maximize the benefit in each hour for every generator separately based on the expectations to the load and how rival suppliers will bid. If this strategy do not succeed, i.e. based on the a priori expectations one realize that a unit will not be dispatched in some hours, then one should follow an alternative strategy for each of these hours. Wen and David call this strategy the "minimum stable output bidding strategy" and the objective of this is to guarantee that the unit can be dispatched at the minimum stable output level. For further information we refer the reader to (Wen & David 2001).

The model presented in (Wen & David 2001) is especially suitable for producers with a marginal cost close to the market price. In the Nordic market, inflow is the principal price driver but the marginal cost levels of the thermal plants are also of high significance (Tjøtta 2006). In periods with considerably inflow and high reservoir levels the water value tends to be low. Hence, in such a situation the marginal cost level of the thermal producers and the load set the price. This is true for all periods where the water value is lower than the thermal marginal cost and this is the usual situation. If the tendency is that the water values are high and at the same time the fuel costs of the thermal plants are low, then periods can arise where the marginal costs of the hydro producers are higher than the marginal costs of the thermal producers. In such a situation the hydropower production and the load set the price, and many of the hydropower producers will therefore have marginal costs close to the market price.

From the discussion above we see that hydropower or thermal power production can set the price depending on which of them having the highest marginal costs. Although, the thermal power production usually have the highest generation costs and therefore are probably most suited for applying such a model that Wen and David propose, we see that the model in (Wen & David 2001) also can be relevant for hydro producers. In addition the consideration of start and stop costs are important for a hydropower producer. Since Wen and David's model reflect this importance, the use of their model for a hydro producer can be justified also when the water value is much lower than the market price.

Chapter 5

Stochastic Programming

5.1 Introduction to Stochastic Programming

5.1.1 Modeling

When investigating a natural system for instance a hydropower plant, one usually makes a model which is applied to give the decision makers a better understanding and overview of the system. This is done because the natural system is too complex, difficult or expensive to study directly.

The accuracy of the model, i.e. how detailed and how close to the real world the model is, does not necessarily measure the quality of the model. Instead one has to look at the purpose of the model to find the right degree of detailing level. As we saw in section 3.2 a long-term model in production scheduling will usually be less detailed than a short-term production scheduling model where more work is done to make the model resemble the real world. This does not mean that a short-term model is "better", it only shows that these models serve different purposes. It is therefore important to remember that a model is never a copy of the real world and that one can never mimic every aspects of a system (Wallace 1999).

There are different ways to categorize different models. A common procedure is to split mathematical programming problems into linear programming, nonlinear programming, networks flow, integer and combinatorial optimization and finally stochastic programming. Such classification can be confusing because it indicates that stochastic programming is different from linear programming in the same way as nonlinear programming is different from linear programming. The truth is that the counterpart of stochastic programming is deterministic programming, and that we therefore have stochastic linear programming, stochastic nonlinear programming and so on (Wallace 1999).

We will in the following emphasize on stochastic linear programming, but the reader

should keep in mind that the stochastic way of thinking could also be used in other model formulation.

5.1.2   An Example of Stochastic Linear Programming

We will in this section present an example of a linear program which will be extended to a stochastic linear program. Many real life problems can be expressed as a linear programming model. Using matrix-vector notation the standard formulation would be

$$\min c^T x \tag{5.1}$$

Subject to

$$Ax = b \tag{5.2}$$

$$x \geq 0 \tag{5.3}$$

This kind of formulation is appropriate when the functions involved are fairly linear in the decisions variables. Next we introduce a very simple stochastic linear programming example from a hydropower plant. As stressed before, profit maximization is more proper to use in the case of hydropower scheduling in a deregulated market. Nevertheless, we choose to illustrate an example where the objective is to minimize the production cost and at the same time cover load. With a simple example of cost minimization in hydropower scheduling we hope that the reader can more intuitively understand the importance of stochastic programming. Using the notation from (5.1), this will correspond to that c_j and a_j are respectively the water value and the energy equivalent at station j. The load is represented by b and the decision variable x_j gives the water flow in station j. Notice that this is a very simplified example. In a linear problem all the parameters, i.e. c, A and b are assumed known and the problem is to find the optimal combination of the decisions variables x that satisfies the constraints.

Many real life situations deal with uncertainty and depending on the situation this uncertainty cannot always be ignored by inseting the mean values or some other fixed estimates of the parameters. Thus, the model needs to reflect that some of the parameters are unknown. Stochastic programming is a framework for modeling optimization problems that involve uncertainty. The uncertain parameters are characterized by probability distributions.

Let us consider our example further and assume that the hydropower plant consists of two reservoirs that are not connected. Our model can then be formulated as

$$\min c_1 x_1 + c_2 x_2 \tag{5.4}$$

Subject to:

$$a_1 x_1 + a_2 x_2 = b \tag{5.5}$$

$$x \geq 0 \tag{5.6}$$

In our simple example it is for instance unreasonable to assume that the load is known in advance. Hence, instead of a linear problem we are now faced with a stochastic linear program.

$$\min c_1 x_1 + c_2 x_2 \tag{5.7}$$

Subject to

$$a_1 x_1 + a_2 x_2 = \tilde{b} \tag{5.8}$$

$$x \geq 0 \tag{5.9}$$

Notice that we in the stochastic linear program have used the notion $\tilde{b}$ to represent the uncertain load. Since we do not know the realization b of $\tilde{b}$ we can not merely minimize the objective function, hence the equation (5.7) is not a well defined problem. To solve this problem let us introduce the possibility that there exists a balancing market were the producer can buy electricity if he do not cover the load our example. Such a market gives the producer the possibility to cover up his obligations after the uncertain load is revealed. Hence, the producer first has to decide how much to produce, then the load is revealed. From this it is given how much he has to buy from the market. The costs due to shortage of production are determined after the observation of the random load and are generally denoted recourse costs. We assume for simplicity that the price the producer has to pay in the market is higher than his own production cost. If we let $y(\tilde{b})$ denote the amount of energy the producer buys in the market and p the price he has to pay we can formulate our problem

$$\min c_1 x_1 + c_2 x_2 + E_{\tilde{b}}[py(\tilde{b})] \tag{5.10}$$

Subject to

$$a_1 x_1 + a_2 x_2 + y(\tilde{b}) = \tilde{b} \tag{5.11}$$

$$x \geq 0 \tag{5.12}$$

When solving this problem we find a production plan, i.e. water flows through the stations that minimize the sum of our original production costs and the expected recourse costs which in our case is the cost of buying energy in the market.

5.2   Mathematical Formulation of Stochastic Programming

5.2.1   General Formulation of the Stochastic Programming Model

In the example above we demonstrated a simple stochastic linear problem in a hydropower plant. Now, we state a more general formulation of a stochastic programming problem. This can be viewed as a mathematical programming model with uncertain parameters. Since the parameters are volatile they are described by distributions $\tilde{\xi}$, in the single-period case, and by stochastic processes $\tilde{\xi}_t$, in the multi-period case. A single-period stochastic programming model can thus be formulated as

$$"min" g_0(x, \tilde{\xi}) \tag{5.13}$$

subject to

$$g_i(x, \tilde{\xi}) \leq 0 \quad i = 1, ..., m \tag{5.14}$$

$$x \in X \subset \mathbb{R}^n$$

Here, $\tilde{\xi}$ describes the random vector of the volatile parameters. The distribution of this vector must be independent of the decision vector x. Usually, the stochastic programming model formulated above can not be solved with continuous distributions. Hence, most solution methods require discrete distributions of the uncertain parameters. Therefore, in most practical applications the "true" stochastic process $\tilde{\xi}_t$ is approximated by a discrete stochastic process $\check{\xi}_t$ with limited number of outcomes. The discrete stochastic distribution and the discrete stochastic process have been denoted respectively $\check{\xi}$ and $\check{\xi}_t$ where $t \in T$. The number of outcomes from the discrete distribution or process is limited by the available computer power.

Given that we are faced with continuous variables in (5.13), we stress that the discrete distribution is only an approximation of the real continuous distribution of the stochastic parameters. Hence, we solve only an approximation of (5.13) (Kaut & Wallace 2003).

5.2.2   Deterministic Equivalent

The general stochastic program as shown in equation (5.13) may be formulated as a deterministic equivalent if the problem can be formulated as a stochastic program with recourse. That is, the problem should be formulated in such a way that one for each constraint could provide a recourse activity $y_i(\tilde{\xi})$ that after observing the realization ξ of the stochastic distribution $\tilde{\xi}$, is chosen such as to compensate its constraint's violation. These recourse activities are assumed to cause an extra cost or a penalty and constitute the recourse function

$$Q(x,\xi) = \min_{y} \sum_{i=1}^{m} q_i y_i(\xi), \quad i = 1, ..., m \tag{5.15}$$

where q_i denotes the cost per unit. Note that the recourse function does not have to be linear as here.

Hence the total cost of both the first stage and the recourse activities can be expressed as

$$f_0(x,\xi) = g_0(x,\xi) + Q(x,\xi) \tag{5.16}$$

If it is meaningful and acceptable to the decision maker to minimize the expected value of the total costs, then one could consider the deterministic equivalent to (5.13) instead of (5.13) itself. The deterministic equivalent would be

$$\min_{x \in X} E_{\tilde{\xi}} \left\{ f_0(x,\tilde{\xi}) \right\} = \min_{x \in X} E_{\tilde{\xi}} \left\{ g_0(x,\tilde{\xi}) + Q(x,\tilde{\xi}) \right\} \tag{5.17}$$

A deterministic equivalent could be applied to multi-stage problems (Kall & Wallace 1994).

5.2.3   Multi-stage Stochastic Programming

The example in section 5.1.2 is denoted a two-stage stochastic linear program with recourse. Such a problem is characterized by that the decision maker takes some action under uncertainty in the first stage, after which a random event occurs, i.e. the actual value of ξ gets known. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision. First stage decisions are chosen by taking their future effects into account. These future effects are measured by the expected value of the recourse costs (Birge 1997).

A multi-stage problem is an extension of a two-stage problem. Instead of two decisions to be taken at stages 1 and 2 we are now faced with K sequential decisions, $x_1, x_2, ..., x_K$, to be taken at the subsequent stages $\tau = 1, 2, ..., K$. The term "stages" can, but need not, be interpreted as "time periods" (Kall & Wallace 1994). In a multi-stage setting the outcome of the uncertain data is gradually revealed. Between each decision i.e. between each stage, new information on the uncertain data arrives. That is, one takes successively a first stage decision x_1, then after observing the realization of ξ_2, one takes a second stage decision x_2. This continues until one reaches stage K.

5.3   Importance of Scenario Tree in Stochastic Programming

5.3.1   Scenario Tree

At present time we know that the uncertain parameters presented by a continuous probability distribution have to be made discrete to fit the stochastic programming model. One way to approximate this probability information is by the use of a so-called scenario tree. A scenario tree is used to assigning states to nodes and it is a discrete approximation of a continuous distribution. It consist of nodes $n \in N$ and the root node corresponds to stage 1. The remaining nodes all have a set of immediate successors and a unique predecessor. For node n the immediate predecessor is denoted n_{-1} and the probability that n is the descendant of n_{-1} i.e. the transition probability, is termed $\pi^{n/n_{-1}}$. The immediate descendants of node n are $N_{+1}(n)$ and nodes with $N_{+1}(n) = \emptyset$, which are the "ending" nodes or leaves. The path from the root node to n is denoted by $\text{path}(n)$ and each path from the from the root node to a leaf represents a scenario (Fleten & Kristoffersen 2006).

5.3.2   Measure of Quality in Scenario Trees

The reason for why scenario trees are applied is to solve a stochastic program. Hence, the scenario tree should be judged by the quality of the decision it provides. It should be noted that it is not of importance how well the distribution is approximated, i.e. the goal is not to search for an optimal discrete distribution in the statistical sense. The important feature is whether the scenario tree leads to a good decision or not in sense of the "true" objective solution of the stochastic model (Kaut & Wallace 2003). We look back at equation (5.13) in section 5.2.1 and further denote the problem as

$$\min_{x \in X} F(x; \tilde{\xi}_t) \tag{5.18}$$

As mentioned before, we need to approximate the continuous stochastic process in the problem into a discrete distribution, i.e. make the scenarios. Hence,

$$\min_{x \in X} F(x; \breve{\xi}_t) \tag{5.19}$$

Now, the optimal solution of the scenario-based problem is denoted as

$$\breve{x}^* = argmin_x F(x; \breve{\xi}_t) \tag{5.20}$$

The error that occurs when we make an approximation of a continuous stochastic process into a discrete stochastic process is given by the parameter $e_f(\tilde{\xi}_t, \breve{\xi}_t)$. The error of approximating a stochastic process $\tilde{\xi}_t$ by a discretization $\breve{\xi}_t$ for a given stochastic problem (5.18), is defined as the difference between the value of the true objective function at the optimal solutions of the true and the approximated problem (Kaut & Wallace 2003).

$$
\begin{aligned}
e_f(\tilde{\xi}_t, \breve{\xi}_t) &= F(argmin_x F(x; \breve{\xi}_t); \tilde{\xi}_t) - F(argmin_x F(X; \tilde{\xi}_t); \tilde{\xi}_t) \\
&= F(\breve{x}^*; \tilde{\xi}_t) - \min_x F(x; \tilde{\xi}_t)
\end{aligned}
\tag{5.21}
$$

The error-term $e_f(\tilde{\xi}_t, \breve{\xi}_t)$ is difficult to calculate in practical problems. Therefore, instead of finding the optimal scenario generation method based on a minimization of the error-term, one can make an evaluation of a given scenario generation method based on certain quality requirements. According to Kaut and Wallace in (Kaut & Wallace 2003), there are at least two important properties that should be satisfied for a scenario generation method in order to be qualified for a given stochastic model. The first requirement is stability; if several scenario trees are generated with the same input and the optimization problem is solved with these trees, one should get the same optimal value of the objective function. The second requirement is that the scenario tree should not introduce any bias compared to the true solution.

Stability requirement

This requirement is rather easy to test. Several scenario trees are generated by discretization of a given stochastic process. Further, the stochastic programming problem is solved for each tree. If the stability requirement is satisfied, we should get approximately the same optimal values of the objective function for every solution of the different scenario trees.

There are two different tests of the stability; the in-sample stability and the out-of-sample stability.

In-sample stability:

$$
\min_x F(x; \breve{\xi}_{tk}) \approx \min_x F(x; \breve{\xi}_{tl}), \qquad k, l \in 1, ..., K
\tag{5.22}
$$

Out-of-sample stability:

$$
F(argmin_x F(x; \breve{\xi}_{tk}; \tilde{\xi}_t) \approx F(argmin_x F(x; \breve{\xi}_{tl}; \tilde{\xi}_t))
\tag{5.23}
$$

By in-sample stability, we mean that the stability is only tested on behalf of the scenario-based optimization problem. In the out-of-sample stability, we have to evaluate the "true"

objective function $F(x; \tilde{\xi}_t)$. The latter method necessitate that we have a full knowledge of the distribution of $\tilde{\xi}_y$. If we have out-of-sample stability the real performance of the solution $x_k{}^*$ is stable. More concretely, the solution does not depend on which scenario tree $\breve{\xi}_t$ that is chosen. The in-sample solutions indicate how good a solution is. However, if we have an out-of-sample stability and an in-sample instability, the solution may be good but we do not know exactly how good. The other way around, if we have an in-sample stability and an out-of-sample instability it is more dangerous since the solution we get depends on which scenario tree that is applied. The out-of-sample stability can be tested by a Monte-Carlo-like simulation method given that the distribution of the stochastic process is known. If historical data is used in the scenario generation, back-testing can be used. For further reading on backtesting we refer the reader to (Wallace 1999). Another approach is to use a scenario generation method that we assume to be stable as a reference scenario tree, and evaluate the solution x_k on the tree and compare it with the solution of the method that is to be tested.

It can be expected that in most practical applications either both stabilities occur or none of them. By this one can conclude that the in-stability test is sufficient to state whether or not the scenario generation method fulfills the requirement of stability. However, if feasible, the out-of-sample stability should be tested as an assurance.

Testing for bias

Another important requirement of the scenario generation method is that the method applied should not introduce any bias into the solution of the objective function. The solution of the scenario-based problem $\breve{x}^*$, should almost be an optimal solution of the original problem. Hence,

$$F(\breve{x}^*; \tilde{\xi}_t) = F(argmin_x F(x; \breve{\xi}_t); \tilde{\xi}_t) \approx \min_x F(x; \tilde{\xi}_t) \qquad (5.24)$$

Testing of this property is in most practical problems impossible, since it requires that the optimization problem with the "true" continuous process is solved. If we were able to solve that, then we would not need the scenario trees in the first place. Nevertheless, in some cases an approximate test can be done. One option is to build a "reference" scenario tree and use it as a representation of the true stochastic process. This is of course only an approximation of the true stochastic process. In general, the reference tree should be as big as possible on the condition that we still can solve the optimization problem. To produce such a tree, we need a method that is guaranteed to be unbiased. A natural consequence of this is that we can not use the method we want to test.

5.4 Scenario Generation

5.4.1 Introduction

In section 5.3 we saw that stochastic programs need to be solved with discrete distributions. The process of creating scenario trees is called scenario generation. When generating scenarios we are faced with at least two issues. For the stochastic process to be solvable, the number of scenarios must be small enough. On the other side, there must be enough scenarios to represent the underlying distribution satisfactorily. In addition, as mentioned in section 5.3.2 a scenario generation method should provide scenario trees that satisfy the stability requirement and not show any bias.

In the case of power production scheduling, the scenarios could for instance describe the behavior of the day-ahead market prices and the inflow. In literature one can see examples of scenarios for this based on time series analysis (Fleten & Kristoffersen 2006). Time series models are models where one attempts to predict stochastic variables using only information contained in their own past values and possibly current and past values of an error term. Given a set of observed data, the models capture the empirically relevant characteristics of the data and describe it (Brooks 2004). From these models scenarios for the variables can be generated. One well known time series model that can be used is the ARMA model developed by Box and Jenkins (Box & Jenkins 1976). If such a model is applied, one will let the multivariate stochastic process of the prices and the inflow constitute a time series characterized by seasonal changes, periodic cycles and stochastic variation. But since the ARMA model is not that suited to take into account such effects as sudden changes caused by heavily rainfall and the tendency that the weather conditions stays the same over a time period, other scenario generation methods can be more appropriate (Fleten & Kristoffersen 2006).

There are many different scenario generation methods, but we will in the rest of this chapter only focus on the scenario generation method we have applied in our case study.

5.4.2 A Heuristic for Moment-matching Scenario Generation

Høyland, Kaut and Wallace propose in (Høyland et al. 2003) an algorithm that produces a discrete joint distribution consistent with specified values of the first four marginal moments and correlations. The algorithm will in the rest of the text be referred to as the HKW algorithm.

The HKW algorithm is a moment matching scenario generation method. Such a method do not require that one knows the distribution functions of the marginals, only that one can describe the marginals by their moments, i.e. the mean, variance, skewness, kurtosis etc. In addition one specifies the correlation matrix and depending on the algorithm, other statistical properties. From the statistical data a moment matching algorithm will construct a discrete distribution satisfying those properties (Kaut & Wallace 2003). Since

a moment matching scenario generation method does not require a distribution function, it is well suited if one only has data.

When applying the HKW algorithm the user specifies the first four moments for every marginal distribution, the correlation between the marginals and how many scenarios the algorithm should generate. The HKW algorithm works as follows; one marginal distribution is generated at a time based on the target moments the user has specified. This is done for all marginals and the marginal distributions are all generated with the same number of realizations. The probability of the i'th realization is the same for each marginal distribution. The HKW algorithm then creates the joint distribution by putting the marginal distributions together. The i'th scenario, that is, the i'th realization of the joint distribution is created by using the i'th realization from each marginal distribution, and given the corresponding probability. Then various transformations are applied in an iterative loop to reach the target moments and correlations. For further information about the HKW algorithm we refer the reader to (Høyland et al. 2003).

5.5   Value of Stochastic Programming

5.5.1   Introduction

So far, we have introduced the concept of stochastic programming and emphasized the importance of keeping the uncertain variables stochastic in the modeling of the problem. This is done without much concern about whether or not this is worthwhile to do. In section 5.1.1 we stressed that the art of modeling is to describe the important aspects of a problem and drop the unimportant ones. Although randomness is present in a problem, it may turn out to be unimportant in the modeling of the problem (Kall & Wallace 1994). Next, we will evaluate the importance of randomness.

5.5.2   Comparing the Deterministic and Stochastic Objective Values

Stochastic programming models have the reputation of being computationally difficult to solve (Birge 1997). The question is; can we replace a stochastic model with a deterministic approach?

The focus in this section is not whether or not we have the right or best model. We are more concerned about how important the uncertainty is in a given model. The most straightforward way to check if randomness is important or not in a given model is to compare the optimal value of the stochastic model with the corresponding optimal value of the deterministic model. The comparison can be done by replacing all random variables in the stochastic model by their means, and with that the problem can be tested in the deterministic case.

When we are to compare the optimal objective values in both the stochastic and deterministic cases, one should be aware that what we are observing is composed of several elements. One important aspect is that while the deterministic solution has one decision for each time period, the stochastic solution "lives" on the scenario tree. As explained in section 5.2.3 the information in the stochastic model may be revealed in several stages, in which the modeler is able to make decisions to compensate for the new information. In the deterministic case, no dynamic is integrated in the modeling process. Although the modeling problem has several time periods, all decisions are made here and now, i.e. at the beginning of the time horizon. Therefore decisions that have elements of option in them will not be of any use in a deterministic model.

As stated above, the stochastic and deterministic way of modeling are fundamentally different from each other. Even if these two models conclude with about the same optimal objective value, one does not know if it is wise to work with a stochastic model. These models are simply too different, and with that difficult to compare (Kall & Wallace 1994).

5.5.3 The Value of the Stochastic Solution - VSS

Despite of the reluctance of comparing the optimal objective values of the stochastic and deterministic models, we present a way of measuring the value of the stochastic solution based on a comparison between those two. This is based on the work of (Birge 1997) and (Wallace 1999).

Let the deterministic version of a given optimization problem be called the mean value solution. Here, all random variables are replaced by their expected values. The expected performance of this is called the expected objective of the mean value solution - EMV. The stochastic version on the other hand is respectively called the stochastic solution and the expected objective value of the stochastic solution - ESS. Further, the definition of the value of the stochastic solution can be stated as the difference between the expected objective value of the stochastic solution and of the mean value solution

$$VSS = ESS - EMV \tag{5.25}$$

Equation (5.25) measures the expected increase in value obtained from solving the stochastic solution of the problem instead of the deterministic one. A low VSS indicates that one should reconsider if it is worthwhile to apply the stochastic model. A clear disadvantage with this method is that we need to know the stochastic solution in advance to be able to find VSS. If we request VSS for the purpose to find out whether or not it is worthwhile to formulate the stochastic solution in the first place, the method is meaningless (Wallace 1999).

5.5.4   The Expected Value of Perfect Information - EVPI

Here, we look at the aspect when we have perfect information of the variables in a stochastic problem. This would only be possible if we have the possibility to wait and see what the uncertain variables turn out to be and then make a recourse action based on this information. With this opportunity, the expected objective value is called the wait-and-see solution - EWS. The modeling problem is now reduced to a deterministic one. Finally, we can express the expected value of perfect information - $EVPI$ as the difference between the expected objective value of the wait-and-see solution and the stochastic solution

$$EVPI = EWS - ESS \qquad\qquad (5.26)$$

The $EVPI$ gives a measure of the maximum amount a decision maker would be ready to pay in return for complete and accurate information about the future (Birge 1997).

Chapter 6

Case Study

6.1 Introduction

We have accomplished a case study on a large hydropower producer in Norway with the purpose to make a stochastic optimization model for bidding and short-term scheduling fitted for this case. The model is based on the work by Stein-Erik Fleten and Trine Kristoffersen in (Fleten & Kristoffersen 2006).

From theory in previous chapters we know that price and inflow are subject to uncertainty also in a short-term perspective. Since the practice among the Norwegian hydropower producers today is to use a deterministic model as a tool in short-term scheduling, it is interesting to investigate whether a stochastic model increases profit compared to a deterministic model.

The practice for the producers is to submit the bids for the next day production before 12.00 each day, i.e. the time frame for constructing bids is short. Thus, a condition for a short-term optimization model is that it requires a short computational time. The bidding is often based upon personal skills and experiences. An optimization model provides a tool which solve the problem of bidding in a structured way. Furthermore, the more details taken into consideration, the more complicated is the optimization model and a longer solution time is required. The detailing level is a trade-off between accuracy of information and computational time.

Often the bidding and the unit commitment problems are handled separately in short-term scheduling. In the model presented next we have included both the bidding and a simplified version of the unit commitment in the same optimization model. This again may reduce time used in the scheduling process.

The aim of our model is to obtain the optimal bidding strategy in terms of expected sales and production profit. In other words, we want the model to compute how much to bid for every hour in the day-ahead market. In addition, we want the model to handle

a simplified version of the unit commitment in the same operation. From chapter 5 we know that this problem can be handled as a two-stage stochastic problem, and solved as a deterministic equivalent. In the first stage the spot price is not yet known and the bidding is computed. In the second stage we anticipate that the spot price and the accepted volume are known and the unit commitment is modeled. The recourse cost arises because of the deviation between the bidden volume and the accepted volume.

Modeling is restricted to a linear programming model (LP-model). In the following section we first present the case and then we present a deterministic approach to the model, i.e. price and inflow are known. After this we introduce a stochastic approach to the model where price and inflow are stochastic variables. Finally the computational results from both the models are presented. From theory in chapter 5 we have learned that a stochastic approach provides a more realistic description of the problem given that uncertain parameters exists. We want to see if a stochastic model of the short-term scheduling problem provides a better expected production profit compared to a deterministic modeling approach.

6.1.1   The Case

The case consists of a large Norwegian hydropower plant with several reservoirs, rivers and tunnels linked together with five power stations. From figure 6.1 one can see the complex structure of the hydropower plant.

In our model we have simplified the power plant to consist of three reservoirs with respectively three underlying stations. For simplicity, new names are given the stations and reservoirs. *Blåsjø* is further denoted as reservoir 1. *Saurdal* is the underlying station and is denoted as station 1. The reservoirs above station *Kvilldal* are aggregated and denoted as reservoir 2. *Kvilldal* is named station 2. The last reservoir is *Suldalsvatnet* and the water from this flows into station *Hylen*. In the model they are respectively denoted as reservoir 3 and station 3. Finally, we can model the power plant as three reservoirs in a cascade with a station underneath each of the reservoirs. This is illustrated in figure 6.2.

Assumptions

Although, the power plant is owned by several producers we have chosen to model it as if there was only one owner. In addition the producer is anticipated to be a price taker. The Norwegian electricity market is generally assumed to be competitive. Variations in inflow, temperatures and other fundamental factors explain observed price movements well and market power problems are minor (Johnsen, Verma & Wolfram 1999).

Furthermore, the only market which the producer participates in is the spot market. Thus, other markets such as the financial market and the balancing market are not
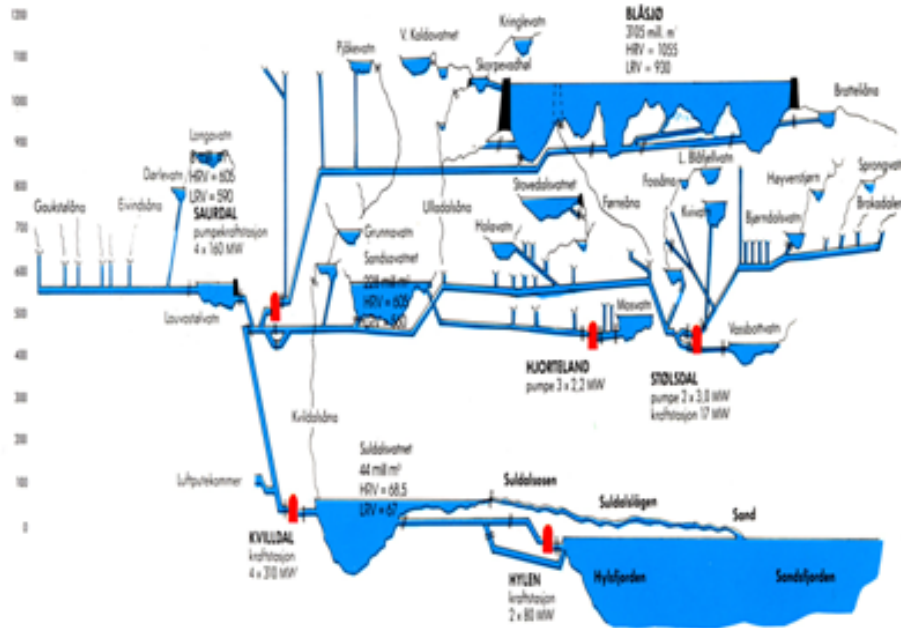
Figure 6.1: Scematic outline of the hydropower plant

regarded in the model. Again we want to stress that the volume in the balancing market is of such a small amount and we have therefore chosen not to model the bidding into the balancing market although this is connected to the short-term scheduling.

6.2   Deterministic Model Formulation

6.2.1   Choice of Time Horizon

The participants at Elspot submit bids for all hours of the next day. Since our model should be used as a tool in the process of constructing bids we choose to use a time horizon of 24 hours. The time horizon is divided into hourly time intervals and denoted $T = 1, ..., 24$.

6.2.2   Modeling the Bidding into Nord Pool

As mentioned in section 2.1.1 there are several types of bids available at Nord Pool. In our model formulation we have included hourly bids and block bids, since these are the most common ones. Hourly bids are bids for a particular hour, and block bids are bids
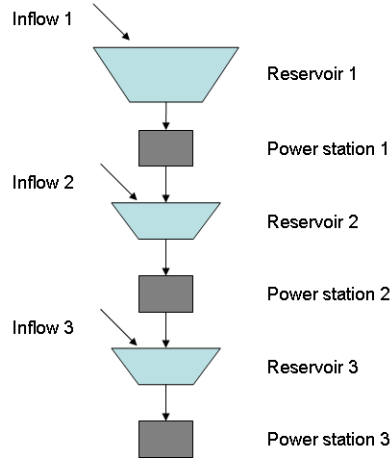
Figure 6.2: Model of the power plant

for minimum four consecutive hours. Thus, the total number of blocks within 24 hours is $B = 231$.

Each bid consists of a price and a corresponding volume. The problem of selecting both bid prices and bid volumes is nonlinear. Based on the work of (Fleten & Pettersen 2005) we avoid these nonlinearities by fixing prices in advance so that only volumes have to be selected. Let p_i, $i \in I$ denote the possible bid prices where $I = 1, ..., 64$. This range is selected because Nord Pool permits the participants to submit maximum 64 price-volume points for each hour. The price points are fixed by choosing equidistant price points which include the possible outcome of the market price. The corresponding bid volumes to p_i are represented by $x_{i,t}$ for hourly bids and $x_{i,b}$ for block bids. Here it is assumed that $i \in I$, $t \in T$ and $b \in B$.

The volume dispatched from hourly bids is denoted y_t and is determined by the point on the bidding curve that corresponds to the spot price, ρ_t. Since the price points are fixed in advance, the relationship between the bid volume and the dispatched volume can be found by a linear interpolation between the price-volume points $(p_i, x_{i,t})$. Now the bidding curve can be expressed as

$$y_t = \frac{\rho_t - p_{i-1}}{p_i - p_{i-1}} x_{i,t} + \frac{p_i - \rho_t}{p_i - p_{i-1}} x_{i-1,t}, \qquad \text{if} \ \ p_{i-1} \le \rho_t < p_i \qquad (6.1)$$

It is natural for a sales bidding curve to be increasing, i.e.

$$x_{i,t} \le x_{i+1,t}, \qquad \forall i \in I, t \in T \qquad (6.2)$$

It should be noted that this constraint may in principle be omitted because Nord Pool does not require such a bidding structure.
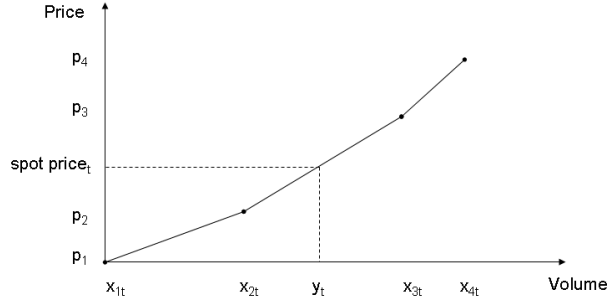
Figure 6.3: Bidding curve of time interval $t \in T$

The so-called mean price condition determines whether a block bid is accepted or not. The price of the block bid must be equal or greater to the average spot price for the hours of the block in order that the block is accepted. From this, one sees that the relation between the volume bid $x_{i,b}$ and the volume accepted y_b for block bids can be expressed as

$$y_b = \sum_{i:p_j \leq \bar{\rho}_b} x_{i,b}, \qquad \forall b \in B \tag{6.3}$$

where

$$\bar{\rho}_b = \frac{1}{|b|} \sum_{t \in b} \rho_t, \qquad \forall b \in B \tag{6.4}$$

Since all transactions are handled at the spot price the revenues from the sales of the hourly accepted bids and block accepted bids accumulate to

$$\sum_{t \in T} \rho_t y_t + \sum_{b \in B} \bar{\rho}_b y_b \tag{6.5}$$

6.2.3 Modeling the Power Stations

To model the power production we index the three reservoirs in our case study $J = \{1, 2, 3\}$. Every station consists of several generators, but because the generators at each station are identical we choose to aggregate the generators to one at each station. Hence, we have a simplified version of the unit commitment. The power generation level from station j at time t is denoted $w_{j,t}$ and the corresponding discharge of water is denoted $v_{j,t}$. Two of the generators at station 1 can also be used as pumps. These will pump water from reservoir 2 up to reservoir 1. We denote the power used to pump the water flow ν_t from reservoir 2 up to reservoir 1 as ω_t.

The water storage level $l_{j,t}$ has to be within its restrictions and we therefore need a variable which keep track of it. If we let the inflow to the reservoirs and the amount of spilled water from the reservoirs be denoted $\delta_{j,t}$ and $r_{j,t}$ respectively, we see from the

discussion above that the water storage level at reservoir 1 at time t can be expressed as

$$l_{1,t} = l_{1,t-1} - v_{1,t} - r_{1,t} + \delta_{1,t} + \nu_t, \qquad \forall t \in T \tag{6.6}$$

That is, the water storage level at time t is equal to the storage level at time $t-1$, minus the water discharged and spilled and plus the natural inflow and the water pumped up from reservoir 2.

Since the reservoirs are cascaded the water flow from an upper reservoir affects the water stored in the underlying reservoirs. This must be allowed for in the expression for the water storage level at reservoir 2 and 3. Since the reservoirs and stations are connected through water gates were water always exists, we do not account for time delays between the upper and lower reservoirs.

For reservoir 2 the water storage level at time t can be formulated as

$$l_{2,t} = l_{2,t-1} - v_{2,t} + v_{1,t} - r_{2,t} + \delta_{2,t} - \nu_t, \qquad \forall t \in T \tag{6.7}$$

Analogous for reservoir 3

$$l_{3,t} = l_{3,t-1} - v_{3,t} + v_{2,t} - r_{3,t} + \delta_{3,t}, \qquad \forall t \in T \tag{6.8}$$

In addition, at time $t = 0$ the water storage level is given as $l_{j,0}$.

All the variables must be within their borders. Thus,

$$l_{\min,j} \le l_{j,t} \le l_{\max,j}, \qquad \forall j \in J, t \in T \tag{6.9}$$

$$w_{\min,j} \le w_{j,t} \le w_{\max,j}, \qquad \forall j \in J, t \in T \tag{6.10}$$

$$v_{\min,j} \le v_{j,t} \le v_{\max,j}, \qquad \forall j \in J, t \in T \tag{6.11}$$

As mentioned in section 3.2.2, the power generated is a function of the water discharge and the net water head of the power station. Whereas the headwater elevation is a function of the reservoir storage level, the tailwater elevation is a function of the discharge (Fleten & Kristoffersen 2006). Because of our short time horizon we assume that the net water head does not vary much, and hence we assume that the power generated is only a function of the water discharged from the reservoir.

The nonlinear relationship between the water flow, $v_{j,t}$ and the power generation may be approximated by a concave function. Since we want to have a linear model we describe the concave fuction as a piecewise linear function with 4 linesegments. That is, we have $q = \{1, ..., 5\}$ discharge-generation points, $(\widehat{v}_{j,q}, \widehat{w}_{j,q})$ for every station. The points represent best-points of different sets of turbines at each station. These points were

found from measurements of the station efficiency. The piecewise linear expression can be stated as

$$w_{j,t} = \widehat{w}_{j,q} + \frac{v_{j,t} - \widehat{v}_{j,q}}{\widehat{v}_{j,q+1} - \widehat{v}_{j,q}} \times (\widehat{w}_{j,q+1} - \widehat{w}_{j,q}), \qquad \text{if } \widehat{v}_{j,q} \leq v_{j,t} \leq \widehat{v}_{j,q+1} \qquad (6.12)$$

The problem when solving equation (6.12) is that one do not know the value of the decision variable $v_{j,t}$ and thus to not know which two points to interpolate between. But because of the concave form this can be solved by replacing (6.12) with

$$w_{j,t} \leq \widehat{w}_{j,q} + \frac{v_{j,t} - \widehat{v}_{j,q}}{\widehat{v}_{j,q+1} - \widehat{v}_{j,q}} \times (\widehat{w}_{j,q+1} - \widehat{w}_{j,q}), \qquad \forall q \in Q, t \in T, j \in J \qquad (6.13)$$

$w_{j,t}$ contributes positively in the objective function and will thus be chosen as high as possible. Since the relationship between $w_{j,t}$ and $v_{j,t}$ is a concave function the optimization model will itself find the right line segment. In figure 6.4, 6.5 and 6.6 the relationship between the generation and the water discharge is shown for station 1, station 2 and station 3, respectively. It should be noted that it is always the lowest line that constitutes the binding constraint, and these line segments form a piecewise linearized concave function.
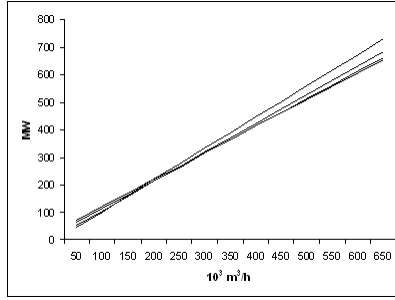

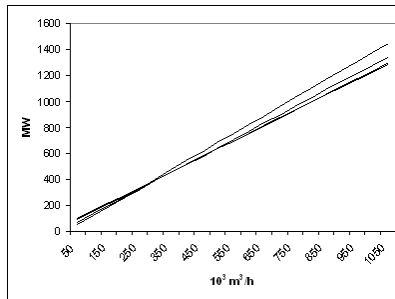Figure 6.4: Power generation function of station 1

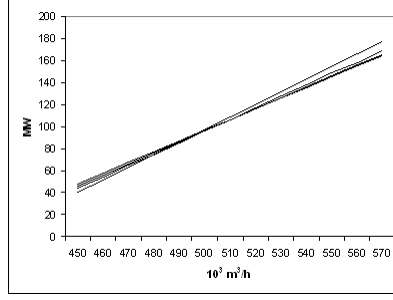Figure 6.5: Power generation function of station 2

Figure 6.6: Power generation function of station 3

For simplicity is the relationship between the energy consumption of the pump, ω_t, and the corresponding water flow from reservoir 2 up to reservoir 1, ν_t, modeled as independent of the consumption level. That is

$$\omega_t = \alpha_{\text{pump}}\nu_t, \qquad \forall t \in T \tag{6.14}$$

where α_{pump} is a constant which says how much energy per cubic meter of water is being used.

Since we assume that the producer sells all the power in the day-ahead market and do not participate in other markets, the total net generation from the stations equals the accepted volumes from the bidding process. That is, we have the constraint

$$\sum_{j \in J} w_{j,t} - \omega_t = y_t + \sum_{b \in B:t \in b} y_b, \qquad \forall t \in T \tag{6.15}$$

6.2.4   Modeling the Water Value

As mentioned in section 4.1, the practice is to make a roughly calculation of the water value in the long- and medium-term production scheduling models, and modify them for the short-term scheduling. Hence, the water values are known to the producer when he is to plan the production in the short-term perspective. Since the focus of our work is on short-term scheduling, we have not put much work into calculating very realistic water values. We have chosen to let the water value be a function only of reservoir levels.

Water Value as a Function of Reservoir Level

When the water level is at its lower boundary, i.e. the reservoir is nearly empty, the water value will be at its highest level. Similar, when the reservoir is full the water value is zero because any new inflow will be disposed off immediately. From this we can describe
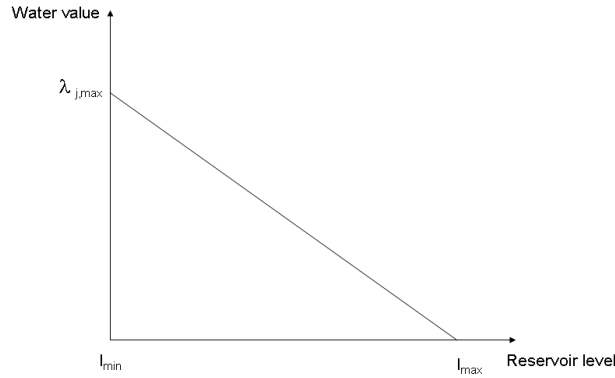
Figure 6.7: Water value as function of the reservoir level

the water value for each reservoir as a linear decreasing function of the reservoir level (Fleten 2006). See figure 6.7.

We have chosen the parameters α_j to denote the energy coefficient for every reservoir j. This has a measurement unit of $[\mathrm{MWh}/m^3]$. After the water has flown through the underlying station, it flows further into the next underlying reservoir and contributes to new power production. Therefore, water in the upper reservoir has an energy coefficient that equals the sum of every energy coefficient from underlying reservoirs in addition to its own.

By multiplying α_j for a given reservoir j with an average of future and forward prices denoted by F, we get the maximum water value, $\lambda_{j,max}$ with denomination $[\text{€}/m^3]$. The water has the value $\lambda_{j,max}$ when the water level is at its lowest boundary. In a situation where the reservoir is nearly empty the probability that the water will be spilled is very low, hence the water is safely stored in the reservoir and can be disposed off at any time in the future. Thus, the water value of the nearly empty reservoir can be approximated by an average of prices of futures and prices of forward contracts (Fleten 2006).

In the Nordic market where the hydropower production stands for a great deal of the total power production, the prices are affected by the water values. Since the water values are affected by the inflow, one can assume that if a great deal of the hydropower producers are located in the same geographical region they are faced with nearly the same inflow, i.e the water values of the different hydropower producers "move" in the same directions. Furthermore, if a great deal of the hydropower producers are faced with pretty equal water values one can expect the market prices to be affected by this by a great deal. Then it is reasonable that in the case of an empty reservoir, the other producers are also nearly empty, and the market price is high. Based on the argument above one can argue that the reflected water value when the reservoir is at its lower boundary should be higher than the average of futures and forward prices.

Reservoir 1 is significantly larger than reservoir 2 and 3. Thus, it takes longer time to

empty this reservoir compared to the other two. Therefore it makes sense to choose future and forward contracts with a longer time horizon to calculate the water value for reservoirs 1's lower boundary. The data material of futures and forward contracts we have used shows that there is little difference in the prices along the time horizon. Therefore, it is a reasonable approximation to use the same future/forward-prices as foundation to quantify the $\lambda_{j,max}$ for all the three reservoirs. Another aspect which can be discussed is whether reservoir 1 should have a constant total water value on a daily basis perspective. This is a rational expectation since this reservoir is significantly larger than the other two. Nevertheless, we have chosen to let the water value vary within the short-term interval which our model operates in. By this, we keep the notations simple and the model is general in the sense that it easier can be applied by other power producer as well.

The maximum water value can now be described as,

$$\lambda_{j,max} = F \times \alpha_j \tag{6.16}$$

Now that we have explained the background for the choice of maximum water value for a given reservoir, we can express the linear water value function as

$$\lambda_j(l) = \lambda_{j,max} \ - \frac{\lambda_{j,max}}{(l_{max} - l_{min})}l \tag{6.17}$$

Total Water Value

The total water value function is the integrated of the water value function expressed in equation (6.17). This will be a concave function of the reservoir level. Now we have a connection between reservoir level and total water value. The total water value is denoted as $\Lambda_j(l)$ and is defined as the integrated of $\lambda_j(l)$.

$$\Lambda_j(l) = \int \lambda_j(l)dl = \lambda_{jmax}l - \frac{\lambda_{jmax}}{2(l_{max} - l_{min})}l^2 + C \tag{6.18}$$

The term C in equation (6.18) is expected to be zero. This follows from the fact that when the water level is at its lowest boundary we regard the total water value to be zero. This is a consequence of the fact that the producer is not able to produce at this water level because of legal or physical restrictions.

A suitable strategy would be to discharge from the reservoir with the lowest water value given that the reservoir underlying the power station is not exceeded by the inflow from the production. We have considered the overflow problem by letting the upper reservoir contain water with a higher value. This forces the lower reservoir to discharge first and the overflow problem is reduced to a certain level. From the introduction of this section we

know that the total water value is a function of the water level in the underlying reservoirs. Hence, the total water values of reservoir 1, 2 and 3 are respectively $\Lambda_1(l_1, l_2, l_3)$, $\Lambda_2(l_2, l_3)$ and $\Lambda_3(l_3)$. It must be noticed that this is a simplification of reality.

As we know from previous chapters, water value is the marginal cost in the hydropower production. Hence, we want to minimize the consumption of water during the time horizon. The difference between the total water value at the end and at the beginning of the period is denoted as the total expense of water during the period. This expense is subtracted from the profit in the objective function and is mathematically explained under

$$\sum_{j \in J} \Lambda_j(l_{j0}) - \Lambda_j(l_{jT}) \tag{6.19}$$
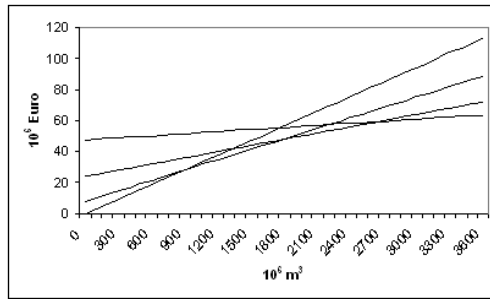
Figure 6.8: Total water value function of reservoir 1
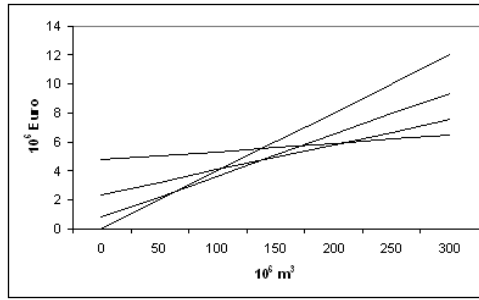
Figure 6.9: Total water value function of reservoir 2

We have to fit the concave total water value function to our linear problem. This is done by a piecewise linearization. Figures 6.8, 6.9 and 6.10 illustrate the total water value as a function of reservoir level by a piecewise linearization based on a few discrete points. The slopes of every point is lined up, such that if we follow the underside of the curve we see that the resulting curve has a concave shape. Because of the way the objective function is formulated we want to maximize the total water value at the end
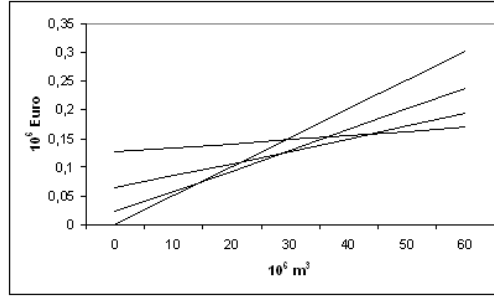
Figure 6.10: Total water value function of reservoir 3

of the time period. By using a constraint which forces the program to choose the slope with the lowest value, we are ensured that the lowest slope is always prevailing. Finally, we describe the concave water value function as a piecewise linear function with 4 line segments. Thus, we have $k = \{1, ...5\}$ points for the relationship between the total water value at the end of the period, $\Lambda_j(l_{jT})$, and the water level, l_{jT}, at the end of the period.

$$\Lambda_j(l_{jT}) \leq \hat{\Lambda}_{k,j} + \frac{\hat{\Lambda}_{k+1,j} - \hat{\Lambda}_{k,j}}{\hat{l}_{k+1,j} - \hat{l}_{k,j}} \times (l_{j,T} - \hat{l}_{k,j}), \qquad \forall k \in K, j \in J \qquad (6.20)$$

6.2.5   Modeling of Start and Stop Costs

An essential factor in unit commitment decisions is the start and stop costs. This is particularly true for thermal plants, but also for hydro plants start/stop costs are relevant. These costs should in a hydro system reflect the fact that whenever there is a start or stop in the production from a unit, water would be lost. In addition altering the production causes unnecessary exhaustion of the plant, increases the risks of component failure and requires more work from the operator. Since these effects of start and stops are hard to measure, assigning a value to the start and stop cost is a difficult task and the costs are usually an estimate. Thus, the important feature in the optimization modeling is to have a model that punishes start and stop in such a way that one avoids getting a result where you stop and then one time period later start the same generator.

Typically start/stop costs are modeled using binary variables saying if the unit is operating or not. Then, when the binary variable changes value, a cost can be imposed. Introducing binary variables in our model causes that we have to solve a mixed integer problem. This is a more complex problem to solve than a model which has a convex linear structure. Thus, if it is possible to avoid integer requirements of the decision variables this would be an advantage. Solving a convex linear problem instead of a mixed integer problem reduces the calculation time and makes it possible to solve a problem with more decision variables and more constraints.

In our model we use the approximated formulation used by Weber in (Weber 2004) to describe start and stop costs. When using this approximation one avoids binary decision variables by defining an additional decision variable, $w_{j,t}^{online}$ which represents the capacity currently online at station j at time step t. Within this model, $w_{j,t}^{online}$ forms an artificial upper bound to the power output and multiplied with the quotient of maximum and minimum output it also forms a lower bound to the power output.

$$w_{j,t} \leq w_{j,t}^{online} \leq w_{max}, \qquad \forall j \in J, t \in T \tag{6.21}$$

$$\left(\frac{w_{min}}{w_{max}} \right) \times w_{j,t}^{online} \leq w_{j,t}, \qquad \forall j \in J, t \in T \tag{6.22}$$

If the capacity online is increasing over time, that is if $w_{j,t}^{online} > w_{j,t-1}^{online}$, start-up costs arise. Thus, the start-up cost function can be defined in the following way:

$$C^{start} \left(w_{j,t}^{online}, w_{j,t-1}^{online} \right) = \begin{cases} c_j^{start} \left(w_{j,t} - w_{j,t-1} \right) & \text{if} \quad w_{j,t}^{online} > w_{j,t-1}^{online} \\ 0 & \text{else} \end{cases} \tag{6.23}$$

On the other and if the capacity online is decreasing over time, i.e. $w_{j,t}^{online} < w_{j,t-1}^{online}$, stop costs arise.

$$C^{stop} \left(w_{j,t}^{online}, w_{j,t-1}^{online} \right) = \begin{cases} c_j^{stop} \left(w_{j,t-1} - w_{j,t} \right) & \text{if} \quad w_{j,t}^{online} < w_{j,t-1}^{online} \\ 0 & \text{else} \end{cases} \tag{6.24}$$

Since $w_{j,t}^{online}$ and $w_{j,t-1}^{online}$ both are decision variables, equation (6.23) and (6.24) are difficult to solve. To cope with this problem we have instead of equation (6.23) and (6.24) used the following equations which states the same

$$C_{j,t}^{start} \geq c_j^{start} \left(w_{j,t} - w_{j,t-1} \right), \qquad \forall j \in J, t \in T \tag{6.25}$$

$$C_{j,t}^{start} \geq 0, \qquad \forall j \in J, t \in T \tag{6.26}$$

$$C_{j,t}^{stop} \geq c_j^{stop} \left(w_{j,t-1} - w_{j,t} \right), \qquad \forall j \in J, t \in T \tag{6.27}$$

$$C_{j,t}^{stop} \geq 0, \qquad \forall j \in J, t \in T \tag{6.28}$$

The start and stop function contributes negatively in the object function. Minimizing $C_{j,t}^{start}$ and $C_{j,t}^{stop}$ requires that the capacity online should be stable for each unit over

time. Since the capacity online variable is linked to the power generation through its restrictions, the model will strive to make as little altering as possible in the power generation from each unit. Thus, in this formulation one does not punish start and stops directly in the traditionally way, but rather says that all altering in the capacity online causes a cost. This formulation can be justified by the fact that altering the production has some of the same negative consequences as start and stops i.e. raising costs of having more people at work, changing parts earlier than else needed and so on.

The efficiency of a station is given by a nonlinear function. For a hydro system, this implies that more water is used to generate the same amount of electricity when the generator is operating at low load compared to when it operates at the best point. Because of this, one wants to avoid that the stations are always kept online. This is taken account for in the relationship between the power output and the water flow.

As mentioned earlier the costs assigned to each start and stop are in a traditionally model just estimates. We have in our model used estimates from EBL (Norwegian Electricity Industry Association). These estimates gives the cost for one start and one stop and are based on the cost of extra work, cost caused by errors when starting/stopping and costs of equipments, maintenances and reinvestments. Since a start not necessary has to be followed by a stop, at least not in a short time frame, we have chosen to divide the start and stop cost in two. Errors when starting occur more frequently than when stopping, therefore the start cost is given a weigh of 55 % and the stop cost a weight of 45% . Since these values are merely an approximation of how starts and stops contribute to of extra costs, we have in our model also used these same figures to assign a value to c_j^{start} and c_j^{stop}. This is an approximation since c_j^{start} and c_j^{stop} in our model reflects the cost of altering generation, instead of the direct costs of start and stop.

6.2.6   Objective Function

Finally, the objective function can be expressed;

$$\max \sum_{b \in B} \sum_{t \in T} (\rho_t y_t + \overline{\rho}_b y_b) - \sum_{j \in J} (\Lambda_j(l_{j0}) - \Lambda_j(l_{jT})) - \sum_{j \in J} \sum_{t \in T} (C_{j,t}^{start} + C_{j,t}^{stop}) \quad (6.29)$$

Equation (6.29) expresses that the revenues are results of the sales of power from both accepted block bids and accepted hourly bids in the day-ahead market. The costs are presented by the usage of water during the time period and the start and stop costs. As mentioned earlier the program aim at maximizing the total value of water at the end of the time period, T. This can easily be seen in the objective function.

6.3 The Stochastic Programming Model

The model described in the last section does not reflect the fact that both day-ahead prices and water inflow are uncertain. To regard the uncertainty we apply a discrete probability distribution of the uncertain data of price and inflow. One way to approximate the continuous distributions into a discrete distribution is to use a scenario tree as described in section 5.3.1. The total number of scenarios is denoted by S. Furthermore, each scenario is denoted by s with a corresponding probability π_s. Finally, we can introduce the two-stage stochastic programming model as a deterministic equivalent

$$\max \sum_{s \in S} \pi_s \left(\sum_{t \in T} \sum_{b \in B} (\rho_{t,s} y_{t,s} + \overline{\rho}_{b,s} y_{b,s}) - \sum_{j \in J} (\Lambda_j(l_{j0,s}) - \Lambda_j(l_{jT,s})) - \sum_{j \in J} \sum_{t \in T} (C_{j,t,s}^{start} + C_{j,t,s}^{stop}) \right) \tag{6.30}$$

subject to

$$\Lambda_{j,s,T}(l_j) \leq \hat{\Lambda}_{k,j} + \frac{\hat{\Lambda}_{k+1,j} - \hat{\Lambda}_{k,j}}{\hat{l}_{k+1,j} - \hat{l}_{k,j}} \times (l_{j,s,T} - \hat{l}_{k,j}), \qquad \forall j \in J, k \in K, s \in S \tag{6.31}$$

$$y_{t,s} = \frac{\rho_{t,s} - p_{i-1}}{p_i - p_{i-1}} x_{i,t} + \frac{p_i - \rho_t}{p_i - p_{i-1}} x_{i-1,t}, \qquad \forall i \in I, t \in T, s \in S \tag{6.32}$$

$$w_{j,t,s} \leq \widehat{w}_{j,q} + \frac{v_{j,t,s} - \widehat{v}_{j,q}}{\widehat{v}_{j,q+1} - \widehat{v}_{j,q}} \times (\widehat{w}_{j,q+1} - \widehat{w}_{j,q}), \qquad \forall q \in Q, t \in T, j \in J, s \in S \tag{6.33}$$

$$C_{j,t,s}^{start} \geq c_j^{start} (w_{j,t,s} - w_{j,t-1,s}), \qquad \forall j \in J, t \in T, s \in S \tag{6.34}$$

$$C_{j,t,s}^{stop} \geq c_j^{stop} (w_{j,t-1,s} - w_{j,t,s}), \qquad \forall j \in J, t \in T, s \in S \tag{6.35}$$

$$\sum_{j \in J} w_{j,t,s} - \omega_{t,s} = y_{t,s} + \sum_{b \in B: t \in b} y_{b,s}, \qquad \forall t \in T, s \in S \tag{6.36}$$

$$l_{1,t,s} = l_{1,t-1} - v_{1,t,s} - r_{1,t,s} + \delta_{1,t,s} + \nu_{t,s}, \qquad \forall t \in T, s \in S \tag{6.37}$$

$$l_{2,t,s} = l_{2,t-1} - v_{2,t,s} + v_{1,t,s} - r_{2,t,s} + \delta_{2,t,s} - \nu_{t,s}, \qquad \forall t \in T, s \in S \tag{6.38}$$

$$l_{3,t,s} = l_{3,t-1} - v_{3,t,s} + v_{2,t,s} - r_{3,t,s} + \delta_{3,t,s}, \qquad \forall t \in T, s \in S \qquad (6.39)$$

$$x_{i,t} \leq x_{i+1,t}, \qquad \forall i \in I, t \in T \qquad (6.40)$$

$$\omega_t = \alpha_{\text{pump}}\nu_t, \qquad \forall t \in T \qquad (6.41)$$

$$l_{\min,j} \leq l_{j,t,s} \leq l_{\max,j}, \qquad \forall j \in J, t \in T, s \in S \qquad (6.42)$$

$$w_{\min,j} \leq w_{j,t,s} \leq w_{\max,j}, \qquad \forall j \in J, t \in T, s \in S \qquad (6.43)$$

$$v_{\min,j} \leq v_{j,t,s} \leq v_{\max,j}, \qquad \forall j \in J, t \in T, s \in S \qquad (6.44)$$

$$w_{j,t,s} \leq w_{j,t,s}^{online} \leq w_{max}, \qquad \forall j \in J, t \in T, s \in S \qquad (6.45)$$

$$\left(\frac{w_{min}}{w_{max}}\right) \times w_{j,t,s}^{online} \leq w_{j,t,s}, \qquad \forall j \in J, t \in T, s \in S \qquad (6.46)$$

$$C_{j,t,s}^{start} \geq 0, \qquad \forall j \in J, t \in T, s \in S \qquad (6.47)$$

$$C_{j,t,s}^{stop} \geq 0, \qquad \forall j \in J, t \in T, s \in S \qquad (6.48)$$

6.4   Scenario Generation

6.4.1   Application of the HKW Algorithm

We choose to use the HKW algorithm for scenario generation. For further information about the HKW algorithm see section 5.4.2 or (Høyland et al. 2003). In our optimization model of the bidding process the day-ahead market price and the inflow to the two lower situated reservoirs are stochastic variables. Inflow to reservoir 1 is anticipated to be deterministic because of reservoir 1´s magnitude. Since we want to construct a price profile for every scenario, we generate scenarios for price and for price standard deviation. In addition we generate scenarios for the inflow to reservoir 2 and 3.

The HKW algorithm requires that the users specify the first four moments of every variable and the correlation between them. The estimators for these data are collected from a historical data set which consists of the spot price in the price area NO1 and inflow to reservoir 2 and 3 for every hour in 2005. First, we calculate the average price within each day. The same is done with the inflows to the two reservoirs in question. To avoid that the algorithm generates scenarios with negative inflow or price all the data is transformed using the natural logarithm. From these four datasets with 365 data each, the input to the algorithm is computed. The input data is summarized in table 6.1 and 6.2.

	Price	*Standard deviation*	*Inflow reservoir 2*	*Inflow reservoir 3*
Mean	3,3628	0,3444	3,1990	4,5867
Variance	0,0183	0,4138	0,5607	0,0478
Skewness	-0,7035	0,3234	-0,7742	-0,2186
Kurtosis	3,2035	3,7888	3,4875	3,2017

Table 6.1: Input data: First four moments

	Price	*Standard deviation*	*Inflow reservoir 2*	*Inflow reservoir 3*
Price	1,0000	0,0118	0,2502	0,0411
St. deviation	0,0118	1,0000	0,1457	0,1400
Inflow res. 2	0,2502	0,1457	1,0000	0,6358
Inflow res. 3	0,0411	0,1400	0,6358	1,0000

Table 6.2: Input data: Correlation matrix

The output from the HKW algorithm is a predefined number of scenarios each with a given probability. We obtain values for price, standard deviation of price and inflow for reservoir 2 and 3 within each scenario. Since the input data were transformed using the natural logarithm, the output data have to be transformed back using Euler's number.

6.4.2 Construction of Price Profiles

The issue is to create price profiles connected to each scenario. To catch the variations during the day, we need to base our different profiles on a standard profile which is made from all the price data of all the hours during 2005. The standard profile should reflect the variations over a "typical" day, i.e. show that the prices tend to have a peak in the morning hours and then again a peak in the evening. This comes from the natural hourly fluctuations in the load. We choose to use the median price instead of the average price, since the standard profile based on the median has a "smoother" curve than the one based on the average price. See figure 6.11.

The standard profile based on the median is calculated by first finding the median price of each hour. From these 24 medians called μ_t, a mean $\bar{\mu}$ is calculated which then is

subtracted from each median so that the curve fluctuates around zero. By dividing with the standard deviation of the 24 medians we get a standard deviation equal to one. Let the standard profile be denoted by Ω_t and the standard deviation of the 24 medians be σ_μ, then we see that

$$\Omega_t = \frac{\mu_t - \bar{\mu}}{\sigma_\mu}, \qquad t = 1, ..., 24 \tag{6.49}$$

A profile for each scenario is constructed by multiplying the standard profile Ω_t with each scenario's standard deviation σ_s. Then the price ρ_s belonging to the scenario is added. If we let $\rho_{t,s}$ be the price profile for scenario s we see that

$$\rho_{t,s} = \Omega_t \times \sigma_s + \rho_s, \qquad t = 1, ..., 24 \tag{6.50}$$

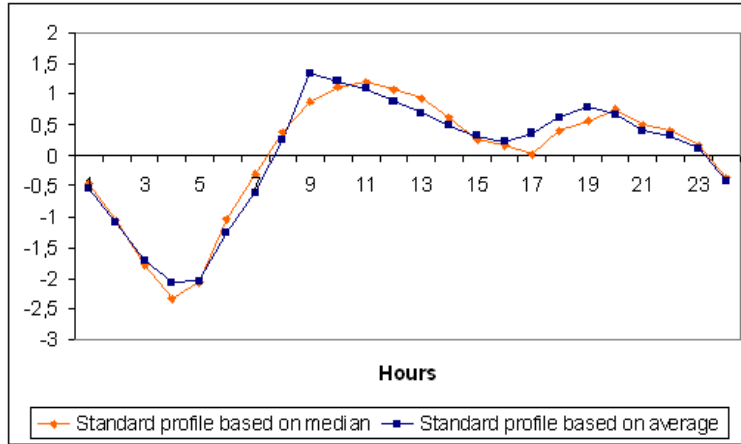and we get price profiles belonging to each scenario $s \in S$.

Figure 6.11: Standard profile based on median and average

Our inflow-data material shows very little variation from hour to hour within a day. Hence, we assume the inflow to be constant for every hour within a day.

6.4.3   Some Results from the Scenario Generation

The number of scenarios within each set generated was either 10, 100 or 250. In this section we will present some results from the scenario generation of a set consisting of 10 scenarios.

Scenarios for the day-ahead market price profile is presented in figure 6.12. Each curve constitutes a price profile in a scenario. Likewise, the scenarios of inflow in reservoir 2 and reservoir 3 are illustrated in figure 6.13 and 6.14 respectively. Each line represents

	Mean	*Std.dev*	*Min.*	*Max.*
Prices [€/MWh]	28,87	1,75	21,61	33,88
Inflow reservoir 1 [m^3/s]	39,00	-	39,00	39,00
Inflow reservoir 2 [m^3/s]	28,11	14,52	6,76	58,24
Inflow reservoir 3 [m^3/s]	98,28	4,92	88,92	107,27

Table 6.3: Descriptive statistics of selected set of scenarios

a scenario of inflow which is assumed to be constant during the day. Our dataset shows that the magnitude of inflow to reservoir 3 is in general higher than the inflow to reservoir 2. This is also reflected in the scenarios in figure 6.13 and 6.14. Notice that reservoir 1 has deterministic inflow and is included in table 6.3 only for consistency. The inflow to reservoir 1 is set to be the mean value of the inflow to reservoir 1 in our dataset.
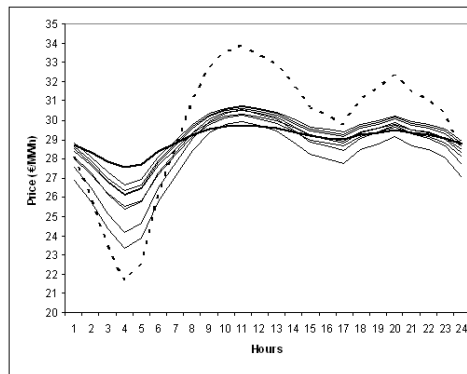

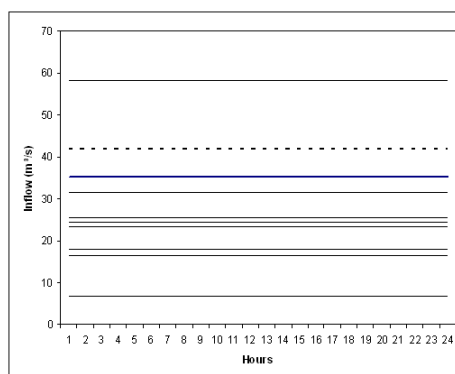Figure 6.12: Hourly day-ahead market price scenarios

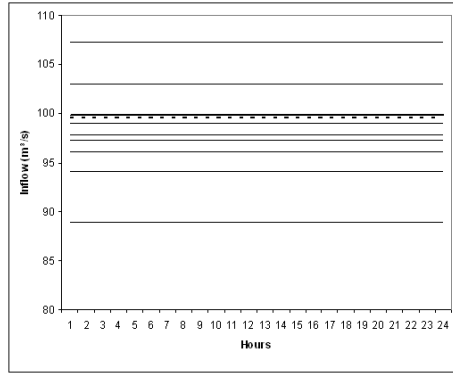Figure 6.13: Hourly water inflow scenarios, reservoir 2

Figure 6.14: Hourly water inflow scenarios, reservoir 3

6.5   Computational Results

We have applied both the deterministic and the stochastic programming model to the case. The deterministic model runs with the average of the five sets of the 10-scenarios, while the stochastic model is tested for 10, 100 and 250 scenarios respectively. The optimization tool applied is Xpress Version 1.6.2. on a Pentium 4 2.4 GHz processor with 256 MB RAM.

6.5.1   Initial Conditions

We have chosen to let the initial conditions for the reservoir levels be 50% of maximum level. The initial value of $w_{j,t=0}^{online}$ is set to maximum production for each reservoir. See appendix A for more information on input data in the model.

6.5.2   General Results

Table 6.4 sum up the computational results when we have applied different numbers of scenarios. An interesting information from the table is that the objective value from the deterministic solution (1 scenario) is consequently lower than the objective values in the stochastic solutions. This can also be confirmed by taking a glance at tables 6.5, 6.6, 6.7 and 6.8. This might indicate that the stochastic model performs better than the deterministic model in general. In addition, the objective value increases with increasing number of scenarios. A reason for this might be that the discrete distribution of the stochastic parameters becomes a better approximation of the real continuous distribution with more scenarios. The computational solution time increases with increasing scenarios. This is an unavoidable disadvantage when the number of scenarios and hence the number of variables increases.

Scenario	Variables	Constraints	Objective value [€]	Solution time [sec]
1	10538	2548	572301	0,02
10	18688	11872	572404	2,6
100	91915	105112	572386	86
250	215102	260512	572560	741,52

Table 6.4: Computational results

In section 6.5.3 one sees that it is optimal to produce at maximum capacity in both scenarios in almost all hours. It should be noted that the model find it optimal to produce at maximum capacity for almost all hours in most of the scenarios, independent of which set of scenarios applied. This might indicate that the water values are set too low or that the costs of altering production, i.e. the approximated start and stop costs are set too high.

6.5.3 Results from a Selected Set of Scenarios

We have chosen a set of 10 scenarios to illustrate some more results. If the reader look back at figure 6.12, the bold line represents scenario 1 and is the scenario with the lowest variance. The dotted line represents scenario 4 which has the greatest variance. Figure 6.15 and 6.16 illustrate how the optimal production progresses during a day when the market price does not fluctuate remarkably, i.e. scenario 1 is prevailing, and when it is volatile as in scenario 4.
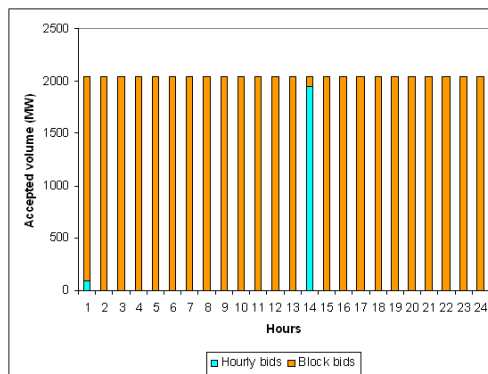
Figure 6.15: Accepted volume, scenario 1

Figure 6.15 states that in scenario 1 the optimal action is to produce at maximum capacity for all the hours during the day. We find this reasonable because the spot price is held steady higher than the water value for all the hours. In figure 6.16 one sees that in scenario 4 one does not want to produce at maximum capacity for all the hours. This is also consistent with how the hourly prices fluctuate as illustrated in figure 6.12. When
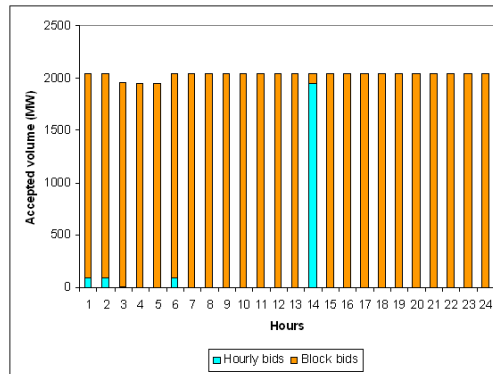
Figure 6.16: Accepted volume, scenario 4

the price moves down as in hour 3, 4 and 5, it is obviously not optimal to produce at maximum capacity.
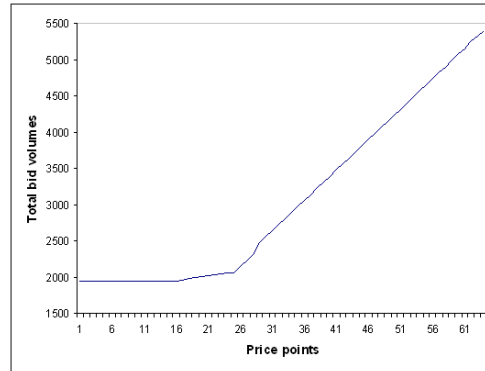
Figure 6.17: Bid volume hour 3

To illustrate the shape of the bidding curve an example hour is chosen. Figure 6.17 shows the optimal bid curve for hour 3 when the model is run with the 10 scenario set. The bid curve is constructed by aggregating the hourly bids for hour 3 and the block bids which apply hour 3. The figure shows that the model finds it optimal to bid approximately 1900 MW at a price as low as zero. This high volume indicates that we probably have set the water values too low. In the model we have not restricted the bid volume and as a consequence one sees that the total bid volume exceeds the maximum generation capacity.

6.5.4   Stability Test

To test for stability five sets of scenarios are made for respectively the 10, 100 and 250 scenario-case. We have carried out the in-stability test described in section 5.3.2.

From the same section we know that if the in-stability test is satisfactorily the out-of-sample stability is also most likely satisfactorily. To test for the in-stability, several scenarios must be generated with the same input data. It is also important that only one scenario generation method is applied. Then the model is run for each of the "equal" scenarios. The in-stability is found satisfactorily if the results from the model do not differ significantly from each other.

We have carried out the test by generating five times a set of the deterministic approach, five times a set of the 10-scenarios, five times a set of the 100-scenarios and five times a set of the 250-scenarios. Further each sample was used as input data in the model. The results from the model are represented in table 6.5, 6.6, 6.7 and 6.8. The maximum percentual deviation from the mean is for all sets below 0,005%. We find that the results within each scenario-set are sufficiently equal, and hence the scenario generation method is stable.

Set of scenarios	*Objective value, [€]*	*Solution time [sek]*
1	572299	0,0
2	572302	0,1
3	572317	0,0
4	572288	0,0
5	572297	0,0
mean	572301	0,02

Table 6.5: Computational results: Deterministic

Set of scenarios	*Objective value, [€]*	*Solution time [sek]*
1	572409	3,1
2	572414	2,4
3	572421	3,0
4	572389	2,0
5	572388	2,5
mean	572404	2,6

Table 6.6: Computational results: 10 scenarios

Value of Stochastic Solution - VSS

To find the value of the stochastic solution (VSS) the deterministic solution is compared to the stochastic solution. The value of the stochastic solution measures the effect of including stochastic variables into the bidding problem rather than simply using the expected values of the variables. In the deterministic case we give the stochastic parameters predefined values which equal the averages of the scenarios, i.e. the expected values of price and inflow. The objective value of the deterministic model is referred to as the

Set of scenarios	Objective value, [€]	Solution time [sek]
1	572391	87,1
2	572382	81,6
3	572365	73,3
4	572392	86,4
5	572402	101,6
mean	572386	86

Table 6.7: Computational results: 100 scenarios

Set of scenarios	Objective value, [€]	Solution time [sek]
1	572560	388,4
2	572584	1328,1
3	572551	1178,8
4	572559	354,5
5	572545	457,8
mean	572560	741,5

Table 6.8: Computational results: 250 scenarios

expected mean value (EMV). Averages of five different runs of the stochastic model are referred to as the expected value of the stochastic solution (ESS). In table 6.9 the results are listed for the different sets of scenarios. The results indicate that there is a slight positive increase in value by applying a stochastic model instead of a deterministic one. The percentual increase is below 1%. Nevertheless, if this model is run every day of the year the total value of applying the model during the year may become significant.

Number of scenarios	ESS	EMV	VSS
10	572404€	572301€	104€
100	572386€	572260€	126€
250	572560€	572259€	301€

Table 6.9: Value of stochastic solution (VSS)

Another way to test the value of the stochastic model is to test it against the situation where we have perfect information. This would only be possible if we had the opportunity to wait until the next days spot prices are published and run the deterministic model with this perfect information and then compare it to the result of our stochastic model.

Block bids reduce the effect of the uncertainty in price by making an "aggregating" bid for several consecutive hours, and the average spot price of those hours decide whether the block bid is accepted or not. By this, one assure oneself against the hourly fluctuations in price. Hence, one expects that block bids are more applied in stochastic models. Figure 6.18 shows the accepted bids when the model is run with the average of the selected set

of scenarios. From a comparison between figure 6.18 and figure 6.15 and 6.16 one sees that block bids are less used in the deterministic case.
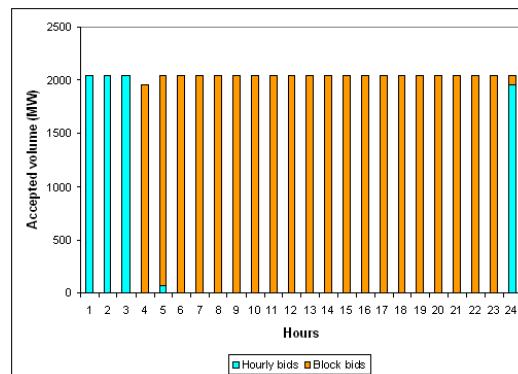

Figure 6.18: Accepted volume deterministic case

At last we want to emphasize the fact that we do obtain a bidding curve that we know is optimal under uncertainty. As long as we know that the price and inflow also are uncertain in the short-term perspective, it is valuable to apply a stochastic programming model in the short-term hydropower scheduling.

Chapter 7

Conclusion and Future Work

7.1 Conclusion

We have presented a short-term scheduling model for a hydropower station which regard the uncertainty in the price and inflow in the short-term perspective.

Our results show that there is a slight improvement in the objective value when the stochastic model is applied instead of the deterministic one. Even though, the percentual increase is rather low one should keep in mind that the day-ahead bidding is done every day. Hence, the accumulated increase in profit may become significant over the year. Stochastic models are often criticized for being computational demanding. The stochastic model presented has a satisfactory solution time, even with 250 scenarios.

It should be noted that the results are preliminary and future work is required to validate the results. We will in this section present some areas were future research could be done.

7.2 Improving the Scenarios

The scenario generation is an important part of making the stochastic model. The quality of the stochastic model is linked to the quality of the scenarios. This follows from the fact that the scenarios provide information to the model of how the stochastic variables vary. If the scenarios are of bad quality, then the stochastic model is necessarily of bad quality too.

Therefore, part of the future work should consist of improving the construction of scenarios. One should carry out more comprehensive testing for stability in the scenarios including testing for different scenario generation methods. A well known problem in making the inflow scenarios is that the inflow is autocorrelated in time. Hence, it is preferable to account for this when choosing scenario generation method. An alternative

way of scenario generation is to base the scenarios on predictions of price and inflow instead of historical data as in the HKW-algorithm. The producers often make their own prognoses of price and inflow which can be utilized in the scenario generation. In addition to the predictions one should also take into account extreme situations which can be weighted with lower probabilities than the other scenarios.

Our results show that the objective value increases with the number of scenarios. From this it is obvious that it is interesting to generate larger sets of scenarios. In addition, the approximation of the continuous "real world" stochastic distribution becomes better the more discrete points in the discrete distribution, i.e. the more scenarios included in the set. With an increasing number of scenarios it is also interesting to see if the objective value finally converges toward a certain value. This indicates how many scenarios that is needed in the set. Furthermore, constructing better standard profiles by either making them more specific to a certain day/season or by letting more stochastic variables describe them is left to future work.

7.3 Improving the model

It is clearly an advantage if the model is programmed more generic. This would make it easy to adjust the model to other hydropower plants. A higher detailing level will describe the physical system better. In our model formulation we do not consider the generation at unit level only at station level. Since we have chosen best-points combination of the turbines to construct the station generation curves, the model finds a too good solution. This would be avoided if one modeled each unit explicitly. In addition it would be interesting to compare our linear model with a mixed integer model. The latter model would have binary variables representing if the units are in operation or not, hence use the standard formulation of start and stop costs.

The modeling of the water values should be improved in further work. The most appropriate would be to use water values from the long- and medium-term scheduling models as input in the short-term model. In addition, a longer time horizon can be chosen to include the coupling to medium-term models. In such a case it would probably be proper to use a multi-stage stochastic model. The time horizon could also be extended by including the hours before the bidding, then one would take into account the fact that no short-term scheduling plan is independent from day to day.

In our model we have fixed the price points by choosing equidistant price points which include the possible outcome of the market price. Another way this could have been done is to have equiprobable price points. Hence, the probability of being dispatched on any of the $I-1$ line segments is the same.

7.4   Validation of the Model

Comparing the model against the regular practice of short-term scheduling is left to future work. That is to run the model over a period and compare the objective values against the real profit earned from bidding in the spot market during the same period. It is only through such a validation that one can throughly reveal whether a stochastic model performs better than common deterministic practice. The EVPI-test could also be accomplished in this setting.

The model can also be run for special designed scenarios to test how the model behaves in "extreme" situations. By this one would get an anticipation if the model acts rational, since one in "extreme" situations intuitively knows how the model optimally would act.

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Appendix A

Input Data - Case Study

Reservoir	Initial reservoir level $[m^3]$	$w_{j,t=0}^{online}[MW]$
1	1761500000	640
2	138185000	1240
3	29150000	160

Table A.1: Input data: Initial conditions in the model

Res.	min. capacity [MW]	max. capacity [MW]	start cost [€]	stop cost [€]	F [€]
1	100	640	206,25	168,8	35
2	200	1240	309,4	253,0	35
3	55	160	151,25	123,8	35

Table A.2: Data input

For more input data to the model, see the enclosed CD-ROM.

Appendix B

CD-ROM

The contents in the enclosed CD-ROM is:

- Xpress codes
- Input data to Xpress
- Input data to HKW algorithm
- Output data from HKW algorithm, five sets of 10 scenarios
- Output data from HKW algorithm, five sets of 100 scenarios
- Output data from HKW algorithm, five sets of 250 scenarios
- The standard profile

Appendix C

Xpress Programming Codes

```
model "stochastic short-term scheduling"
uses "mmxprs", "mmodbc"; !gain access to the Xpress-Optimizer solver

declarations
        S=10 !number of scenarios
        T=24 !number of hours
        B: integer !number of block
        I=64 !number of price-steps
        J=3 !number of reservoirs
        L=5 ! number of line-points
        P=6 !number of efficiency-points
        O=2 !efficieny-points range

        time_24 = 1..T
        scenario = 1..S
        reservoir = 1..J
        block_data =1..3
        price =1..I
        line_points = 1..L
        v_points = 1..P
        factor=1..O

end-declarations

B:= sum(t in time_24)t-24-23-22

declarations
        block = 1..B
        blocks:array(block_data,block)of integer
              !1.row:start,2.row:stop,3.row: #hours
        a:integer
        q:integer

        prob: array(scenario)of real !probability of each scenario to occur
        price_steps: array(price)of real !price-steps
              spotprice_h: array(time_24,scenario)of real !hourly spotprice from the scenarios
        spotprice_b: array(block,scenario)of real !block-prices made from hourly spot prices


        alpha: array(reservoir)of real !energy coefficient
        start_cost: array(reservoir)of real
        stopp_cost: array (reservoir)of real


        vmin: array(reservoir)of real !minimum level of water flow into a station
        vmax: array(reservoir)of real !maximum level of water flow into a station
        wmin: array(reservoir)of real !minimum level of generation
        wmax: array(reservoir)of real !maximum level of generation
        lmin: array(reservoir)of real !minimum level of reservoir
        lmax: array(reservoir)of real !maximum level of reservoir

        l_start: array(reservoir)of real !initial level at reservoirs
        value_start: array(reservoir)of real !initial value of water
        online_initiell: array(reservoir)of real !initial value of online-variable

        v_point1: array(v_points,factor)of real !waterflow points station 1
        v_point2: array(v_points,factor)of real! waterflow points station 2
        v_point3:array(v_points,factor)of real !waterflow points station 3

        inflow:array(reservoir,scenario)of real !inflow
```

```
        tau: array(reservoir)of integer !time delay

        water_level: array(line_points,reservoir)of real !reservoir level to determine value of water
        value_water: array(line_points, reservoir)of real !value of water
        water_value_empty: array(reservoir)of real !maximum water value
        F: real ! future average price [euro]
        k: real

        pump_min: real !pumps at station 1
                    pump_max: real
                    pump_alpha:real
    end-declarations

!Initialization from Excel
!Here one has to adjust of number of scenarios
initialisations from "mmodbc.odbc:input data Xpress.xls"
[lmin,lmax,wmin,wmax,alpha,start_cost,stopp_cost,tau,l_start,vmin,vmax,online_initiell]as "ResRange"

price_steps as "priceStepRange"
pump_min as "pumpRangeMin"
pump_max as "pumpRangeMax"
pump_alpha as "pumpAlphaRange"

v_point1 as "v1Range"
v_point2 as "v2Range"
v_point3 as "v3Range"
F as "FRange"

!deterministic
!inflow as "DetInflowRange"
!spotprice_h as "DetSpotRange"
!prob as "DetProbRange"

!10 scenarioer
inflow as "inflowRange"
spotprice_h as "spotPriceRange"
prob as "probRange"

!100 scenarioer
!inflow as "inflowRange100"
!spotprice_h as "spotRange100"
!prob as "probRange100"

!250 scenarioer
!inflow as "inflowRange250"
!spotprice_h as "spotRange250"
!prob as "probRange250"
end-initialisations


!Initializations of blocks
a:=1
q:=4
forall(b in block)do
  blocks(1,b):=a
  blocks(2,b):=q
      q:=q+1
 if q=25 then
      a:=a+1
      q:=a+3
```

```
 end-if
  blocks(3,b):= blocks(2,b)-blocks(1,b)+1
 end-do

!Initializations of block spot price
forall(b in block, s in scenario)do
     spotprice_b(b,s):=(1/blocks(3,b))*sum(t in time_24|t<=blocks(2,b)and blocks(1,b)<=t)spotprice_h(t,s)
end-do

!Initializing maximum water value
forall(j in reservoir)
water_value_empty(j):= F*sum (i in j..J)alpha(i)


!Initializing the water levels to dermine water values
forall(j in reservoir)do
     k:=0
     forall(lp in line_points)do
          water_level(lp,j):= k*lmax(j)
          k:=k + 1/(L-1)
     end-do
end-do

!Intializing the different values of the water
forall(lp in line_points, j in reservoir)
     value_water(lp,j):= water_value_empty(j)*water_level(lp,j)-
                              ((water_value_empty(j)*water_level(lp,j)*water_level(lp,j))/(2*(lmax(j)-lmin(j))))

!Initalizing value of the water in start
forall(j in reservoir,s in scenario)
     forall(lp in line_points|lp<L)do
     if (water_level(lp,j)<=l_start(j) and l_start(j)<water_level(lp+1,j))then
          value_start(j):= ((l_start(j)- water_level(lp,j))*(value_water(lp+1,j)-value_water(lp,j)))/
                              (water_level(lp+1,j)-water_level(lp,j))+ value_water(lp,j)
     end-if
     end-do



declarations !of decision variables

     z: array(scenario)of mpvar !used in formulation of objective function
     x_h: array(price,time_24)of mpvar !hourly bid
     y_h: array(time_24,scenario)of mpvar !accepted hourly bid
     x_b: array(price,block)of mpvar !block bid
     y_b: array(block,scenario)of mpvar !accepted block bid

     v: array(reservoir,time_24,scenario)of mpvar !water flow
     w: array(reservoir,time_24,scenario)of mpvar !generation

     online: array(reservoir, time_24, scenario)of mpvar !online variable to determine start/stop costs
     start: array(reservoir, time_24, scenario)of mpvar !start costs
     stopp: array(reservoir, time_24, scenario)of mpvar !stop costs

     l: array(reservoir, time_24,scenario)of mpvar !reservoir level
     value_end: array(reservoir,scenario)of mpvar !value of the water at end of period
     r: array(reservoir,time_24,scenario)of mpvar !amount of spill

     pump: array(time_24,scenario)of mpvar !energy used in pump
     pump_flow: array(time_24,scenario)of mpvar !water flow from pump
```

end-declarations

!OBJECTIVE FUNCTION
profit:= sum(s in scenario)(prob(s)*z(s))

!CONSTRAINTS
forall(s in scenario)
    z(s)=((sum(t in time_24)spotprice_h(t,s)*y_h(t,s))+ !revenues hourly bids
        (sum(b in block)spotprice_b(b,s)*blocks(3,b)*y_b(b,s))+ !revenues block bids
        (sum(j in reservoir)value_end(j,s))- !total value of water at end
        (sum(j in reservoir)value_start(j))- ! total value of water at start
        (sum(j in reservoir,t in time_24)start(j,t,s))-!app. start costs
        (sum(j in reservoir,t in time_24)stopp(j,t,s)))!app. stop costs

!the value of the water in the end if the end-level is between 1 and 2
forall(s in scenario,j in reservoir)
value_end(j,s)<=((l(j,T,s)- water_level(1,j))*(value_water(2,j)-value_water(1,j)))/
                   (water_level(2,j)-water_level(1,j))+ value_water(1,j)

!the value of the water in the end if the end-level is between 2 and 3
forall(s in scenario,j in reservoir)
value_end(j,s)<=((l(j,T,s)- water_level(2,j))*(value_water(3,j)-value_water(2,j)))/
                   (water_level(3,j)-water_level(2,j))+ value_water(2,j)

!the value of the water in the end if the end-level is between 3 and 4
forall(s in scenario,j in reservoir)
value_end(j,s)<=((l(j,T,s)- water_level(3,j))*(value_water(4,j)-value_water(3,j)))/
                   (water_level(4,j)-water_level(3,j))+ value_water(3,j)

!the value of the water in the end if the end-level is between 4 and 5
forall(s in scenario,j in reservoir)
value_end(j,s)<=((l(j,T,s)- water_level(4,j))*(value_water(5,j)-value_water(4,j)))/
                   (water_level(5,j)-water_level(4,j))+ value_water(4,j)

!Relationship between accepted volume and bid volume
 forall(t in time_24,s in scenario, i in 1..I-1|
    price_steps(i) <= spotprice_h(t,s) and price_steps(i+1) > spotprice_h(t,s))!)

 y_h(t,s)<=((spotprice_h(t,s)-price_steps(i))/(price_steps(i+1)-price_steps(i)))*x_h(i+1,t)+
             ((price_steps(i+1)-spotprice_h(t,s))/(price_steps(i+1)-price_steps(i)))*x_h(i,t)

 forall(b in block, s in scenario)
 y_b(b,s)= sum(i in price|price_steps(i)<=spotprice_b(b,s))x_b(i,b)

!Realation between generation and water flow at station 1
 forall(j in reservoir,t in time_24,s in scenario|j=1)
 w(j,t,s)<=v_point1(1,1)+((v(j,t,s)-v_point1(1,2))/(v_point1(2,2)-v_point1(1,2)))*(v_point1(2,1)-v_point1(1,1))

 forall(j in reservoir,t in time_24,s in scenario|j=1)
 w(j,t,s)<=v_point1(2,1)+((v(j,t,s)-v_point1(2,2))/(v_point1(3,2)-v_point1(2,2)))*(v_point1(3,1)-v_point1(2,1))

 forall(j in reservoir,t in time_24,s in scenario|j=1)
 w(j,t,s)<=v_point1(3,1)+((v(j,t,s)-v_point1(3,2))/(v_point1(4,2)-v_point1(3,2)))*(v_point1(4,1)-v_point1(3,1))

 forall(j in reservoir,t in time_24,s in scenario|j=1)

w(j,t,s)<=v_point1(4,1)+((v(j,t,s)-v_point1(4,2))/(v_point1(5,2)-v_point1(4,2)))*(v_point1(5,1)-v_point1(4,1))

forall(j in reservoir,t in time_24,s in scenario|j=1)
w(j,t,s)<=v_point1(5,1)+((v(j,t,s)-v_point1(5,2))/(v_point1(6,2)-v_point1(5,2)))*(v_point1(6,1)-v_point1(5,1))

!Realation between generation and water flow at station 2
forall(j in reservoir,t in time_24,s in scenario|j=2)
w(j,t,s)<=v_point2(1,1)+((v(j,t,s)-v_point2(1,2))/(v_point2(2,2)-v_point2(1,2)))*(v_point2(2,1)-v_point2(1,1))

forall(j in reservoir,t in time_24,s in scenario|j=2)
w(j,t,s)<=v_point2(2,1)+((v(j,t,s)-v_point2(2,2))/(v_point2(3,2)-v_point2(2,2)))*(v_point2(3,1)-v_point2(2,1))

forall(j in reservoir,t in time_24,s in scenario|j=2)
w(j,t,s)<=v_point2(3,1)+((v(j,t,s)-v_point2(3,2))/(v_point2(4,2)-v_point2(3,2)))*(v_point2(4,1)-v_point2(3,1))

forall(j in reservoir,t in time_24,s in scenario|j=2)
w(j,t,s)<=v_point2(4,1)+((v(j,t,s)-v_point2(4,2))/(v_point2(5,2)-v_point2(4,2)))*(v_point2(5,1)-v_point2(4,1))

forall(j in reservoir,t in time_24,s in scenario|j=2)
w(j,t,s)<=v_point2(5,1)+((v(j,t,s)-v_point2(5,2))/(v_point2(6,2)-v_point2(5,2)))*(v_point2(6,1)-v_point2(5,1))

!Realation between generation and water flow at station 3
forall(j in reservoir,t in time_24,s in scenario|j=3)
w(j,t,s)<=v_point3(1,1)+((v(j,t,s)-v_point3(1,2))/(v_point3(2,2)-v_point3(1,2)))*(v_point3(2,1)-v_point3(1,1))

forall(j in reservoir,t in time_24,s in scenario|j=3)
w(j,t,s)<=v_point3(2,1)+((v(j,t,s)-v_point3(2,2))/(v_point3(3,2)-v_point3(2,2)))*(v_point3(3,1)-v_point3(2,1))

forall(j in reservoir,t in time_24,s in scenario|j=3)
w(j,t,s)<=v_point3(3,1)+((v(j,t,s)-v_point3(3,2))/(v_point3(4,2)-v_point3(3,2)))*(v_point3(4,1)-v_point3(3,1))

forall(j in reservoir,t in time_24,s in scenario|j=3)
w(j,t,s)<=v_point3(4,1)+((v(j,t,s)-v_point3(4,2))/(v_point3(5,2)-v_point3(4,2)))*(v_point3(5,1)-v_point3(4,1))

forall(j in reservoir,t in time_24,s in scenario|j=3)
w(j,t,s)<=v_point3(5,1)+((v(j,t,s)-v_point3(5,2))/(v_point3(6,2)-v_point3(5,2)))*(v_point3(6,1)-v_point3(5,1))

!Increacing bid curve
forall(t in time_24,i in 1..(I-1))
    x_h(i,t)<=x_h(i+1,t)

!online constraints
forall(j in reservoir, t in time_24, s in scenario)
    online(j,t,s)<=wmax(j)
forall(j in reservoir, t in time_24, s in scenario)
    w(j,t,s)<=online(j,t,s)
forall(j in reservoir, t in time_24, s in scenario)
    (wmin(j)/wmax(j))*online(j,t,s)<=w(j,t,s)

!start/stop costs
forall(j in reservoir, t in time_24, s in scenario)
    start(j,t,s)>=0

```
forall(j in reservoir, t in time_24, s in scenario|t=1)
     start(j,t,s)>=start_cost(j)*(online(j,t,s)-online_initiell(j))

forall(j in reservoir, t in time_24, s in scenario|t<T)
     start(j,t+1,s)>=start_cost(j)*(online(j,t+1,s)-online(j,t,s))

forall(j in reservoir, t in time_24, s in scenario)
     stopp(j,t,s)>=0

forall(j in reservoir, t in time_24, s in scenario|t=1)
     stopp(j,t,s)>=stopp_cost(j)*(online_initiell(j)-online(j,t,s))

forall(j in reservoir, t in time_24, s in scenario|t<T)
     stopp(j,t+1,s)>=stopp_cost(j)*(online(j,t,s)-online(j,t+1,s))


!generation equals accepted volume
forall(t in time_24,s in scenario)
     sum(j in reservoir)w(j,t,s)-pump(t,s)=y_h(t,s)+
               sum(b in block|t<=blocks(2,b)and blocks(1,b)<=t)y_b(b,s)


!water flow within its restrictions
forall(j in reservoir, t in time_24,s in scenario)
vmin(j)<=v(j,t,s)

forall(j in reservoir, t in time_24,s in scenario)
v(j,t,s)>=vmax(j)

!generation within its restrictions
forall(j in reservoir, t in time_24, s in scenario)
wmin(j)<=w(j,t,s)

forall(j in reservoir, t in time_24, s in scenario)
wmax(j)>=w(j,t,s)

!reservoir within its restrictions
forall(j in reservoir,t in time_24,s in scenario)
lmin(j)<=l(j,t,s)

forall(j in reservoir,t in time_24,s in scenario)
lmax(j)>=l(j,t,s)

!waterlevel balance at reservoir 1
forall(j in reservoir,t in time_24,s in scenario|j=1 and t=1)
l(j,t,s)-l_start(j)+v(j,t,s)+r(j,t,s)=inflow(j,s)+ pump_flow(t,s)

forall(j in reservoir, t in time_24,s in scenario|j=1 and t>1)
l(j,t,s)-l(j,t-1,s)+v(j,t,s)+r(j,t,s)=inflow(j,s)+ pump_flow(t,s)

!waterlevel balance at reservoir 2
forall(j in reservoir, t in time_24, s in scenario|j=2 and t=1 and t>tau(j))
l(j,t,s)-l_start(j)+v(j,t,s)+ r(j,t,s)=v(j-1,t-tau(j),s)+inflow(j,s)- pump_flow(t,s)

forall(j in reservoir, t in time_24, s in scenario|j=2 and t>1 and t>tau(j))
l(j,t,s)-l(j,t-1,s)+v(j,t,s)+ r(j,t,s)=v(j-1,t-tau(j),s)+inflow(j,s)-pump_flow(t,s)

forall(j in reservoir, t in time_24, s in scenario|j=2 and t=1 and t<=tau(j))
l(j,t,s)-l_start(j)+v(j,t,s)+ r(j,t,s)=inflow(j,s)-pump_flow(t,s)
```

```
forall(j in reservoir, t in time_24, s in scenario|j=2 and t>1 and t<=tau(j))
l(j,t,s)-l(j,t-1,s)+v(j,t,s)+ r(j,t,s)=inflow(j,s)- pump_flow(t,s)

!waterlevel balance at reservoir 3
forall(j in reservoir, t in time_24, s in scenario|j=3 and t=1 and t>tau(j))
l(j,t,s)-l_start(j)+v(j,t,s)+ r(j,t,s)=v(j-1,t-tau(j),s)+inflow(j,s)

forall(j in reservoir, t in time_24, s in scenario|j=3 and t>1 and t>tau(j))
l(j,t,s)-l(j,t-1,s)+v(j,t,s)+ r(j,t,s)=v(j-1,t-tau(j),s)+inflow(j,s)

forall(j in reservoir, t in time_24, s in scenario|j=3 and t=1 and t<=tau(j))
l(j,t,s)-l_start(j)+v(j,t,s)+ r(j,t,s)=inflow(j,s)

forall(j in reservoir, t in time_24, s in scenario|j=3 and t>1 and t<=tau(j))
l(j,t,s)-l(j,t-1,s)+v(j,t,s)+ r(j,t,s)=inflow(j,s)


!Relation between energy used to pump and water flow
forall(t in time_24,s in scenario)
pump(t,s)=pump_alpha*pump_flow(t,s)


!maximize profit
maximize(profit)

!print out
writeln ("Profit: ", getobjval)
writeln("")

forall(i in price, t in time_24)
        if getsol(x_h(i,t))>0 then
                writeln ("Hourly bid at hour " ,t, " : ",getsol(x_h(i,t)),"MW at pricepoint ", i)
        end-if
writeln("")

forall(t in time_24, s in scenario)
        if getsol(y_h(t,s))>0 then
                writeln ("Accepted volume: ",getsol(y_h(t,s)),"MW at hour ",t," in scenario ",s)
        end-if
writeln("")


forall(i in price, b in block)
        if getsol(x_b(i,b))>0 then
                writeln ("Blockbid " ,b, " :",getsol(x_b(i,b)),"MW at pricepoint ", i)
        end-if
writeln("")

forall(b in block, s in scenario)
        if getsol(y_b(b,s))>0 then
                writeln ("Accepted volume: ",getsol(y_b(b,s)),"MW as block ",b," in scenario ",s)
        end-if


end-model
```