Term Structure of Futures Prices in the Nordic Power Market

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Preface

This thesis was performed during the spring of 2003 at the Norwegian University of Science and Technology, NTNU, Department of Industrial Economics and Technology Management, section of Managerial Economics.

We would like to thank our teaching supervisor, associate professor Stein-Erik Fleten, for valuable comments throughout the process and help on the theoretical and practical issues developed in the thesis

We would also like to thank Kenneth Andreassen and Jan Fredrik Foyn at the Statistics and analysis department of Nord Pool ASA for providing access to the FTP-server and other data sources that have been crucial to our empirical investigations.

Trondheim, 13 June 2003

Roald Maudal Kristian Solum

Summary

In this thesis, we present a spot price model for the Nordic hydro-thermal power system. The model is based on an open market situation, where the participants are price takers. The problem is solved following a social planner's problem, where the social surplus is maximized over the planning horizon. The optimal price reflects the marginal cost of thermal production. The optimal price also reflects the expectations about the price for the coming periods, corrected for active shadow prices. This results in a forward curve somehow less variable than what is actually observed in the market. Several assumptions and factors influencing on the model are discussed in order to obtain an understanding of the difference between the forward curve predicted by the model and the one actually observed in the market.

This thesis also studies the optimal position taken by different producers and retailers in the forward market. Starting from Bessembinder & Lemmon (2002), we developed analytical expressions for the optimal hedge positions. For the producers, the expressions obtained explain the optimal hedge position from the risk aversion, the ability to benefit from high prices and variations in the prices and the production technology. We show substantial differences between flexible hydropower producers with reservoir capacity and thermal base-load producers. The flexible hydropower producers should optimally take a smaller position in the forward market.

For the retailers, we find that the optimal hedge position varies with the price structure of their customers. Today, the customers can choose between a fixed retail price, a variable price or a spot price structure following the system price with a mark-up. The optimal hedge position will also depend on the mix of retail customers, the risk aversion and the flexibility of the customers. I.e. retailers with customers living in areas depending on electricity as the source of heating will be more exposed to price and demand peaks than retailers with more flexible customers, resulting in a larger position in the forward market.

Based on the optimal hedge positions obtained for the different types of participants, the mix of the participants and other factors discussed, we develop hypotheses regarding the risk premium in the forward market. In the short run, we predict the forward price to be an upward bias to the expected spot price on average, indicating a negative risk premium. This contradicts the findings from other commodity markets. We also predict seasonal variations in the risk premium, with a very high negative premium in the high demand, high price winter periods and a positive premium in the spring periods of unregulated discharge. In the long run, we expect a positive risk premium. Empirical results indicate a negative risk premium in the short run. We also find significant differences over the year with a very high negative risk premium in January and a positive premium in June. In addition, we find a significant negative risk premium in July. The latter was not expected. In the long run, empirical results are hard to obtain. But for a time horizon of one to two years, we find a negative risk premium. The magnitude of this risk premium is less than what was found for the short horizon and the two year premium is less than the one year premium.

1 Introduction

This thesis was performed during the spring of 2003 as a final project for the “sivilingeniør” or Master of Technology degree.

Chapter 2 introduces the Nordic Power Market, its organization, products and price structure. A brief comparison of the liquidity in the Nordic power market and the other Nordic financial markets is performed. Chapter 3 describes the general futures pricing theory and its application on electricity futures and forwards. Chapter 4 contains a spot price model and a discussion of its impacts on the forward market. This includes the differences between the implications of the model and what is actually observed in the forward market. In addition, different aspects and limitations of the model are discussed. Chapter 5 contains a derivation of different optimal hedge positions for participants in the market. Three hypotheses for the risk premiums are proposed. Chapter 6 contains an empirical analysis of the risk premiums, while chapter 7 is a discussion of the results obtained in the empirical analysis while chapter 8 concludes.

2 The Nordic Power Market

The Norwegian power market was deregulated in 1991 as a consequence of the Norwegian Energy act of July 29, 1990. This law introduced marked-based principles for production and consumption in Norway. During the first years, the organized trade in the open market was handled by Statnett Marked, a department of the system operator (TSO) Statnett. After England and Wales, Norway was the second country to deregulate the electricity market. In 1996 Sweden followed, resulting in the establishment of the Nordic Power Exchange Nord Pool ASA. This was the world’s first multinational energy exchange. In 1998 Finland joined the common exchange. Denmark joined the market in two steps. Western Denmark (Jutland and Fyn) joined in 1999, while eastern Denmark (Zeeland) joined the market in October 2000 [Koekebakker & Ollmar, 2001].

The Nordic Power Exchange is divided into two separate markets. A physical spot market named Elspot and a financial market, Eltermin, for hedging and speculation. In addition, there exists OTC markets where both physical and financial contracts are traded.

As of January 2002, the Elspot market is operated by Nord Pool Spot AS, which is owned by the system operators Svenska Kraftnät in Sweden, Statnett in Norway, Fingrid in Finland and Nord Pool ASA with 20% of the shares each. The remaining 20% of the shares are to be distributed equally between the two Danish TSO companies Eltra on Jutland and Elkraft System on Zeeland. Svenska Kraftnät and Statnett own Nord Pool ASA with 50% each [Nord Pool, 2003 a].

2.1 The Elspot market

In 2002 123,6 TWh was traded through the Elspot market, equivalent to 32% of the total consumption in the four countries. The total turnover in the spot market in 2002 was NOK 27 billion [Nord Pool, 2003 a].

On Elspot, hourly power contracts are traded daily for physical delivery in the next day's 24-hour period. It could be argued that the Elspot market is actually a day-ahead forward market. We will refer to Elspot as a spot market.

Price calculation is based on the balance between bids and offers from all market participants. Elspot's price mechanism is used to regulate the flow of power where there are capacity restrictions between the various countries and between different areas in Norway. Thus, Elspot may be viewed as a combined energy and capacity market. The spot price is calculated assuming no active restrictions in the power grid. The average spot price over the 24-hour period is called the System Price. When constraints in the grid between the countries are taken into consideration, local prices are obtained. Norway stands out from the four other countries with several local prices due to the physical state of the Norwegian grid. The number of Norwegian price areas varies over time. During the winter of 2002/2003 Norway was divided into four local price areas, but as of early June only two price areas are used. This varies with the available power production capacity and water resources.

Figure 2-1 shows the System Price from 1993 to February 2003. The lowest price was registered on Saturday August 14 at 14,80 NOK/MWh, while the highest price was registered on Monday January 6, 2003 at 831,41 NOK/MWh.

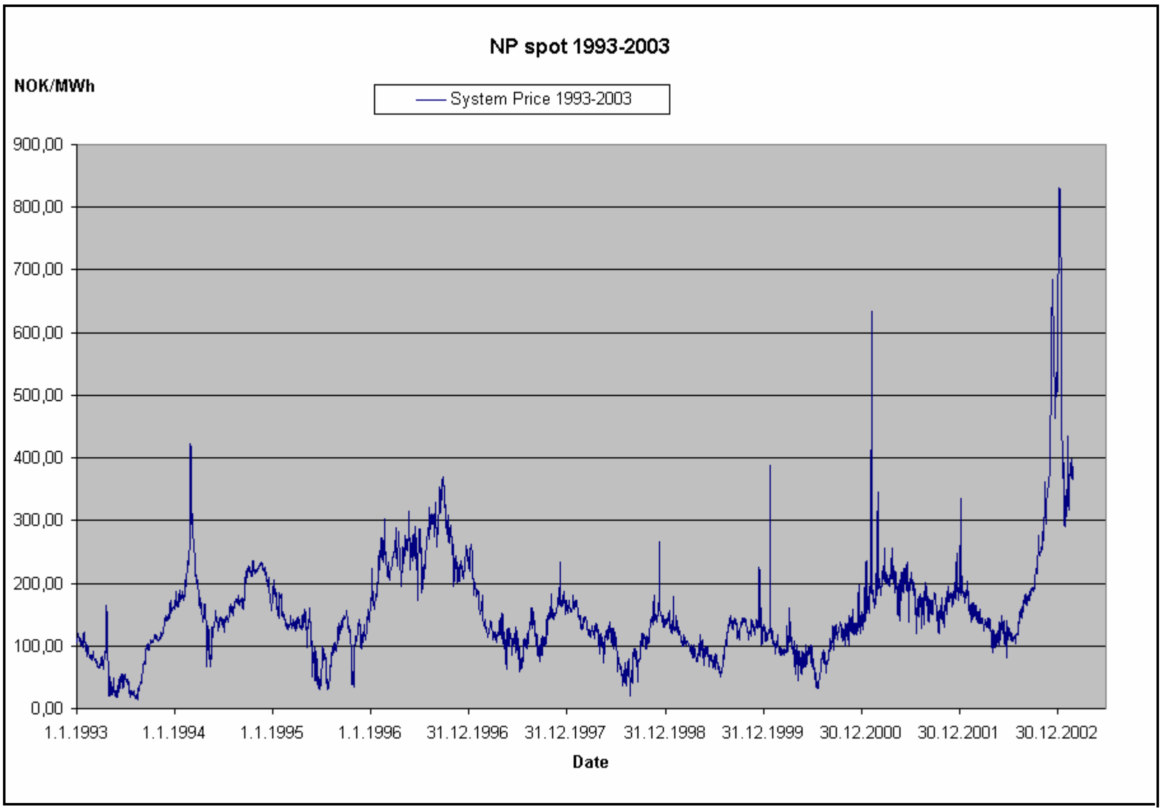

Figure 2-1 The system price at the Nordic Power Exchange Nord Pool from 1993 to Feb. 2003 [source Nord Pool]

The price varies over the year, week and day. This follows from changes in inflow and demand over the different periods [Lucia & Schwartz, 2002]. The demand for electricity follows a somehow noticeable regular pattern within the year, mainly driven by the temperature. The inflow is connected to the rainfall and snow melting over the year. Figure 2-2 displays the pattern in the System Price over the year and the inflow and demand in Norway. The figure indicates high prices in periods of high demand and low inflow and low prices in periods of high inflow and low demand.

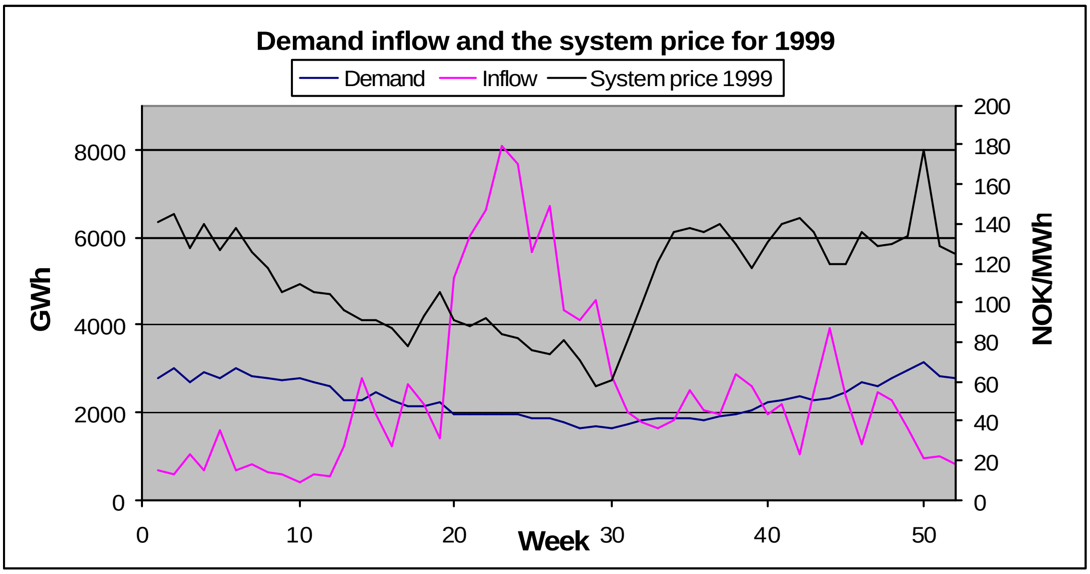

Figure 2-2 The System Price, the Norwegian demand and inflow over the year of 1999. [Source Nord Pool].

2.2 The Eltermin market

In 1993 a standard term futures market based on an auction trade system was established at Statnett Marked, the precursor to the Nordic power exchange, Nord Pool. The contracts were designed to satisfy different needs of the participants, like:

- The need of generators and retailers who apply the products as tools in the risk management.
- The needs of traders who profit from volatility in highly volatile power markets and contribute to high liquidity and trade activity.

In the early days Nord Pool traded futures contracts including base-load contracts, peak-load contracts and off-peak load contracts, all with a time horizon of 6 months. As the market evolved, only base load contracts were pursued [Nord Pool, 1998].

The contracts in the term market took physical delivery of electricity until September 29, 1995. From this point the contracts have been financial contracts based on cash delivery [Nord Pool, 1998]. The contracts are settled using the system price in the spot market as a reference. The transition from a physical to a financial market has improved the possibility for speculators to take positions in the market and has improved the liquidity and promoted trade. Figure 2-3 shows how the liquidity has changed over the years since the development of the Eltermin market.

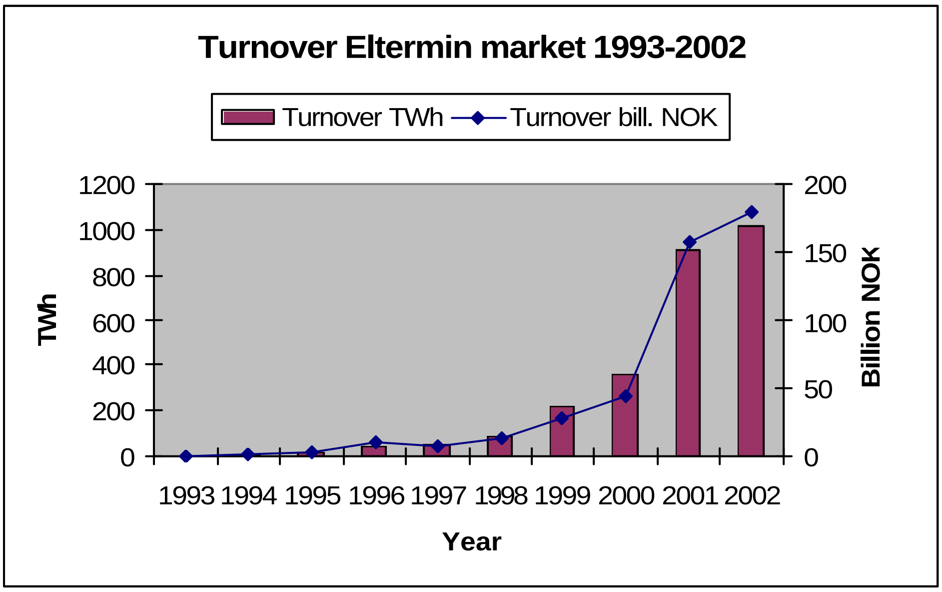

Figure 2-3 Turnover in the financial market at Nord Pool in the period 1993-2003

During the winter and spring of 2003, the liquidity has fallen. In the beginning of May 2003, 658 TWh was traded and cleared at Nord Pool Clearing[1] (NPC), combining Nord Pool's financial market and cleared contracts from the OTC market. At the same time in 2002 the volume was 1142 TWh. An extreme market situation of 2002/2003 is mostly blamed for the fall in liquidity as the players have found it to risky to trade large positions with extreme prices and volatility. In addition, most American trading companies that used to have much risk willing capital have pulled out. On top of this, three Norwegian producers pulled out as market makers during the winter [Montel, 2003]

Comparing Nord Pool with other Nordic financial markets, we find that Oslo Stock Exchange had a turnover of 444 billion NOK in the stock market and 41 billion NOK in the derivatives market for 2002 [Oslo Stock Exchange, 2003]. At the same time Nord Pool and Nord Pool Spot had a total turnover of 461 billion NOK [Nord Pool, 2003 a]. The Swedish stock market, Stocholmsbörsen, is a more liquid market place with a turnover in stocks of 2701,8 billion SEK for 2002 [Stockholmsbörsen, 2003]. Comparing the derivatives markets in the different Nordic countries with the Eltermin market, we find large variations in the traded volume and turnover from exchange to exchange:

[1] Nord Pool Clearing ASA (NPC) is 100% owned by Nord Pool ASA from March 2002. Previously, the tasks of NPC was performed by Nordic Electricity clearing (NEC), an unit of Nord Pool ASA.

Table 2-1 The turnover in the derivatives market for the Nordic stock exchanges and Nord Pool

Exchange	Contracts traded	Underlying value of trading in millions of euro
Nord Pool Eltermin	144 518	23 977
Copenhagen	624 554	Not available
Helsinki	2 644 358	1 202
Oslo	3 177 464	5 515
Stockholm	55 107 653	147 892

Stockholmsbörsen is the leading stock exchange in the Nordic countries and the most active derivatives market, hardly comparable with the others. For Copenhagen Stock Exchange only the numbers of traded contracts are available. The number is significantly lower than for the other countries. Comparing Nord Pool with the stock exchanges, we see that the trade in derivatives at Nord Pool is larger than for most of the markets, only beaten by Stockholmsbörsen. The numbers of contracts are quite low compared to the financial exchanges, but due to high prices for the contracts the total turnover is significant.

When comparing the derivatives market at Oslo Stock Exchange (OSE) with the financial market at Nord Pool we find that Nord Pool has experienced an increase in the turnover over the years. The number of contracts traded is higher for the traditional financial market of OSE, but the total turnover in monetary units is higher for Nord Pool. We also see that the OTC market trades a lower amount of contracts than Eltermin. Still, the turnover is higher for the cleared OTC market. This is due to the fact that contracts with long delivery periods are more traded in the OTC market. We will describe the different products in the Eltermin market in chapter 2.3.

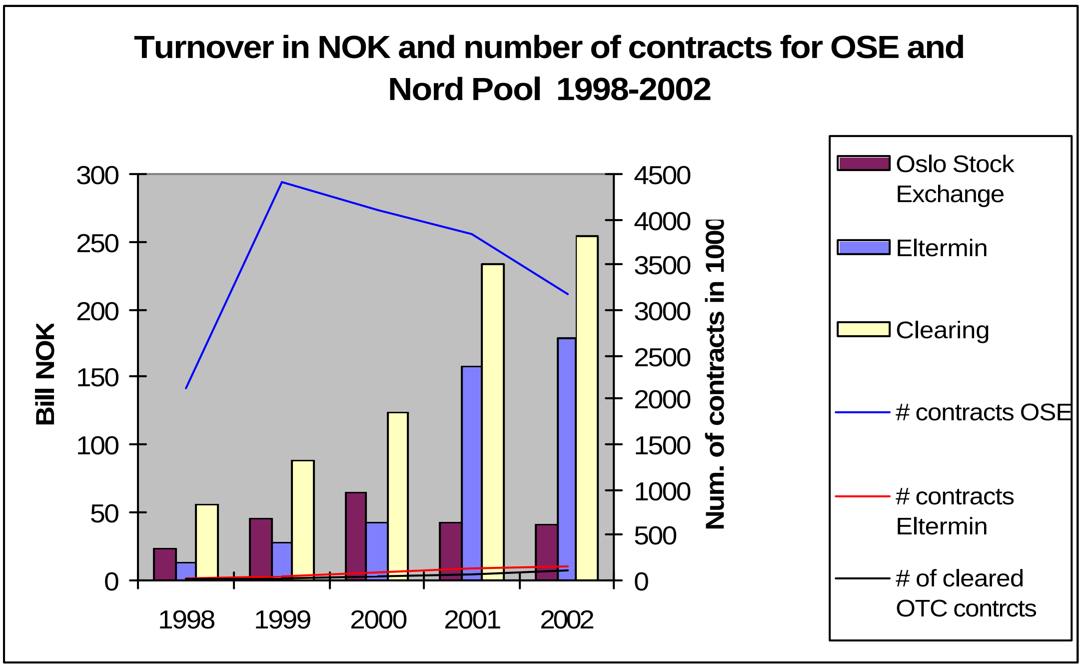

Figure 2-4 The numbers of derivatives contracts and the turnover in billion NOK at Oslo Stock Exchange, the Eltermin market and for OTC contracts cleared through Nord Pool Clearing (NPC) [Source Oslo Stock Exchange and Nord Pool ASA].

2.3 The products in the term market

We will here give a brief description of the contracts traded in the Eltermin market and the OTC market. This involves standardized futures and forward contracts and other types of contracts only traded in the OTC market.

2.3.1 Futures contracts

The futures market is standardised in day, week and block contracts. Each block consists of 4 weeks. When the due date approaches, blocks are split into week contracts. Simultaneously, new blocks are added to maintain a total time horizon of 8-12 months for the blocks. In a normal year of 52 weeks 4-7 weeks are traded daily. Similarly 3 to 9 day contracts are traded daily.

Futures are settled with daily mark to market settlement during the trading period and a final delivery settlement which starts on the due date. The final settlement starting at the due date is based on the difference between the contracts' closing price and the market clearing price (the System Price) in the spot market. Figure 2-5 illustrates the settlement procedure for the futures contracts.

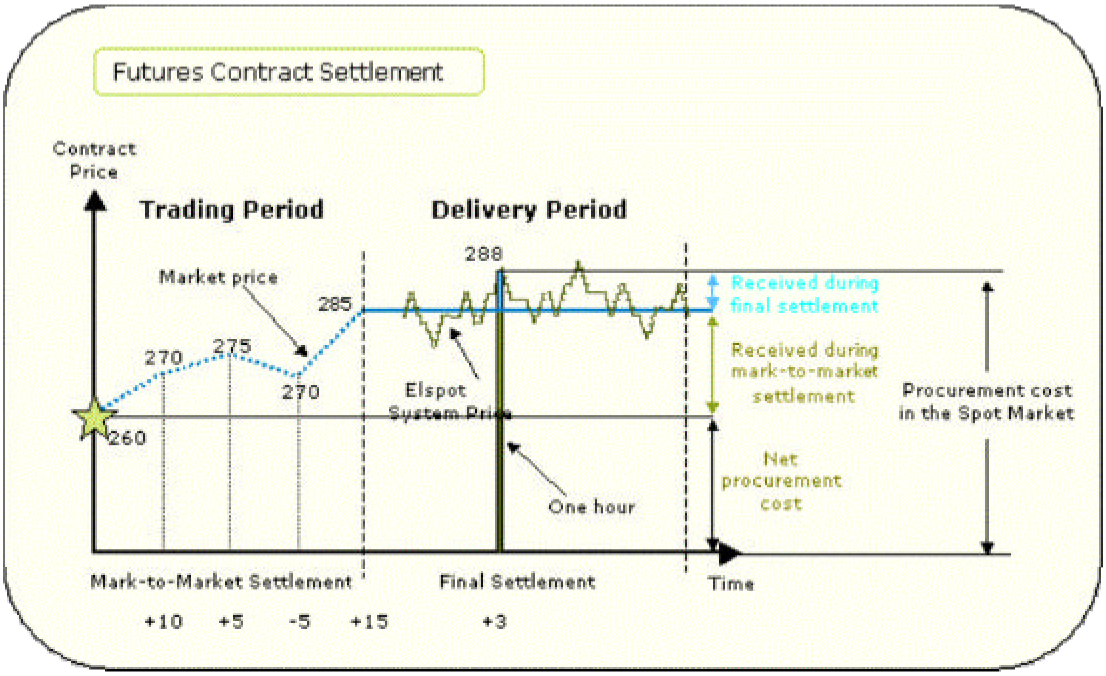

Figure 2-5 The settlement during the trading and delivery period for futures contracts [source Nord Pool]

2.3.2 Forward contracts

The contracts with the longest trading periods and the longest settlement periods are the forward contracts. In contrast to other markets, the forward contracts in the Nordic Power market are standardized in volumes, trading periods and delivery periods.

The products in the forward market are included in table 2-2.

Table 2-2 The forward contracts traded at Nord Pool. xx represents the year of the delivery period.

Season	Ticker	Delivery period	No. of hours
Winter1	FWV1-xx	01. Jan. – 30 April	2879
Summer	FWSO-xx	01. May – 30. Sept.	3672
Winter 2	FWV2-xx	01. Oct. – 31. Dec	2209
Year	FWYR-xx	01. Jan – 31. Dec	8760

There is no splitting of the forward contracts during the trading period, but the year contract is split into seasons in the delivery period [Nord Pool, 2001].

The forward contracts differ from the futures contracts in the settlement procedure. In the trade period prior to the due date there is no mark to market settlements. This makes cash requirements on margin accounts redundant.

2.3.3 Contracts for difference

Nord Pool also lists a forward called Contract for Difference (CfD), which is a forward on the difference between the System Price and the different area prices. The financial contracts listed at Nord Pool have the System Price as underlying, but the participants take physical delivery in their respective area prices. This creates a demand for an instrument that enables a perfect hedge. The CfD satisfies this demand. The market price of these contracts reflects the markets' expectations about the difference between the System Price (MCP) and the area price over the contract period.

$$\text{CfD} = \text{AreaPrice} - \text{MCP}$$

By taking a position in the futures market and making a corresponding trade in the spot market during the delivery period, the participants are completely hedged for the contractual volume. Still, there is uncertainty concerning future load, or volume risk, which cannot be hedged in the Eltermin market. This is further discussed later in the thesis.

2.4 Future development of the Eltermin market

The early deregulation of the Nordic power market has given Nord Pool an advantageous position as the leading power pool. As other countries have followed the deregulation process, the energy markets in Europe are increasingly integrated. Nord Pool is trying to secure its competitive advantages by introducing euro as trading currency. This happens as euro is becoming the main currency for a growing proportion of the exchange members. This includes the Nordic members and the growing number of non-Nordic exchange members.

The transition from NOK to the euro will be a gradual process. The process began with a euro-denominated year contract for 2006, which was listed for trade from January 2, 2003. According to Nord Pool, the common currency will make it easier to introduce new products meeting European market needs, giving the exchange members a broader range of risk management tools [Nord Pool, 2002].

The year contract for 2006, ENOYR-06, will be cascaded into quarters, instead of seasons. The quarters will be listed in groups of 4, the first starting from January 2, 2004. The quarters will be settled as forward contracts.

The Block contracts will be replaced with Month contracts from the spring of 2003. Listing started on April 7, 2003. All contracts will have the prefix ENO, which stands for Electricity Nordic. This means that new listed contracts will have different ticker codes. The month contracts will be listed as forward contracts. This will make spread trading between quarters and months more easy. The cash flow will be equal and thus make a more perfect hedge. The transition from block contracts to monthly contracts will make the exchange more in line with the German exchange, EEX, in which Nord Pool takes proprietary interest.
The Week contracts will be listed with 8 consecutive contracts, in a continuous rolling cycle, as opposed to the current cycle that implies listing of Weeks in groups of 4, after cascading of the Block contracts. The new listing cycle will start from September 2003. Only standardised contracts traded at Nord Pool can be cleared trough NPC by the participants in the OTC market [Nord Pool, 2003 b and Nord Pool, 2003 c]. The week and day contracts will remain futures contracts.

2.5 The OTC market

The OTC market, also known as the bilateral market, has a larger traded volume than Nord Pool. During 2001 910 TWh was traded through Nord Pool's financial market. The same year 1748 TWh was traded in the OTC market and cleared through Nor Pool Clearing (NPC). For 2002 the numbers were 1019 TWh traded at Nord Pool and 2089 TWh cleared through NPC.

The OTC market includes standardised forward and futures contracts similar to those traded at Nord Pool and different types of swing contracts with flexibility in the load profile. Only standardized contracts also traded at Nord Pool can be cleared through NPC. A typical load factor contract has a one-year maturity, 5000 hours of maximum load, with the additional constraint that 2/3 of the energy must be utilized in the summer season, and 1/3 in the winter season. [Fleten et al, 2002] The OTC market also trades contracts with delivery periods of 5, 10 and 20 years [Syvertsen, 2001].

The OTC market has a market share of almost 100% for options. European options are traded at Nord Pool but the liquidity is very low. The OTC market is trading European, American and Asian options.

3 Futures pricing theory

There are two ordinary views of commodity futures pricing. The first is the standard no-arbitrage or cost of carry models. This is also known as the theory of storage. Classical literature on these models includes Kaldor (1939), Working (1948), Brennan (1958) and Telser (1958). The second approach used in literature is based on equilibrium considerations. This alternative view splits a futures price into the expected risk premium and a forecast of future spot price. The classical literature on this approach includes Keynes (1930) and Hicks (1939). Recent work applied on the field of electricity includes Routledge et al (2001) and Bessembinder & Lemmon (2002).

3.1 The theory of storage

This traditional view explains the difference between futures prices and the spot price in terms of interest foregone in storing the commodity, warehousing costs and convenience yield on inventory. The convenience yield can be explained as the premium a holder is willing to pay to benefit from having the commodity instead of the futures. These benefits may include the ability to profit from temporary local shortages or the ability to keep a production process running [Hull, 2000].

Following the theory of storage, the futures price at time t for a contract with maturity at time T is given by the range [McDonald, 2003]

$$S_t e^{r(T-t)} + U - Y \le F_{t,T} \le S_t e^{r(T-t)} + U \qquad (3.1)$$

where S_t is the spot price at time t, r is the interest rate, U is the storing cost from t to T and Y is the convenience yield for the period. If the convenience yield and storing costs are expressed as proportions of the spot price, the futures price can be expressed by the range

$$S_t e^{(r+u-y)(T-t)} \le F_{t,T} \le S_t e^{(r+u)(T-t)} \qquad (3.2)$$

The concept of convenience yield gives a no-arbitrage region for the forward price rather than a no-arbitrage price [McDonald, 2003]. This is because the average investor will not necessarily be able to earn the convenience yield, i.e. participants benefiting from holding the commodity physically are likely to hold the optimal amount already.

Pindyck (1994) concludes that convenience yield is highly convex in inventories for commodities as copper, heating oil and lumber. The convenience yield becomes very large when inventories become small. This prevents stock-outs from occurring.

3.2 The equilibrium approach

This approach is also known as the theory of risk premium. A speculator with a long position hopes that the price of the asset will be above the futures price at maturity. Suppose the speculator puts the present value of the futures price into a risk free investment at time t while simultaneously taking a long futures position. The cash flows to the speculator are

Time t: $-F_{t,T}e^{-r(T-t)}$
Time T: S_T

Where S_T is the value of the asset at time T. The present value of the investment is

$$-F_{t,T}e^{-r(T-t)} + E[S_T]e^{-k(T-t)}$$

where k is the discount rate appropriate for the investment and E denotes the expected value. Assuming that all investment opportunities in securities markets have zero net present value gives

$$F_{t,T} = E[S_T]e^{(r-k)(T-t)} \tag{3.3}$$

The difference between k and r is the risk premium. A typical equilibrium approach to commodity pricing is a model or theory that calculates or explains the size of k, the unknown discount rate. A well-known equilibrium asset prising model is the capital asset pricing model, CAPM. According to the CAPM there are two types of risk, the systematic risk and the non-systematic risk. Holding a well-diversified portfolio can eliminate non-systematic risk. Systematic risk cannot be diversified away. It arises from correlation between returns from the investments and returns from the stock market as a whole. If the systematic risk in an investment is negative, the investor will be prepared to accept returns lower than the risk-free rate [Hull, 2000]. Although power price-dependent instruments have been introduced in the savings market[2], we doubt that the behaviour of investors outside the electricity industry significantly affect power derivative prices. Thus, we do not think that the CAPM can be used to explain k.

Other pricing models introduced to explain k are the consumption CAPM and the arbitrage pricing theory, APT. The APT starts by assuming that each stock's given return depends partly on macroeconomic influences and "noise" - events that are unique to the company. The arbitrage pricing theory does not say what the factors are. It could be an oil price factor or some other fuel factor relevant to the electricity producers, changes in the forecast of real GNP, changes in long government bonds or the inflation [Brealy & Myers, 2000]. It is difficult to see to what extent the APT can determine k in electricity pricing and we find the theory not relevant for this purpose.

The consumption CAPM defines risk as a stock's contribution to the uncertainty about consumption. The electricity prices influence the consumption. The high prices of the winter

[2] The financial institution Nordea has an index linked bond on power prices

of 2002/2003 influenced the consume price index significantly. We do not believe this to be relevant for the participants in the electricity market, thus we believe the consumption CAPM to be unable to explain the discount rate k for electricity prices.

The equilibrium approach, or risk premium approach, and the theory of storage are not competing point of views. Variation in the expected premium or expected change in the spot price translates into variations in the interest rate, the marginal storage cost or the marginal convenience yield in the theory of storage [Fama & French, 1987].

The forward premium is often defined as the difference between the expected spot price and the forward price

$$FP = E\left[S_T\right] - F_{t,T} \tag{3.4}$$

3.3 Futures prices and the expected future spot price

Several authors have discussed whether the futures price is a biased estimate of the expected future spot price or whether it equals the expected future spot price. In the 1930's Hicks and Keynes argued that if hedgers tend to hold short positions and the speculators tend to hold long positions, the forward price will be lower than the expected spot price.

$$F_{t,T} < E\left[S_T\right] \tag{3.5}$$

This happens as speculators require a premium or compensation for taking risk, implying that $k > r$, when following (3.3). The hedgers are prepared to take positions reducing their expected payoff, since they at the same time are reducing their risk. This situation, where the futures price is lower than the expected spot price is called normal backwardation. According to the CAPM this happens if S_T is positively correlated with the stock market.

If the hedgers hold long positions and the speculators hold short positions, the futures price will be above the expected spot price

$$F_{t,T} > E\left[S_T\right] \tag{3.6}$$

This situation is known as contango. According to (3.3) this should happen if $k < r$. Following the CAPM approach, this happens if S_T is negatively correlated with the stock market, i.e. the investment has negative systematic risk.

If the price of electricity S_T is uncorrelated with the level of the stock market, the investment in the forward market has zero systematic risk and $k = r$. Equation (3.3) then shows that the futures price equals the expected spot price

$$F_{t,T} = E\left[S_T\right] \tag{3.7}$$

This is also known as the expectation hypothesis.

Some authors refer to the terms contango and backwardation somewhat different from that above. E.g. Pilipovic (1998) and McDonald (2003) say the market is in contango when the forward curve is upward sloping and in backwardation when the forward curve is downward sloping. This is what Hull (2000) characterizes as normal market and inverted market respectively. We hereby emphasis that when we refer to contango and backwardation, we follow the framework given by (3.6) and (3.5). The following figure is included for clarifying and visual purposes [Copeland & Weston, 1988]

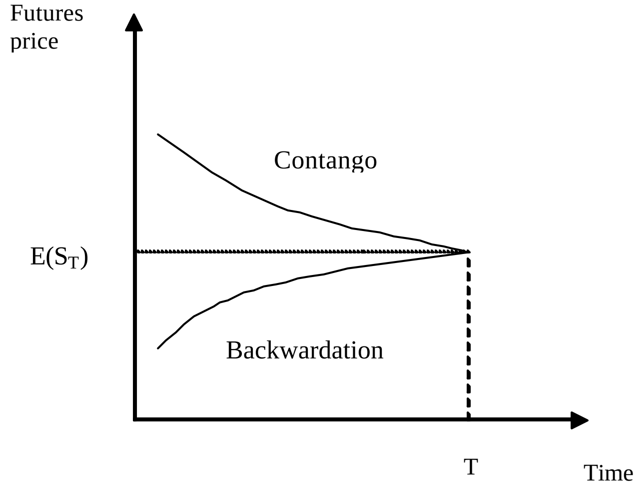

Figure 3-1 The definition of a contango and backwardation market

Syvertsen (2001) shows that for long-term futures and forward contracts, the contract price will be under the expected spot price, i.e. a backwardation situation. This happens as the expected cash flow from a new electricity production facility will be discounted with a higher discount rate than the risk free rate of return. At the same time, the production could be hedged by taking positions in the forward market. The cash flow from the derivatives will be discounted using the risk free rate of return. The discounted cash flows should have the same net present value, and this indicates the forward or futures price to be lower than the expected spot price.

3.4 Futures pricing theory for electricity

Electricity has certain characteristics that make it differ from other commodities. Electricity can be considered as a flow commodity strongly characterized by its very limited storability and transportability. Both limits to the possibility of carrying electricity across time and space are crucial in explaining the behaviour of electricity spot and derivatives prices compared to other commodities [Lucia & Schwartz, 2002]. These limitations reduce arbitrage possibilities, which are based on links across time and space. This will affect the spot–futures relationship, thus the theory of storage or cost-of-carry models do not really apply for pricing power forwards [Bessembinder & Lemmon, 2002]. Longstaff & Wang (2002) draw the same conclusions and focus on how electricity forward prices are related to the expected future spot prices. These two papers both focus on the Pennsylvania, New Jersey, Maryland market (PJM), USA. This is a market dominated by thermal production capacity. However, the Nordic market is largely based on hydropower in which reservoirs play a substantial role. In Norway hydropower constitutes 99% of the total electricity production [Nordel, 2002]. Hydropower constitutes a large share of the electricity production in Sweden as well. According to Gjølberg & Johnsen (2001) this allows the producer to effectively store electricity by keeping water in the reservoirs. The consumers, on the other hand, have no possibility for storing electricity. This results in an asymmetry between the producers' and consumers' possibilities to arbitrage spot-futures using storage. According to Gjølberg & Johnsen (2001), the producers can apply the cost-of carry method. The storage cost will be zero when reservoir levels are low and increase as the reservoir levels increase and the probability for spillage of water increases. The cost of spillage is the value of the electricity that could have been sold.

The non-storability of electricity is also likely to affect derivative pricing significantly, notably influencing on the shape of the forward curve and its behaviour [Lucia & Schwartz, 2002].

3.4.1 Empirical results on electricity markets

Longstaff & Wang (2002) have conducted an empirical analysis of electricity forward pricing at the PJM market, USA, using a high-frequency data set of hourly spot and day-ahead forward prices. Due to differences in terminology, this equals a comparison of the day-ahead spot prices and the regulating power prices in the Nordic market. Longstaff & Wang find significant risk premiums in electricity forward prices at PJM. These premiums vary systematically throughout the day and are directly related to economic risk factors such as the volatility of unexpected changes in demand as well as the risk of price spikes.

4 A spot price model for the Nordic power market

We here develop a spot price model for a system combining hydropower including reservoir capacity with thermal production capacity. The model is based on Johnsen (2001) who develops a supply-demand model for the Norwegian electricity market.

4.1 Model formulation for the hydropower system

Given a hydropower system without storage possibilities, the price will be determined by the inflow and demand, so that supply equals demand

$$D\left(p_j\right) = I_j, \qquad j = t,... \tag{4.1}$$

D( ) = demand equation, D` < 0, and p = price. I = inflow of water measured in energy units. This implies that high inflow results in low prices and low inflow leads to high prices. However, the Nordic hydropower system has quite some water storage capacity. In Norway this capacity is about 75% of the annual generation capacity (Johnsen, 2001). The producers are presumed to generate the highest possible income from their power production by managing their water reservoirs actively. As a consequence water is stored throughout the summer for use in the winter when prices are high.

Figure 4-1 is made to help understanding the physical relationship between inflow, storage and consumption. The figure shows the median inflow and storage aggregated for Norway and Sweden, while the consumption is the real data from 2001.

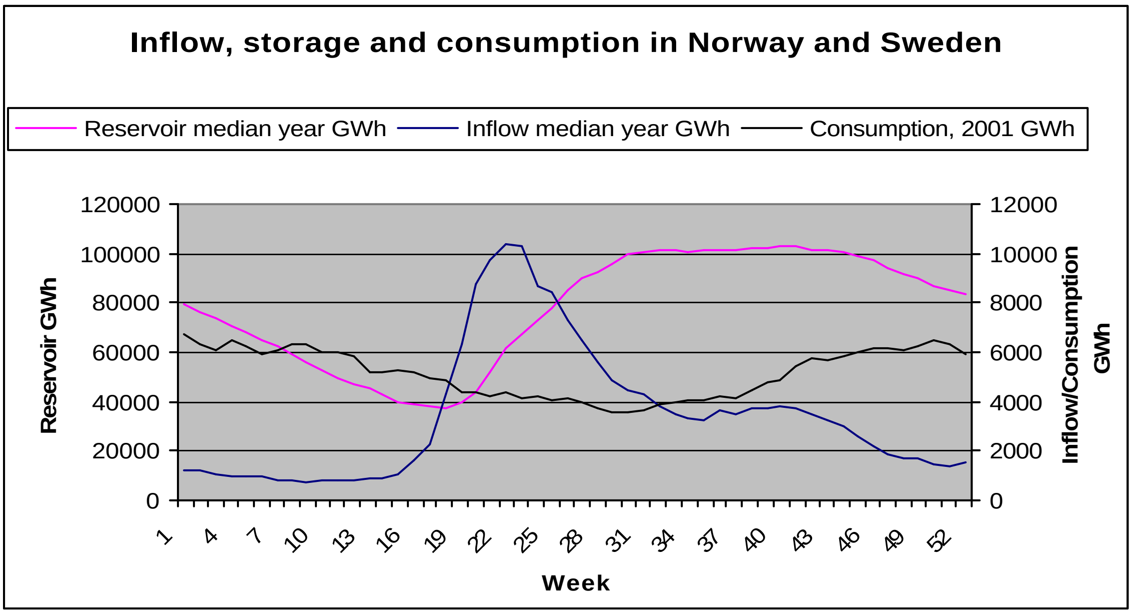

Figure 4-1 The consumption, inflow and reservoir level for Norway and Sweden over the year.

Total consumption for Sweden and Norway was about 268 TWh in 2001. The total water reservoir capacity of Norway and Sweden is approximately 115,5 TWh, while the median yearly inflow adds up to 179 TWh. By introducing a capacity factor as total resources divided by the generation capacity we add some understanding to the flexibility of the system. The total installed capacity for hydropower producers in Norway and Sweden is approximately 43,5 GW [Nordel, 2002]. Total reservoir capacity divided by total capacity gives a factor of 2655 hours. Assuming no overflow such that all the inflow is eventually used for power generation, we obtain the capacity factor of 4115 hours. If we apply the median inflow and the lowest median reservoir filling as the resources, indicating the resources available over a year, and divide by the generation capacity, we obtain a capacity factor of 4989 hours. By not taking the time aspect into consideration the storage capacity of Norway and Sweden is about 43%. This indicates a high degree of flexibility in the power system.

The variation in reservoir levels over time is determined by the water budget. The water budget is given by

$$r_j \le r_{j-1} + I_j - y_j \qquad j = t,... \tag{4.2}$$

where r_j is the water reservoir filling at time j, and y_j is the production of hydropower at time j, both measured in energy units. The inequality is caused by potential overflow. Both rainfall and snow melting amount to the inflow in equation (4.2):

$$I_j = I_j^W + \psi_j s_{j-1} \qquad j = t,... \tag{4.3}$$

Here I_j^W equals the inflow from rainfall and s_j is the snow volume at time j, both measured in energy units. ψ is a variable determining the degree of snow melting, thus $0<\psi<1$.

The snow budget is given by

$$s_j = s_{j-1} + I_j^S - \psi_j s_{j-1} \qquad j = t,... \tag{4.4}$$

Here I_j^S is the snowfall at time j, given in energy units.

There will be several physical constraints for the hydropower system. These are related to reservoir levels, restrictions on draining of the reservoirs and production capacity.

The upper and lower reservoir filling constraints are given by

$$r_j \le \bar{r} \qquad j = t,... \tag{4.5}$$

and

$$r_j \ge \underline{r} \qquad j = t,... \tag{4.6}$$

$\bar{r}$ and $\underline{r}$ represent the upper reservoir capacity and the lower feasible reservoir level respectively. The lower feasible level is normally defined with respect to environmental conditions and is set by public agencies. According to Gjølberg and Johnsen (2001) a reservoir filling of 85 % is interpreted as a typical maximum level. A higher filling than this would give a high probability of spillage of water and thereby a loss of income. At the same time a reservoir filling of 10 % could be seen as the minimum level. At this point the soil conditions would make it difficult to exploit the water.

The generation capacity sets the upper limit of weekly generation

$$y_j \leq \bar{k} \qquad\qquad j = t,... \tag{4.7}$$

$\bar{k}$ represents the maximum weekly power generation measured in energy units.

4.2 Model formulation for the thermal system

By combining thermal production capacity with the hydropower system, we modify the framework given by Johnsen. A thermal production system will be restricted by its total production capacity

$$q_j^{term} \leq q_{max}^{term} \tag{4.8}$$

A cost function for thermal production can be given in different ways. From MatPower, a power system simulation package developed by Zimmermann and Gan (1997) of PSERC at Cornell University, USA, we have that the cost data can be modelled as a piecewise linear function or as a polynomial function on the form

$$C(P) = a_0 + a_1 P + a_2 P^2 + ... + a_n P^n \tag{4.9}$$

where P is the output in MW.

Following Bessembinder and Lemmon (2001) a cost function for the thermal production capacity in the market is given by

$$TC = F + \frac{a}{b}(Q_P)^b \tag{4.10}$$

where F is the fixed cost, Q_P is the total output and b is a constant greater than or equal to two. This implies that the marginal production cost increases with output and is consistent with what is observed in most energy systems. These are characterised by a convex supply curve, which reflects the fact that the industry employs an array of different production technologies and fuel sources. This includes coal, nuclear power, oil and gas. If b is greater than two, marginal cost increases with an increasing rate of output. This leads to a positively skewed price distribution, even when the distribution of power demand is symmetric.

The marginal cost for thermal production at time j is then given by

$$MC_j^{term} = a\left(Q_j^{term}\right)^{b-1} \tag{4.11}$$

Modelling the marginal cost curve as a smooth function is a simplification. Figure 4-2 shows an estimate of the marginal cost curve in the Nordic power system presented in an official report to the Norwegian government [NOU, 1998].

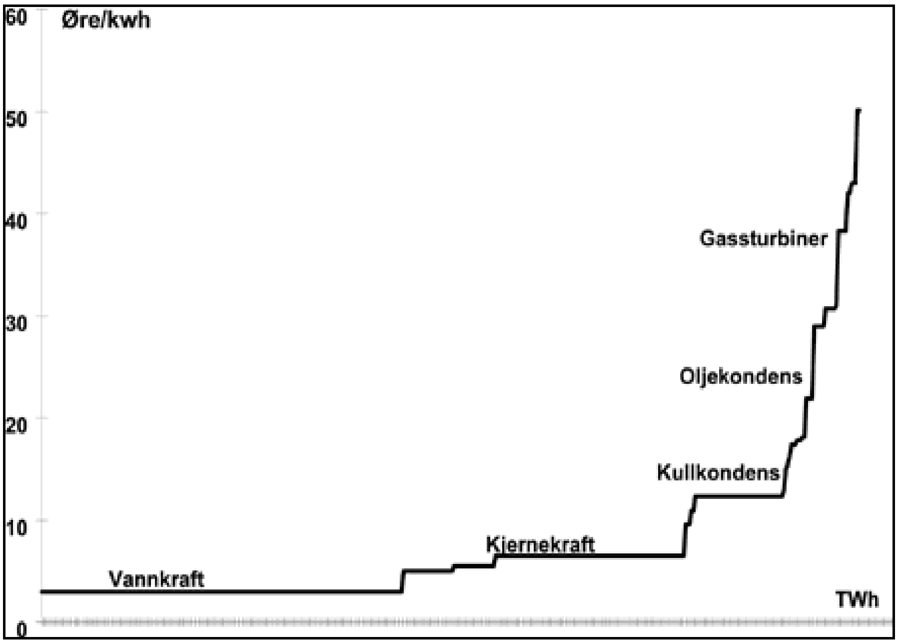

Figure 4-2 The marginal cost curve for the Nordic Power market. From left to right the power sources are hydropower, nuclear power, coal, oil and gas.

From the figure we see that a piecewise constant marginal cost curve would possibly be more appropriate for the Nordic power market.

4.3 Aggregated model formulation

As Johnsen (2001) we assume that the producers observe the inflow, snowfall and temperature of the week before the production decision is made. This is of course a simplification, but due to quite reliable weather forecasts three days ahead this does not necessarily cause big errors in the model. We focus on short-term price movements and use an annual planning period of 52 weeks. The final week of the planning period is the last week before the snow melting starts. We assume this week to be deterministic, for instance in week 17, but for some years this may be delayed for one to three weeks. Each individual producer is assumed to take the day-ahead spot price as given, while at the market level the price is endogenous.

We derive a general model for the spot price of electricity by reformulating the problem as a social planner's problem [Williams & Wright, 1991]. This is in line with the famous principle of the "invisible hand" introduced by the economist Adam Smith in the book "the Wealth of Nations". A perfectly competitive market where individuals seek to optimise their own benefit also results in more wealth for the nation as a whole. It is almost as if an "invisible hand" sets the price optimally, and the hand could represent a planner aiming for the best interests of the society.

We assume the cost of storage as negligible and the weekly interest rate to be zero. The planner's problem in the current week is to select a water storage level and thermal production that will maximize the flow of expected future surplus.

The social surplus, V_t, consists of the area under the demand curve and above the marginal cost curve in each of the future periods, including the present period, plus some terminal value of the water reservoir and the snow volume.

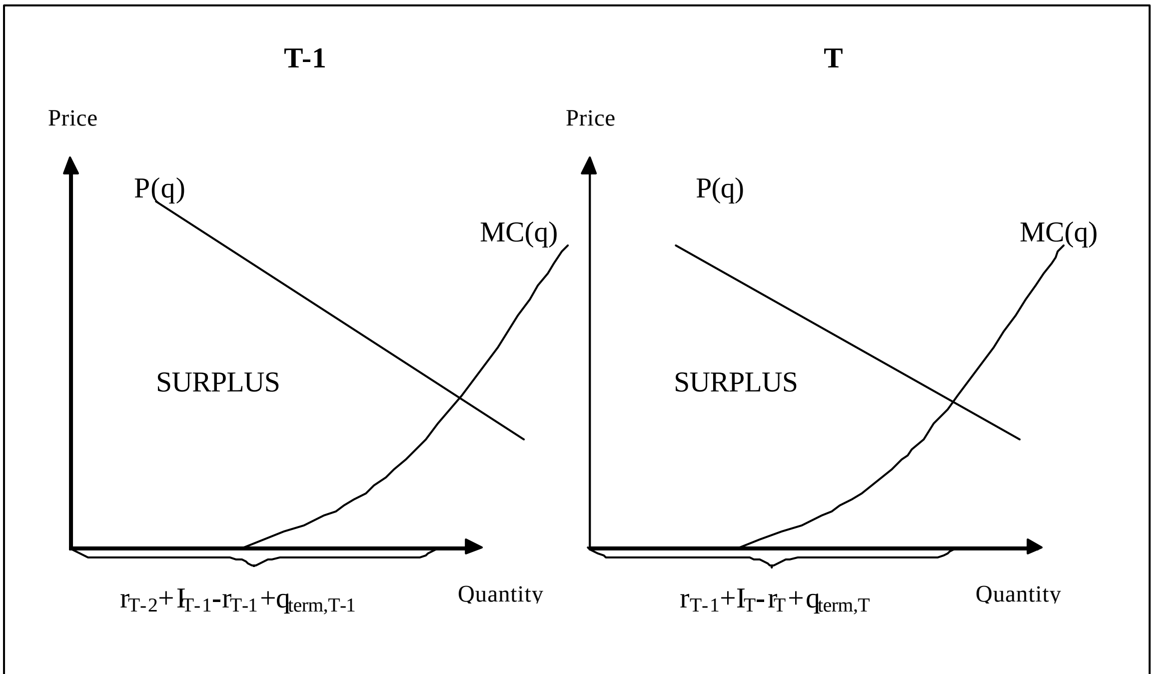

Figure 4-3 The social surplus for the two periods is given by the area above the marginal cost curve MC(q) and under the demand curve P(q)

Hence, V_T consists of the surplus in the right part of the figure above plus some terminal value, and V_{T-1} consists of the sum of the two surpluses in the figure above plus the terminal value.

The terminal value is denoted V^* and the idea is to reflect that the producers have a longer horizon than the final week before the snow melting starts. The terminal value is given as a function of the reservoir and snow levels at the end of the planning horizon and is given by

$$V^* = Ar_T + Bs_T + C\left(r_T + g_s s_T\right)^2 \tag{4.12}$$

where A, B, C and g_s are coefficients and T is the last week before the snow melting starts. The complete value at time t is given by

$$V_t = \sum_{j=t}^{T} E_t \int_0^{r_{t-1}+I_t-r_t+q_t^{term}} P(q)dq + E_t V^* - \sum_{j=t}^{T} E_t \int_0^{q_t^{term}} a\left(Q_{term}\right)^{b-1} dQ_{term} \tag{4.13}$$

The total production at time j, is given by $r_{j-1} + I_j - r_j + q_j^{term}$, assuming no overflow. E_t represents expectations made in week t. When determining r_t and q_t^{term} all future period values of these variables must be considered too. The solution of the problem is found by backward induction.

The storage of week T is determined by differentiation of

$$\begin{aligned} V_T = & \int_0^{r_{T-1}+I_T-r_T+q_T^{term}} P\left(q\right)dq + Ar_T + Bs_T + C\left(r_T + g_s s_T\right)^2 - \int_0^{q_T^{term}} a\left(Q_{term}\right)^{b-1} dQ_{term} \\ & - l_T^5\left(r_T - \bar{r}\right) - l_T^6\left(\underline{r} - r_T\right) - l_T^7\left(r_{T-1} + I_T - r_T - \bar{k}\right) - l_T^8\left(q_T^{term} - q_{\max}^{term}\right) \end{aligned} \tag{4.14}$$

with respect to r_T and q_T^{term}. The λ^i-values are shadow prices of the constraints given by equation (4.i) above, where i = 5,…,8.

Assuming that the thermal cost function is a monotonic smooth, increasing function i.e. $b \geq 2$, the first order condition for maximum is given by

$$\frac{\delta V_T}{\delta r_T} = -P\left(r_{T-1} + I_T - r_T + q_T^{term}\right) + A + 2C\left(r_T + \gamma s_T\right) - \lambda_T^5 + \lambda_T^6 + \lambda_T^7 = 0 \tag{4.15}$$

and

$$\frac{dV_T}{dq_T^{term}} = P\left(r_{T-1} + I_T - r_T + q_T^{term}\right) - a\left(q_T^{term}\right)^{b-1} - \lambda_T^8 = 0 \tag{4.16}$$

From the first equation, we see that the price in period T is a function of the terminal reservoir level, snow levels and the shadow prices related to the hydropower production system

$$P_T = P\left(r_{T-1} + I_T - r_T + q_T^{term}\right) = A + 2C\left(r_T + \gamma s_T\right) - \lambda_T^5 + \lambda_T^6 + \lambda_T^7 \tag{4.17}$$

From the second equation we see that the price is related to the marginal cost for the thermal production system, which is a function of the amount produced by the thermal system. This can be expressed as

$$P_T = a\left(q_T^{term}\right)^{b-1} + \lambda_T^8 \tag{4.18}$$

Equating (4.17) and (4.18) gives

$$A + 2C\left(r_T + \gamma s_T\right) - \lambda_T^5 + \lambda_T^6 + \lambda_T^7 = a\left(q_T^{term}\right)^{b-1} + \lambda_T^8 \tag{4.19}$$

Assuming the constraints not to be binding, the reservoir level can be expressed as

$$r_T = \frac{a}{2C}\left(q_T^{term}\right)^{b-1} - \frac{A}{2C} - \gamma s_T \tag{4.20}$$

For T-1 the storage and thermal production are found by maximization of

$$\begin{aligned} V_{T-1} = & \sum_{j=T-1}^{T} E_{T-1} \int_0^{r_{j-1}+I_j-r_j+q_j^{term}} P(q)\,dq + E_{T-1}V_T^* - \sum_{j=T-1}^{T} \int_0^{q_T^{term}} MC\left(Q_{term}\right) dQ_{term} \\ & - l_{T-1}^5\left(r_{T-1} - \bar{r}\right) - l_{T-1}^6\left(\underline{r} - r_{T-1}\right) - l_{T-1}^7\left(r_{T-2} + I_{T-1} - r_{T-1} - \bar{k}\right) - l_{T-1}^8\left(q_{T-1}^{term} - q_{\max}^{term}\right) \end{aligned} \tag{4.21}$$

with respect to r_{T-1} and q_{T-1}^{term} and the terminal condition given by equation (4.20). The first order conditions for maximum are

$$\begin{aligned} \frac{\delta V_{T-1}}{\delta r_{T-1}} = & -P\left(r_{T-2} + I_{T-1} - r_{T-1} + q_{T-1}^{term}\right) + E_t\left[P\left(r_{T-1} + I_T - r_T + q_T^{term}\right)\right] \\ & -\lambda_{T-1}^5 + \lambda_{T-1}^6 + \lambda_{T-1}^7 = 0 \end{aligned} \tag{4.22}$$

and

$$\frac{\delta V_{T-1}}{\delta q_{T-1}^{term}} = P\left(r_{T-2} + I_{T-1} - r_{T-1} + q_{T-1}^{term}\right) - a\left(q_{T-1}^{term}\right)^{b-1} - \lambda_{T-1}^8 = 0 \tag{4.23}$$

These equations can be rewritten such that

$$\begin{aligned} P_{T-1} &= P\left(r_{T-2} + I_{T-1} - r_{T-1} + q_{T-1}^{term}\right) \\ &= E_t\left[P\left(r_{T-1} + I_T - r_T + q_T^{term}\right)\right] - \lambda_{T-1}^5 + \lambda_{T-1}^6 + \lambda_{T-1}^7 \end{aligned} \tag{4.24}$$

and

$$P_{T-1} = P\left(r_{T-2} + I_{T-1} - r_{T-1} + q_{T-1}^{term}\right) = a\left(q_{T-1}^{term}\right)^{b-1} + \lambda_{T-1}^8 \tag{4.25}$$

Equating the two expressions gives

$$E_{T-1}\left[P\left(r_{T-1} + I_T - r_T + q_T^{term}\right)\right] = a\left(q_{T-1}^{term}\right)^{b-1} + \lambda_{T-1}^8 + \lambda_{T-1}^5 - \lambda_{T-1}^6 - \lambda_{T-1}^7 \tag{4.26}$$

or

$$P_{T-1} = a\left(q_{T-1}^{term}\right)^{b-1} + \lambda_{T-1}^8 = E_{T-1}\left(P_T\right) - \lambda_{T-1}^5 + \lambda_{T-1}^6 + \lambda_{T-1}^7 \tag{4.27}$$

The general condition for week j is then given by

$$P_j = a\left(q_j^{term}\right)^{b-1} + \lambda_j^8 = E_j\left[P_{j+1}\right] - \lambda_j^5 + \lambda_j^6 + \lambda_j^7 \tag{4.28}$$

Rational hydropower producers will drive the price up to a level where no net gain from storage is expected. At this level the current price equals the expected price in the next period corrected for positive shadow prices of binding constraints.

4.3.1 Implications

Based on the equation for the general conditions, there are several implications:

1. The price in a given period reflects the marginal cost of thermal production in that time period. If the thermal production is at the maximum capacity the price equals the marginal cost of thermal production plus the shadow price of thermal production.

2. The price at a given period reflects the expected price at later periods, corrected for active shadow prices.

3. Given no active constraints, the price today equals the expected price for later periods, i.e.

$$p_t = E_t p_{t+1} = E_t p_{t+2} = \ldots\ldots = E_t p_{T-1} = E_t p_T \tag{4.29}$$

Equation (4.29) implies, considering the fact that the prices reflects thermal marginal cost, that thermal production will be constant for all periods and that hydropower is used as a swing producer to keep prices constant as the demand equation varies over time. Assuming risk neutrality, this implies that the forward price equals today's spot price until time T, i.e. when the snow melting period starts. In other words the forward curve is a constant function of time from t to T if the shadow prices are not active, assuming the forward curve to be an unbiased estimate of the expected future spot price.

To show how this condition can be achieved for a perfectly competitive market, or for the social planner, we will draw on the theory of microeconomics. Due to the fact that thermal production has to be constant, this condition can only be achieved by managing the water resources. We base this section on a two-period framework, but this can easily be extended to more periods. For the purpose of visualization we use the fall and the winter. F refers to the fall period, while W refers to the winter period.

Consider the two following graphs showing the supply and demand curves for the two periods:

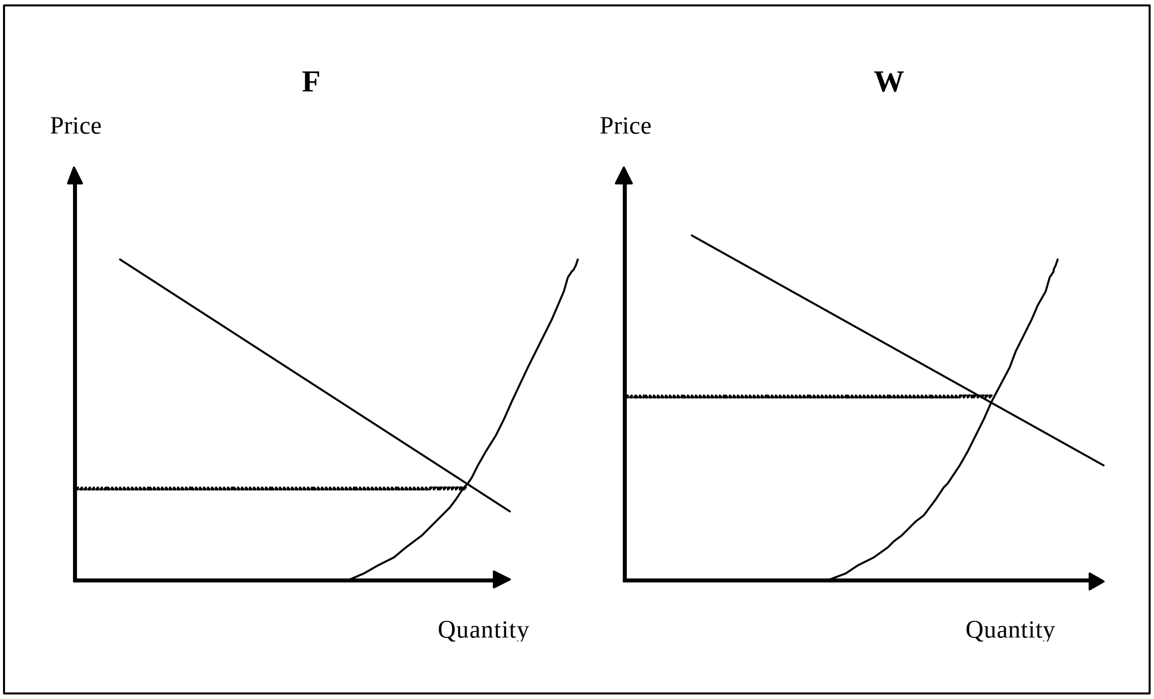

Figure 4-4 The supply and demand curves for the fall and winter period given no hydro storage possibilities.

Note that the marginal cost for hydro energy is regarded to be zero, while the marginal cost of thermal production are assumed to be convex. This is in line with the theoretical framework presented in the preceding section. From this picture we can read two important points. First, for the winter period the demand curve is shifted rightwards relative to the fall period. Cold weather is the main reason for this. Consumers demand more power at each price level. Second, the supply curve for the winter period is shifted to the left relative to the fall period. This is due to less inflow from snow melting in the winter period. This results in different prices for the two periods when the social planner is not able to store water from one period to the other. Also, the amount of thermal production will differ between the two periods.

Now, consider the following graphs:

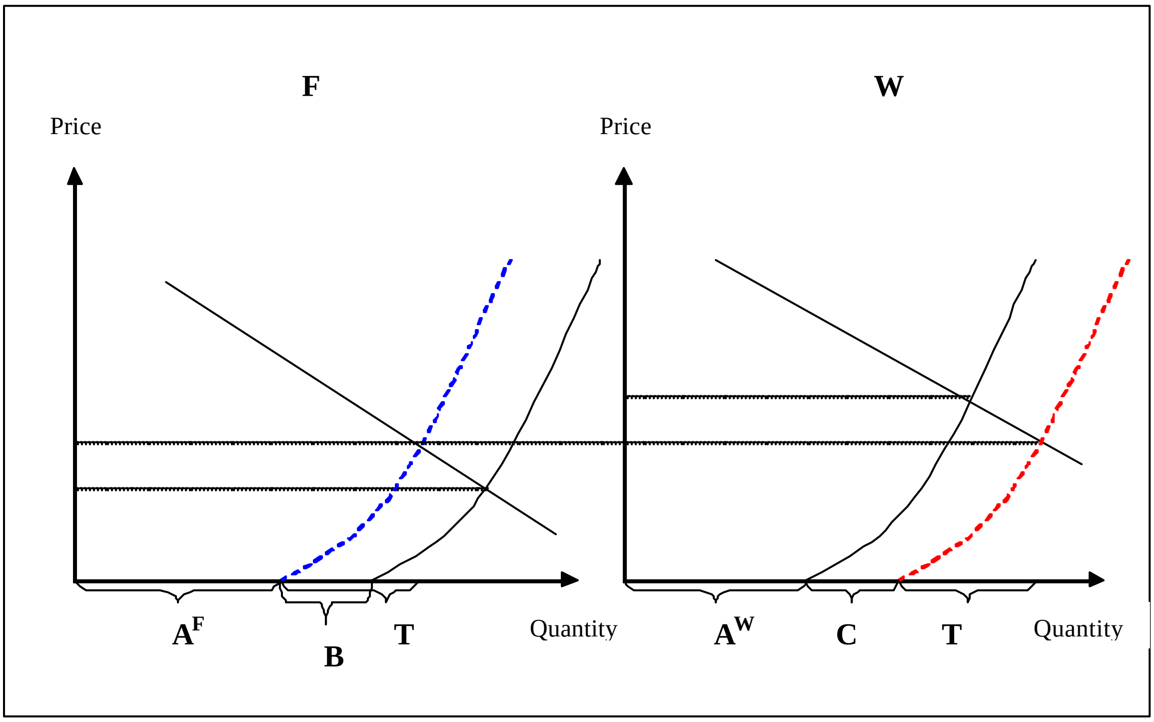

Figure 4-5 The supply and demand curve for the fall and winter period given storage possibilities

By storing water in the fall period, the supply curve will shift to the left. This is reflected by B in the figure above. Furthermore, the supply curve is shifted to the right in the winter period by using stored water. C depicts this situation. When the prices in the two periods are the same, the same amount of thermal production takes place in both periods. T reflects this in the figure. A^F and A^W depicts the hydropower production in the two periods.

The physical conditions that enable the social planner to manage water as shown in the picture above are water reservoirs and different levels of inflow. Consider the water reservoir budget, given by equality when no overflow occurs

$$r_j = r_{j-1} + I_j - y_j \qquad j = t, \ldots \tag{4.30}$$

Reordering this to reflect the amount of water stored for each period, we obtain

$$r_j - r_{j-1} = I_j - y_j \qquad j = t, \ldots \tag{4.31}$$

In the fall period when inflow is high and production is low relative to the winter period, storage can effectively take place. For the opposite reasoning, the storage ability is more limited in the winter period.

Maximizing social surplus derived the result from the preceding section. It seems reasonable that social surplus is decreased for the fall period and increased for the winter period in order to maximize the two-period joint social surplus.

4.3.2 The case of active constraints

If the deterministic model has active constraints, there will no longer be a horizontal forward curve. Consider a three period model where the periods are denoted T-2, T-1 and T. Assume that the model has no active constraints in the first and last period, but that the demand for electricity is very high in period T-1 and that the production capacity limit for hydropower is reached in the period, i.e. $\lambda^7_{T-1}>0$. From the general equation given by (4.28) we have that the price in period T-1 is given by

$$P_{T-1} = E_{T-1}\left[P_T\right] + \lambda^7_{T-1}$$

and the price in period T-2 is given by

$$P_{T-2} = E_{T-2}\left[P_{T-1}\right]$$

since there are no active constraints for period T-2. This indicates that the active constraint in period T-1 affects the price of period T-2, illustrated by

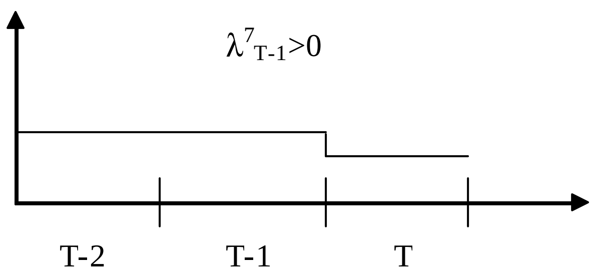

Figure 4-6 The price structure given an active constraint in period T-1 increasing the price in period T-2 and T-1

Lets now assume that the optimal solution is the same for the two last periods, but that for period T-2 the optimal solution gives a maximum reservoir level, thus $\lambda^5_{T-2}>0$.

We now have that

$$P_{T-2} = E_{T-2}\left[P_{T-1}\right] - \lambda^5_{T-2}$$

The connection between the price in period T and T-2 is now given by

$$
\begin{aligned}
\mathrm{P}_{T-2} &= \mathrm{E}_{T-2}\left[\mathrm{P}_{T-1}\right]-\lambda^{5}_{T-2} = \mathrm{E}_{T-2}\left[\mathrm{E}_{T-1}\left[\mathrm{P}_{T}\right]+\lambda^{7}_{T-1}\right]-\lambda^{5}_{T-2} \\
&= \mathrm{E}_{T-2}\left[\mathrm{P}_{T}\right]+\lambda^{7}_{T-1}-\lambda^{5}_{T-2}
\end{aligned}
$$

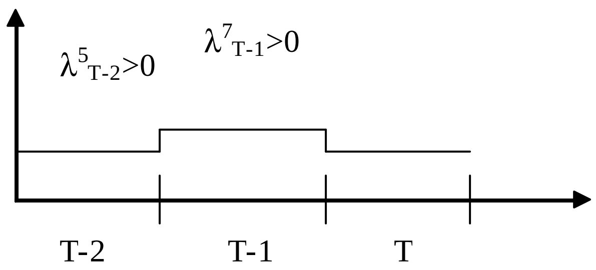

Figure 4-7 The price structure of a three period model given an active constraint in period T-1 increasing the price and an active constraint in T-2 decreasing the price.

As a result of the active constraints on the hydropower production, the prices and thereby the thermal production, are changed from period to period. If no restrictions are active for the period, the price will equal the price of the subsequent period, which is already determined trough the backward induction. In this case the thermal production will be constant for the two periods, using hydropower as a swing producer.

To further explain this, we will investigate the interpretation of shadow prices, the λ's, in an economic sense. The expression for the complete value of social surplus as seen from period T-3 is given by

$$
\begin{aligned}
V_{T-3} = &\sum_{j=T-3}^{T} E_{T-3} \int_{0}^{r_{j-1}+I_j-r_j+q_j^{term}} P(q)\,dq + E_{T-3}V_T^{*} - \sum_{j=T-3}^{T} \int_{0}^{q_T^{term}} MC\left(Q_{term}\right) dQ_{term} \\
&- l^{5}_{T-3}\left(r_{T-3}-\bar{r}\right) - l^{6}_{T-3}\left(\underline{r}-r_{T-3}\right) \\
&- l^{7}_{T-3}\left(r_{T-4}+I_{T-3}-r_{T-3}-\bar{k}\right) - l^{8}_{T-3}\left(q^{term}_{T-3}-q^{term}_{\max}\right)
\end{aligned}
\qquad (4.32)
$$

The shadow prices are the resulting effect on V_{T-3} by changing the right hand side of the physical restrictions they correspond to. Hence, if restriction (4.5) is active in period T-3, the value of l^{5}_{T-3} is the gain in total social surplus from period T-3 to period T if we were provided with an extra unit of reservoir capacity. As shown earlier in this chapter, the shadow prices affect the prices directly when optimising the social surplus by the storage of water in the reservoir and by the thermal production.

This is also the reason why shadow prices for a period, for instance T-2, affects the preceding periods, i.e. T-3, T-4,...,t. A physical restriction in period T-2 that limits the social surplus from period T-2 to period T also limits the social surplus from period T-3 to T. The economic interpretation can be seen by repeating the general condition for week j as given by equation (4.28)

$$P_j = a\left(q_j^{term}\right)^{b-1} + \lambda_j^8 = E_j\left[P_{j+1}\right] - \lambda_j^5 + \lambda_j^6 + \lambda_j^7$$

By assuming the water reservoir capacity constraint, (4.5), will be active in period T-3, then the shadow price l_{T-3}^5 will affect the price in the preceding periods. One can interpret this as hydroelectric producers using more water in the preceding periods thereby lowering the prices for all those periods. This also means that thermal production will no longer be constant. To build on the same example as above, the positive l_{T-3}^5 reflects the lower utilization of thermal production, which lowers the marginal thermal price from the situation where no such constraint appears. And in the same manner as more water is used for hydroelectric production in period T-2 and the periods preceding period T-2, the marginal price of thermal production is decreased for the period T-2 and the periods preceding period T-2.

Now we will take a look at a shadow price that causes the preceding prices to increase as in period T-1 above where the maximum hydro production is reached. Here the shadow price λ^7_{T-1} will take on a positive value and affect the price in the preceding periods. The interpretation of this is that hydroelectric producers reach their limit on generation capacity resulting in more thermal production entering. In this situation the constraint

$$r_{T-2} + I_{T-1} - r_{T-1} \leq \bar{k}$$

will be on its upper limit and the consumption of water is large. To prepare for this situation these producers will tend to save water in the preceding periods. This again results in the level of thermal production to increase for these periods.

The value of the shadow prices would be determined in the optimisation process. What is important here is that the ”social planner” smoothes the effect of the active constraints over the preceding periods in order to maximize social surplus.

As the time goes by, the social planner will work with a rolling horizon. As the model is deterministic he will observe deviations in inflow from a week to the next, resulting in a different reservoir level for the time period than originally planned. If this happens close to the end of the planning horizon T, this can influence the forward curve until T quite a lot. Deviations earlier in the planning period will be smoothened out over the planning horizon.

Figure 4-8 shows the changes in the reservoir levels over the year in the period 1995 to 2002.

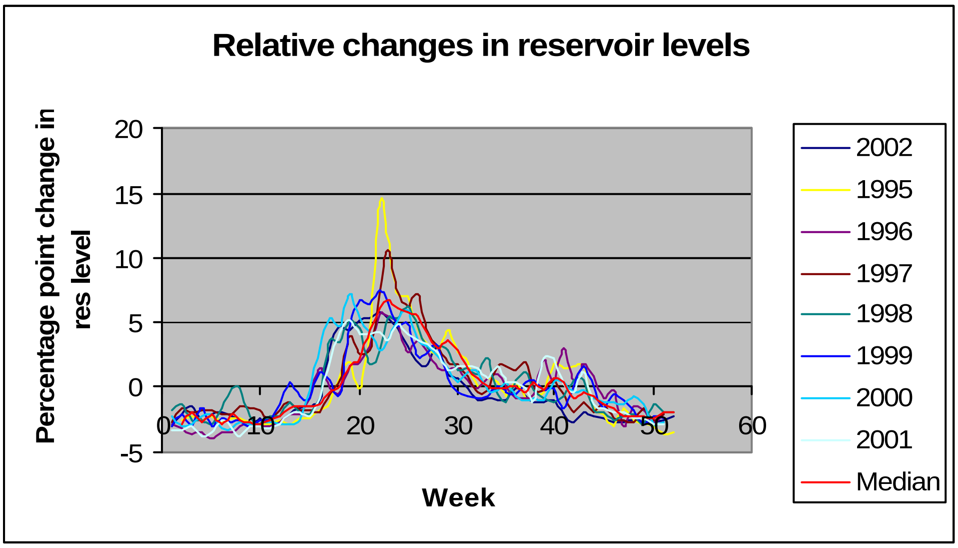

Figure 4-8 Relative changes in the reservoir level from week to week in percentage points in the period 1995 to 2002. The deviation from the median is largest in the summer implying a larger uncertainty in this period. [source Nord Pool]

From figure 4-8 we see that the changes in the reservoir filling have highest standard deviation in the summer period. This means that the unexpected deviations from the deterministic inflows used in the model is at its highest in this period. Because of that, we should expect the forward curve to be very volatile in the summer period.

4.3.3 The choice of time horizon and its implications

As stated in chapter 4.3, Johnsen (2001) focus on an annual planning period of 52 weeks and where the final week of the period is the last week before the snow melting starts. The purpose of Johnsen (2001) is to model the supply and demand in order to obtain the spot price, hence the short horizon. Using backward dynamic programming requires a terminal period, and a time horizon of one year including the seasons in the market should be sufficient for that purpose. When using the same framework to investigate the term structure we should assume a longer time horizon. As pointed out in Williams & Wright (1991), it is then appropriate to apply the same ending point, but some cycles further ahead.

It is often the practice to apply the point where inventories is on their lowest as the ending point in planning horizons, hence the last week before the snow melting in the case of reservoirs. This is also the practice when calculating the water values in the Nordic market. The water value is the expected future value from storing water today rather than spending it.

By choosing the end of the planning period to be the last week before the snow melting 3 years ahead, we may be able to get a clue about the term structure through the backward dynamic programming.

The term structure for the last periods will depend on average expected conditions. That are with respect to inflow and restrictions expected to be activated. The present conditions with respect to resources (snow and water in the reservoirs) will however affect the term structure from the present and up to 2 years ahead depending on the situation for the two- and three-year reservoirs. If the present conditions are marked by over average snow and water volumes, this will result in a lowered close end of the term structure. These conditions will also have an effect in the year after the snow melting period because of over average levels of well regulated reservoirs. For this scenario we would then expect an upward sloping term structure from the present, via the snow melting, and to the winter period of next year.

4.4 Comparing the model with the market

Figure 4-9 shows the forward curve observed in the market on the second Monday of October in the years 1999-2002. The curve is designed by using the shortest contracts for periods covered by more than one contract

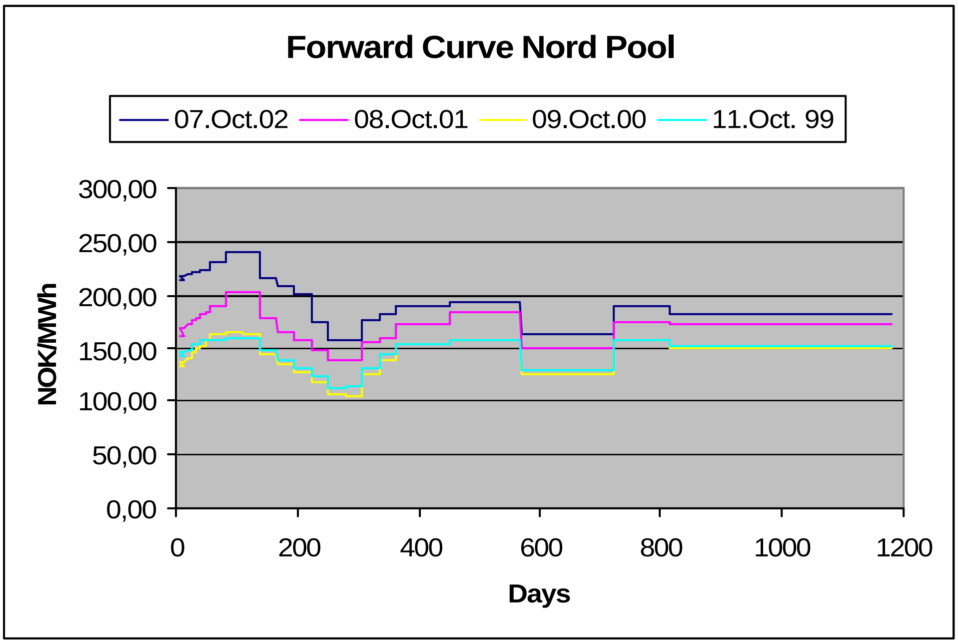

Figure 4-9 The forward curve at Nord Pool in October for the periods 1999-2002

Comparing the forward curve given by our model with the one observed in the market indicates that our model is not as detailed as the market. The restrictions causing the prices for different time horizons to differ treat an aggregated version of the Nordic Market. For aggregated production capacity and reservoir levels the restrictions is not often binding and the term structure would therefore entail less variations than what we observe. Figure 4-9 clearly indicates frequent changes in the prices, with one peak and one bottom per year and regular changes in the price over the period. Lucia and Schwartz (2002) has inspected the term structure in the Nordic power market over almost two years, concluding that the forward curve has a seasonal component and regular changes over time.

The arbitrage equation given by our model shows that the prices are changed as constraints are becoming active. This could be interpreted in three ways: The market is either adjusting to other factors than the active constraints, the number of restrictions in the model is not sufficient or that the market is inefficient in arbitraging away price differences over different time intervals.

4.4.1 The Deterministic model versus a stochastic model

The model presented in this thesis is based on the assumption of a deterministic inflow. If a deviation from the deterministic inflow from one week to the next is observed, this will shift the forward curve. The assumption of deterministic inflow is not realistic, and will affect the model. Most producers use stochastic models with statistical series of 30-60 hydrological years when planning their production. The EMPS model (EFI's Multi Area Power Simulator) is an example of such a model. The EMPS model is used by most of the largest producers in the Nordic power market [www.sintef.com].

Figure 4-10 shows the inflow in Norway and Sweden in the period 1995-2002 and for a mean year, indicating large variations from year to year.

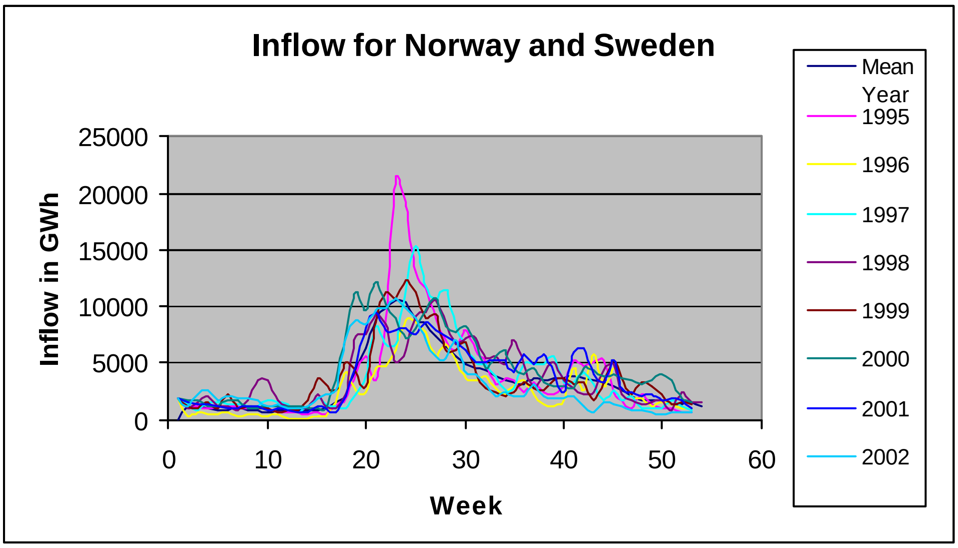

Figure 4-10 Inflow in Norway and Sweden in GWh for every week from 1995 to 2002 and for the mean year. [source Nord Pool]

1996 was an extremely dry year with high prices, while 2000 was a very wet year with a total Norwegian hydropower production of 142 TWh. The production in a normal year is 118 TWh. 2002 was a very special year, with a 18 TWh of excess inflow compared to the normal situation in the first six months and an inflow 33 TWh under the normal level in the last six months.

Given k as the index for the inflow scenario and k=1,…,K, an optimal solution can be found for each scenario k. The social surplus will be given as the expected value over the scenarios

$$E[V_t] = \sum_{k=1}^{K} {}^k p \cdot {}^k V_t \qquad (4.33)$$

where ${}^{k}p$ is the probability of scenario k and ${}^{k}V_t$ is the social surplus under scenario k. The price or arbitrage equation will be given by

$$\begin{aligned} P_j &= \sum_{k=1}^{K} {}^{k}p \cdot {}^{k}P_j = \sum_{k=1}^{K} {}^{k}p \cdot \left(a\left({}^{k}q_j^{term} \right)^{b-1} + {}^{k}\lambda_j^8 \right) \\ &= E_j\left[\sum_{k=1}^{K} {}^{k}p \cdot {}^{k}P\left(r_j + I_{j+1} - r_{j+1} + q_{j+1}^{term} \right) \right] - \sum_{k=1}^{K} {}^{k}p \cdot \left({}^{k}\lambda_j^5 + {}^{k}\lambda_j^6 + {}^{k}\lambda_j^7 \right) \end{aligned} \tag{4.34}$$

where the variables and shadow prices are defined as before, but where the index k indicates that the variable is under scenario k.

If it is unrealistic that the optimal solution under the deterministic model will have positive shadow prices for many time steps, we will expect the shadow prices to be active more frequently under certain of the extreme wet or dry scenarios. This will affect the forward curve and result in a more detailed curve with changes from week to week more often than what we expect in the deterministic model. In reality, the number of possible inflow scenarios is infinite, giving an infinite number of restrictions.

4.4.2 The number of constraints

The model has relatively few constraints. Several new constraints could be added to achieve a more detailed and realistic model. An increase in the number of constraint would increase the decomposition of the forward curve, since the curve is affected by the active constraints.

Most hydroelectric power plants have restricted possibilities in changing their discharge of water. This is restricted in the licence given by the authorities. In Norway, the Norwegian Water Resources and Energy Directorate (NVE), which is a directorate under the Ministry of Petroleum and Energy, gives the licenses. This is done in order to preserve the natural life in the river courses and for avoiding inundations that can result in damages on the crop land, roads and bridges. This type of regulations influences the flexibility of using the water resources as the swing producer in the power system and might affect the systems ability to smooth the forward curve. The restrictions can be functions of time and can be modelled as

$$r_j + I_j - r_{j-1} < y_j^{max} \tag{4.35}$$

$$r_j + I_j - r_{j-1} < y_j^{min} \tag{4.36}$$

where y_j^{max} and y_j^{min} are the upper and lower hydro production in energy units in week j.

Johnsen et al (1999) use changes in production over the day as a constraint. There will probably not be any such constraints from week to week, except those already covered by (4.35) and (4.36), implying that the change in production from a week to another will be below $y_{j-1}^{max} - y_j^{min}$. As for the discharge restrictions, most of the other restrictions could probably be modelled as functions of time.

For the thermal production systems only one restriction is included in the model. This one is connected to the total production capacity of thermal power. In operational planning for a thermal system, also known as Unit Commitment or UC, several restrictions must be considered. Larsen (2001) is considering restrictions like minimum downtime and minimum uptime, ramping, prohibited operational zones, crew limitations, emission requirements and reserve requirements. These restrictions should be considered important when planning for a single production unit or for a single producer, but they seem unlikely to affect the system as a whole considering the fact that each producer is assumed to be a price taker.

Even though overflow almost never occurs for the case of large reservoirs, it is more common for the case of small river reservoirs. When reservoirs are about to reach high levels, the water values of the reservoir owners will decrease and they will tend to spend more water for generation of power. Even though no reservoir level constraints are active, this will cause decreasing prices and we can expect the participants in the market to view constraints as binding before they actually reach such conditions. This may explain parts of the term-structure. Gjølberg & Johnsen (2001) investigates the relationship between basis and reservoir filling and states that large positive basis should only be observed when reservoirs are very full. For their data set they find the basis to become positive on average when the total reservoir capacity reaches 55% of the maximum level. They claim this may be a result of an inefficient market. However, in this context we use their findings to support our previously mentioned expectation about participants in some occasions viewing constraints as binding before they physically become binding.

4.4.3 Market Power

One assumption in this model is that each producer is a price taker and that he is taking the spot price for the next period as given. The fact that producers are price takers means that they are not able to manipulate the prices by increasing or decreasing their own production. The validity of this assumption for all producers can be discussed. In the Nordic region every outage and planned maintenance for production units with installed capacity over 200 MW have to be reported to Nord Pool, who makes the information available for the market participants. This is done in order to let the participants have the same information about the available production capacity in the market at all times.

Since the production capacity for disclosure requirements is set to 200 MW, it might be assumed that units with larger capacity might influence the prices and operating conditions, at least at a regional level. For the units with more than 400 MW of installed effect, planned outages are reported 3 years in advance.

The biggest production units in the Nordic market are the Swedish and Finnish nuclear power plants. Sweden has four nuclear power plants (NPP) with 11 active reactors, while Finland has two nuclear power plants, with two reactors each. Table 4-1 and 4-2 shows the capacity of the Nordic nuclear reactors.

Table 4-1 Nuclear power plants and their capacity for Sweden [source Nord Pool]

Nuclear Power Plant	Reactor	Installed capacity [MW]	Producer
Ringhals	R1	835	Vattenfall
	R2	872	Vattenfall
	R3	920	Vattenfall
	R4	915	Vattenfall
Forsmark	F1	961	Vattenfall
	F2	959	Vattenfall
	F3	1155	Vattenfall
Barsebäck	B2	605	Vattenfall
Oscarshamn	G1	470	Sydkraft
	G2	605	Sydkraft
	G3	1160	Sydkraft

Table 4-2 Nuclear power plants and their capacity for Finland [Source Nord Pool]

Nuclear Power Plant	Reactor	Installed capacity [MW]	Producer
Loviisa	1	500	Fortum
	2	500	Fortum
Olkiluoto	OL1	850	PVO pool OY
	OL2	850	PVO pool OY

From this, we see that Vattenfall's nuclear power plants amount to approximately 8,1% of the total Nordic installed effect of 88971 MW [Nordel Yearly Report, 2002].

Each year every nuclear power plant will have to recharge about 20% of the nuclear fuel, and such operations last for six to eight weeks. This happens in the summer periods, when the demand for electricity is at its lowest. Outages like these will affect the parameters of the thermal cost function given by equation (4.10). This means that the marginal cost curve will be a function of time. This will affect the optimal solution and thus the forward curve.

Using a two stage stochastic Cournot model of the Scandinavian electricity market, Fleten & Lie (2000) conclude that Vattenfall has incentives to reduce its thermal production in order to increase the market spot price. Fleten & Lie focus on the power market as an oligopoly, while our model focus on the market as a perfect competitive market. The potential market power in the Nordic electricity market is beyond the scope of this thesis and will not be discussed any further.

4.4.4 Other factors

This model aggregates all the production capacity and reservoir capacity in the market. The spot price, on which the values of the forward and futures contracts are determined, is calculated assuming no restrictions on transmission capacity between the price areas. The producers will be paid according to their local area price, which is affected by the transmission capacity between the areas. Thus, we can assume that the producers focus on the local area price in their production planning. This will affect their use of water and fuel, which again will affect the spot price. Naturally, the detailing level of the model will influence the optimal solution and the prices.

Our model is based on the assumption of the expected inflow and temperature for an arbitrary week to be independent of which week the expectations are made. This means that the weather of a week does not influence the expectations about the inflow and temperature in the next week. Talking to the metrology competence in Norway, we find that the assumption of the weather in one week not affecting the weather of the coming week seems to be incorrect. The weather conditions have a degree of autocorrelation. This phenomenon is known as climatologic persistence. The climatologic persistence varies from place to place and over the year. For instance, a sunny day in Sahara is likely to be followed by another sunny day. The probability of this to happen in the Nordic countries is lower, approximately 60 %, but this is also depending of the time of the year [Saltbones, 2003]. Skartveit (2003) studying time series of sun radiation conclude that “rather than occurring at random, sunny or cloudy weather thus occur in spells having a duration distribution which in general varies with location and season". In addition, this fact will influence the inflow, since the inflow is dependent on the soil conditions. In reality the soil has a limited capability of absorbing water, meaning that precipitation when the soil is saturated will increase the inflow compared to when the soil is dry. This excess inflow, which to a large extent can be assumed to be non-regulated will affect the producers’ reservoir content and the volume of discharge. This will influence the spot price and thus the forward curve.

These considerations about the weather will influence the short-term spot prices, but will probably have a very small effect on the prices in the further periods to come.

5 Hedging with futures and forwards in the Nordic Power Market

We divide the electricity market's participants into producers, retailers and large electricity consumers. Even though some of the largest firms in the energy sector entail business that relates to all three participant groups, these lines of business is separated as different business units due to the energy act of 1991 [Wangensteen, 2001].

5.1 The participants' total risk

The participants can be risk neutral, risk averse or risk seeking. However, it is reasonable to believe that most participants are risk averse as they would probably prefer a sure outcome to an unsure outcome with the same expected value. Risk averse participants have concave utility functions, which means that the marginal utility of income is decreasing. While a risk neutral participant only considers expected income, a risk averse participant takes both expected income and risk into consideration. Even though most participants are expected to be risk averse, it's important to note that there exist different degrees of risk aversion.

The fact that a company's utility is a non-linear function of income results in risk and risk analysis always having to be connected to the company's complete income and complete risk. The corporate management's evaluation of goals and strategy also entails an evaluation of the company's attitude towards risk. From such an analysis it is decided on which risk profile the contract portfolio should have. A company with large risk factors beyond the purchase of power will probably accept a lower level of risk in its contract portfolio than a company with low risk beyond the purchase of power. The share of the budget used for power purchase will also influence the risk acceptable for contracts.

In the financial community the portfolio management aims at investing in a combination of securities that jointly results in high expected return while keeping the risk within the desired level. For the power market this means buying and selling contracts in order to ensure low purchasing price of power for the case of a consumer, and to ensure high selling price for the case of a producer, based on the chosen level of risk. Diversifying the investments among a number of different companies is a strategy for reducing risk in the financial community. A pure electric power company has not this opportunity to reduce its risk if it operates in the power industry exclusively. For consumers, the share of the total power consumption purchased at fixed prices and the share purchased at spot prices, tells something about their risk exposure.

5.2 Risk in the power market

According to Wangensteen (2001), there are three main sources of riks in the Nordic electricity market. These are strategic risk, market risk and technical risk. The strategic risk is closely linked to political decisions and covers changes in external conditions. Examples are changes in the energy laws, concessions, rules for power exchange, interest rates and foreign currency. Technical risk relates to outages in production and distribution facilities. Market risk relates to the price movements, which again relates to the supply and demand of electric power. Following Wangensteen, the market risk can be divided into four components

- Price risk
  This is associated with the uncertainty connected to future spot prices.

- Volume risk
  This is related to the future volume of power and is often caused by temperature dependent consumption.

- Counter party risk
  This is the risk of the opposite party of the contract not being able to pay or deliver.

- Liquidity risk
  This type of risk arises from the fact that some markets periodically experience low liquidity, which makes it more difficult to close or change positions at desired moments of time.

The volume risk and price risk is not totally independent, since the price tend to increase as the demand increases.

In most cases the greatest risk is connected to the price risk. Bessembinder & Lemmon (2001) and Longstaff & Wang (2001) show that price risk is a major risk for both buyers and sellers of electricity. Longstaff & Wang (2001) point out that the complexity of the market makes it difficult to argue that the participants always take the same position in the term market, whether long or short.

There can be considerable volume risk for hydropower producers as production varies with the precipitation and demand varies with the temperature. The latter is also relevant for the retailers. In the short term, electricity demand can be fairly well forecasted, but deviations are almost certain to take place. A power retailer that contracts to buy power in the bilateral market may experience that demand turns out to be less than anticipated and will not be able to sell his contracted volume to end-users. If the spot price drops, which is likely to happen when demand is less than anticipated, the retailer will lose by selling his excess volume in the market. On the other hand, unexpected cold weather might force a retailer to buy more power in the spot market. Spikes in demand are very often associated with high spot prices [Knittel & Roberts, 2000].

In summary, both volume risk and price risk are important to market participants. Their profits are driven by the total cost or revenue associated with power which again is driven by the product of quantity (volume) and price. Participants can hedge price risk by entering into derivative contracts in the organised market, Nord Pool, or in the bilateral market. These contracts only hedge the contractual volume.

Longstaff & Wang (2001) and Bessembinder & Lemmon (2001) also discuss a related source of risk, the risk of total demand approaching or exceeding the physical limits of power generation. These extreme situations will cause extreme prices, also known as price spikes. Quoting Longstaff & Wang (2001):

"The risk of price spikes as demand approaches system capacity is an extreme type of price risk which may have important implications for the relation between spot and forward prices in the PJM market."

We will come back to this issue for the Nordic market later in the thesis.

The counter party risk can be perfectly hedged by entering standard contracts at Nord Pool. For these kind of contracts Nord Pool Clearing handles the counter party risk. NPC can also handle clearing for bilateral contracts and this is highly utilized by the participants.

The liquidity risk is more severe for other electricity markets. The Nordic electricity market has experienced satisfying liquidity, but in the winter and spring of 2003 the liquidity has fallen. This was discussed in chapter 2.

5.3 Participants in the power market

Power producers have some kind of storing ability in the form of water reservoirs or in the form of thermal fuel. Hydropower producers are quite flexible in their ability to regulate power production. Producers are in this sense able to arbitrage the spot-futures market via storage [Gjølberg & Johnsen, 2001]. Here, we assume power producers to sell power in the wholesale market and to large electricity consumers.

According to Bessembinder and Lemmon (2002) the ex-post profit of producer i, π_i, is given by

$$\pi_i = P_w Q_{Pi}^W + P_F Q_{Pi}^F - C\left(Q_{Pi}\right) \tag{5.1}$$

where P_W is the wholesale spot price, $Q_{Pi}{}^W$ denotes the quantity sold by producer i in the wholesale spot market and $Q_{Pi}{}^F$ is the quantity producer i has previously agreed to deliver (purchase if negative). Q_{Pi} is the physical production for producer i and equals $Q_{Pi}{}^W+ Q_{Pi}{}^F$. In the Nordic market the futures and forward contracts are financial instruments, implying that the profit is given by

$$\pi_i = P_w Q_{Pi}^W + Q_{Pi}^F\left(P_F - P_W\right) - C\left(Q_{Pi}^W\right) \tag{5.2}$$

$Q^F{}_{Pi}$ must then be interpreted as the volume hedged in the forward market for this time step. We here exclude the possibility of speculating in the forward market. $Q^F{}_{Pi} > 0$ indicates a net short position for the producer and $Q^F{}_{Pi} < 0$ indicates a long position. Here $Q_{Pi}{}^W$ is the volume sold in the spot market and equals the physical production.

From conversations with participants in the market, we have the impression that the retailers operate with a quite short time horizon in their hedging decisions [Hydro Energy, 2003]. On the website of Montel Internet Service on April 7, 2003, the CEO of the Norwegian retailer

Eidsiva Energi explains that their routine for buying power is a 14 days horizon in the forward market. In other cases it is regular practice to hedge the purchase of energy for a contract with a large customer, according to the customers' energy consumption profile, at the time the contract is agreed upon [Mo ,2003].

In the PJM market, retailers buy the difference between realized retail demand and previous forward purchases. The ex-post profits for each retailer j is given by [Bessembinder & Lemmon, 2002]

$$\pi_{Rj} = P_R Q_{R_j} + P_F Q_{Rj}^F - P_W \left(Q_{R_j} + Q_{Rj}^F \right) \tag{5.3}$$

where P_R is the fixed retail price and Q_{Rj}^F is the quantity sold forward by retailer j (purchased if negative). For the Nordic market with pure financial futures and forward contracts this should be rewritten as

$$\pi_{Rj} = P_R Q_{Rj} + \left(P_F - P_W \right) Q_{Rj}^F - P_W Q_{Rj} \tag{5.4}$$

The first expression on the right side indicates the income. The second term indicates the income or cost from holding a financial position, while the third term indicates the cost of purchasing power in the spot market.

Large electricity consumers include all kinds of industry, but power demanding companies operating in the ferroalloy industry and in manufacture of paper are among the largest. Traditionally these companies have had their own power production capacity. Examples of Norwegian companies still in this position are Norsk Hydro and Elkem. The paper manufacturer Norske Skog used to have their own production capacity, but this was sold out during 2002 in order to focus on the core activities [Norske Skog, 2002].

We can write the cost of purchasing power for such a firm as the sum of energy bought spot corrected for gains or losses in the forward market

$$Cost_{Li} = P_W Q_{Li}^W + Q_{Li}^F \left(P_F - P_W \right) \tag{5.5}$$

To be consistent, the financial position is defined as positive if the net position is short.

Longstaff & Wang (2002) mention indications of firms in the PJM market to appear on both sides of the term market over time. They divide the market participants into five sectors, the generation owner sector, the transmission owner sector, the electric distribution sector, retail end users and power trading firms. Of course some of the participants can be viewed as natural buyers or sellers of electricity, although not as buyers or sellers exclusively. For instance, a producer that experiences generation failure might have to buy electricity in the spot market to fulfil bilateral contracts. Also, retailer firms might have excess capacity from bilateral contracts and might have to sell in the spot market as the need of their retail customers is already fulfilled. In summary, they find that the PJM market participants change their trading motives over time and with market conditions.

This is also true for the participants in the Nordic market. However, it makes sense to assume that on average producers are hedging by taking short positions in the futures market, while retailers and large consumers are hedging by taking long positions in the futures market in the majority of time.

5.4 Aspects from the literature on risk premium

The futures pricing literature explains premium or bias in futures prices in relation to hedging pressure by producers. This means that upward or downward bias (contango or backwardation) in the futures price depends on whether the aggregate position by producers is long or short.

By introducing trading costs, proposition 3 in Hirshleifer (1990) states the following in the case of consumption goods:

“Under the assumption of good-bad information structure, a fixed setup cost of participation in the futures market and that the spot price declines with aggregate output, if demand is inelastic/elastic, the futures price is a downward/upward biased predictor of the futures price at any later time, and of the later spot price”.

His procedure differs from the standard hedging pressure approach in that demand for the futures-traded commodity is determined as an optimizing consumption choise among different goods, and in selecting futures positions, individuals take into account that the relative prices of the goods they consume are changing. Hirschleifer also assumes additive logarithmic preferences when deriving his proposition. The costs of trading are here assumed to be reflected in minimum contract sizes, brokerage commissions and the time and intellectual costs of learning how to trade intelligently. The producers are assumed to sell futures to consumers when demand is inelastic, as is typical for agricultural products. This indicates backwardation in these commodity markets in line with the findings of Keynes and Hicks. Also, this proposition assumes that consumers rather than producers are driven by the fixed cost from the futures market. The reasoning goes as follows. Many consumers relative to producers result in very small positions of consumers, and sufficiently small transaction costs will deter only consumer.

Fama & French (1987) employ univariate tests for expected premiums and find evidence of positive returns from a long futures position in 19 of 21 commodities on a 1 to 3 months horizon. However, they stress the fact that the validity of the statistical tests employed are discussable. Chang (1985) also finds support for the theory of normal backwardation in wheat, corn and soybeans futures markets. In the case of corn and soybeans, Chang (1985) finds that the speculators were rewarded a risk premium for the bearing of risk rather than for their favourable forecasting ability, and emphasizes that the theory of backwardation is ideal for explaining this.

The kind of reasoning for backwardation as given by Hirshleifer (1990) is not valid for the Nordic power market, especially not on a short time horizon. First, demand for electricity has historically been viewed as inelastic in the case of small residential consumers [Wangensteen, 2001]. However, these are not participating in the futures market. Retailers and large electricity consumers do participate on the buyer side of the futures market and may appear very elastic to prices. Second, because retailers and large consumers constitute the demand side, transaction costs will not necessarily cause the demand side to deter the futures market. Hence the classic backwardation situation for many commodities does not have to occur in the Nordic electricity market.

5.5 Optimal positions in the futures market

Rolfo (1980) derives the optimal hedge position for a cocoa producer who is exposed to both price and quantity risk. In his framework, p is the price in the physical market and p_f is the price in the futures market. f is the futures price on which the producer enters the futures market before the harvest. Q is total production. By holding n futures contracts the producer can modify his income (in the harvest) from pQ to $W = pQ + n(f - p_f)$. Here a positive n corresponds to a short position in the futures market. As mentioned earlier, risk averse participants take both expected income and risk into consideration. In a mean variance framework the utility function of participants is a function of only expected income, E(W), and variance of income, var (W). The utility function is given by

$$U = E(W) - m\left[\operatorname{var}(W)\right] \tag{5.6}$$

where m is the risk parameter. To explain the meaning of m a bit further, the utility function can be interpreted as the Lagrange function in a maximization problem where expected income is maximized subject to a given level of variance of income. If it is assumed a constant absolute risk aversion at all levels of wealth, m is the price, measured in units of expected income, paid to maintain the same expected utility [Rolfo, 1980].

Rolfo (1980) finds the optimal hedge as a solution of the first-order condition dU/dn = 0:

$$n^* = \frac{\operatorname{cov}\left(pQ, p_f\right)}{\operatorname{var}\left(p_f\right)} + \frac{f - E\left(p_f\right)}{2m \cdot \operatorname{var}\left(p_f\right)} \tag{5.7}$$

The first fraction of this expression is the coefficient of p_f in a linear regression where PQ is the dependent variable, which is b in the expression $pQ = bp_f$. The first fraction of (5.7) can be viewed as the optimal hedge position for the producers income given no bias in the futures price.

The second fraction of (5.7) is proportional to the bias in the futures price and inversely proportional to the risk parameter, m. This fraction disappears if the futures price is unbiased or if the participant is extremely risk avers.

The use of a mean-variance framework has disadvantages, though it makes it possible to derive closed form expressions for the optimal hedge position. Using this framework means assuming either constant risk aversion or that preferences are given only by expected value and variance of expected value. This implies risk aversion increasing with wealth. A more realistic utility function is suggested by Rolfo (1980). This is a logarithmic utility function that allows for decreasing absolute risk aversion and constant relative risk aversion. However, it is not possible to derive closed form expressions in that case.

To guide the use of, and to help understanding the optimal positions in the forward market later in the thesis, building on the work of Hirshleifer & Subrahmanyam (1993), we show in appendix A the derivation of the optimal futures position for the following condition:

q is the quantity of a commodity and P is the spot price of the commodity at time T. This makes Pq the revenue of a participant that sells q of the commodity at time T. P_F is the futures price at time t for delivery at time T. E is the expectation operator and C is consumption. Also, assume the common mean-variance preference for all participants to be

$$E(C)-\frac{A}{2}\operatorname{var}(C) \tag{5.8}$$

A is a risk aversion parameter reflecting absolute risk reversion. Now, letting x represent the optimal hedge (short if positive) and W initial wealth, we show in appendix A that

$$x=\frac{P_F-E(P)}{A\operatorname{var}(P)}+\frac{\operatorname{cov}(Pq,P)}{\operatorname{var}(P)} \tag{5.9}$$

This is the same result as in Rolfo (1980) given the price in the futures market equals the spot price at time T.

Bessembinder & Lemmon (2001) have used the same framework as Hirshleifer & Subrahmanyam (1993) and ends up with the following expression for the optimal forward position

$$Q^F_{\{P,R\}i}=\frac{P_F-E(P)}{AVar(P)}+\frac{Cov(\rho_{\{P,R\}i},P)}{Var(P)} \tag{5.10}$$

where P is the spot price in the wholesale market and $Q^F_{\{P,R\}i}$ corresponds to the quantity sold in the forward market (purchased if negative) for producers and retailers respectively, given as MWh/h over the period. $r_{\{P,R\}i}$ is the “but-for-hedging” profits of producers and retailers. The interesting thing here is that the difference in optimal positions for the different participants in this framework results from the covariation of the “but-for-hedging” profits with the spot price. The “but-for-hedging” profits of the different participants can be found from the expressions for power producers, retailers and large consumers by neglecting the terms involving financial positions. As before, we observe one term that reflects the position taken in response to the bias in the forward price as compared to the expected spot price. The second term reflects the position taken to minimize the variance of profits [Rolfo, 1980].

It is a familiar fact that the covariation between the "but-for-hedging" profit and spot price is positive for producers and negative for retailers. Thus, assuming an unbiased forward market, producers hedge short while retailers and large consumers hedge long. If the forward market is biased, the participants take positions to exploit this fact according to their risk preference.

5.6 Application of optimal hedge positions in the Nordic power market

5.6.1 Producers

The covariation between "but-for-hedging" profit of producers and spot price is given by

$$\operatorname{cov}\left(PQ - C\left(Q\right), P\right) \tag{5.11}$$

In appendix B we use this and show that the optimal hedge position for a producer i can be written as

$$Q_{Pi}^{F} = \frac{P_F - E\left[P\right]}{AVar\left(P\right)} + E\left[Q\right] + \frac{\operatorname{cov}\left(P^2, Q\right)}{Var\left(P\right)} - \frac{E\left[P\right]\operatorname{cov}\left(P, Q\right)}{Var\left(P\right)} - \frac{\operatorname{cov}\left(P, C(Q)\right)}{Var\left(P\right)} \tag{5.12}$$

Q refers to the production of producer i, and C(Q) refers to producer i's cost function. The hedging volume for different producers will differ according to their risk aversion parameter A, their expected production, and the covariance between the producers' costs and the prices, which is connected to the production technology. It will also be affected by the covariance between prices and production volume. The expression is given for a certain period of time. This could be on a weekly basis for the short time horizon, following the model given in chapter 4, or for a longer period of time on a longer horizon. For instance, for the nearest weeks, futures week contracts will be used. For a longer time horizon, futures blocks, forward seasons or forward year contracts are used. Thus E[Q] is the producers expected average output [MWh/h] for the time step, E[P] is the expected average system price over the period. The expressions for variances and covariances will also be functions of time. C(Q) is assumed to be constant over time for thermal producers, i.e. the parameters in the functions are unchanged over the year.

If we ignore the first term of equation (5.12) by assuming the participants expect the forward price to be unbiased, we see that producers hedge their total expected production over a given period of time adjusted for the three terms involving the covariance between prices and production volume and between price and production cost.

On average, the covariances between the price and production volume and the covariance between the price-squared and the production volume is positive for thermal producers and negative for hydropower producers. This happens, as the prices tend to be high when hydro reservoirs are at low levels. Thus, hydropower production will be limited. As the prices increase, new thermal production capacity will enter, giving a positive covariance for the price and the thermal production. In periods of high inflow, hydropower production will increase, lowering the prices. Thus, the third term on the right hand side of (5.12) will reduce the short position for hydropower producers and increase the short position for a base load thermal producer. The fourth term of (5.12) will increase the short position of hydropower producers and reduce the position of thermal producers. The covariance between the production volume and the cost will be positive for both types of producers, but will be very small for hydropower producers. This term reduces the optimal hedge position of the producers.

To distinguish between different producers with different production technology and flexibility in their production, we have to take a closer look at the expressions. We rewrite the covariance expressions as below

$$\operatorname{cov}\left(P^2,Q\right)=corr\left(P^2,Q\right)\cdot\sqrt{\operatorname{var}\left(P^2\right)}\cdot\sqrt{\operatorname{var}\left(Q\right)} \tag{5.13}$$

$$\operatorname{cov}\left(P,Q\right)=corr\left(P,Q\right)\cdot\sqrt{\operatorname{var}\left(P\right)}\cdot\sqrt{\operatorname{var}\left(Q\right)} \tag{5.14}$$

$$\operatorname{cov}\left(P,C\left(Q\right)\right)=corr\left(P,C\left(Q\right)\right)\cdot\sqrt{\operatorname{var}\left(P\right)}\cdot\sqrt{\operatorname{var}\left(C\left(Q\right)\right)} \tag{5.15}$$

Hydropower producers differ from another in their capacity factor, i.e. in their ability to store water and their flexibility to drain off their reservoirs over a short period of time, or in other words in production capacity. Studying the third term of the expression for the hedge position, or (5.13), we find that a hydropower producer with a large degree of flexibility will have a larger covariance between P^2 and Q than a producer with a lower degree of flexibility. The correlation will be slightly lower, but the increase in the standard deviation in the production volume will increase the covariance. This, together with the negative correlation between P and Q reduces the amount of production being hedged for flexible producers. Studying the fourth term of (5.12), we find that the covariance is larger for flexible hydropower producers than for less flexible producers. Due to the negative sign before the term, this will increase the position being hedged for a flexible hydropower producer. When jointly comparing the third and fourth term of (5.12), we find the third to be the largest. The correlation coefficient will not differ significantly and the variance of the production capacity will be equal for the two terms. Now, the main difference in magnitude can be traced in the difference between the expressions

$$\sqrt{\operatorname{var}\left(P^2\right)} \tag{5.16}$$

and

$$E\left[P\right]\cdot\sqrt{\operatorname{var}\left(P\right)} \tag{5.17}$$

We know the value of E[P] roughly to be in the interval [0 – 300] depending on the time of the year, while the value of the square root of the variance of spot price (standard deviation) is around 66 [Lucia & Schwartz, 2002]. By doing some testing on real data the magnitude of the standard deviation of squared spot prices are typically around 50000. Thus the absolute value of the third term of (5.12) will be greater than the fourth term. Altogether, we will summarize the interpretation of these two terms as increased flexibility of hydropower producers reducing their short positions.

For thermal producers, the system price as a function of quantity is convex, hence the correlation between P^2 and Q will be weaker than the correlation between P and Q. The third and fourth term of (5.12) tend to be more like in absolute values than for hydropower producers. The third term regarding extreme prices makes thermal producers increase their short hedge position, due to the positive correlation between P and Q. Thermal producers do not benefit from extreme prices to the same extent as the more flexible hydropower producers. Hence, we believe the last term of (5.12) is the most important one for reducing thermal producers' hedging position from hedging of the total expected production volume. The last term express the covariance between production costs and prices. Different thermal production technology will differ in this size.

For a flexible peak-load gas fired production unit, the expected production will be low for most periods. Even though the third term of the right hand side of (5.12) increases the hedge position, both the fourth and fifth term reduces the position considerably. Especially the last term will be large compared to base-load producers due to the possibility of rapidly changing the production volume. This reduces the hedge position of a peak-load gas fired plant compared to the general base-load production unit. This leads to a very small hedge position for the peak-load gas fired installation, probably 0 or close to 0. From intuition, this seems reasonable as the peak-load gas fired plant has the flexibility to exploit the periods of high prices. Frayer & Uludere (2001) show how such a flexible peak-load plant can be more valuable than a mid-merit coal-fired plant from valuation by real options.

To summarize, we have that the hedged volume depends on the expected production, the flexibility of production and the cost structure of production.

5.6.2 Retailers

In the case of retailers, the covariation between the "but-for-hedging" profit and the spot price is given by

$$\operatorname{cov}\left(P_R Q - PQ, P\right) \tag{5.18}$$

Retailers offer contracts with different price structures to its end-users. These are fixed price, spot price or a variable price, also known as a market price. The first type of contract is normally entered for a period of one to five years. Spot price means that the customers pay the spot price at Nord Pool plus a mark-up. The mark-up varies from approximately 0,015 to 0,05 NOK/kWh. The variable price is set for a period of time and adjusted at regularly intervals of time. In periods when the spot prices vary considerably this is done more often than in stationary periods, but the adjustments can only be done a period of time after the customers have been notified. Traditionally, Norwegian retailers have offered market prices. In Sweden, a fixed price structure has been more common. Today, most retailers offer all three types of contracts, but the pattern of the customers' choice of contracts has not changed significantly.

In appendix B we show that the position taken in the forward market, for a given time period, for a retailer with a portfolio of customers with a fixed retail price is given by

$$Q_{Rj}^{F} = \frac{P_F - E[P]}{A\,\text{var}(P)} + \frac{P_R \,\text{cov}(P,Q)}{\text{var}(P)} - E[Q] - \frac{\text{cov}(P^2,Q)}{\text{var}(P)} + \frac{E[P]\,\text{cov}(P,Q)}{\text{var}(P)} \qquad (5.19)$$

If we ignore the first term of equation (5.19), by assuming the participants expect the forward price to be unbiased, we see that the retailers hedge their total expected volume adjusted for the three terms involving the covariance between prices and volume. Q refers to the retail quantity, i.e. the quantity demanded locally by the customers. Note that a negative value refers to a long position.

Due to positive correlation between spot price and retail quantity, the covariance terms in (5.19) have positive absolute values. It is a familiar fact that high spot prices tend to appear in periods of high customer demand and low prices tend to appear in periods of low demand.

The second term on the right hand side of (5.19) reflects the retailer's risk due to the fixed retail price. Given the correlation between retail quantity and spot price retailers reduce their long positions according to the level of the preset retail price. Naturally, the retail price must be above the spot price on average. The higher the retail price, the lower the need for taking long positions in the futures market.

Expected price level is related to expected retail volume. The two last terms says something about how extreme prices affect retailers. The stronger the relationship between extreme prices and retail volume, the larger the long position. Due to the expected system price and the relationship between system price and retail quantity this effect is reduced. As argued in the section for producers, it is reasonable to assume the fourth term on the right hand side to be larger than the fifth term. Thus, the sum of these terms increases the optimal hedge position of retailers. This can also explain differences between retailers. The degree to which retail customers' demand correlates with spot price is very dependent on the energy supply structure of the bulk of the customers. In areas where the customers have the flexibility to change between electricity and other sources like oil, gas, wood and biological fuel, the correlation is not that strong as for areas largely based on electrical heating.

For retailers with a portfolio of customers following the spot price, we show in appendix B that the expression for the hedge position for a given time period is

$$Q_{Rj}^{F}=\frac{P_F-E[P]}{A\operatorname{var}(P)}-E[Q]-\frac{\operatorname{cov}(P^2,Q)}{\operatorname{var}(P)}+\frac{E[P]\operatorname{cov}(P,Q)}{\operatorname{var}(P)}+\frac{E[Q]\operatorname{cov}(P_R,P)}{Var(P)} \\ +\frac{\operatorname{cov}(P_RP,Q)}{Var(P)}-\frac{E[P]\operatorname{cov}(P_R,Q)}{\operatorname{var}(P)} \tag{5.20}$$

For a retailer offering its end-users a spot contract, we will have a correlation coefficient of 1 for P_R and P. From this it follows that the fifth term on the right hand side equals E[Q]. The covarians between the product P_RP and the quantity will be equal to the covarians between P^2 and the quantity. We also have that cov(P_R,Q) equals cov(P,Q). This means that the hedge position for a retailer offering spot contracts to its customers is given by

$$Q_{Rj}^{F}=\frac{P_F-E[P]}{A\operatorname{var}(P)} \tag{5.21}$$

I.e. the retailers will only invest in the forward market in order to make money from the difference between the forward/futures price and the expected future spot price. If the futures price equals the expected spot price, i.e. if the expectation hypothesis holds, the retailer will stay away from the forward market. The stronger the degree of risk aversion, the lower the speculation position in the forward market.

For a retailer having a portfolio of customers with a variable price, we show in appendix B that the optimal hedge position for this portfolio can be written as

$$Q_{Rj}^{F}=\frac{P_F-E[P]}{A\operatorname{var}(P)}+\frac{E[P_R]\operatorname{cov}(P,Q)}{\operatorname{var}(P)}-E[Q]-\frac{\operatorname{cov}(P^2,Q)}{\operatorname{var}(P)}+\frac{E[P]\operatorname{cov}(P,Q)}{\operatorname{var}(P)} \\ +\frac{\operatorname{cov}(P_R,PQ)}{Var(P)}-\frac{E[P]\operatorname{cov}(P_R,Q)}{Var(P)} \tag{5.22}$$

We recognize the first five terms on the right hand side from our earlier discussions. The second term on the right hand side in (5.22) will equal the second term in (5.19) for the nearest time horizon, when the retailer is not allowed to change his variable price. For a longer time horizon this term indicates that the higher the retailer expects his retail price to be over that period of time, the smaller the hedge position.

Studying the sixth and the seventh term on the right hand side of (5.22) we note that the sixth term reduces the long position while the seventh term increases the long position. The covariation in the sixth term is positive, since on average, the retail price is adjusted up as the retailers' costs, PQ, increases and down when the retailers cost declines. The correlation between P_R and Q is more difficult to interpret. The retail price tends to be high when the demand is high. This happens as the retailers increase the retail price before periods of high expected demand and high expected cost. It makes sense to assume that the sixth term is larger than the seventh term as retailers adjust the retail price with a stronger correlation between retail price and retail costs, PQ, than with the correlation between retail price and quantity. This implies that the net value of the two expressions will reduce the long position taken by the retailer.

For the shortest time horizon, when the retailers are unable to change their variable retail price, we will have cov(P_R,PQ) and cov(P_R,Q) to be zero. This, combined with E[P_R] being fixed, thus equals P_R, leads to expression (5.22) being equal to the expression for the hedge position of a retailer with a fixed retail price, (5.19).

When comparing the three different scenarios for a retailer and excluding the speculation position, we find that a retailer having a portfolio of customers following the spot price should avoid taking positions in the term market. Retailers with fixed retail price will have the largest hedge position, while retailers with variable price will take a slightly lower position given the same expected retail demand E[Q] for the period.

Most producers offer all the three types of retailer contracts. For such retailers, the total hedge position will be the sum of the three expressions, where each expression hedges the corresponding customers.

5.6.3 Large power consumers

In this context, we do not elaborate any further on the case of large electric power consumers. We believe the consumption profile of these participants to be quite fixed and thereby varying in a familiar manner. Due to this, we regard large power consumers to hedge long their total known power quantity.

5.6.4 Producers versus retailers

As in the preceding parts for producers and retailers, we will for now not focus on the term including the speculation position.

First, we focus on the two common covariance terms of producers and retailers. This is the third and the fourth term of (5.12). These terms reduce the hedge position of flexible hydropower producers. The same terms increase the long position for the retailers. The lack of flexibility for the retailers make these terms dominate in the expressions for the retailers' hedge positions. Comparing these covariance expressions for the retailers with fixed price, retailers with a variable price and the producers, we believe the correlation part of the terms to be higher for the retailers than for the producers, since consumption of the end-users are believed to be the dominant reason for extreme prices in the market [Johnsen et al, 1999]. The remaining terms in the expressions for the optimal hedge position are in sum reducing the need for hedging the total quantity for the participants.

Thus, we believe the expression for the covariances between prices and volume to be the dominant term in the hedge expressions. Considering the fact that approximately 53 % of the Nordic production capacity is hydropower facilities [Nordel, 2002], most of them quite flexible, it is reasonable to assume a contango situation.

5.7 *Hypotheses about the risk premium*

Based on the model developed here, we state the following hypotheses for the risk premium in the Nordic Power Market.

1. The forward market is on average contango in the short-term.

2. The forward market experience variations in the short-term premium, with contango for the cold season and backwardation in the spring season.

3. The forward market is in backwardation in the long-term.

Here, we define short-term as a time horizon from 1 week until 1 month. By long-term we mean a horizon of 1 to 20 years. The time horizon refers to the time period until the expiration date of the forward and futures contracts in the market, i.e. the period available for trading ahead of the delivery period.

5.7.1 Explanation for hypothesis 1 - short-term contango

We will here briefly summarize the indications for an average contango situation in the short-term premium.

First, the conditions leading to a backwardation situation for certain commodities are often the opposite of the conditions that prevail in the Nordic power market. This was discussed in chapter 5.5.

Secondly, we have found the difference in flexibility to be important. Hydropower producers, with storage capabilities represented by hydro reservoirs and quite flexible production regarding regulating operations, are generally more flexible and more able to arbitrage the spot-futures market than the retailers and the thermal base-load producers. Our description of the optimal hedge position for the different types of producers and retailers have shown differences in the ability to benefit from the variability in prices and demand.

The hydropower producers are better suited to take advantage of production during periods with high spot prices by increasing their production rapidly. During periods of low spot prices they are able to store water and buy in the spot market in order to cover their bilateral commitments. Thermal producers are much less flexible due to constraints regarding regulation of output and costs related to this matter. For this reason, the thermal base load producers will have a larger position of their expected production hedged in the forward market than the flexible hydropower producers. While hydropower producers experience low or zero marginal production costs, the marginal production costs of thermal producers are convex and exponentially increasing with production. In this respect, profits for thermal producers change more closely with spot prices than for hydropower producers. For hydropower producers the changes are more extreme when spot prices are high and less extreme when spot prices are low.

On the other hand, retailers and large consumers are unable to store power in any sense. We believe the covariation between "but-for-hedging" profits and spot price should in this way be stronger for retailers. This was further detailed in chapter 5.7. The composition of the retailers' end-users will also affect the retailers' demand for hedging. We have shown that a retailer with a fixed price will hedge a large part of his expected retailer quantity. Also retailers with a variable price structure will hedge significant parts of its expected retail quantity, while retailers that offer a spot price with a mark-up will only take positions in the forward market if he can find a significant premium in the market. Traditionally, the use of a variable price structure or a fixed price structure has been widespread. This indicates a high demand of hedging from the retailers.

Third, the retailers short horizon in the futures market indicates an increased demand for a long position in the weeks before maturity. This means that the pressure is driven towards a contango situation in the weeks before maturity.

This would lead to an unbalanced hedging pressure, indicating a contango situation. If all the participants in the market had been risk neutral, and such a situation was expected, they should optimally take long positions in the futures market to exploit this fact. However, we believe the participants are too risk averse to exploit the full potential of this and thereby this effect is not strong enough to balance the hedging pressure, giving an equilibrium contango situation.

5.7.2 Explanation for hypothesis 2 - seasonal variations in the short-term risk premium

In our discussion, we find the covariation between "but-for-hedging" profits and spot price to be stronger for retailers than for the producers. This is particularly severe in the cold seasons when demand is high and price spikes potentially can occur. The effect of price spikes can be measured by the kurtosis of the distribution of the changes in prices. This is found to be more than 4,5 times higher in cold seasons than in warm seasons in the Nordic market [Lucia & Schwartz, 2002]. According to the second term of expression (5.10), retailers and large consumers should then optimally incline to hedge larger positions than hydropower producers leading to a contango situation. This was further discussed in chapter 5.6.1. Here, we also explained how extreme prices are less important for hydropower producers and thermal producers when deciding hedging positions. Lucia & Schwartz (2002) finds positive sign of the skewness estimates for the price series and stresses that this reveals that high extreme values are more probable than low extreme values. This positive skewness of the price distribution is especially beneficial to hydropower producers, and their short hedging demand due to this phenomenon is decreasing. This is because of the zero or constant low marginal production costs compared to thermal producers. Thermal producers on the other hand, can to some extent experience skewness in fuel prices for the same reasons as the distribution of changes in electricity prices is skewed.

During the spring period of snow melting, some of the hydropower producers experience overflow in reservoirs and some experience reservoirs approaching their maximum capacities. At the same time we will observe non-regulated discharge. This reduces the flexibility of these producers substantially. As a result it is reasonable to believe that the covariation between these producers' "but-for-hedging" revenues and spot price is strengthening. This again optimally (expression (5.10)) induces producers to take larger short positions for these periods. At this time of year demand is decreasing, making especially retailers reduce their long positions. It is reasonable to believe the effect of this situation to be strong enough to shift the hedging pressure to a situation of backwardation, and therefore expresses this as a hypothesis.

If participants expect the seasonal variations, they should optimally take positions in the futures market to exploit this fact. However, we believe the participants are too risk averse to exploit the full potential of this and thereby this effect is not strong enough to balance the hedging pressure.

5.7.3 Explanation for hypothesis 3 - long-term backwardation

As mentioned, information given by the participants indicates that retailers have a quite short horizon in their hedging. Industrial participants like companies in the ferroalloy industry and paper manufacturers will normally have a longer time horizon when it comes to hedging. In the quarterly report for the first quarter of 2003 we find that: "Elkem's power coverage for 2003 and 2004 was mainly established prior to 2001" [Elkem, 2003]. Large producers will probably have a quite long hedging position. In the bilateral market 5- 10- and 20-years contracts are traded.

Producers have stronger incentives to hedge on a long time horizon than retailers. Retailers change their fixed retail price on a regular basis and are thus able to transfer costs to their customers in the long run. This is probably the reason why producers have strong resources dealing with analysis. Being able to predict prices in the long term well is also important for valuing producers' long term investments and to make decisions on whether to invest or not.

Also, as mentioned in chapter 3, Syvertsen (2001) draws on the investment perspective of power assets and claims that forward prices must be lower than expected spot prices when using a discount rate higher than the risk free rate. Investments in power assets are mostly long-term investments and to the extent of the validity of Syvertsen's theory, we would expect to observe this in the long term.

6 Empirical analysis

6.1 Changes in the forward curve – empirical results

The spot model in chapter 4 is based on deterministic inflow, but we have shown that the inflow is a stochastic factor and that we expect deviations to give changes in the spot price and thus the forward curve. The degree of change is expected to be biggest in the weeks before the snow melting starts. Also, we know that the variability in inflow from year to year is at its maximum in the summer period. This should make the forward curve more volatile in the summer period.

Due to the differences in contracts over time, with a non-constant time to maturity for different types of contracts, empirical results for changes in the forward curve are hard to obtain. Koekebakker & Ollmar (2001) perform empirical tests on the forward curve dynamics in the Nordic Power Market using smoothed data, computed using the software package ELVIZ, which uses a sinusoidal continuous forward curve. In this way they can approximate weekly futures prices for all time horizons, although weekly contracts in reality are traded at Nord Pool with maturity for only 3 to 7 weeks. More advanced models for smoothing the forward curve is also developed. Fleten & Lemming (2001) combine the information in observed bid-ask prices with information from forecast generated by bottom-up models. This improves the shape of the seasonal variations in the forward price curve.

Koekebakker & Ollmar (2001) find that correlations between short- and long-term forward prices are lower than for other markets. They find that short-term forward prices are more volatile than long-term forward prices. In their study, Koekebakker and Ollmar implicitly assume that volatility dynamics have been constant. In order to test this assumption, they plot the volatility series for the shortest maturity. To compute the series, they calculate the annualised volatility on the price returns of the one-week forward price using a 30-day moving window. Using a 30-day moving window requires a smoothed forward curve, since not all futures weeks are traded for more than four weeks after a block contract is split. This means that for some of the week contracts a maximum of 20 trading days can be observed, given that none of the days in the period are public holidays.

Based on the results of Koekebakker & Ollmar (2001), with the shortest contracts being most volatile, we focus only on the nearest end of the forward curve. Since we have not generated smoothed forward curves, we use a four-week moving window for the closest futures week contracts. The number of trading days over the four weeks varies between 14 and 20, depending on the number of public holidays in the period. Multiplying with the square root of 250, which is the total number of trading days over the year, annualises the volatility.

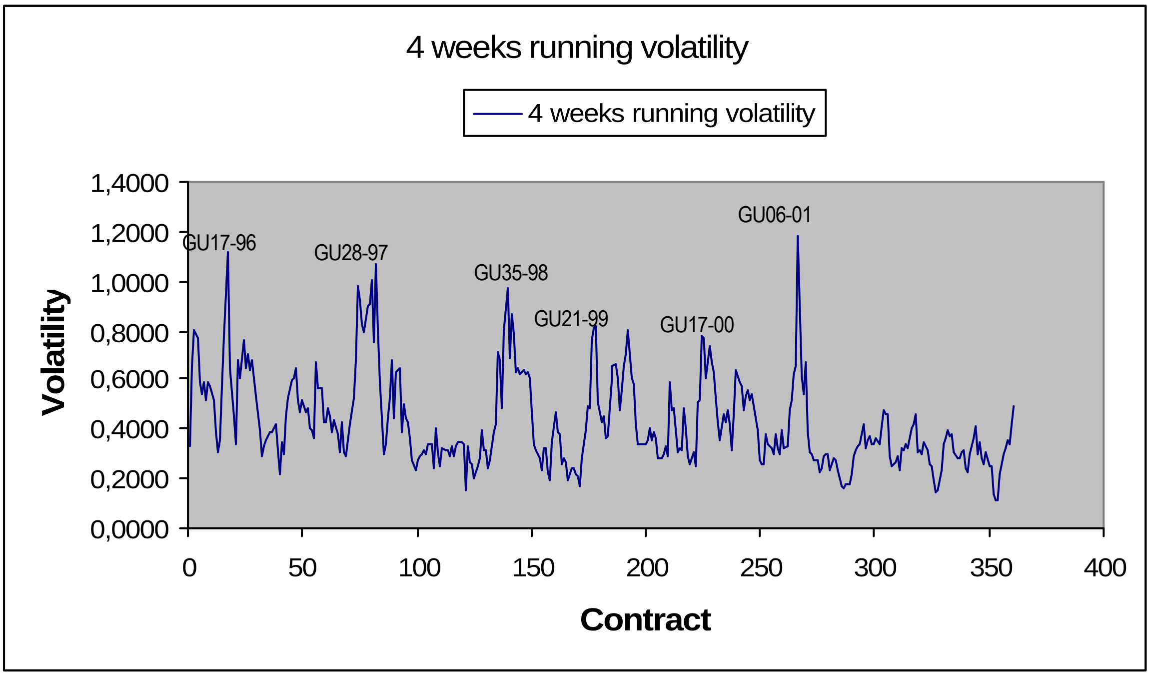

Figure 6-1 Four weeks running volatility for closest futures week contracts from 1996-2002. A volatility of 0,2 means a volatility of 20%. The x-axis shows the number of weeks after the first observation.

We see that the volatility peaks during the summer period. In the figure, we have omitted the observations for the last four contracts of 2002. This is done because of the very special happenings in this period, which naturally drove the volatility up to extreme levels. In our theory, we predicted the forward curve to be quite volatile in the periods before the snow melting starts. This should result in a high volatility for the weekly futures contracts just around week 17. We observe that this is the case for 1996, 1997, 1999 and 2000.

The peak for week 28 in 1997 can be explained by extremely high inflow in the four preceding weeks with a total excess of 18,4 TWh to the normal inflow. For the futures contract of week 35 of 1998 we observe an excess inflow of 8 TWh in the last four trading weeks.

We note that the highest volatility is reached for week 06 in 2001. The excess inflow for the last four weeks of trading is here approximately 890 GWh. We know that the inflow in this period is normally quite stable, but this excess inflow is not big enough to describe the situation. When investigating the time series of prices for this futures week contract, we find extreme variations in the price over the last days of trading. The largest correction happened on the second last day of trading. At this day the price of the futures changed almost 22 %. This happened as the spot price for the last day of trading was estimated, resulting in the third highest system price and the highest spot price for one hour observed until then. When observing what happened to the system price during the delivery week, we find that a new price record was set on Monday February 5 at 633,36 NOK/MWh. The highest hourly spot price on this day was set in hour 9 with 1951,76 NOK/MWh. This is still the highest hourly spot price observed in the Nordic Power market. There was also a consumption record set this day. In hour 10, the total Nordic consumption was 69327 MWh/h. The Norwegian consumption set a record of 23 054 MWh/h during hour 9.

From this, we conclude that the market is most volatile during the summer period. This is in line with the results of Koekebakker & Ollmar (2001). We also see an increase in the volatility in the snow melting period for most of the years, but this seems to happen later than we originally predicted.

6.2 The short-term risk premium

The relative risk premium can be estimated as the excess return over the expected spot price. From the futures pricing theory we have

$$F_{t,T} = E[S_T]e^{-p(T-t)}$$

where p equals k-r in (3.3). This can be rewritten

$$p(T-t) = \ln\left(\frac{E[S_T]}{F_{t,T}}\right)$$

For the empirical work we use

$$PREM = \ln\left(\frac{E_t[S_T]}{F_{t,T}}\right) \quad (6.1)$$

where PREM is the risk premium over the period in percent given continuous compounding, $E_t[S_T]$ is the expected spot price for time period T (here over week T) and $F_{t,T}$ is the futures price at time t for delivery in time period T.

The expected spot price for time period T will change over time. For a short time horizon, the realized spot price over the delivery period could have been used as a proxy. But the spot price in the case of electricity can be exposed to shocks from for example technical incidents in production facilities or incidents in the power grid system. This is impossible to allow for in advance and also affects the realized spot price. Instead we use the closing price of the futures week contract on the last day of trading as a proxy for the expected spot price for the corresponding week. Thus, an estimator for the relative forward premium is given by the test statistic

$$\widehat{PREM} = \ln\left(\frac{F_{T-1,T}}{F_{t,T}}\right) \quad (6.2)$$

T-1 indicates the last day of trading for the contract.

For this to be true, we must know whether the futures price of the last day of trading for the week contracts is an unbiased estimator for the spot price in the corresponding week. This assumption is investigated in the following chapter.

6.2.1 Test of forecasting ability of the last closing price for futures week contracts

Whether the closing price is an unbiased estimator of the expected spot price can be tested by different methods. We start by comparing the descriptive statistics for weekly data of spot prices with the closing prices for the corresponding weekly futures data. The weekly spot price is the average of the spot prices for the days of the week.

Figure 6-2 shows descriptive statistics from Minitab for the weekly spot prices for the period from September 1995 through December 2002. Minitab is a statistical software package delivered by Minitab Inc.

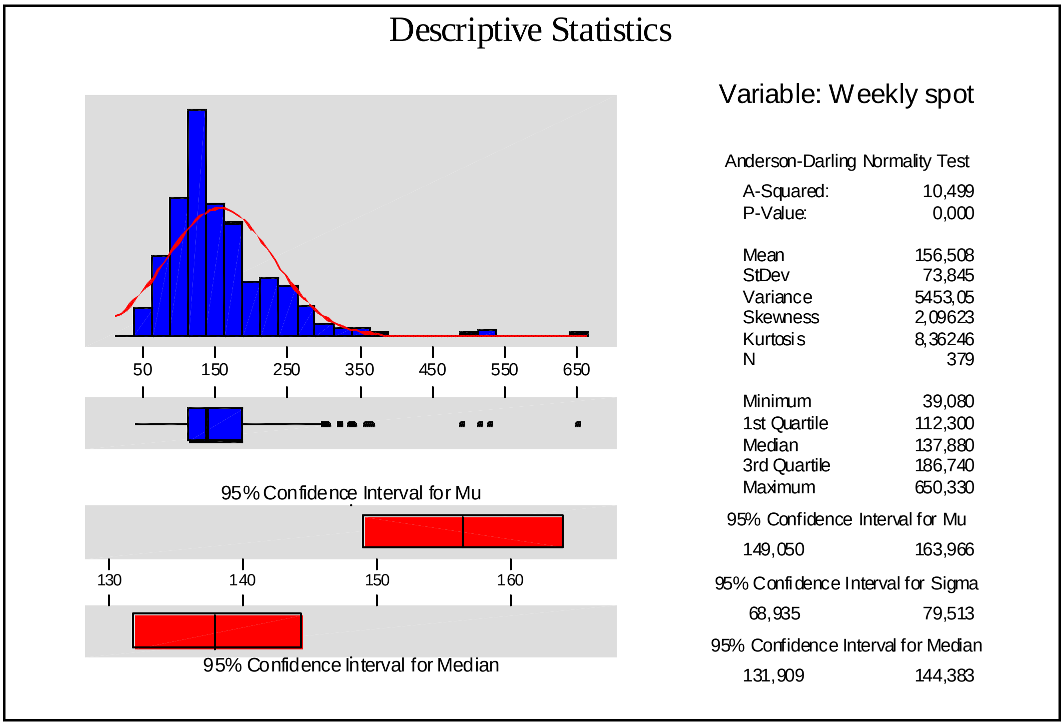

Figure 6-2 Descriptive statistics for the weekly spot price in the period September 1995 to the end of 2002.

We specially note the high kurtosis and the positive skewness. The box plot shows the distribution of the weekly prices. We observe four significant outliers. These correspond to week 49-52 in the year 2002.

Descriptive statistics for futures week contracts for the same period as the weekly average spot prices are given below. The futures prices shown are the closing prices on the last day of trading.

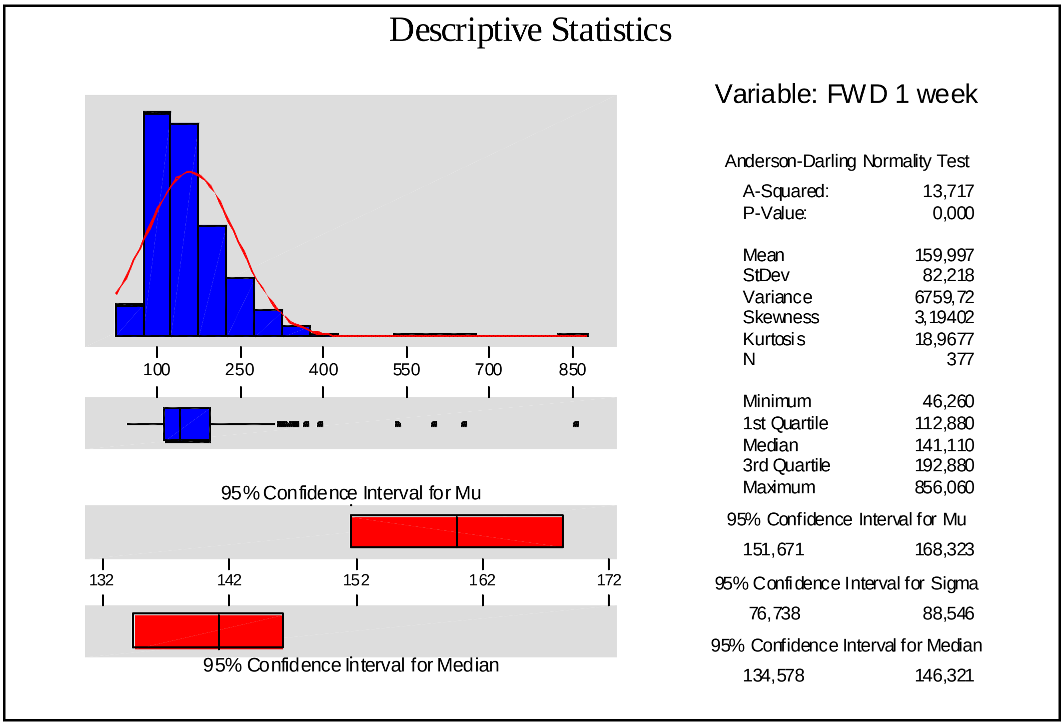

Figure 6-3 Descriptive statistics for the futures week contracts on the last day of trading.

As for the weekly spot prices, we observe four significant outliers corresponding to week 49-52 for 2002. We note that the kurtosis for the futures prices is 18,97 compared to the kurtosis of 8,36 for the average spot prices. This is due to the four extreme outliers. The median and the mean are both higher for futures prices than for average spot prices over the delivery period.

Plotting the time series of the difference between the closing price of the futures week contracts and the realized spot price for the corresponding weeks gives

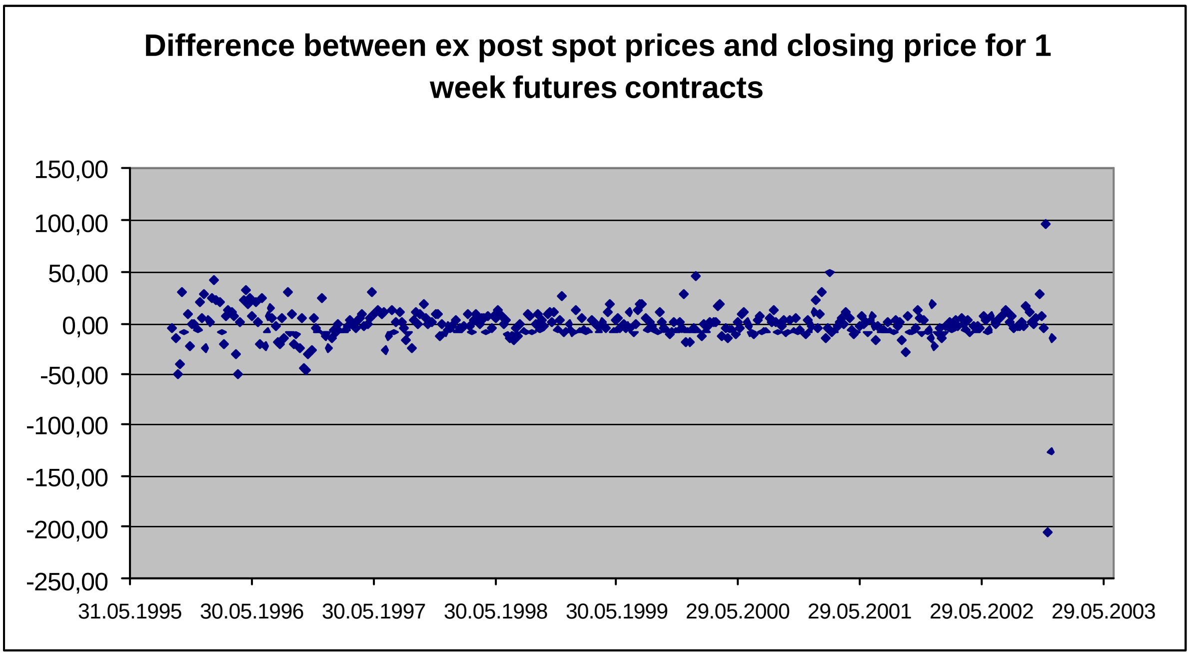

Figure 6-4 The difference between the closing price of the one week futures contract and the spot price for the corresponding week

A positive value means a gain from a long position in the futures contract, while a negative value means a loss from a long position. As for the two preceding figures there are outliers at the end of the data set. The biggest discrepancy was found for the futures contract GU50-2002. This closed at 856,06 NOK/MWh while the average spot price over the week was 650,33 NOK/MWh. We know that this happened in a period when the market experienced an extreme situation with large spreads.

Descriptive statistics from Minitab for the difference between the last closing price for futures week contracts and weekly average spot prices gives the following summary:

Variable	N	N*	Mean	Median	TrMean	StDev
1 w dev	378	1	-1,46	-1,32	-1,12	19,45

Variable	SE Mean	Minimum	Maximum	Q1	Q3
1 w dev	1,00	-205,73	95,65	-8,52	6,17

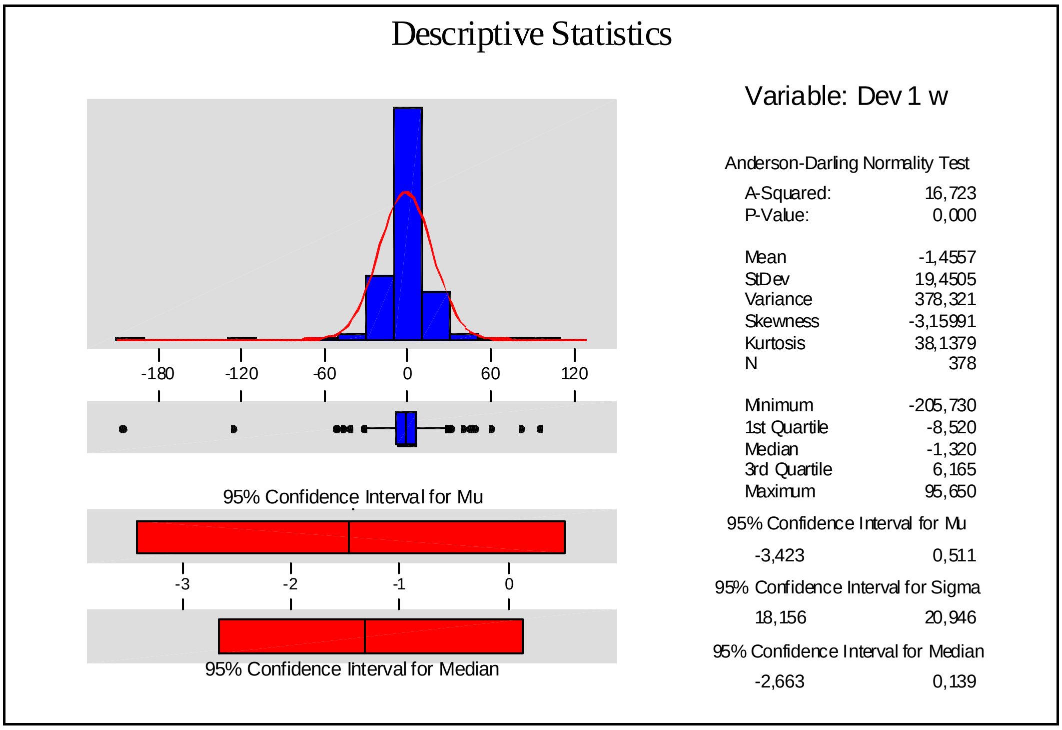

Figure 6-5 Descriptive statistics for the deviation between the realized spot price and the futures closing price

The mean is –1,46 and the median is –1,32 indicating a loss of a long position in the futures contract. The trimmed mean, neglecting the extreme outliers, is negative as well. From the output we have the 95% confidence interval of [-3,423; 0,511] indicating that the futures price is an unbiased estimator of the expected spot price. The result is also given in relative terms.

Descriptive Statistics: Rel 1 w dev

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Rel 1 w dev	378	-0,00726	-0,00941	-0,00775	0,09607	0,00494

Variable	Minimum	Maximum	Q1	Q3
Rel 1 w dev	- 0,46246	0,45229	-0,05946	0,04238

The one week futures price has on average exceeded the spot price with 0,7%. We see that sometimes the futures price has overshot the spot price dramatically, with maximum errors of 46,2%. This happened in week 43 of 1995, in other words in the earliest days of the futures market. The following week, the realized spot price was 45,2% above the closing price of the futures week contract, which is the maximum value observed.

It is interesting to see to what degree the last four observations influence the result. Removing the four observations reduces the standard deviation for the difference between the prices from 19,45 NOK/MWh to14,25 NOK/MWh. The mean and median are also reduced giving a 95 % confidence interval of [-2,243; 0,655], and the conclusion about the futures price as an unbiased estimator remains true.

A commonly employed test for futures forecasting ability of the spot price relies on the R^2 in a regression of the realised spot price on the futures price

$$S_t = \alpha + \beta F_{t,t-1} + \varepsilon_t \tag{6.3}$$

where t=1,….,N, and N is the number of observations and ε_t is an error term. The higher R^2, the more the futures prices explain the spot prices.

The regression equation is given in the print out from Minitab:

```
Spot/Week = 12,6 + 0,909 FWD close

Predictor        Coef     SE Coef          T        P
Constant       12,594       1,992       6,32    0,000
FWD close      0,90862     0,01125      80,76    0,000

S = 17,29       R-Sq = 94,5%     R-Sq(adj) = 94,5%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         1     1948829     1948829   6522,15    0,000
Residual Error   376      112349         299
```

If the futures prices are unbiased predictors of the subsequent spot prices, α should be 0 and β should be equal to 1 [Williams & Wright, 1991]. According to Williams and Wright (1991) a β-value of 0,9 is not uncommon. β is often judged to be statistically significantly less than 1,0, which indicates that our futures data are unbiased estimates of the spot price.

The time series properties of prices influenced by storage should give several reasons to pause before application of conventional statistical tests to equation (6.3). Heteroskedacity, which reflects different variances of the price distribution to different levels of the price, is one problem. A plot of the residuals versus the fitted values indicates this as a problem for four of the observations in our time series.

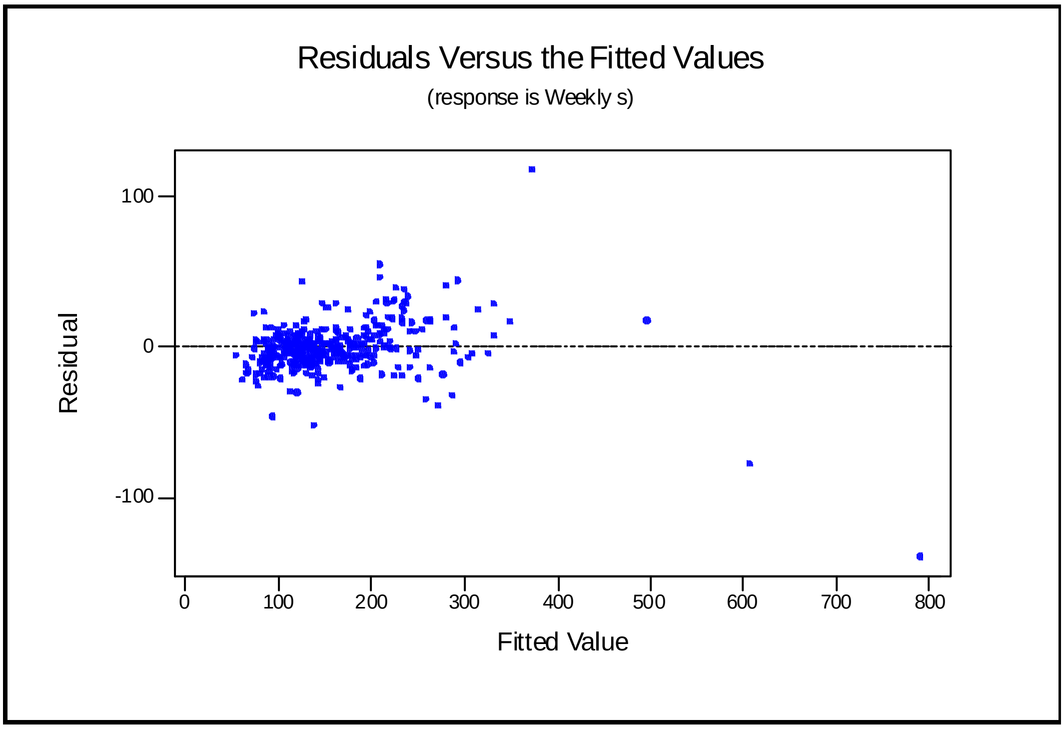

Figure 6-6 The residuals versus the fitted values for the regression of weekly spot prices versus the close for weekly futures prices

We have four observations that set apart from the rest of the observations. These are the prices from week 49-52 of the year 2002. Neglecting these four still indicates heteroscedacity, especially for prices over 200 NOK.

The confidence intervals are based on the assumption that the differences in the prices being normally distributed. Figure 6-7 shows a normality test for the differences between the prices.

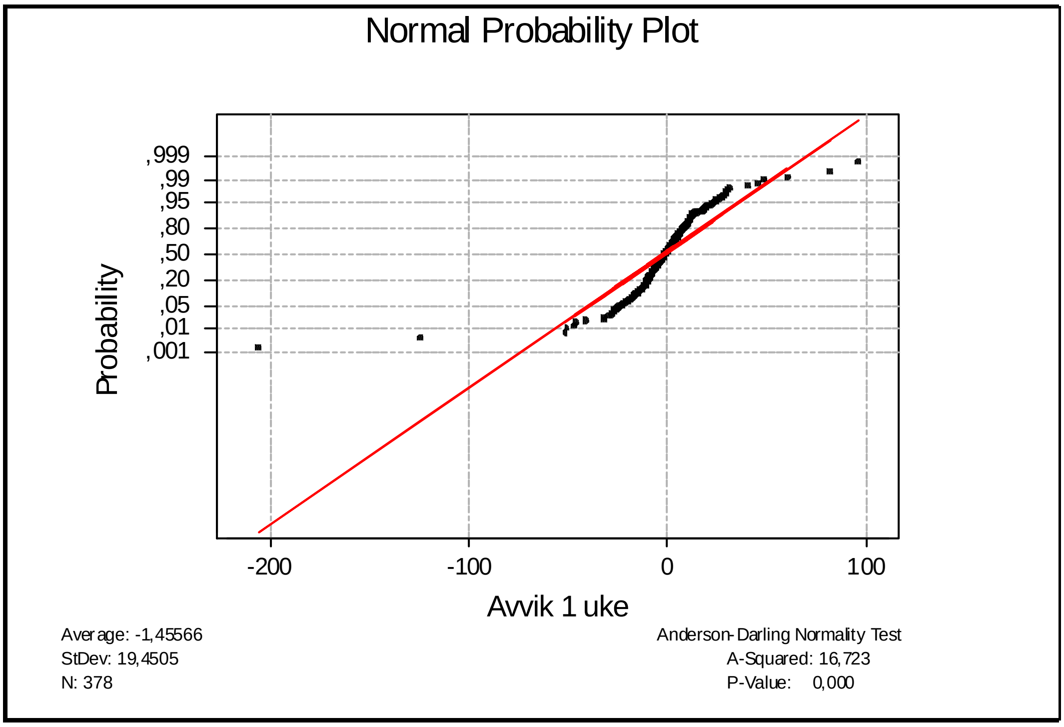

Figure 6-7 A normality test for the differences between realised spot prices over a week and last closing prices for the corresponding 1-week futures.

The input data (the differences) are plotted as the x-values. Minitab calculates the probability of occurrence, assuming a normal distribution, and plots the calculated probabilities as y-values. The grid on the graph resembles the grids found on normal probability paper, with a log scale for the probabilities. A least-squares line is fit to the plotted points and drawn on the plot for reference. The line forms an estimate of the cumulative distribution function for the population from which data are drawn. Minitab also displays the sample mean, standard deviation, and sample size of the input data on the plot. This is also known as a Q-Q plot.

The normality test indicates the data not being normally distributed. The number of observations is high, indicating that the t-test is a good proxy for the estimation. Still, we use non-parametric statistics to support or reject our conclusions. From figure 1-4 we see that the distribution is not symmetric, and thus we use the non-parametric sign test. The sign test focuses on the median. Under the null hypothesis the median is 0, and the chance of being under or above 0 is both 50% and constant. Thus we have a binominal distribution, and the sign test focuses on the probability of observing a number of observations under 0 from the total data set, given the 50% chance of being under 0. The sign test does not focus on the magnitudes of the deviations [Warpole et al, 1998.]

Sign Test for Median: 1 w dev

```
Sign test of median = 0,00000 versus  not =  0,00000

                N      N*  Below  Equal  Above          P      Median
1 w dev       378      1    206      0    172     0,0896      -1,320
```

There is almost 9% chance of observing a number of observations (206 of 378) under 0 given the null hypothesis of a median of 0.

Neither of the different tests performed, the t-test, the non-parametric test or the regression analysis supports rejection of the hypothesis that the last closing price of the 1-week futures contract is an unbiased estimator of the expected spot price for the corresponding week. We find extreme variations in the data. This shows that the physical nature and technology of electricity makes price forecasting quite difficult.

6.2.1.1 Theil's U statistic

To see whether the spot price of one week is a better estimator for the spot price the following week than the closing price of the futures contract for the week, Theil's U test was performed. The Theil's U statistic is given by

$$U = \sqrt{\frac{\left(\frac{1}{n}\right)\sum_i (y_i - \hat{y})^2}{\left(\frac{1}{n}\right)\sum_i (y_i)^2}} = \sqrt{\frac{MSE}{\left(\frac{1}{n}\right)\sum_i (y_i)^2}} \tag{6.4}$$

Comparing two predictors, the one with the lowest U-value is the best predictor. Combining the U estimators for the two predictors give

$$U = \sqrt{\frac{MSE_F}{MSE_S}} = \sqrt{\frac{\sum_i \frac{(F_t - S_{t+1})}{n}}{\sum_i \frac{S_t - S_{t+1}}{n}}} = \sqrt{\frac{353{,}77}{476{,}93}} = 0{,}86 \tag{6.5}$$

The value is below 1 implying that the futures prices have better forecasting ability than the spot prices of the preceding week [Greene, 2000]. The level of significance for this test is unknown.

In reality, when observing the spot prices for the given week before trading the futures contract for the preceding week, a maximum of 6 of the 7 spot prices are available. This supports the assumption of the last closing price of the close of the week futures contract being a better estimator than the spot price for the preceding week.

6.2.2 Estimation of the short-term risk premium

Here we concentrate on the short-term horizon, one to three weeks into the future. When estimating the risk premium following (6.1) for the short-term forward we use futures week contracts. As mentioned in the beginning, the last closing price of the futures contract for the week is used as a proxy for the expected spot price over the week. The futures prices is the closing price of the week contract one, two and three weeks ahead. The printout from Minitab is given below:

Descriptive Statistics: Prem 1 w; Prem 2 w; Prem 3 w

Variable	N	N*	Mean	Median	TrMean	StDev
Prem 1 w	377	2	-0,01225	-0,00784	-0,01228	0,10402
Prem 2 w	376	1	-0,01850	-0,01390	-0,01828	0,14784
Prem 3 w	375	2	-0,02037	-0,02597	-0,01937	0,17517

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Prem 1 w	0,00536	-0,47634	0,71209	-0,06661	0,04352
Prem 2 w	0,00762	-0,77960	0,71507	-0,10030	0,06934
Prem 3 w	0,00905	-0,80576	0,83884	-0,11715	0,08701

We observe that both the mean and the median are negative for all the time intervals. A 5% trimmed mean is calculated. Minitab removes the smallest 5% and the largest 5% of the values (rounded to the nearest integer), and then averages the remaining values. The trimmed mean is also negative, indicating a mean negative risk premium, i.e. contango. Note that the premiums are the actual premiums for the periods. They are here not annualized and hence not directly comparable.

For the one, two and three week risk premium a two-sided hypothesis test given by

H_0: PREM=0 vs H_1: PREM$\neq$ 0

was performed. The printout from Minitab is given below:

One-Sample T: Prem 1 w; Prem 2 w; Prem 3 w

Test of mu = 0 vs mu not = 0

Variable	N	Mean	StDev	SE Mean
Prem 1 w	377	-0,01225	0,10402	0,00536
Prem 2 w	376	-0,01850	0,14784	0,00762
Prem 3 w	375	-0,02037	0,17517	0,00905

Variable	95,0% CI	T	P
Prem 1 w	(-0,02279;-0,00172)	-2,29	0,023
Prem 2 w	(-0,03349;-0,00351)	-2,43	0,016
Prem 3 w	(-0,03815;-0,00258)	-2,25	0,025

H_0 is rejected, implying a negative risk premium in the Nordic power market for the futures contracts with one, two and three weeks to maturity with P-values of 0,023, 0,016 and 0,025 respectively.

To justify the use of the t-tests we will have to verify that the data are normally distributed and random. We inspect whether this is true by performing a time series plot and a normality test.

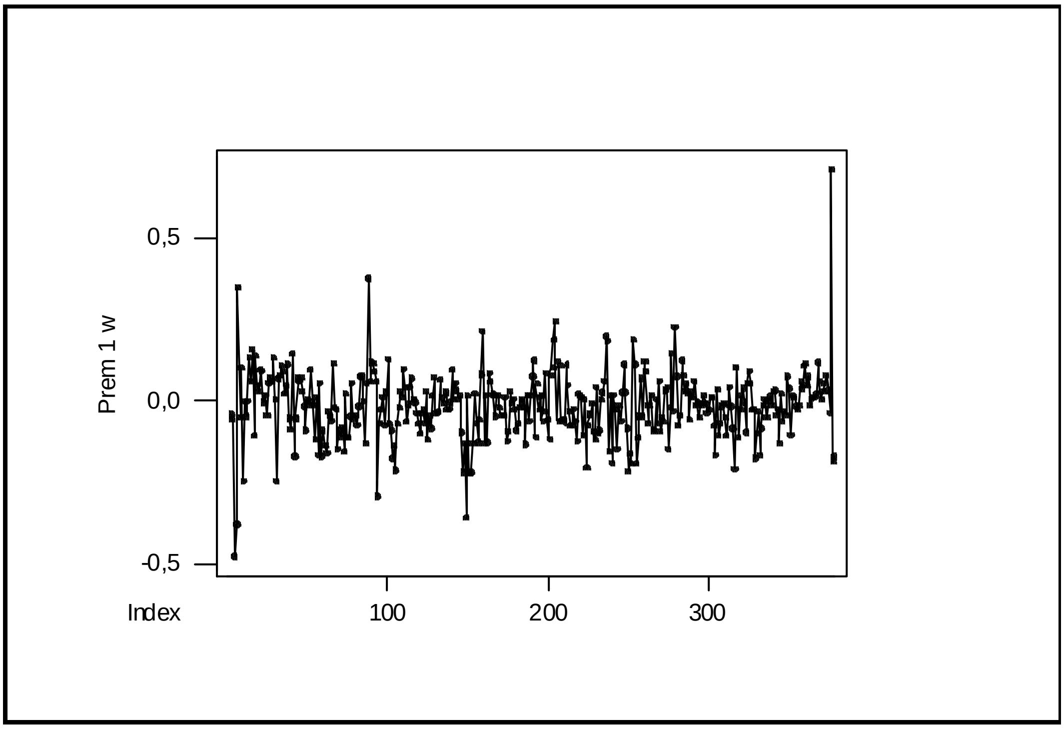

Figure 6-8 Time series plot of the risk premium for forward contracts with one week to maturity over the period September 1995 to December 2002. The premium is given in percent, i.e. 0,1 indicates a 10 % premium.

By inspection, the data seem to be random, but there might be a seasonal component. A seasonal component will be in line with our hypothesis, predicting a larger premium in the high demand periods.

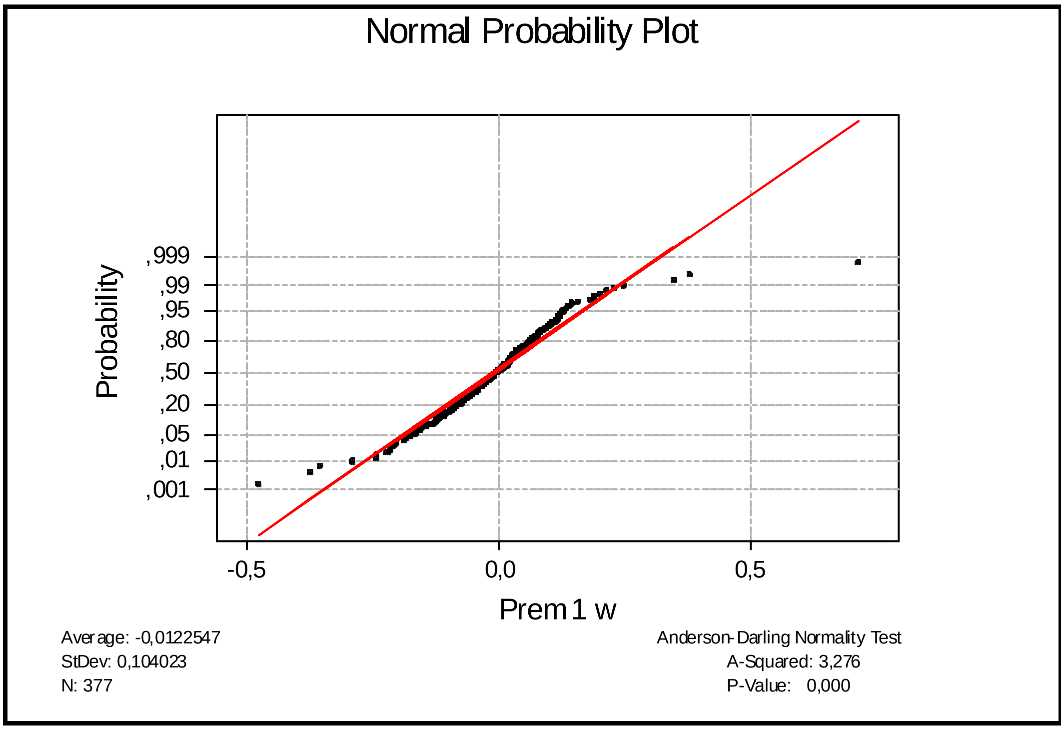

Figure 6-9 Normal probability plot for the 1 week risk premium.

We observe fatter tails than what we should expect from the normal distribution. Corresponding tests performed for the 2- and 3-week risk premiums indicate the same results as for the 1-week risk premium.

According to the central limit theorem you can do a t-sample test and have increasing confidence in the result as the number of observations increases. We have approximately 370 observations that should be satisfying. But since we observe deviations, we will try to do the estimations from an alternative point of view.

When the distribution is not normally distributed, nonparametric statistics can be applied. The signed-rank test, also known as the Wilcoxon signed-rank test, utilizes both the sign and magnitude. For the test to be valid, the population should be approximately symmetric [Walpole et al, 1998]. We test whether this assumption holds for our sample by performing descriptive statistics.

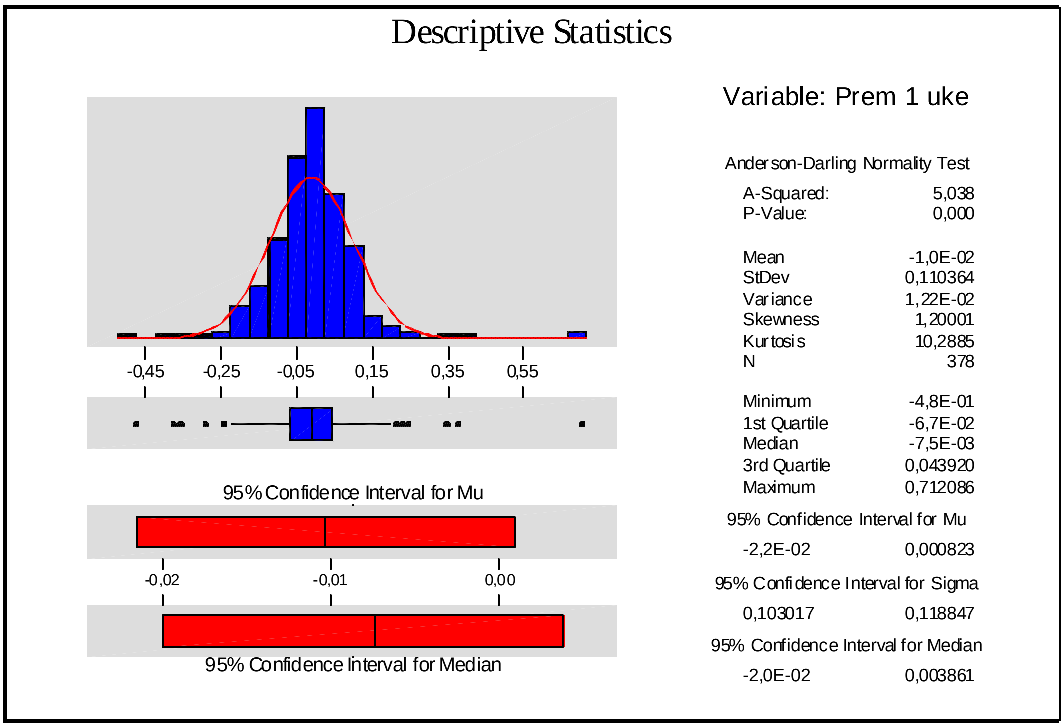

Figure 6-10 The one week forward premium. The skewness is quite low, indicating the Wilcoxon signed rank test to be valid.

The distribution is quite symmetric, with a skewness of 1,2 and especially for the 2- and 3-week premium with 0,07 and 0,03 respectively.

The signed rank test performed in Minitab gives

Wilcoxon Signed Rank Test: Prem 1 w; Prem 2 w; Prem 3 w

Test of median = 0,000000 versus median not = 0,000000

	N	N Missing	N for Test	Wilcoxon Statistic	P	Estimated Median
Prem 1 w	378	3	378	30788,0	0,009	-0,01092
Prem 2 w	376	3	376	29830,0	0,008	-0,01815
Prem 3 w	375	4	375	30154,0	0,015	-0,02010

The P-values are under 0,05 and Ho is rejected on a 95 % level of significance. The medians are negative and support the results from the t-test giving an average negative risk premium on a short time horizon.

In all the calculations of the risk premiums performed so far, we have had a couple of empty observations in our data set, marked as N^* or "N missing" in the Minitab print outs. These empty observations do not influence on the calculations of the standard deviations, the mean or the median. They will influence on the calculations of the confidence intervals, as the standard deviation is divided on the numbers of total observations, the empty observations included. They also affect the Wilcoxon signed-rank test in the same way. Still, 3 or 4 empty observations in the total data set of approximately 370 observations is very small, making the errors in the calculations extremely small.

6.3 Estimation of seasonal variations in the short-term risk premium

A very common method dealing with testing population means is called the analysis of variance, ANOVA. We will try to use this method to see whether there are significant differences between the mean risk premiums from month to month, i.e. if there are seasonal effects.

The ANOVA tests the hypothesis

$H_0 : RP_{Jan} = RP_{Feb} = \ldots = RP_{Dec}$ vs. H_1: At least two different from each other

where RP is the abbreviation for risk premium. Part of the print out from Minitab is given below:

One-way ANOVA: Prem 1 w versus Month

Analysis of Variance for Prem 1 w

Source	DF	SS	MS	F	P
Month	11	0,2323	0,0211	2,00	0,027
Error	364	3,8355	0,0105		
Total	375	4,0678			

Table 6-1 summarises the ANOVA with the 95 % confidence intervals and the P-value for the 1-week risk premium of the different months.

Table 6-1 ANOVA for the 1-week risk premium

Level	N	Mean	Stdev	SE MEAN	95,0% CI	P-value
Jan	31	-0,0450	0,0858	0,0154	(-0,0764;-0,0135)	0,007
Feb	28	-0,0196	0,0875	0,0165	(-0,0535; 0,0144)	0,247
March	32	-0,0005	0,058	0,0103	(-0,0215; 0,0204)	0,959
April	29	0,0012	0,0909	0,0169	(-0,0334; 0,0357)	0,946
May	32	0,0093	0,1026	0,0181	(-0,0276; 0,0463)	0,610
June	30	0,0216	0,0581	0,0106	(0,0000; 0,0433)	0,051
July	30	-0,0729	0,1128	0,0206	(-0,1150;-0,0308)	0,001
Aug	32	0,0118	0,116	0,0205	(-0,0300; 0,0536)	0,570
Sept	29	-0,0164	0,0875	0,0162	(-0,0497; 0,0168)	0,321
Oct	33	0,0244	0,1192	0,0204	(-0,0665; 0,0167)	0,232
Nov	35	-0,134	0,1126	0,019	(-0,0521; 0,0253)	0,485
Dec	35	-0,0011	0,1484	0,0251	(-0,0520; 0,0499)	0,967

From the P-value of the ANOVA we see that there is a 2,7% chance of the mean being the same for all months. The P-values and the confidence intervals indicate a negative risk premium for January and July. June seems to be the only week with a positive risk premium.

The two-week ANOVA yields

```
Analysis of Variance for Prem 2 w
Source     DF        SS        MS        F        P
Month      11    0,4958    0,0451     2,13    0,018
Error     364    7,7009    0,0212
Total     375    8,1967
```

Table 6-2 summarises the ANOVA with the 95 % confidence intervals and the P-value for the 2-week risk premium of the different months

Table 6-2 The ANOVA for the 2-week risk premium

Level	N	Mean	Stdev	SE MEAN	95,0% CI	P-value
Jan	31	-0,0623	0,1125	0,0202	(-0,1035;-0,0210)	0,004
Feb	28	-0,0354	0,1474	0,0279	(-0,0926; 0,0217)	0,214
March	32	-0,0096	0,0932	0,0165	(-0,0432; 0,0240)	0,563
April	29	0,0099	0,1182	0,022	(-0,0351; 0,0549)	0,656
May	32	0,0107	0,1253	0,0222	(-0,0345; 0,0559)	0,633
June	30	0,0358	0,0994	0,0181	(-0,0014; 0,0729)	0,058
July	30	-0,106	0,1591	0,029	(-0,1655;-0,0466)	0,001
Aug	32	0,0043	0,2001	0,0354	(-0,0679; 0,0764)	0,905
Sept	29	-0,0209	0,1466	0,0272	(-0,0767; 0,0348)	0,448
Oct	33	-0,0267	0,1457	0,0254	(-0,0783; 0,0250)	0,301
Nov	35	-0,0354	0,1738	0,0294	(-0,0951; 0,0243)	0,237
Dec	35	0,0098	0,173	0,0292	(-0,0496; 0,0692)	0,739

The P-value of the means being the same for all months is 0,018 indicating seasonal variations for the risk premium. As for the one-week premium, January and July are significantly negative. Here the levels of significance are 99,6% and 99,9% respectively . June has significant positive risk premium with a significance level of 94,2%, while for the rest of the months there are no indications of significant premiums.

The three-week ANOVA yields

One-way ANOVA: Prem 3 w versus Month

```
Analysis of Variance for Prem 3 w
Source     DF        SS        MS        F        P
Month      11    0,8290    0,0754     2,57    0,004
Error     363   10,6466    0,0293
Total     374   11,4756
```

Table 6-3 summarises the ANOVA with the 95 % confidence intervals and the p-value for the 3-week risk premium of the different months

Table 6-3 The ANOVA for the 3-week risk premium

Level	N	Mean	Stdev	SE MEAN	95,0% CI	P-value
Jan	31	-0,0622	0,1353	0,0243	(-0,1118; -0,0125)	0,016
Feb	28	-0,0591	0,1896	0,0358	(-0,1326; 0,0144)	0,111
March	32	-0,0223	0,1172	0,0207	(-0,0646; 0,0199)	0,29
April	29	0,0271	0,1345	0,025	(-0,0240; 0,0783)	0,287
May	32	0,0066	0,1223	0,0216	(-0,0375; 0,0507)	0,762
June	30	0,0538	0,1101	0,0201	(0,0127; 0,0949)	0,012
July	30	-0,132	0,1754	0,032	(-0,1975;-0,0665)	0,000
Aug	32	-0,0032	0,2397	0,0424	(-0,0897; 0,0832)	0,939
Sept	29	-0,0161	0,1977	0,0367	(-0,0913; 0,0590)	0,663
Oct	32	-0,0236	0,1704	0,0301	(-0,0850; 0,0379)	0,440
Nov	35	-0,0448	0,1985	0,0336	(-0,1129; 0,0234)	0,191
Dec	35	0,0257	0,2008	0,0339	(-0,0433; 0,0947)	0,454

From the P-value we see that there is a 0,4 % chance of the mean being the same for all months.

A negative risk premium is found for January and July with P-values of 0,016 and 0,000. A positive risk premium is indicated for June with a p-value of 0,012. For the 9 other months the data indicates no risk premium different from zero. This means contango for January and July and normal backwardation for June. The contango for July is not in accordance with our hypothesis.

The ANOVA shows that there are differences for the risk premiums over the year. As the data have shown deviations from the assumption of normal distribution, a non-parametric statistics test is performed.

The Kruskal-Wallis test is a non-parametric test for the differences in values [Warpole et al, 1998]. This test was performed in order to compare the different means of the different months. Using the Kruskal-Wallis test we assume that the populations have approximately the same shape or at least approximately the same standard deviations. From the ANOVA analysis we have that the standard deviations have a range from 0,0581 to 0,1284 for the 1-week premium. The variation is even larger for the 2- and 3-week premiums, indicating that the premiums are not perfectly fulfilled. The print out from the Kruskal-Wallis test on the 1-week premium is given below:

Kruskal-Wallis Test: Prem 1 w versus Month

```
Kruskal-Wallis Test on Prem 1 w

Month        N    Median    Ave Rank         Z
 1          31  -4,2E-02       137,5     -2,71
 2          28  -2,0E-02       174,9     -0,67
 3          32  5,86E-03       203,1      0,83
 4          29  3,41E-03       204,6      0,86
 5          32  -7,4E-04       209,3      1,16
 6          30  1,97E-02       234,2      2,44
 7          30  -5,8E-02       128,9     -3,12
 8          32  2,81E-02       218,3      1,66
 9          29  -9,3E-03       182,7     -0,27
10          32  -1,3E-02       185,3     -0,15
11          35  6,22E-03       190,3      0,13
12          35  -1,6E-03       184,0     -0,23
Overall    375                 188,0

H = 26,73  DF = 11  P = 0,005
H = 26,73  DF = 11  P = 0,005 (adjusted for ties)
```

The P-value is 0,005. This means that the chance of observing 12 samples as separated as these, when the months in fact have the same median is only 0,005. We therefore have statistical evidence for that the months differ, given that the assumptions are fulfilled. The same test is performed for the 2- and 3-week premiums and given in the print outs below:

Kruskal-Wallis Test: Prem 2 w versus Month

```
Kruskal-Wallis Test on Prem 2 w

Month        N    Median    Ave Rank         Z
 1          31  -8,3E-02       144,0     -2,36
 2          28  -6,5E-02       161,1     -1,37
 3          32  -9,2E-04       197,7      0,53
 4          29  1,05E-02       208,4      1,05
 5          32  -1,1E-02       204,3      0,89
 6          30  2,71E-02       234,6      2,45
 7          30  -5,2E-02       136,1     -2,73
 8          32  6,08E-03       210,6      1,23
 9          29  1,57E-02       189,0      0,05
10          32  -1,1E-02       187,2     -0,05
11          35  -2,8E-02       186,5     -0,09
12          35  -3,4E-03       193,2      0,30
Overall    375                 188,0

H = 22,71  DF = 11  P = 0,019
```

Kruskal-Wallis Test: Prem 3 w versus Month

Kruskal-Wallis Test on Prem 3 w

Month	N	Median	Ave Rank	Z
1	31	-8,2E-02	151,0	-1,98
2	28	-1,2E-01	152,2	-1,82
3	32	-4,5E-02	183,5	-0,25
4	29	0,031751	219,6	1,63
5	32	-1,5E-02	205,4	0,95
6	30	0,025356	243,1	2,90
7	30	-8,9E-02	119,7	-3,60
8	32	0,064762	208,4	1,12
9	29	-1,9E-02	192,2	0,22
10	32	0,014140	194,9	0,38
11	35	-3,5E-02	180,3	-0,44
12	35	-9,7E-03	202,1	0,81
Overall	375		188,0	

H = 31,73 DF = 11 P = 0,001

The Kruskal-Wallis test indicates that the risk premium varies form month to month. As for the ANOVA, the assumptions for the Kruskal-Wallis test are not perfectly fulfilled. Still, the test strengthens the hypothesis of variations in the short-term risk premium over the year.

6.4 Estimation of the long-term risk premium

When estimating the long-term risk premium we use data for a one and two year horizon due to lack of sufficient data for the longer term. Following (6.1) for the long-term forward we build the framework on approximations due to the classification of futures/forward products and their relatively long delivery periods. When estimating the 1- and 2-year forward premium, we use week contracts. The last closing price of the weekly futures contract is used as a proxy for the expected spot price over the week. This is the same procedure as we used when estimating the short-term forward premium. Week contracts are not traded one and two years ahead. A rough proxy is to use the price of the season contract the week belong too one and two years ahead as a proxy for the futures price of the week one and two years ahead of the delivery period. Obviously, this will not be the correct procedure for any single week. However, it is reasonable to believe that the errors we make for the weeks early in the seasons to some extent is made up for by the errors we make for the weeks in the end of the seasons. After all, the forward price of the season is the average forward price of the weeks the season includes.

The print out from Minitab is given below:

Descriptive Statistics: Premium 1 year; Premium 2 year annualized

Variable	N	Mean	Median	TrMean	StDev
Premium	272	-0,1555	-0,2409	-0,1667	0,4168
Premium	134	-0,1032	-0,1433	-0,1101	0,2358

Variable	SE Mean	Minimum	Maximum	Q1	Q3
Premium	0,0231	-1,0322	1,5904	-0,4240	0,1131
Premium	0,0147	-0,5721	0,8610	-0,2670	0,0611

Both the mean and the median premium is negative. The mean of the annualized 1– and 2-year premiums are -0,1555 and -0,1032 respectively. Also the trimmed mean is more negative indicating a negative risk premium, i.e. contango. Note that the negative two year premium is less than the one year negative premium.

Testing the two sided hypothesis of

H_0: PREM=0 VS H_1: PREM≠0

One-Sample T: Premium 1 year; Premium 2 year annualized

```
Test of mu = 0 vs mu not = 0

Variable            N       Mean     StDev   SE Mean
Premium 1 ye      272    -0,1555    0,4168    0,0253
Premium 2 ye      134    -0,1032    0,2358    0,0204

Variable              95,0% CI            T        P
Premium 1 ye  ( -0,2053; -0,1057)     -6,15  0,000
Premium 2 ye  ( -0,1433; -0,0631)     -5,07  0,000
```

The test shows zero P-values and the hypothesis is rejected. This t-test strongly indicates a negative risk premium for the 1- and 2-year risk premium.

The normal probability plots for the 1- and 2-year premiums are given below:

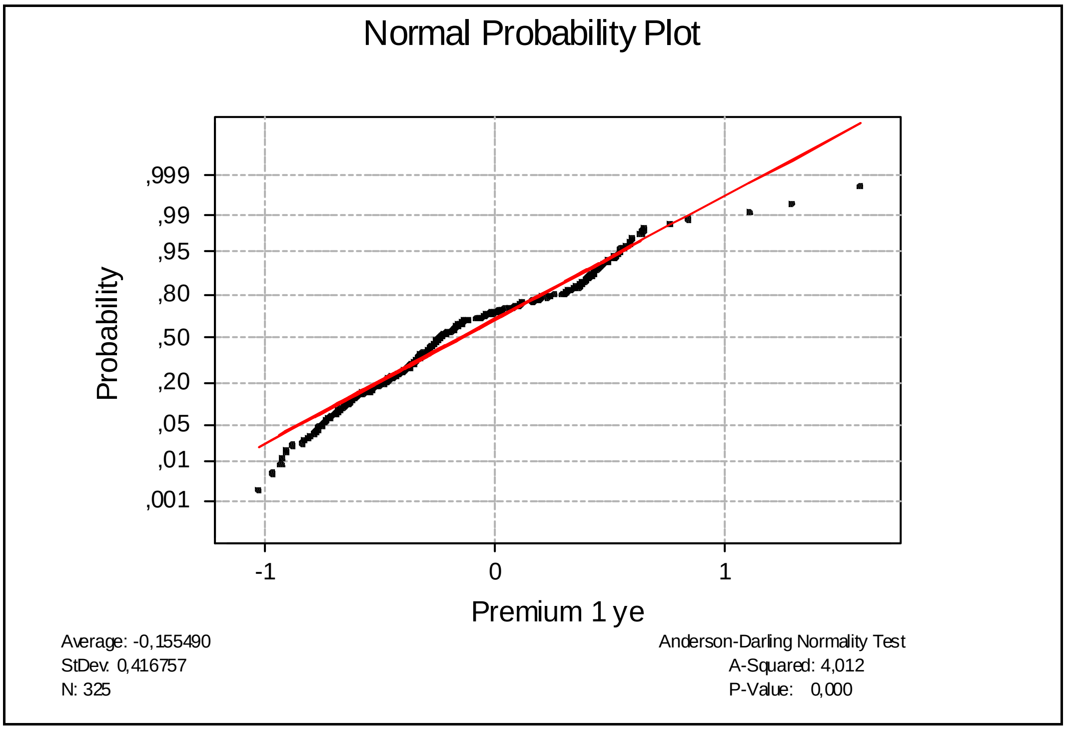

Figure 6-11 Normality plot of the 1-year risk premium

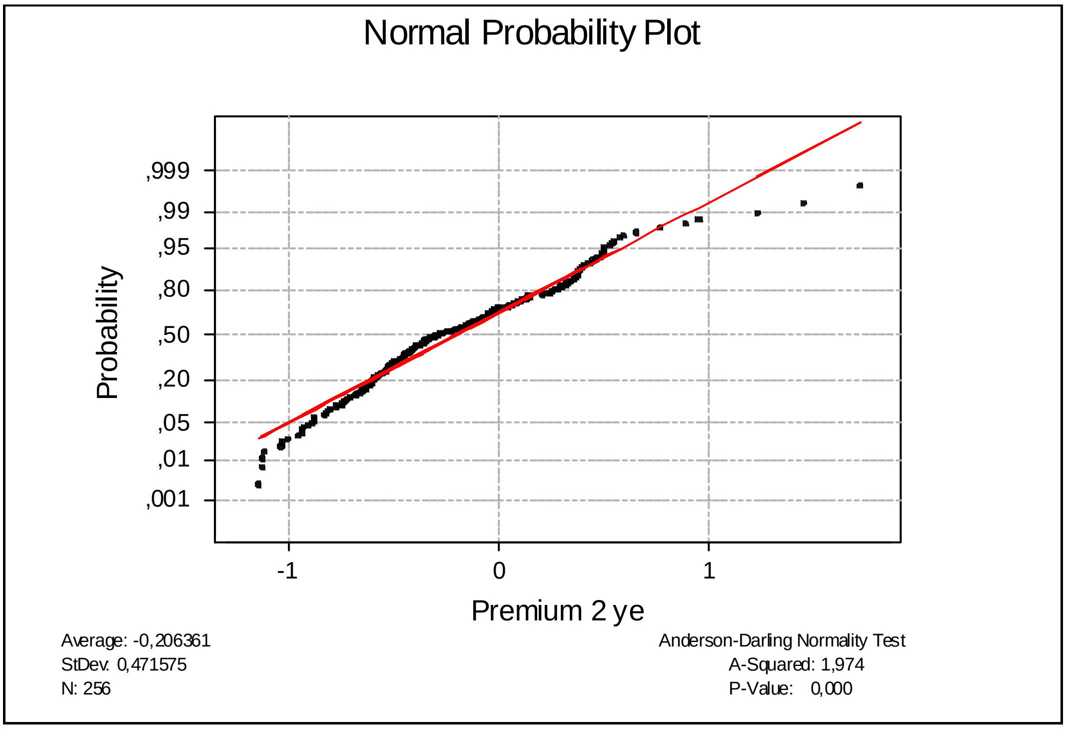

Figure 6-12 Normality plot of the 2-year risk premium

We clearly spot deviations from normal distribution in the normal probability plots. However, 272 observations of the one-year premium and 134 observations of the two year premium should be sufficient for performing t-tests on our data.

7 Discussion of empirical results on the risk premium

This chapter discusses the results of our empirical findings compared to the hypotheses in chapter 5.

7.1 Hypothesis 1

"The forward market is on average contango in the short term"

For the 1-week forward premium we find a significant negative premium for the total sample period. The same is found for the 2- and 3-week premiums. The significance of the tests as measured by the P-values are 0,023, 0,016 and 0,025 respectively. The mean values for the premiums are -0,01225, -0,01850 and -0,02037 respectively. This supports our underlying assumption that retailers and large consumers are less flexible than producers and overall results in retailers and large consumers paying a premium in order to hedge their purchase of power. In order to compare the premiums we annualise the premiums and obtain -0,6370, -0,4810 and -0,3531 for the annualised 1-, 2- and 3-week premiums. The negative premium thus seems to decrease with the time horizon, something that goes well together with our hypothesis.

7.2 Hypothesis 2

"The forward market experiences variations in the short-term premium, with a high degree of contango for cold season and backwardation in the spring season"

In chapter 6.3 we find a significant negative premium for January in the 1-, 2- and 3-week horizon. The mean values of the premiums for January are -0,0450, -0,0623 and -0,0622 respectively, all significant with P-values below 0,016. For February the results are not at all significant, but the mean values are anyway negative. The same is true for March. We do not find a similar trend for December. Although the market is young for the case of providing us with a sufficient amount of data, the analysis indicates support for hypothesis 2 as regards contango in the cold seasons. When looking at the mean for April and May we do not find any significant results for rejecting the null hypothesis of a zero premium. However, the mean values are all slightly positive, but we do not see any indications for the hypothesis on backwardation to be true besides that.

On the other hand, we observe a significant positive premium for June for all the short-term premiums. The P-values are 0,051, 0,058 and 0,012 for the 1-, 2- and 3-week premiums respectively. The mean values for the premiums are 0,0216, 0,0358 and 0,0538 respectively. Based on this, it seems like the snow melting and the high inflows manifest themselves somewhat later than we expected.

Surprisingly, we find a significant negative premium for July for the 1-, 2- and 3-week premiums. The mean value for July is also more negative than the value for January. This might be a holiday effect due to the general staff holiday in July. However, it should be a familiar fact that the demand and the spot price will be low in the general staff holiday, and we note that this should thereby not be the reason for a negative premium. We do not investigate this any further except mentioning the potential for profiting by holding short futures positions for the weeks in July.

7.3 Hypothesis 3

"The forward market is in backwardation in the long term"

There is not enough data available to perform reliable tests on this issue. Long term contracts of different durations above 5 years are traded in the bilateral market, but prices of such contracts are not available. However, our tests for the forwards with one and two years to maturity resulted in annualized premiums of -0,1555 and -0,1032. Although they are both negative, the premium is decreasing from the one year horizon to the two year horizon. We also note that these premiums are considerable below the annualized 3 week premium of -0,3531. If this is the trend we might expect positive premiums and backwardation for a ten year horizon, but we can not reliably state this based on our present research.

7.4 Sources of error when estimating the risk premium

The expectations about the spot price a year and some further into the future are very likely to be based on assumptions about average reservoir levels, inflow and consumption. Thus, the extent to which reservoir levels, inflow and consumption deviate from average conditions may affect the spot price considerably. The expected spot price for time period T at t, $E_t\left[S_T\right]$, is an important uncertain factor when estimating the risk premium. As explained in the section dealing with short-term forward premium, it is not possible to observe the participants' expectations and the closing price of the contract on the last day of trading is used as a proxy for the expected spot price. In the case of the 1- and 2-year premiums, we use the same procedure. For this to be true we assume that the expectations about the spot price over period T will not alter much from date t to the last closing price of the contract for time period T. This is reasonable for the short-term, especially for the case of one week. However, when using the same procedure for the 1- and 2-year premiums, this is a rather crude assumption.

In order to achieve a better understanding of how this works, we plotted reservoir levels and inflow jointly with their median levels, the spot price and the futures price one year ahead. When estimating the one year risk premium we have employed data for the period starting from week 40 1996 to week 52 2002. We also extended the plot to include the first 21 weeks of 2003 to illustrate why we expelled these data from our data set, when estimating the risk premiums. In addition to strongly deviating resource levels, the spot prices reached extreme levels during these first weeks of 2003. Risk premium for the rest of this section refers to the 1-year risk premium.

Looking at the plot of reservoir levels and prices in appendix C, we note the tendency of above average reservoir levels to appear simultaneously with low spot prices, and the other way around. The reservoir levels are the aggregated level of Norway and Sweden which constitutes 96% of the water reservoir capacity in the Nordic area. For 1998, the majority of 1999 and 2000, the above average level may explain parts of the negative risk premium we find (as reflected by the difference of the spot price and the price of the season one year ahead) due to low levels of the actual spot prices. However, in 1996, 1997 and parts of 2001, below average reservoir levels appear and spot prices in 1996 and 2001 are relatively high. This may explain parts of the positive risk premium for these years. Still, we observe negative risk premium for all of 1997. In the first half of 2002, above average levels may explain parts of the negative premium for that period, while below average levels for the second half may explain parts of the positive premium for that period. Noting that the positive premiums in

2002 are quite large, the situation of 2002 should rather work in favour of our findings than against them.

The plot of the inflow and prices in appendix D, broadly speaking shows the same trends as the plot of reservoir levels and prices. However, while low reservoir levels in 1997 could not explain the negative risk premiums for that year, the above average inflow may explain parts of that fact.

While reservoir levels and inflow deviating from their normal (median) values may explain parts of the negative risk premium we find, the consumption does not seem to have the same degree of explanatory power. The plot in appendix E shows the total consumption in the Nordic area and the prices. Apparently it is not the low consumption in 1996 that causes the positive premium, but we could argue to some extent that large consumption during the winter 2000/2001 could explain parts of the positive premiums there. Consumption would be more important for the case of a pure thermal energy market.

It is important to emphasise that even though the above mentioned physical conditions partly can explain our findings for the 1- and 2-year premiums, they have very limited explanation power for the short horizon premium. The participants are quite aware of the reservoir levels and this will be comprised in the futures prices on such a short horizon. The same is true for inflow from snow aggregated in the mountains. It is also quite good weather forecasts regarding high and low pressures and the levels of precipitation on a week's horizon. Hence, we conclude this discussion by marking that while the physical conditions regarding reservoir levels, inflow and consumption to some extent can explain our findings in the long horizon, they have very limited influence on our findings for the short horizon.

Here, we will briefly mention an approximation that could possibly result in a better proxy for the expected spot price for the one and two year horizon. On this horizon, the expectation is fundamentally based on average conditions as regards inflow of water, reservoir levels and demand. Some kind of approach adjusting the realised spot price (or the last closing price of the contract) for deviations from normal conditions in inflow, reservoirs and demand would be helpful.

The spot price model of chapter 4 draws on the theory of Johnsen (2001). Given no physical constraints active, the price will be the same over all periods. Our model results in this price being equal to the marginal cost of a constant thermal production.

Johnsen (2001) suggests the following linear auto-regressive distributed-lag demand function

$$\begin{aligned}\Delta y_j &= a_0 + a_1 \Delta p_j + a_2 p_{j-1} + a_3 y_{j-1} + a_4 \Delta w_j + a_5 w_{j-1} \\ &\quad + a_6 \Delta t_j + a_7 t_{j-1} + e_j \qquad j = t,\dots,T\end{aligned} \tag{7.1}$$

Production y equals demand, and the producers are assumed to be rational with respect to the way physical conditions result in altering the price. For instance they are aware of the fact that high inflow increases the supply, leads to lower prices and increases the demand. They also know that low temperatures result in higher demand and higher prices. In equation (7.1) a_i = unknown coefficients, w = vector of exogenous explanatory variables including the price of alternative fuels, activity level and day-length, t = heating degree days and e = error term.

Johnsen (2001) finds expressions for $E_t p_T$ and $E_{t-1} p_T$ and assumes $E_t I_j^S = E_{t-1} I_j^S$, $E_t I_j^W = E_{t-1} I_j^W$ and $E_t t_j = E_{t-1} t_j$ to hold. That is, the expected weather, here snowfall, inflow and temperature, for an arbitrary week is assumed to be the same and independent of which week t the expectations are made. The physical restrictions are also assumed not to be binding. This results in the following expression for the weekly change in the spot price

$$\Delta p_t = \frac{1}{(T_1 - t - 1)a_2 - a_3 b_0}\left\{\begin{matrix} \left(1-(1+a_3)L\right)\times\left[\left(I_t^W - EI_t^W\right) - b_1\left(I_t^S - EI_t^S\right)\right] \\ -\left(a_6 + (a_7 - a_6)L\right)\left(t_t - Et_t\right) - e_t \end{matrix}\right\} + u_t \qquad (7.2)$$

where u_t is another error term and L indicates the lag-operator. This expression explains some important properties of the price in this framework. First, the price will remain unchanged when factors influencing demand are equal to their expected values. Second, unexpected changes in these factors changes the price more heavily the closer the end of the season the changes take place. This is reasonable since unexpected rain – or snowfall far ahead of the start of the snow melting period is stored and spread out over a large number of weeks. Closer to the end of the planning period, this strengthens the risk of overflow. This is also reasonable as the volatility of spot price changes is found to be higher for the summer season [Lucia & Schwartz, 2002] and our empirical results of the forward curve being more volatile in the snow melting and summer period.

Now, Johnsen (2001) estimates the parameters for an econometric version of (7.2) using data for the period (week 34/1994 – 52/1995). When simulating the prices for the period (week 34/1994 -52/1996) using the econometric version, the demand and prices from the model reproduces the actual figures quite well.

The idea, in the case of estimating the long-term risk premium, is to use the framework of Johnsen backwards. In this way, the expected spot price for a period of time can be found for one to two years in advance by considering deviations from the factors in (7.2). The evolution of the Nordic market from the periods used by Johnsen (2001) has been great, and the above model only includes hydro power producers. In order to use this method the parameters should be estimated for a longer time period and include factors due to the hydro-thermal Nordic power supply.

8 Conclusion

In this thesis we have presented a spot price model, the optimal hedge positions to be taken by the different types of participants in the market and the risk premiums observed in market.

The spot price model presented, combining hydropower with thermal production, indicates a smoother forward curve than what is observed in the market. We point out the deterministic nature of the model, the number of constraints and aggregation level as the main reasons for the differences.

We find that for the producers, the optimal positions depend on the ability to benefit from variations in the prices, the production technology, the cost structure and the expected production. For the retailers, the optimal position depends on the exposure to high demand in periods of high prices and the retail price structure. Based on this theory, we present three hypotheses regarding the risk premium. We expect a negative risk premium on average in the short term, regular variations in the short term premium over the year and a positive long-term risk premium.

Based on more than seven years of data we have found strong indications of a negative risk premium in the short run. This is done by hypothesis testing and the results have been supported by the use of non-parametric tests. We also find that the short-run risk premium varies over the year. The premium increases in magnitude during January and is positive in June. This is in accordance with our hypotheses. We also observe a significant negative risk premium in July. We are not able to explain the latter, based on the theory presented for the risk premiums. For the long-term risk premium it is difficult to obtain empirical data, due to the non-constant horizon of the traded products and the nature of the future and forward products. We try to explain the risk premium for the long term by studying the variations in inflow and resource levels.

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Appendix A

P – spot price of commodity at time T (End of period)
q- quantity of commodity to be sold at T
Pq- revenue of particpant selling the commodity at time T
P_F - futures price at time t (today) of commodity futures with expiration at time T
C – consumption
x -quantity hedged in the futures market (short if positive)
W- initial wealth

Utility, U = E(C) – (A/2)var(C)

$$C = W + Pq + z\,(P_F - P) \tag{A.1}$$

$$\text{var}(C) = \text{var}(Pq) + z^{2}\,\text{var}(P) - 2z\,\text{cov}(Pq, P) \tag{A.2}$$

$$E(C) = W + E(Pq) + x\,(P_F - E(P)) \tag{A.3}$$

The first order condition with respect to x is given by (sufficient because second derivative is negative)

$$\frac{\partial U}{\partial x} = P_F - E(P) - Ax\,\text{var}(P) + A\,\text{cov}(Pq, P) = 0 \tag{A.4}$$

Reordering this gives

$$x = \frac{P_F - E(P)}{A\,\text{var}(P)} + \frac{\text{cov}(Pq, P)}{\text{var}(P)} \tag{A.5}$$

Appendix B

Producers

The ”but-for-hedging” profit for producers is given by

$$r_P = PQ - C(Q) \qquad \text{(B.1)}$$

where P is the spot price and Q is the quantity produced and sold. C(Q) is the cost of producing the quantity Q. Now, the covariance between “but-for-hedging” profit and spot price is given by

$$\mathrm{cov}(r_p, P) = \mathrm{cov}(PQ - C(Q), P) \qquad \text{(B.2)}$$

Rewriting this expression

$$\begin{aligned}\mathrm{cov}(PQ - C(Q), P) &= E\left[P^2Q - PC(Q)\right] - E\left[PQ - C(Q)\right]\cdot E[P] \\ &= E[PPQ] - E[PC(Q)] - E[P]\cdot E[PQ] + E[P]E[C(Q)]\end{aligned} \qquad \text{(B.3)}$$

Note the following

$$E[PPQ] = E[P]\cdot E[PQ] + \mathrm{cov}(P, PQ) \qquad \text{(B.4)}$$

$$E[PC(Q)] = E[P]\cdot E[C(Q)] + \mathrm{cov}(P, C(Q)) \qquad \text{(B.5)}$$

Substituting (B.4) and (B.5) into (B.3) yields

$$\begin{aligned}\mathrm{cov}(PQ - C(Q), P) &= E[P]\cdot E[PQ] + \mathrm{cov}(P, PQ) - E[P]\cdot E[C(Q)] \\ &\quad - \mathrm{cov}(P, C(Q)) - E[P]\cdot E[PQ] + E[P]\cdot E[C(Q)] \\ &= \mathrm{cov}(P, PQ) - \mathrm{cov}(P, C(Q))\end{aligned} \qquad \text{(B.6)}$$

The first term can be rewritten as

$$\begin{aligned}\mathrm{cov(P,PQ)} &= \mathrm{E[PPQ] - E[P]E[PQ]} \\ &= \mathrm{E\left[P^2Q\right] - E[P]\left(E[P]\cdot E[Q] + cov(P,Q)\right)} \\ &= \mathrm{E\left[P^2\right]E[Q] + cov\left(P^2,Q\right) - E[P]\left(E[P]\cdot E[Q] + cov(P,Q)\right)} \\ &= \mathrm{E[Q]\left(E\left[P^2\right] - E[P]^2\right) + cov\left(P^2,Q\right) - E[P]cov(P,Q)} \\ &= \mathrm{E[Q]\cdot Var(P) + cov\left(P^2,Q\right) - E[P]cov(P,Q)}\end{aligned} \qquad \text{(B.7)}$$

Thus

$$\mathrm{cov}(PQ - C(Q), P) = E[Q]\cdot Var(P) + \mathrm{cov}\left(P^2, Q\right) - E[P]\mathrm{cov}(P, Q) - \mathrm{cov}(P, C(Q)) \qquad \text{(B.8)}$$

Equation (5.10) gives us the optimal hedge position

$$Q_{Pi}^{F} = \frac{P_F - E[P]}{AVar(P)} + E[Q] + \frac{\text{cov}(P^2, Q)}{Var(P)} - \frac{E[P]\text{cov}(P,Q)}{Var(P)} - \frac{\text{cov}(P, C(Q))}{Var(P)} \tag{B.9}$$

Retailers with fixed retail price

The "but-for-hedging" profit for retailers is given by

$$r_R = P_R Q - PQ \tag{B.10}$$

where P_R is the fixed retail price (for customers of the retailers), P is the spot price and Q is the retail quantity. Now, the covariance between the "but-for-hedging" profit and spot price is given by

$$\text{cov}(r_R, P) = \text{cov}(P_R Q - PQ, P) \tag{B.11}$$

Rewriting this

$$\begin{aligned} \text{cov}(P_R Q - PQ, P) &= E\left[P P_R Q - P^2 Q\right] - E[P_R Q - PQ] \cdot E[P] \\ &= E[P P_R Q] - E[PPQ] - E[P] \cdot E[P_R Q] + E[P] \cdot E[PQ] \end{aligned} \tag{B.12}$$

Note the following

$$E[P P_R Q] = P_R \cdot E[PQ] = P_R \cdot \left(E[P] \cdot E[Q] + \text{cov}(P,Q)\right) \tag{B.13}$$

$$E[PPQ] = E[P] \cdot E[PQ] + \text{cov}(P, PQ) \tag{B.14}$$

$$E[P] \cdot E[P_R Q] = E[P] \cdot \left(P_R E[Q] + \text{cov}(P_R, Q)\right), \quad \text{cov}(P_R, Q) = 0 \tag{B.15}$$

Substituting (B.13), (B.14) and (B.15) into (B.12) yields

$$\begin{aligned} \text{cov}(P_R Q - PQ, P) &= P_R \cdot \left(E[P] \cdot E[Q] + \text{cov}(P,Q)\right) - E[P] \cdot E[PQ] - \text{cov}(P, PQ) \\ &\quad - E[P] \cdot P_R E[Q] + E[P] \cdot E[PQ] \\ &= P_R \text{cov}(P,Q) - \text{cov}(P, PQ) \end{aligned} \tag{B.16}$$

Using (B.7) we obtain

$$\text{cov}(P_R Q - PQ, P) = P_R \text{cov}(P,Q) - E[Q] \cdot Var(P) - \text{cov}(P^2, Q) + E[P]\text{cov}(P,Q) \tag{B.17}$$

Equation (5.10) gives us the optimal hedge position

$$Q_{Rj}^{F} = \frac{P_F - E[P]}{A\,\mathrm{var}(P)} + \frac{P_R \,\mathrm{cov}(P,Q)}{\mathrm{var}(P)} - E[Q] - \frac{\mathrm{cov}(P^2,Q)}{\mathrm{var}(P)} + \frac{E[P]\,\mathrm{cov}(P,Q)}{\mathrm{var}(P)} \tag{B.18}$$

Retailers with variable retail price

For retailers with a more frequently moving retail price, the ”but-for-hedging” profit is still given by

$$r_R = P_R Q - PQ \tag{B.19}$$

The only difference is due to the moving P_R which also introduces more co variance terms. Now the co variance between the “but-for-hedging” profit and spot price is given by

$$\begin{aligned} \mathrm{cov}(P_R Q - PQ, P) &= E\left[P P_R Q - P^2 Q\right] - E[P_R Q - PQ]\cdot E[P] \\ &= E[P P_R Q] - E[PPQ] - E[P]\cdot E[P_R Q] + E[P]\cdot E[PQ] \end{aligned} \tag{B.20}$$

Note the following

$$\begin{aligned} E[P P_R Q] &= E[P_R]\cdot E[PQ] + \mathrm{cov}(P_R, PQ) \\ &= E[P_R]\cdot\left(E[P]\cdot E[Q] + \mathrm{cov}(P,Q)\right) + \mathrm{cov}(P_R, PQ) \end{aligned} \tag{B.21}$$

$$E[PPQ] = E[P]\cdot E[PQ] + \mathrm{cov}(P,PQ) \tag{B.22}$$

$$E[P]\cdot E[P_R Q] = E[P]\cdot\left(E[P_R]\cdot E[Q] + \mathrm{cov}(P_R, Q)\right) \tag{B.23}$$

Substituting (B.21), (B.22) and (B.23) into (B.20) yields

$$\begin{aligned} \mathrm{cov}(P_R Q - PQ, P) = {} & E[P_R]\cdot\left(E[P]\cdot E[Q] + \mathrm{cov}(P,Q)\right) + \mathrm{cov}(P_R, PQ) - \\ & E[P]\cdot E[PQ] - \mathrm{cov}(P,PQ) - E[P]\cdot\left(E[P_R]\cdot E[Q] + \mathrm{cov}(P_R,Q)\right) \\ & + E[P]\cdot E[PQ] \\ = {} & E[P_R]\,\mathrm{cov}(P,Q) + \mathrm{cov}(P_R, PQ) - \mathrm{cov}(P,PQ) - E[P]\cdot\mathrm{cov}(P_{R,} Q) \end{aligned} \tag{B.24}$$

The second term can be rewritten as

$$\begin{aligned} \mathrm{cov}(P_R, PQ) &= E[P_R PQ] - E[P_R]\cdot E[PQ] \\ &= E[P_R PQ] - E[P_R]\cdot\left(E[P]\cdot E[Q] + \mathrm{cov}(P,Q)\right) \\ &= E[P_R P]\cdot E[Q] + \mathrm{cov}(P_R P, Q) - E[P_R]\cdot\left(E[P]\cdot E[Q] + \mathrm{cov}(P,Q)\right) \\ &= E[Q]\cdot\left(E[P_R P] - E[P_R]\cdot E[P]\right) + \mathrm{cov}(P_R P, Q) - E[P_R]\cdot\mathrm{cov}(P,Q) \\ &= E[Q]\cdot\mathrm{cov}(P_R, P) + \mathrm{cov}(P_R P, Q) - E[P_R]\cdot\mathrm{cov}(P,Q) \end{aligned} \tag{B.25}$$

Using (B.7) and (B.25) we obtain

$$\begin{aligned}\operatorname{cov}(P_RQ-PQ,P) &= E[P_R]\cdot\operatorname{cov}(P,Q)+E[Q]\cdot\operatorname{cov}(P_R,P)+\operatorname{cov}(P_RP,Q)-E[P_R]\cdot\operatorname{cov}(P,Q)\\ &\quad -E[Q]\cdot Var(P)-\operatorname{cov}(P^2,Q)+E[P]\operatorname{cov}(P,Q)-E[P]\cdot\operatorname{cov}(P_R,Q)\\ &= -E[Q]Var(P)-\operatorname{cov}(P^2,Q)+E[P]\cdot\operatorname{cov}(P,Q)+\\ &\quad E[Q]\cdot\operatorname{cov}(P_R,P)+\operatorname{cov}(P_RP,Q)-E[P]\cdot\operatorname{cov}(P_R,Q)\end{aligned} \tag{B.26}$$

Equation (5.10) gives us the optimal hedge position

$$\begin{aligned}Q_{Ri}^F &= \frac{P_F-E[P]}{A\cdot Var(P)}-E[Q]-\frac{\operatorname{cov}(P^2,Q)}{Var(P)}+\frac{E[P]\cdot\operatorname{cov}(P,Q)}{Var(P)}+\frac{E[Q]\cdot\operatorname{cov}(P_R,P)}{Var(P)}+\\ &\quad \frac{\operatorname{cov}(P_RP,Q)}{Var(P)}-\frac{E[P]\cdot\operatorname{cov}(P_R,Q)}{Var(P)}\end{aligned} \tag{B.27}$$

In order to directly read the changes between the situation with fixed retail price and this situation (with moving retail price), we manipulate the sixth term of (B.27) below

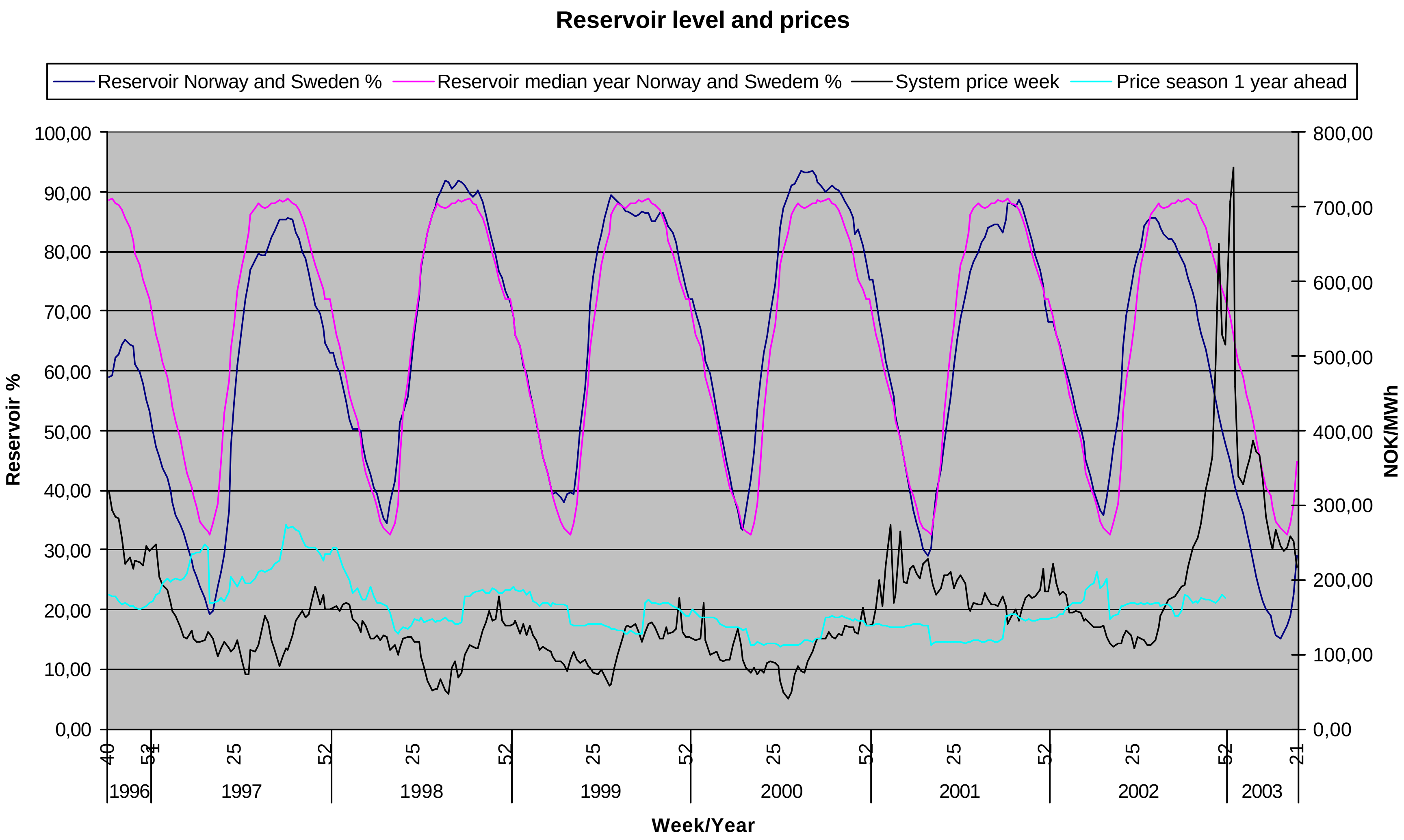
$$\begin{aligned}\operatorname{cov}(P_RP,Q) &= E[P_RPQ]-E[P_RP]\cdot E[Q]\\ &= E[P_R]\cdot E[PQ]+\operatorname{cov}(P_R,PQ)-E[Q]\cdot\left(E[P_R]\cdot E[P]+\operatorname{cov}(P_R,P)\right)\\ &= E[P_R]\cdot\left(E[P]\cdot E[Q]+\operatorname{cov}(P,Q)\right)+\operatorname{cov}(P_R,PQ)-E[Q]\cdot\left(E[P_R]\cdot E[P]+\operatorname{cov}(P_R,P)\right)\\ &= E[P_R]\cdot\operatorname{cov}(P,Q)+\operatorname{cov}(P_R,PQ)-E[Q]\cdot\operatorname{cov}(P_R,P)\end{aligned} \tag{B.28}$$

Substituting (B.28) into (B.27) yields

$$\begin{aligned}Q_{Ri}^F &= \frac{P_F-E[P]}{A\cdot Var(P)}-E[Q]-\frac{\operatorname{cov}(P^2,Q)}{Var(P)}+\frac{E[P]\cdot\operatorname{cov}(P,Q)}{Var(P)}+\frac{E[P_R]\cdot\operatorname{cov}(P,Q)}{Var(P)}+\\ &\quad \frac{\operatorname{cov}(P_R,PQ)}{Var(P)}-\frac{E[P]\cdot\operatorname{cov}(P_R,Q)}{Var(P)}\end{aligned} \tag{B.29}$$

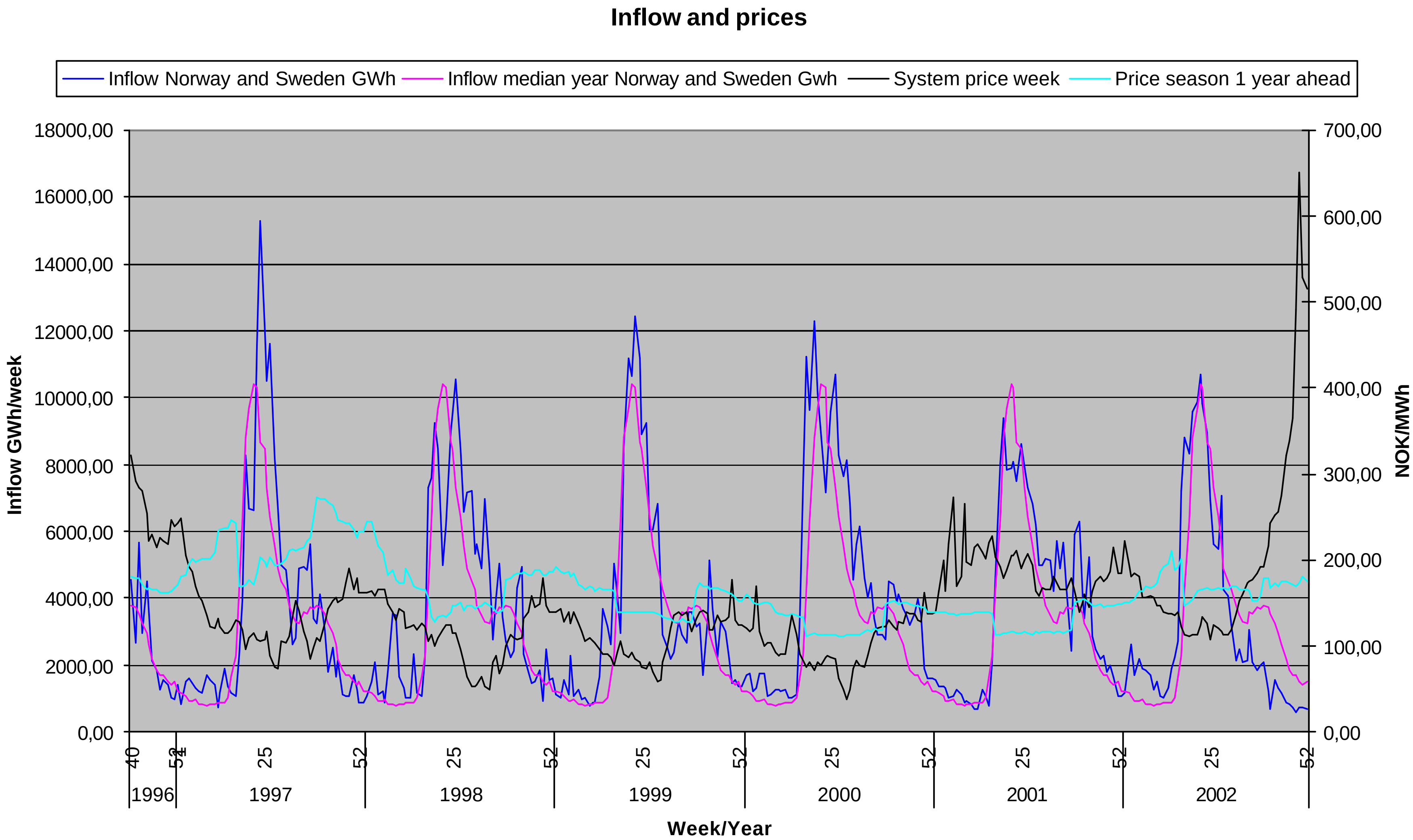
When the retail price is fixed, the co variances in the 2 last terms of (B.29) is zero and we are left with the same expression as in (B.18), that this the 5 first terms of (B.29).

Appendix C



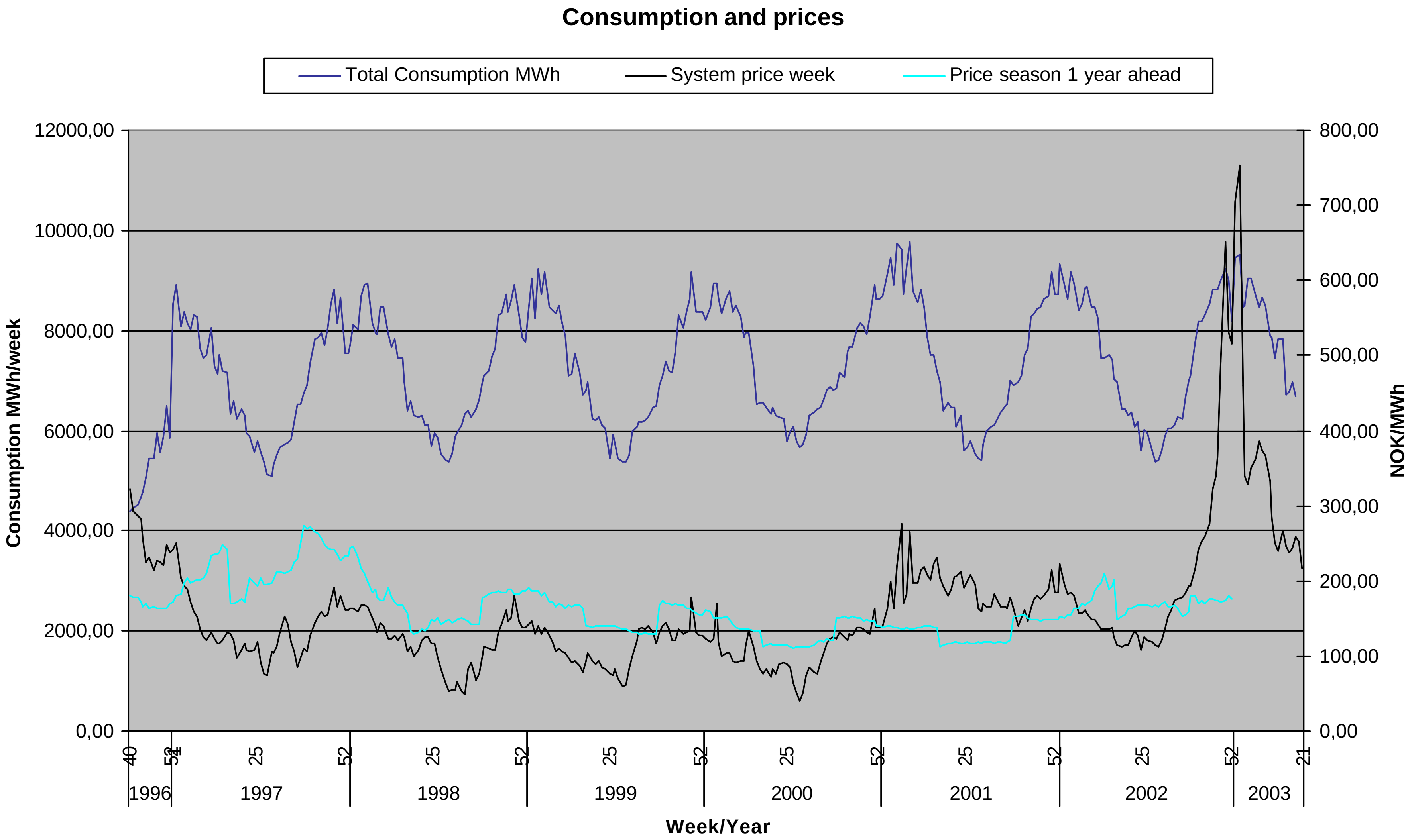
Reservoir level and prices
Reservoir Norway and Sweden %
Reservoir median year Norway and Swedem %
System price week
Price season 1 year ahead
100,00
90,00
80,00
70,00
60,00
50,00
40,00
30,00
20,00
10,00
0,00
Reservoir %
800,00
700,00
600,00
500,00
400,00
300,00
200,00
100,00
0,00
NOK/MWh
40
52
1
25
52
25
52
25
52
25
52
25
52
25
52
25
52
21
1996
1997
1998
1999
2000
2001
2002
2003
Week/Year

Appendix D



Inflow and prices
Inflow Norway and Sweden GWh
Inflow median year Norway and Sweden Gwh
System price week
Price season 1 year ahead
Inflow GWh/week
18000,00
16000,00
14000,00
12000,00
10000,00
8000,00
6000,00
4000,00
2000,00
0,00
NOK/MWh
700,00
600,00
500,00
400,00
300,00
200,00
100,00
0,00
40
52
1
25
52
25
52
25
52
25
52
25
52
25
52
25
52
1996
1997
1998
1999
2000
2001
2002
Week/Year

Appendix E



Consumption and prices
Total Consumption MWh
System price week
Price season 1 year ahead
Consumption MWh/week
12000,00
10000,00
8000,00
6000,00
4000,00
2000,00
0,00
NOK/MWh
800,00
700,00
600,00
500,00
400,00
300,00
200,00
100,00
0,00
40
52
1
25
52
25
52
25
52
25
52
25
52
25
52
25
52
21
1996
1997
1998
1999
2000
2001
2002
2003
Week/Year