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Tittel	**Verdsetting av langsiktige forwardkontrakter i kraftmarkedet** Valuation of long term electricity forward contracts
Formål	Utarbeide metoder for å prissette langsiktige kraftkontrakter.

Følgende hovedpunkter skal behandles:

1. Undersøke risikopremien i tidligere omsatte kontrakter. Videre undersøke muligheten for å bruke informasjon om tilbud og etterspørsel etter langsiktige kraftkontrakter i prissettingen.
2. Analysere volatiliteten til spot- og forwardprisen.

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___________semester 2005

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Preface

This master thesis was performed during the spring of 2005 at the Norwegian University of Science and Technology, NTNU, Department of Industrial Economics and Technology Management. The thesis is accomplished in cooperation with M3kraft in Stavanger.

I would like to thank my supervisor, associate professor Stein-Erik Fleten, for help on the theoretical and practical issues developed in the thesis. I would also like to thank Kristian Lenning at M3kraft for valuable comments during the process.

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Trondheim, 10th of June, 2005

Torun Revdal

Summary

In this master thesis, a method for valuation of power contracts with a delivery period of ten years is presented. This contract is from now on named *ENO10*. Few such contracts have been realized lately and a pricing approach where the supply and demand of different market participants is considered is therefore hard. The approach chosen here is based on analyses of different kinds of historic data.

Prices of contracts with one, two and three years to maturity and a delivery period of one year traded at Nord Pool between 1998 and 2004 are used in the analysis. These contracts are subsequently called *ENOYR1, ENOYR2* and *ENOYR3*. Price estimates on *ENO10* received from Trønder-Energi are also used in the development of the model. These estimates are available in the period between week 48 in 2000 and week 33 in 2002. In addition, oil forward prices from EcoWin, interest rates from Norges Bank and from the European Central Bank and data on the hydrologic balance from Trønder-Energi are used.

In the development of the model, the expected growth of the long end of the forward curve, *b*, is estimated for each available data point. Trønder-Energi's price estimates on *ENO10* and the prices of *ENOYR1*, *ENOYR2* and *ENOYR3* traded at Nord Pool are used to achieve this. The estimates are calculated by assuming that the present value of *ENO10* should equal the present value of the estimated forward curve, consisting of the prices of *ENOYR1*, *ENOYR2* and *ENOYR3* in the short end and *b* in the long end. By using the assumption that the forward curve crosses through the price of *ENOYR3*, an equation for *b* is found. Next, the correlation between *b* and different variables is tested. These variables are chosen based on evaluation of six hypotheses.

The significant variables included in the regression equation for *b* are the price of *ENOYR3* and the growth between the prices of *ENOYR2* and *ENOYR3* in percentages. Neither the interest rate, the hydrologic level nor the oil forward price is included in the regression. The reason for this is presumably the high correlation between these and the price of *ENOYR3*. Including these other variables in addition to the price of *ENOYR3* therefore gives insignificant results.

The forward curve is then approximated by the regression equation for *b* and the contracts with respectively one, two and three years to maturity that are traded at Nord Pool. The price of *ENO10* is found by estimating the present value of the contracts approximated by the forward curve and adjusting for when settlement is expected to occur.

The estimated model should however be used with care. The analysed data are old, and even though the model somehow is adjusted for that, new market relations may be relevant today. Nevertheless, the uncertainty regarding future conditions will always be high. Every model that tries to estimate long term prices will therefore be characterized with high uncertainty.

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1 Introduction

1.1 Objective and background

The objective of this master thesis is to find a model for valuation of long term electricity forward contracts. Developing forecasts for electricity prices far into the future is a relatively new area. The electricity market used to be a monopoly in most countries, and the prices used to reflect the governments social policy where the main thought was to cover production costs. After the liberalization of the Nordic market in the early 1990s, the focus however changed, and the connections of importance are now more and more studied. Even though long term contracts have low liquidity, development of such forecasts for long term prices has become very important. Primarily these prices are important in investment analysis and to make value assessments easier.

Different kinds of long term contracts exist. At Nord Pool contracts with up to three years to maturity are realized. These contracts are quite liquid[1]. Contracts with longer time to maturity are only traded bilaterally, and these contracts have lower liquidity. In this report, the following terminology is used: *ENOYRy*-contracts are contracts that have delivery periods of one year. $y = 1$ means that these are contracts with delivery period next year, $y = 2$ means that the delivery period is the year after that and so on.

Long term contracts are in this report considered to be contracts with longer delivery periods and time to maturity than the contracts that are traded at Nord Pool. M3kraft requests a model for valuation of electricity forward contracts with a delivery period of 10 years. The main objective of this report is therefore to develop a model for pricing such contracts, which from now on are called *ENO10*. Similar approaches can be followed for pricing contracts with delivery periods of for instance 5, 15 and 20 years.

1.2 Challenges

Most of the power production in the Nordic market comes from hydroelectric power. As a result of great yearly variation in rainfall, the prices fluctuate. Very dry years can lead to very high prices, just as cold winters increase demand and then again prices. The volatility in the Nordic market is therefore extremely high, something that makes future prices very uncertain.

Many models for valuation of power contracts are used today. However, the majority of these are models that price short term contracts[2]. Short term contracts are liquid contracts and are traded bilaterally and at Nord Pool every day. Valuation of such contracts is a rather different approach than valuation of long term contracts. The short term contracts depend to a great degree of the hydrologic balance[3]. Long term contracts do not to the same degree depend on this, and to price such contracts one therefore has to consider other approaches.

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[1] Contracts with 3 years to maturity may have lower liquidity in periods.

[2] Short term contracts are in this context contracts with less than a year to maturity.

[3] The hydrologic balance is a function of water levels in the reservoirs and expected inflow and snow melt.

There is currently no technology by which electricity can be stored effectively once generated. Electricity supply and demand therefore has to be balanced to keep the network from collapsing. This lack of storage opportunities also implies that electricity cannot be considered a financial product to be held in a hedge portfolio. The usual cash-and-carry relationship is therefore more difficult to apply to electricity markets, leading to the fact that the traditional relation between spot- and forward prices does not exist. Other pricing methods must therefore be established for valuation of electricity contracts than the methodology that is used to price other commodities.

Audet, Heiskanen, Keppo and Vehviläinen (2004) in addition show that a forward contract is less and less correlated with the spot price the further we get from maturity. Koekebakker and Ollmar (2001) on the other hand find that correlations between short- and long term contracts are lower in electricity markets than in other markets. While short term contracts are correlated with expectations on inflow, long term contracts correlate more with other factors. These contracts will probably correlate with expectations on the supply of other commodities, such as coal, oil, gas and beliefs on potential development of new wind and hydro power plants. Expectations on future demand are also important and the prices on CO_2-quotas have become an important factor.

In addition, one now has to allow for foreign currency. The contracts with the longest time to maturity that are traded at Nord Pool have up till recently been denoted in NOK, but now they are denoted in EURO. To price a long term contract, the euro-forward therefore also has to be taken into consideration.

1.3 Structure

In chapter two, rules of thumb regarding M3kraft's current model is discussed briefly, before some existing models for valuation of long term power contracts are considered in chapter three. This chapter includes a discussion of advantages and disadvantages of the different models. Chapter four presents a qualitative analysis of the market structure and the different participants' demand for long term contracts, before the hypotheses of interest are presented in chapter five. These hypotheses are evaluated in chapter six and the recommended model is presented based on the evaluation of the hypotheses in chapter seven. A discussion concerning the variables in the model is performed in chapter eight. This chapter also includes a qualitative analysis of other relevant conditions in the market.

In chapter nine an estimation of the risk premium on *ENO10*-contracts is performed, according to the first point in the problem formulation. "Realized contracts" is here interpreted as *ENO10*-contracts. Since there are few such realized contracts, the estimation is based on the estimated forward curve of chapter seven and is rather uncertain. In accordance with the second point in the problem formulation a volatility analysis is then performed in chapter ten before some concluding remarks are given at last, in chapter eleven.

2.3 Rules of thumb

Different rules of thumb for estimation of the price of *ENO10* are used in practice. An analyst in Skagerak Energi suggests that an estimate for *ENO10* is given by adding 2-4% to the price of *ENOYR3.* An analyst in Elkem Energi, suggests an approach where the price of *ENO10* is given by the average of the price of *ENOYR2* and *ENOYR3* plus 10NOK/MWh. These are both rather occasional methods. In this thesis more on the relations between the price and different factors are examined. In the end this results in a model that can be used to price *ENO10*.

3 Existing models for valuation of long term electricity forward contracts

In this chapter some theoretical models for valuation of long term electricity forward contracts are discussed. Their advantages and disadvantages are also considered.

3.1 Spot price based models

Spot price models are models that try to describe the dynamics in forward prices by means of the spot price.

3.1.1 Cost-of-carry models

These are models that attach the forward- and the spot price by means of storage cost[5] and convenience yield[6]. The spot price is modelled by a stochastic relation and the forward is priced by arbitrage. The forward price is then given by

$$F(t,T) = S(t) \cdot e^{(r-y)(T-t)} \tag{5}$$

where *T* is time to maturity, *S(t)* is the spot price, *r* is the risk-free rate of return and *y* is the convenience yield.

The approach above suggests that if one knows the spot price at *t*, one can price the forward with delivery at *T* by arbitrage. The main problem with this model is that forward prices are given endogenously from the parameters governing the spot price dynamics. This link will not be present in electricity markets, and is a result of that the spot price today reflects the relationship between supply and demand today, and says nothing about what this relationship will be in the future (Bunn and Karakatsani, 2003). Such a model can be relevant in the valuation of forward contracts with short time to maturity, since these to a certain degree are correlated with reservoir levels and the hydrologic balance. Valuation of contracts with ten years to maturity will on the other hand be more difficult.

3.1.2 The risk premium approach

If one is able to estimate the expected spot price at *T*, one should also be able to price the forward contract. Several approaches has been developed, see e.g. Schwarz (1997). If one is able to create a model for the spot price over time, then one just needs an estimate for the risk premium to estimate a proper forward price.

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[5] The cost of storing a commodity.

[6] A measure of the benefits from ownership of an asset that are not obtained by the holder of a long futures contract on the asset.

The forward-/spot price relationship would then be:

$$F(t,T) = E[S_T] \cdot e^{(r-k)(T-t)} \tag{6}$$

where *k-r* is the risk premium.

Estimation of the risk premium for long term contracts is however not easy, due to the low liquidity in such markets.

3.2 Modelling the forward curve

One class of approaches abandons the modelling of spot price dynamics and focuses instead on the term structure of forward commodity prices across different maturities. These models can be more credible than the spot price based models. Next a model by Cortazar and Schwarz (1994) is considered. The forward dynamics are now given by:

$$\frac{dF(t,T)}{F(t,T)} = \sum_{i=1}^{n} \sigma_i(t,T) dw_i(t) \tag{7}$$

This leads to:

$$d \ln F(t,T) = -\frac{1}{2}\sum_{i=1}^{n} \sigma_i^2(t,T)dt + \sum_{i=1}^{n} \sigma_i(t,T) dw_i(t) \tag{8}$$

where *F(t,T)* denotes the forward price at time *t* for delivery at *T*, w_i are independent Brownian motions under the equivalent-martingale measure and σ_i are volatility functions of spot prices. The number of volatility components is suggested from eigenvalue decomposition of the covariance matrix of forward returns. Many variations of σ_i have been suggested for the Nordic market.

Lucia and Schwarz (2002) considers the volatility function

$$\sigma_1(t,T) = \sigma e^{-\kappa(T-t)} \tag{9}$$

where σ og κ are positive constants. Implied in this is that when *T* grows, the volatility converges to zero.

Audet et al. (2004) generalizes the one-factor model above. They assume that all the contracts in the model are driven by separate Brownian motions. These Brownian motions are given by a parametric correlation structure, adding flexibility to the model considered by Lucia and Scwarz (2002). Bjerksund et al. (2000) suggests a one-factor model, where the volatility is given by

$$\sigma_1(t,T) = \frac{a}{T-t+b} + c \tag{10}$$

where a, b and c are positive constants. This model implies that the volatility approaches a constant c when T grows. Bjerksund et al. (2000) also suggests a three-factor model where

$$\sigma_1(t,T)=\frac{a}{T-t+b} \tag{11}$$

$$\sigma_2(t,T)=(\frac{2ac}{T-t})^{\frac{1}{2}} \tag{12}$$

$$\sigma_3(t,T)=c \tag{13}$$

The models above describe the dynamics in the forward prices. Whether they can be used for pricing purposes or not is another issue. The dynamics in the forward prices is greatly affected by relations in the market, such as supply and demand. These models do not include this, and the use of historical prices and historical volatility can therefore be a source of error.

3.3 Equilibrium models

A lot of equilibrium models that try to estimate future supply and demand and thereby electricity prices has also been developed. These are based on the fact that supply and demand always are in instantaneous balance, and divide the forward price in one expected spot price-component and one risk premium-component. By knowing the expected spot price and the risk premium, the forward contract can be priced.

3.3.1 Expected spot price – parametric model

One such model is presented by Koekebakker and Sødal (2003). They let aggregate supply be iso-elastic, given by the function

$$M\cdot S^{\gamma} \tag{14}$$

where γ is the price elasticity of supply, S is the energy price and M, which is a rough measure of aggregate capacity, follows the mean-reverting diffusion

$$dM=(\beta_M-\kappa_M\ln(M))Mdt+\sigma_M MdB_M \tag{15}$$

Likewise the aggregate demand is given by the function

$$N\cdot S^{-\varepsilon} \tag{16}$$

Where ε is the price elasticity of demand and N, which is a rough measure of aggregate market size, follows the diffusion

$$dN=(\beta_N-\kappa_N\ln(N))Ndt+\sigma_N NdB_N \tag{17}$$

The two Brownian motions are correlated with $dB_M dB_N=\rho_{MN}dt$.

Supply and demand must equal, leading to

$$S = (\frac{N}{M})^{\alpha} \tag{18}$$

where $\alpha = \dfrac{1}{\varepsilon + \gamma}$

Itos lemma then gives

$$\begin{aligned}
dS &= (\beta_M - \kappa_M \ln(M))M(-\alpha\frac{S}{M})dt + (-\alpha\frac{S}{M})\sigma_M M dB_M + (\beta_N - \kappa_N \ln(N))N(\alpha\frac{S}{N})dt + (\alpha\frac{S}{N})\sigma_N N dB_N \\
&+ \frac{1}{2}\alpha(\alpha+1)\frac{S}{M^2}\sigma_M^2 M^2 dt + \frac{1}{2}\alpha(\alpha-1)\frac{S}{N^2}\sigma_N^2 N^2 dt + \frac{1}{2}(-2\alpha^2\frac{S}{MN})\sigma_M\sigma_N\rho_{MN} MN dt \\
&= (\beta_S - \alpha(\kappa_N \ln(N) - \kappa_M \ln(M)))Sdt + \alpha S(\sigma_N dB_N - \sigma_M dB_M)
\end{aligned} \tag{19}$$

where

$$\beta_S = \alpha(\beta_N - \beta_M) + \frac{\alpha}{2}[(\alpha+1)\sigma_M^2 + (\alpha-1)\sigma_N^2 - 2\alpha\sigma_M\sigma_N\rho_{MN}] \tag{20}$$

It is further assumed that $\kappa_N = \kappa_M \equiv \kappa_S$ and that $\rho_{MN} = 1$. (19) then reduces to

$$dS = (\beta_S - \kappa_S \ln(S))Sdt + \sigma_S S dB_N \tag{21}$$

where

$$\sigma_S = \alpha(\sigma_N - \sigma_M) \tag{22}$$

If one has a model for future supply and demand, this can be used to estimate future spot prices. An obvious advantage with this model compared to the ones earlier mentioned, is that it takes expectations on changes in supply and demand into consideration. A disadvantage is that the number of variables in this kind of model is extremely large, making a satisfactory empirical analysis hard.

3.3.2 Expected spot price – Long run marginal cost (LRMC)

Other models are based on the fact that the price in the long run will approach LRMC. In the short run fundamental factors such as production capacity and demand will not change. On the long view, however, changes in demand, transfer capacity and government regulations will to a large extent affect prices. These factors should therefore be allowed for in the valuation of long term contracts.

Long run marginal cost for the production technologies that depend on such factors, change slowly. A perfect market will converge towards long run equilibrium; in such a way that long run marginal cost will converge towards the most competitive technology. At present this is gas power. Considering this, long term prices will converge towards 250-300NOK/MWh, the sum of the annuity of the investment cost and the gas price (Vogstad, 2004). On the longer view other technologies may be cheaper, possibly wind power.

Expectations on gas prices and future prices on other energy carriers are therefore of great importance for the expectations on long run marginal cost and the long term price. The relation between the different expectations is important. Expectations on large gas prices can make competing alternatives, such as wind power, more relevant. Consequences of the introduction of a market for green certificates in the Nordic market and the market for CO_2-quotas are also of great importance.

3.3.3 Expected spot price – Different models

Public authorities have also made models to estimate future spot prices. These are based on expectations on the European power market, growth in demand, new production capacity, gas power plants and CO_2-quotas. The price is set in the market cross between expected future supply and demand.

Econs model is made by Tennbakk and Torgersen (2003) and considers several distinct scenarios. The results are given in table 3.1. For assumptions, see Tennbakk and Torgersen (2003).

	Reference	**Low growth in demand**	**Small growth in production capacity**	**Medium CO_2-price**	**High CO_2-price**
2005	200				
2008	220	200	246	233	261
2012	223	210	273	233	260

Table 3.1: Econ's expectations on future power prices. All numbers in NOK/MWh.

The Frisch center has developed another model. For assumptions, see Dønnum, Golumbek, Jespersen and Kverndokk (2002). The results are given in table 3.2. The four alternatives; ”Business as usual” (BAU), ”Business as usual in a dry year” (BAUT), ”No new capacity in Norway” (BUP) and ”No new capacity in Norway in a dry year” (BUPT) are considered.

	BAU	**BAUT**	**BUP**	**BUPT**
2010	270	310	300	350
2020	310	330	370	550

Table 3.2: The Frisch center's expectations on future power prices. All numbers in 2002 NOK/MWh.

Finn Roar Aune in SSB has also written a report on these issues. The assumptions are given in Aune (2003) and the results are summarized in table 3.3. The alternatives are:

- A reference with no new cables abroad and modest development of gas power after 2010.
- 1 new 600 MW cable to Germany in 2005 (CABLE1)
- One additional 600 MW cable to Germany in 2008 (CABLE2)
- 1 new 1200 MW cable to England in 2005 (CABLE3)
- 6 TWh gas power accelerated to 2005 (GAS1)
- Additional 6 TWh gas power accelerated to 2008 (GAS2)

	Reference	**CABLE1**	**CABLE2**	**CABLE3**	**GAS1**	**GAS2**
2006	180	178	178	184	177	177
2008	190	189	190	198	185	180
2010	215	217	218	223	211	199
2012	250	250	250	253	247	246
2014	250	250	250	253	246	246
2016	250	251	251	253	248	248
2018	250	251	250	252	249	249
2020	250	250	250	251	249	249

Table 3.3: SSB's expectations on future power prices. All numbers in 2002 NOK/MWh.

The examples above show that there are large differences in expected prices, as a result of own expectations and which model is used. Valuations of long term contracts will therefore in the end become a matter of judgement, where one values own expectations against others.

3.3.4 Risk premium

The risk premium depends on the risk preferences of producers, retailers and speculators in the market. The market is in so called backwardation if the market is dominated by risk-averse producers who are willing to pay a premium to reduce their own risk. As a result of this, the forward price will be smaller than the expected spot price. If the market is in contango, the opposite occurs. Retailers will now pay to avoid risk, and the forward price becomes larger than the expected spot price. In the following the risk premium is defined as positive in the first case and negative in the other.

Some participants in the Nordic market often assume that consumers hedge using short-maturity contracts, while producers hedge using the long end of the curve (Mork, 2004). If this is the case the risk premium will be positive in the long run. Others (Ollmar, 2003) believe that producers rarely hedge their long term risk, and that this results in a negative risk premium in the long run.

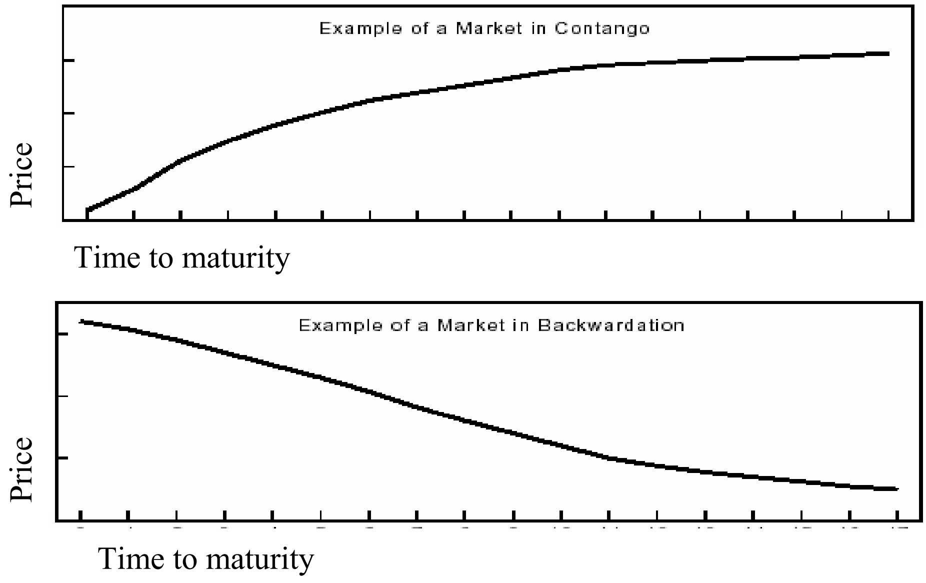

Figure 3.1: Market in contango and backwardation.

Figure 3.1 shows how the prices develop for respectively a market in contango and a market in backwardation. It shows that when the market is in contango, the long term power prices are higher than the short term prices. When the market is in backwardation, the opposite occurs.

3.3.4.1 Model for the risk premium

The relationship between the forward and expected spot price can be described as

$$F(t,T) = E_t[S_T] \cdot e^{(r-k)(T-t)} \tag{6}$$

as described in section 3.1.

(6) can also be written as

$$PREMIUM = k - r = \ln\left(\frac{E_t[S_T]}{F(t,T)}\right) \tag{23}$$

where the premium is given over *T-t* and is continuously compounded. Expected spot prices can be found by the equilibrium models given above.

3.3.4.2 Non-parametric model for the risk premium

While the theory says what to do once we have a model for the underlying variable, it gives little guidance in choosing the right model in the first place (Stanton, 1997). That the model fits historical data well is in addition no guarantee that the model will fit future data. Many models on price dynamics have been made, however empirical tests have given various results. As a result of this, non-parametric models have grown more usual. Then one avoids having to specify functional forms of μ and σ.

In the following, Stanton (1997) and Ollmar (2003) are used as a theoretical base to estimate the risk premium non-parametrically. The starting point is the following process:

$$dF_t = \mu(F_t)dt + \sigma(F_t)dZ_t \tag{24}$$

Conditional expectation $E_t[f(X_{t+\Delta}, t+\Delta)]$ can then be written by a Taylor-series

$$E_t[f(F_{t+\Delta}, t+\Delta)] = f(F_t,t) + \zeta f(F_t,t)\Delta + \frac{1}{2}\zeta^2 f(F_t,t)\Delta^2 + \ldots + \frac{1}{n!}\zeta^n f(F_t,t)\Delta^n + O(\Delta^{n+1}) \tag{25}$$

where

$$\zeta f(x,t) = \lim_{\tau \to t} \frac{E(f(F_\tau,\tau) | F_t = x) - f(x,t)}{\tau - t} = \frac{\partial f(x,t)}{\partial t} + \frac{\partial f(x,t)}{\partial x}\mu(x) + \frac{1}{2}\frac{\partial^2 f(x,t)}{\partial x^2}\sigma^2(x) \tag{26}$$

The most common use of (25) is to construct approximations to the expectations on the left hand side, given known functions for μ and σ. The intention here is quite the opposite; to estimate the expectation on the left hand side. This can then be used to construct approximations for μ and σ.

(25) can be written as

$$\zeta f(F_t,t)=\frac{1}{\Delta}E_t[f(F_{t+\Delta},t+\Delta)-f(F_t,t)]-\frac{1}{2}\zeta^2 f(F_t,t)\Delta-\frac{1}{6}\zeta^3 f(F_t,t)\Delta^2-.... \tag{27}$$

Ignoring all terms except the first on the right hand side gives:

$$\zeta f(F_t,t)=\frac{1}{\Delta}E_t[f(F_{t+\Delta},t+\Delta)-f(F_t,t)]+O(\Delta) \tag{28}$$

The approximation is uncertain in the first place, so this should be a reasonable approach.

What now is needed is a function that satisfies

$$\zeta f(x,t)=g(x,t) \tag{29}$$

The function

$$f_{(1)}(x,t)\equiv x \tag{30}$$

is therefore considered. (26) gives

$$\zeta f_{(1)}(x,t)=\mu(x) \tag{31}$$

(29) gives the following approximation of μ

$$\mu(F_t)=\frac{1}{\Delta}E_t[F_{t+\Delta}-F_t]+O(\Delta) \tag{32}$$

To find an approximation of σ

$$f_{(2)}(x,t)\equiv(x-F_t)^2 \tag{33}$$

is considered. This leads to

$$\zeta f_{(2)}(x,t)=2(x-F_t)\mu(x)+\sigma^2(x) \tag{34}$$
$$\zeta f_{(2)}(F_t,t)=\sigma^2(F_t) \tag{35}$$

Substitution into (28) gives the following approximation of σ:

$$\sigma^2(F_t)=\frac{1}{\Delta}E_t[(F_{t+\Delta}-F_t)^2]+O(\Delta) \tag{36}$$

This gives the following approximation for the risk premium

$$PREMIUM = \frac{\mu(F_t)}{\sigma(F_t)} \tag{37}$$

Since *ENO10* barely have been traded in the market the past few years, there are few data to analyze. One possibility is however to use predicted prices of *ENO10*. If there is some kind of correlation between the risk premium of contracts with shorter time to maturity and *ENO10* this can also be a possible approach.

4 Supply and demand of long term contracts

In the end the value of consumption goods depends on the supply and demand of the market participants. Producers want to sell, consumers want to buy and traders buy and sell depending on their expected profits.

To build a model based on this, one has to evaluate power producers' and retailers' net demand for forward contracts. In theory the price is set in the market cross between supply and demand, as shown in figure 4.1.

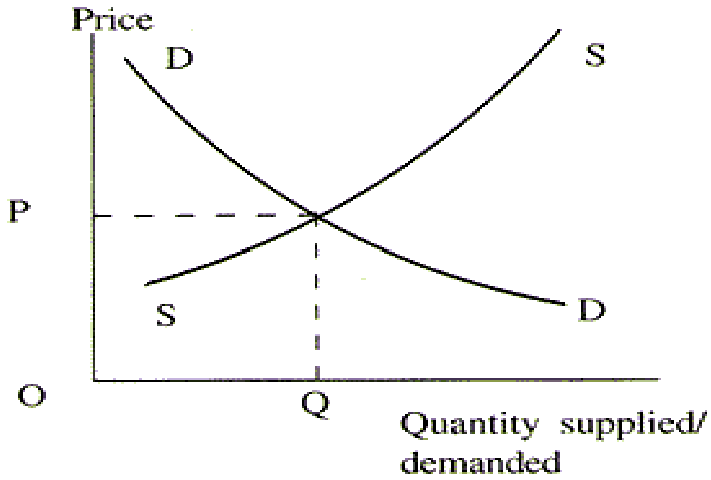

Figure 4.1: Price formation in a perfect market.

The problem is however that *ENO10* barely is traded at all. According to market participants few *ENO10*-contracts have been realized in the market the past years. As a result it is impossible to determine the price of *ENO10* directly based on supply and demand of the contract. An analysis of the market participants can however give some ideas on the preferences regarding supply and demand for long term contracts.

4.1 Market participants

It is a common assumption that the participants in the Nordic countries operate on an integrated market. However, for a large proportion of the time, the market is divided into smaller markets as a result of limitations in transfer capacities. The size of the market will therefore vary across time, and can change rapidly. In this thesis, however, the market is analysed as a whole.

4.1.1 Producers

Table 4.1 and figure 4.2 shows how the production portfolios differ between the Nordic countries. It is clear that Norway to a great extent rely on hydro power, whereas especially Sweden and Finland have more diversified portfolios.

[TWh]	**Norway**	**Sweden**	**Denmark**	**Finland**	**Total**
Hydro power	106,00	52,98	0,02	9,30	168,30
Nuclear power		65,46		21,82	87,28
Thermal power	0,9	13,48	38,17	48,65	101,20
Wind power	0,22	0,63	5,56	0,86	7,27
Total	**107,12**	**132,55**	**43,75**	**80,63**	**364,05**

Table 4.1: Electricity generation in the Nordic countries in 2003 (Nordel's annual report, 2003).

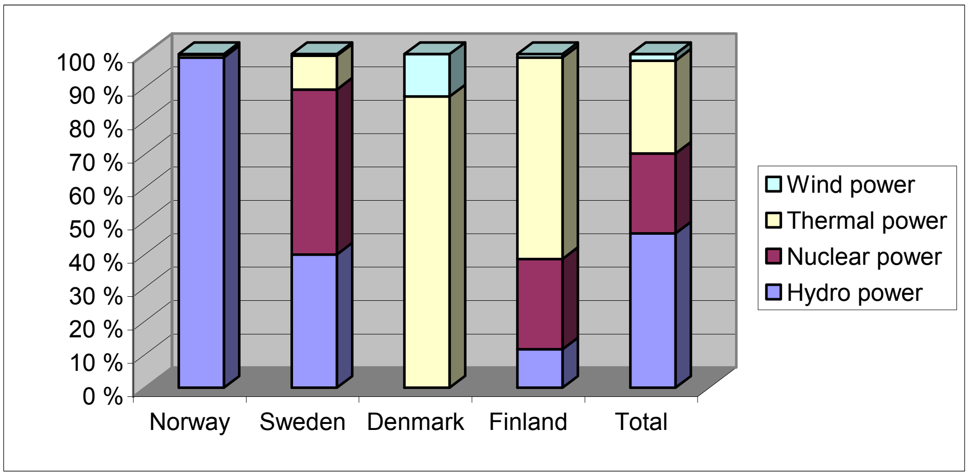

Figure 4.2: Electricity generation in the Nordic countries in 2003 (Nordel's annual report, 2003).

In the Nordic market, there are many small power producers, and a few large ones. The Swedish Vattenfall, the Finish Fortum and the Norwegian Statkraft are the largest ones. If annual production is considered, these three had a market share of 47% in 2001 (Bye et al., 2003). However, the production numbers do not take into consideration that the companies cooperate, by cross-ownership, shared ownership of power plants and shared sale- and business functions.

Table 4.2 shows the largest producers in the Nordic market in 2001 (Bye et al., 2003).

	Company	**Production [TWh]**	**Market share [%]**
1.	Vattenfall	75,2	19
2.	Fortum	60,6	16
3.	Statkraft	44,8	12
4.	Sydkraft	33,2	8
5.	Teollisuuden Voima (TVO)	15,1	4
6.	Elsam	14,6	4
7.	Energi E2	11,8	3
8.	E-CO	10,2	3
9.	Norsk Hydro	9,8	3
10.	Phjolan Voima (PVO)	8,0	2
11.	BKK	8,0	2
12.	Agder Energi	7,9	2
13.	Lyse Energi	5,9	2
14.	Helsingin Energi	5,4	1
15.	Vannkraft Øst	4,9	1
	15 largest power producers	**315,4**	**81**
	Total Nordic market	**388,0**	**100**

Table 4.2: The largest power producers in the Nordic market in 2001.

The market structure of producers is relatively clear. There are certainly a large number of producers, but they are well-established and the market structure changes slowly (Bye et al., 2003).

The four largest producers cover approximately 55% of the market when production in 2001 is considered. The distribution in figure 4.3 is based on numbers published on the different participants' home pages.

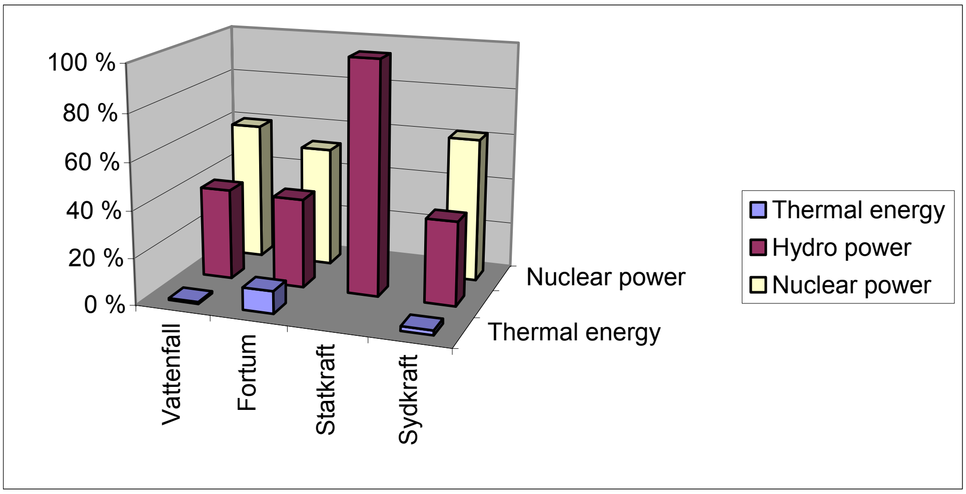

Figure 4.3: The production portfolio of the four largest electricity producers in the Nordic countries.

From the figure it is obvious that Statkraft has a less diversified production portfolio than the other three. Nearly all of Statkraft's production comes from hydro power. Vattenfall and Sydkraft's portfolios are quite similar to each other with the cornerstones being hydro and nuclear power. Fortums's production portfolio is also well diversified, and their portfolio includes a larger part of thermal energy.

4.1.2 Consumers

On the demand side, the participants are not as dominating as the case is on the supply side. However, there are some very large consumers. In Norway the three largest consumers are Hydro, Elkem and Norske Skog (Stortingsmelding 29, 1998-1999). They cover a large part of the electricity consumption of the industry in Norway. The consumers of electricity can be divided into three groups; households, service industry and industry. The distribution for the Nordic countries is given in table 4.3 and figure 4.4.

[TWh]	**Norway**	**Sweden**	**Denmark**	**Finland**	**Total**
Households	35,22	41,86	9,60	20,45	107,13
Service (incl. transport)	22,36	25,93	10,82	15,34	74,45
Industry	43,61	59,24	9,55	45,21	157,61
Other (incl. agriculture)	1,70	6,97	2,78	0,86	12,31
Total	**102,89**	**134,00**	**32,75**	**81,85**	**351,49**

Table 4.3: Net electricity consumption in the Nordic countries in 2003 (Nordel's annual report, 2003).

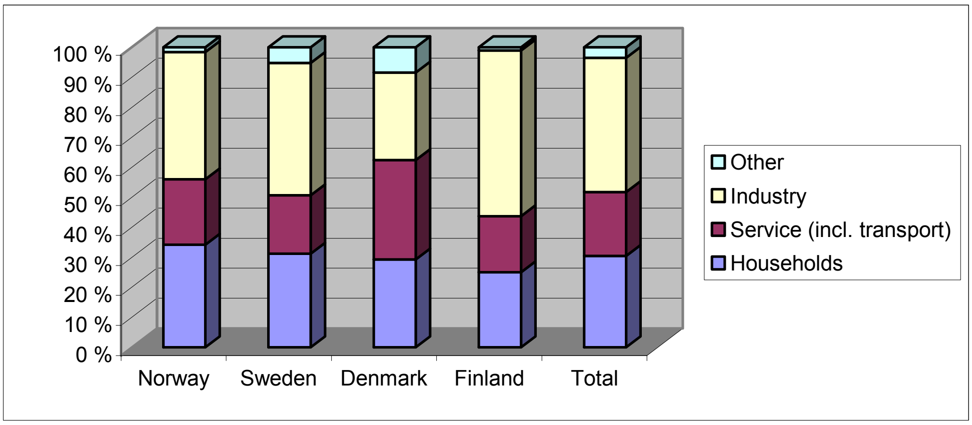

Figure 4.4: Distribution of electricity consumption in the Nordic countries in 2003 (Nordel's annual report, 2003).

Norway and Sweden have quite similar distributions, since electricity is used for many of the same purposes (Botterud et al., 2002). In Finland a much larger part of the electricity is used in industrial companies, while in Denmark a smaller part is used for these purposes.

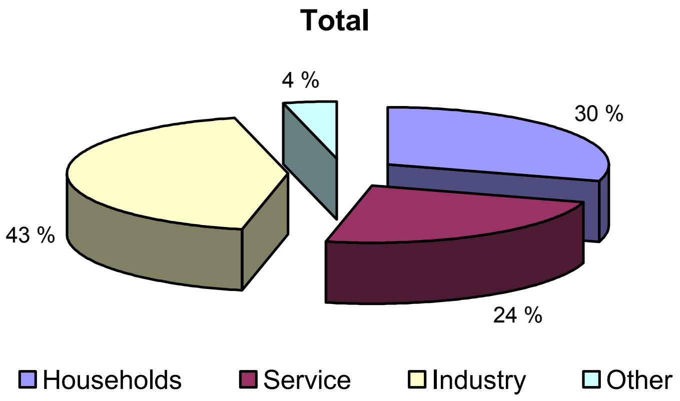

Figure 4.5: Aggregated electricity consumption in 2003 (Nordel's annual report, 2003).

As is shown above, the industry stands for 43% of the total electricity consumption in the Nordic countries, while households and service industry constitutes 54%. These last two groups buy their electricity from retailers, while industrial companies generally buy their electricity in the wholesale market. Some of the smaller industrial companies however buy their electricity from retailers. Based on this it is reasonable to assume that retailers cover 50-60% of the physical market for electricity, whereas the industrial companies cover approximately 40-50%.

4.1.3 Demand for long term contracts

Power producers, large consumers and retailers trade in the wholesale market. The trading either takes place bilaterally or through the spot market at the Nordic power exchange, Nord Pool. It is estimated that about 70% of the trade in the wholesale market takes place bilaterally, while 30% go through the spot market at Nord Pool (Stortingsmelding 15, 2004-2005). As mentioned earlier only contracts with 3 years to maturity are traded at Nord Pool, while contracts with longer time to maturity can be traded bilaterally.

4.1.3.1 Producers

Vattenfall, Fortum and Sydkraft have quite similar production portfolios. Nuclear power represents the largest part; while hydro power represents the second largest part. It is therefore reasonable that these companies have other preferences regarding long term contracts than for instance Statkraft.

Statkraft experiences more uncertainty concerning future conditions, since nearly all of their production comes from hydro power. There exists a high volume risk, since future rain falls are hard to predict. Producers with only hydro power, such as Statkraft, are therefore probably more reluctant to enter into long term contracts than producers with more diversified portfolios. Such producers, including Vattenfall, Fortum and Sydkraft, can therefore sell more of their production forward. The uncertainty regarding the supply to be used in the nuclear and thermal power plants is much smaller than the uncertainty regarding hydrologic conditions.

As is shown earlier, Norway relies on hydro power, while the other Nordic countries have more diversified portfolios. Producers in Norway may therefore be more reluctant when it comes to entering into long term contracts than producers in the other Nordic countries. However, size does also matter. Larger companies, such as Statkraft, have higher safety margins than smaller companies. Their production facilities are spread over a larger area than the facilities of many of the small companies, and they therefore benefit from different weather conditions across the country. This advantage results in that large companies, such as Statkraft, have higher incentives to enter into long term contracts than smaller companies.

4.1.3.2 Consumers

Producers can control parts of the generation by storing water in the reservoirs. The situation on the demand side is different. Consumers therefore wish to lock in the prices of future expected demand, given that they are risk-averse. Next the preferences of industrial companies and retailers are considered.

A good deal of the retailers' demand can be predicted relatively certain. To avoid price risk, long term contracts are therefore requested. However, before the liberalisation of the market in 1991, every end-user had to buy their electricity from the local retailer. After the liberalisation, this is no longer a demand. The end-users can now freely choose which retailer to buy their electricity from. As a result, the retailers will now face a higher volatility regarding future demand. The uncertainty regarding how much they will sell to end-users in the future is higher than before, and as a result it is reasonable that the retailers are more reluctant when it comes to entering into long term contracts than they used to be.

The industry probably has higher incentives for entering into long term contracts than the retailers. They can, as opposed to the retailers, influence their own consumption and thereby their demand for electricity. Assuming that they are risk-averse, they want to reduce price risk. In addition, they want to be sure that there is a sufficient amount of electricity available for them in the future. Actually, in 1997, about half of the industry's power demand was covered by long term contracts. These were however contracts where the terms were set by the Norwegian Parliament (OED, 1997) and not by market principles. Few long term contracts have been traded in the market the past years. After the high prices in the winter

2002/2003, the prices of the long term contracts that have been offered by Statkraft and Vattenfall in the market have been too expensive for many industry participants. This is especially a problem for the smaller industry companies. As a result, few new market based contracts have been entered into. With lower prices, more long term contracts would be entered into. These thoughts are in accordance with beliefs in the market, among others by PIL[7].

4.2 Pricing approach

A next step may be to try to estimate future supply and demand based on the analysis of the market participants. One such model is presented in chapter 3.3. Another model is developed by Bessembinder and Lemmon (2000). They present an equilibrium model of forward markets that applies when prices are determined by industry participants, and not speculators. Power producers' and retailers' demands for forward contracts are examined, and solutions for the equilibrium forward power price and optimal forward positions are found. The equilibrium power price will then depend on expected market demand and on demand volatility. In general the forward prices decrease when expected power demand is low and demand volatility is modest. When expected demand is high or demand volatility is higher, the forward prices increase.

Finding a proper size of these factors requires a more thorough analysis of the participants in the market than the one performed in chapter 4.1. One also has to make considerations regarding future market conditions. There is a great deal of uncertainty connected with this approach. In this report another approach is preferred. Nevertheless, this approach indirectly takes the expectations on future supply and demand into consideration, since the model includes the price of *ENOYR2* and *ENOYR3*. The price of *ENO10* depends on these prices, which are set in the market cross between supply and demand. This approach is presented in the following chapters.

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[7] Prosessindustriens landsforening (PIL). See reference list.

5 Hypotheses

Based on expectations and earlier introduced arguments, some hypotheses on the relationship between *ENO10* and different factors are presented. Arguments for each of the hypotheses are also given.

5.1 Hypothesis 1

"The difference in the prices of ENOYR2 and ENOYR3 traded at Nord Pool represents the growth in the forward curve and can be used to price ENO10."

ENOYR2 and *ENOYR3* are relatively liquid contracts traded at Nord Pool, whereas *ENO10* barely is traded at all. Hypothesis 2 is based on a belief that the difference between *ENOYR2* and *ENOYR3* would reflect the growth in the forward curve in a reasonable way.

If this is the case, the forward curve would simply become an extension of the curve crossing through the price of *ENOYR2* and *ENOYR3*.

5.2 Hypothesis 2

"ENOYR2 and ENOYR3 are not correlated with today's hydrologic balance[8]*. The hydrologic balance is therefore of minor importance in the valuation of long term contracts."*

The hydrologic balance is of great importance in pricing short term contracts. This is shown by different authors, among others Gjolberg and Johnsen (2001). It is therefore likely that there is a relation between *ENOYR1* and the hydrologic balance. The time to maturity for this contract is so short (especially in the last months of the year) that it is a reasonable assumption that the hydrologic balance will influence the price of this contract. Prices 2-3 years into the future however experience greater uncertainty regarding future conditions. Time to maturity on *ENOYR2* and *ENOYR3* is long and it is therefore reasonable that these contracts are less correlated with the hydrologic balance.

Since the price of *ENOYR1* influences the price of *ENO10* to some degree, the hypothesis emphasizes that the hydrologic balance is of "minor importance" in pricing long term contracts. This means that the hydrologic balance only influences the short end of the forward curve.

5.3 Hypothesis 3

Hypothesis 3 depends to a certain degree on the results achieved when analysing hypothesis 2. If hypothesis 2 is rejected, the hypothesis is more uncertain.

"The price level of ENOYR3 does not influence the growth of the forward curve."

---

[8] See footnote 3 on page 1.

Assuming that the price of *ENOYR3* is relatively independent of short term effects such as the hydrologic balance, hypothesis 3 is proposed. In that case, the price of *ENOYR3* reflects many of the future expectations in the economy. The growth function should then just reflect the expected change in the price level, and not be influenced by short term conditions.

If hypothesis 2 is rejected, the opposite is more likely to occur. Then the growth of the forward curve should be adjusted for the price level of *ENOYR3*. If the price of *ENOYR3* is higher than normal, the growth in the forward curve should probably be smaller to adjust for that. The opposite should happen when the price is lower than normal.

5.4 Hypothesis 4

"Increasing oil forward prices leads to increasing power forward prices and vice versa."

This hypothesis is based on a belief that the prices of different energy carriers influence each other. This is reasonable because the different energy carriers are substitutes in such a way that they can be used for the same purposes and therefore are demanded by many of the same participants. If the price of one energy carrier increases, it is therefore likely that the price of the other also increases.

5.5 Hypothesis 5

"The level of the interest rates on long term government bonds influences the long term power prices. Increasing interest rates results in decreasing forward prices"

Today's hydrologic balance is probably of minor importance for prices several years into the future. The amount of water in the reservoirs today will mostly influence prices in the short run. Prices on the longer view are likely to depend on factors that to a larger extent reflect the state of the economy. Interest rates are examples of such factors. Different interest rates may be important, both the rate of return on 3-, 5- and 10-year government bonds.

When the rate of return increases, the present value of a future commitment decreases. When the present value decreases, the price is likely to do the same.

5.6 Hypothesis 6

"The prices of long term contracts depend to a high degree on the prices of CO_2-quotas. Increasing CO_2-prices results in higher power prices."

The price of the CO_2-quotas will probably influence the power price. Much of the power production in Europe is based on fossil fuels. For instance it is estimated that in Denmark a quote-price of 150NOK/ton CO_2 will increase the marginal production costs of the fossil power production with about 120DKK/MWh (Løvdal, 2004). The numbers for other countries with fossil power production will probably increase similarly. Such quotas can in addition stimulate new development of renewable energy. Today it is more expensive to produce energy from renewable energy sources than from fossil fuels. Together, these effects will presumably result in higher power prices.

6 Empirical analyses

6.1 Assumptions

To develop the model, price estimates on *ENO10* from Trønder-Energi from week 48 in 2000 to week 33 in 2002 are used. These are not realized contracts, but predicted prices. In addition the weekly forward prices of *ENOYR2* and *ENOYR3* are used. Time series plots of the prices, differences between them and a scatter plot of the price of *ENO10* versus the price of *ENOYR3* is given in appendix 1.

A first assumption is that the forward curve should cross through the price of *ENOYR3*. This is the contract with longest time to maturity and should therefore be the one that says the most about future conditions. The second assumption is that the net present value of *ENO10* should equal the net present value of the corresponding year contracts. By using these two assumptions the growth of the forward curve, subsequently called *b*, can be found.

Another important issue is that the data in the analysis period is old. The equations estimated in this chapter are therefore not directly transferable to today's situation. This is especially a result of that the price level changes over time. However, even though some of the equations are doubtful for today's situation, the connections that are found will more probably be relevant also in the future.

6.2 Model

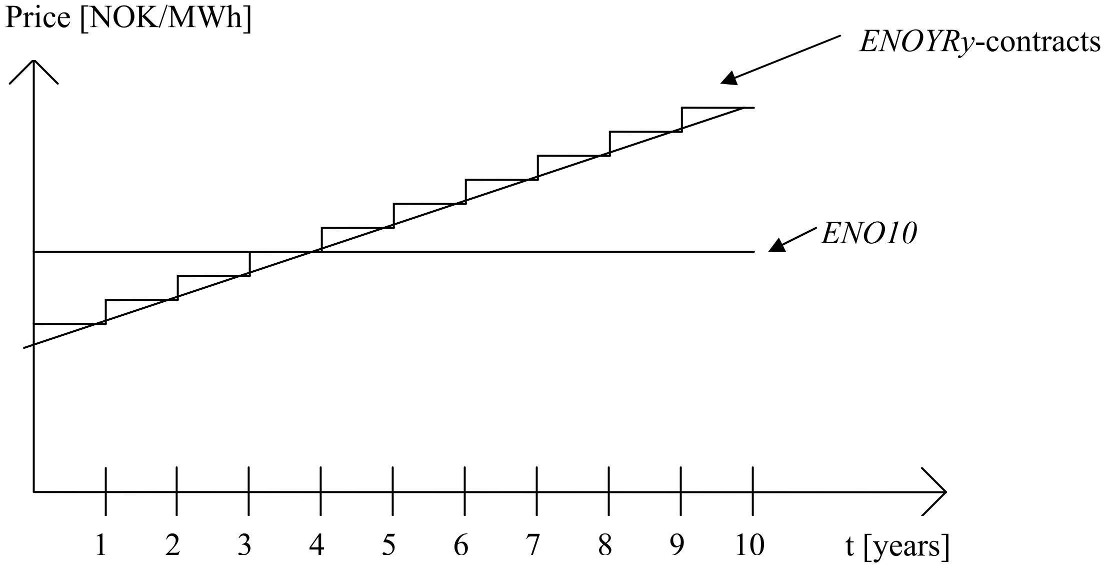

Figure 6.1: Straight-line approximation for *ENOYRy*-contracts.

Figure 6.1 shows the flat price of *ENO10* and an assumed pattern of the prices of the *ENOYRy*-contracts. The straight line is an approximation to the stepwise function that illustrates the prices of the *ENOYRy*-contracts.

For simplicity, in this chapter it is assumed that the contract period starts immediately. This is necessary to calculate some values for *b* that can be used in the analyses. In later sections it is also taken into consideration that this generally not is the case.

The net present value of *ENO10* when the contract period starts immediately is given by

$$\int_0^{10} p_{10} e^{-r_{10} t} dt \tag{38}$$

The net present value of the corresponding year contracts can be given by

$$\int_0^1 ENOYR1 e^{-r_{10} t} dt + \int_1^2 ENOYR2 e^{-r_{10} t} dt + \int_2^3 ENOYR3 e^{-r_{10} t} dt + \int_3^{10} (a+bt) e^{-r_{10} t} dt \tag{39}$$

This follows from the assumption that the stepwise function in figure 6.1 is approximated with a straight line:

$$y = a + bt \tag{40}$$

a is given by *b* and *ENOYR3*, since the line should go through the price of *ENOYR3*. *a* is therefore given by

$$a = ENOYR3 - 3b \tag{41}$$

Equating (38) and (39) and using (41) gives an expression for *b*. Calculations and assumptions are shown in appendix 2. *b* is then solved on each data point. Regression analyses can then be performed with *b* as the dependent variable.

6.3 Evaluation of hypothesis 1

"The difference in the prices of ENOYR2 and ENOYR3 traded at Nord Pool represents the growth in the forward curve and can be used to price ENO10."

To test this hypothesis, the relationship between *b* and *(ENOYR3-ENOYR2)* has to be studied. *b* is now, and in the chapters that follow, given in NOK/(MWh year) and the prices of *ENOYR2* and *ENOYR3*[9] are given in NOK/MWh. A regression analysis on *b* versus this growth gives the results in table 6.1.

---

[9] *ENOYR2* and *ENOYR3* are given as weekly prices, which mean that they are computed as averages of the daily prices the corresponding week. This is done so that the prices will match the other available data that generally are given per week. *ENOYR-06* and *ENOYR-07* are given in EURO. These prices are converted to NOK by multiplying the price with the spot exchange rate, which January 2005 was 8,21. This give minor errors, since the EURO rate of return is quite similar to the rate of return in NOK (See chapter 7.5 for an ideal approach).

```
Regression Analysis: b versus (ENOYR3-ENOYR2)

The regression equation is
b = 4,35 + 0,357 (ENOYR3-ENOYR2)


88 cases used, 1 cases contain missing values


Predictor          Coef   SE Coef      T       P
Constant         4,3531    0,6393   6,81   0,000
(ENOYR3-ENOYR2)0,3566    0,1116   3,20   0,002


S = 1,62066   R-Sq = 10,6%   R-Sq(adj) = 9,6%


Analysis of Variance

Source           DF        SS       MS       F       P
Regression        1    26,831   26,831   10,22   0,002
Residual Error   86   225,881    2,627
Total            87   252,712
```

Table 6.1: Regression analysis of *b* as a function of (*ENOYR3-ENOYR2).*

The table shows that there is a correlation between *b* and *(ENOYR3-ENOYR2)*. This is shown by R-Sq(adj)[10] and the P-values. The graphs in figure 6.2 indicate the same thing. In addition, the Pearson correlation coefficient[11] is 0,326, something that indicates an existing correlation.

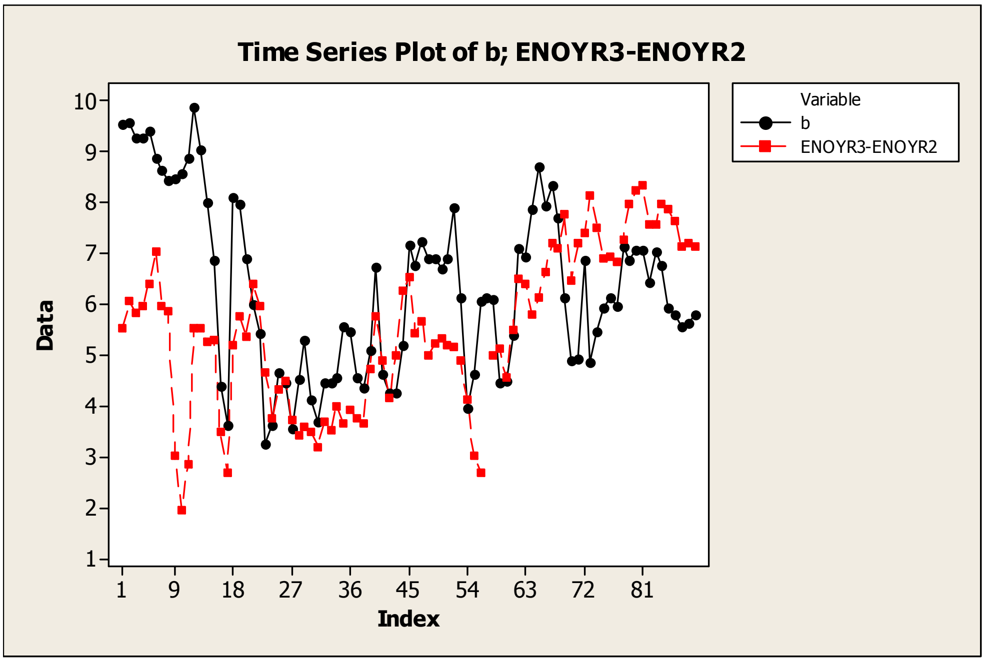

Figure 6.2: Time series plot of *b* versus (*ENOYR3-ENOYR2).*

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[10] R-Sq(adj) is the coefficient of determination and indicates how much variation in the response that is explained by the model. The higher the R-Sq(adj), the better the model fits the data. It accounts for the number of predictors in the model, and is therefore useful for comparing models with different numbers of predictors. See e.g Walpole, Myers and Myers (1998) for more.

[11] Pearson correlation coefficient measures the degree of linear relationship between two variables. The correlation coefficient assumes a value between -1 and +1. If one variable tends to increase as the other decreases, the correlation coefficient is negative. Conversely, if the two variables tend to increase together, the correlation coefficient is positive. The closer the coefficient is to -1 or 1 the higher is the correlation.

When *b* is tested versus the percentage growth the results in table 6.2 and figure 6.3 appear. This correlation is even higher:

```
Regression Analysis: b versus (ENOYR3-ENOYR2)/ENOYR2

The regression equation is
b = 3,69 + 78,2 (ENOYR3-ENOYR2)/ENOYR2


88 cases used, 1 cases contain missing values


Predictor                  Coef  SE Coef     T      P
Constant                 3,6894   0,5617  6,57  0,000
(ENOYR3-ENOYR2)/ENOYR2    78,17    15,98  4,89  0,000


S = 1,51623   R-Sq = 21,8%   R-Sq(adj) = 20,9%


Analysis of Variance

Source          DF       SS      MS      F      P
Regression       1   55,001  55,001  23,92  0,000
Residual Error  86  197,711   2,299
Total           87  252,712
```

Table 6.2: Regression analysis of *b* as a function of *(ENOYR3-ENOYR2)/ENOYR2.*

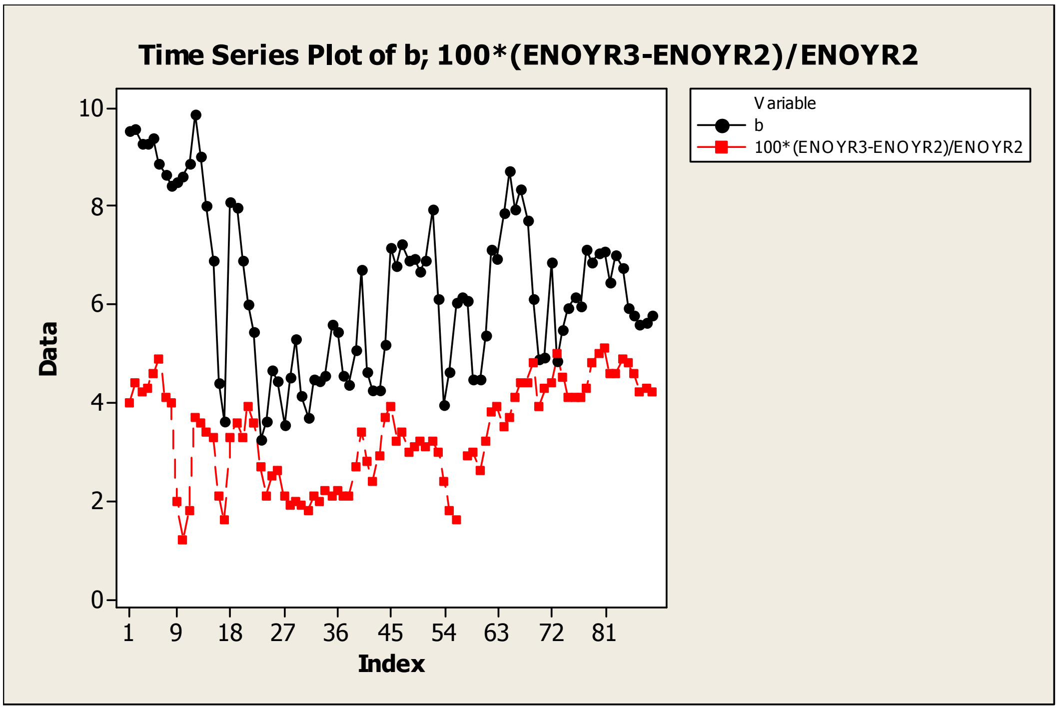


Figure 6.3: Time series plot of *b* versus 100(ENOYR3-ENOYR2)/ENOYR2.***

The Pearson correlation coefficient is now 0,467, which indicates an even better correlation.

The hypothesis is therefore reasonable and it is possible that the growth between the prices of *ENOYR2* and *ENOYR3* in percentages could be a variable in a regression equation for *b*.

6.4 Evaluation of hypothesis 2

“ENOYR2 and ENOYR3 are not correlated with today’s hydrologic balance. The hydrologic balance is therefore of minor importance in the valuation of long term contracts.”

Historical prices on *ENOYR2* and *ENOYR3* should be tested against the level of the hydrologic balance. Data on the hydrologic balance is available in the period 2000 to 2004. The data are from Trønder-Energi. The results of the regression analysis are given in table 6.3. The hydrologic balance is given in GWh.

Regression Analysis: ENOYR2 versus Hydrologic balance

```
The regression equation is
ENOYR2 = 165 - 0,00180 Hydrologic balance


258 cases used, 107 cases contain missing values


Predictor                       Coef    SE Coef       T      P
Constant                     165,271      1,934   85,45  0,000
Hydrologic balance        -0,0017961  0,0001099  -16,34  0,000


S = 24,8385   R-Sq = 51,0%   R-Sq(adj) = 50,8%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  164639  164639  266,86  0,000
Residual Error  256  157940     617
Total           257  322579
```

Regression Analysis: ENOYR3 versus Hydrologic balance

```
The regression equation is
ENOYR3 = 171 - 0,00151 Hydrologic balance


257 cases used, 108 cases contain missing values


Predictor                       Coef    SE Coef       T      P
Constant                     170,768      1,957   87,26  0,000
Hydrologic balance        -0,0015127  0,0001110  -13,62  0,000


S = 25,0422   R-Sq = 42,1%   R-Sq(adj) = 41,9%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  116384  116384  185,59  0,000
Residual Error  255  159914     627
Total           256  276297
```

Table 6.3: Regression analysis of the prices of *ENOYR2* and *ENOYR3* as functions of the hydrologic balance.

Pearson's correlation coefficient between the hydrologic balance and respectively *ENOYR2* and *ENOYR3* is -0,714 and -0,649, indicating a negative, but high, correlation. When the hydrologic balance decreases, the prices therefore increase. The P-values are close to zero, something that indicates that the results are significant. There is therefore some evidence that both *ENOYR2* and *ENOYR3* actually are correlated with the hydrologic balance. *ENOYR2* seems to be slightly more dependent on the hydrologic balance than *ENOYR3*. There is therefore reason to believe that the hypothesis is false.

These results indicate that the hydrologic balance may be a factor that should be considered in the valuation of *ENO10*. The hydrologic balance should therefore be tested as a variable in a regression analysis for *b*.

6.5 Evaluation of hypothesis 3

"The price level of ENOYR3 does not influence the growth of the forward curve."

Since there are indications that hypothesis two is false, hypothesis 3 is more questionable. However, the hypothesis is tested by testing the expected growth of the forward curve, *b*, against the price of *ENOYR3*. As table 6.4 shows, there is a strong correlation.

Regression Analysis: b versus ENOYR3

```
The regression equation is
b = 31,5 - 0,147 ENOYR3


88 cases used, 1 cases contain missing values


Predictor       Coef  SE Coef       T      P
Constant      31,540    1,817   17,36  0,000
ENOYR3      -0,14734  0,01060  -13,90  0,000


S = 0,951300   R-Sq = 69,2%   R-Sq(adj) = 68,8%


Analysis of Variance

Source          DF      SS      MS       F      P
Regression       1  174,88  174,88  193,25  0,000
Residual Error  86   77,83    0,90
Total           87  252,71
```

Table 6.4: Regression analysis of *b* as a function of the price of *ENOYR3*.

Table 6.4 suggests that when the price of *ENOYR3* increases, *b* decreases. The price level of *ENOYR3* therefore becomes important in the estimation of *b*. This is probably a result of that the price of *ENOYR3* is influenced by short term effects, such as the hydrologic balance. When the price of *ENOYR3* is high compared to the average price level, *b* should decrease as a result. When the price is low, b should be higher. This probably happens because *b* has to adjust for that the price of *ENOYR3* not correctly represents the price level in the future. Hypothesis 3 is therefore doubtful, and the price of *ENOYR3* should therefore be tested as a regression variable in the regression equation for *b*.

6.6 Evaluation of hypothesis 4

"Increasing oil forward prices lead to increasing power forward prices and vice versa."

In the evaluation of this hypothesis a regression analysis of *ENOYR3* versus oil forward prices is performed. Different oil forwards are used. The data are from EcoWin, and both Brent Crude Future Positions[12] 1 and 18 are considered. The oil prices are given in USD/bbl and the results are given in table 6.5.

Regression Analysis: ENOYR3 versus Oil price (1pos)

```
The regression equation is
ENOYR3 = 107 + 2,75 Oil price (1pos)


322 cases used, 43 cases contain missing values


Predictor            Coef  SE Coef      T      P
Constant          106,918    5,214  20,51  0,000
Oil price (1pos)   2,7514   0,1889  14,56  0,000


S = 25,3314   R-Sq = 39,9%   R-Sq(adj) = 39,7%


Analysis of Variance

Source          DF      SS      MS       F      P
Regression       1  136116  136116  212,12  0,000
Residual Error 320  205338     642
Total          321  341453
```

Regression Analysis: ENOYR3 versus Oil price (18pos)

```
The regression equation is
ENOYR3 = 87,1 + 4,57 Oil price (18pos)


196 cases used, 169 cases contain missing values


Predictor            Coef  SE Coef      T      P
Constant           87,080    5,756  15,13  0,000
Oil price (18pos)  4,5682   0,2301  19,85  0,000


S = 15,9063   R-Sq = 67,0%   R-Sq(adj) = 66,8%


Analysis of Variance

Source          DF      SS     MS       F      P
Regression       1   99707  99707  394,08  0,000
Residual Error 194   49084    253
Total          195  148790
```

Table 6.5: Regression analysis of the price of *ENOYR3* as a function of oil forward prices.

---

[12] 1st position here indicates a forward contract sold today with delivery period next month. 18th position is a forward contract sold today with delivery period 2,5-3 years from now.

The table shows that there is a high correlation between *ENOYR3* and both the future positions. Especially between *ENOYR3* and the 18th position the correlation is high. The regression equation indicates that the price of *ENOYR3* increases when the oil price increases. This is in accordance with the hypothesis. Since there is a high correlation between the long term prices three years into the future, it is possible that there is a relation even further in the future.

6.7 Evaluation of hypothesis 5

"The level of the interest rates on long term government bonds influences the long term power prices. Increasing interest rates results in decreasing forward prices"

To evaluate this hypothesis data on *ENOYR3* against data on r_3, the rate of return on 3-year government bonds is tested. These data go back to 1998 and should therefore give credible results. The rate of return is given as a number between 0 and 1.

Regression Analysis: ENOYR3 versus Rate of return (3-year gov)

```
The regression equation is
ENOYR3 = 285 - 1948 Rate of return (3-year gov)


322 cases used, 43 cases contain missing values


Predictor                       Coef  SE Coef       T      P
Constant                     284,619    4,089   69,61  0,000
Rate of return (3-year gov)  -1947,76    73,72  -26,42  0,000


S = 18,3141   R-Sq = 68,6%   R-Sq(adj) = 68,5%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  234124  234124  698,03  0,000
Residual Error  320  107329     335
Total           321  341453
```

Table 6.6: Regression analysis of the price of *ENOYR3* as a function of rate of return on 3-year government bonds.

The table above considers the forward prices as a linear function of the interest rate. Other functions of the rate of return explain the data even better. In table 6.7 the dependence of other functions of the interest rate is examined.

Function of r_3	**R-Sq(adj)**	
	ENOYR2	***ENOYR3***
r_3	65%	68,5%
r_3^2	58,6%	62,5%
$Ln(r_3)$	69,6%	72,7%
$1/r_3$	71,2%	74,3%

Table 6.7: R-Sq(adj) of the price of *ENOYR3* as functions of r_3.

Using *1/r₃* as the explanatory variable, over 74% of the variation in the data is explained, considering *ENOYR3*. The regression analyses are shown in appendix 3.

Table 6.7 shows that there is a high correlation between r_3 and the price of *ENOYR3*. In addition, table 6.6 shows that the price of *ENOYR3* is expected to fall when the rate of return increases. This is in accordance with the hypothesis. The analysis says on the other hand nothing about this relation when time to maturity grows further. Nevertheless, since there is a relation between r_3 and the price of *ENOYR3* it is possible that a similar relation will exist between r_{10}, the rate of return on 10-year government bonds, and the price of *ENO10.* There are however no evidence that this is true.

6.8 Evaluation of hypothesis 6

"The prices of long term contracts depend to a high degree on the prices of CO_2-quotas. Increasing CO_2-prices results in higher power prices."

The market for trading CO_2-quotas opened January 2005. Since then the prices of such quotas have rapidly increased. This is shown in figure 6.4. According to market participants, the prices of *ENOYR-06* and *ENOYR-07* have also increased as a result of this. The price development of these contracts after the opening of the market is shown in figure 6.5.

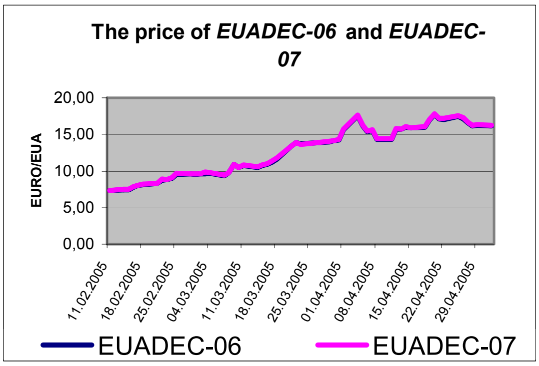


Figure 6.4: EUA Closing values[13]

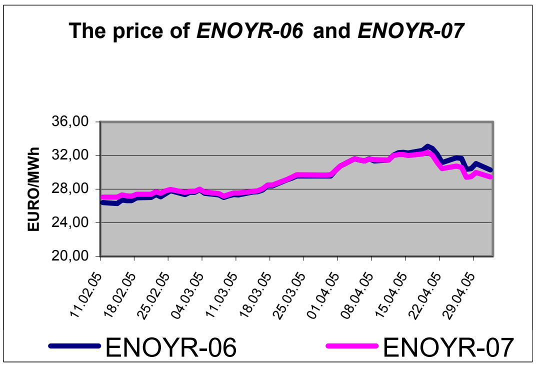


Figure 6.5: Development of forward prices

Comparing figure 6.4 and figure 6.5, there seems to be some kind of correlation. Performing a regression analysis with *ENOYR-06* and *ENOYR-07* as the dependent variables and *EUADEC-06* and *EUADEC-07* as regression variables gives the results of table 6.8. *EURO-06* and *EURO-07* is here given in EURO/MWh whereas *EUADEC-06* and *EUADEC-07* is given in EURO/EUA.

---

[13] 1 EUA = 1 ton CO_2.

```
Regression Analysis: ENOYR-06 versus EUADEC-06

The regression equation is
ENOYR-06 = 21,8 + 0,596 EUADEC-06


Predictor     Coef  SE Coef      T      P
Constant   21,7709   0,3469  62,76  0,000
EUADEC-06  0,59609  0,02643  22,56  0,000


S = 0,657654   R-Sq = 90,7%   R-Sq(adj) = 90,5%


Analysis of Variance

Source          DF      SS      MS       F      P
Regression       1  220,06  220,06  508,80  0,000
Residual Error  52   22,49    0,43
Total           53  242,55


Regression Analysis: ENOYR-07 versus EUADEC-07

The regression equation is
ENOYR-07 = 23,1 + 0,482 EUADEC-07


Predictor     Coef  SE Coef      T      P
Constant   23,1427   0,4098  56,47  0,000
EUADEC-07  0,48163  0,03104  15,52  0,000


S = 0,773082   R-Sq = 82,2%   R-Sq(adj) = 81,9%

Analysis of Variance

Source          DF      SS      MS       F      P
Regression       1  143,87  143,87  240,73  0,000
Residual Error  52   31,08    0,60
Total           53  174,95
```

Table 6.8: CO_2-prices' influence on power prices.

As the table shows there is a high correlation between power prices and prices on CO_2-quotas. Even though the results are based on very limited data, there is reason to believe that there is a correlation.

The reason for this impact on the price of electricity is reasonable. All producers of electricity based on fossil fuels may face additional costs of purchasing emission permits. In order to stay within their specified quota, they are faced with higher marginal cost of power production because they will either have to change the production or buy permits to cover their emissions. Thus the allocation of permits for the electricity sector and the prices of the CO_2-quotas will influence the price.

Chapter 6 has given some indications on which variables that might be relevant in a regression equation for *b*. In chapter 7, these insights are used to develop such an equation.

7 Recommended model

A good model is both easy to interpret and easy to use. It also explains much of the variation in the historical data. Based on the calculation of the formula for b and examination of the hypotheses in chapter 6, a model can be created.

An analysis across different factors with b as the dependent variable is performed in this chapter. These variables are based on the hypotheses in the previous chapter. Both plain regression models and autoregressive models are considered. The best models are chosen based on the P-values of the included variables and the R-Sq(adj) of the total model. The strength of the models is then tested in different ways.

7.1 Analysing b

7.1.1 Regression analysis

b, the growth of the forward curve, may depend on different factors. Some possible variables are listed below. These are based on the evaluation of the hypotheses of chapter 6. In addition, the spot price, S, and the prices of *ENOYR1* and *ENOYR2* are tested.

1. Spot price, S
2. The price of *ENOYR1*
3. The price of *ENOYR2*
4. The price of *ENOYR3*
5. The growth between the price of *ENOYR2* and *ENOYR3* in NOK/MWh, g_t
6. The growth between the price of *ENOYR2* and *ENOYR3* in %, g_p
7. The rate of return on 3-, 5- and 10-year government bonds, r_3, r_5, r_{10}
8. Oil forward price, p_{oil}
9. Hydrologic balance, h_{bal}

In table 7.1 the linear dependency between *b* and these variables are tested.

Regressor variables	**R-Sq(adj)**
S	Insignificant[14]
ENOYR1	70,4%
ENOYR2	72,3%
ENOYR3	68,8%
g_t	9,6%
g_p	20,9%
r_3	Insignificant
r_5	10,1%
r_{10}	17,5%
p_{oil} *(1pos)*	7,9%
p_{oil}*(18pos)*	Insignificant[15]
h_{bal}	12,4%

Table 7.1: Linear dependency between *b* and different factors.

The correlation between *b* and the price of both *ENOYR1*, *ENOYR2* and *ENOYR3* is high. This indicates that also the price of *ENOYR1* and *ENOYR2* could be relevant variables in pricing procedures. Table 7.2 shows the correlation between the prices of *ENOYR1*, *ENOYR2* and *ENOYR3*.

Correlations: ENOYR1; ENOYR2; ENOYR3

```
        ENOYR1  ENOYR2
ENOYR2   0,920
         0,000

ENOYR3   0,883   0,993
         0,000   0,000


Cell Contents: Pearson correlation
               P-value
```

Table 7.2: Correlations between *ENOYR1*, *ENOYR2* and *ENOYR3*.

Because of this correlation, including all these three variables in a regression equation gives insignificant results. Even though both *ENOYR1* and *ENOYR2* have a higher R-Sq(adj) than *ENOYR3*, it is probably more reasonable to include *ENOYR3* in the model. This contract is further from maturity and would therefore better reflect the future state of the economy. The difference in R-Sq(adj) can be occasional. In the regressions the price of *ENOYR3* is therefore chosen as the one of these three variables to be considered.

The best model when different functions of the variables above is considered, as R-Sq(adj) regards, is given by

$$b = 28{,}3 - 0{,}135 \cdot ENOYR3 + 33{,}1 \cdot g_p \tag{42}$$

---

[14] A significance level of 5% is chosen.

[15] Even though the evaluation of hypothesis 3 seemed to suggest that the oil forward prices with the longest time to maturity were more correlated with the long term power prices than the oil forward prices with shorter time to maturity, *b* correlates more with p_{oil}*(1pos)* than p_{oil}*(18pos)*. Probably *b* would correlate more with the price of an oil forward contract with longer time to maturity.

where $g_p = \dfrac{ENOYR3 - ENOYR2}{ENOYR2}$

The stepwise regression[16] choosing this model is given in appendix 4. An analysis of variances and relevant P-values is also given here. This model fits the data quite well, R-Sq(adj) is 72,0%. Figure 7.1 shows the residual plots for *b*. The residuals are relatively normally distributed and there is no trend in the data whatsoever. The error variance can therefore be considered homogenous. This is also supported by the P-value of the Anderson Darling test[17] (the AD-value in the figure). The model is therefore reliable.

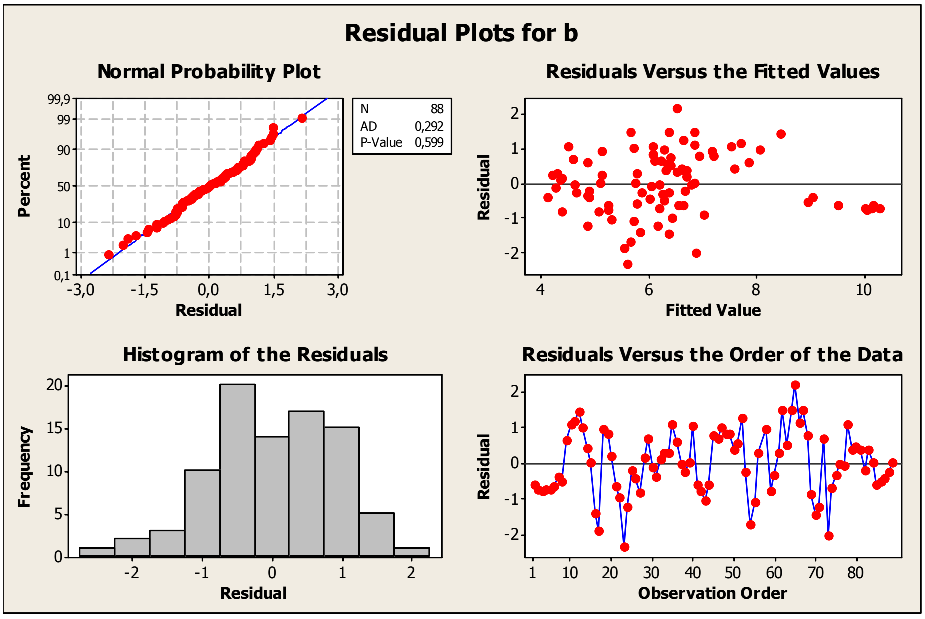


Figure 7.1: Residual plots for *b*.

An additional test of how strong the model is is performed by analyzing the change of *b* versus the change in the included variables. This test is given in appendix 4. The change in the price of *ENOYR3* is significant and the change in g_p is significant at an 8%-level. This means that the model is quite good.

7.1.2 Autoregressive models

Next autoregressive models are considered. These are models where the current value of the variable, here *b*, depends upon the values the variable took in previous periods plus an error term. An autoregressive model of order p, denoted by AR(p) can be described as

$$b_t = \mu + \varphi_1 b_{t-1} + \varphi_2 b_{t-2} + \ldots + \varphi_p b_{t-p} + u_t \tag{43}$$

---

[16] Backward elimination is here chosen. This is a method for determining which variables to retain in a model. It starts with the model that contains all the predictors and then removes one variable at a time. The method deletes the predictor from the model that results in the largest decrease in SSE. No variables can re-enter the model. The elimination procedure ends when none of the variables have a P-value greater than the value specified, here 0,05. For more, see e.g. Walpole, Myers and Myers (1998).

[17] A P-value higher than the significance level indicates that the residuals are normally distributed. The smaller the Anderson-Darling value, the greater the distribution fits the data.

following the terminology of Brooks (2002). u_t is here a white noise disturbance term. An AR(1) model, gives the following results

$$b_t = 1,28 + 0,791 b_{t-1} \tag{44}$$

R-Sq(adj) is now 64,8%. AR(2) models give insignificant results.

7.1.3 Combination of autoregressive - and regression models

Different combinations of the models of section 7.1.1 and 7.1.2 is possible. The backward elimination procedure when lags of the variables are included suggests that b_{t-1}, $ENOYR3$, $ENOYR3_{t-1}$ and g_p should be included in such a model. However, some of the variables are not significant in this model when a regression is performed. By removing $ENOYR3_{t-1}$ and including $g_{p,t-1}$ a model with significant variables is established.

$$b_t = 17,9 + 0,405 \cdot b_{t-1} - 0,0844 \cdot ENOYR3 + 81,7 \cdot g_p - 73,9 \cdot g_{p,t-1} \tag{45}$$

The variance analysis and P-values are given in appendix 5. R-Sq(adj) is 78,8% and all of the variables are significant at a 1%-level. Figure 7.2 shows the residual plots for b. The residuals for this model are also normally distributed and there is no trend in the data, making the error variance homogenous and the model reliable. The Anderson-Darling value is lower than for model (42). According to this test, this model therefore fits the data slightly better.

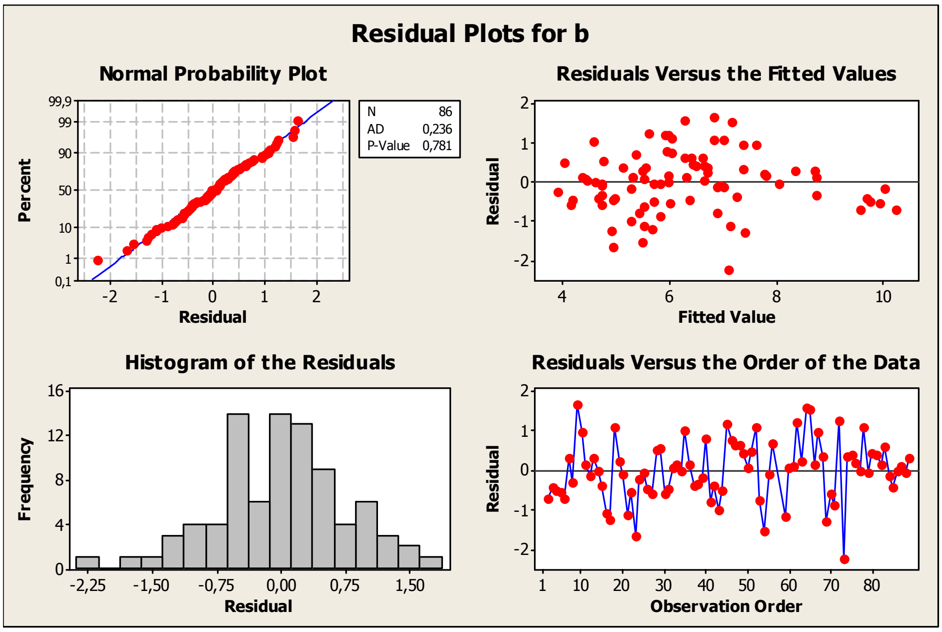


Figure 7.2: Residual plots for b.

This model is also tested by investigating the change in b versus the change in the variables in the model. The results are given in appendix 5. All the variables are significant, except for $g_{p,t-1}$. Still, the model is quite strong. Creating a model without $g_{p,t-1}$ could however be an alternative. Such a model is also given in appendix 5.

7.2 Model adjustment

Price levels change across time. A change in supply and demand for instance causes prices to change. Inflation is also important.

NVE's energy status points out that demand have increased more than supply the past years. As a result prices have increased. If this trend continues, prices will continue to increase. Figure 7.3 shows the trend in the weekly price of *ENOYR3* between 1998 and 2004.

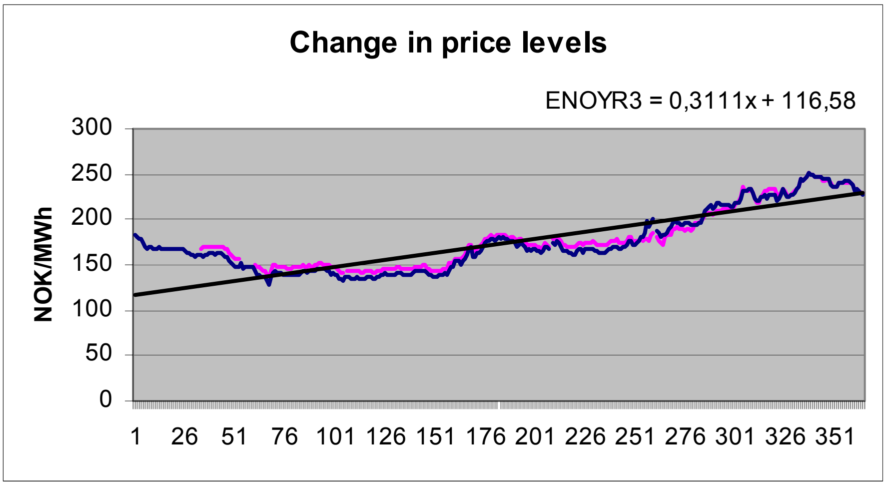


Figure 7.3: Change in price levels.

The models for *b* developed in chapter 7.1 include the price of *ENOYR3* as a regression variable. However, the prices in the period that is analysed ranged from 140NOK/MWh to 190NOK/MWh. The prices in the spring of 2005 were much higher; up to 32-33EURO/MWh or about 260-270NOK/MWh. Using the models of section 7.1 without adjusting the price of *ENOYR3* will therefore result in an underestimated growth.

To fix this, the models have to be adjusted so that they can be used for today's data. This can be done by using the equation for the change in price level given in figure 7.3. This trend line is estimated based on the prices of *ENOYR3*.

The average price level for the data analysed is computed for week 40 in 2001. This is the week in the middle of the analysis period.

The price level of *ENOYR3* was then

$$= 116,58 + 0,3111 \cdot (52 \cdot 3 + 40) \approx 178 NOK / MWh$$

The price level when the contract is to be priced is given by

$$= 116,58 + 0,3111 \cdot (52 \cdot 7 + week\ number) = 116,58 + 0,3111 \cdot 52 \cdot 7 + 0,3111 \cdot week\ number$$
$$\approx 230 + 0,31 \cdot week\ number$$

where the week number is given as the number of weeks since January 1st, 2005.

The adjusted price of *ENOYR3* is then given by

$$ENOYR3_{adj} = \frac{ENOYR3 \cdot 178}{(230 + 0,31 \cdot week\ number)} = \frac{ENOYR3}{1,3 + 0,002 \cdot week\ number} \tag{46}$$

ENOYR3 in (42) and (45) should then be replaced with $ENOYR3_{adj}$. g_p and $g_{p,t-1}$ does not have to be adjusted because such an adjustment hardly affects the price at all.

The adjusted equations for *b* are then given by (42b) and (45b). These are the expressions for *b* that should be used subsequently for pricing purposes.

$$b_{adj} = 28,3 - 0,135 \cdot ENOYR3_{adj} + 33,1 \cdot g_p \tag{42b}$$

$$b_{t,adj} = 17,9 + 0,405 \cdot b_{t-1} - 0,0844 \cdot ENOYR3_{adj} + 81,7 \cdot g_p - 73,9 \cdot g_{p,t-1} \tag{45b}$$

The trend line in figure 7.3 should be recalculated regularly, for instance once a year. This is important because the price growth not necessarily will stay at today's level. Future price levels will depend on future supply, demand and changes in for instance CO_2-prices and prices on green certificates.

7.3 Valuation of ENO10

7.3.1 The basic model

As mentioned earlier, prices on contracts for the next three years are available at Nord Pool. To set a correct price on *ENO10* these should be used as well as the function for the estimated growth.

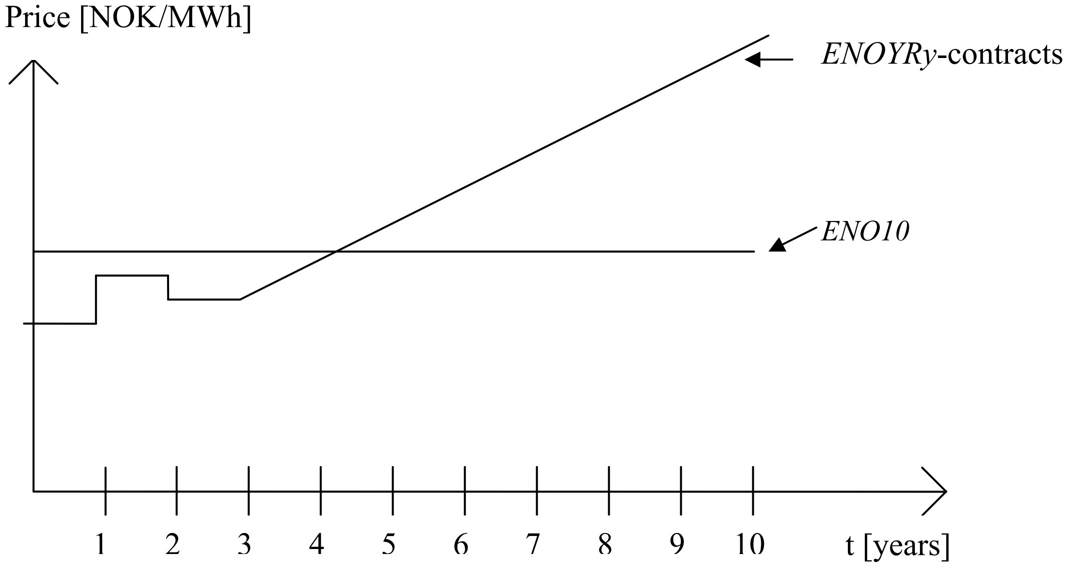


Figure 7.4: Hypothetical forward curve.

The forward function then might look like the curve in figure 7.4. A formula to price *ENO10* is then given by (47). This formula is appropriate when the time between the settlements is small. This is the case for instance if settlement happens every month. The formula is derived in appendix 2.

$$p_{10} = \frac{r_{10} \cdot [\int_{s}^{1+s} ENOYR1 e^{-r_{10}t} dt + \int_{1+s}^{2+s} ENOYR2 e^{-r_{10}t} dt + \int_{2+s}^{3+s} ENOYR3 e^{-r_{10}t} dt + \int_{3+s}^{10+s} (a+bt) e^{-r_{10}t} dt]}{e^{-sr_{10}} - e^{-(10+s)r_{10}}} \tag{47}$$

s is here the time to the start of the contract period given in years. If delivery starts immediately, $s = 0$. p_{10} is given in NOK/MWh.

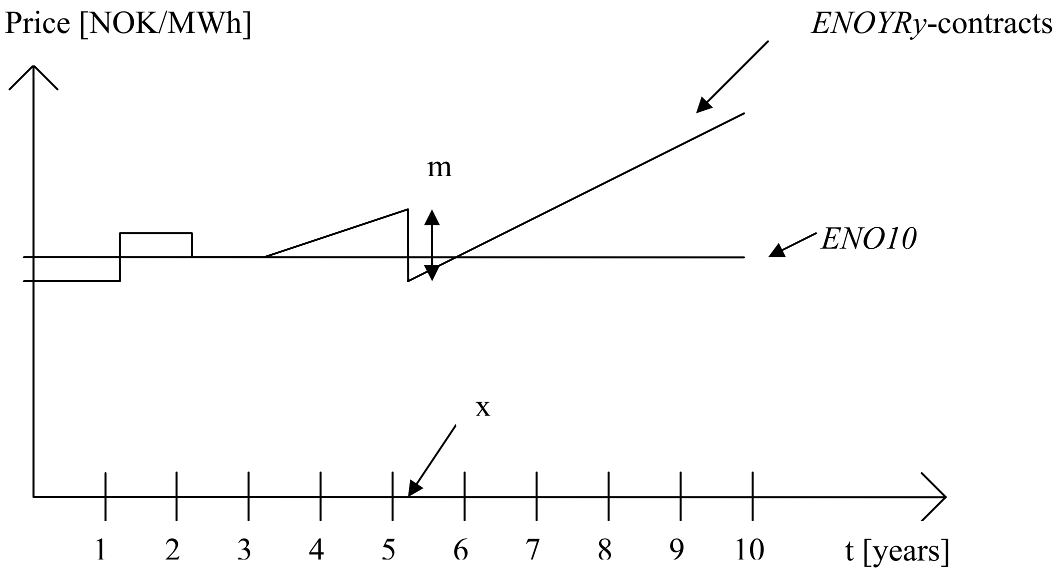
If annual settlement is assumed, (48) is more appropriate:

$$p_{10} = \frac{\int_{s}^{1+s} ENOYR1 e^{-r_{10}t} dt + \int_{1+s}^{2+s} ENOYR2 e^{-r_{10}t} dt + \int_{2+s}^{3+s} ENOYR3 e^{-r_{10}t} dt + \int_{3+s}^{10+s} (a+bt) e^{-r_{10}t} dt}{\frac{1}{(1+\mathrm{r}_{10})^{k}} + \frac{1}{(1+\mathrm{r}_{10})^{1+k}} + ... + \frac{1}{1+\mathrm{r}_{10})^{9+\mathrm{k}}}} \tag{48}$$

This is based on (3). The numerator represents the net present value of the contracts and the denominator adjusts for when settlement occurs. As opposed to (47) that assumes continually settlement in the delivery period, (48) assumes that there is one payment per year in ten years. k in the equation is here the time to the first settlement day, given in years.

7.3.2 Extended model

The models above do not take into consideration that sudden incidents can occur in the future. Consider an example where large gas power plants are planned built five years from now. The price of *ENOYR3* will not be influenced by this, since the settlement of the contract ends before the development is expected to occur. It is however likely that the forward curve after t=3 will experience a decrease in price. The model should therefore be modified for such cases.



Figur 7.5: Hypothetical forward curve with sudden change in price.

For such cases one can add or subtract the expected change in price, as figure 7.5 shows. *b* stays the same for this new line, whereas *a* experiences an increase or decrease of the same size as the expected increase or decrease in price, here denoted by m.

If the forward curve follows figure 7.5, a more proper formula would be

$$p_{10} = \frac{r_{10} \cdot [\int_{s}^{1+s} ENOYR1 e^{-r_{10}t} dt + \int_{1+s}^{2+s} ENOYR2 e^{-r_{10}t} dt + \int_{2+s}^{3+s} ENOYR3 e^{-r_{10}t} dt + \int_{3+s}^{x+s} (a+bt) e^{-r_{10}t} dt + \int_{x+s}^{10+s} (a+bt) e^{-r_{10}t} dt]}{e^{-sr_{10}} - e^{-(10+s)r_{10}}} \tag{49}$$

or if annual settlement is assumed:

$$p_{10} = \frac{\int_{s}^{1+s} ENOYR1 e^{-r_{10}t} dt + \int_{1+s}^{2+s} ENOYR2 e^{-r_{10}t} dt + \int_{2+s}^{3+s} ENOYR3 e^{-r_{10}t} dt + \int_{3+s}^{x+s} (a+bt) e^{-r_{10}t} dt + \int_{x+s}^{10+s} (a+bt) e^{-r_{10}t} dt}{\frac{1}{(1+\mathrm{r}_{10})^{k}} + \frac{1}{(1+\mathrm{r}_{10})^{1+k}} + \ldots + \frac{1}{(1+\mathrm{r}_{10})^{9+k}}} \tag{50}$$

where x is the time that the price increase or decrease is expected to happen and m is the size of the expected increase or decrease. As before, *k* is the time to the first settlement date and *s* is the time to the contract period starts, both given in years.

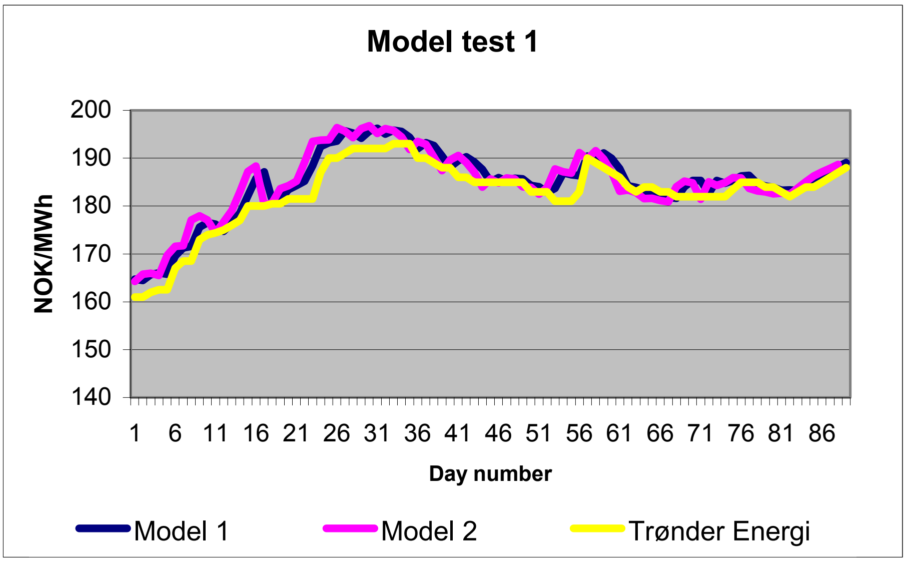
7.3.3 Comment

The models given in chapter 7.3.1 and 7.3.2 would be most correct if the delivery period starts January 1st. This is because the prices of *ENOYR1*, *ENOYR2* and *ENOYR3* are prices of contracts with delivery period between January 1st and December 31st. The model therefore assumes that the first year of the contract starts at January 1st. The effects caused by this error should nevertheless presumably be small.

7.4 Testing the models

In figure 7.6 Trønder-Energi's estimated prices of *ENO10* are plotted against the prices of *ENO10* that are estimated by model (42) and (45) of section 7.1[18]. These two models are chosen rather than (43), based on R-Sq(adj). The data in the figure are from week 48 in 2000 to week 33 in 2002.

Pricing model (48) with s = 0 and k = 1 is used here, but model (47) gives quite similar results.



Figur 7.6: Trønder-Energi's *ENO10*-estimates versus model-estimates.

As the figure shows, both models give quite good fits.

An additional model test might be to create two new models that are based on only the 60% first data points. $s = 0$ and $k = 1$ is also chosen here. The objective is to see whether the model fits the rest of the data points or not. If there is a good fit, the model is good. The same variables that are included in (42) and (45) should be included in the model. The regression equations and variance analysis is given in appendix 6. Figure 7.7 shows the results. Both models fit the data quite good in the longest end, even though only the 60% first observations are analysed. The created models are therefore believable.

---

[18] As an initial value for b_{t-1} the formula b=28,3-0,135*ENOYR3*+33,1g_p (model 1) is used as an approximation. As an initial value for g_{t-1} the average growth between 1998 and 2004 is used as an approximation.

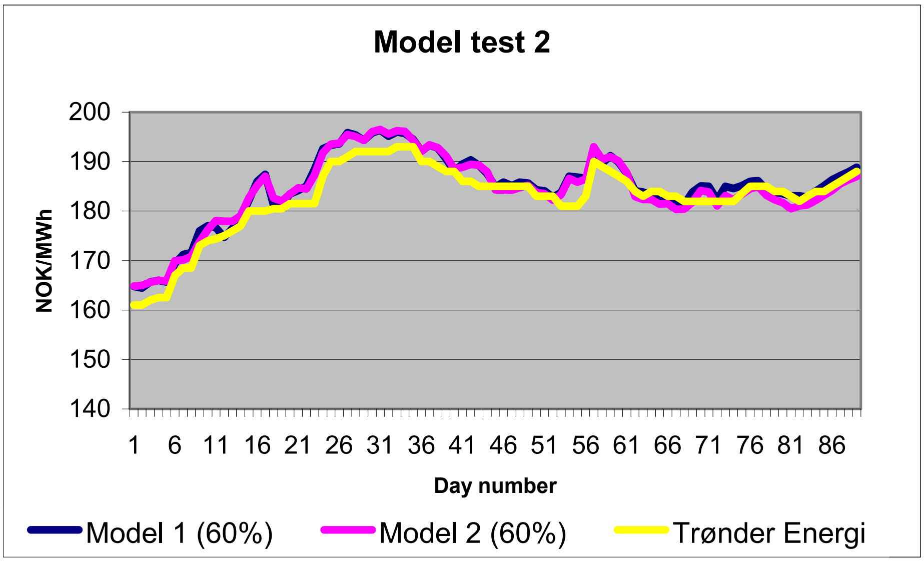


Figure 7.7: Model test based on a regression with only 60% of the data.

To choose the best of these two models, R-Sq(adj) should be considered. One should at the same time consider which model that seems most reasonable and which model is easiest to use.

The model in (45) has the highest R-Sq(adj), whereas the model in (42) is a bit more easy to use. For instance, the estimation of b_{t-1} can be difficult the first times that the model in (45) is used[19]. After using the model for some time, this problem vanishes. According to the Anderson Darling test, (45) is also the model that fits the data best. In addition, since it is reasonable to believe that the forward curve not changes that much from week to week, it is also logic that b_{t-1} is a parameter in the model. Based on this (45b) is therefore the recommended equation for b. (42b) can be used as an estimation of b_{t-1} the first times the model is used.

7.5 Adjusting the model for foreign currency

Today, the prices of *ENOYR2* and *ENOYR3* are denoted in EURO at Nord Pool. When these prices are used in the valuation of *ENO10* they therefore first have to be converted to NOK.

To find the euro-forward, interest rate parity is used (Brealey and Myers, 2003). The euro-forward is then given by

$$FW_{NOK/EURO} = \frac{1 + r_{NOK}}{1 + r_{EURO}} \cdot S_{NOK/EURO} \qquad (51)$$

where FW is the forward rate and S is the spot rate. The rates of return are the relevant rates of return the year the forward is computed.

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[19] As an approximation the first time (45) is used, b_{t-1} can be approximated by (42).

7.6 An example of contract valuation

Consider the date 14.04.2005. The following prices are given by Nord Pool's FTP-server:

$ENOYR06: \ 32{,}37 \ EUR/MWh$
$ENOYR07: \ 32{,}11 \ EUR/MWh$
$ENOYR08: \ 32{,}20 \ EUR/MWh$

On 13.04.2005 the prices were:

$ENOYR06: \ 32{,}33 \ EUR/MWh$
$ENOYR07: \ 32{,}10 \ EUR/MWh$
$ENOYR08: \ 32{,}03 \ EUR/MWh$

$S_{NOK/EUR}$ was April 13th 8,1910 NOK/EUR and April 14th 8,2465NOK/EUR.

r_{NOK} for the three different years are found on Norges Bank's home pages. r_{1NOK} was march 2005 2,45% whereas r_{3NOK} was 2,95%. Since there are no information on the rate of return on 2-year government bonds, the average of these two is used, which means that r_{2NOK} is 2,69%. r_{10NOK} is 4,02%.[20]

r_{EUR} is found on the European Central Bank's home pages. r_{1EUR} was march 2005 2,34%, whereas r_{2EUR} was 2,49% and r_{3EUR} was 2,74%[21].

By (51) the forward exchange rates on April 13th are

$$FW_{NOK/EURO}(y=1) = \frac{1+r_{1NOK}}{1+r_{1EURO}} \,.\, S_{NOK/EURO} = 8{,}200 NOK/EUR$$
$$FW_{NOK/EURO}(y=2) = \frac{1+r_{2NOK}}{1+r_{2EURO}} \,.\, S_{NOK/EURO} = 8{,}207 NOK/EUR$$
$$FW_{NOK/EURO}(y=3) = \frac{1+r_{3NOK}}{1+r_{3EURO}} \,.\, S_{NOK/EURO} = 8{,}208 NOK/EUR$$

The prices of the *ENOYRy*-contracts in NOK on April 13th then becomes

$$ENOYR06_{NOK}: \ 32{,}33 EUR/MWh \cdot 8{,}200 NOK/EUR = 265{,}10 NOK/MWh$$
$$ENOYR07_{NOK}: \ 32{,}10 EUR/MWh \cdot 8{,}207 NOK/EUR = 263{,}44 NOK/MWh$$
$$ENOYR08_{NOK}: \ 32{,}03 EUR/MWh \cdot 8{,}208 NOK/EUR = 262{,}89 NOK/MWh$$

The forward exchange rates on April 14th are also estimated by (51):

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[20] r_{3NOK} and r_{10NOK} represents the effective interest rate on government bonds of respectively 3 and 10 years. r_{1NOK} is the effective 12 month NIBOR rate.

[21] r_{1EUR} is a 12 month euro interbank offered rate (EURIBOR). r_{2EUR} and r_{3EUR} are government bond yields.

$$FW_{NOK/EURO}(y=1)=\frac{1+r_{1NOK}}{1+r_{1EURO}}\ .\ S_{NOK/EURO}=8,255NOK/EUR$$

$$FW_{NOK/EURO}(y=2)=\frac{1+r_{2NOK}}{1+r_{2EURO}}\ .\ S_{NOK/EURO}=8,263NOK/EUR$$

$$FW_{NOK/EURO}(y=3)=\frac{1+r_{3NOK}}{1+r_{3EURO}}\ .\ S_{NOK/EURO}=8,263NOK/EUR$$

The prices of the *ENOYRy*-contracts in NOK on April 14th then becomes

$$ENOYR06_{NOK}:\ 32,37EUR/MWh\cdot 8,255NOK/EUR=267,21NOK/MWh$$

$$ENOYR07_{NOK}:\ 32,11EUR/MWh\cdot 8,263NOK/EUR=265,32NOK/MWh$$

$$ENOYR08_{NOK}:\ 32,20EUR/MWh\cdot 8,263NOK/EUR=266,07NOK/MWh$$

In addition *ENOYR08adj* is estimated to

$$ENOYR08_{adj}=\frac{266,07NOK/MWh}{1,3+0,002\cdot 15}=200,05NOK/MWh$$

b is estimated to 1,96 NOK/(MWh year) by (45b). *s* is set to 0, since the estimation of *b* in chapter 6 assumed that $s = 0$. The price is first calculated to 274,8NOK/MWh by using (48) with $k = 1$. Second, if $k = 0$ instead of $k = 1$, the price is estimated to 264,2NOK/MWh. Third, if (47) is used instead of (48), the price estimate is 270,4NOK/MWh. Which is the correct price, therefore depends on when settlement is to occur. If there are multiple settlement dates with short intervals in between, the price would converge to (47). If monthly settlement is the case, (47) would therefore be the most correct formula.

These price estimates are quite high. The reason for this is that the prices around April 14th were historically high. However, the prices are reasonable compared to the beliefs of M3kraft.

8 Model discussion

In this chapter a discussion regarding the factors in the model is performed. In addition some other relevant market factors are discussed.

8.1 Regression equation for b

One of the regression variables that are included in the model is the price of *ENOYR3*. It is reasonable that *b* should be adjusted for the price level of *ENOYR3*. If the price is high compared to normal levels, *b* should be smaller and if the price is low, *b* should be larger. This is probably a result of that the price of *ENOYR3* to a certain degree depends on short term effects such as the hydrologic balance. As a result the price does not always reflect future price levels very well. It is therefore logic that b is adjusted for that.

The second regression variable included in the model is the difference between the price of *ENOYR2* and *ENOYR3* in percentages. This variable serves as a valuable supplement to the price of *ENOYR3*. In general, *b* becomes smaller when the difference between these two prices is small and larger when the difference is larger. This is in accordance with the beliefs of M3kraft.

In addition it is logic that b_{t-1} is a variable in the model. Since long term relations are being predicted, the forward curve should not change that much from week to week. Inclusion of last week's value for *b* is therefore reasonable.

8.2 Correlations between relevant factors

The evaluation of the hypotheses in chapter 6 showed that the rate of return, the oil forward price and the hydrologic level could be relevant regression variables in the equation for *b*. None of these variables are nevertheless included in the model.

It is however possible to say that the effects of changes in these variables are indirectly included in the model. The reason for this is the high correlation between these variables and the price of *ENOYR3*. Including the variables directly in the model in addition to the price of *ENOYR3* will on the other hand give insignificant results.

The same reasoning can presumably also be used for the prices on CO_2-quotas. This can however not be tested, since this marked opened 2005 and the prices of *ENO10* are available from 2000-2002. Because the price of *ENOYR3* is one of the regression variables in the equations for *b*, the effect of increasing CO_2-prices is therefore included in the model. It is therefore not necessary to take this variable into consideration in addition to the ones already included.

8.3 Other factors

Other market relations are also relevant for the development of the prices. Some factors are considered in this chapter.

8.3.1 Tradable Green Certificates

A new type of market-based instrument is "Tradable Green Certificates". The purpose of this instrument is to create incentives for more electricity production from renewable sources. The major characteristic of a green certificate system is that producers of renewable electricity receive certificates. These certificates can then be traded within a national market. Demand for the certificates can originate from several sources. In the first place, there may be a demand from environment-conscious customers. In the second place, demand can be imposed by the government or other actors in the electricity supply chain.

The price of the certificates will depend on the market. When the supply of certificates is low, the price will be high, something that gives an incentive for new producers to provide renewable energy. In theory, renewable energy will therefore be provided efficiently because the most cost efficient producers will enter the market. Figure 8.1 shows how the market will work. *MC renewables* is marginal production cost from renewable energy sources. P_E is the market price of electricity, Q is the target quantity and mc^* is the corresponding marginal cost. The certificates will then be sold for $P_C = mc^* - P_E$. The figure indicates that as long as the marginal production cost is below P_E, producers of renewable energy will produce independent of the market for green certificates. When prices increase, the producers produce when their marginal cost is below mc^*. (Schaeffer et. al, 1999)

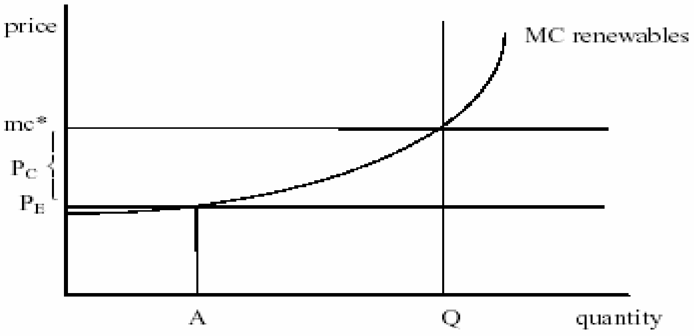

Figure 8.1: Pricing green certificates.

The trade of green certificates will probably reduce the demand for CO_2-quotas. This is a result of that the green certificate system will contribute to more electricity production from renewable sources. When the demand for CO_2-quotas decreases, so will the price. In a system where one both trades CO_2-quotas and green certificates, the market for certificates will contribute to a lower price on CO_2-quotas, whereas the CO_2-market will contribute to lower prices on green certificates. This will again influence the power price. Whereas the introduction of both the market for CO_2-quotas and the market for green certificates puts an upward pressure on prices, the interactions between the two markets can put a downward pressure on the price. The extent of these effects also depends on the power price. When the power price is high, the marginal costs of most producers are lower than the market price, leading to a low price on the certificate. The opposite occurs for low power prices. Demand for the certificates increases and so does therefore also the price.

This will probably not affect the pricing model to any large extent. The effects will be included in the price of *ENOYR3* and needs no special consideration. It is at the same time important to remember to adjust the model for the changes in the price levels that the introduction of such a market would cause, cf. chapter 7.2.

8.3.2 Risk preferences

The different market participants may have different attitudes towards risk. There are risk-averse, risk neutral and risk seeking participants. That being said, it is a reasonable assumption that most of the participants want to avoid risk, and that most of them therefore are risk-averse. According to Brealey and Myers (2002) this is consistent with the existence of bankruptcy costs and market incompleteness. In addition many of the participants in the Nordic market are governmental with risk-averse managements.

Large yearly variations in demand and the fact that electricity cannot be stored leads to high price volatility in the electricity market. Bessembinder and Lemmon (2002) show that this is of great importance, both to sellers and buyers of electricity. Increased market volatility gives greater incentives for hedging, both for producers and retailers. As a result, producers, retailers and consumers demand methods for reducing risk.

Changes in risk aversion over time can influence the forward curve. More risk-averse producers will decrease the forward price and thereby increase the risk premium. More risk-averse retailers will increase the forward price and thereby decrease the risk premium.

The extreme prices the winter 2002/2003 lead to increased volatility. Mork (2004) examines whether the risk premium changed after the high prices and volatilities in the winter 2002/2003. According to Mork (2004) it would be reasonable that many customers, both industry and end-users would react by purchasing more fixed-price contracts. The statistics however indicates that the opposite has occurred. However, there is not enough data to make any conclusions regarding this. The implications on the price are therefore hard to obtain. On the other hand, the effect of changed risk preferences will be included in the price of *ENOYR3*. The model does therefore not have to be adjusted for changes in the market participants' risk preferences.

8.3.3 Political risk

Political decisions affect the framework conditions in the power market. Examples of political risks are changes in regulations on taxes and charges, changes in rules regarding minimum releases of water in addition to impositions by NVE. Besides, most companies have bank loans and are therefore vulnerable to changes in the interest rate level that are decided by Norges Bank. Many of the producers in the Nordic market are in addition partly owned by the government. They are therefore exposed to a political risk as a result of that the government influences the size of the share dividends.

These factors may influence the power price. However, the price of *ENOYR3* will reflect these beliefs. On the other hand, if some changes are expected to occur further into the future, (49) or (50) should be used instead of (47) or (48).

9 Risk premium

9.1 Uncertain approach

Estimation of the risk premium on long term contracts is a very uncertain approach. In the first place the expected spot prices vary dependent on who you ask. In the second place the forward price estimates differ because the market participants use different models. In addition the lack of data on realized *ENO10*-contracts is a problem when the risk premium of *ENO10*-contracts is to be estimated. Another approach for estimation of the size of the risk premium is therefore chosen here. Next the expected spot prices from SSB are used together with the estimated forward curve of chapter 7. (45b) is here used to estimate *b*. SSB's reference alternative is chosen, but the same procedure can be done for the other alternatives.

9.2 Estimation of the risk premium

Figure 9.1 shows the expected spot prices and the estimated forward prices from 2006-2015. It shows that the expected spot prices are lower than the estimated forward prices in the whole period. This indicates that the risk premium is negative in the whole period and that the market therefore is in contango. Consumers are more willing to enter into long term contracts than producers.

The difference is however largest in the shortest run, something that suggests that the difference between the risk preferences of producers and consumers is larger for the closest years than on the longer view. This is in accordance with the beliefs of Mork (2004) in chapter 3.3.4; producers are more interested in hedging on the longer view. However, the figure still shows that the consumers are more willing to hedge than the producers; also when time to maturity grows. This is in conformity with Ollmar (2003).

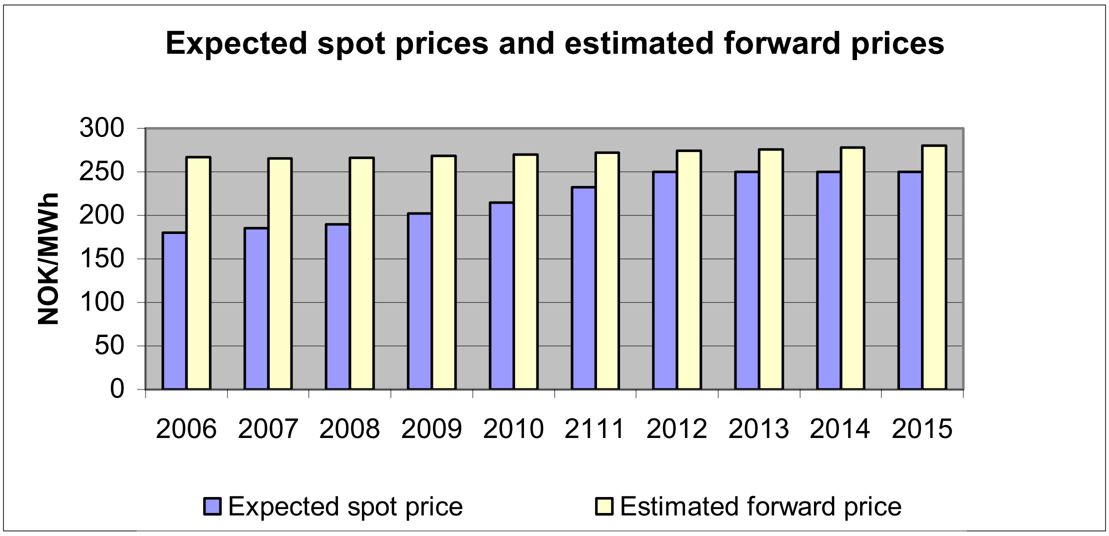

Figure 9.1: Expected spot prices and estimated forward prices.

To estimate the risk premium (23) is used:

$$PREMIUM = k - r = \ln(\frac{E_t[S_T]}{F(t,T)}) \tag{23}$$

The average risk premium for the ten years is then used as an approximation for the risk premium of *ENO10*. This is estimated to -0,22. This indicates that retailers are more willing to enter into *ENO10*-contracts than producers.

9.3 Valuation of ENO10 by the risk premium approach

Given the risk premium and the expected spot price, the forward price can be found. *F(t,T)* is then given by

$$F(t,T) = E_t[S_T] \cdot e^{-(k-r)(T-t)} \tag{6}$$

The expected average spot price over the ten relevant years can be used as the expected spot price together with the risk premium estimated in section 9.2.

This approach is rarely used in practise. As mentioned earlier the expectations on future spot prices vary among market participants. The risk premium will therefore vary dependent on who you ask.

9.4 Empirical studies on the risk premium

The risk premium has been studied by several authors. Ollmar (2003) has performed a non-parametric estimation of the risk premium of contracts with 1 day to 3 years to maturity. His results suggest that the risk premium is negative in the short run, but grows when time to maturity increases. The risk premium is however also negative when time to maturity is 1000 days (ca 3 years) and Ollmar (2003) estimates this value to -0,05. He does not estimate the risk premium on contracts with longer time to maturity, due to the lack of data, but assumes that this also will be negative, since power producers rarely hedge their long term risk. This analysis is based on Nord Pool data from 1995-2002.

Maudal and Solum (2003) estimates the risk premium on contracts with one and two years to maturity in their master thesis. Their analysis is performed using prices on season contracts as expected spot prices. They estimate the risk premium on *ENOYR1* and *ENOYR2* to respectively -0,1555 and -0,1032. The risk premium on *ENOYR2* is higher than the risk premium on *ENOYR1* and they suggest that if this trend continues, the risk premium might become positive in the long run. There are however no analysis supporting this claim.

Botterud et al. (2002) has estimated the risk premium on contracts with a delivery period of one year that is held to maturity (*ENOYR1* held to maturity). The data considered are Nord Pool data from 1995-2001. The risk premium on this contract is estimated to -0,183.

All of these analyses indicate that the risk premium is negative even for contracts with up to three years to maturity. None of the analyses however make any conclusions regarding the sign or the size of the risk premium of contracts that have maturities in the more distant future.

10 Volatility analysis

10.1 Formulas

Historical volatility can be estimated by (52)

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}} - \frac{1}{n}\sum_{i=1}^{n}\frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}}\right)^2} \tag{52}$$

where n is the number of observations and log is the natural logarithm.

t_i-t_{i-1} is here inserted as a fraction of the year. σ then becomes annual volatility. The length of the interval between observations can both be computed in calendar days or in so called business days. The approximate number of business days per year, 250, is chosen in this report. Several authors instead use 365, the actual number of calendar days per year. Whatever the choice, it has to be applied consistently to avoid errors.

The formula above can be used if the volatility is homoscedastic. If this is not the case, (53) should be used:

$$\sigma(t_k) = \sqrt{\frac{1}{m-1}\sum_{i=k-m+1}^{k}\left(\frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}} - \frac{1}{m}\sum_{i=k-m+1}^{n}\frac{\log P_i - \log P_{i-1}}{\sqrt{t_i - t_{i-1}}}\right)^2} \tag{53}$$

where m is the width of the moving window, that is, a specified number of observations preceding t_k used to estimate volatility. One here assumes that the volatility is constant at least for a relatively short period of time. This approach is often called the moving window method, since the data set used for estimation moves together with the time at which volatility is estimated. These formulas and arguments are given by Eydeland and Wolyniec (2003) among others.

10.2 Spot price volatility

To estimate spot price volatility, (52) is used. Homogenous volatility is here assumed[22]. Using (52) on daily spot price data from 1998 to 2004 gives an annual volatility of 164,6%. This number is computed by assuming that the number of trading days per year is 250.

Lucia and Schwartz (2002) estimate spot price volatility in the period 1993-1999 and find a volatility of 189%. They however assume that the length of the interval between observations is computed in calendar days. To represent t_i-t_{i-1} as a year fraction, they therefore divide by 365. To be able to compare the volatility estimated by Lucia and Schwartz (2002) with the volatility estimated here, the same number of days must be assumed. By assuming that the number of days is 365, the spot price volatility between 1998 and 2004 is estimated to

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[22] In reality the volatility depends on the season of the year. This is however not important in this report, hence the simplification.

198,9%. This estimate is slightly higher than the one estimated by Lucia and Schwartz (2002). The reason for this is probably that the winter 2002/2003 now is included. Prices and volatilities then reached extreme levels.

Several authors have analyzed spot price volatility. For more on spot price volatility, see e.g. Lucia and Schwarz (2002).

10.3 *Forward price volatility*

Figure 10.1 shows the historical volatility of the prices of *FWYR01*, *FWYR02*, *FWYR03*, *FWYR04*, *FWYR05*, *ENOYR06* and *ENOYR07* that were traded at Nord Pool between 1998 and 2004. It is here assumed that the volatility changes with time. $m = 20$ is chosen as the size of the mowing window, since this is usual according to several authors, among others Eydeland and Wolyniec (2003). The figure shows that the volatility differs from contract to contract. One contract, however, stands out. *FWYR03* has an extremely high volatility in the short end. The reason for this is presumably the extreme prices in the winter 2002/2003.

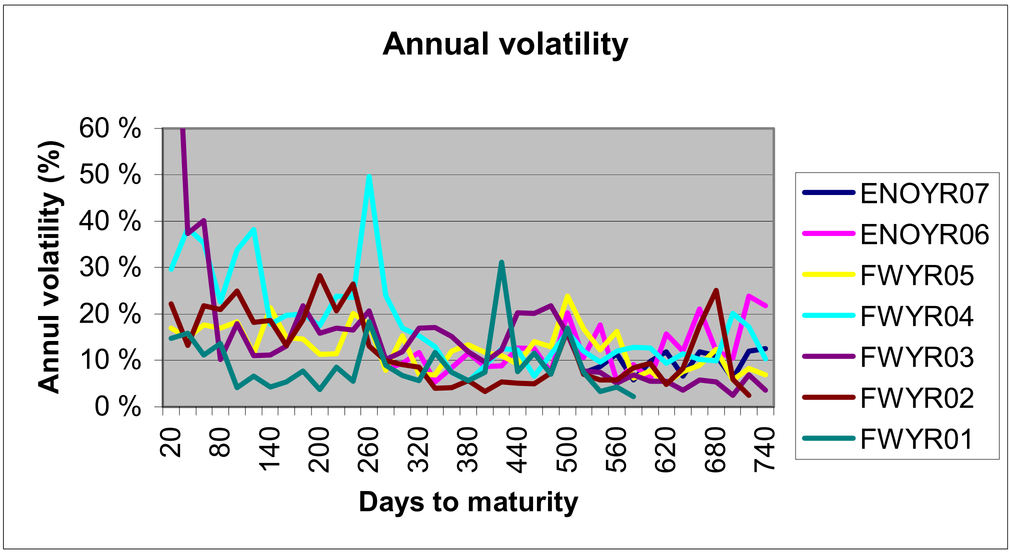

Figure 10.1: Annual volatility of different contracts

Figure 10.2 shows the development of the volatility when the seven years are analyzed together. It is here assumed that contracts with the same time to maturity have the same volatility. Since seven years of data now are analyzed, 140 observations[23] are assumed to have the same volatility.

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[23] 7 years * 20 days * 1 observation/day= 140 observations

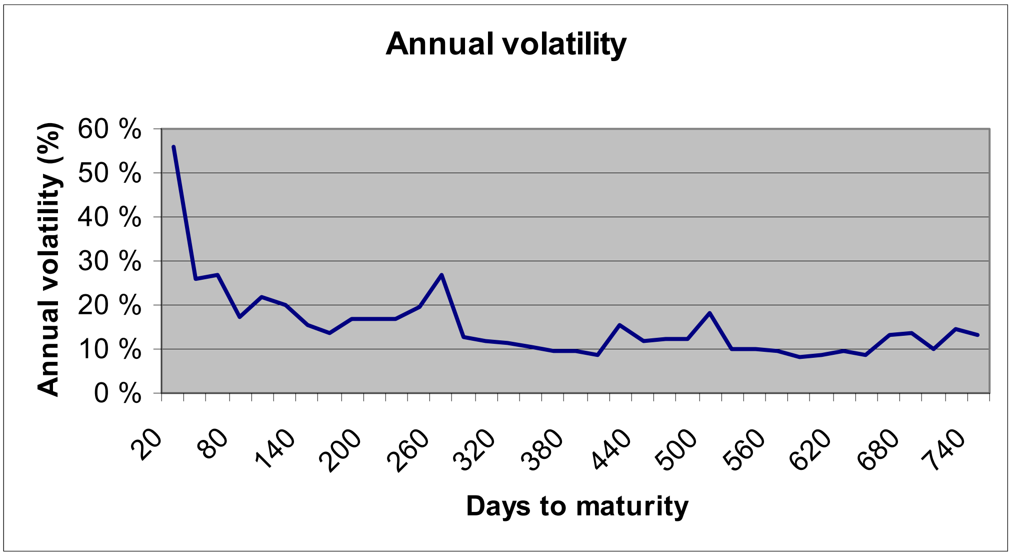

Figure 10.2: Estimated annual volatility

Both figures show that the volatility of the *ENOYR*-contract is highest when time to maturity is short. When time to maturity grows, the volatility decreases.

When time to maturity exceeds 1 year (250 days) the volatility stays at about the same level. An assumption when time to maturity grows further is that the volatility will stay at quite the same level or be a little bit lower. This is natural since the correlation between the prices of the different year contracts is so high, cf. table 7.2. This correlation will probably not be that strong for contracts with longer time to maturity since these will be less dependent on short term changes, but seeing that the future prices depend on many of the same conditions, it is likely that these prices also are quite correlated. When the price of one contract increases, the prices of the others do the same.

Several authors have tried to estimate parametric volatility functions. Two of the most common approaches are the ones by Bjerksund et al. (2000) and Lucia and Schwarz (2002), respectively

$$\sigma_1(t,T) = \frac{a}{T-t+b} + c \tag{10}$$

and

$$\sigma_1(\mathrm{t,T}) = \sigma e^{-\kappa(T-t)} \tag{9}$$

Both these models assume heteroscedastic volatility. Figure 10.2 indicates that the volatility is quite constant and > 0 in the long run. (9) will therefore probably be more doubtful for long term volatility estimates since it suggests that the volatility converges to zero in the long run. Model (10) is more believable, since it assumes a constant volatility in the long run.

The volatility of *ENO10* can be approximated by the average of the volatility of the *ENOYRy*-contracts. The volatility over the last two of the three years[24] is 11,4%. In the shorter run, the

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[24] This is computed as an average of the volatility of the contracts with 280-740 days to maturity. This is approximately the two last years in figure 10.2.

volatility is a little higher and on the longer view it might be a little lower. In the long run an annual volatility of 10-15% can therefore be assumed, compensating for higher volatility in the short run and possibly a little lower volatility on the longer view. These numbers are however quite uncertain.

11 Conclusions and suggestions to M3kraft

In this thesis a possible approach for valuation of *ENO10*, a power contract with a delivery period of 10 years, has been presented. The main focus has been to estimate the expected growth of the forward curve. This is called *b*. Different kinds of data have been analyzed; both power prices, oil forward prices, interest rates and data on hydrologic levels.

Six hypotheses have been evaluated. The evaluation of these is then used as a mean to estimate a regression equation for *b*. This regression equation then becomes an estimation of the forward curve in the long end. In the short end, the available prices of *ENOYR1*, *ENOYR2* and *ENOYR3* are used. The price of *ENO10* is then calculated by estimating the present value of the contracts approximated by the forward curve and adjusting for when the delivery period starts and for the time of the settlement.

b should be estimated by (45b). The price of *ENO10* should then be estimated by (47) or (48). These formulas depend on the contract period, regulated by *s*, and the time of the settlement, regulated by *k*. If there are annual settlements, (48) is preferred. However, if the time between the settlements is little (47) would be more correct for pricing purposes. For monthly settlements (47) is therefore recommended.

Market conditions change constantly; supply and demand changes through time and new methods to influence the market participants' actions are developed. The market for CO_2-quotas and green certificates are examples in that respect. The uncertainty regarding future conditions will therefore always be high. In addition, most of the production of electricity in the Nordic market is based on hydroelectric power, and this leads to large yearly variations in power production and thereby prices. All models that try to estimate long term prices will therefore be characterized with high uncertainty.

The method suggested in this thesis should therefore not be used uncritically. It should be used as a guide, but own expectations of future conditions should be considered just as important. In addition, price levels change across time, and this should be taken into consideration. As a result, the suggested equations should not be considered as stationary, but should be adjusted regularly. Price estimates should therefore be stored to be used for further investigations.

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13 Appendix

Appendix 1

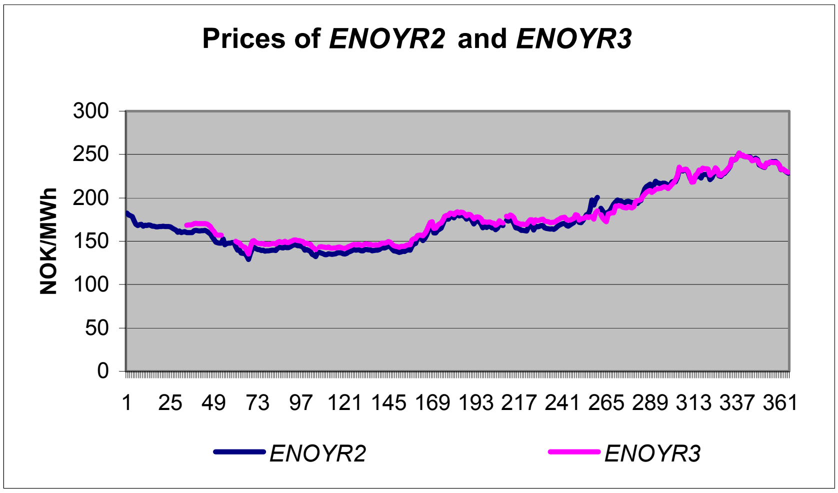

Figure A.1: The development of the prices of *ENOYR2* and *ENOYR3* between 1998 and 2004.

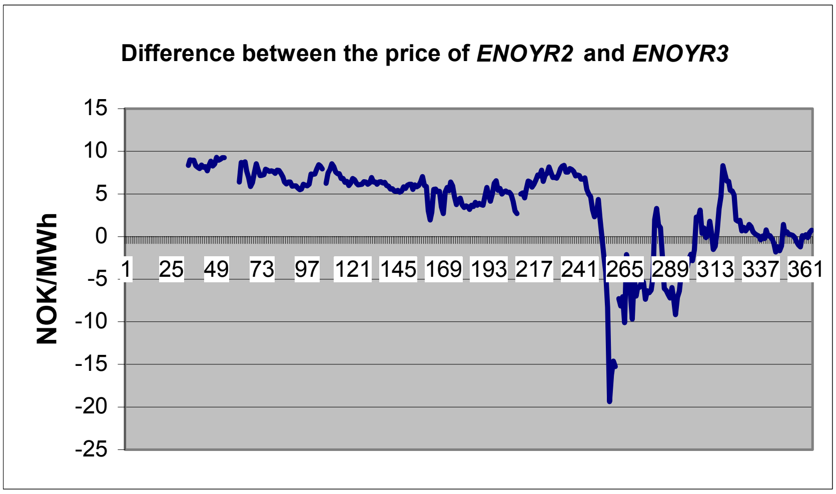

Figure A.2: The difference between the prices of *ENOYR2* and *ENOYR3.*

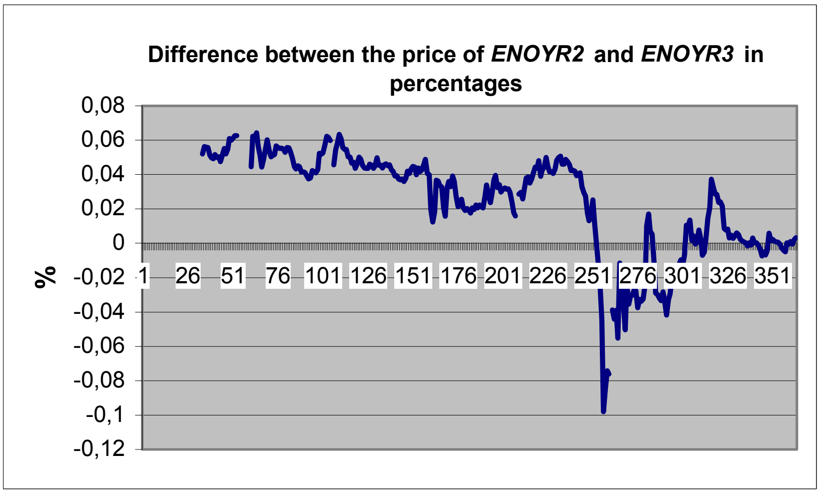

Figure A.3: The difference between the prices of *ENOYR2* and *ENOYR3* in percentages.

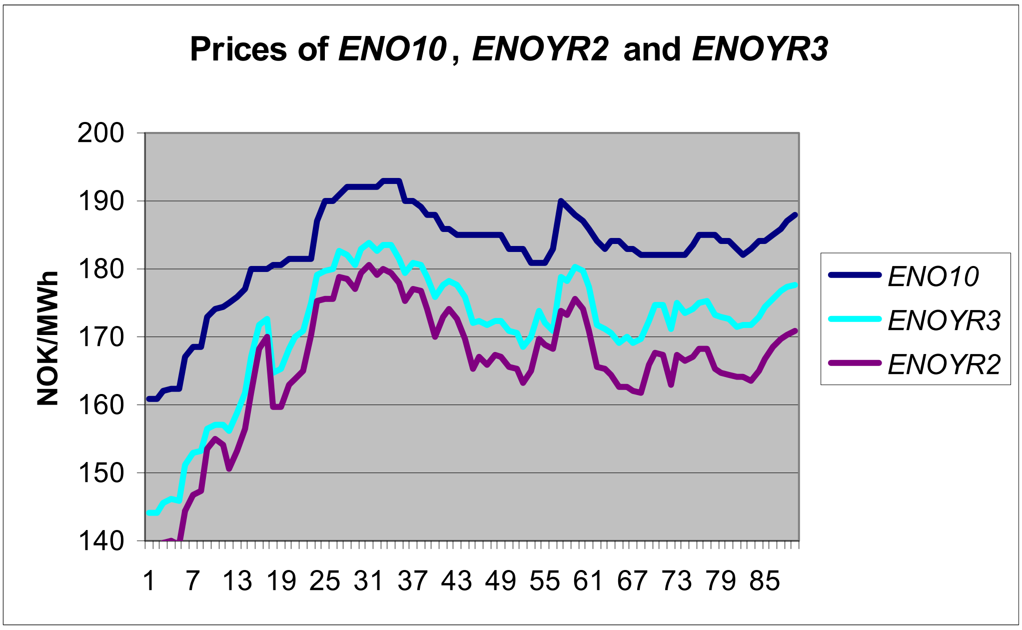

Figure A.4: Prices of *ENOYR2*, *ENOYR3* and *ENO10* between week 48 in 2000 and week 33 in 2002.

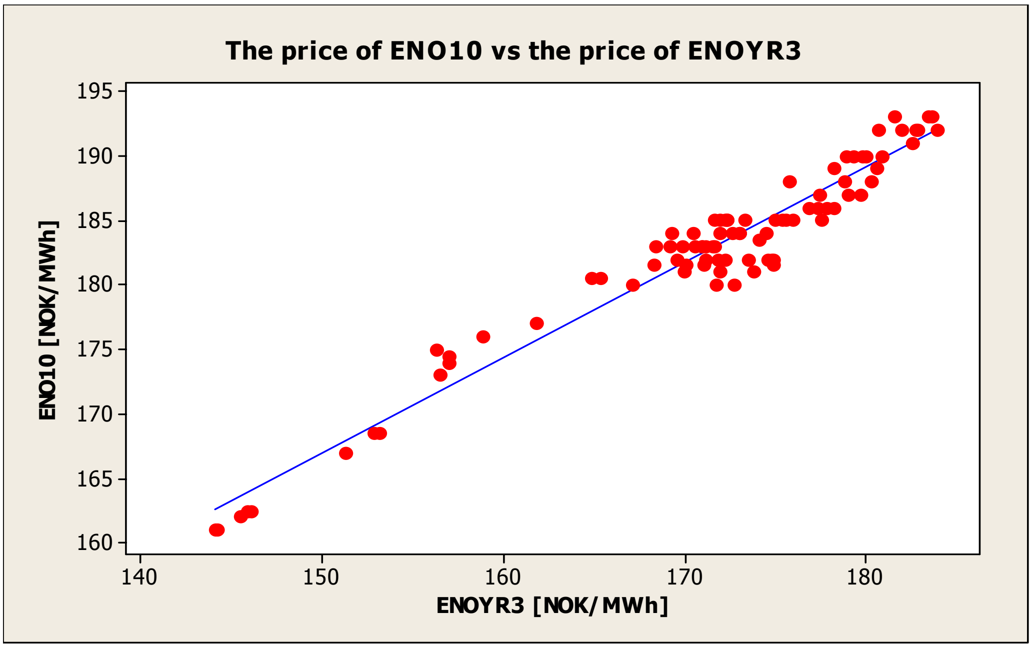

Figure A.5: Prices on *ENO10* versus prices on *ENOYR3 (in NOK/MWh) between week 48 in 2000 and week 33 in 2002.*

Appendix 2

s=0

In the estimation of b in chapter 6 it is assumed that the contract period starts immediately. The present value of the flat *ENO10*-contract when t is given in years is then given by

$$\int_0^{10} p_{10} e^{-r_{10}t} dt$$

where r_{10} is the rate of return on 10-year government bonds and p_{10} is the price of the contract.

This gives

$$\int_0^{10} p_{10} e^{-r_{10}t} dt = [\frac{-p_{10} \cdot e^{-r_{10}t}}{r_{10}}]_0^{10} = \frac{p_{10}(1-e^{-10r_{10}})}{r_{10}}$$

In the calculation it is assumed that there is one price per year. The net present value of the *ENOYRy*-contracts is given by

$$\int_0^1 ENOYR1 e^{-r_{10}t} dt + \int_1^2 ENOYR2 e^{-r_{10}t} dt + \int_2^3 ENOYR3 e^{-r_{10}t} dt + \int_3^{10} (a+bt) e^{-r_{10}t} dt$$

This gives

$$\begin{aligned} &\int_0^1 ENOYR1 e^{-r_{10}t} dt + \int_1^2 ENOYR2 e^{-r_{10}t} dt + \int_2^3 ENOYR3 e^{-r_{10}t} dt + \int_3^{10} (a+bt) e^{-r_{10}t} dt = \\ &[\frac{-ENOYR1 \cdot e^{-r_{10}t}}{r_{10}}]_0^1 + [\frac{-ENOYR2 \cdot e^{-r_{10}t}}{r_{10}}]_1^2 + [\frac{-ENOYR3 \cdot e^{-r_{10}t}}{r_{10}}]_2^3 + [\frac{-(b \cdot r_{10} \cdot t + r_{10} \cdot ENOYR3 - b(3 \cdot r_{10} - 1)) \cdot e^{-r_{10}t}}{r_{10}^{\ 2}}]_3^{10} = \\ &\frac{ENOYR1(1-e^{-r_{10}})}{r_{10}} + \frac{ENOYR2(e^{-r_{10}} - e^{-2 \cdot r_{10}})}{r_{10}} + \frac{ENOYR3(e^{-2r_{10}} - e^{-3r_{10}})}{r_{10}} + \frac{e^{-10r_{10}}[(r_{10}(e^{7r_{10}} - 1) \cdot ENOYR3 + b(e^{7r_{10}} - 7r_{10} - 1)]}{r_{10}^{\ 2}} \end{aligned}$$

a is here replaced with

$$a = ENOYR3 - 3b$$

Equating the present value of the flat *ENO10*-contract with the present value of the *ENOYRy*-contracts gives (by multiplying each side with r_{10}^2*)*

$$\begin{aligned} r_{10} p_{10}(1-e^{-10r_{10}}) = r_{10}[&ENOYR1(1-e^{-r_{10}}) + ENOYR2(e^{-r_{10}} - e^{-2r_{10}}) + ENOYR3(e^{-2r_{10}} - e^{-3r_{10}})] \\ &+ e^{-10r_{10}}[r_{10}(e^{7r_{10}} - 1) \cdot ENOYR3 + b(e^{7r_{10}} - 7r_{10} - 1)] \end{aligned}$$

Solving for b gives

$$b = \frac{r_{10}[p_{10}(1-e^{-10r_{10}}) - ENOYR1(1-e^{-r_{10}}) - ENOYR2(e^{-r_{10}} - e^{-2r_{10}}) - ENOYR3(e^{-2r_{10}} - e^{-3r_{10}})] - e^{-10r_{10}} \cdot r_{10} \cdot ENOYR3 \cdot (e^{7r} - 1)}{e^{-10r_{10}}(e^{7r_{10}} - 7r_{10} - 1)}$$

b is calculated on each available data point and is given in NOK/(MWh year). The calculated values for *b* are then used as the dependent variable in the regression analyses.

Solving for p_{10} gives

$$p_{10} = \frac{r_{10} \cdot [\int_0^1 ENOYR1 e^{-r_{10}t} dt + \int_1^2 ENOYR2 e^{-r_{10}t} dt + \int_2^3 ENOYR3 e^{-r_{10}t} dt + \int_3^{10} (a+bt) e^{-r_{10}t} dt]}{1 - e^{-10r_{10}}}$$

or

$$p_{10} = \frac{r_{10}[ENOYR1(1-e^{-r_{10}}) + ENOYR2(e^{-r_{10}} - e^{-2r_{10}}) + ENOYR3(e^{-2r_{10}} - e^{-3r_{10}})] + e^{-10r_{10}}[r_{10}(e^{7r_{10}} - 1) \cdot ENOYR3 + b(e^{7r_{10}} - 7r_{10} - 1)]}{r_{10}(1 - e^{-10r_{10}})}$$

p_{10} is given in NOK/MWh. This is the price if continually settlement is assumed. A formula for annual settlement is given in chapter 7.3.

s≠0

If the delivery period does not start immediately, the present value of the flat ENO10-contract is given by

$$\int_s^{10+s} p_{10} e^{-r_{10}t} dt = [\frac{-p_{10} \cdot e^{-r_{10}t}}{r_{10}}]_s^{10+s} = \frac{p_{10}(e^{-sr_{10}} - e^{-(10+s)r_{10}})}{r_{10}}$$

where *s* is the time to the start of the delivery period given in years.

The present value of the year contracts is then given by

$$\int_s^{1+s} ENOYR1 e^{-r_{10}t} dt + \int_{1+s}^{2+s} ENOYR2 e^{-r_{10}t} dt + \int_{2+s}^{3+s} ENOYR3 e^{-r_{10}t} dt + \int_{3+s}^{10+s} (a+bt) e^{-r_{10}t} dt$$

Equating these two and solving for p_{10} gives

$$p_{10} = \frac{r_{10} \cdot [\int_s^{1+s} ENOYR1 e^{-r_{10}t} dt + \int_{1+s}^{2+s} ENOYR2 e^{-r_{10}t} dt + \int_{2+s}^{3+s} ENOYR3 e^{-r_{10}t} dt + \int_{3+s}^{10+s} (a+bt) e^{-r_{10}t} dt]}{e^{-sr_{10}} - e^{-(10+s)r_{10}}}$$

Here continually settlement is assumed. A model for annual settlement is given in chapter 7.3.

Appendix 3

The rate of return used in the analysis beneath is the rate of return on 3-year government bonds.

Regression Analysis: ENOYR2 versus Rate of return

```
The regression equation is
ENOYR2 = 286 - 2075 Rate of return


363 cases used, 2 cases contain missing values


Predictor                        Coef  SE Coef       T      P
Constant                      285,836    4,388   65,13  0,000
Rate of return               -2074,86    79,88  -25,98  0,000


S = 19,9959   R-Sq = 65,1%   R-Sq(adj) = 65,0%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  269772  269772  674,71  0,000
Residual Error  361  144341     400
Total           362  414112
```

Regression Analysis: ENOYR2 versus Rate of return^2

```
The regression equation is
ENOYR2 = 235 - 19916 Rate of return^2


363 cases used, 2 cases contain missing values


Predictor             Coef  SE Coef       T      P
Constant           235,266    2,889   81,42  0,000
Rate of return^2  -19916,1    879,3  -22,65  0,000


S = 21,7673   R-Sq = 58,7%   R-Sq(adj) = 58,6%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  243064  243064  512,99  0,000
Residual Error  361  171048     474
Total           362  414112
```

Regression Analysis: ENOYR2 versus ln(Rate of return)

```
The regression equation is
ENOYR2 = - 120 - 99,4 ln(Rate of return)


363 cases used, 2 cases contain missing values
```

```
Predictor              Coef  SE Coef       T      P
Constant            -119,86    10,29  -11,65  0,000
ln(Rate of return)  -99,414    3,452  -28,80  0,000


S = 18,6515   R-Sq = 69,7%   R-Sq(adj) = 69,6%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  288529  288529  829,40  0,000
Residual Error  361  125584     348
Total           362  414112
```

Regression Analysis: ENOYR2 versus 1/Rate of return

```
The regression equation is
ENOYR2 = 86,9 + 4,34 1/Rate of return


363 cases used, 2 cases contain missing values


Predictor           Coef  SE Coef      T      P
Constant          86,934    3,095  28,09  0,000
1/Rate of return  4,3430   0,1450  29,96  0,000


S = 18,1409   R-Sq = 71,3%   R-Sq(adj) = 71,2%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  295310  295310  897,35  0,000
Residual Error  361  118802     329
Total           362  414112
```

Regression Analysis: ENOYR3 versus Rate of return^2

```
The regression equation is
ENOYR3 = 238 - 18918 Rate of return^2


322 cases used, 43 cases contain missing values


Predictor             Coef  SE Coef       T      P
Constant           238,205    2,751   86,59  0,000
Rate of return^2  -18918,1    817,9  -23,13  0,000


S = 19,9838   R-Sq = 62,6%   R-Sq(adj) = 62,5%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  213660  213660  535,01  0,000
Residual Error  320  127793     399
Total           321  341453
```

Regression Analysis: ENOYR3 versus ln(Rate of return)

```
The regression equation is
ENOYR3 = - 94,6 - 92,6 ln(Rate of return)


322 cases used, 43 cases contain missing values


Predictor              Coef  SE Coef       T      P
Constant            -94,603    9,441  -10,02  0,000
ln(Rate of return)  -92,625    3,168  -29,24  0,000


S = 17,0486   R-Sq = 72,8%   R-Sq(adj) = 72,7%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  248443  248443  854,77  0,000
Residual Error  320   93010     291
Total           321  341453
```

Regression Analysis: ENOYR3 versus 1/Rate of return

```
The regression equation is
ENOYR3 = 97,8 + 4,04 1/Rate of return


322 cases used, 43 cases contain missing values


Predictor           Coef  SE Coef      T      P
Constant          97,776    2,854  34,26  0,000
1/Rate of return  4,0382   0,1326  30,45  0,000


S = 16,5447   R-Sq = 74,3%   R-Sq(adj) = 74,3%


Analysis of Variance

Source           DF      SS      MS       F      P
Regression        1  253860  253860  927,42  0,000
Residual Error  320   87593     274
Total           321  341453
```

Appendix 4

Stepwise Regression: b versus Spot price; ENOYR1; ...

```
Backward elimination.  Alpha-to-Remove: 0,05


Response is b on 9 predictors, with N = 76
N(cases with missing observations) = 13 N(all cases) = 89


Step                                1        2        3        4        5
Constant                        19,50    26,54    27,37    28,67    28,28

Spot price                     0,0043   0,0042   0,0043
T-Value                          0,61     0,62     0,65
P-value                         0,542    0,540    0,519

ENOYR1                          0,038    0,038    0,038    0,034
T-Value                          0,75     0,77     0,78     0,71
P-value                         0,456    0,441    0,439    0,478

ENOYR2                            0,1      0,2
T-Value                          0,07     0,08
P-value                         0,942    0,934

ENOYR3                         -0,319   -0,336   -0,185   -0,187   -0,146
T-Value                         -0,17    -0,18    -2,79    -2,84    -4,47
P-value                         0,864    0,854    0,007    0,006    0,000

(ENOYR3-ENOYR2)/ENOYR2             74       77       52       45       33
T-Value                          0,24     0,25     1,86     1,75     1,71
P-value                         0,812    0,802    0,068    0,085    0,091

Rate of return (10-year gov)       55
T-Value                          0,05
P-value                         0,960

1/r10                            0,44     0,22     0,21     0,25     0,20
T-Value                          0,10     0,81     0,82     1,02     0,84
P-value                         0,922    0,423    0,415    0,310    0,402

Hydro balance                 0,00005  0,00005  0,00005  0,00004  0,00002
T-Value                          1,24     1,29     1,30     1,14     0,94
P-value                         0,218    0,201    0,198    0,258    0,352

Oil forward price (1pos)       -0,049   -0,049   -0,049   -0,056   -0,043
T-Value                         -1,09    -1,10    -1,11    -1,32    -1,13
P-value                         0,278    0,273    0,271    0,191    0,263

S                               0,910    0,904    0,897    0,893    0,890
R-Sq                            62,33    62,33    62,33    62,09    61,81
R-Sq(adj)                       57,20    57,83    58,45    58,80    59,09
Mallows C-p                      10,0      8,0      6,0      4,4      2,9

Step                                6
Constant                        35,00

Spot price
T-Value
P-value

ENOYR1
T-Value
P-value

ENOYR2
```

```
T-Value
P-value

ENOYR3                       -0,166
T-Value                       -7,26
P-value                       0,000

(ENOYR3-ENOYR2)/ENOYR2           23
T-Value                        1,51
P-value                       0,136

Rate of return (10-year gov)
T-Value
P-value

1/r10
T-Value
P-value

Hydro balance               0,00002
T-Value                        1,04
P-value                       0,303

Oil forward price (1pos)     -0,038
T-Value                       -1,00
P-value                       0,319

S                             0,888
R-Sq                          61,43
R-Sq(adj)                     59,25
Mallows C-p                     1,6


Step                              7       8
Constant                      33,74   32,76

Spot price
T-Value
P-value

ENOYR1
T-Value
P-value

ENOYR2
T-Value
P-value

ENOYR3                       -0,164  -0,162
T-Value                       -7,20   -7,07
P-value                       0,000   0,000

(ENOYR3-ENOYR2)/ENOYR2           26      40
T-Value                        1,72    3,31
P-value                       0,090   0,001

Rate of return (10-year gov)
T-Value
P-value

1/r10
T-Value
P-value

Hydro balance               0,00003
T-Value                        1,43
P-value                       0,157
```

```
Oil forward price (1pos)
T-Value
P-value

S                               0,888   0,895
R-Sq                            60,88   59,77
R-Sq(adj)                       59,25   58,67
Mallows C-p                       0,5     0,5
```

Regression Analysis: b versus ENOYR3; (ENOYR3-ENOYR2)/ENOYR2

```
The regression equation is
b = 28,3 - 0,135 ENOYR3 + 33,1 (ENOYR3-ENOYR2)/ENOYR2


88 cases used, 1 cases contain missing values


Predictor                   Coef  SE Coef       T      P
Constant                  28,313    1,988   14,24  0,000
ENOYR3                  -0,13499  0,01074  -12,56  0,000
(ENOYR3-ENOYR2)/ENOYR2     33,08    10,16    3,25  0,002


S = 0,902312   R-Sq = 72,6%   R-Sq(adj) = 72,0%


Analysis of Variance

Source          DF       SS      MS       F      P
Regression       2  183,508  91,754  112,70  0,000
Residual Error  85   69,204   0,814
Total           87  252,712
```

Regression Analysis: Change in b versus Change in EN; Change in (E

```
The regression equation is
Change in b = - 0,240 Change in ENOYR3 + 36,1 Change in (ENOYR3-ENOYR2)/ENOYR2


86 cases used, 3 cases contain missing values


Predictor                            Coef  SE Coef      T      P
Noconstant
Change in ENOYR3                 -0,24016  0,04945  -4,86  0,000
Change in (ENOYR3-ENOYR2)/ENOYR2    36,14    20,17   1,79  0,077


S = 0,837369


Analysis of Variance

Source          DF      SS      MS      F      P
Regression       2  36,639  18,319  26,13  0,000
Residual Error  84  58,900   0,701
Total           86  95,538
```

Appendix 5

Regression Analysis: b versus Lag b; ENOYR3; (ENOYR3-ENOY; Lag (ENOYR3-

```
The regression equation is
b = 17,9 + 0,405 Lag b - 0,0844 ENOYR3 + 81,7 (ENOYR3-ENOYR2)/ENOYR2
    - 73,9 Lag (ENOYR3-ENOYR2)/ENOYR2


86 cases used, 3 cases contain missing values


Predictor                      Coef  SE Coef      T      P
Constant                     17,944    3,254   5,51  0,000
Lag b                       0,40525  0,09329   4,34  0,000
ENOYR3                     -0,08444  0,01629  -5,18  0,000
(ENOYR3-ENOYR2)/ENOYR2        81,66    16,21   5,04  0,000
Lag 1 (ENOYR3-ENOYR2)/ENOYR2 -73,94    16,87  -4,38  0,000


S = 0,777924   R-Sq = 79,8%   R-Sq(adj) = 78,8%


Analysis of Variance

Source          DF       SS      MS      F      P
Regression       4  193,148  48,287  79,79  0,000
Residual Error  81   49,018   0,605
Total           85  242,167
```

Regression Analysis: Change in b versus Change in Ia; Change in EN; ...

```
The regression equation is
Change in b = - 0,214 Change in lag b - 0,273 Change in ENOYR3
              + 40,9 Change in (ENOYR3-ENOYR2)/ENOYR2 - 5,1 Change in lag (ENOYR3-
ENOYR2)/ENOYR2


84 cases used, 5 cases contain missing values


Predictor                            Coef  SE Coef      T      P
Noconstant
Change in lag b                   -0,2144   0,1020  -2,10  0,039
Change in ENOYR3                 -0,27329  0,05020  -5,44  0,000
Change in (ENOYR3-ENOYR2)/ENOYR2    40,91    19,56   2,09  0,040
Change in lag (ENOYR3-ENOYR2)/ENOYR2-5,12    18,65  -0,27  0,784


S = 0,801092


Analysis of Variance

Source          DF      SS      MS      F      P
Regression       4  41,590  10,397  16,20  0,000
Residual Error  80  51,340   0,642
Total           84  92,930
```

Regression Analysis: b versus Lag b; ENOYR3; (ENOYR3-ENOYR2)/ENOYR2

```
The regression equation is
b = 19,3 + 0,321 Lag b - 0,0923 ENOYR3 + 22,8 (ENOYR3-ENOYR2)/ENOYR2
```

```
87 cases used, 2 cases contain missing values


Predictor                   Coef  SE Coef      T      P
Constant                  19,306    3,531   5,47  0,000
Lag b                    0,32140  0,09951   3,23  0,002
ENOYR3                  -0,09227  0,01762  -5,24  0,000
(ENOYR3-ENOYR2)/ENOYR2     22,81    10,15   2,25  0,027


S = 0,857838   R-Sq = 74,8%   R-Sq(adj) = 73,9%


Analysis of Variance

Source          DF       SS      MS      F      P
Regression       3  181,130  60,377  82,05  0,000
Residual Error  83   61,079   0,736
Total           86  242,209
```

Appendix 6

Regression equations when only the 60% first observations are analyzed:

Regression Analysis: b versus ENOYR3; (ENOYR3-ENOYR2)/ENOYR2

```
The regression equation is
b = 29,3 - 0,139 ENOYR3 + 23,3 (ENOYR3-ENOYR2)/ENOYR2


Predictor                  Coef  SE Coef       T      P
Constant                 29,286    2,709   10,81  0,000
ENOYR3                 -0,13915  0,01362  -10,21  0,000
(ENOYR3-ENOYR2)/ENOYR2    23,27    18,43    1,26  0,212


S = 0,863012   R-Sq = 81,9%   R-Sq(adj) = 81,2%


Analysis of Variance

Source          DF       SS      MS       F      P
Regression       2  169,058  84,529  113,49  0,000
Residual Error  50   37,239   0,745
Total           52  206,297
```

Regression Analysis: b versus Lag b; ENOYR3; (ENOYR3-ENOY; Lag (ENOYR3-ENOYR2)/ENOYR2

```
The regression equation is
b = 20,2 + 0,410 Lag b - 0,0949 ENOYR3 + 71,0 (ENOYR3-ENOYR2)/ENOYR2
    - 83,2 Lag (ENOYR3-ENOYR2)/ENOYR2


52 cases used, 1 cases contain missing values


Predictor                        Coef  SE Coef      T      P
Constant                       20,210    4,008   5,04  0,000
Lag b                          0,4104   0,1094   3,75  0,000
ENOYR3                       -0,09492  0,01944  -4,88  0,000
(ENOYR3-ENOYR2)/ENOYR2          71,00    19,05   3,73  0,001
Lag 1 (ENOYR3-ENOYR2)/ENOYR2   -83,16    18,93  -4,39  0,000


S = 0,716338   R-Sq = 87,7%   R-Sq(adj) = 86,7%


Analysis of Variance

Source          DF       SS      MS      F      P
Regression       4  172,019  43,005  83,81  0,000
Residual Error  47   24,118   0,513
Total           51  196,136
```