An optimization-based model of the Nordic power market

Caroline Tandberg

TIØ4500 - Specialization Project
Managerial Economics and Operations Research
Supervisor: Stein-Erik Fleten
Department of Industrial Engineering and Technology Management
Norwegian University of Science and Technology
December 14, 2011

Preface

This work has been done as a part of the subject TIØ4505 Managerial Economics and Operations Research, Specialization Project at the Norwegian University of Science and Technology (NTNU) during the autumn of 2011.

I would like thank my supervisor, Associate Professor Stein-Erik Fleten, for advice and support during the report-writing. Special thanks also goes to Erlend Torgnes and Ole Lofsnes at Econ Pöyry for providing both data and guidance throughout the semester.

Abstract

In this paper, an optimization-based model of the Nordic power market is developed. The objective of the proposed method is to minimize the total generation cost, and the model is formulated as a deterministic linear program. The dual values for the power balance constraints can be seen as the power price, and these power prices are forecasted in each of the Nordic countries for a two-year horizon. The model successfully allocates the different generating units and transmission lines according to the demand, minimizing the total system cost during the planning horizon.

An important issue discussed in this paper is how to handle reservoirs in a long-term perspective for a hydrothermal system. In the proposed method, the reservoirs are aggregated in the respective countries in addition to the inflows being treated as deterministic parameters. This is a modeling oversimplification, and the results indicate that the power prices are very sensitive to a change in inflow-scenario. Other approaches to long-term hydrothermal scheduling where there is stochasticity involved are studied, and it is suggested that such an approach will better suit reality.

Contents

List of Figures

List of Tables

1 Introduction

Operations research (OR) is currently used actively in power generation scheduling, and it is important that these tools are continuously improved. As computers are getting better and power systems are expanding, there is much to gain by using advanced optimization software to aid the decision-making process. The main focus of this paper is long-term hydrothermal scheduling (LTHS), with an emphasis on the treatment of hydropower. The objective of the LTHS is to determine a generation schedule which minimizes the expected generation costs along the planning period. Because there is a limited amount of water available stored in reservoirs, the optimal operation is very complex. A system with a given load and an endogenous price is considered, and the aim is to capture the main properties of electricity prices through the model. While the Nordic power system is the target area for investigation in this paper, the results are universally applicable, especially in other hydro-dominated systems.

1.1 The power system

In order to better understand the scheduling problem, it is important to grasp how a hydrothermal power system works. Electricity is generated and consumed continuously and simultaneously, and the role of the transmission system is to connect the location where the power is generated to substations located near consumers.

1.1.1 Supply

There are many forms of electricity generation, and they can be categorized into nuclear, thermal and renewable sources. Thermal sources run on fossil fuels such as coal and gas, and CO_2 is released in the generation process. Renewable sources include hydro-, solar-, wind- and wave-power, and is characterized by that the sources for it will never run out. These energy sources build the supply curve according to their marginal costs, and in optimal system operation, the demand is always covered with the cheapest possible combination of generators.

1.1.2 Demand

Demand for power fluctuates during each 24-hour period and during each year. Since electricity is used for heating in most Scandinavian homes, the demand for electricity is temperature dependent, and at its highest during the winter months. The demand can be predicted in the short-term with a very high accuracy according to daily load curves and forecasts. In the long-term however, there is a greater uncertainty about the load, and the load is usually divided into elastic and inelastic demand.

1.1.3 Deregulation

A trend in power markets show a move from regulated markets toward deregulated markets. In a deregulated market there is a separation between the potentially competitive functions of generation and retail from the natural monopoly functions of transmission and distribution. Introduction of a deregulated market in the Nordics came in the beginning of the 90s, and there is now competition between the different power producers. The objective of the power production scheduling has therefore gone from a cost minimization to a profit maximization objective. In a liberalized market there is no effect of these different objectives, because they result in the same production plans. A liberalized market requires that a large number of buyers can buy from a large number of suppliers or producers. Nord Pool is the Nordic power exchange and selects the bids so that welfare is maximized (Wangensteen, 2007). Figure 1 shows a map of the Nordic region with the physical transmission lines and the price areas used by Nord Pool.

1.2 Generation planning

The task of planning the power generation is complicated by the presence of uncertainties. In the future very little is known for sure, and there is uncertainty associated with many factors, the most important being; inflows, power prices, consumption, fuel costs and CO_2 prices. In Norway most of the electricity generation comes from hydropower, but because of the connections to the continent, a hydrothermal system must be modeled. Seen from a socio-economic point of view, the objective of optimal operation of such a system is to determine a strategy which, for each stage in the planning period, given the system state, produces generation targets for each plant that will minimize the expected value of generation costs. These costs consists of fuel costs for

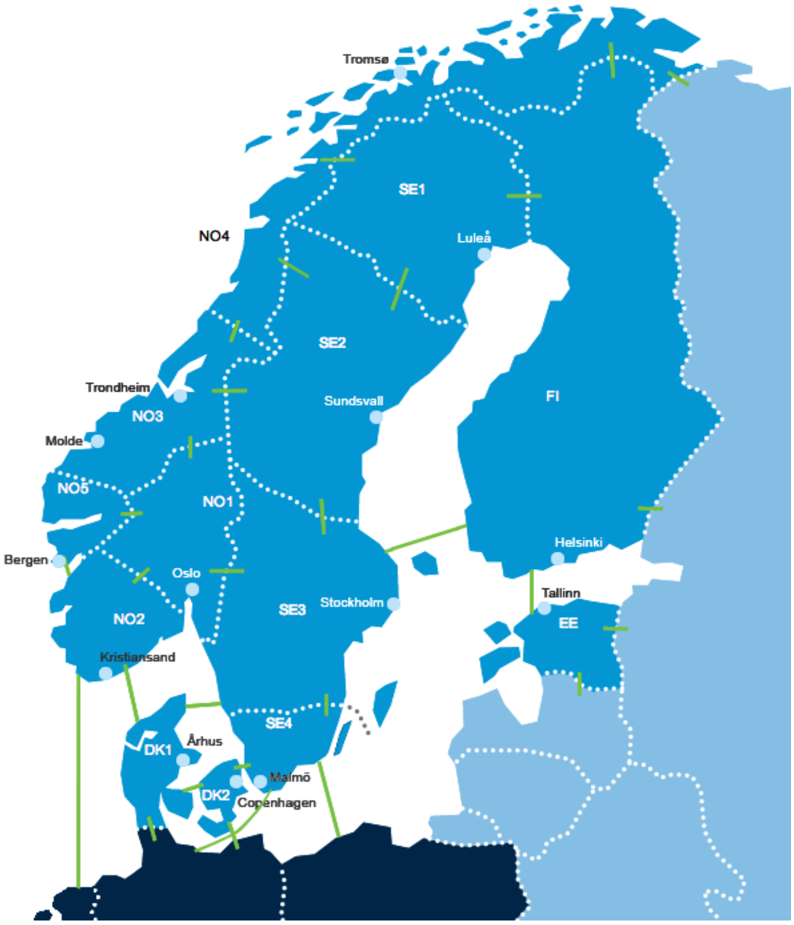


Figure 1: Picture of the Nordic power market with connections to Europe (*Source: Nord Pool*)

thermal units, purchase costs from neighboring systems, plus the penalties for failure of load supply. Everything ranging from generation, transmission and reservoir levels must be decided, and the ideal solution would be one single optimization process, taking into account everything and resulting in the optimal solution. However, due to the span in space and time, this is impossible to do with the required level of detail. The result is that the scheduling is divided into three stages; long-, medium-, and short-term.

From the long-term scheduling, a price forecast for the next 1-5 years is found. Although the time horizon can be much longer, up to 20-30 years, the definition of LTHS in this report is 1-5 years. The inputs into the model is everything from generation capacities in the different areas to demand forecasts. The areas used can be the same as the Nord Pool areas, or they could be even more detailed and based on river systems and bottlenecks. The main goal of the long-term scheduling is to find out how the water resource should be used during the whole period, and for hydro-producers this serves as strategic management. This is important to find out since there is only a limited amount of water, and this amount available is not known exactly due to uncertain inflows. Examples of models used for this purpose includes ECON BID and the EMPS model. In these models, the Nordic system is divided into a number of areas, and the reservoirs in each area is aggregated. The uncertainty of inflow is taken into account.

The seasonal scheduling serves as a link between the long-term and the short-term scheduling. The time horizon ranges from 3 - 18 months, and the model uses a simplified representation of uncertainty, and can even be deterministic. The main goal is to optimize the use of water within the period, while the end of the modeling period values must match with the corresponding long-term values. These values are can be either water values or reservoir levels, depending on how the models are coupled. The seasonal scheduling includes a more detailed representation of the system, and water values for the individual reservoirs are calculated. Water values are explained further in Section 3.2.1

In the short-term scheduling, a deterministic model is used where inflow and and prices are assumed known. A detailed description of the system is used, and a unit commitment problem is solved where the number and type of generators in use are decided.

1.3 Hydropower as an energy source

The combination of very high mountains and large plains in the Nordics make perfect conditions for reservoir-building. Collecting millions of tons of water in huge dams provides a means for controlling both the timing and the scale of hydro-generation. Many of the large reservoirs in Norway are situated in places where precipitation falls as snow during the winter, and this means low inflow into the reservoirs during winter. A snow reservoir builds up in the mountains, and when this starts to melt in spring, the inflow into the reservoirs is high. The melting season can continue well into the summer months.

Reservoirs provide a means for storing energy, and from an operations research point of view there are many decisions that must be made so that the reservoir is used in an optimal way depending on what goals one is pursuing. Reservoirs can be modeled mathematically and a set of assumptions, constraints, objectives and decision variables are specified. There are many factors which complicate the operational decisions of a hydro-plant owner, and the model should strive to include these factors. The most important complications are given in this section.

For hydro plants, a discharge capacity in m^3/s and an energy equivalent e in kWh/m^3 is usually specified (Doorman, 2009):

$$e = \frac{1}{3.6 \times 10^6} \cdot \gamma g H \eta \qquad [kWh/m^3] \tag{1.1}$$

where

γ : water density $[kg/m^3]$
g : gravity acceleration $[m/s^2]$
H : plant head $[m]$
η : plant efficiency

In reality, the efficiency of the plant η will depend on the discharge and the plant head. This results in a nonlinear relation between turbine discharge and power output. In the EMPS model, this is represented by a piecewise linear curve. In other models, a linear relationship is assumed in order to simplify the model. When reservoirs are aggregated and one large reservoir is modeled, assumptions of constant efficiency is usually accepted. However in short-term scheduling, this nonlinearity has a larger effect due to larger variations in head, and should be taken into account for the plant efficiency η.

Reservoirs are usually hydraulically coupled with other reservoirs, and large river systems should be modeled. If aggregation is used in the model, disaggregation methods are applied in order to get a feasible solution. Heuristics can be used for this purpose, although optimality can not be guaranteed.

1.4 Structure of this report

In the next section relevant literature is presented. Section 3 gives an overview of the approaches developed for long-term hydrothermal scheduling. The methods presented are deterministic and stochastic, and the strengths and weaknesses of each mettod is discussed. In Section 4, an example of a deterministic linear program is described. This model and the results are then analyzed and evaluated in Section 5, and the final conclusions and future work are given in Section 6.

2 Related work

In this section, related work regarding reservoir handling and hydrothermal scheduling is reviewed. Yeh (1985) classified the methods available for optimal reservoir handling at the time into four categories; linear programming (LP), dynamic programming, nonlinear optimization and simulation. Several of these methods can be combined in different ways, but few models deal simultaneously with all the aspects of the scheduling problem such as multiple periods, multiple reservoirs and stochastic inflows.

2.1 The development of stochastic solution methods

One common simplification is to use a deterministic model by replacing stochastic components by their expected values as in Soares and Carneiro (1991). It was implied early on by Massé (1946), and later argued by Gjelsvik (1982), that deterministic models are not well suited for the planning problem as they are not flexible enough. Stochastic dynamic programming and dynamic programming has been used in the scheduling problem for a long time, an early reference being Buras (1963). Yakowitz (1982) reviews the evolution of these models, and of these papers Turgeon (1980) is of particular importance. It aims to develop a method that deals with multi-reservoir systems by breaking up the original problem into a series of subproblems that are solved by dynamic programing. Egeland et al. (1982) is another example where decomposition methods are used to solve the multi-reservoir problem. In Gjelsvik et al. (1992) some stochastic methods are reviewed together with some important applications. A more recent review of dynamic programming applied to reservoir operation is given in Nandalal and Bogárdi (2007). In Wolfgang et al. (2009) an application of stochastic dynamic programming called the water value method is explained in relation to the EMPS model. The reservoirs are aggregated and heuristics are used in the de-aggregation procedure. Early investigations into the water value approach can be found in Stage and Larsson (1961) and in Lindqvist (1962).

2.2 Dimensionality reduction approaches

Because the dimensionality of stochastic dynamic programming can quickly get very large due to the curse of dimensionality, it has limited use for real-world problems. Birge (1985) develops decomposition and partitioning methods that are able to handle multidimensional problems, and these methods are further developed and applied to the generation scheduling problem by Pereira and Pinto (1985). This method is called stochastic dynamic dual programming, and was successfully applied to a 37-reservoir system, although larger systems have been solved more recently. Ferrero et al. (1998) aims at reducing the dimensionality of dynamic programming by limiting the time period to a two-stage algorithm that can handle multiple reservoirs.

2.3 Methods for the deregulated market

Yu et al. (1998) proposes a method that maximizes profit for hydroelectric plants in a deregulated system. The method breaks down the long-term problem into monthly based mid-term problems without using an exact cost function. Market power of producers in a deregulated market is discussed in Scott and Read (1996), where a multistage programming algorithm is employed called the dual dynamic programming. Their focus is in the New Zealand market where there are relative few suppliers, and each sub-problem is solved using a Cournot duopoly model. An evaluation of the efficiency of the Nordic market is performed by Halseth (1999), where a special case of supply curve equilibrium is used to describe possible strategies for suppliers.

3 Approaches to reservoir management

Reservoirs used for power generation should be managed in such a way that is optimal, and as discussed in the introduction, an optimal solution is one that minimizes operational costs of generation. Each single company must decide on how much water should be stored and how much should be released for generation in all periods. There are many factors that makes this a very difficult task (Soares and Carneiro, 1991):

- Long horizon of analysis (usually 2-3 years, but in reality to infinity)
- Stochastic nature of water inflows and load
- Operational inter-dependence between hydro plants in the same cascade
- Nonlinearity of thermal costs and hydro generation function.

In order to make the LTHS problem easier to solve, many simplifications can be done. Deterministic models assume that future inflows and demands are known along the planning period. Inflows can for example be replaced by their expected values. Linear Programming (LP) models linearize the nonlinear cost functions and hydro generation functions. In LP-models, the reservoirs are measured in MWh, and each unit of water has an energy equivalent value. Constant marginal cost is assumed for the thermal generators. Another simplification that is done in order to avoid the hydraulic coupling between reservoirs, is to aggregate the reservoirs into one large reservoir in each area. A simple deterministic, one-area, LP-model is given here for reference:

$$min\ \mathcal{W} = \sum_{t \in T} \Big[\sum_{g \in G} C_{gt} x_{gt} \Big]$$

subject to:

$$\sum_{g \in G} x_{igt} = D_t \qquad \forall t \in T \tag{3.1}$$

$$y_{t+1} - y_t + x_{hydro,t} \leq I_t \qquad \forall t \in T \tag{3.2}$$

$$y^{Min} \leq y_t \leq y^{Max} \qquad \forall t \in T \tag{3.3}$$

$$\sum_{g \in G} x_{gt} \leq GC_g \quad \forall g \in G, \forall t \in T \tag{3.4}$$

Here, t is the index for the time period, and T is the set of all time periods. The set of generators in the area is denoted G with index g. C_{gt} is the cost associated with generation, and x_{gt} gives the generation. The objective is to minimize costs in all periods. Constraint (3.1) states that supply must equal demand D_t in all time periods, also known as the power balance constraint. (3.2) gives the relationship of the reservoir levels y_t, the generation, and inflow I_t, this is known as the water balance constraint. Constraints (3.3) and (3.4) gives the reservoir level and generation capacities.

Obviously this model is a major simplification of reality. However it shows some of the challenges associated with LTHS, and this will be taken as a point of origin when explaining different solution procedures, and also when expanding the model into a Nordic model in Section 4.

3.1 Dynamic programming

Dynamic programming (DP) is a solution procedure that can be used to solve many structured optimization problems (Lundgren et al., 2010). It follows from Bellmans Principle of Optimality:

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. (Bellman, 1957)

For deterministic problems, this can be put into the following equation form called the Bellman backward relationship:

$$V_t(S_t) = \min_{x_t} \left\{ C_t(S_t, x_t) + V_{t+1}(S_{t+1}) \right\} \tag{3.5}$$

where S_{t+1} is the state we transition into if we are currently in state S_t and take action x_t. V_{t+1} is the accumulated suboptimal costs for all the stages following $t+1$, and $C_t(S_t, x_t)$ is the cost of the decision x_t given state S_t at stage t. V_{t+1} is also referred to as the future cost function or the cost-to-go function.

The LTHS problem separates easily into time stages, where the decision taken in one stage affects the decisions you can make in future stages. DP is therefore very well suited for solving reservoir operational problems. Another advantage with DP is that it handles nonlinearities in both constraints and objective function, and as discussed before, the LTHS is nonlinear of nature.

The elements of dynamic programming, together with the implications for optimal reservoir handling, are as follows (Powell, 2007):

The state variable This captures all the information we need to make a decision, as well as the information we need to describe how the system evolves over time. In our case the state variable is the reservoir level, $S_t = y_t$. In the multiple reservoir case this will be a vector containing all the reservoir levels, $S_t = (y_{1t}, ..., y_{nt})$, where n is the number of reservoirs.

The decision variable Decisions represent how we control the process. In the case of LTHS the decision in each stage is how much water to release from the reservoir for power generation, $x_{hydro,t}$. It also involves deciding how much thermal power to produce.

Exogenous information This is data that first becomes known each time period, which in our case is the inflow into each reservoir, I_t.

The transition function This function determines how the system evolves from the state S_t to the state S_{t+1} given the decisions that was made at time t and the new information that arrived between t and $t + 1$. This is the physical relationship between the reservoir level in $t + 1$ and t, which is equation (3.2) above.

The contribution function This determines the costs incurred or the rewards received during each time interval, ie. the cost of producing x amount of energy in period t.

The objective function Here we formally state the problem of minimizing the cost over a specified time period. The minimization is subjected to constraints in storage volume and release.

The solution process starts at the final stage T where the cost $V_T(S_T)$ is supposed to be known. In this way, it is possible to work backwards and find the optimal decisions at every stage and state. Decision tables providing optimal water discharge and operational costs for each possible discrete state of the system are given by this procedure. By using Bellman's principle and working backwards in time using the fact that the optimal decisions are known in the future states, the number of decision variables are radically reduced.

In DP, the states must be discretized. That means that at each stage the reservoir levels are evaluated across a range of possible levels. The problem becomes more realistic the more reservoir levels one operates with. However the problem also becomes very large, especially when the state variable is a vector (in the multiple-reservoir case). This is known as the "curse of dimensionality" and prohibits large problems from being solved using DP.

3.2 Stochastic dynamic programming

Uncertainty is an important factor in reservoir management. There is uncertainty in all the factors discussed in Section 1.2, but our focus here will be on stochastic nature of inflow. In Stochastic dynamic programming (SDP), the present decision is optimized with this uncertainty taken into account. A probability distribution of possible inflow scenarios is considered in each stage, and a multi-stage stochastic optimization is performed for each possible outcome. The SDP optimization process derives the optimal operating strategy from Bellman's backward recursive relationship (given here for a single reservoir optimal operation) (Nandalal and Bogárdi, 2007):

$$V_t(S_t) = \min_{x_t} \Big\{ C_t(S_t, x_t) + E\{V_{t+1}(S_{t+1})\} \Big\} \tag{3.6}$$

The objective is to minimize the expected sum of costs over the whole time period. The difference between SDP and DP is that SDP takes stochasticity into account.

3.2.1 The water value method

The water value method is a special version of SDP, and can be used to solve the LTHS problem (Wolfgang et al., 2009). The more water that is used for generation in the present period reduces the availability of water in the future. The value of using water in the present period must therefore be balanced against the possibility of using that water in the future. The value of the water is a function of future development depending on load, inflow and market prices. The "water value", which is actually the *expected marginal value* of the energy stored in the reservoir, is calculated at each decision stage. These are calculated recursively, starting at the end of the period. Some starting values for the last time period, T, must be used in order to be able to calculate the water values in time period $T-1$. These are usually estimated. The more water the reservoir has, the less the value of the water will be. The water values at stage $T-1$ is then found by finding the optimal operation of the reservoir considering the inflow probabilities and the water values in period T. One can calculate the water values recursively, and in the end a table with water values at all stages and reservoir levels has been found. These can then be used to help to find the optimal decisions in the current stage.

Both DP and SDP have the disadvantage of having to discretize all the future states. The expected cost is calculated at each possible state, and this in turn causes the problem to grow exponentially in size when more variables and states are added. Although they handle nonlinearities very well, the curse of dimensionality limits their use. Aggregation of reservoirs is a method used to lower the number of variables needed, but the dimensionality of the problem is still very large. The use of SDP is usually limited to handling only a handful of reservoirs. (Gjelsvik et al., 1992)

3.3 Approximate dynamic programming

Approximate dynamic programming (ADP) is similar to DP, but instead of the backward recursive relationship, the solution procedure steps forward through time. In DP, the solution requires that we loop over all possible states, exactly computing the value function which we then use to produce optimal decisions. In ADP, the future value function is not known, and hence it cannot use the algorithm for DP. The value function is instead approximated in an iterative manner for all possible states. In each iteration only some of the value functions are updated.

Moving forward in time, the exogenous information is estimated in each stage, and a *sample path* is followed. In the LTHS problem this sample path represents a unique sequence of inflows, and these can be estimated by using the probablitity distribution together with a Monte Carlo simulation.

Once we have the approximated value functions $\bar{V}_t(S_t)$ and the sample path, optimal decisions can be taken moving forward through the sample path. After each iteration a new sample path is made, and the approximated value functions in iteration n, are improved by using to the following equation:

$$\hat{v}_t^n = \min_{x_t \in \mathcal{X}_t^n} \left\{ C_t(S_t^n, x_t) + \gamma \sum_{\hat{\omega} \in \hat{\Omega}_{t+1}^n} p_{t+1}(\hat{\omega}) \bar{V}_{t+1}^{n-1}(S_{t+1}) \right\} \tag{3.7}$$

The improved approximated value function, $\bar{V}_t^n$, is then computed by using a weighting between $\hat{v}_t^n$ and $\bar{V}_t^{n-1}$, known as "smoothing". This equation is similar to the backward recursive equation of DP, but the expected value function is replaced by its approximation $\bar{V}_{t+1}^{n-1}(S_{t+1})$ that was found in the previous iteration. χ_t^n is the feasible region for the decisions x_t, $\hat{\Omega}_{t+1}^n$ are all the possible sets of outcomes in the inflow, while $p_{t+1}(\hat{\omega})$ is the probability of outcome $\hat{\omega} \in \hat{\Omega}_{t+1}^n$. $\gamma \leq 1$ is a discount factor.

In ADP, there is no need to loop over all possible states, and there is also no requirement that the inflow scenarios are independent from the previous period. However there are a few downsides with ADP that has to be dealt with; We only update the values of states we visit, but the states we have not visited gets the same value as in the last iteration. This can eventually cause these states to look uninviting even though they can produce better value functions. There is still a need to find the set of possible inflows, and the states of the system needs to be discretized.

3.4 Stochastic dual dynamic programming

Stochastic Dual Dynamic Programming (SDDP) is a very powerful method to solve the hydrothermal problem without the need for discretization of future states. It is described in Pereira and Pinto (1991), and it is able to handle multi-reservoir systems. SDDP is based on approximation of the expected cost-to-go function of SDP by piecewise linear functions. These piecewise functions are obtained from the dual values of the optimization problem.

We first study a two-stage, one-reservoir deterministic example. This is equivalent to a Benders decomposition algorithm (Benders, 1962), which can be stated as follows:

$$\min_{x,y} w = f(x) + c^T y \tag{3.8}$$

$$s.t. \quad Ay + Bx \geq I \tag{3.9}$$

This problem can be regarded as a general form of the reservoir optimization problem where x is the discharge decision taken here and now, and $c^T y$ is the future cost function at reservoir level y. The constraint (3.9) is a general form of the water balance constraint (3.2). The other constraints are left out here for simplicity. For a given discharge decision $\bar{x}$, the second stage problem will be:

$$\min \alpha = c^T y \tag{3.10}$$
$$s.t. \quad Ay \geq I - B\bar{x} \tag{3.11}$$

And the first-stage optimization problem is:

$$\min w = f(x) + \alpha(x) \tag{3.12}$$

By taking the dual of the second-stage problem we get a maximization problem:

$$\max \quad \pi(I - B\bar{x}) \tag{3.13}$$
$$s.t. \quad A^T \pi \leq c \tag{3.14}$$

If we knew all the extreme points of the constraint set $A^T\pi \leq c$, the objective value of (3.13) is the same as that of (3.10). However this would be computationally demanding, so instead an approximation of the future cost function is found. For an initial set of n trial decisions $\{x_1, .., x_n\}$ we can calculate the set of associated dual values $\{\pi^1, ..., \pi^n\}$. The approximated future cost is then found as:

$$\min \hat{\alpha}(x) = \alpha \tag{3.15}$$
$$s.t. \quad \alpha \geq \pi^i(I - B\bar{x}) \quad \forall i = 1, ..., n \tag{3.16}$$

The value for $\hat{\alpha}(x)$ can then be substituted into the first-stage objective function (3.12), to minimize w as a function only of x. This will be a *lower bound* for the future cost-function, and an *upper bound* can be found by evaluating (3.8) for $\bar{x}$. The idea behind SDDP is that for each trial decision x, $\hat{\alpha}(x)$ will improve in value, i.e, the lower bound will increase. This implies that the future cost function can be evaluated without the need for discretization of x. Each new constraint $\pi^i(I - B\bar{x})$ can be seen as a linear approximation of the future cost function. In the multi-reservoir case, a set of π^{ik} are found in each iteration, where k is the number of reservoirs. These represent the cost-to-go functions for each reservoir as a set of hyperplanes. The hyperplane approximations are built iteratively, to give increasing accuracy. By choosing the initial discharge decisions with care, we can quickly get a very good representation of the future cost-to-go function. In this way, the many-dimensional water value tables used in SDP are avoided.

Extensions to this model are used to represent the multistage, stochastic problems. For the multistage problem with T time periods, the algorithm runs iteratively forwards and backwards through all the time periods. In the *forward* run, the optimization problem is solved with a set of trial decisions $\bar{x}_{t-1}$ for every stage t. In the *backward run*, additional hyperplanes are constructed in the previous stage by using multipliers found in the current stage. To extend the problem into the stochastic one, we assume that the inflow vector I_t is discretized into m scenarios $\{I_{tj}, j = 1, ..., m\}$. The idea is then to determine "good" trial decisions at each stage. Ideally the forward and backward iteration is run for every combination of scenarios I_{tj}, but more realistically, a Monte Carlo simulation is carried out.

4 Mathematical formulation

An outline of a deterministic LP model that can be used to solve the LTHS is given in this section. It is an extension of the model outlined in the start of Section 3, and the goal of the model is to forecast the spot price in each country of the Nordic region. The countries consists of Norway (NO), Sweden (SE), Finland (FI) and Denmark (DK). The outside world has been modeled according to their power prices, and the most important countries to model is the exchange with Germany (GE), Poland (PO) and the Netherlands (NL). The data used in the model together with sources are given in Section 4.4, while the mathematical model is given in Section 4.5.

4.1 Sets and indices

A : set of areas, indexed by i :
T : set of time periods, indexed by t :
L : set of transmission lines, indexed by l :
F : set of fuel types, indexed by f :
G : set of generator types, indexed by g :
K : set of generator categories, indexed by k :
J : set of outside regions, indexed by j :

4.2 Parameters

α :	curtailment cost (constant)
γ :	losses in the lines in percent
C_{gt} :	running cost of generator type g in time period t (€/MWh)
D_{it} :	firm demand in area i in period t
TC_l :	transmission capacity of flow in line l (MWh/h)
GC_{ki} :	capacity of generator type k in area i (MW)
CHP_t :	CHP generation in time period t (MWh/h)
I_{it} :	inflow in area i in period t (MWh/h)
y_i^{Max} :	reservoir maximum in area i
y_i^{Min} :	reservoir maximum in area i
y_i^{Start} :	start reservoir level in area i
y_i^{End} :	end reservoir level in area i
ToA_l :	gives the area/region that line l goes to
FrA_l :	gives the area/region that line l goes from
PP_{jt} :	power prices in outside region j in time period t
PF_{ft} :	fuel prices of fuel type f in time period t
PC_t :	price of CO_2-quota in time period t
N_f :	carbon content in fueltype f
E_f :	energy content in fueltype f
V_g :	variable operating cost of generator type g t
η_g :	efficiency of generator g
$Gcat_g$:	gives the generator category of generator type g

4.3 Variables

x_{igt} :	generation of generator type g in area i in time period t
y_{it} :	reservoir level in area i in beginning of time period t
z_{it} :	curtailment in area i in period t
b_{lt} :	cross border flow in transmission line l period t

4.4 Data

The granularity of the model is hourly, and the time horizon is two years. Data for the model is from the years 2008 and 2009, and these are the years to be forecasted. The data that is too large to present here can be found in the file "data.xls".

4.4.1 Generation capacities

Generation capacities are assumed to be constant for the whole of the modeling period. Capacities are given for hydro, nuclear, thermal and gas turbines in each area. The category "thermal" consists of a few generator types that are grouped into one category. The capacities can be seen in Table 1, and the data is from Nordel.

Table 1: Generation capacities

	NO	**SE**	**FI**	**DK**
Hydro	29474	16195	3097	0
Nuclear	0	8938	2646	0
Thermal	0	2271	2935	784
Gasturbine	699	1607	840	412

Wind generation is stochastic and is left out of the model due to lack of hourly data. The curtailment cost is constant and equal to 1000 €/MWh.

4.4.2 CHP generation

Combined heat and power (CHP) units can produce both heat and electrical power, and is divided into Industry and District generation. The power generation depends on both fuel prices and outside temperature. As a simplification, CHP is modeled as a fixed generation profile, i.e. one that is not optimized on the basis of prices. The data is from Finnish Energy Industry, Svensk Energi and Energinet, and is the combined production from CHP Industry and CHP District. The hourly production is simulated from the weekly generation.

4.4.3 Generation costs

The following equation was used to calculate the running cost of the generator types in all time periods:

$$C_{gt} = \frac{PF_{ft}}{E_f \eta_g} + V_g + \frac{PC_t N_f}{\eta_g} \tag{4.1}$$

The cost of generation depends on the cost of the fuel used, the variable cost of production, and the CO_2 quota prices. The efficiency of a generator type is also relevant for generation costs, and a table containing important characteristics for the different generator types can be found in Table 2. The fuel types used in the model are coal and gas, and the hourly fuel prices are derived from weekly average prices. While the data for the gas prices are from Nord Pool Gas, the coal prices are from Vattenfall. Oil prices have minimal effect on the generation costs as there are few generators running on oil in the Nordics. Therefore oil prices are not included in this model. The CO_2 quota prices only change once daily, but in the model they are specified hourly. The quota prices are also from Vattenfall. Data for carbon and energy content for each fuel type is presented in Table 3. The data in Tables 2 and 3 are from Econ Pöyry.

Table 2: Generator information

Generator type	Category	Fuel type	Efficiency	Variable Cost [€/MWh]
CoalCondensing	Thermal	Coal	42 %	1.5
CoalExtraction	Thermal	Coal	41 %	1.5
GasExtraction	Thermal	Gas	39 %	1.5
CCGT	Thermal	Gas	54 %	0.8
GasTurbine	Gasturbine	Gas	38 %	0.8
Hydro	Hydro	Water	100 %	0
Nuclear	Nuclear	Uranium	100 %	15

Table 3: Fuel characteristics

Fuel type	Carbon content [ton/MWh]	Energy content [MWh/ton]
Coal	0.34	7.17
Gas	0.20	13.33

4.4.4 Demand

The hourly demand in each area is found from the aggregated demand together with the daily demand distribution between the areas. The demand distribution is assumed constant throughout the day, hence we can calculate the hourly demand in each area. The daily consumption data is from Nord Pool, and the total hourly demand is from Vattenfall.

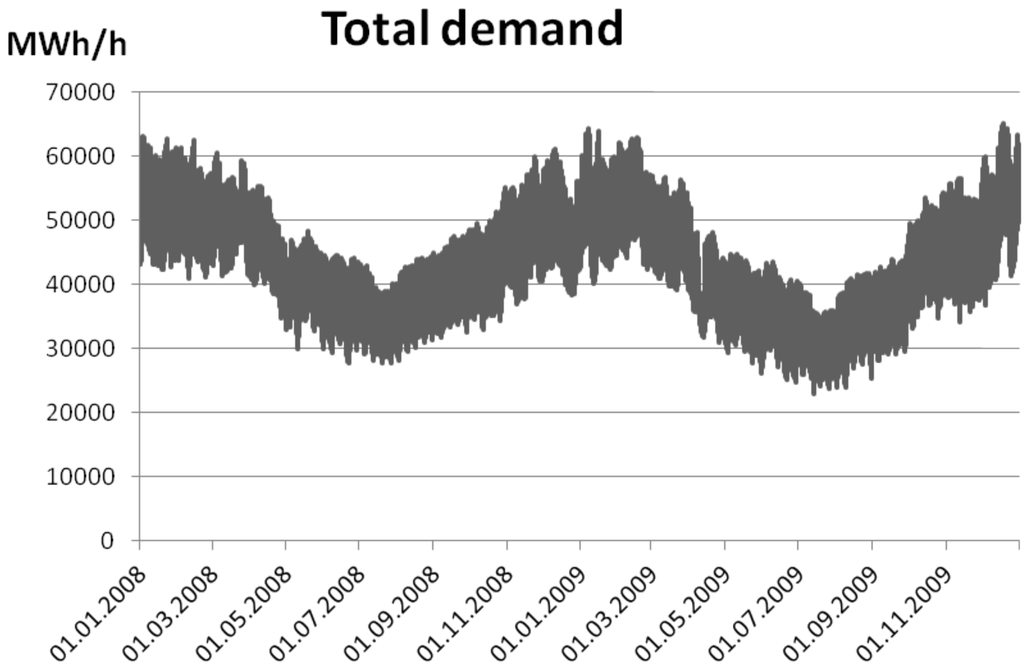


Figure 2: Aggregated demand in the Nordics

In Figure 2, it can be seen how the demand for power fluctuates during the year, and there are also major daily variations.

4.4.5 Transmission capacities

Transmission capacities between regions are found by aggregating area-capacities to a country level. The transmission capacities are modeled as constant throughout the entire period, and they can be seen in Table 4. The data is from Econ Pöyry. Losses are assumed to be 1% of the transmitted amount.

Table 4: Transmission capacities

From area	To area	Capacity [MW]
NO	SE	3550
NO	DK	950
NO	FI	100
SE	NO	3900
SE	DK	1980
SE	FI	2050
DK	NO	950
DK	SE	2440
FI	NO	100
FI	SE	1650
NO	NL	700
SE	GE	600
SE	PO	600
DK	GE	2085
GE	SE	600
GE	DK	1550
NL	NO	700
PO	SE	600

4.4.6 Reservoir data

The reservoir capacities in GWh are aggregated for each area as if there was one large reservoir in each country. The starting level in each country is given, and the final reservoir level is not allowed to be smaller than a certain amount. The data is given in Table 5, and is from NVE, Svensk Energi and Finnish environment inst. The minimum reservoir level in all periods is 1000 GWh in all countries.

Table 5: Reservoir data [GWh]

	Norway	**Sweden**	**Finland**
Reservoir capacity	81888	33758	5530
Start value	60901	23587	4113
End value	55342	19636	3155

4.4.7 Inflow

Hourly inflow into the reservoirs in MWh/h in each area is derived using weekly accumulated values and assuming constant inflow each week. The aggregated inflow data is from Vattenfall, while the disaggregated values are from Nord Pool Spot. The inflows can be seen in Figure 3, and it can be seen that the inflows are highest during the spring and summer months. The inflow in 2008 was higher than the inflow in 2009.

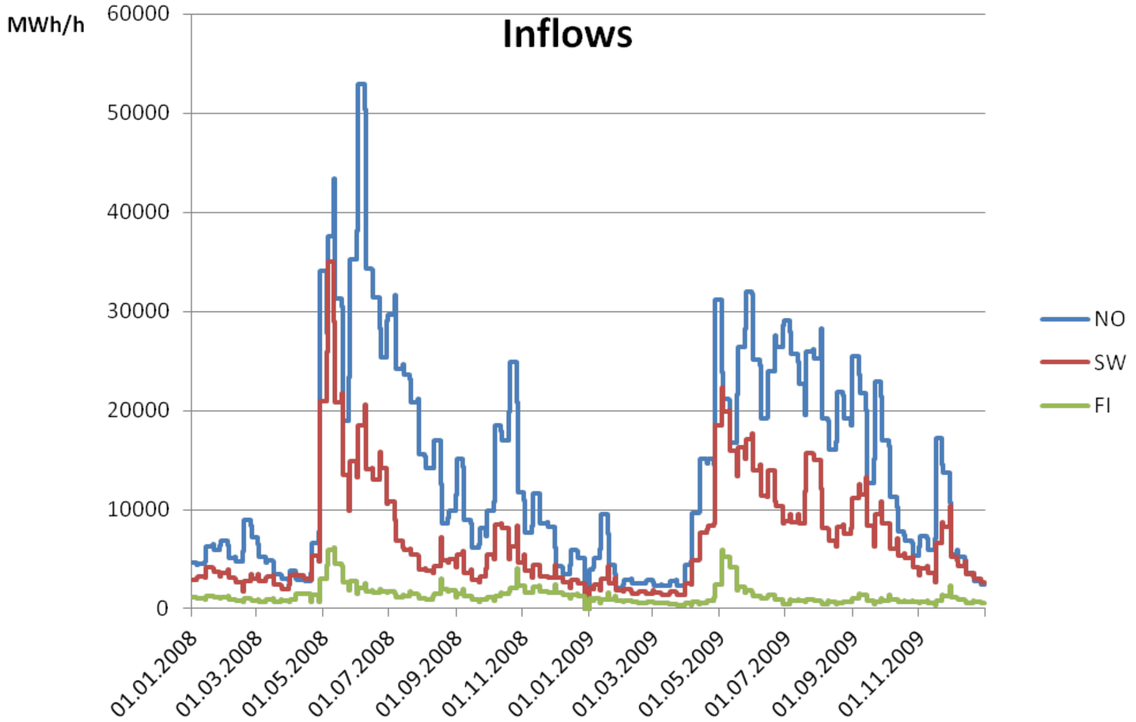


Figure 3: Inflows in respective areas

4.4.8 Outside region power prices

A daily profile of power prices in the outside areas together with weekly average power prices are used to simulate the hourly prices. The daily distribution of prices is from Econ Pöyry, and the weekly averages are from the Power Exchanges in the respective outside regions. Where the weekly averages were not available, the monthly average was used. All prices are in €/MWh.

4.5 Mathematical model

$$min\mathcal{W} = \sum_{t \in T} \Big[\sum_{i \in A} \sum_{g \in G} C_{gt} x_{igt} \tag{4.2a}$$

$$+ \sum_{i \in A} \alpha z_{it} \tag{4.2b}$$

$$+ \sum_{j \in J} \Big(\sum_{l \in L | ToA_l = j} PP_{jt} b_{lt} - \sum_{l \in L | FrA_l = j} PP_{jt} b_{lt} \Big) \Big] \tag{4.2c}$$

subject to

$$\sum_{g \in G} x_{igt} + z_{it} + \sum_{l \in L | ToA_l = i} b_{lt} - \sum_{l \in L | FrA_l = i} b_{lt} = D_{it} - CHP_t \tag{4.3}$$

$$\forall i \in A, \forall t \in T$$

$$y_{i,1} = y_i^{Start} \tag{4.4}$$

$$\forall i \in A$$

$$y_{i,t+1} - y_{it} + x_{i,hydro,t} \leq I_{it} \tag{4.5}$$

$$\forall i \in A, \forall t \in T$$

$$y_i^{Min} \leq y_{it} \leq y_i^{Max} \tag{4.6}$$

$$\forall i \in A, \forall t \in T$$

$$y_{i,nT} \geq y_i^{End} \tag{4.7}$$

$$\forall i \in A$$

$$\sum_{g \in G | Gcat_g = k} x_{igt} \leq GC_{ki} \tag{4.8}$$

$$\forall i \in A, \forall k \in K, \forall t \in T$$

$$b_{lt} \leq TC_l \tag{4.9}$$

$$\forall l \in L, t \in T$$

4.6 Discussion of the model

4.6.1 Objective function

The objective is to minimize total cost in all areas and time periods, and these consists of generation cost (4.2a) and curtailment costs (4.2b). In addition there is the possibility to trade power with the outside regions. This is represented in the objective function as a negative cost when selling and a positive cost when buying as seen in the part (4.2c).

4.6.2 Constraints

Constraint number (4.3) is the power balance constraint, making sure the demand is covered in all areas and time periods. The demand is either covered by their own generation, by import/export, or by demand curtailment. The z_{it} functions as a slack variable, and an alternative to having such a slack variable with a high cost is to change the equality sign with a greater than or equal to sign. Then the problem would be infeasible if the system was unable to cover the demand, so the current formulation provides a greater flexibility. Constraints (4.4), (4.6) and (4.7) are reservoir constraints on the minimum and maximum level of water and the start and end values. (4.5) is the reservoir balance constraint giving the link between inflow, hydro generation and the reservoir levels. A spill variable could have been used here to give equality, but it is not necessary in this formulation as we have a maximum reservoir level. (4.8) gives the capacity constraint of each generator category in each period and area, while (4.9) gives the transmission capacity constraint for each line in all periods.

4.7 Number of variables and constraints

If all the variables are created, the number of variables would be:

$$|A| \times |G| \times |T| + |A| \times |T| + |A| \times |T| + |L| \times |T|$$

And the number of constraints:

$$|A| \times |T| + |A| + |A| \times |T| + |A| \times |T| + |A| + |A| \times |K| \times |T| + |L| \times |T|$$

With $|T| = 17544$ time periods, $|G| = 7$ generator types, $|A| = 4$ areas and $|L| = 18$ transmission lines, the number of variables is 947 373 and the number of constraints are 807 024. However we know that some of the variables will not take any values so we can choose not to create these variables. For example there are no reservoirs in Denmark, to the y_{it}-variables for denmark do not need to be created. In Xpress^{MP}, this is done by creating the variables dynamically as we need them. By doing this, the number of variables is reduced to 877 197. This is still a large number of variables, and the solution time is 270 seconds.

By having an hourly resolution in the model the number of variables is very large. In a deterministic model it is solvable, but in stochastic models the number of variables created would be very large. It is therefore common to have a larger time resolution for stochastic models, for example weekly or monthly. The challenge is then to secure load capacity for peak load. This is done by using a load curve that shows the distribution of power consumption during the time period.

4.8 Implementation

The solver package used is Xpress^{MP} and the implementation is written in the MOSEL language. The optimizer solves the model with the data presented in Section 4.4 with regards to the constraints and objective presented in Section 4.5. The MOSEL-code is attached in Appendix A.

5 Results

Results from running the model is given in this section together with some analysis of these results. In Section 5.1, power prices, water values and reservoir levels are discussed. In Section 5.2, a glimpse of what is happening in a given time period is analyzed, while in Section 5.3 other inflow scenarios are discussed.

5.1 Prices and costs

In optimization, the dual value of the constraint is called the shadow price. This is the marginal cost of strengthening the constraint, or the marginal utility of relaxing the constraint. These are studied here in order to better understand the results of the model.

5.1.1 Power balance

The dual value of the power balance constraint (4.3) is the marginal cost of consuming one more unit of power, and this can be seen as the power price in €/MWh. The power prices are shown in Figure 4.

5.1.2 Water balance

The dual values of the water balance constraint (4.5) can be seen as the marginal value of having one more unit of water available. According to the theory presented in 3.2.1, this is known as the water value and is the marginal cost of using water. The water values can be seen in Figure 5. Because the objective function decreases as the constraint is relaxed, the sign is negative. The reservoir levels are shown in Figure 6. Although the reservoirs never reach the maximum level in any of the areas during the scheduling period, they all reach the minimum reservoir level around the same time. This time is around the beginning of April 2009, and if we compare these results with the inflow data in Figure 3 we can see that it is at this time in 2009 that the melting season starts.

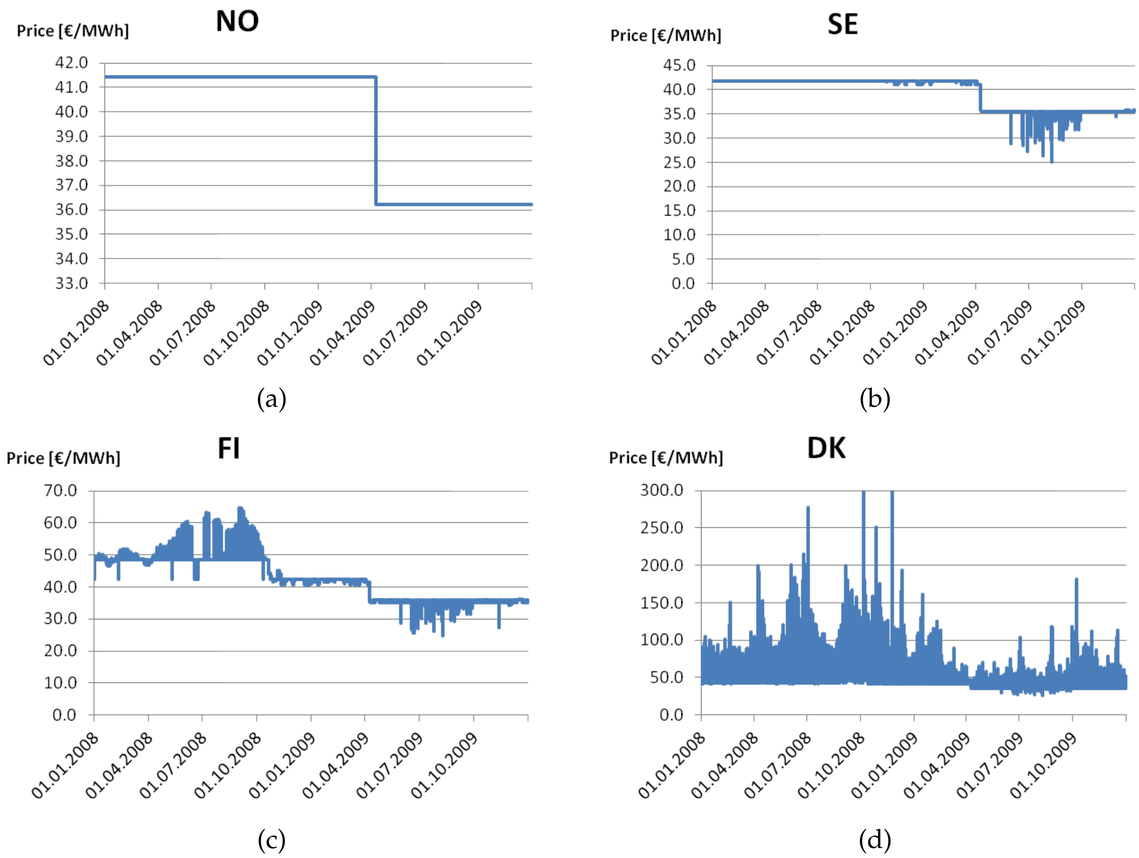
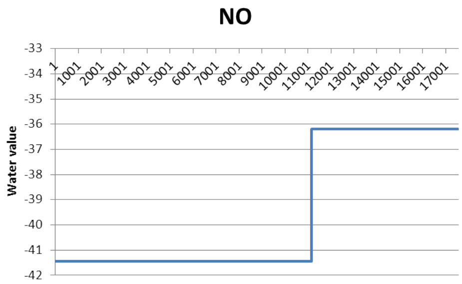
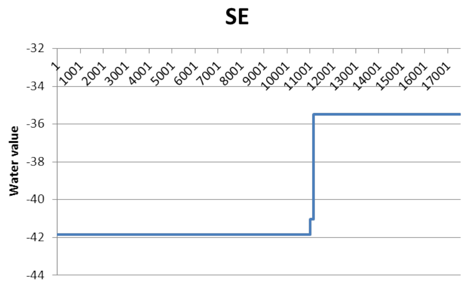


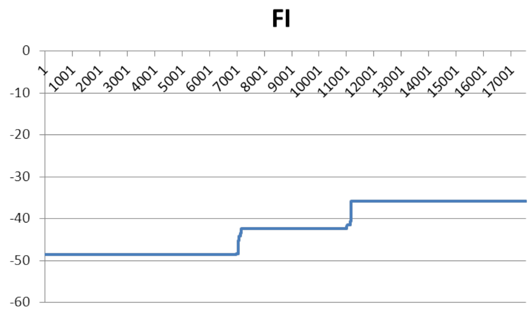
Figure 4: Power prices



(a)

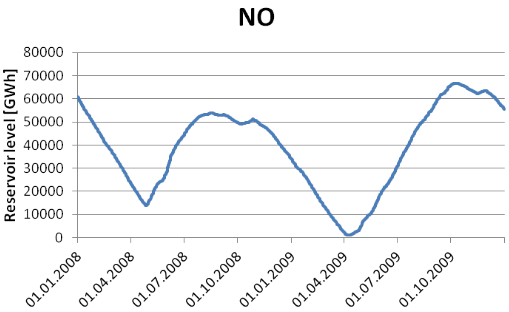


(b)

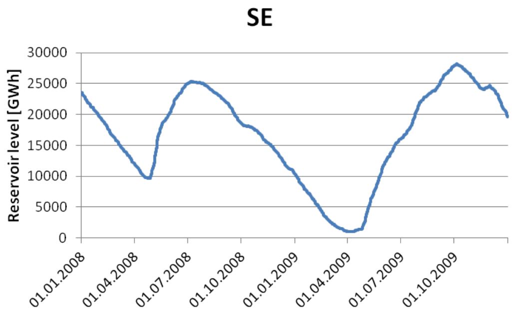


(c)

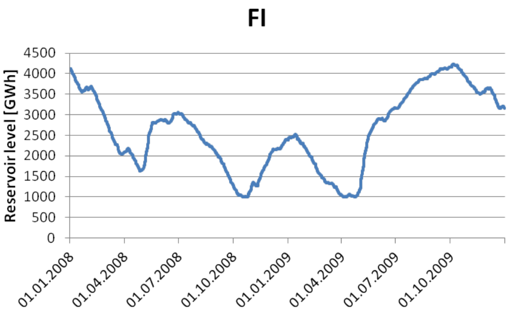
Figure 5: Water values



(a)



(b)



(c)

Figure 6: Reservoir levels

5.1.3 Analysis

The optimal generation is found by the optimizer using a merit order list. This includes ranking the available generation units according to their marginal cost, and engaging them in that order until demand is covered or their capacities are met. The power price in the outside regions can be seen as a marginal cost, and is ranked by the optimizer in the same way as a generator with the transmission capacity as the limit. The price of the most expensive generation unit or outside region price sets the dual value of the power balance constraint which can be seen as the power price. The water values are used as the marginal cost of hydropower.

By comparing the power prices in Figure 4 and the water values in Figure 5 in the three hydro-producing countries, we can see a strong connection. Especially in Norway, the power prices and the water values are exactly the same (but with opposite sign). This means that it is always the water value that governs the power price in Norway. By comparing this to the generation capacities in Table 1, we see that most of the generation capacity is from hydropower in Norway. In Sweden and Finland there are more fluctuations, but we can still see a strong connection. The water values are major price-drivers, but at times there are other units in these areas determining the power price. For Denmark, the price fluctuates during the whole scheduling period. They do not have hydropower, but they have strong transmission capacity from Norway, Sweden and Germany. So the price in Denmark is more vulnerable to changes to the German power prices than the water values in Norway and Sweden.

The minimum reservoir levels of 1000 GWh are reached in the beginning of April in 2009. So in the model, it is optimal to empty the reservoirs as much as possible right before the melting season starts in 2009. At the same time the water values in Norway and Sweden drop, and hence the power prices drop. This can be explained in relation to the model being deterministic. Because inflow values are perfectly predictable, the optimizer chooses to decrease the value of the water right at the beginning of the melting season when the water is at the minimum level. In reality, because of the stochastic nature of inflows, the water value is inversely proportional to the reservoir level. This points to a major flaw in using deterministic values.

It can also be noted that there is never any gap between the production and the consumption. There is always enough power to satisfy demand, and the gap cost of 1000 €/MWh is never used. The highest power prices are seen in Denmark during December 2008, and the powe price reaches more than 300 €/MWh. However because we have left out wind power completely, these prices would not be reached in reality.

5.2 Analysis of a single time period

To better understand the model, we study a single time period to see what is going on. The period chosen is period 10 000, and this corresponds to the hour 4pm-5pm on the 20th of February 2009.

The generation costs are seen in Table 6, while the actual generation is seen in Table 7. From Table 8, we see that in all the three hydro-producing countries, it is the water value that sets the power price in this particular time period. The power prices in outside areas are seen in Table 9, and from this we can see that it is the German power price that sets the Danish power price in this hour. This price is slightly higher than in the rest of the Nordic region. In each area, only the units which are below the power price are set into use, and by comparing to their respective capacities from Table 1 we can see that the Nuclear and CoalCondensing units are producing on max capacity. Ofcourse the 0 €/MWh for hydro generation is misleading, as the water value is actually the marginal cost.

Table 6: Generation costs

Generator type	Cost [€/MWh]
Hydro	0.0
Nuclear	15.0
CoalCondensing	28.2
CoalExtraction	28.8
GasExtraction	47.4
CCGT	34.0
GasTurbine	48.0

Table 7: Generation by type (average hourly production)

Generator type	Area	Generation [MW]
Hydro	NO	20139
Hydro	SE	8735
Hydro	FI	2324
Nuclear	SE	8938
Nuclear	FI	2646
CoalCondensing	SE	2271
CoalCondensing	FI	2935
CoalCondensing	DK	784

Table 8: Shadow prices

	NO	SE	FI	DK
Dual costs	41.4	41.9	42.3	43.6
Water values	-41.4	-41.9	-42.3	

Table 9: Outside power prices [€/MWh]

GE	NL	PO
43.6	42.7	39.3

From Table 10, we see which transmission lines are in use and the amount transferred in these lines. We see that the power flow in the transmission lines are at their maximum values when we compare to the capacities in Table 4 in all the transmission lines except for in the line from Denmark to Germany. This line is shown in italic text and is not transmitting at the limit.

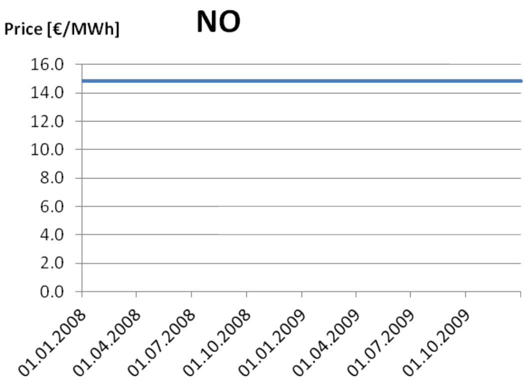
Table 10: Border flow

From area	**To area**	**Border flow [MW]**
NO	DK	950
NO	FI	100
SE	DK	1980
NO	NL	700
SE	GE	600
DK	*GE*	*160*
PO	SE	600

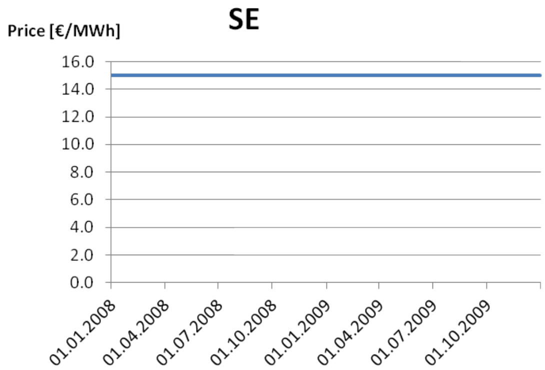
It is typical in linear programming that variables are at their limits, so it is not surprising to see that this is the case for the border flow and generation variables.

5.3 Other inflow scenarios

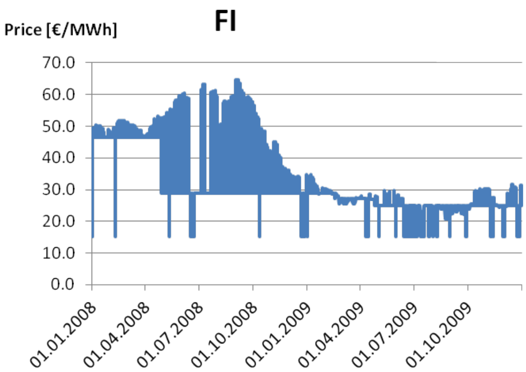
To investigate the effect that the inflow has on the output, two other inflow scenarios are run in the model; one where the inflow is 25% higher than in the original case for all time periods, and one where the inflow is 25% lower. The results on the power prices are shown in Figure 7 (high inflow) and in Figure 8 (low inflow).



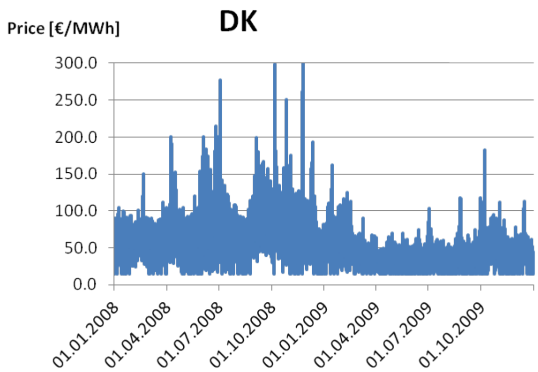
(a)



(b)



(c)



(d)

Figure 7: Power prices in case of high inflow

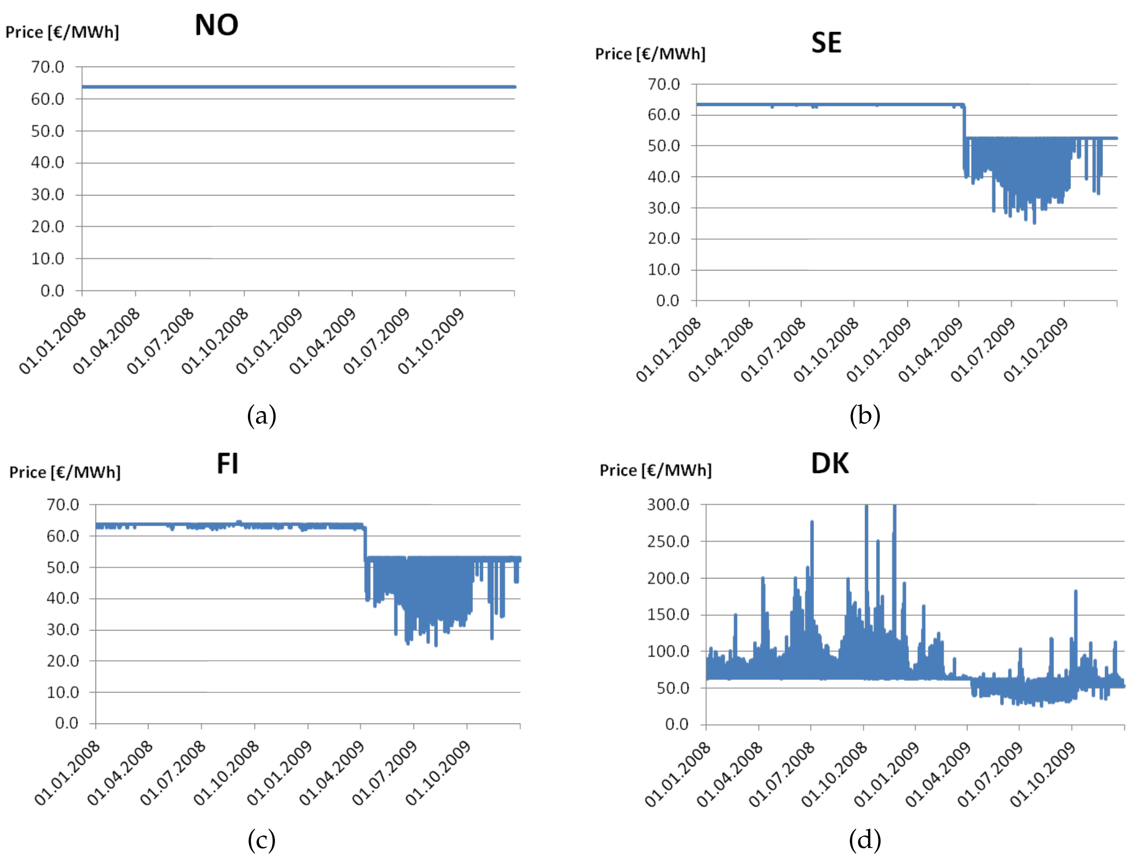


Figure 8: Power prices in case of low inflow

From these figures we see that the power prices are very much affected by the inflow levels. The objective function value has not been discussed earlier because it has little relevance in this model. The factors that we are interested in are the dual values of the constraints and the values of the variables. However it was noted that the objective value (that is, the total cost of the system operation) increased by more than 100 % in the low inflow scenario. In the high inflow case, the objective function value decreased by more than 100%.

6 Conclusion and further work

6.1 Conclusion

This paper has analyzed the optimal operation of a hydrothermal power system. The particular area of study has been the Nordic power system, and this is characterized by a large amount of hydropower. The handling of reservoirs is therefore of great interest, and different approaches to reservoir-handling have been studied and evaluated. A linear deterministic single-reservoir model was formulated and implemented in Xpress^{MP}. The model successfully allocated the different generating units and transmission according to the demand in each period, minimizing the total system cost during the planning horizon. Power prices for the Nordic countries together with reservoir levels were forecasted on an hourly resolution for a two-year period. Three inflow scenarios were tested in the model, and the results from running the deterministic model on the different inflow scenarios showed the major influence that inflow scenario has on the power price. According to the results, reservoirs are completely emptied to the minimum level just before the second inflow season starts.

The above conclusions address the issue that deterministic representation of stochastic factors will not give a reliable result. Only if the expected scenario turns out to materialize, can the model guarantee optimality. The chance for this is minimal, and a stochastic modeling approach should be preferred.

6.2 Further work

In this section, improvements to the model are suggested.

6.2.1 Improving the deterministic model

A list of issues that should be considered for further development of the model is given here:

- In the model, it is assumed that all generators are available at all times during the planning period. In reality however, this is not the case as generators need to be disconnected from the system due to maintenance. This can be both planned maintenance or a fault forcing them to shut down.

- Shutting down and starting up generators does not come without a cost as implicitly implied in the model, and the start-up costs for the different generators should be included in the model.
- Windpower generation is left out completely of the model, and as seen in the results, this has major implications on the forecasted power price in Denmark.
- Transmission losses should be proportional to the power transferred, not a constant percentage as is used in the model.
- Exchange with Estonia and Russia is left out of the model.

The improvements listed above are all possible to implement into the model without much difficulties. Another weakness of the model that is not simple to improve, is the use of aggregated reservoirs for each country. Localized spillage can occur that is not captured by the model. Also, with the minimum reservoir level of 1000 GWh, many smaller reservoirs would run dry. The way to improve this in the model would be to use smaller price areas based on river systems instead of countries. There is much data that must be collected if the areas used were to change, but the same MOSEL-code can in principle be used.

6.2.2 Stochastic programming

The results from running the model with different inflow scenarios implied that deterministic inflows represent a serious flaw in the model. If the proposed generation plan was used in reality, there would be a high risk of shortage and spillage because the risks for these are not represented in the model. A model that handles uncertainty should be preferred, with the most important uncertainty being the inflow. Relevant models are presented in Section 3, and a large task with implementing a stochastic model is to get different inflow scenarios.

References

R. Bellman. Dynamic programing. *Princeton, New Jersey*, 1957.

J.F. Benders. Partitioning procedures for solving mixed-variables programming problems. *Numerische Mathematik*, 4(1):238–252, 1962.

J.R. Birge. Decomposition and partitioning methods for multistage stochastic linear programs. *Operations Research*, 33:989–1007, 1985.

N. Buras. Conjunctive operation of dams and aquifers. *American Society of Civil Engineering*, 89(6):111–131, 1963.

G. Doorman. *Hydro Power Scheduling*. NTNU, 2009.

O. Egeland, J. Hegge, E. Kylling, and J. Nes. The extended power pool model—operation planning of multi-river and multi-reservoir hydrodominated power production system—a hierarchial approach. In *1982 CIGRE Meeting, Paris*, 1982.

R.W. Ferrero, J.F. Rivera, and S.M. Shahidehpour. A dynamic programming two-stage algorithm for long-term hydrothermal scheduling of multireservoir systems. *Power Systems, IEEE Transactions on*, 13(4):1534–1540, 1998.

A. Gjelsvik. Stochastic seasonal planning in multireservoir hydroelectric power systems by differential dynamic programming. *Modeling, Identification and Control*, 3(3):131–149, 1982.

A. Gjelsvik, TA Røtting, and J. Røynstrand. Long-term scheduling of hydrothermal power systems. In *Hydropower*, volume 92, pages 539–546, 1992.

A. Halseth. Market power in the nordic electricity market. *Utilities Policy*, 7(4): 259–268, 1999.

J. Lindqvist. Operation of a hydrothermal electric system: A multistage decision process. *Power Apparatus and Systems, Part III. Transactions of the American Institute of Electrical Engineers*, 81(3):1–6, 1962.

J. Lundgren, M. Rönnqvist, and P. Varbrand. *Optimazation*. Studentlitteratur, 2010.

P. Massé. *Les réserves et la régulation de l'avenir dans la vie économique*. Hermann & Cie., 1946.

K.D.W. Nandalal and J. Bogárdi. *Dynamic programming based operation of reservoirs: applicability and limits*. Cambridge Univ Pr, 2007.

M.V.F. Pereira and L. Pinto. Stochastic optimization of a multireservoir hydroelectric system: A decomposition approach. *Water Resources Research*, 21(6): 779–792, 1985.

M.V.F. Pereira and L. Pinto. Multi-stage stochastic optimization applied to energy planning. *Mathematical Programming*, 52(1):359–375, 1991.

W.B. Powell. *Approximate Dynamic Programming: Solving the curses of dimensionality*, volume 703. Wiley-Blackwell, 2007.

T.J. Scott and E.G. Read. Modelling hydro reservoir operation in a deregulated electricity market. *International Transactions in Operational Research*, 3(3-4):243–253, 1996.

S. Soares and A.A.F.M. Carneiro. Optimal operation of reservoirs for electric generation. *Power Delivery, IEEE Transactions on*, 6(3):1101–1107, 1991.

S. Stage and Y. Larsson. Incremental cost of water power. *Power Apparatus and Systems, Part III. Transactions of the American Institute of Electrical Engineers*, 80 (3):361–364, 1961.

A. Turgeon. Optimal operation of multireservoir power systems with stochastic inflows. *Water Resources Research*, 16(2):275–283, 1980.

I. Wangensteen. *Power system economics: the Nordic electricity market*. Tapir academic press, 2007.

O. Wolfgang, A. Haugstad, B. Mo, A. Gjelsvik, I. Wangensteen, and G. Doorman. Hydro reservoir handling in norway before and after deregulation. *Energy*, 34(10):1642–1651, 2009.

S. Yakowitz. Dynamic programming applications in water resources. *Water Resources Research*, 18(4):673–696, 1982.

W.W.G. Yeh. Reservoir management and operations models: A state-of-the-art review. *Water resources research*, 21(12):1797–1818, 1985.

Z. Yu, F.T. Sparrow, and B.H. Bowen. A new long-term hydro production scheduling method for maximizing the profit of hydroelectric systems. *Power Systems, IEEE Transactions on*, 13(1):66–71, 1998.

Appendix A - Mosel code