Norwegian University of Science and Technology NTNU

Faculty of Social Science and Technology Management

Department of Industrial Economics and Technology Management

MASTER THESIS

for

STUD.TECHN. MARCUS NAVJORD WINNEM

Field of study	**Accounting and Finance** Investering, finans og økonomistyring
Start date	15.01.2006
Title	**Hedging Hydroelectric Generation** Risikostyring av vannkraftproduksjon
Purpose	To develop efficient methods for risk management of hydroelectric generation assets based on stochastic optimization techniques

Main contents:

1. Modelling and estimation of inflow and price processes.
2. Development and testing of stochastic methodology for production planning and calculation of hedges.
3. Comparison of results with results from a scenario based (deterministic) methods.

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NTNU
Norges teknisk-naturvitenskapelige universitet

STANDARDAVTALE

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faglærer ved NTNU Stein-Erik Fleten

bedrift/virksomhet Elkem Energi Handel

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DECLARATION

Stud.techn. ____

Institute of Industrial Economy and Technology Management

I hereby declare that I have written the above mentioned
thesis without any kind of illegal assistance

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Place Date

____

Signature

Preface

This thesis has been written for the degree of Master of Technology, at Institute of Industrial Economy and Technology Management, NTNU, 2006.

Several people have helped in the process. Thanks to Stein Erik for good advice, Håvard for being my academic anchor at Elkem, Thomas and Bjarte for giving me some hands on experience in the power market and Bjørn for giving me a job.

And Margrethe, yes, I am coming home now.

Trondheim, June 2006

Abstract

The current thesis represents an effort to develop an efficient and flexible framework for medium to long term hedging and production scheduling of hydroelectric generation in deregulated markets. The hydroelectric scheduling problem is a highly dynamic, stochastic problem, with future price and inflow representing the main uncertainties. A price taking producer is assumed, and the scope is limited to a single reservoir, single station system.

The production scheduling problem is analysed in an option pricing context, and it is shown how the generation asset can be interpreted as a complex derivative on the spot electricity price. Viewing the generation asset as an option to produce electricity, a framework for hedging price risk is presented, assuming negligible transaction costs. The framework is based on calculating hedge signals by finite difference methods.

Price and inflow uncertainty is modelled seperately as stochastic processes. Two price models are estimated. The Schwartz (1997) mean reverting spot model is estimated and calibrated to the Nord Pool term structure. The residuals from estimation show large excess kurtosis. The second price model is a Bjerksund et al. (2000) type forward curve model. The model is estimated by calibration to the term structure of swaps and implied volatilities of at the money options. A thorough time series analysis of inflow data is undertaken in order to investigate inflow behaviour. It is found that Box-Cox transformation and deseasonalization produces a stationary underlying series, which can be modelled as a low order ARMA process. As expected the inflows are strongly autocorrelated However, the residuals from the estimation show excess kurtosis and positive skewness.

In order to solve the production scheduling problem and calculate hedges a multi stage stochastic algorithm, based on the least-squares Monte Carlo technique is proposed. The algorithm has been implemented for testing in Matlab and results are compared to a deterministic model. The Elkem Energi Siso AS system is used a test case. Results are somewhat disappointing, leaving hypotheses regarding expected hedges unanswered. However, under the Schwartz model the objective value is relatively close to the upper bound of the deterministic model. A discussion of reasons for failure is provided, and possible solutions are proposed.

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1 Introduction

After the deregulation of the Nordic electricity market during the 1990's there has been a need to consider price risk when scheduling hydroelectric generation. The power market is proving highly volatile, but also offers access to financial products that can be used for risk management purposes. However, the producer also faces a considerable volume risk in the form of uncertain future inflows. Following, it is not obvious what market transactions to undertake to minimize the variance of future cash flows. In order to set up efficient and consistent hedges the producer is in need of production scheduling tools, which explicitly accounts for both price and inflow uncertainty. The current thesis seeks to develop medium to long term methods aiding the producer in making the right hedging decisions.

As a preparation for the current study a project work (Pedersen & Winnem, 2005) was carried out during the autumn, 2005. This thesis is independent of the project, but further develops the concepts. As with the project, the current work is connected to Elkem Energi Handel AS (Elkem). The Elkem Energi Siso AS (Siso) hydroelectric plant is used as a test case. Siso is a single reservoir, single station plant, which also defines the scope of the thesis.

In connection with the thesis relevant literature has been studied. Concerning algorithm development Longstaff & Schwartz (2001) and Glasserman (2004) are extensively used. For programming formulations Wallace & Fleten (2003) provides an excellent overview, regarding the hydroelectric production scheduling problem. For modelling of price processes Schwartz (1997) and Bjerksund et al. (2000) are used. The text of Brockwell & Davis (2002) provides the foundation for the modelling of inflows.

The thesis is organized as follows. Chapter 2 introduces basic concepts of hydroelectric production scheduling. The power market is treated in chapter 3. Chapter 4 interprets the generation asset in a financial context, and chapter 5 develops a financially based risk management framework. Mathematical programming formulations are provided in chapter 6, along with a proposed solution algorithm. Chapter 7 includes estimation of price processes and a time series analysis of inflow data. Chapter 8 discusses implementation issues. Results, which are discussed in chapter 10, are presented in chapter 9. Chapter 11 concludes and suggests a possible extension of the current thesis.

2 The Hydroelectric Production Scheduling Problem[1]

Prior to deregulation the goal of scheduling was to meet demand at minimum cost. In a deregulated market the objective is different; the producer[2] should seek to maximize the expected market value of the generation asset (Fleten & Wallace, 2003). According to financial theory and the law of one price maximization of market value implies that one must schedule in accordance with market prices. Participants with limited accumulated production capacity will generally not influence the market price, and can hence treat price as an exogenous variable. For the remainder of the thesis the preceding statements are fundamental and are therefore stated as the following explicit assumptions:

> *Assumption 2.1*
> *The goal of production scheduling is to maximize the expected market value of the generation asset.*

> *Assumption 2.2*
> *The proprietor of the generation asset does not posses power to influence the market price of electricity.*

Facing uncertain future inflow and a volatile market the producer must in principle, continuously determine the instantaneous release, and thereby the generated output that maximizes the market value of the asset. In practice the bidding into the Nordic market happens on a one day ahead basis. In this respect the scheduler only needs to update the production plan every 24 hours.

The main challenge in scheduling hydroelectric generation is clearly the management of the water reservoir. In order to achieve optimal management one must explicitly account for both price risk and volume risk, in the form of volatile electricity prices and uncertain future inflow respectively. Physical relations and restrictions add to the complexity of the problem. Combined with the need to construct a schedule with fine resolution the dimensionality is of such magnitude that there is a need to decompose the problem. The following sections seek to

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[1] Hereafter abbreviated as HPSP.
[2] Throughout the thesis the term *producer* will be used in the meaning of *hydroelectric producer*.

establish the basic concepts of hydroelectric production scheduling, with regard to physical, economical and decomposition principles.

2.1 Physical Principles

The concepts of hydroelectric generation are quite simple. Basically water at a higher elevation is released through a turbine, which in turn drives a generator to produce power. The concept is illustrated in Figure 2.1, and a more detailed view of a possible turbine-generator setup is provided in Figure 2.2.

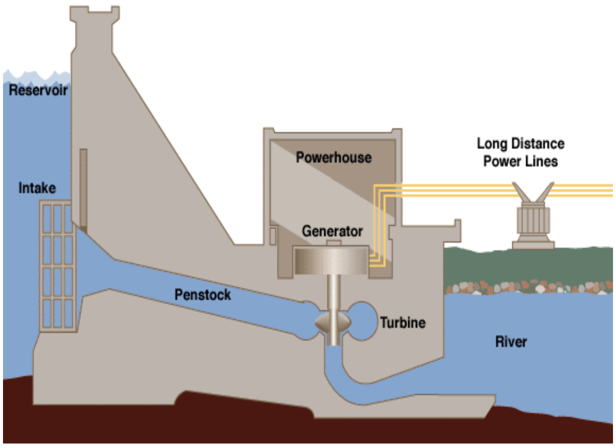

Figure 2.1: Schematic of hydroelectric plant

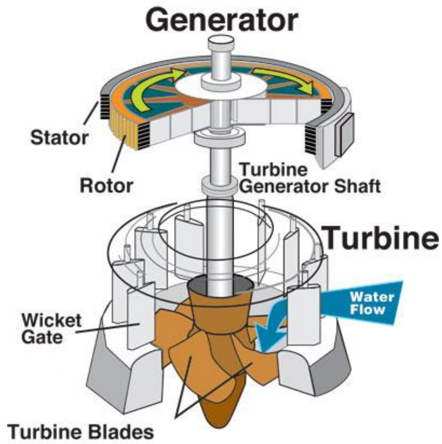

Figure 2.2: Possible turbine-generator setup[3]

For scheduling purposes there is a need to know how the power output depends on physical variables. For medium and long term scheduling it is often assumed that the generated amount of power is proportional to the release (Näsäkkälä & Keppo, 2005). In the real world the situation is more complex, as the output depends non-linearly on the release and the water head. Also the maximum release will not be independent of the head, but decrease as the reservoir level decreases.

For the Siso reservoir an empirical production function is in use. A specification of the functional form is provided in Appendix 1. The function is plotted in Figure 2.3, the non-linearities clearly visible.

[3] The picture shows a so called Kaplan turbine.

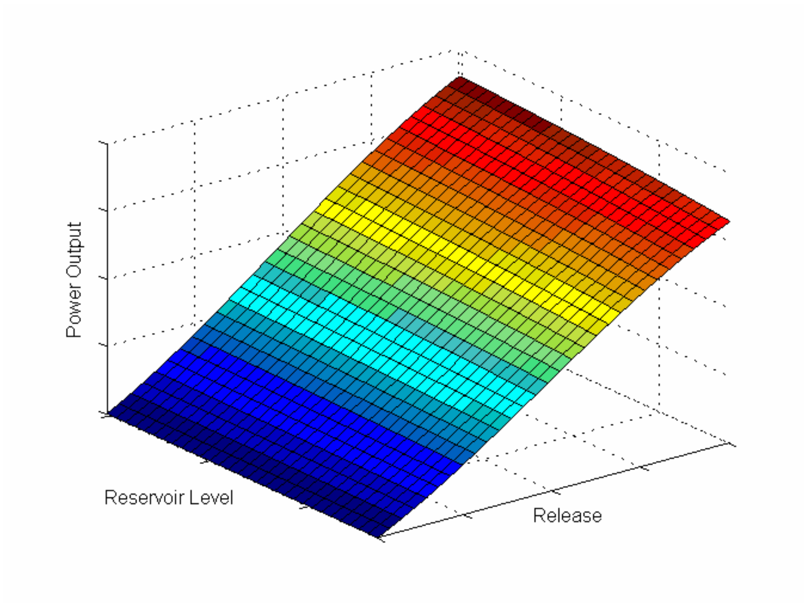

Figure 2.3: Siso Production Function

2.2 Economical principles

Intuitively the producer must balance the benefits from producing immediately against the value of storing water for later generation. The water thus has an opportunity cost. This opportunity cost is often referred to as the water value, of which the understanding is essential to the analysis of HPSP. In order to formalize the concept of water value let us introduce the following notation for some key quantities and variables:

t	Index of period
V_t	Period t expected value of production
q_t	Period t release
m_t	Reservoir level at the beginning of period t
$w(m,q)$	Generated amount of electricity as a function of reservoir level and release
Π_t	Period t stochastic electricity price
π_t	Period t realized electricity price
Ψ_t	Period t stochastic inflow
ψ_t	Period t realized inflow
r	Appropriate hurdle rate

The value of the production can now be formulated as

$$V_t = \max_{q_t, m_t} E\left[\sum_{i=t}^{\infty} \frac{\Pi_i w(m_i, q_i)}{(1+r)^{i-t}}\right]. \tag{2.1}$$

As formally stated the value of the production is the sum of the discounted production revenues. It should be noted that all costs are neglected. (2.1) can obviously also be formulated recursively as

$$V_t = \max_{q_t, m_t} \left[E\left[\Pi_t w(m_t, q_t)\right] + \frac{E\left[V_{t+1}\right]}{1+r} \right]. \tag{2.2}$$

Given that the current period is *t* and assuming that the outcome for the price in period *t* is known (2.2) reduces to

$$V_t = \max_{q_t, m_t} \left\{ \pi_t w(m_t, q_t) + \mu_{V_{t+1}} \right\}, \text{ where} \tag{2.3}$$

$$\mu_{V_{t+1}} = \frac{E\left[V_{t+1} \middle| \Pi_0, \Pi_1, \ldots, \Pi_t, \Psi_0, \Psi_1, \ldots, \Psi_t\right]}{1+r}.$$

To establish a criterion for the optimal production strategy (2.3) is differentiated with respect to the release:

$$\frac{d}{dq_t}\left(\pi_t w(m_t, q_t) + \mu_{V_{t+1}}\right) = \frac{\partial}{\partial q_t}\left(\pi_t w(m_t, q_t)\right) + \frac{\partial \mu_{V_{t+1}}}{\partial m_{t+1}} \frac{\partial m_{t+1}}{\partial q_t},$$

According to the water balance[4] the derivative of the reservoir level with respect to the release must equal minus one. In order to fulfil the first order optimality conditions we obtain the following control criterion:

$$\frac{\partial \mu_{V_{t+1}}}{\partial m_{t+1}} = \pi_t \frac{\partial w(m_t, q_t)}{\partial q_t}. \tag{2.4}$$

[4] See Section 6.1

Economically equation (2.4) signifies that the producer, in any given period, should produce such that the marginal change of the discounted expected future value equals the marginal revenues from producing immediately. The variable $\partial\mu_{V_{t+1}} / \partial m_{t+1}$ is what is often referred to as the marginal water value. For the remainder the marginal water value will be referred to merely as the water value. When referring to the total discounted expected future value the term future value will be used.

The preceding analysis does not account for the physical restrictions on the system. In practice there will be restrictions on both the release and the reservoir. If the reservoir is full and the immediate inflow is larger than the production capacity, water will be spilled. In this case the system restrictions yield the above derivation of the control criterion invalid. In an optimization sense one can alternatively define the water value as the Lagrange multiplier associated with the water balance. Clearly the Lagrange multiplier must equal zero in this situation. This signifies the important insight that the opportunity cost of water decreases as we move towards the upper bound on the reservoir level, as a consequence of increased risk of spilling. Another special case arises when the price in one period is of such a magnitude that it is optimal to increase the release to the upper bound, even if there is no risk of spilling. (2.2) will in this case generally not be satisfied as an equality. Following the preceding discussion a more general form of the control criterion can be formulated. Let us first define the water value as

$$\xi_t = \begin{cases} \dfrac{\partial\mu_{V_{t+1}}}{\partial m_{t+1}}, & \text{When spilling can be avoided} \\ 0, & \text{When spilling can not be avoided} \end{cases}$$

The control criterion can now be stated more generally as:

$$\begin{aligned} &\max q_t \\ &s.t. \\ &\xi_t \leq \pi_t \frac{\partial w(m_t, q_t)}{\partial q_t} \end{aligned}$$

In words the producer should increase the release as long as the marginal revenue is larger than the water value. Hence, if the water value can be determined, the production schedule can be constructed with relative ease. Note however that the preceding analysis considers the first order optimality condition only. In the case of a nonlinear production function the situation will generally be more complex.

2.3 Decomposition principles

When managing a well regulated reservoir it is necessary to consider planning horizons of several years. A useful quantity for comparing different systems with respect to regulation is the relative regulation, R:

$$R = \frac{M_{\max} - M_{\min}}{\bar{\Psi}}$$

$M_{\max}$ and $M_{\min}$ are respectively the upper and lower bound on the reservoir level and $\bar{\Psi}$ is the average annual inflow. According to A. Gjelsvik The higher the relative regulation the longer the planning horizon needs to be (counselling appointment, October, 2005). The reason is that with a high relative regulation dispositions may affect the state of the reservoir far into the future. On the opposite if the relative regulation is such that it is highly probable that the reservoir will be spilling at a certain time[5], T^*, regardless of the prior state, it is not necessary to consider planning horizons beyond T^*. The water value will in the period prior to T^* equal zero regardless of the reservoir level, which is a sufficient boundary condition.

Hydroelectric generation assets are often complex systems of numerous linked reservoirs, several stations and a host of restrictions that are more or less difficult to model. At the same time the producer must make bids into the market with hourly resolution. In combination with production decisions influencing up to several years into the future, the computational complexity is great at least. Because of the dimensionality of the problem it is common, and necessary, to decompose HPSP into two or three coupled sub problems in a hierarchical fashion (Flatabø et al, 2002). Table 2.1 lists approximate horizons, typical resolutions and associated solution methods.

[5] In the Nordic system typically in spring during the snow melt.

Horizon	*Resolution*	*Solution Methods*
Long Term 1-5 Years	Week	Aggregation of reservoirs and systems. Stochastic optimization and simulation based methods.
Medium Term 3-18 Months	Week	Multi reservoir scenario based deterministic methods, or aggregation of reservoirs and stochastic optimization.
Short Term 1-2 Weeks	Hour	Multi reservoir deterministic optimization.

Table 2.1: HPSP Hierarchy

The long term analysis seeks to capture long term fluctuations in price and inflow. It is customary to make simplifications such as aggregation of several reservoirs and stations into one hypothetical reservoir and one station. Output from the long term analysis is used as a boundary condition in the medium term analysis. The medium term analysis serves merely as a link, increasing the detail level, between the long term and short term models. (Flatabø et al., 2002). In the near future the price and inflow is often assumed deterministically known. The short term model should be sufficiently detailed to yield feasible schedules with resolution of an hour or shorter. The short term schedule is usually simulated to ensure that restrictions not included in the model are not violated.

2.3.1 Coupling Methods

Appropriate boundary conditions are essential to obtain a credible schedule and consistency between the different models. Different solutions to coupling the levels in the scheduling hierarchy are listed in Table 2.2.

The time of year for linking the models is also important. The uncertainty in the boundary condition may be quite different in different seasons. As an example the long term and medium term models are usually linked in spring or autumn when the reservoir is expected to culminate.

Link	*Issues*
Reservoir Level	The end reservoir is specified. This is an easy to implement solution in deterministic models, but may prove inflexible. Regarding stochastic models it generally will be hard to specify an end reservoir, since the reservoir level itself is a stochastic variable.
Penalty Function	The objective is punished if a target reservoir is missed. Might be an adequate solution for simple systems, but it may be difficult to specify a sensible penalty function in multi reservoir systems.
Water Value	The models are linked by specifying the water value function as a boundary condition. Increases the flexibility in optimization and might make it easier to gain consistency between models with different level of detail.

Table 2.2: Coupling Methods

2.4 Existing Commercial HPSP Tools

In the Norwegian system the SINTEF developed models EMPS and EOPS are extensively used for long and medium term scheduling. In the EMPS model the electricity price is internalized (i.e. the price is an output from the model) and hence can not be expected to be in accordance with market prices. EMPS is a model of the entire system and is used for price forecasting and system analysis as well as for production scheduling by large market participants. Most smaller producers use the EOPS model for local scheduling, which treat the electricity price as an external variable. Finally SHOP is another SINTEF developed tool intended for short term scheduling.

3 The Nordic Power Market

Over some years after deregulation, electricity contracts started to trade OTC and the electricity exchange Nord Pool was established. During the period 1996 to 1999 the Swedish, Finnish and Danish markets were also deregulated. Today Nord Pool remains the largest trading place for electricity contracts in the Nordic countries, and the Nordic model is often looked to in developing power markets (Haug, 2006). In addition to providing a market place for both physical and financial delivery Nord Pool also provides clearing services for OTC trades. It should be noted that the bilateral markets are substantial[6]. In the following the main focus will nevertheless be exchange traded contracts.

3.1 The Physical Market

For physical delivery of power there exists both a non-mandatory day ahead market, Elspot, run by Nord Pool and a regulating market for real time balancing of supply and demand. The responsibility of ensuring system balance lies with the system operator[7], Nord Pool merely provides a market place for trading of electricity contracts.

Every day participants at Elspot submit bids, in the form of a bid curve, for each of the 24 hours of the following day. When all bids are submitted aggregate bid curves for the supply and demand side are created. The intersections of the 24 curve pairs determine the cleared *system prices* for the following day. In absence of transmission constraints the system price would equal the realized spot price. The introduction of transmission constraints leads to different *area prices* within the system. In the following we will frequently be referring to the hourly prices and their daily averages as the *spot price*. The term is somewhat misleading as the contracts are in fact swaps for short time periods.

The system operator ensures security of supply through market mechanisms. Quite similar to the bidding process in the day-ahead market the participants submit price-volume bids for upward or downward regulation in the regulating market. When regulating units are needed the price is set at the highest of the bids from the units called upon.

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[6] In 2004, 757 TWh were turned over at the exchange and 1207 TWh were cleared OTC by Nord Pool alone. (Source: nordpool.no, 2006)

[7] In Norway the system operator is Statnett.

3.2 Spot Price Behaviour

Electricity price time series exhibit several unique characteristics compared to other financial time series. To understand the differences one must look to the fundamentals of the commodity and to the fundamentals of the price formation process. Arguably the single most important fundamental factor to distinguish the electricity price is the non-storability of power. Electricity is a flow commodity which must be generated and consumed continuously; there is no economical way for a consumer (or arbitrageur) to buy and hold the commodity.

Due to the relative youth of the deregulated power markets the historical data is limited. Still a few studies, such as Lucia & Schwartz (2002) and Eydeland & Wolyniec (2003) have investigated electricity price behaviour. According to Lucia & Schwartz (2002) aspects that have been discovered in various studies include seasonality, spikes, mean reversion, non-stationary volatility and resulting fat tails in the distribution of returns.

3.2.1 Seasonality

Perhaps the most important consequence of the non-storability of power is the existence of a seasonal pattern in electricity prices. Due to low temperatures, demand is in Scandinavia typically larger during the winter months as heating to a large extent is based on electricity. The system can become quite strained, and costly gas fired power is frequently price setting. Also there is much less inflow to hydro-electric units during the winter, leading to even tighter supply in the hydro dominated system. In spring the amount of inflow to the reservoirs usually increase dramatically forcing water values down, and at the same time demand drop as temperatures rise towards summer. The seasonality in the system price can be seen in Figure 3.1. Clewlow & Strickland (2000) deem seasonal variations in the Nordic power market to be significant.

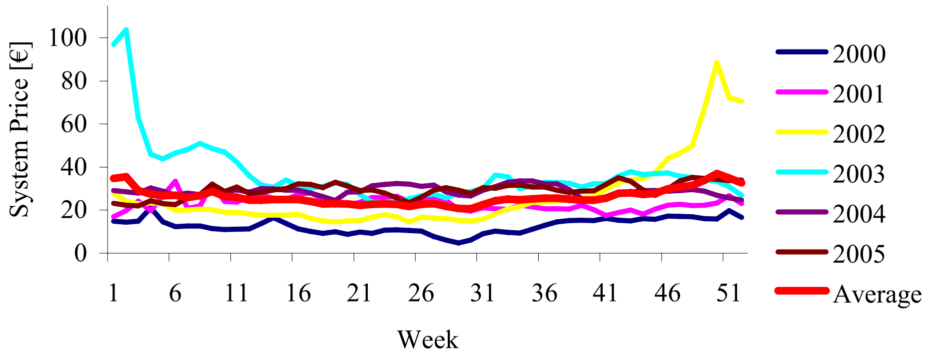

Figure 3.1: Seasonality in the weekly average system price

There is not only variation over the year, but also within the week and within the day (Lucia & Schwartz, 2002). Typically the price is lower during the weekend, and there are also peak hours in the morning and afternoon.

3.2.2 Spikes

Figure 3.2 is a plot of the average daily system price and the corresponding log returns in 2005. It is evident that large, discontinuous price moves are frequent. Often the jumps in the price take the form of a spike. The behaviour is common in power markets in general (Eydeland & Wolyniec, 2003).

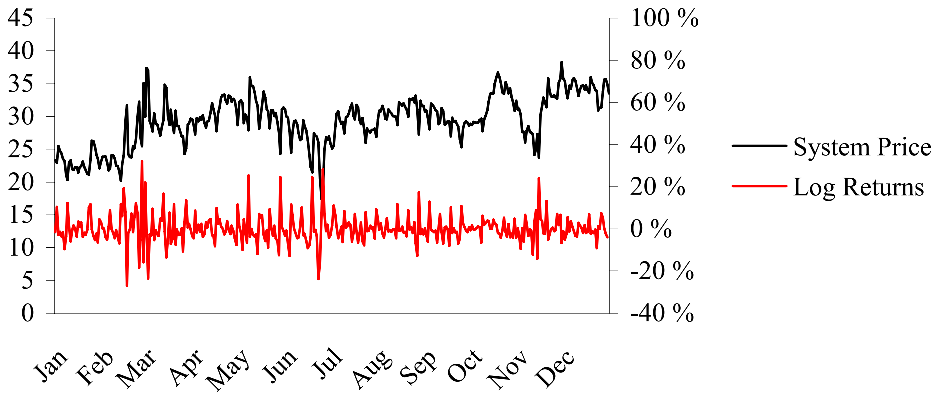

Figure 3.2: Daily system price 2005

Eydeland & Wolyniec (2003) explain the frequent spikes in electricity prices by the limited quantity of production capacity at different marginal cost levels. There is indeed a clear difference in the marginal cost of power plants fired by gas, coal and oil. In the Nordic market, the marginal cost of hydroelectric generation is not merely determined by the actual cost of producing, but also by the water values, adding to the complexity.

3.2.3 Mean Reversion

In commodity price modelling, mean reversion is often assumed. Mean reversion basically signifies that prices tend to revert towards some expectation. According to Eydeland & Wolyniec (2003) applying a naive model to estimate mean reversion in power prices will in most cases give significant results, but might well be caused by the presence of spikes and stochastic volatility. Fundamentally one can still argue that mean reversion should be present over long time horizons. The fundamental idea being that in an equilibrium setting relatively high prices will attract high cost producers, putting a downward pressure on prices, and conversely low prices will force some producers out of the market, putting an upward pressure on prices (Schwartz, 1997).

3.2.4 Non-Stationary Volatility and a Resulting Fat Tailed Distribution

Lucia & Schwartz (2002) find positive skewness, strong excess kurtosis and evidence of non-stationary volatility in Nord Pool spot price returns. In other words the returns are not well represented by a normal distribution. In all the spot price returns show extremely fat tails compared to standard financial time series. Combined with spikes and mean reversion the spot price is inherently difficult to model.

3.3 The Financial Market

A number of different financial future, forward and option contracts trade at Nord Pool. The financial forward contracts do not have physical delivery, but are settled against the realized system price. The options have forwards as underlying and expire before delivery of the underlying contract. In addition to futures, forwards and options there are also contracts on the difference between area prices and the system price, so called *contracts for difference*.

3.3.1 Term structure of swap contracts

The contracts referred to as futures, forwards and options are in reality future swaps, forward swaps and options. The forward swap is settled by marking to market during the delivery period; the daily payoff is the forward price less the daily average system price. In other words a long position in a forward swap effectively exchanges a floating price against a fixed price. The distinction between future swaps and forward swaps is that the future swaps have daily mark-to-market settlement from the day they are entered. Future swaps are traded for days and weeks, and forward swaps are traded for months, quarters and years. For simplicity we will treat future swaps as forward swaps and merely refer to both as swaps. A graphical example of the term structure of swaps is provided in Figure 3.3.

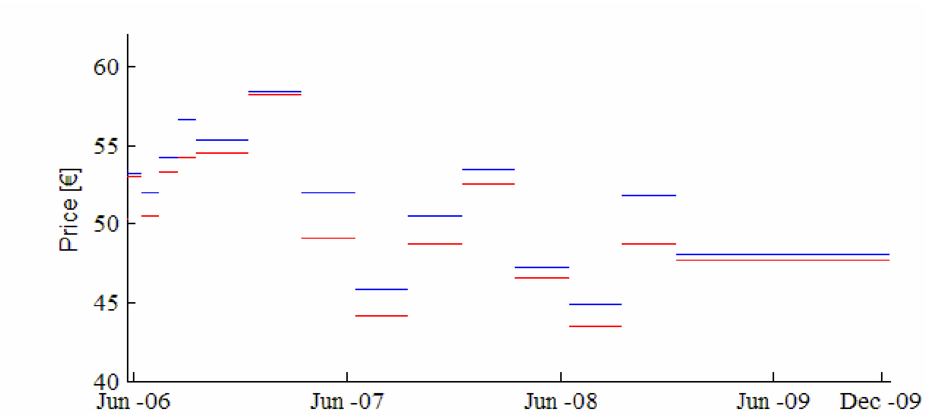

Figure 3.3: Term structure of forward swaps April 20, 2006

As is clearly seen the specific term structure exhibits seasonality. According to Lucia & Schwartz (2002) the seasonality in spot prices is indeed included by market participants in the valuation of swaps. Haug (2006) argues that mean reversion effects in the spot price should also be included in the valuation of swap contracts in an efficient market. In all swap returns are more comparable to other financial time series than the spot returns. That being said Lucia & Schwarz (2002) find evidence of strong excess kurtosis and also positive skewness in historical swap returns, as is also reported by Benth and Koekebakker (2005).

3.3.3 Spot-Swap Relation

Assuming complete markets and no arbitrage opportunities the price of a forward contract on a financial asset will satisfy the equation

$$f(0,t) = s_0 e^{(r_f - d)t}, \tag{3.1}$$

where $f(0,t)$ is the forward price of a contract with delivery at time t, s_0 is the underlying spot price and d is an equivalent dividend payout rate. For commodities d might include convenience yield and storage costs. Assuming a forward market for electricity the relation (3.1) is not directly applicable. The reason is that electricity can not be stored and hence the arbitrage arguments underlying the relation are not valid. Still it is possible to infer information about the future electricity price from the forward market. The relation

$$f(0,t) = E\left[\Pi_t\right] e^{(r_f - \alpha)t}, \tag{3.2}$$

where $r_f - \alpha$ is a risk premium holds also in electricity markets (McDonald, 2003). (3.2) signifies that the forward price of electricity delivered at time t is the expected spot price at time t discounted by a risk adjusted rate. The observed swap prices are thus best interpreted as a weighted sum of the expected spot price during the delivery period.

3.3.2 Term Structure of Volatility

Only European options on quarterly and yearly swaps are traded at Nord Pool, and the current volumes trading on the exchange are quite low[8]. The exchange traded volumes are however misleading regarding liquidity as options frequently are traded in the OTC market, and the producer typically can get quotes from brokers. The prices of traded options can give an indication of the markets expectation of future volatility. This information might be useful for the producer, considering the option like features of the generation asset[9].

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[8]In fact most days no options trade on Nord Pool, but a small number of OTC trades cleared by Nord Pool are reported.

[9] See Chapter 4 for a discussion of the option characteristics of the hydroelectric plant.

Haug (2006) argues that the Black 76 option pricing formula might be used for pricing power options, pointing to stock returns showing higher kurtosis than Nord Pool swap returns. The Samuelson effect, which means decreasing volatility of contracts with increasing time to maturity, is frequently observed in empirical studies of energy price data (Eydeland & Wolyniec, 2003). According to Benth & Koekebakker (2005) the Samuelson effect is extremely pronounced in the electricity market. One can hence not expect the Black 76 formula to yield realistic prices using the same volatility for different maturities, but it can be used in analyzing the term structure of implied volatility.

4 Interpreting the Generation Asset as a Derivative

From a financial perspective the combination of price uncertainty and flexibility suggests a real option approach to the production scheduling problem. Especially since a relatively liquid market for power contracts exists, one may be tempted to adapt option pricing techniques for valuation and risk management of hydroelectric generation. This section will seek to develop the intuition behind a real option approach to HPSP and explain how the generation asset may be viewed as a complex derivative on the spot price. Also issues that weigh against the use of standard financial tools will be elaborated. Recent contributions to the analysis of HPSP using financial methods include Barz & Tseng (2002), Davison et al. (2004) and Näsäkkälä & Keppo (2005). Eydeland & Wolyniec (2003) discuss real option valuation of a variety of energy assets. For a treatment of real option valuation in general the reader is referred to Dixit & Pindyck (1994) or Trigeorgis (1996).

4.1 The Reservoir as a Source of Added Value

Consider a run-of-river power plant and an otherwise equal plant with storage capacity. In an optimization sense the water balance constraint is less strict for a plant with a reservoir. Since enforcing tighter constraints on the solution never can yield an increase in objective value, the run-of-river plant can never be worth more than the plant with storage. From a financial perspective the run-of-river plant does not yield any optionality regarding the production schedule[10]. In other words there is not room for practicing active management. In the case of available storage the producer has the option to postpone production; there is increased flexibility compared to the run-of-river plant. Considering two otherwise equal projects where one of the projects has an embedded option, the project with the embedded option will have a greater value. Thus the plant with storage is more valuable than the run-of-river plant. Under the assumption that the producer maximizes the market value, the decision not to produce is equivalent to optimally exercising the option to postpone (or reduce) production.

4.2 Analogy to Gas Storage

As has been discussed power prices exhibit seasonality, since the commodity can not easily be stored. Essentially a reservoir proves a mean to store electricity. In this respect the resrevoir effectively gives the producer the possibility to undertake arbitrage in time, by taking advantage of the seasonal spreads in the financial market. Eydeland & Wolyniec (2003) develop a method for hedging and valuation of gas storage based on replicating the asset as a static portfolio of American spread options. They further claim the same method can be used with modifications in valuation of hydroelectric assets. In the Nordic power market American spread options are not traded. Since we do not observe such prices applying some modified version of the Eydeland & Wolyniec (2003) method would be difficult and computationally expensive. Some insights can nevertheless be gained. Assume that the movements of electricity swap contract prices reflect the movement of the expected spot electricity price. If a hydroelectric generation asset can be viewed as a complex portfolio of spread options on the spot electricity price the volatility and correlation between swap contracts should influence the value and optimal production schedule. Hence the producer should consider not only swap contract prices when scheduling, but also prices of options on swaps and the correlation of swap prices.

[10] The statement ignores the option to actively manage the timing and frequency of revisions.

4.3 Analogy to the Spark Spread Option

Quite stylized, ignoring physical restrictions, a gas fired power plant can be represented as a spread option with payoff

$$\max\left[\Pi_{el} - \eta_{gas}\Pi_{gas}, 0\right], \tag{4.1}$$

where η_{gas} is the efficiency of the gas fired power plant (Eydeland & Wolyniec, 2003).

Davison et al. (2004) consider production scheduling of a single reservoir hydroelectric plant in continuous time. The operating objective is

$$\max_{q} E\left[\int_0^T e^{-rt}\dot{w}(m,q)\Pi dt\right].$$

In a quite general universe with regard to price and inflow processes Davison et al. (2004) derive the following criterion for the optimal production strategy

$$\max_{\dot{q}}\left[\dot{w}(m,\dot{q})\Pi - \dot{q}\frac{\partial V}{\partial m}\right], \tag{4.2}$$

where $\dot{q}$ and $\dot{w}$ denote the instantaneous release and instantaneous power output respectively. Interpreting (4.2) in the light of the spark spread payoff the water value can be viewed as the unit fuel cost. The large difference between (4.1) and (4.2) is that in the latter case both the water value (fuel price) and the marginal output depend on the production rate. Even if the marginal output is constant the water value still will not generally be constant in release. In the spark spread case we observe market prices for gas; in hydroelectric scheduling we can not look only to the market to determine the water value.

4.4 Challenges in Applying Financial Methods to HPSP

Since the producer can choose the timing of storage and production HPSP has, in an option pricing context, American features. In a discrete setting the problem may be viewed as Bermudan. Further since immediate production decisions will affect future water values, and since inflow is uncertain HPSP is an inherently path dependent problem. Inflow uncertainty is probably what most weighs against naively applying financial methods to the problem. Since the market for inflow risk is non-existing it is difficult to robustly apply risk neutral pricing[11]. Also physical restrictions and relations are significant and can not be overlooked.

On the bright side most existing solutions for HPSP involve some sort of price simulation and the theory of real option valuation is continuously developed. Financial models are also becoming increasingly suited to describe electricity market prices. Applying insights from the field of finance may thus help in producing better planning and hedging decisions, as long as the temptation to make oversimplifying assumptions is overcome.

5 Hedging Hydroelectric Generation

As electricity prices are proving highly volatile risk management is a wide spread concern. Both standard exchange traded contracts and more exotic OTC contracts are currently used for hedging. There is however not an established standard or framework on how to undertake transactions in the market to reduce risk. In the following the hedging problem is analysed in the light of financial theory.

The Black & Scholes (1973) option pricing theory is based on the assumption that it is possible to continuously trade the underlying asset in order to eliminate all risk. In practice one can only trade discretely of course, but the theory still is extensively used. Along the lines of Black & Scholes, real asset hedging is undertaken by trying to identify a portfolio of

[11] See section 5.1 for a discussion of market price of inflow risk and incomplete markets.

financial contracts replicating the asset cash flows. The real asset hedge then consists of a negative position in the replicating portfolio.

Assuming complete markets production planning and contract risk management can be separated (Fleten & Wallace, 2003). Thus in complete markets the production plan may give signals to risk management, but risk management should not affect the production plan. Under less strict assumptions Fleten & Wallace (2003) still argue that there may be benefits from separation of planning and hedging, and it is that path that is pursued here. For the further analysis the following assumption is made:

> *Assumption 5.1*
> *The financial electricity market is sufficiently liquid to neglect transaction costs.*

Assumption 5.1 may be criticized on the basis of considerable bid-ask spreads and transaction fees in the Nordic power market. Large producers are on the other hand often involved in trading activities. Some are even market makers enjoying lower fees. For such players the overall transaction costs will be lower, making the assumption more reasonable. What really determines the validity of the assumption is the stability of the hedges; if the hedges are relatively stable, minor bid-ask spreads should not greatly affect the solution. In the literature models integrating production scheduling and contract management, in the case of significant transaction, costs are proposed by Mo et al. (2001) and Fleten et al. (2002).

5.1 Implications of an Incomplete Market

The electricity market can not be viewed as complete in the sense that the producer can hedge all price risk. Considering medium to long term production scheduling a resolution of one week is customary. Since weekly contracts only exist for the first 5-6 weeks price risk can not be eliminated far into the future. Also the realized area price generally is not perfectly correlated with the system price, implying that even if a continuous forward curve was traded all price risk could not be hedged[12]. Still price risk can be hedged quite well compared to inflow risk.

[12] Contracts for difference are not available for all price areas.

In the hydro dominated Nordic system the hydrological balance affects power prices (Fleten et al., 2002). When the total system inflow turns out to be less than expected the prices tend to go up, and opposite the prices tend to go down when the total inflow is large. As water shortage often is regional (Fleten et al., 2002) one can expect a dependence relationship between local inflow and the hydrological balance. A negatively correlated dependence relationship between inflow and price provides the hydroelectric producer with a natural hedge. If the inflow falls short the producer can expect to be compensated by higher prices and vice versa. One can however not expect this dependence relationship to totally eliminate inflow risk. Since there does not exist a market for inflow risk there will always be some basis risk that can not be hedged. The residual inflow risk is a volume risk; the producer does not know for certain the future amount of potential energy in the reservoir. If the producer does not account for this risk when making hedging decisions, the resulting volatility of cash flows can not generally be expected to decrease. Fleten & Wallace (2003) illustrate the problem with a simple example, where selling the expected production forward increases the variance of cash flows.

As was explained in section 4 holding a hydroelectric generation asset can be interpreted as a complex derivative position on the underlying spot price. One can not expect the payoffs from such a position to be symmetrical. Classical discounting techniques can therefore not be applied in a consistent way[13]. Since all sources of risk can not be fully hedged risk neutral pricing can not be bluntly applied either. In such a situation Eydeland & Wolyniec (2003) suggest adjusting the drift of the price process with the market price of the residual risk.

Let α be expected return, σ volatility and r_f the risk free interest rate. The Sharpe ratio , λ, can be understood as the price of risk per volatility unit and is defined as

$$\lambda = \frac{\alpha - r_f}{\sigma}.$$

[13] See Chapter 11 in McDonald (2003).

The capital asset pricing model (see for instance Brealey & Myers, 2003) can be used to find the risk premium for a portfolio consisting of pure hydroelectric producers as

$$\alpha_{hydro} - r_f = \beta_{hydro}\left(\alpha_{market} - r_f\right),$$

where α_{hydro} is the expected return on the portfolio of hydroelectric stocks, α_{market} is the expected return on the market as a whole and β_{hydro} is the beta of the hydroelectric portfolio. Further the Sharpe ratio of two assets equally correlated with the market will in absence of arbitrage opportunities be identical (McDonald, 2003). Assume that the risk premium of stocks on hydroelectric producers mainly is caused by inflow risk. Further assume that the market value of the production of the company in question is equally correlated with the market as the portfolio of hydroelectric producers. The premium for the inflow risk, α_{inflow}, of the company in question will then equal

$$\alpha_{inflow} - r_f = \frac{\sigma_{production}}{\sigma_{hydro}}\left(\alpha_{hydro} - r_f\right), \tag{5.1}$$

where $\sigma_{production}$ is the volatility of the market value of the production. $\sigma_{production}$ is of course not easily observable, but (5.1) does give some intuition. With a qualified guess of the ratio $\sigma_{production} / \sigma_{hydro}$ (5.1) can at least prove a starting point for pinning down a realistic risk premium. A serious objection is that hardly any pure hydroelectric generation stocks are exchange traded in Norway. Most companies are either publicly owned or have extensive activities beyond hydroelectric generation. As a last resort Eydeland & Wolyniec (2003) propose one might plainly use the corporate risk premium. Since one ultimately is not interested in pricing non-existent markets the approach may suffice, even if it is not theoretically elegant.

5.2 Calculating Sensitivities by Finite Difference Methods

For hedging purposes the generation asset will be viewed as an option to produce power. Following standard financial procedures (see for instance McDonald, 2003) the sensitivities of the asset value to the underlying uncertainties will yield signals to which hedging transactions should be undertaken in the market.

Let the dynamics of a hypothetical forward curve, *f*, be driven by a single source of risk with volatility σ, i.e. assume that only parallel shifts in the forward curve occur. Further let the resolution of *f* be equal to the scheduling resolution, for instance one week for medium term planning, and assume that the forward price converges to the spot price. Also let *f* be arbitrage free with respect to the corresponding swap term structure dynamics. Express the time 0 expected market value of the period *t* uncertain discounted cash flows from the production as $C_{0,t} = C_{0,t}\left(f, \sigma, \mathbf{x}\right)$, where **x** is a relevant vector of variables. We will in the following be concerned with the sensitivities

delta: $\Delta_{0,t} = \dfrac{\partial C_{0,t}}{\partial f}$,

gamma: $\Gamma_{0,t} = \dfrac{\partial^2 C_{0,t}}{\partial f^2}$ and

vega: $vega_{0,t} = \dfrac{\partial C_{0,t}}{\partial \sigma}$.

Following Glasserman (2004) suppose we have some simulation based mechanism for estimating $C_{0,t}$ by solving HPSP for a set of *S* price-inflow scenarios. Further assume that we can individually vary the variables of $C_{0,t}$. Delta, gamma and vega can then be estimated by the finite difference estimators

$$\Delta_{0,t} \approx \hat{\Delta}_{0,t} = \frac{C_{0,t}\left(f + h_\Delta, \sigma, \mathbf{x}\right) - C_{0,t}\left(f - h_\Delta, \sigma, \mathbf{x}\right)}{2h_\Delta},$$

$$\Gamma_0^t \approx \hat{\Gamma}_0^t = \frac{C_0^t\left(f + h_\Gamma, \sigma, \mathbf{x}\right) - 2C_0^t\left(f, \sigma, \mathbf{x}\right) + C_0^t\left(f - h_\Gamma, \sigma, \mathbf{x}\right)}{h_\Gamma^{\ 2}} \text{ and}$$

$$vega_{0,t} \approx \widehat{vega}_{0,t} = \frac{C_{0,t}\left(f, \sigma + h_{vega}, \mathbf{x}\right) - C_{0,t}\left(f, \sigma - h_{vega}, \mathbf{x}\right)}{2h_{vega}} \text{ (Glasserman, 2004).}$$

Glasserman (2004) also provides methods for reducing bias and variance of the estimators with regard to the magnitude of $h_{(\cdot)}$ relative to *S*. In the case of HPSP the methods seem quite intractable however. The magnitude of $h_{(\cdot)}$ and N must therefore probably be determined by a combination of qualified guessing and simulation experiments.

5.3 Delta Hedging Hydroelectric Generation

Assume that $\hat{\Delta}_{0,t}$ has been determined for all scheduling periods. For simplicity consider only a single period τ and assume that a traded swap with price $F_{0,\tau} = f_{0,\tau}$ covers the period exactly. $\hat{\Delta}_{0,\tau}$ then signifies the number of swaps to short in order to hedge the price risk in period τ[14]. Let the value of the portfolio of swaps and future production in period τ at time t be denoted by $V_{t,\tau}$. Suppose Δt time passes and the forward curve shifts by a small amount. The change in the value of the portfolio will then be

$$V_{0,\tau} - V_{\Delta t,\tau} = C_{0,t} - C_{\Delta t,\tau} - \hat{\Delta}_{0,\tau}\left(F_{0,\tau} - F_{\Delta t,\tau}\right) \approx C_{0,t} - C_{\Delta t,\tau} - \frac{C_{0,t} - C_{\Delta t,\tau}}{F_{0,\tau} - F_{\Delta t,\tau}}\left(F_{0,\tau} - F_{\Delta t,\tau}\right) = 0 .$$

As can be seen the producer is effectively hedged against price risk in period τ. Note also that the hedging strategy is dynamic; the producer must recalculate delta and adjust the position accordingly on a regular basis.

In the real world the forward curve dynamics are not well approximated by a one factor model. Models with more sources of risk should therefore be used, especially for risk management purposes (Bjerksund & Stensland, 2000). Clewlow & Strickland (2000) discuss finite difference methods for delta hedging when the forward curve is driven by multiple sources of risk, the extension being relatively simple.

5.4 Hedging with swaps: Hypotheses

The producer has a natural long position in future spot power. It is therefore expected that the resulting hedge will be a short position in swaps. Assuming that there is a natural negative dependence relationship between price and inflow the producer is partially naturally hedged.

[14] Notice that the unit of delta is MWh as $[C_{0,t}]$ = € and $[f]$ = € / MWh.

It is therefore suspected that the delta should be less than the expected production, $E\left[w_t\right]$. The above contemplations are formulated as Hypothesis 1 below.

Hypothesis 1

a. $\Delta_t \geq 0 \qquad \forall t$

b. $E\left[w_t\right] - \Delta_t > 0 \qquad \forall t$

If the scheduling resolution is less than the length of the delivery period of the available hedge products it seems reasonable that the optimal hedge position should be less than the expected cumulative production in the delivery period[15]. Moreover since delivery periods of traded swaps increase with time to delivery the hedge position should be adjusted accordingly. The resolution problem should however not affect the delta directly since we are assuming an underlying continuous forward curve, but rather the way delta is used to create a hedging strategy. Considering scheduling periods that are shorter than the available hedge product the producer can merely minimize the price risk. If, say two scheduling periods are only covered by a single two-period hedge product, the producer will be overhedging one period and underhedging the other.

5.5 Including Options in Hedging: Hypotheses

As explained in section 0 the reservoir yields an option to postpone production. If the volatility of the electricity price increases it can be expected that the value of the option to postpone will increase:

Hypothesis 2

$vega_t \geq 0 \qquad \forall t$

[15] As long as the expected production is not constant in the period.

Option values are generally nonlinear in the price of the underlying. Since a generation asset has option characteristics we can not expect the value of the production to be linear in swap prices either:

> *Hypothesis 3*
> Γ_t *is not generally equal to zero.*

Assuming hypotheses 2 and 3 hold it is reasonable to believe that options can be used fruitfully in hedging. If one is able to reduce gamma and vega, hedges will be more stable (McDonald, 2003). The reason is that the hedge will be a better approximation of the value function. Hence a last hypothesis is proposed:

> *Hypothesis 4*
> *Incorporating options in hedging will yield more stable hedges.*

5.6 A Conceptual Hedging Framework

Based on the discussion so far a step by step framework for hedging price risk is proposed below. The proposed framework implies a dynamic hedging strategy. Thus stability of hedges must decide how often the portfolio needs to be rebalanced.

1. Estimate a realistic price process that conditions on available market information.
2. Estimate the market price of risk, and adjust the price process accordingly.
3. Solve the production planning problem to identify the optimal production schedule.
4. Calculate sensitivities by finite difference methods.
5. Calculate sensitivities of the set of available hedge products (swaps and options).
6. Calculate the optimal hedge positions in the least squares sense.
7. Scrutinize the resulting hedge positions and hedge away, if the results are sensible.

Steps 1 and 3 are treated more carefully in section 7.1 and chapter 6 respectively. Step 5 is dependant on the chosen price model, which should match market prices. Regarding step 6 we can not expect to eliminate price risk with the available hedge instruments, it must instead be minimized in some fashion. Clewlow & Strickland (2000) propose ordinary unconstrained least squares procedures when hedging energy derivatives. One might consider other approaches, for example restricting the number of instruments. If options are included, one can for instance first minimze gamma and vega and secondly use swaps to reduce delta. Step 7 should not be underestimated. One must always bear in mind that the generation asset can not be replicated perfectly with financial instruments. A hedging model for hydroelectric generation should be viewed as a decision support tool, not the truth.

5.7 Hedging To Modify the Company's Risk Exposure

Risk management of hydroelectric generation may be interpreted in a wider context than just minimizing electricity price risk. The fundamentals of the power market make the power price highly dependant on a number of factors such as the gas, coal and CO_2 price, all of which are traded in financial markets. If one is able to robustly model the dependence of the power price on the underlying factors it should be possible to identify possible hedge positions relative to the underlying drivers. If the producer say, is uncomfortable with CO_2 prices it might be possible to reduce the exposure to CO_2 risk and at the same time keep exposure to other sources of risk. Hedging hydroelectric generation can in this wider sense be viewed as modifying the risks to better suit the capabilities of the company.

6 HPSP Solution Framework

In order to identify the optimal production strategy and the resulting hedge positions a framework for solving HPSP is needed. In the following subsections the problem is formulated mathematically and a solution algorithm is proposed. The attention is restricted to medium to long term scheduling of a well regulated, single reservoir, single station system. For an extensive treatment of programming formulations the reader is referred to Fleten & Wallace (2003).

6.1 Deterministic HPSP formulation

In the case of perfect knowledge of future inflow and price development production scheduling is reduced to a deterministic optimization problem. In the case of a linear production function the problem is easily solved as a linear program. As the production function in reality depends non-linearly on both the water head and the flow rate, a realistic model does not present convexity. Consequently even the deterministic problem is non-trivial. Following is a mathematical formulation of the deterministic HPSP.

Set

$\mathbf{P}$	The set of scheduling periods. $\mathbf{P} = \{0,1,...,T\}$

Index

t	Index for period

Data

π_t	Electricity price in period t
ψ_t	Inflow in period t
M_{max}	Upper bound on the reservoir level
M_{min}	Lower bound on the reservoir level
M_0	Initial reservoir
M_{T+1}	End reservoir
Q_{max}	Upper bound on the release
r	One period interest rate

Variables

V_0	Present value of production
m_t	Initial reservoir in period t
l_t	Spill in period t
p_t	Generated electricity in period t
q_t	Released volume in period t

Objective

(1) $$V_0 = \max_{q_t, m_t, l_t} \sum_{t=0}^{T} \frac{\pi_t}{(1+r)^t} p_t$$

Constraints

(2) $p_t = w(m_t, q_t)$ $\forall t \in \mathbf{P}$

(3) $m_{t+1} - m_t + q_t + l_t = \psi_t$ $\forall t \in \mathbf{P}$

(4) $m_0 = M_0$

(5) $m_{T+1} = M_{T+1}$

(6) $M_{\min} \leq m_t \leq M_{\max}$ $\forall t \in \mathbf{P}$

(7) $q_t \leq Q_{\max}$ $\forall t \in \mathbf{P}$

(8) $q_t, l_t \geq 0$ $\forall t \in \mathbf{P}$

The objective function, (1), is the sum of the discounted production revenues in each period. Equation (2) yields the generated amount of power as a function of the reservoir level and the release. Constraint (3) is the water balance, which states that the amount of stored water transferred from one period to the next must equal the initial reservoir in the preceding period plus the net inflow in the preceding period. Constraints (4) and (5) are restrictions on the initial and the end reservoir respectively. The significance of constraint (5) is that without it an optimal solution will seek to empty the reservoir towards the end of the planning horizon, which is often not optimal in a wider perspective. The upper and lower bounds on the reservoir level are formulated in constraint (6). Constraint (7) restricts the maximum release and finally (8) is the non-negativity constraint on the release and the spill. Note that production costs are not included in the objective. The assumption is that costs are too small too affect the solution. According to Nilsson & Sjelvgren (1997) costs related to start-ups are mainly caused by wear, but the authors point to loss of water also being a problem. Nilsson & Sjelvgren (1997) claim that costs can be important in short term scheduling. In medium and long term scheduling, with a weekly resolution, costs are likely to be rather small compared to production revenues.

6.1.1 Extensions and Variations of the Deterministic Formulation

There are some obvious extensions to the above model that are easily incorporated into the formulation. For one it is often necessary to include a strictly positive lower bound on the release due to environmental or other reasons. Planned revisions or environmental regulations might necessitate time varying upper and lower bounds on the release and reservoir variables. Due to technical and safety reasons there will sometimes be constraints on the ramp rates and on the rate of change of the reservoir level. Ramp constraints can be formulated as

$$x_{t+1} - x_t \le D_x^+ \text{ and } x_t - x_{t+1} \le D_x^- ,$$

where x is the variable in question, D_x^+ is the maximum allowed increase and D_x^- is the maximum allowed decrease. From a technical point of view the upper bound on the release should also be restricted by a function of the reservoir level.

Perhaps the most obvious variation of the model is to omit the constraint on the end reservoir. To avoid emptying the reservoir one might instead define a function for the value of the end reservoir or penalize deviations from a target reservoir.

6.1.2 Applications of the Deterministic Model

In the case that future prices and inflows are deterministically known a solution to the program will yield the optimal production strategy and the correct value. In the real world the future is uncertain, and one might want to apply the model to expected inflow and expected price. The most realistic application of the deterministic model is in combination with some kind of Monte Carlo simulation of the price and inflow processes. The problem might then be solved for a set of joint price-inflow scenarios to yield the expected value of the production, by averaging over the solutions. The optimal value, V_0^u, will then be an upper bound to the true value (Näsäkkälä & Keppo, 2005), since the deterministic solution is effectively conditioning on unknown information:

$$V_0^u = V_0 \big| \Pi_t, \Psi_t \qquad \forall t \in \mathbf{P} .$$

It should be noted that even though this approach yields an upper bound for the total value, it is not really possible to infer much about the true, optimal production strategy. According to O. B. Fosso blindly following the expected production plan proposed by the deterministic solution is generally not recommended (lecture, October, 2005). Fleten & Wallace (2003) argue that deterministic solutions underestimate the risk of spilling water and do not see any value in waiting with releasing water, in order to learn more about the future. Concerning risk management applications one should exercise great care in interpreting results from a deterministic model. If one chooses to analyse HPSP in a financial context it is obvious that setting up hedges based on a deterministic model is, at best, theoretically unfounded. The upper bound might however be used in the evaluation of the performance of other models, the idea being that a realistic (stochastic) model yielding values close to the upper bound is better than one yielding values further from the bound.

6.2 Stochastic HPSP formulation

Following the discussion of the importance of explicitly incorporating price and inflow uncertainty, a stochastic programming formulation is provided. Price and inflow are now stochastic variables, and the program is formulated recursively.

Objective

(1) $$V_t = \max_{q_t, m_t, l_t} E\left[\Pi_t p_t + \frac{V_{t+1}}{1+r}\right] \qquad \forall t \in \mathbf{P}$$

Constraints

(2) $$p_t = w(m_t, q_t) \qquad \forall t \in \mathbf{P}$$

(3) $$m_{t+1} - m_t + q_t + l_t = \Psi_t \qquad \forall t \in \mathbf{P}$$

(4) $$m_0 = M_0$$

(5) $$\mu_{V_{T+1}} = g_T(\pi_T, \psi_T, m_{T+1})$$

(7) $$M_{\min} \le m_t \le M_{\max} \qquad \forall t \in \mathbf{P}$$

(8) $$q_t \le Q_{\max} \qquad \forall t \in \mathbf{P}$$

(9) $$q_t, l_t \ge 0 \qquad \forall t \in \mathbf{P}$$

The objective (1) is identical to equation (2.2), apart from the introduction of the spill variable as an argument. The constraint on the end reservoir in the deterministic formulation is replaced with constraint (5), giving the conditional discounted value of the end reservoir as a function of the terminal price, inflow and end reservoir. Note that the functional form of g_T implies the assumption that the period T price and inflow exhibit all information of the price and inflow history. The remaining constraints correspond to the constraints in the deterministic formulation. A solution to the stochastic formulation will yield the optimal production schedule and the true value of the production.

6.3 Stochastic Solution Algorithm

In order to solve the stochastic programming formulation a multistage stochastic algorithm is called for. The application to medium to long term scheduling and hedging imply several requirements of the solution algorithm:

- Inflow and price uncertainty must be part of the state space.
- The solution method must prove means for calculating hedge signals.
- Since end effects can be pronounced in the case of a well regulated system, the solution algorithm must be able to handle a relatively long analysis horizon.
- For obtaining realistic hedges there is a need for a time resolution of not less than a week; a solution that has constant expected production for long time periods might be in conflict with market incentives.
- Implementation and maintenance of the system should be within reach for the practitioner.
- Electricity price processes are not yet fully understood. As a consequence a method where the price description can be replaced with relative ease would be particularly sought after.
- The problem must be solvable within reasonable time.

In the literature different solution approaches have been suggested. Standard stochastic dynamic programming (SDP) techniques are frequent. SDP is based on representing the uncertainty on trees or as a Markov chain. The tree representation leads to dimensionality problems when considering problems with a large number of stages. Stochastic dual dynamic programming (SDDP) (Pereira (1989), Pereira et al. (1999)) has received much attention by SINTEF Energy Research. According to S.E. Fleten however “SDDP is best suited for problems with uncertainty in the right hand side only, in our case inflow, and not in the objective, in our case price” (e-mail correspondence, June 9, 2006). Zhang & Ponnambalam (2006) develop the Fletcher-Ponnambalam model (Fletcher & Ponnambalam, 1998) accounting for price uncertainty. The model avoids scenario generation altogether, which could mean reduced computing times. However, the price description used is highly unrealistic and results remain inconclusive. Davison et al. (2004) consider a single reservoir system and derive nonlinear partial-integro-differential equations based on real options theory for valuation and the optimal operating strategy. The equations are solved via sophisticated numerical methods for short term scheduling problems. Näsäkkälä & Keppo (2005) suggests an approach based on specifying a parameterized production threshold function. The method shows promising results applied to a Norwegian system.

Of the methods proposed in the literature Näsäkkälä & Keppo (2005) seems to be closest to satisfying the aforementioned requirements. What poses the greatest challenge is identifying the parameters of the threshold function. Näsäkkälä & Keppo (2005) reports that 150 different parameter combinations were chosen from a larger set of parameters based on qualitative judgement and simulations. Further extensive simulation was undertaken to identify the final parameters. Maintenance of the model may thus prove a challenge. Relative to the aforementioned requirements, the existing algorithms that have been identified are either quite complicated, pose difficult implementation issues or have infeasible computing time.

6.4 Least-Squares Monte Carlo Algorithm

Inspired by advances in pricing of American options by simulation, originating from Longstaff & Schwartz (2001), a HPSP solution algorithm based on least-square Monte Carlo (LSM) techniques has been developed. LSM is closely related to what is referred to as the stochastic mesh. Intuitively the LSM method is quite simple, but the relation to the stochastic mesh is important as a theoretical basis for the approach.

6.4.1 Estimating Continuation Values by Regression

Figure 6.1 show the principles of a stochastic mesh. Solid lines represent independently drawn scenarios, and dashed lines represent a possibility to move from one scenario to another. Let $n_{s,t}$ denote the node defined by the intersection of scenario *s* and period *t*. Since the scenarios are generated independently the appropriate weight to assign to the transition from an arbitrary node $n_{i,t}$ to a following node $n_{j,t+1}$ is not obvious. Glasserman (2004) discuss general conditions on the mesh and how the distribution of the generating process can be used to determine the transition weights. The calculation is generally non-trivial as the transition densities may be unknown or fail to exist (Glasserman, 2004). The main drawback of the stochastic mesh is obvious. The determination of weights is likely to be difficult and time consuming. Considering realistic price-inflow dynamics the calculation might even turn out to be infeasible.

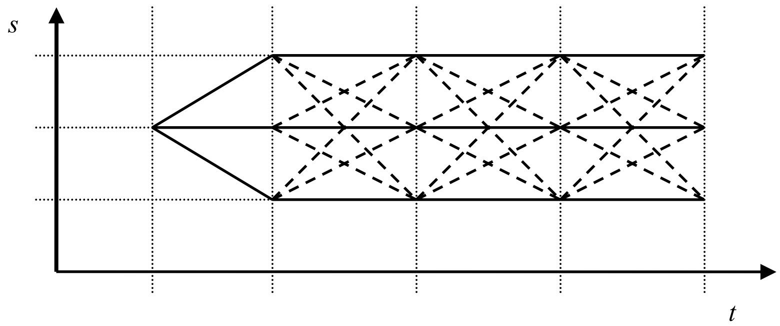


Figure 6.1: Stochastic Mesh

In practice the goal is not however to calculate transition weights. Glasserman (2004) discuss applications of the technique to evaluate continuation values of American style options. Concerning HPSP, the expected future value of the reservoir as a function of the state variables is needed, in order to identify the optimal operating strategy. The LSM method uses least-squares regression to estimate continuation values for American style options. According to Glasserman (2004) the regression leads to an implicit choice of weights in the stochastic mesh. In other words one can avoid calculation of weights altogether by using regression estimators as an approximation to the continuation value. Longstaff & Schwartz (2001) apply the LSM approach to value American style, path dependent options in multifactor settings and obtain accurate approximations. In later years the LSM approach has been extended to real option problems by Gamba (2002). Thanawalla (2005) apply LSM to value swing options and claim that the methodology has relevance to physical assets that derive their value from cash flows resulting from flexible supply. The choice to extend LSM to HPSP is thus based on the apparent flexibility of the approach.

6.4.2 Description of the LSM Algorithm

The current section will explain the LSM algorithm. For a detailed pseudo code the reader is referred to Appendix 2. In the algorithm the intraperiod price and inflow is assumed revealed at the beginning of each period. The assumption implies that at the beginning of period t the end reservoir in the period, m_{t+1}, reduces to a deterministic variable.

The concepts of the algorithm are quite simple and somewhat similar to traditionally used SDP, the main exception being that independent parallel scenarios are used instead of trees or restrictive Markov chains. Inputs to the algorithm are a set of joint price-inflow scenarios, a specification of the function for the value of the end reservoir[16] and a discrete set of reservoir levels. The set of reservoir levels should approximate the continuous set of all possible reservoir levels. The algorithm has two main parts. The first part works recursively to produce an approximated function for the conditional expected future value in every period. Secondly the conditional expectations are used in stepwise forward simulation to construct the production paths. The operating strategy is found as the expectation over the resulting paths.

Part 1: Recursion

Beginning in the last period, T, the one stage problem is solved for all possible combinations of scenarios and possible reservoir levels defined by the discrete set. Since the value of the end reservoir is specified by a predetermined function the optimal values, conditioned on the initial reservoir in the period, are found. A hypothetical reservoir development in an arbitrary scenario is shown in Figure 6.2.

[16] The function g_T in the stochastic programming formulation.

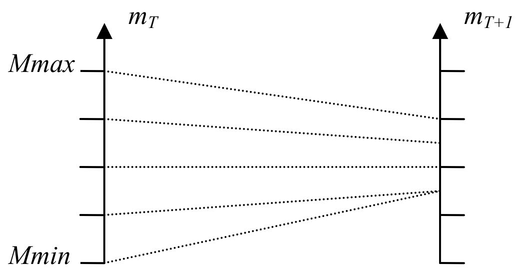


Figure 6.2: Possible one stage reservoir development

For each discrete initial reservoir level a function for the conditional expected value[17] in *T-1* is estimated by regressing the corresponding optimal values on a set of basis functions of the state variables, price and inflow, in the previous period. To create a scenario specific future value function in *T*-1 all regression functions are evaluated with the particular realizations of price and inflow. The scenario specific future value function is approximated as a linear interpolation between the resulting values. When all scenario specific future value functions are created the one stage sub problem in period *T*-1 can be solved in the same manner, utilizing the scenario specific future value function. The procedure is repeated until all future value functions are estimated.

Note that the defining values of the interpolation functions need not be stored for the whole process. Merely the coefficients of the regression functions need to be stored, as they are used in creating the scenario specific future value functions.

End Part 1

Part 2: Stepwise Forward Optimization

To eliminate possible regression bias new scenarios are drawn. Starting in the first period, in the current reservoir, the scenario specific future value functions are used in solving the one stage problems. If the future value functions have not been stored the necessary part of the function can be created through the regression functions. In the case of a well regulated system it is often only possible to release a small part of the available water during one period, and hence only a small part of the scenario specific future value function is needed. After determining the optimal strategy in the first period the uncertainty in the next period is

[17] Conditional on ending in a specific reservoir.

revealed. The next subproblem can then be solved while enforcing the water balance as a link to the previous period. When all sub problems in all scenarios are solved, the operating strategy is found as the expectation over all paths.

End Part 2

In the case that g_T is a concave function with respect to the reservoir value and also assuming a linear production function, the future value functions should be concave. This follows from the fact that an increased future reservoir level must at least yield the same expected value of future production, but at the same time increase the risk of spilling. The scenario specific future value function should thereby look something like Figure 6.3. The shape of the resulting future value functions should hence be able to convey some information about the stability of the regressions.

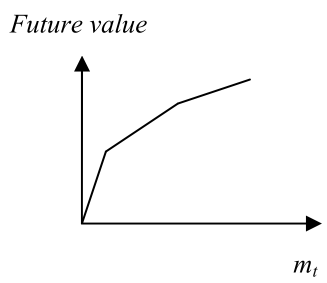


Figure 6.3: Reasonable shape of future value function

6.4.3 Basis functions

According to Longstaff & Schwartz (2001) numerical results indicate that simple powers of the state variables give accurate results in determining the optimal exercise strategy for American style options. For the application to HPSP it is therefore chosen to use the basis functions π, π^2, π^3, ψ, ψ^2, ψ^3 and $\pi\psi$ as well as a constant. Longstaff & Schwartz (2001) also suggest a number of other possible basis functions such as Laguerre polynomials, Jacobi polynomials and trigonometric series.

7 Representation of Uncertainty

The proposed LSM algorithm necessitates a mechanism to create synthetic price and inflow series. In previous sections arguments for a fundamental dependence of price on inflow have been advanced. Estimation of a joint price-inflow distribution would therefore be preferable. Considering the non-stationary and complex features of price and inflow processes, estimating a joint distribution would require extensive studies. Probably one would need to introduce additional explanatory variables and apply advanced statistical methodology to achieve satisfactory results. Considering the limited time frame of the current work, price and inflow processes are estimated separately. However, the models should allow for correlation in order to investigate the qualitative influence on operating strategy and hedges.

7.1 Price model

Commodity price processes have been extensively studied, with electricity price processes receiving increased attention in later years. Three main approaches can be distinguished in the literature. Obviously the modelling of spot prices, along with the derivation of pricing formulas for derivative contracts, has received attention. Among others, Schwartz (1997) is an important contribution in this area. On the other hand the Heath-Jarrow-Morton (HJM) approach (Heath et al., 1992) of modelling the entire forward curve has also been studied for modelling energy prices. Bjerksund & Stensland (2000) and Benth & Koekebakker (2005) develop term structure models with a basis in the Nordic electricity market. Clewlow & Strickland (2000) and Eydeland & Wolyniec (2003) provide good overviews of both the spot modelling approach and the term structure approach. Common to both the spot modelling approach and the HJM approach is that models based on Brownian motion tend to have problems fitting the data[18]. Finally Eydeland & Wolyniec (2003) suggests a modelling approach combining fundamental models and fuel price models with calibration to electricity market prices. The approach is labelled hybrid modelling.

[18] The problem is more pronounced for spot prices than for swap prices.

7.1.2 Considerations When Choosing a Price Model

Hydroelectric scheduling is affected not only by the short term dynamics of the spot price, but also by the long term dynamics of the forward curve. Especially when considering a system with high relative regulation, the state and dynamics of the long end of the forward curve may influence heavily on the immediate production decision. As a hydroelectric system may be considered a derivative directly on the spot price, one might be tempted to model the spot electricity price only. For hedging purposes it is not possible to use the spot, as it can not be stored. Thus, there is a need for a model that conditions on current market information. A good model for hydroelectric scheduling could possibly be a combination of a term structure model and a spot model. In other words a term structure model with a joint spot model on the short end, allowing for short term spikes in the spot price. Obviously the development of a realistic price model would be quite a study in itself, considering the deviations from standard assumptions pointed out in chapter 3. Some quite simplifying assumptions with regard to the price process are hence made.

7.1.3 Choice of Price Model

In order to incorporate market information there is a need for the price model to match quoted market prices of swap contracts and preferably also the prices of traded options. In this respect it seems natural to choose a HJM model. On the other hand Benth & Koekebakker (2005) point out several problematic issues regarding the HJM approach in modelling spot electricity prices, one of the most important being that swap prices do not in general converge to the spot price. Pure spot models on the other hand often require extensive calibration to match market prices, which would be impractical in simulation of hedges. Also the price model chosen should allow for correlation with the inflow model, to facilitate testing of possible effects of price inflow dependence on hedges and production strategy.

Following the preceding discussion it is chosen to implement a one factor term structure model, as well as a one factor spot model calibrated to the term structure. The difference between the spot model and the term structure model being that the latter can better be calibrated to match the implied volatilities of quoted options. The spot model chosen is the

Schwartz (1997) one factor mean reverting model, and the chosen term structure model is of the Bjerksund et al. (2000) type.

7.1.4 Schwartz One Factor Model

The Schwartz (1997) one factor mean reverting model is defined by the stochastic differential equation

$$\frac{d\Pi(t)}{\Pi(t)} = \kappa\left(\theta(t) - \ln\Pi(t)\right)dt + \sigma dZ(t), \tag{7.1}$$

where $dZ(t)$ is a standard Browninan motion, κ is the mean reversion rate, $\theta(t)$ is a time varying mean and σ is the standard deviation of the Brownian motion. In simulation and parameter estimation of (7.1) it is convenient to do the transformation

$$\mathrm{X}(t) = \ln\Pi(t).$$

Applying Itô's formula yields

$$d\mathrm{X}(t) = \kappa\left(\hat{\theta}(t) - \mathrm{X}(t)\right)dt + \sigma dZ(t), \qquad \hat{\theta}(t) = \theta(t) - \frac{\sigma^2}{2\kappa}. \tag{7.2}$$

The simulation of (7.1) may be accomplished by discretizing (7.2) as

$$\Delta\mathrm{X}_t = \kappa\left(\hat{\theta}_t - \mathrm{X}_t\right)\Delta t + \sigma\sqrt{\Delta t}\,\mathrm{Z}_t,$$

where Z_t is a standard normal variate. Clewlow & Strickland (2000) point to the fact that the deterministic drift is a function of the price, and as a consequence the discretization is only correct in the limit $\Delta\mathrm{X}_t \to d\mathrm{X}(t)$. The time steps should for this reason be chosen small relative to the mean reversion rate. Schwartz (1997) shows that the model can be calibrated to the term structure of forward contracts by letting

$$\theta_t = \frac{1}{\kappa}\frac{d\ln f(0,t)}{dt} + \kappa\ln f(0,t) + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa t}\right),$$

where $f(0,t)$ denotes the current forward price for a contract with maturity at time *t*. Calibrating the process in this fashion will yield a risk neutral spot price process with regard to forward prices. However, the volatility structure of the model will not generally correspond with market price of options. According to Clewlow & Strickland (2000) the Schwartz model is equivalent to a one factor lognormal forward curve model with volatility structure $\sigma(t,s) = \sigma e^{-\kappa(t-s)}$, where $\sigma(t,s)$ is the volatility of a forward contract at time *t* with delivery at time *s*.

7.1.5 Estimation of Schwartz Model Parameters

As we do not observe forward prices for electricity there is a need to create a forward curve from observed swap prices to calibrate the model. This issue is treated in section 7.2. Further Schwartz (1997) suggests using Kalman filtering techniques to estimate the parameters κ and σ. Though more crude, the parameters are in this case estimated by a simple least squares procedure. Specifically the estimation procedure is based on the fact that when observations of the process (7.2) are regularly spaced, the joint density is the same as the joint density of the AR(1) process

$$\left(1 - e^{-\kappa}B\right)\mathrm{Y}_n = \left(1 - e^{-\kappa}B\right)\hat{\theta}_n + \varepsilon_n, \qquad \varepsilon_n \sim N\left(0, \frac{\sigma^2\left(1 - e^{-2\kappa}\right)}{2\kappa}\right), \tag{7.3}$$

where *B* denotes the back shift operator (Brockell & Davis, 2002).

Utilizing the discretization (7.3), the parameters κ, σ, and $\hat{\mu}_n$ were simultaneously estimated by minimizing the mean square of the prediction errors. Weekly system prices, obtained from Nord Pool, for the period 1995-2005 were used in estimation. Descriptive statistics of the prices and log returns are provided in Appendix 3. The minimization was undertaken using the Solver Add-in in Microsoft® Excel 2003. As the solver is not particularly robust towards local minima starting values were carefully chosen. The logarithm of the weekly, historical mean was used as a starting value for $\hat{\theta}_n$, and starting values for κ and σ were found by a linear regression procedure.

The final, annualized parameter estimates are provided below.

$$\hat{\kappa} = 1.8835$$
$$\hat{\sigma} = 0.2387$$

An augmented Dickey-Fuller test was undertaken to test for a unit root. The value of the statistic was -3.22, implying that the null hypothesis of a unit root is rejected at level .05 (see Brockwell & Davis, 2003). A regression plot resulting from the Dickey-Fuller test, and descriptive statistics of the residuals are provided in Appendix 3. Following the discussion in section 3.2 it should not come as a surprise that the model fit is not very good. The excess kurtosis of the residuals is of such magnitude that the lognormal model does not prove a decent fit to the data.

7.1.7 Bjerksund et al. One Factor Term Structure Model

Bjerksund et al. (2000) adapt the Black 76 model to describe swap dynamics, and provide analytical pricing formulas for European options. The current section is based on the work of the aforementioned authors.

Let $f(t,s)$ be the price at time t for a forward contract with delivery over an infinitesimal delivery period at time *s,* and let $\sigma(t,s)$ be the volatility of $f(t,s)$. Assuming lognormal forward prices, Bjerksund et al. (2000) propose the following general model for the forward dynamics

$$\frac{df(t,s)}{f(t,s)} = \sigma(t,s)\,dZ(t)\,. \tag{7.4}$$

The forward price at a future time u is then given by

$$f(u,s) = f(t,s)\exp\left(\int_t^u \sigma(v,s)\,dZ(v) - \frac{1}{2}\int_t^u \sigma(v,s)^2\,dv\right). \tag{7.5}$$

Assuming that the forward price converges to the spot price, the future spot price is given by letting $u \to s$ in (7.5), i.e. $f(t,t) = \Pi(t)$. Since we do not observe prices of options on forwards we need an associated swap model in order to calibrate (7.4) to observed option prices. Following arbitrage arguments the price of a swap, $F(t;T_1,T_2)$, at time *t* with delivery in the period T_1 to T_2 must be

$$F(t;T_1,T_2) = \int_{T1}^{T2} \hat{\omega}(s;r,T_1,T_2) f(t,s) ds, \qquad \hat{\omega}(s;r,T_1,T_2) = \frac{e^{-rs}}{\int_{T_1}^{T_2} e^{-rv} dv}$$

(Benth & Koekebakker, 2005).

Bjerksund et al. (2000) approximate the swap dynamics as

$$\frac{dF(t;T_1,T_2)}{F(t;T_1,T_2)} \approx \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \frac{df(t,s)}{f(t,s)} ds = \Sigma(t;T_1,T_2) dZ \text{ , where} \tag{7.6}$$

$$\Sigma(t;T_1,T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \sigma(t,s) ds \tag{7.7}$$

Benth & Koekebakker (2005) derive an expression for the true swap dynamics, and discuss the implications of the approximation (7.6). In the following it is assumed that the approximation will suffice for our purposes. To calibrate the swap model to traded options let the Black 76 volatility be denoted by σ_{B76} and let the time to expiration be denoted by τ. Following Bjerksund et al. (2000)

$$\sigma_{B76} = \sigma_{B76}(\tau - t, T_1 - t, T_2 - t) = \sqrt{Var_t\left[\ln \frac{F(\tau;T_1,T_2)}{F(t;T_1,T_2)}\right] \frac{1}{\tau - t}} \text{ , where} \tag{7.8}$$

$$Var_t = \int_t^{\tau} \left(\Sigma(s;T_1,T_2)\right)^2 ds . \tag{7.9}$$

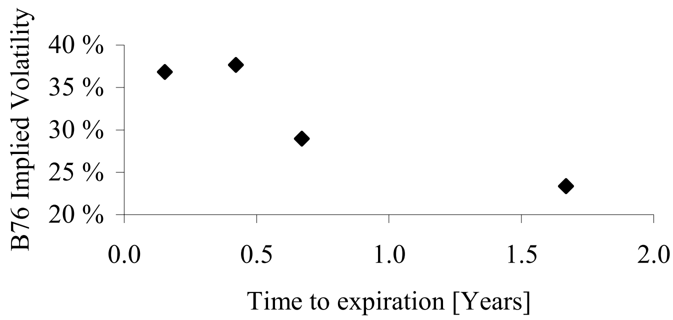
Assuming a parameterized form of $\sigma(t,s) = \sigma(t,s;\mathbf{x})$ one can through the relations (7.7), (7.8) and (7.9) calibrate the parameters $\mathbf{x}$ in order to match observed option prices.

7.1.8 Calibration of Volatility to Market Prices

There is not an established industry standard in the sense of the best volatility function. There are some effects the volatility function should be able to capture however. First of all there is the pronounced Samuelson effect. Also volatility tends to level off at a nonzero level for long maturity contracts. Here the function (7.10), proposed by Rebonato (1999), is assumed.

$$\sigma(t,s) = \left(a + b\left(s-t\right)\right)\exp\left(-c\left(s-t\right)\right) + d \tag{7.10}$$

Often it is observed that the Black 76 implied volatility for the second option contract to mature actually is higher than for the first contract to mature. An example is provided in Figure 7.1. (7.10) can also capture this feature.



Figur 7.1: Black 76 implied volatility of at the money option contracts 20 April, 2006

The estimation of the parameters , $\mathbf{x}$, was undertaken by minimizing the mean square error of the prices predicted by the model. The regression was conducted in Matlab® and the prices used were closing prices for at the money call options on 20 April, 2006. It is assumed that the observed prices represent the markets expectation of volatility on the specific date. Regression results are provided in Table 7.1. The resulting volatility function is plotted in Figure 7.1, along with the volatility structure implied by the estimated Schwartz model.

Product	*Price*	*Model Price*	*Parameters*	
ENOC53Q3-06	3.31	3.37	a	0.2510
ENOC57Q4-06	5.34	5.22	b	0.7774
ENOC50YR-07	4.80	4.87	c	1.8017
ENOC48YR-08	5.64	5.61	d	0.0990

Table 7.1: Regression results

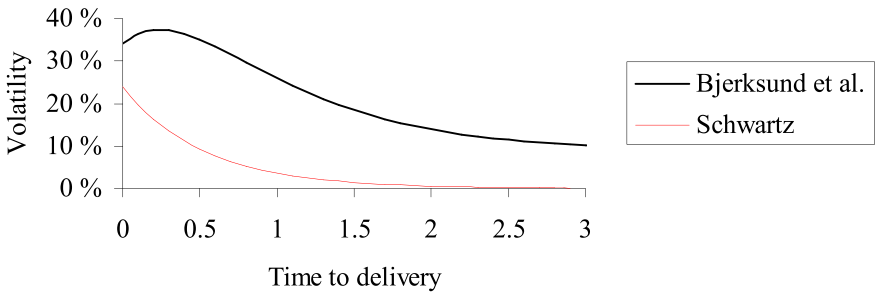


Figure 7.1: Forward volatility function

7.1.9 Discussion

The Rebonato volatility function does give a decent picture of the at the money volatility, implied by the traded options. In general one should expect the parameters to be quite unstable, since only four different delivery period option contracts currently trade at Nordpool. Estimation based on historical volatility would generally be far more robust, but might not represent the current market price of volatility. Further, because of the non-Gaussian behaviour of swap returns, one can not expect the forward curve model to price options with relatively high or low strikes correct. The estimated volatility of the Schwartz model is clearly below market expectations. Compared to the Schwartz model it can be expected that the Bjerksund et al. model will perform relatively well. With that said, referring to 5.3, a multifactor forward curve model would be preferable in real life hedging applications.

7.2 Constructing a Smooth Forward Curve

In order to implement the above described price models there is need to construct a continuous forward curve from observed swap price data. The approach of Ollmar (2003) has been chosen in the current case. For a different approach see for instance Fleten & Lemming (2003). Ollmar's approach is based on the fitting of a fifth order polynomial spline to the observed swap prices. The resulting curve can easily be integrated to produce swap prices with arbitrary delivery periods. Following is a short description of the method and a presentation of the results obtained from an application to Nord Pool price data.

Let $f_i(0,t;\mathbf{y}_i)$, where $\mathbf{y}_i$ are the polynomial coefficients, denote the smooth curve piece corresponding to the swap price $F(0,T_i,T_{i+1})$. Assume we observe N non-overlapping, adjacent swap contracts. To find the parameters of the spline solve the minimization problem:

(1) $$\min_{\mathbf{y}_1,\ldots,\mathbf{y}_N} \sum_{i=1}^{N} \int_{T_i}^{T_{i+1}} [f_i''(0,t;\mathbf{y}_i)]^2\, dt$$

s.t.

(2) $f_i(0,T_i;\mathbf{y}_i) = f_{i+1}(0,T_{i+1};\mathbf{y}_{i+1})$ $\quad i = 1,\ldots,N-1$

(3) $f_i'(0,T_i;\mathbf{y}_i) = f_{i+1}'(0,T_{i+1};\mathbf{y}_{i+1})$ $\quad i = 1,\ldots,N-1$

(4) $f_i''(0,T_i;\mathbf{y}_i) = f_{i+1}''(0,T_{i+1};\mathbf{y}_{i+1})$ $\quad i = 1,\ldots,N-1$

(5) $f'(0,T_N;\mathbf{y}_N) = 0$

(6) $\int_{T_i}^{T_{i+1}} e^{-rt} f_i(0,t;\mathbf{y}_i)dt = F(0,T_i,T_{i+1})$ $\quad i = 1,\ldots,N$

The objective, (1), represents the maximum smoothness criterion by minimization of the change in the first derivative along the curve. Constraints (2), (3) and (4) ensure that the curve is continuous and differentiable in the second order sense. The fifth constraint forces the spline to level out at the end of the horizon. The last constraint ensures that there are no arbitrage opportunities between the smoothed curve and the original term structure. Ollmar (2003) shows how the problem can be solved as a system of linear equations, and also extends the model to include bid-offer spreads.

When contracts with long delivery periods are included, the resulting smooth curve will not necessarily exhibit the expected seasonal characteristics. Ollmar (2003) proposes to include a prior function in the optimization to adjust for seasonal variations. The adjustment is undertaken by subtracting the integral of the adjustment function over the respective delivery period from all swap prices before the optimization, and adding the prior function to the spline after the optimization. In Figure 7.2 the result from an application of the method is shown. The figure illustrates how the resulting seasonal variation is quite unrealistic for the contract for the year 2009 when we do not include a prior function.

The prior function included for the correction of this problem is the simple sinusoidal function

$$a\cos\left(\frac{2\pi}{365}(t-b)\right),$$

where a and b are constants and t is measured in days. The specific choice of a and b is done to give a peak in mid January and an amplitude of € 10. In real applications one should strive to find good prior functions. Both Ollmar (2003) and Fleten & Lemming (2003) suggest using the results from bottom up models.

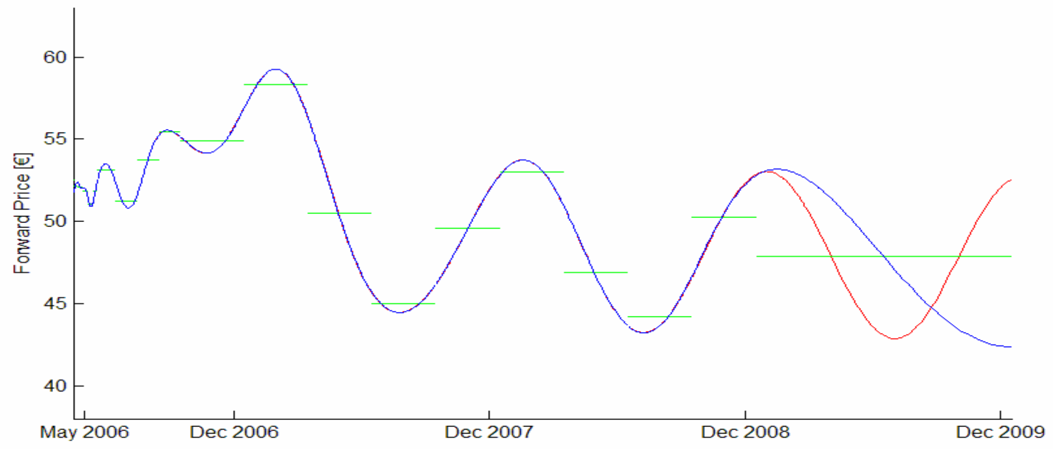


Figure 7.2: Smoothed term structure of closing prices on 20 April, 2006

7.3 Time Series Analysis of Inflow Data

In connection with the current work a time series analysis of the Siso reservoir inflow series has been undertaken. Tools used in the analysis include the statistical computing environment R and the Matlab® toolboxes Econometrics Toolbox (LeSage, 2005) and the UCSD GARCH Toolbox (Sheppard, 2005).

River flow time series are in the literature analyzed using a host of different models. The current text will not attempt to give an overview, but rather restrict attention to fairly basic approaches. Despite the fact that hydrological time series are intrinsically continuous only discrete autoregressive moving average (ARMA) type models will be considered. In some cases hydrological time series exhibit the Hurst effect (Hurst, 1951), which is long range dependence in the time series. The main explanations for the Hurst phenomenon are either that the time series exhibits long term memory or that the mean of the series changes with time (Montanari, Rosso & Taqqu, 1997). To analyze long range dependence fractionally integrated ARMA (FARIMA) models are applicable (Brockwell & Davis, 2002).

7.3.1 Definitions and terminology

$\{\Psi_n\}$ is an ARMA(*p,q*) process if $\{\Psi_n\}$ is stationary and satisfy the general equation

$$\Psi_n - \phi_1\Psi_{n-1} - \ldots - \phi_p\Psi_{n-p} = Z_n + \theta_1 Z_{n-1} + \ldots + \theta_q Z_{n-q} \,,$$

where Z_n is white noise and the polynomials $\left(1-\phi_1 z - \ldots - \phi_p z^p\right)$ and $\left(1+\theta_1 z + \ldots + \theta_q z^q\right)$ have no common factors. (Brockwell & Davis, 2002)

An ARMA classical decomposition model is defined by $X_n = s_n + Y_n$, where Y_n is an ARMA process and s_n is a seasonal component.

7.3.2 The Inflow Data Set

Weekly accumulated inflow data from the period 1960-2005 was obtained for the analysis. The inflow has been measured using the weekly measurements of the reservoir level as a proxy. The time series is plotted in Figure 7.3.

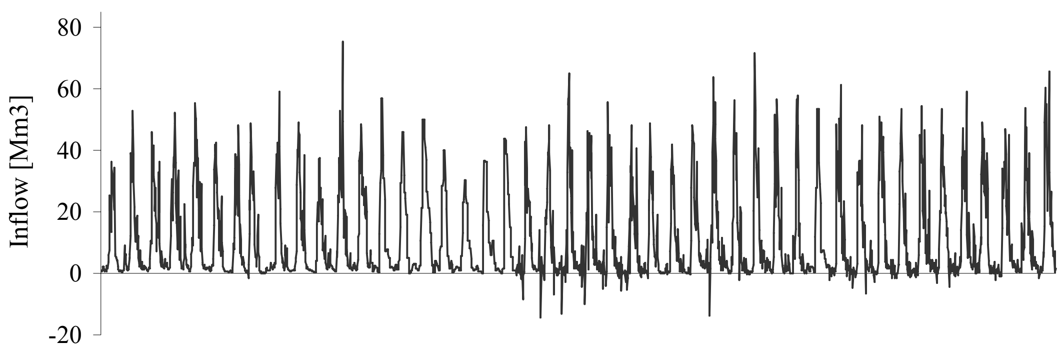


Figure 7.3: Siso inflow time series

The time series clearly exhibits strong seasonality with a distinct peak in the melting period and small values in winter. A closer inspection of the series reveals that the melting period arrives in different weeks from year to year resulting in one or a few weeks of extreme inflows. Figure 7.4 illustrates the seasonal component as the average weekly inflow (black) and a fitted sum of five harmonics (red).

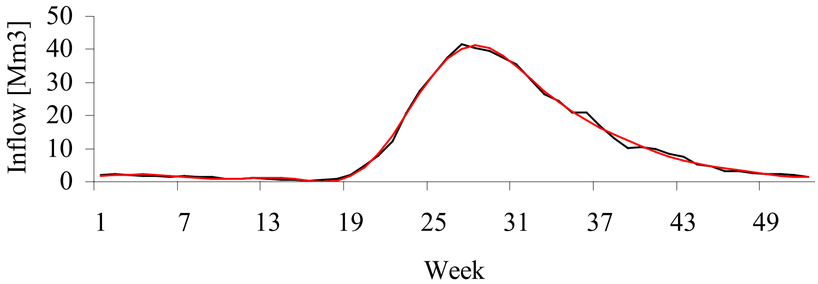


Figure 7.4: Seasonel component of Siso inflow time series

From Figure 7.3 we further see that some observation values are in fact negative. Small negative values could be caused by evaporation, but some are of such magnitude that evaporation does not seem to give an adequate explanation. A more plausible explanation is that measurement errors in one week have lead to negative values in a later week. This is indeed the case as extreme weather conditions and equipment failure has lead to a need for estimating inflow from time to time. Unfortunately information about which values are estimated is unavailable. Also the visual inspection reveals that in the years 1973-1979 measurements were conducted only every fourth week with the weekly inflow reported as the monthly mean. In all the quality of the data is questionable. Further descriptive statistics of the data set are provided in Table 1 and Table 2 in Appendix 4.

7.3.4 Filtering of the Data

Concerning the negative observation values a simple filtering procedure was applied to leave only strictly positive values. It was assumed that a negative value was due to measurement error in the previous week, and the negative observation value, ψ_n and the preceding observation ψ_{n-1} was replaced such that the following equations were satisfied:

(1) $\psi_n^* = \frac{\psi_{n+1} + \psi_{n-1}^*}{2}$ and

(2) $\psi_{n-1}^* + \psi_n^* + \psi_{n+1} = \sum_{i=-1}^{1} \psi_{n-i}$.

Star denotes a replacement value, and ψ_{n+1} denotes the observation following the negative value. Equation (1) forces ψ_n to be a linear interpolation between the neighboring values, and equation (2) preserves the amount of cumulative inflow. The procedure was applied iteratively until every value of the time series was strictly positive. A plot of the filtered approximation is provided in Figure 1 in Appendix 4. Note that this filtering procedure is unfounded theoretically, and it is difficult to say how well the approximated series resembles the true process. At least the sample autocorrelation functions (SACF) of the filtered and unfiltered series (plotted in Figures 2 and 3 in Appendix 4) are nearly identical.

With regard to the years where only monthly measurements were made, it was decided to do a preliminary analysis of a truncated series with data from the years 1980-2005 only. Unfortunately such a limited data set is not sufficient to do a robust analysis of a possible Hurst effect (Montanari, Rosso & Taqqu, 1997). Still, the existence of a Hurst effect is fundamentally plausible. The Siso reservoir receives a great deal of its inflow from surrounding glaciers. Since Norwegian glaciers tend to shrink and grow over time (see for instance Feaney & Sweeney (2005)) one might have reason to suspect that the mean inflow show some dependence on this phenomenon.

7.3.5 Measures to Produce an Underlying Stationary Time Series

As expected the sample ACF suggested a seasonal pattern with period 52. Differencing did however not produce a stationary time series. An investigation of the variance structure of the time series by applying a moving window strongly suggested seasonal variance, as can be seen in Figure 7.5.

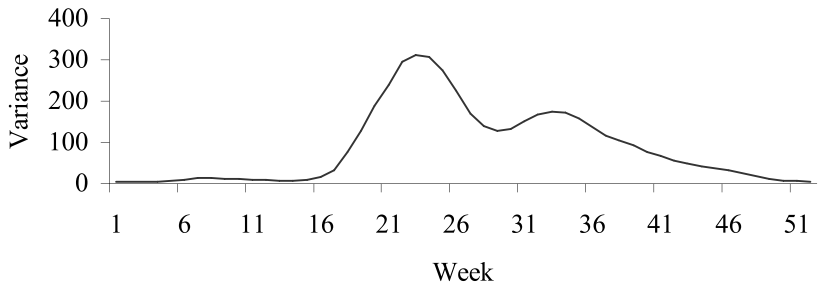


Figure 7.5: Average 10 week moving window sample variance

A Box-Cox transformation was applied in an effort to stabilize the variance of the series. The Box-Cox transformation is defined by

$$f_\lambda(\Psi_n) = \lambda^{-1}\left(\Psi_n^\lambda - 1\right), \qquad \Psi_n \geq 0, \lambda > 0 .$$

The optimal value for λ was found to be 0.1241. For simplicity define $Y_n \equiv f_\lambda(\Psi_n)$ The transformed series was differenced at lag 52. Plots of the SACF and the sample partial autocorrelation function (SPACF) of the transformed, differenced series are provided in Figures 4 and 5 in Appendix 4. Both plots indicate that the series still exhibits periodicity. Further differencing did not produce stationarity.

A classical decomposition model (of the form $Y_n = s_n + X_n$) was also investigated. The seasonal component (Figure 7.6) was modelled as a sum of five harmonics fitted to the sample average of the Box-Cox transformed series. The resulting SACF and SPACF (Figures 6 and 7, Appendix 4) of the underlying series decay quickly indicating stationarity.

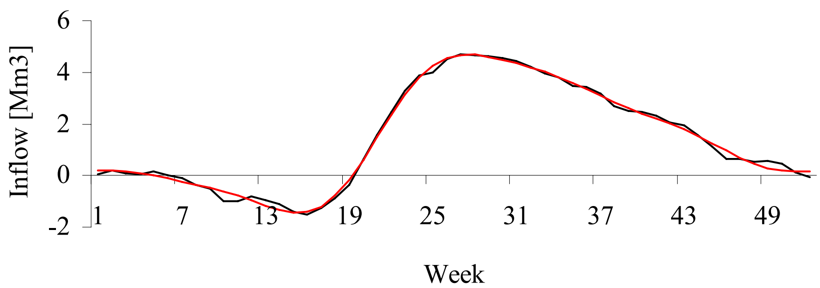


Figure 7.6: Seasonal component of Box-Cox transformed series

7.3.6 Parameter Estimation

Preliminary analyses suggested to investigate further an AR(1) model or a low order ARMA process in modelling X_n. The estimation procedure undertaken is based on maximization of the Gaussian log likelihood of the ARMA process. According to Brockwell & Davis (2002) using the Gaussian likelihood for parameter estimation and model selection is well founded even if the noise is not Gaussian. Table 7.2 displays the log likelihood and the Akaike information criterion (AIC) for different choices of p and q.

(p,q)	*Log Likelihood*	*AIC*
(1,0)	-1756.58	3519.15
(1,1)	-1756.57	3521.15
(1,2)	-1756.11	3522.22
(1,3)	-1752.44	3516.88
(1,4)	-1747.69	3509.37
(2,1)	-1756.58	3523.15
(2,2)	-1755.88	3523.15
(2,3)	-1748.34	3510.68
(2,4)	-1747.68	3511.37
(3,1)	-1756.10	3524.21
(3,2)	-1753.58	3521.15
(3,3)	-1753.73	3523.45
(3,4)	-1747.68	3513.37

Table 7.2: Log likelihood and AIC of fitted ARMA models

According to theory the model with the smallest AIC value should be chosen (Brockwell & Davis, 2002). Further it seems sensible to consider the magnitude of the standard error (SE) of the parameter estimates to the value of the estimated parameters, especially considering the quality of the data. Table 7.3 lists parameter estimates and standard errors of the five most promising models.

(p,q)		$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\phi}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_4$	σ^2
(1,0)	Estimate	0.644	-	-	-	-	-	-	0.787
	SE	0.021	-	-	-	-	-	-	
(1,3)	Estimate	0.899	-	-	-0.278	-0.165	-0.177	-	0.782
	SE	0.044	-	-	0.056	0.046	0.038	-	
(1,4)	Estimate	0.939	-	-	-0.306	-0.179	-0.176	-0.087	0.777
	SE	0.0251	-	-	0.037	0.033	0.031	0.028	
(2,3)	Estimate	1.289	-0.326	-	-0.657	-0.071	-0.118	-	0.777
	Se	0.114	0.101	-	0.114	0.046	0.044	-	
(3,4)	Estimate	0.895	0.048	-0.007	-0.262	-0.199	-0.182	-0.094	0.777
	SE	0.435	0.542	0.172	0.434	0.288	0.072	0.072	

Table 7.3: Parameter estimates

As can be seen from the table the estimated parameters of the ARMA(3,4) model have large standard errors. In fact the AR(1) model performs quite well based on its AIC value and the low variance of the parameter estimate. Both the ARMA(1,4) and the ARMA(2,3) model have one parameter that has a relatively large standard error compared to the parameter. Finally the ARMA(1,3) model performs quite well with respect to the AIC value and has parameter estimates with relatively low standard errors. Also the parameters of the ARMA(1,3) model are quite close to the parameters of the ARMA(1,4).

7.3.7 Model Residuals

Plots of the SACF up to lag 104 for the residuals from the estimation of the AR(1) model and the ARMA(1,3) model are provided in Figures 8 and 9 in appendix 4. Less than five values fall outside the $1.96/\sqrt{\text{number of observations}}$ boundary for the ARMA(1,3), which is acceptable (Brockwell & Davis, 2002). For the AR(1) model the situation is less clear as six autocorrelation values fall outside the bounds. Concerning the normality hypothesis the situation is far worse.

Higher order sample descriptive statistics of the residuals are provided below

	Skewness	*Excess Kurtosis*
AR(1)	0.885	1.51
ARMA(1,3)	0.887	1.45

The UCSD GARCH toolbox was used to perform a Lilliefors and a Shapiro-Francia test, both of which opposed the normality hypothesis.

7.3.8 Discussion

As the quality of the underlying data seems to be quite poor the application of advanced models does not seem well founded. It appears that a low order ARMA model can represent the transformed and deseasonalized Siso inflow time series quite well, even an AR(1) model might be sufficient for hydroelectric scheduling purposes. The large value of ϕ_1 for the AR(1) signifies the important insight that weekly inflows are strongly autocorrelated; inflows in one week strongly influence the outcome of inflows in the following week, which is reasonable.

Finally, several weaknesses in the analysis should be noted. First of all the elimination of negative values was undertaken in a crude way, possibly changing the characteristics of the time series. The deseasonalization could have been more sophisticated. Also better testing of the residuals should have been undertaken, especially with respect to whiteness and possible hetereoscedasticity.

7.4 Correlation of Residuals

As pointed out one should really make assumptions about a joint distribution in order to analyse price-inflow dependence. There will be no attempt to estimate a multivariate distribution, but rather an ad hoc investigation to scratch the surface regarding possible correlation. What is interesting is how the noise terms of the processes are correlated. Let $\varepsilon_{\text{price},i}$ and $\varepsilon_{\text{inflow},i}$ respectively denote the residuals from estimation of the Schwartz model and residuals from estimation of the AR(1) inflow model. Further, since spot prices were only obtained for the period 1995-2005, let $i = 0$ denote week 1, 1995, and let $i = N$ denote week 52, 2005. The sample correlation is calculated by

$$\hat{\rho}(\varepsilon_{price,i}, \varepsilon_{inflow,i}) = \frac{\sum_{i=1}^{N}\left(\varepsilon_{price,i} - \overline{\varepsilon}_{price}\right)\left(\varepsilon_{inflow,i} - \overline{\varepsilon}_{inflow}\right)}{\left(N-1\right)s_{price}s_{inflow}},$$

where $\overline{\varepsilon}$ denotes the sample mean and s denotes the sample standard deviation[19]. The resulting overall correlation of the residuals was $\hat{\rho} = -0.16$. A 30-week centered, moving window was also applied to investigate the stability of the correlation. The result can be viewed in Figure 7.7, where the black line is the estimated local correlation and the red line is $\hat{\rho}$.

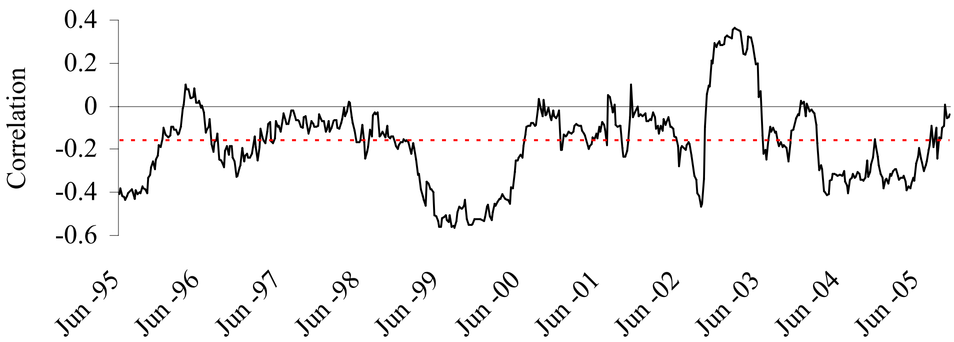


Figure 7.7: 30 day moving window correlation of residuals

It is emphasized that the results must not be given too much weight. Since correlation only is an appropriate measure of dependence for the elliptical family of joint distributions (Eydeland & Wolyniec, 2003) the results may not really be applicable at all, considering the deviations from normality of both sets of residuals. The purpose of the results will merely be to serve as a reference point for investigating the effect of price-inflow dependence on the operating strategy.

[19] Sample in the meaning the sample of residuals.

8 Implementation Issues

The proposed LSM-algorithm, along with the estimated price models, has been implemented in Matlab®. Time issues did not allow any of the estimated inflow models to be implemented for use in the LSM algorithm[20]. However, an inflow model estimated by Elkem has been implemented, also in Matlab®. The Elkem inflow model is based on the arithmetic Ornstein-Uhlenbeck process, but has deterministic, seasonally varying mean and variance. Appendix 5 provides details about the model. Matlab code is provided on the CD accompanying the thesis.

To allow for an easy comparison with the deterministic model the value of the end reservoir was set equal to zero. The solution should then seek to empty the reservoir towards the end of the planning horizon, regardless of solution methodology. Hence the comparison of the LSM model value with the upper bound should be unbiased. Further a linear production function was assumed, allowing the deterministic formulation to be solved as a linear program. The assumption of a linear production function also implies that the single period sub problems, in the LSM algorithm, can be solved by a simple line search, without any risk of ending up in a local maximum[21]. The solver for the single stage problem had to be specifically developed for the application.

The market price of residual risk was assumed equal to zero, making discounting at the risk free rate possible. The risk free rate was assumed constant and approximately equal to the yield on Norwegian government bonds. Correlation between the price and inflow processes was introduced by Cholesky factorization.

8.1 Theoretical Computing Time

Let K be the number of discrete reservoirs in the LSM recursion, S the number of scenarios and T the number of scheduling periods. Since the LSM-algorithm works on parallel, independently drawn scenarios, the computing time is linear in T. The linearity makes a scheduling resolution of one week possible for relatively long analysis horizons. The number

[20] Challenges in estimation lead to the results not being available until the end of the work.
[21] At least as long as the regression procedure is stable and results in a concave future value function.

of LSM single stage sub problems, N, that must be solved is the number of problems in the backward recursion plus the number of problems in the forward simulation: $N = KST + ST = (1+K)ST$. Hence the solution time should increase linearly for an increase in either of K, S or T.

9 Results

Preliminary testing of the proposed LSM algorithm has been undertaken. Due to somewhat disappointing and unreliable results extensive documentation is not provided. The instability of the algorithm did not allow for accurate sensitivities to be estimated, and hence hedges have not been calculated. For the same reasons correlation issues could not be addressed either. Following is a presentation of scheduling results obtained with the Schwartz spot model (Section 9.1) and the Bjerksund et al. forward curve model (Section 9.2). Both models were calibrated to the Nord Pool term structure on 20 April, 2006. The volatility function used in the forward curve model is the function estimated in section 7.1.8. The initial inflow was set to the historical weekly mean. Table 9.1 lists additional model inputs.

Model input	*Value*
Start date	21 April, 2006
End date	20 May, 2009
Number of periods (T)	162
Initial Reservoir (M_0)	50 %
Number of scenarios (S)	5000
Number of reservoirs (K)	30
Annualized discounting rate (r_f)	3.5%
Correlation ($\hat{\rho}$)	-0.16

Table 9.1: Model data and parameters used in Case 1 and Case 2

9.1 Case 1: Schwartz Spot Model

The objective values from the LSM model and the deterministic model along with the relative difference are provided below.

Total value LSM model: V_0^{LSM} =120.14 M€

Total value deterministic model: V_0^{DET} =123.83 M€

Relative difference in value: $\frac{V_0^{DET} - V_0^{LSM}}{V_0^{LSM}} = 3.07\%$

The following two graphs seek to illustrate the reservoir management strategy (Figure 9.1) and the release strategy (Figure 9.2). In Figure 9.1 the expected development of the reservoir is plotted along with reservoir bounds and percentiles for the LSM solution. Notice that the LSM expected reservoir for the better part of the period is below the deterministic expected reservoir. Considering Figure 9.2, the LSM model clearly yields the more aggressive strategy, alternating between relatively high and low expected releases.

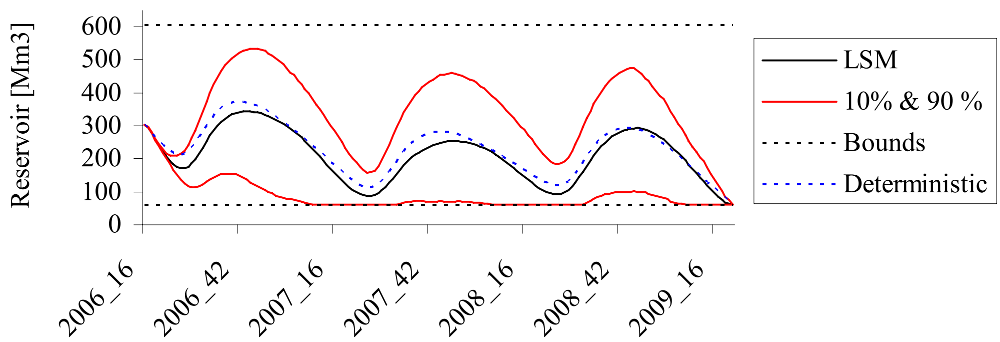


Figure 9.1: Case 1 reservoir management strategy

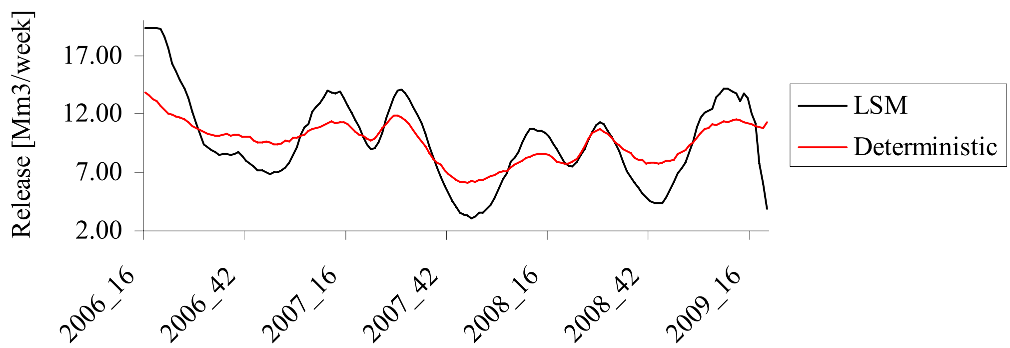


Figure 9.2: Case 1 release strategy

In Figure 9.3 the joint behaviour of the different variables in the LSM solution is illustrated by a plot of the development of the expectations. For ease of exposition the expectations have been standardized. As can be seen the solution saves water to periods where the expected price is high. Also the solution starts producing more in spring in order to accommodate possible large inflows.

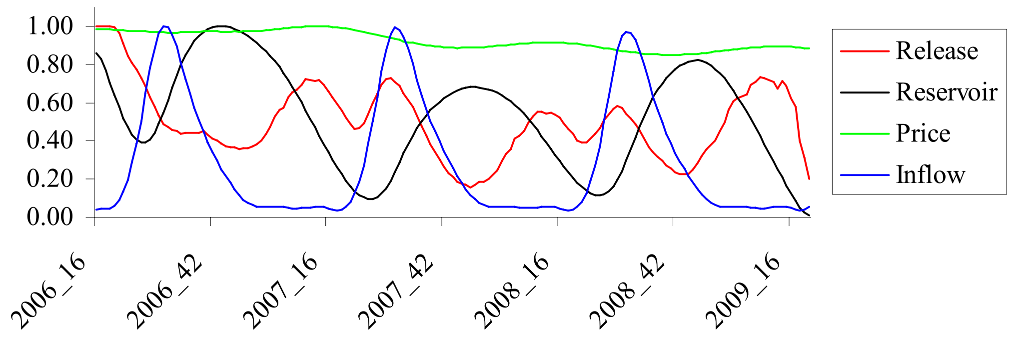


Figure 9.3: Case 1 joint behaviour of variables

9.2 Case 2: Bjerksund et al. Forward Curve Model

The objective values from the LSM model and the deterministic model along with the relative difference are provided below.

Total value LSM model: V_0^{LSM} =119.93 M€

Total value deterministic model: V_0^{DET} =130.68 M€

Relative difference in value: $\frac{V_0^{DET} - V_0^{LSM}}{V_0^{LSM}} = 8.96\%$

Notice that the LSM model yields a lower value in Case 2 than in Case 1. For the deterministic model the opposite is the case. Figure 9.4 compares the operating strategies resulting from the LSM model and the deterministic model. The aggressiveness of the LSM solution is the most striking feature, yielding a strategy for a large part alternating between close to zero and the upper bound.

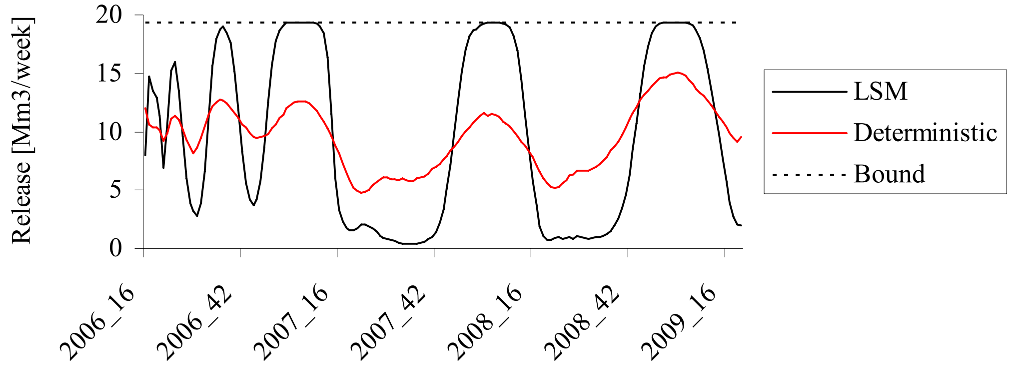


Figure 9.4: Case 2 release strategy

9.3 Convergence

Ad toc tests were run to investigate the convergence of the algorithm with regard to the number of scenarios and the number of discrete reservoirs in the backward recursion. Equally spaced reservoirs were used, and tests were only run with the Schwartz price model.

The total value changed less than 1 % when increasing the number of reservoirs beyond 30. Increasing the number of scenarios from 5000 to 10000 also changed the objective value by less than 1 %. The expected release in individual weeks converged slower. It was not possible to determine the number of scenarios required to calculate sensitivities and analyse correlation issues in reasonable computing time. Table 9.2 lists solving times with $K = 30$, $T = 162$ with different choices for S on a 3.2 GHz, 504 MB RAM computer.

Number of Scenarios (S)	*Number of Sub Problems* (N)	*Solving Time* [Seconds]
100	5.02×10^5	9.47
1000	5.02×10^6	721
5000	2.51×10^7	6.84×10^3
10000	5.02×10^7	6.38×10^4

Table 9.2: Solving time

9.4 Future Value Functions

Referring to the discussion towards the end of 6.4, concerning the expected shape of the future value functions, plots are provided in Appendix 6. The plots are for a random selection of scenarios resulting from the application of the LSM model with input as in Case 1 above. The provided plots are generally qualitatively representative, also under the forward curve model. As can be seen the future value functions are quite linear for low reservoir values, but exhibit concavity as the reservoir level approaches the upper bound. Notice also the difference between the future value functions in July (Figure 1) and in late October (Figure 2).

10 Discussion

Under the Schwartz price model the LSM algorithm produces somewhat realistic results. The resulting objective value is relatively close to the upper bound. Also the operating strategy seems realistic from a qualitative point of view, considering the price incentives and the expected inflow pattern. On the other hand the strategy is quite aggressive, suggesting maximum production for the first couple of weeks. A priori one would expect the operating strategy suggested by a stochastic model to see more value in postponing production, relative to a deterministic solution. Especially since there is no apparent risk of spilling the production strategy in the first weeks is probably too aggressive.

Under the forward curve model the LSM algorithm does not produce a trustworthy operating strategy. The objective value is relatively far from the upper bound. Further, the upper bound is larger under the forward curve model, but the LSM objective value is lower than under the Schwartz model. As the forward curve model has larger variance than the Schwartz model, one should expect the option to postpone production to be worth more under the forward curve model. The LSM algorithm does not seem to be able to capture this value. A single positive feature of the solution under the forward curve model is that the solution at least allocates the water to the peaks of the initial forward curve.

Remember that the regression is equivalent to choosing implicit transition weights in the stochastic mesh. The most plausible explanation for the failure of the LSM algorithm is thus that the approximation through the regression is simply not good enough. The implicit weights are probably off target, leading to poor approximations of the future value functions. There is a possibility that increasing the number, or choosing a different type, of basis functions can relieve the instability.

Assuming that the algorithm will work with the proper basis functions, a serious drawback of the proposed algorithm is still that it is difficult to calculate hedge sensitivities in reasonable time. As the expected operating strategy converges slower than the objective value, one must solve for a large number of scenarios in order to calculate finite difference hedge signals. It must be noted that the Matlab code involves numerous for and while loops. Looping is well known to be slow in Matlab®, and vectorization is recommended, when possible. Optimization of the code may thus reduce computing time drastically. Also, the unexpected non-linear increase in computing time is probably due to memory issues. Reprogramming the algorithm, specifically addressing the memory issue, in a lower level language can further lead to a veritable decrease in computing time. Quasi Monte Carlo or variance reduction techniques can further help in reducing the number of required scenarios.

Finally, the future value functions seem to exhibit a qualitatively, reasonable shape. The fact that the future value functions are quite linear at low reservoir levels indicates that the regressions at least are able to estimate future prices consistently. Further the concavity at high reservoir levels show that the algorithm is able to comprehend the risk of spilling. Naturally the concavity of the future value functions is more pronounced in summer than in late autumn. In summer, the risk of spilling increases dramatically at high reservoir levels, while in late autumn the risk of spilling is minimal.

11 Conclusion and Suggestions for Further Work

The proposed LSM algorithm does not produce stable, credible results. As a consequence the hypotheses regarding expected hedge signals remain unanswered, and the effect of price-inflow correlation on the operating strategy and hedges, can not be quantitatively addressed. The failure of the algorithm is most likely due to the regressions not being able to provide an adequate approximation of the future value functions. On the bright side, the results indicate that the LSM approach is not totally off track. With appropriate basis functions the LSM algorithm can possibly yield stable results, closely approximating the optimal strategy. If so, the LSM approach will yield an easy to implement, efficient and flexible stochastic algorithm.

The most obvious extension to the current thesis is to further investigate the LSM approach, using different basis functions. To find better basis functions analyses should be undertaken in order to get an understanding of the shape and behaviour of the future value function. Statistical measures can be used in order to compare different basis functions with regard to closeness of fit. Regarding operating strategy, it is recommended to compare an improved LSM algorithm with an established stochastic algorithm, since merely comparing with the deterministic solution only conveys information about closeness to the upper bound of the objective value.

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Appendix 2: LSM Algorithm Pseudo Code

The main part of the algorithm, Part B and Part C below, take scenarios as inputs. The scenarios consist of joint price and inflow scenarios where π_t^s and ψ_t^s denote the realized price and inflow in period t in scenario s. Let further $\boldsymbol{\pi}_\mathbf{t}$ and $\boldsymbol{\psi}_\mathbf{t}$ be the vector of prices and the vector of inflows in period t.

Part B in the algorithm also takes a discrete ordered set of possible reservoirs as an input. The set should be a good approximation of the continuous set of all possible reservoir levels. Let an element of the set of discrete reservoir levels be denoted by x_k, where index k is defined as below.

Indexes

t: Period
s: Scenario
k: Reservoir level

Quantities

T: The last scheduling period (The initial period is defined to be period 0)
S: The number of scenarios
K: The number of elements in the set of discrete reservoirs

Notation for the future value functions

Express the approximate function for the expected value of the end reservoir as

$$g_{T+1} \equiv g_{T+1}(\pi_T, \psi_T, m_{T+1}) \approx E\left[V_{T+1} \middle| \Pi_T, \Psi_T\right]$$

For all t = 1...T-1 and all s = 1...S let

$g_t^s = g_t^s(m_{t+1}) \approx E\left[V_{t+1} \middle| \Pi_t = \pi_t, \Psi_t = \psi_t\right]$ and

$h_t^k = h_t^k(\pi_t, \psi_t) \approx E\left[V_{t+1} \middle| \Pi_t, \Psi_t, m_{t+1} = x_k\right]$,

where g_t^k is a piecewise linear approximation to the expected future value function in period t in scenario s.

Part A: Simulation

Simulate S price and inflow scenarios to produce all $\boldsymbol{\pi_t}$ and $\boldsymbol{\psi_t}$ for Part B.

Part B: Estimation of water value functions

Step 1

Lock $t = T$

Go to **Step 2**

Step 2

If $t = T$

For $s = 1 \ldots S$

$$g_{t+1}^s = g_{T+1}(m_{t+1}; \pi_t = \pi_t^s, \psi_t = \psi_t^s)$$

End for

End if

For $s = 1 \ldots S$

For $k = 1 \ldots K$

Solve:

$$V_t^{s,k} = \max_{q_t^s, l_t^s} \left[\pi_t^s w(x_k, q_t^s) + \frac{g_{t+1}^s}{1+r} \right]$$

$s.t.$

$$\begin{aligned} m_{t+1}^s &= x_k - q_t^s - l_t^s + \psi_t^s \\ Q_{\min} &\le q_t^s \le Q_{\max} \\ M_{\min} &\le m_{t+1}^s \le M_{\max} \\ l_t^s &\ge 0 \end{aligned}$$

End for

Store all objective function values in previous loop as the vector $\mathbf{V_t^k}$.

End for

Go to **Step 3**

Step 3

For $k = 1...K$

Regress $\mathbf{V}_t^k$ on a set of basis functions of price ($\boldsymbol{\pi}_t$) and inflow ($\boldsymbol{\psi}_t$) to produce the function $h_t^k(\pi_t, \psi_t)$.

End for

For $s = 1...S$

Estimate g_{t-1}^s as a piecewise linear function by interpolating between the values produced by evaluating $h_t^i(\pi_t^s, \psi_t^s)$ for $i = 1...K$.

End for

Go to **Step 4**

Step 4

Lock $t = t - 1$

If $t > 0$

Go to **Step 2**

Else

Go to **Part C**

End if

Part C: Simulation

Simulate S price and inflow scenarios to produce all $\boldsymbol{\pi}_\mathbf{t}$ and $\boldsymbol{\psi}_\mathbf{t}$ for Part C.

Part D: Production Scheduling

Step 1

For $s = 1...S$

Lock $m_0^s = M_0$

End for

Go to **Step 2**

Step 2

For $s = 1 \ldots S$

Solve

$$V_t = \max_{q_t^s, l_t^s} \left[\pi_t^s w(m_t, q_t^s) + \frac{g_{t+1}^s}{1+r} \right]$$

$s.t.$

$$\begin{aligned} m_{t+1}^s &= m_t^s - q_t^s - l_t^s + \psi_t^s \\ Q_{\min} &\leq q_t^s \leq Q_{\max} \\ M_{\min} &\leq m_{t+1}^s \leq M_{\max} \\ l_t^s &\geq 0 \end{aligned}$$

End for

Go to **Step 3**

Step 3

If $t < T$

Lock $t = t + 1$

Go to **Step 2**

Else

Go to **Part D**

End if

Part D: Valuation

Step 1

For $t = 1 \ldots T$

For $s = 0 \ldots S$

$$C_{0,t}^s = \frac{\pi_t^s \times w(m_t^s, q_t^s)}{(1+r)^t}$$

End for

$$C_{0,t} = \frac{1}{S} \sum_{i=0}^{S} C_{0,t}^i$$

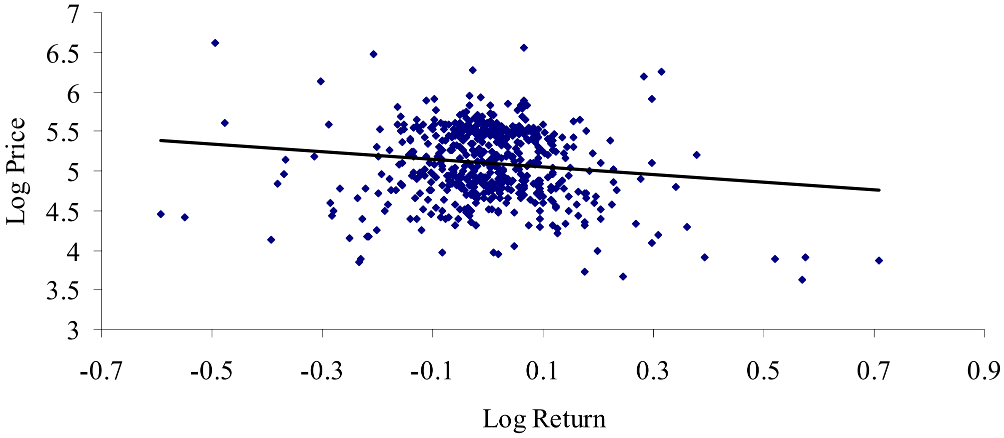
End for

$$V_0 = \sum_{i=0}^{T} C_{0,i}$$

End **Step 1**

Appendix 3: Spot Price Descriptive Statistics and Regression Plot

Statistic	*Prices*	*Log Returns*	*Residuals*
Mean	181.25	0.00	0.00
Median	165.51	0.00	0.00
Standard Deviation	84.76	0.13	0.12
Sample Variance	7183.48	0.02	0.01
Excess Kurtosis	7.45	5.16	4.28
Skewness	1.69	0.21	0.01
Range	714.21	1.30	1.21
Minimum	37.50	-0.59	-0.58
Maximum	751.72	0.71	0.63
Observations	571.00	570.00	570.00



Regression plot from Dickey-Fuller test

Appendix 6: Plots of Future Value Functions

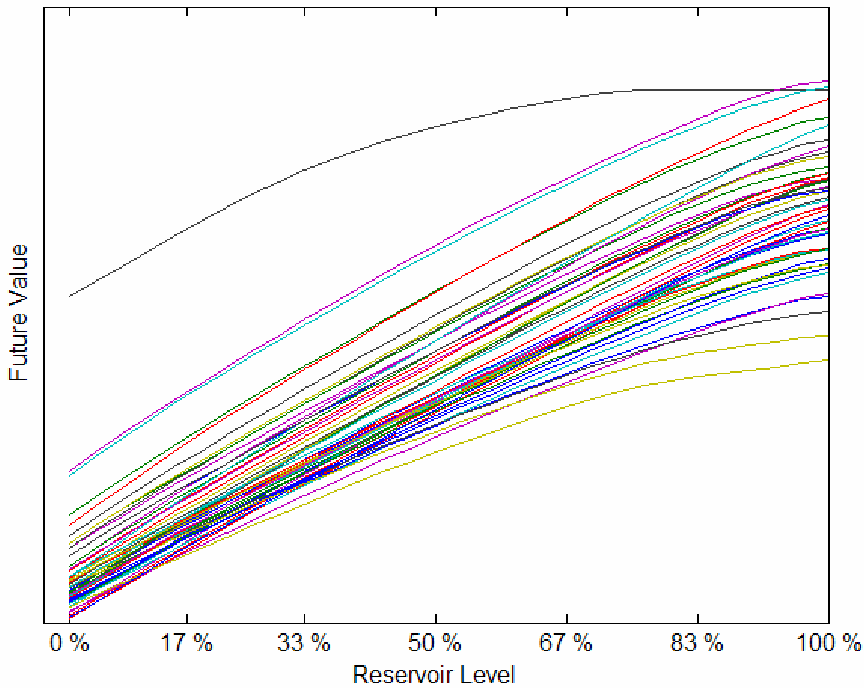


Figure 1: Future value functions week 44, 2007 (29 October – 4 November)

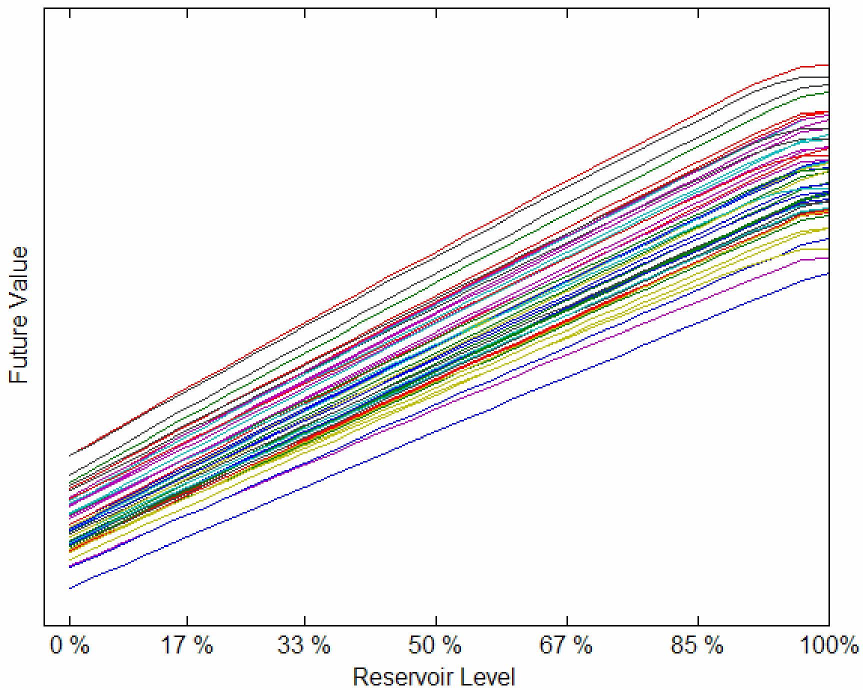


Figure 2: Future value functions week 28, 2007 (9 July – 15 July)