An Empirical test of Option Based Default Probabilities using Payment Behaviour and Auditor notes.

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July 8, 2004

Abstract

This paper empirically tests the Black and Scholes, Merton framework for bankruptcy, based on *a priori* hypotheses from the comparative statics of the model, using payment behavior and auditor notes as a proxy for financial distress. The results indicate that the standard deviation of equity is the most significant parameter in the model, but also equity ratio seems to have a significant influence on the probability of default. The results show that an increase in the volatility of equity increases the probability of default, while an increase in the equity ratio or drift of equity reduces the probability of bankruptcy. The coefficient of the time horizon of debt is close to zero, aligned with the ambigous effect of the time horizon of debts influence on the porbability of bankruptcy. This is in line with the *a priori* hypothesis derived from the comparative statics from the model.

JEL: G12, G33

Keywords: Option theory, Bankruptcy prediction, Empirical tests

1 Introduction

Models of corporate bankruptcy predicting the probability of business failure with a high degree of accuracy have previously been developed. During the decade prior to 1977, several failure prediction studies were conducted for both large and small nonfinancial firms. Starting with Beaver (1966) univariate analysis of 30 ratios and culminating with the multiple discriminant analysis (MDA) Zeta model developed by Altman et al. (1977)[1], researchers attempted to improve the accuracy of multiratio predictive models by optimizing a set of predictor variables.

After the mid-1970s, researchers focused increasingly on the difficulties associated with the then-prevailing methodological approaches [e.g. see Eisenbeis

[1] Examples of other studies from this period are Altman (1968), Beaver (1968), Edmister (1972) and Wilcox (1973).

(1977), Mesah (1984) and Zmijewski (1984)]. Despite the criticisms explored in these later studies, the overriding conclusion from the body of research conducted to date is that financial ratios provide a significant indication of the likelihood of financial distress. However, efforts to overcome the methodological difficulties associated with MDA resulted in a shift to logit analysis [e.g. see Ohlson (1980) and Zavgren (1985)], which produces an estimate of the probability of failure under less restrictive assumptions relative to MDA.

Of course, all firms in financial distress do not end in business failure. Bankruptcy after all, represent only an extreme result. Thus, financial distress is best depicted as a continuum ranging from being "financial weak" to bankrupt with the possibility of various degrees of financial weakness. For example, Lau (1987) extend the traditional failure / nonfailure dichotomy to a financially stable state and four states of financial distress[2]. There is also studies on comparisons between multistate models and two state models like Johnsen and Melicher (1994). They use binary and multinominal logit models in classifying US firms during the 1970-1983 period. They find a significant reduction in misclassification error rates for the multinominal model. Results also suggest that secondary classification information can be used to augment primary classifications to improve the ability to correctly predict bankrupt firms, as well as predict financially weak firms that will suffer severe financial distress in the future.

Current empirical literature on bankruptcy prediction lacks unanimity, however. The overview over the results of empirical studies given in Dimitras et. al. (1996) shows the wide dispersion. Particularly in the area of inter-temporal and inter-industry validity of the empirical bankruptcy prediction models there remain problems. As the highly relevant models often are developed given a certain data set, the models are dependent on the period of time, country and sectors included in the data set. A comprehensive theory of bankruptcy prediction has not been developed thus far. Using a theoretical model of bankruptcy prediction to guide the empirical research could, however, provide a clear frame of reference in which the results can be evaluated and directions for future research can be specified.

The purpose of this paper is to contribute to an option based default theory by testing the empirical implications of a simple theoretical model on default probability, based on the Black and Scholes, Merton framework, henceforth BSM. Merton (1974) introduced a framework for valuating corporate capital categories using the Black and Scholes (1973) and Merton (1973) option pricing theory, where the combination of limited liability and a leveraged capital structure forms option like payoff structures. Hence, the value of equity can be seen as a call option on the firm's assets, with the face value of debt being the exercise price and the time horizon of debt equivalent to the option's time to maturity. Despite the existing large number of models based on this structural relationship, few of these models has actually been empirically tested. Moreover, the predicted signs of the variables in the models tested, in. i.e. Charitou and Trigeorgis (2002) and Hillegeist et. al. (2002) have not been adequately theoretically derived. Furthermore, previous models have been restricted to testing risk neutral probabilities of default, or at best adjusting the risk neutral probabilities of default by some accounting measure. An emphasis of this paper has

[2] A continuum of financial distress from weak to severe was represented by firms reducing or omitting dividend payments, technical default, or loan payment default, firms in chapter X or XI of the Bankrupty Act, and firms in bankruptcy or liquidation.

therefore been given to the derivation of "actual" or "objective" probabilities of default and the derivation of theoretical hypotheses for the signs used in the regression analysis. The derivation shows that the signs of the coefficients from the BSM framework are not clear-cut.

The organization of this paper is as follows. Section 2 shows the testable hypotheses formulated on the basis of the comparative statics of the bankruptcy model. These hypotheses are tested empirically on Norwegian firm data from 1999 and 2000. Section 3 describes the data and methodology, section 4 presents and discusses the results. Conclusions are formulated in section 5. Complete derivations of the comparative statics is given in the Appendix.

2 Contingent Claims Approach to default modeling

The Contingent Claims Approach to default modeling was developed from the pioneering works of Black and Scholes (1973) and Merton (1973, 1974). Its roots in traditional option theory is conspicuous, and its parallels are many. As opposed to modeling the price of an option as a function of the price of the underlying asset, the BSM framework uses a credit derivative modeled as a contingent claim on the value of assets of the firm. The framework thereby provides an intuitive approach to the prediction of financial distress, where corporate default occurs when the firms value hits a boundary or barrier equal to the face value of debt. That is, in the event of the firm not having sufficient values to service its debt obligations. If the value of a firms assets is below the specified barrier, the call option is not exercised and the remainder of its assets is transferred to the debtholders. This is analogous to the event of a call option not being exercised, which probability is given by $1 - N(d_2^*)$ or simply $N(-d_2^*)$.

The BSM framework is subject to the following assumptions. Markets are liquid and frictionless with no transaction costs or taxes. Assets are perfectively divisible and there are no arbitrage opportunities. We assume the existence of a risk free asset and the existence of only one debt class, a zero-coupon bond. Since the framework is an analogy to an European call-option, bankruptcy can only occur at maturity. Under these assumptions, the BSM framework can then be stated as follows:

$$E_0 = V_0 N(d_1^*) - De^{-rT} N(d_2^*) \tag{1}$$

where

$$d_1^* = \frac{\ln\left(\frac{V_0}{D}\right) + \left(r + \frac{\sigma_V^2}{2}\right)T}{\sigma_V \sqrt{T}}$$

$$d_2^* = \frac{\ln\left(\frac{V_0}{D}\right) + \left(r - \frac{\sigma_V^2}{2}\right)T}{\sigma_V \sqrt{T}} = d_1^* - \sigma_V \sqrt{T}$$

where

V_0: the firms value of assets

D: the face value of debt
r: riskfree rate
σ_V: volatility of assets
T: time horizon of debt

For empirical purposes, we substitute the riskfree rate, r, with an estimate of the expected drift, μ_V (growth rate) of the value of assets in order to transform the calculated probability of bankruptcy from a risk neutral to an "actual" or "objective" probability measure. From this we have that N(-d2) is the probability of bankruptcy under the "objective" probability measure. Also, note that since not all corporate claims are publicly traded, the values of the firms assets, V_0 and σ_V are not directly observable in the market[3].

Regardless of the large and constantly growing number of studies of corporate distress, few attempts have been carried out with the aim to derive the sensitivities with respect to the different underling parameters of the many structural models. The lack of sensitivity measures creates yet another empirical problem. Namely the problem of forming sensible hypotheses *a priori* to the empirical testing of the structural models. Comparative static proposes the following relationship, shown in Table 1.

[PLEASE INSERT TABLE 1 ABOUT HERE]

The probability of default increases with firms volatility and decreases with the firms drift and value. The effect of time to maturity has a ambiguous effect on the default probability depending on the relationship between the volatility, the drift and the face value of debt. For most cases, the probability of default increases with T. But for firms with very low equity ratios and high equity volatility, with an initial state near bankruptcy, time seems to be the only factor that can move the firm away from bankruptcy.

Further,The comparative statistics indicate that the capital structure and risk/returns characteristics of the firm may have a strong combined effect on default probability. The Comparative statics proposed in Table 1 gives a theoretical basis for empirical analyses of default probability. Since the majority of empirical default studies is neither guided nor restricted by financial theory, the results in this paper may be a valuable contribution to future studies.

3 Data and Empirical Approach

3.1 Data set and selection

The data used in this study consists of Norwegian public limited companies listed on the Oslo Stock Exchange for the years 1999 and 2000. Financial distress among exchange listed firms are characteristically a thorny process. Financial distress and following default are by itself far from common, financial distress among exchange listed firms are consequently even rarer.

[3] A transformation from risk neutral to "actual" or "objective" probabilities is shown in the Appendix together with a practical approach for estimating the value and volatility of the firms assets, V_0 and σ_V respectively.

The sample in our dataset consisted of 86 companies in 1999 and a total of 105 companies in 2000. A database containing company specific data on payment behavior, auditor notes and accounting data from Dun & Bradstreet Company where mapped against market data from Datastream. All financials and primary capital certificates were then excluded due to the structural difference with respect to regulatory capital requirements, and hence, different bankruptcy environment. Out of our sample of 86 (1999) and 105 (2000), 28 (1999) and 37 (2000) where found to have payment or auditor remarks. The occurrences of payment remarks where then scaled by their severity and the size by turnover, joined with auditory notes, forming our dependent variable indicating a financial distress rate of approximately 13% (11 in 1999) and 11,5 % (12 in 2000) of our sample.

3.2 Empirical model and proxy variables

A logit analysis is used to estimate the influence of the variables in the BSM framework for bankruptcy prediction using payment behavior and auditor remarks as a proxy for financial distress. Further, since V and σ_V are unobservable variables, we derive proxies from the partial derivations shown in Appendix 1. The proxy variables used in the analysis are:

- Volatility of equity, σ_E: volatility of the market value of equity
- Equity ratio, E/TA: book-value of equity divided by the book value of total assets
- Drift of equity, μ_E: last years return on equity
- Debt time horizon, STD/LTD: short term debt divided by long term debt

Where volatility of equity is calculated using daily closing prices from the Oslo Stock Exchange. The Equity ratio, E/TA is a proxy for the firms capital structure, using the book value of equity divided by the book value of total assets. Various forms of this variable has previously been used in the prediction of financial distress. Altman (1968) uses market value of equity divided by book value of total debt. Altman et. al. (1977) uses capitalization measured by common equity divided by total capital, where as Izan (1984) uses market value of equity divided by total liabilities.

Various forms of equity drifts has also been used to adjust the drift in the option framework from risk neutral to "objective" probabilities. Charitou and Trigeorgis (2002) uses the difference between the 3-month US Treasury-bill rate as riskless return and the firm's payout yield, $r - D$, as a proxy for equity drift. Hillegeist et. al. (2002), on the other hand, uses the difference between the one-year Treasury bill rate and the prior year's adjusted return on book value of assets, $\mu - D$. Similarly, we use the last years return on equity adjusted for dividends as a proxy for the drift of equity, μ_E. Finally, short term debt divided by long term debt is used as a proxy for the debt time horizon. This is similar to Charitou and Trigeorgis (2002) using the average duration of all outstanding debt.

In accordance with table 1, our hypothesis states the following. An increase in the volatility of equity, σ_E, ultimately increases the probability of bankruptcy. The effect of an increase in the equity ratio, E/TA, will decrease the probability of financial distress. Similarly, an increase in the drift of equity also reduces the probability of financial distress. The debt time horizon has a more ambiguous influence on the probability of financial distress[4]. The ambiguous effect is similar to the one hypothesized in Charitou and Trigeorgis (2002).

The advantage of basing empirical proxies on a theoretical model is that the results are not found due to sample specific reasons. This is in contrast with previous empirical studies of bankruptcy prediction like Beaver (1966), Altman (1968) and Ohlson (1980) where the variables included are based on popularity and performance in previous studies. Also the possibility of statistical overfitting is reduced. This occurs when, given a certain data set, a small number of variables from a large list are found to be significantly explaining default. In another sample other variables could have been found, making the significant variables from the first data set less significant or even without explanatory power. In most empirical studies no hypothesis concerning the expected relation between variables and the probability of default is formulated.

4 Logistic regression results

The results of the analysis are presented in Table 2, where the estimated coefficients and their level of significance are given. The significance level is calculated using Wald statistic.

[PLEASE INSERT TABLE 2 ABOUT HERE]

Using a 5% level of significance, the volatility of equity, σ_E, is statistically significantly positive for both samples, supporting the hypothesis that an increase in the volatility of equity increases the probability of financial distress. Further, the Equity ratio, E/TA, is significant on a 10% level in 2000. None of the other coefficients are statistically significant. However, all coefficients have the correct hypothesized sign. The debt time horizon that where suggested to have an ambiguous effect on the probability of bankruptcy is approximately equal to zero. These results indicate a support for the strong dominating effect originating from the volatility of equity that seems to drive the BSM framework for default prediction[5].

[PLEASE INSERT TABLE 3 ABOUT HERE]

Note that unlike the R^2 measures of OLS regressions, the R^2 shown in Table 3 are not goodness-of-fit tests but measures of strength of association. The Cox & Snell R^2 is designed as an interpretation of multiple R-Square based on the

[4] The analytical derivation of the effect of a change in the debt time horizon is given in the Appendix.

[5] As described in Appendix 1

likelihood. However, maximum values proposed by the Cox and Snell R^2 can be less than 1.0. The interpretation of the values 0.111 (1999) and 0.055 (2000) therefore difficult. The Nagelkerke R^2 shown in Table 3 is a modification of the Cox & Snell measure, dividing the Cox & Snell coefficient by its maximum value, yielding a coefficient between 0 and 1. Consequently, the Nagelkerke R^2 is higher than the Cox and Snell R^2.

[PLEASE INSERT TABLE 4 ABOUT HERE]

Table 4 shows a classification of the percentages of correctly classified firms based on the logistic regression. The table shows that 71.4% (1999 and 2000) of companies in financial distress where captured by the model, with an overall correct classification of 47.7% (1999) and 63.5% (2000). Compared to earlier studies this might seem low. Altman (1968) correctly classifies 94% of bankrupt and 97% of non-bankrupt companies correctly. Altman et. al. (1977) correctly classifies 92.5% of bankrupt and 91.4% of non bankrupt companies. An important factor contributing to a lower classification percentage in this study is the fact that the logistic regression model was not designed to fit the data but directly derived from a theoretical model. Results from a theoretically based model will however have the advantage of providing consistent results without regards to time period, country and sectors included in the datasets.

5 Summary and conclusions

The purpose of this paper is twofold. First to contribute the growing number of empirical and theoretical studies of corporate default by deriving *a priori* testable hypotheses. Second, the paper empirically tests the hypotheses based on the comparative statics of the BSM framework using historical payment behavior and auditor notes as a proxy for financial distress. Collectively, this helps us develop a deeper understanding of the underlying parameters influencing the probability of default in structural bankruptcy models. The logistic regression model supports the hypothesis that an increase in the volatility of equity increases the probability of bankruptcy. Our model also give partial support for the hypothesis that an increase in capital structure, $\frac{E}{TA}$, decreases the probability of bankruptcy. Moreover, all coefficients have the correct hypotesized sign, although only the volatility of equity is statistically significant for both samples. The coefficient of the time horizon of debt is close to zero. This is aligned with the hypothesis and previous research proposing an ambiguous effect of the time horizon of debts influence on the probability of bankruptcy.

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Tables

Table 1
Comparative Statics

Table 1 reports a summary of the comparative statics for the probability of default in a BSM framework.

Comparative Statics	Effects on P
Volatility of Equity	> 0
Equity ratio	< 0
Drift of Equity	< 0
Debt time horizon	$\lesseqgtr 0$

See Appendix 1 for derivations of the comparative statics.

Table 2
Regression results

Table 2 reports the estimated parameters of the logit model. Significance levels are reported in brackets.

		Hyp.	1999	2000
Constant			-3.514**	-2.175**
			(7.292)	(6.072)
Volatility of Equity	σ_E	> 0	2.264**	2.265**
			(3.423)	(3.493)
Equity ratio	E/TA	< 0	-0.572	-5.482*
			(0.072)	(5.074)
Drift of Equity	μ_E	< 0	-0.582	-0.541
			(0.551)	(0.599)
Debt time horizon	STD/LTD	$\lesseqgtr 0$	-0.001	-0.001
			(0.026)	(0.026)

* and ** denotes statistical significance at 10 and 5 percent levels respectively. Wald statistic is given in parentheses.

Table 3
Goodness of fit

Table 3 reports goodness of fit.

Year	-2 Log Likelihood	Cox & Snell R^2	Nagelkerke R^2
1999	39.075	0.111	0.285
2000	74.548	0.055	0.093

Table 4
Classification Table

Table 4 reports the number of correct classifications for the year 1999 and 2000.

Year	not financial distress	financial distress	Overall
	% correct	% correct	% correct
1999	45.6	71.4	47.7
2000	61.9	71.4	62.5

Cutoff 0.05.

APPENDIX

Recall, the BSM framework for bankruptcy prediction where:

$$E_0 = V_0 N(d_1) - De^{-rT} N(d_2) \tag{2}$$

and

$$d_1 = \frac{\ln\left(\frac{V_0}{D}\right) + \left(\mu + \frac{\sigma_V^2}{2}\right)T}{\sigma_V \sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{V_0}{D}\right) + \left(\mu - \frac{\sigma_V^2}{2}\right)T}{\sigma_V \sqrt{T}} = d_1 - \sigma_V \sqrt{T}$$

where
V_0: the firms value of assets
D: the face value of debt
r: riskfree rate
σ_V: volatility of assets
T: time horizon of debt

Using Ito calculus, it can also be shown that:

$$\sigma_E E = \sigma_V \times V \times \frac{\partial E}{\partial V} = \sigma_V \times V \times N(d_1)$$

Let us now define the following sensitivities for the equity:

- Equity Gamma measures the change in the equity value w.r.t. a change in the equity delta and is given by::

$$\begin{aligned}
\Gamma^E &= \frac{\partial^2 E}{\partial V^2} \\
&= \frac{\partial [N(d_1)]}{\partial V} = n(d_1) * \frac{\partial (d_1)}{\partial V} \\
&= n(d_1) * \frac{1}{\left(\frac{V}{F}\right)} * \frac{1}{F} * \frac{1}{\sigma \sqrt{T}} \\
&= \frac{n(d_1)}{V \sigma \sqrt{T}} > 0
\end{aligned}$$

- Equity Theta $\Theta^E = \frac{\partial E}{\partial t}$. Theta measures the time decay or the rate of change of the equity value w.r.t. the passage of time, all other things equal. This means that ∂t is always negative, so that we can write $\Theta^E = -\frac{\partial E}{\partial t}$.First we give some elements that will be used in the calculations of equity theta:

$$\begin{aligned} d_2 &= d_1 - \sigma\sqrt{T} \\ d_2^2 &= d_1^2 - 2d_1\sigma\sqrt{T} + \sigma^2 T \\ &= d_1^2 - 2\left[\ln\left(\frac{V}{F}\right) + \left(r + \frac{1}{2}\sigma^2\right)T\right] + \sigma^2 T \\ &= d_1^2 - 2\ln\left(\frac{Ve^{rT}}{F}\right) \end{aligned}$$

Next:

$$n(d_2) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_2^2}{2}} = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}+\ln\left(\frac{Ve^{rT}}{F}\right)}$$

$$= \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}e^{\ln\left(\frac{Ve^{rT}}{F}\right)} = n(d_1)\frac{Ve^{rT}}{F}$$

Similarly:

$$n(d_1) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}} = n(d_2)\frac{F}{Ve^{rT}}$$

Finally:

$$\frac{\partial d_2}{\partial T} - \frac{\partial d_1}{\partial T} = \left[-\frac{\ln(\frac{V}{F})}{2\sigma T^{3/2}} + \frac{r}{2\sigma\sqrt{T}} - \frac{\sigma}{4\sqrt{T}}\right] - \left[-\frac{\ln(\frac{V}{F})}{2\sigma T^{3/2}} + \frac{r}{2\sigma\sqrt{T}} + \frac{\sigma}{4\sqrt{T}}\right] = -\frac{\sigma}{2\sqrt{T}}$$

The equity theta can now be found as:

$$\Theta^E = -\frac{\partial E}{\partial t}$$

$$= -\left(V\frac{\partial N(d_1)}{\partial T} + \underbrace{\frac{\partial V}{\partial T}}_{=0}N(d_1) + rFe^{-rT}N(d_2) - Fe^{-rT}\frac{\partial N(d_2)}{\partial T}\right)$$

$$= -\left(Vn(d_1)\frac{\partial d_1}{\partial T} + rFe^{-rT}N(d_2) - Fe^{-rT}n(d_2)\frac{\partial d_2}{\partial T}\right)$$

Substituting and rearranging terms we get:

$$= -\left(Vn(d_1)\frac{\partial d_1}{\partial T} - Fe^{-rT}n(d_1)\frac{Ve^{rT}}{F}\frac{\partial d_2}{\partial T} + rFe^{-rT}N(d_2)\right)$$

$$= -\left(Vn(d_1)\left[\frac{\partial d_1}{\partial T} - \frac{\partial d_2}{\partial T}\right] + rFe^{-rT}N(d_2)\right)$$

$$= -\left(Vn(d_1)\left[\frac{\sigma}{2\sqrt{T}}\right] + rFe^{-rT}N(d_2)\right)$$

$$= -\frac{Vn(d_1)\sigma}{2\sqrt{T}} - rFe^{-rT}N(d_2)$$

- The sensitivity of the default probability with respect to a change in the value of the firm V is:

$$\frac{\partial p}{\partial V} = \frac{\partial N\left(-\hat{d}_2\right)}{\partial V} = n\left(-\hat{d}_2\right) \times \frac{\partial\left(-\hat{d}_2\right)}{\partial V}$$

$$= n\left(-\hat{d}_2\right) \times \frac{\partial\left[-\left(\frac{\ln\left(\frac{V}{F}\right)+\left(\mu_V - \frac{\sigma_V^2}{2}\right)T}{\sigma_V\sqrt{T}}\right)\right]}{\partial V}$$

$$= n\left(-\hat{d}_2\right) \times \frac{\partial}{\partial V}\left[\frac{-\ln(\frac{V}{F})}{\sigma_V\sqrt{T}}\right] = -n\left(\hat{d}_2\right) \times \left[\frac{1}{V\sigma_V\sqrt{T}}\right] < 0$$

- The sensitivity of the default probability with respect to a change in the volatility of the value of the firm σ_V:

$$\frac{\partial p}{\partial \sigma_V} = \frac{\partial N\left(-\hat{d}_2\right)}{\partial \sigma_V} = n\left(-\hat{d}_2\right) \times \frac{\partial\left(-\hat{d}_2\right)}{\partial \sigma_V}$$

$$= n\left(\hat{d}_2\right) \times \frac{\partial\left[-\left(\frac{\ln\left(\frac{V}{F}\right)+\left(\mu_V - \frac{\sigma_V^2}{2}\right)T}{\sigma_V\sqrt{T}}\right)\right]}{\partial \sigma_V}$$

$$= -n\left(\hat{d}_2\right) \times \frac{\partial\left[\frac{\ln\left(\frac{V}{F}\right)}{\sigma_V\sqrt{T}} + \frac{\mu_V\sqrt{T}}{\sigma_V} - \frac{\sigma_V}{2}\sqrt{T}\right]}{\partial \sigma_V}$$

$$= -n\left(\hat{d}_2\right) \times \left[-\frac{\ln\left(\frac{V}{F}\right)}{\sigma_V^2\sqrt{T}} - \frac{\mu_V\sqrt{T}}{\sigma_V^2} - \frac{1}{2}\sqrt{T}\right]$$

$$= n\left(\hat{d}_2\right) \times \left[\frac{\ln\left(\frac{V}{F}\right) + T\mu_V + \frac{1}{2}T\sigma_V^2}{\sigma_V^2\sqrt{T}}\right] > 0$$

- The sensitivity of the default probability with respect to a change in the drift of the firm μ_V:

$$\frac{\partial p}{\partial \mu_V} = \frac{\partial N\left(-\hat{d}_2\right)}{\partial \mu_V} = n\left(-\hat{d}_2\right) \times \frac{\partial\left(\hat{d}_2\right)}{\partial \mu_V}$$

$$= n\left(\hat{d}_2\right) \times \frac{\partial\left[-\left(\frac{\ln\left(\frac{V}{F}\right)+\left(\mu_V-\frac{\sigma_V^2}{2}\right)T}{\sigma_V\sqrt{T}}\right)\right]}{\partial \mu_V}$$

$$= -n\left(\hat{d}_2\right) \times \frac{\sqrt{T}}{\sigma_V} < 0$$

- The sensitivity of the default probability with respect to a change in the time to maturity T:

$$\frac{\partial p}{\partial T} = \frac{\partial N\left(-\hat{d}_2\right)}{\partial T} = n\left(-\hat{d}_2\right) \times \frac{\partial\left(\hat{d}_2\right)}{\partial T}$$

$$= n\left(\hat{d}_2\right) \times \frac{\partial\left[-\left(\frac{\ln\left(\frac{V}{F}\right)+\left(\mu_V-\frac{\sigma_V^2}{2}\right)T}{\sigma_V\sqrt{T}}\right)\right]}{\partial T}$$

$$= -n\left(\hat{d}_2\right) \times \frac{\partial\left[\frac{\ln\left(\frac{V}{F}\right)}{\sigma_V\sqrt{T}} + \frac{\mu_V\sqrt{T}}{\sigma_V} - \frac{\sigma_V}{2}\sqrt{T}\right]}{\partial T}$$

$$= -n\left(\hat{d}_2\right) \times \left[-\frac{\ln\left(\frac{V}{F}\right)}{2\sigma_V T^{3/2}} + \frac{\mu_V}{2\sigma_V\sqrt{T}} - \frac{\sigma_V}{4\sqrt{T}}\right]$$

$$= n\left(\hat{d}_2\right) \times \left[\frac{\ln\left(\frac{V}{F}\right)}{2\sigma_V T^{3/2}} - \frac{\mu_V}{2\sigma_V\sqrt{T}} + \frac{\sigma_V}{4\sqrt{T}}\right] \gtrless 0$$

To sum up, we have now shown that:

$$\frac{\partial p}{\partial V} < 0,\ \frac{\partial p}{\partial \sigma_V} > 0,\ \frac{\partial p}{\partial \mu_V} > 0 \text{ and } \frac{\partial p}{\partial T} \gtrless 0$$

However, since V, σ_V and μ_V are unobservable, we use market traded estimates of Equity as equivalent measures. That is, E, σ_E and μ_E

It can be shown that:

$$\frac{\partial E}{\partial V} = N(d_1) > 0$$

and that

$$\frac{\partial \sigma_E}{\partial \sigma_V} = \frac{\partial (\sigma_V \times V \times N(d_1))}{\sigma_E}$$

$$= 1 \times V \times N(d_1) + \sigma_V \times V \times n(d_1) \times \frac{\sigma_V \times T \times \sigma_V \times \sqrt{T} - \left(\ln\left(\frac{V}{F}\right) + \mu_V + \frac{\sigma_V^2}{2}\right)\sqrt{T}}{\sigma_V^2 \times T}$$

$$V \times N(d_1) + \sigma_V \times n(d_1) \times \frac{2\sigma_V \times T^{3/2} - \ln\left(\frac{V}{F}\right) \times \sqrt{T} - \left(\mu_V + \frac{\sigma_V^2}{2}\right) \times T^{3/2}}{\sigma_V^2 \times T}$$

$$V \times N(d_1) + \sigma_V \times n(d_1) \times \frac{\frac{3}{2}\sigma_V \times T^{3/2} - \ln\left(\frac{V}{F}\right) \times \sqrt{T} - \mu_V \times T^{3/2}}{\sigma_V^2 \times T} \lesseqgtr 0$$

Howeverfor relevant ranges, $\frac{\partial \sigma_E}{\partial \sigma_V} > 0$

Hence, since

$$\frac{\partial E}{\partial V} > 0 \text{ and } \frac{\partial p}{\partial V} > 0,\ \frac{\partial p}{\partial E} < 0$$

and since

$$\frac{\partial \sigma_E}{\partial \sigma_V} > 0 \text{ and } \frac{\partial p}{\partial \sigma_V} > 0,\ \frac{\partial p}{\partial \sigma_E} > 0$$

This gives us the comparative statics presented in Table 1.