Energy Procurement Portfolio Optimization

Paula Rocha and Daniel Kuhn

Imperial College London

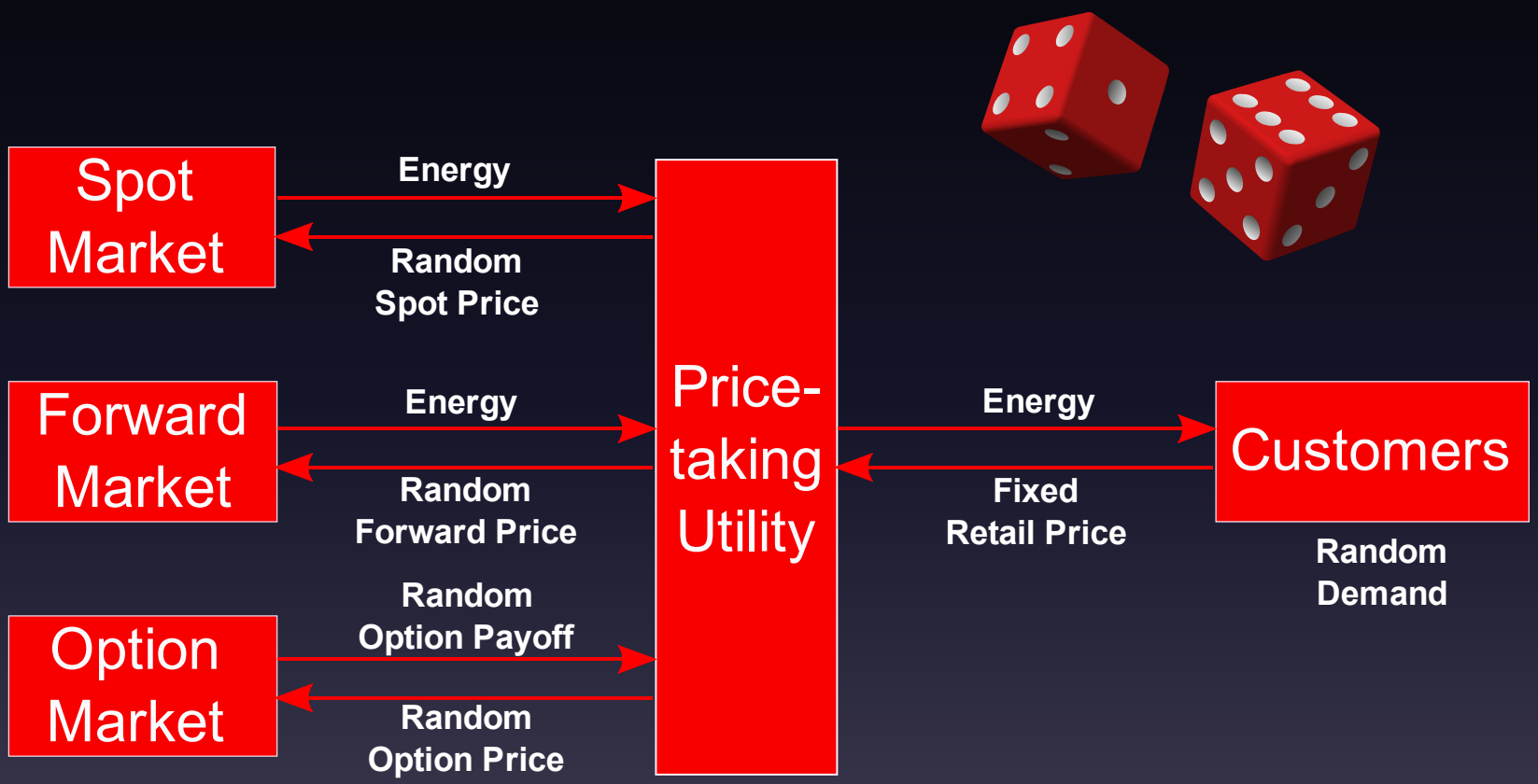
March 9, 2010

Risk Management for Utilities

Market

Utilities

Sources of Randomness



Electricity Market

Forwards

* Fixed delivery profile (base, peak, off-peak)
* Fixed volume
* Physically settled

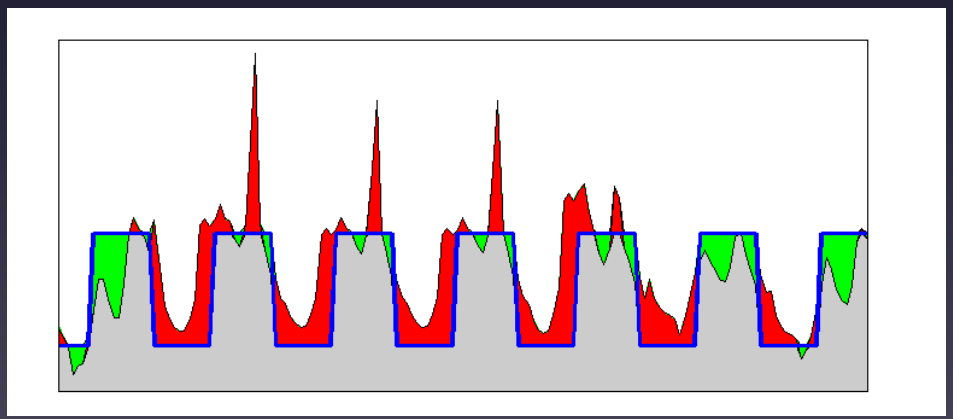
European Call Options

* Forwards as underlying instruments
* Financially settled

Financial Transactions

Costs (Uncertainties; Decisions)

Spot Trading	$S_t(D_t - \sum_i v_t^i x_{\mathsf{F},t}^i)$
Forward Trading	$\sum_i F_t^i v^i x_{\mathsf{f},t}^i$
Option Trading	$\sum_j C_t^j v^{i(j)} x_{\mathsf{c},t}^j$
Option Exercising	$-\sum_j \max\{F_t^{i(j)} - K^j, 0\} v^{i(j)} x_{\mathsf{C},t}^j$



Financial Transactions

Constraints (Decisions)

Budget Constraints

$$x^i_{\mathsf{F},t} = x^i_{\mathsf{F},t-1} + x^i_{\mathsf{f},t}$$
$$x^j_{\mathsf{C},t} = x^j_{\mathsf{C},t-1} + x^j_{\mathsf{c},t}$$

No-Short-Sales Constraints

$$x^i_{\mathsf{F},t} \geq 0$$
$$x^j_{\mathsf{C},t} \geq 0$$

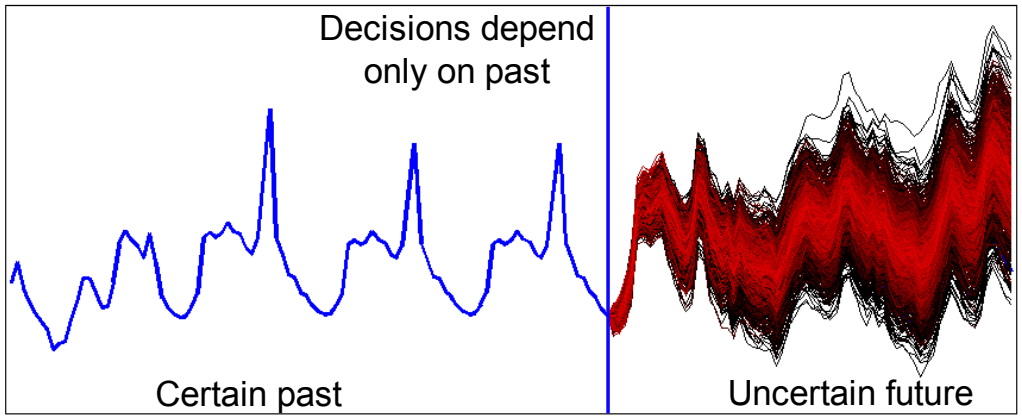
No-Trading Constraints

$$x^i_{\mathsf{f},t} = 0 \quad \text{if forward } i \text{ has expired}$$
$$x^j_{\mathsf{c},t} = 0 \quad \text{if option } j \text{ has expired}$$

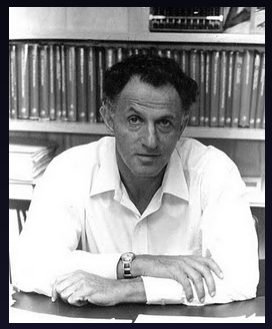
Mean-Risk Model

minimize γ Expected Costs $+(1-\gamma)$ Risk

subject to Budget Constraints
No-Short Sales Constraints
No-Trading Constraints
Non-Anticipativity Constraints



Problem Complexity

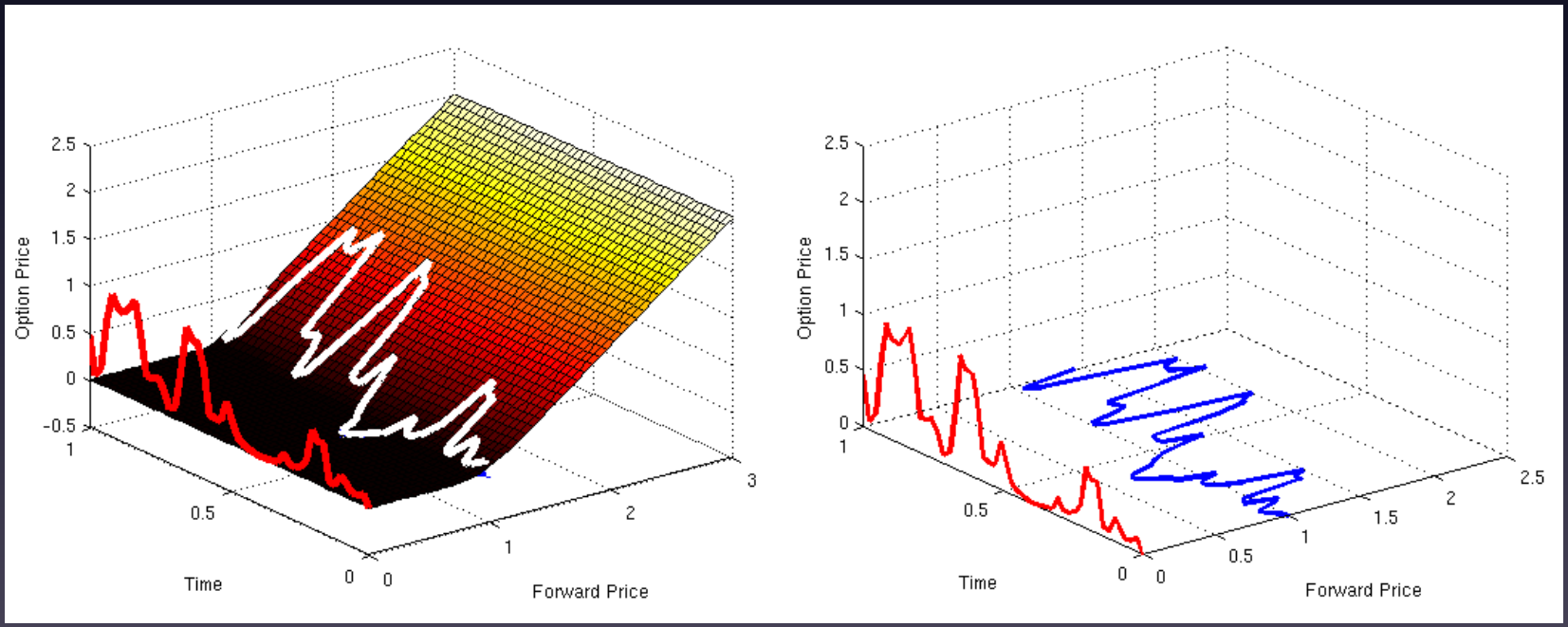


* # State variables $\propto$ # contracts $\gg 4$
  $\Longrightarrow$ DP problematic

* Arbitrage-free scenario tree
  requires (# contracts $+1)^T$ scenarios
  $\Longrightarrow$ SP problematic

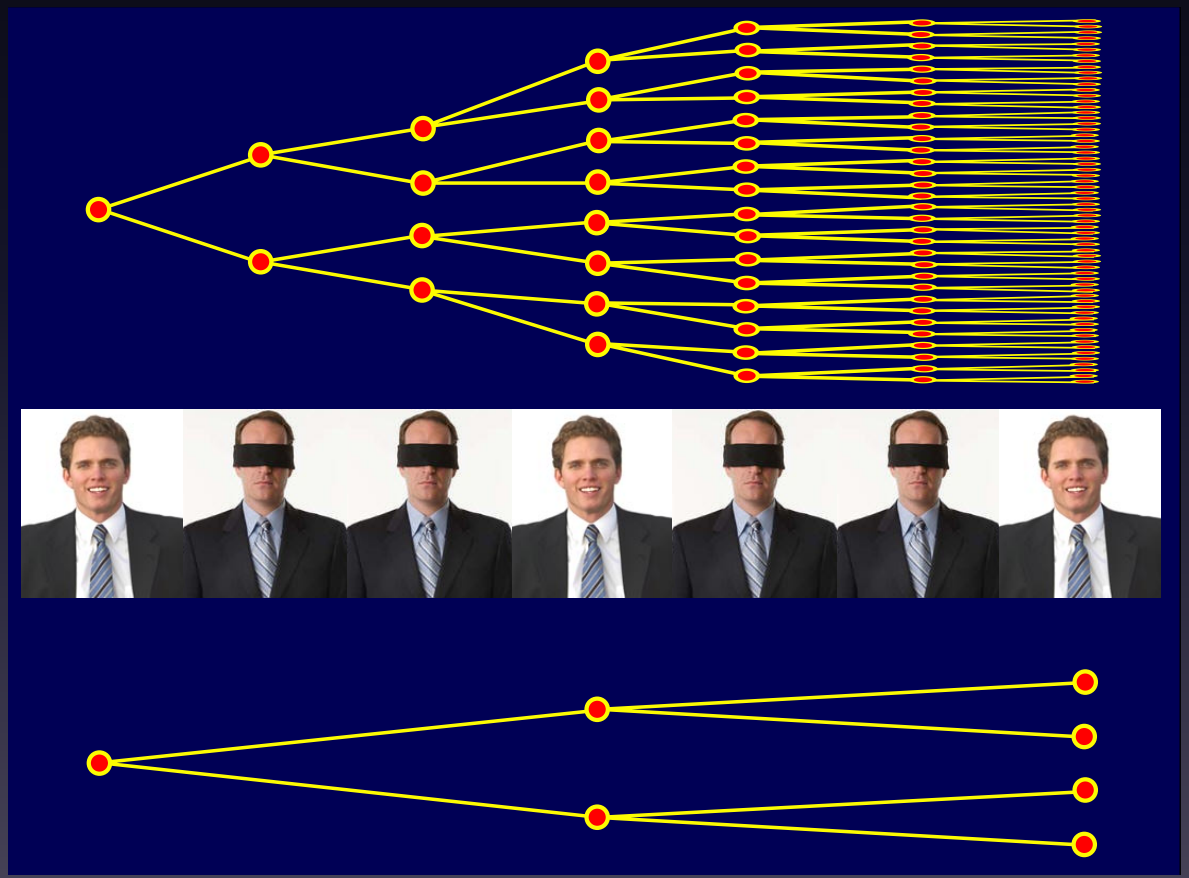
Dimensionality Reduction

* All uncertainties explained by few risk factors $\{\xi_t\}_{t=1}^T$
* Eliminate perfect dependencies
* Use principal component analysis



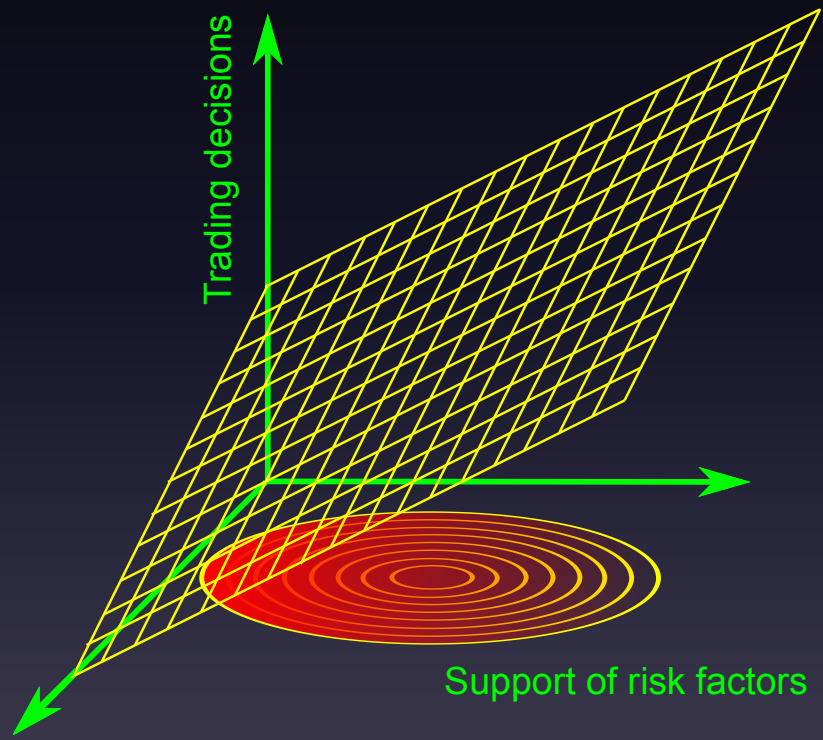
Stage Aggregation

* Observe uncertain parameters only every third (nth) stage
* Trading only reasonable when new information is revealed



Linear Decision Rules

* Trading decisions: linear decision rules of the risk factors



Problem Data

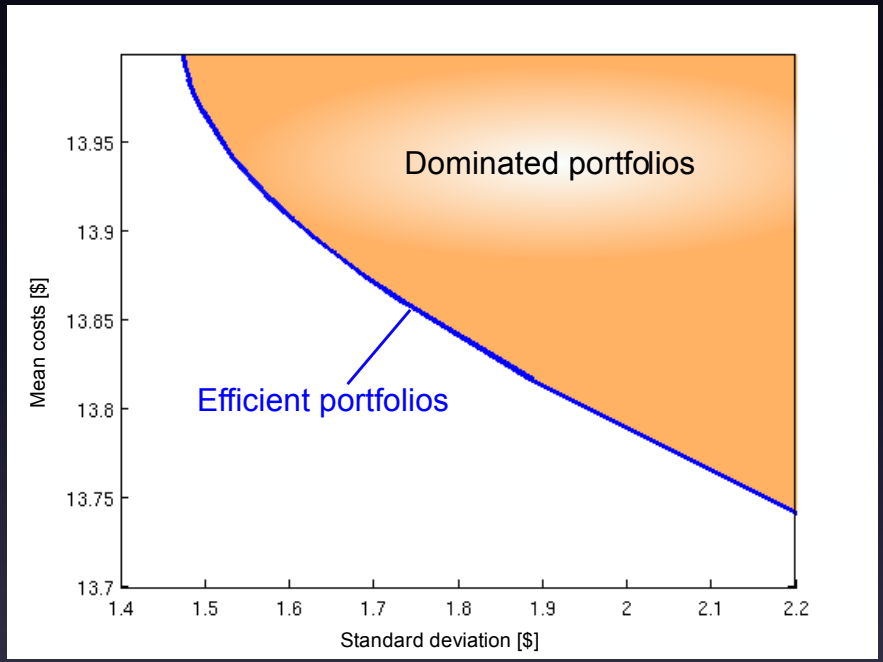
# stages	28
# forwards	3
# options	3
# risk factors	2 (demand, spot price)

$\Rightarrow$ SP would need $> 10^{21}$ scenarios

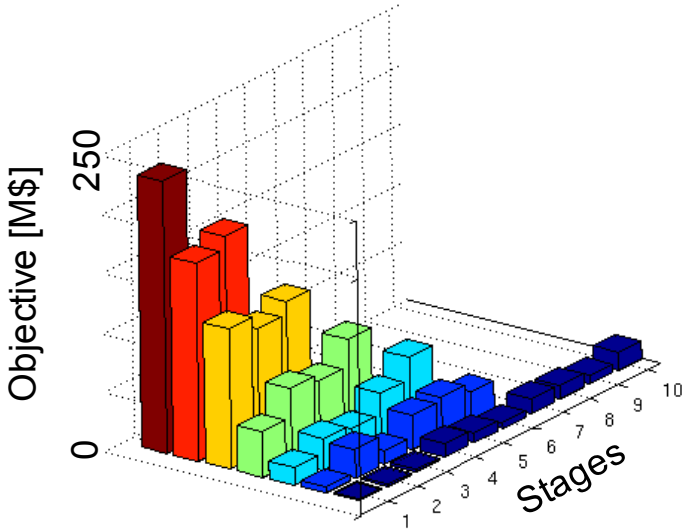
Uncertainty modelling:

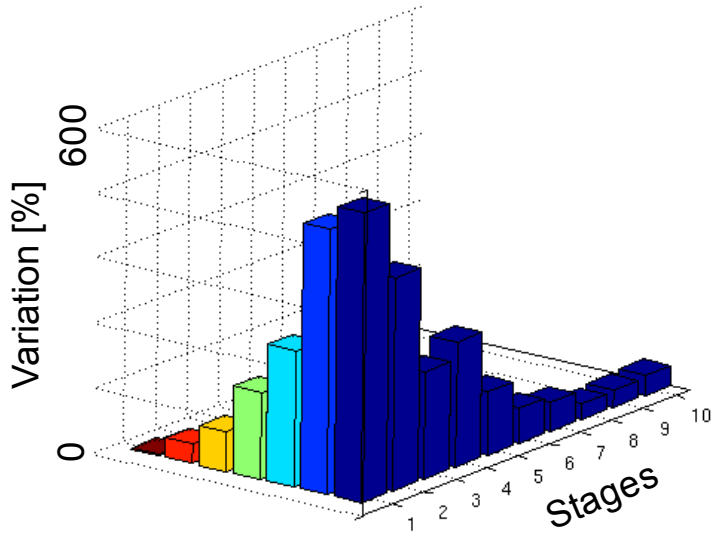
* Spot price — One-factor model w seasonality (Lucia & Schwartz, 2002)
* Demand — Log-Vasicek model w seasonality

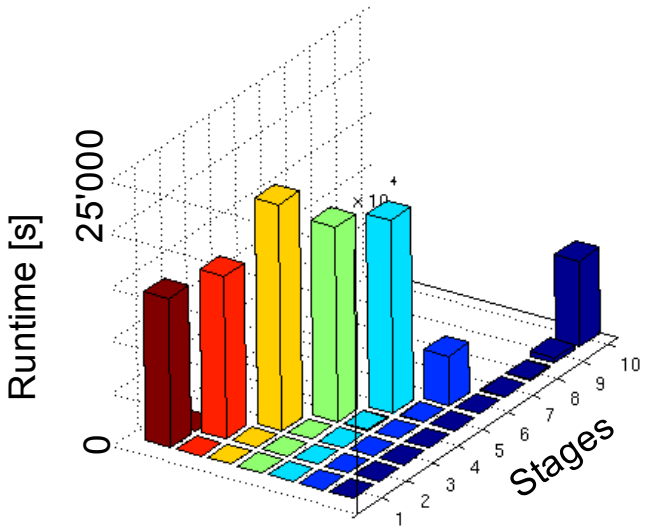
Efficient Frontier

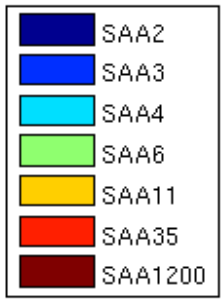


SAA Results

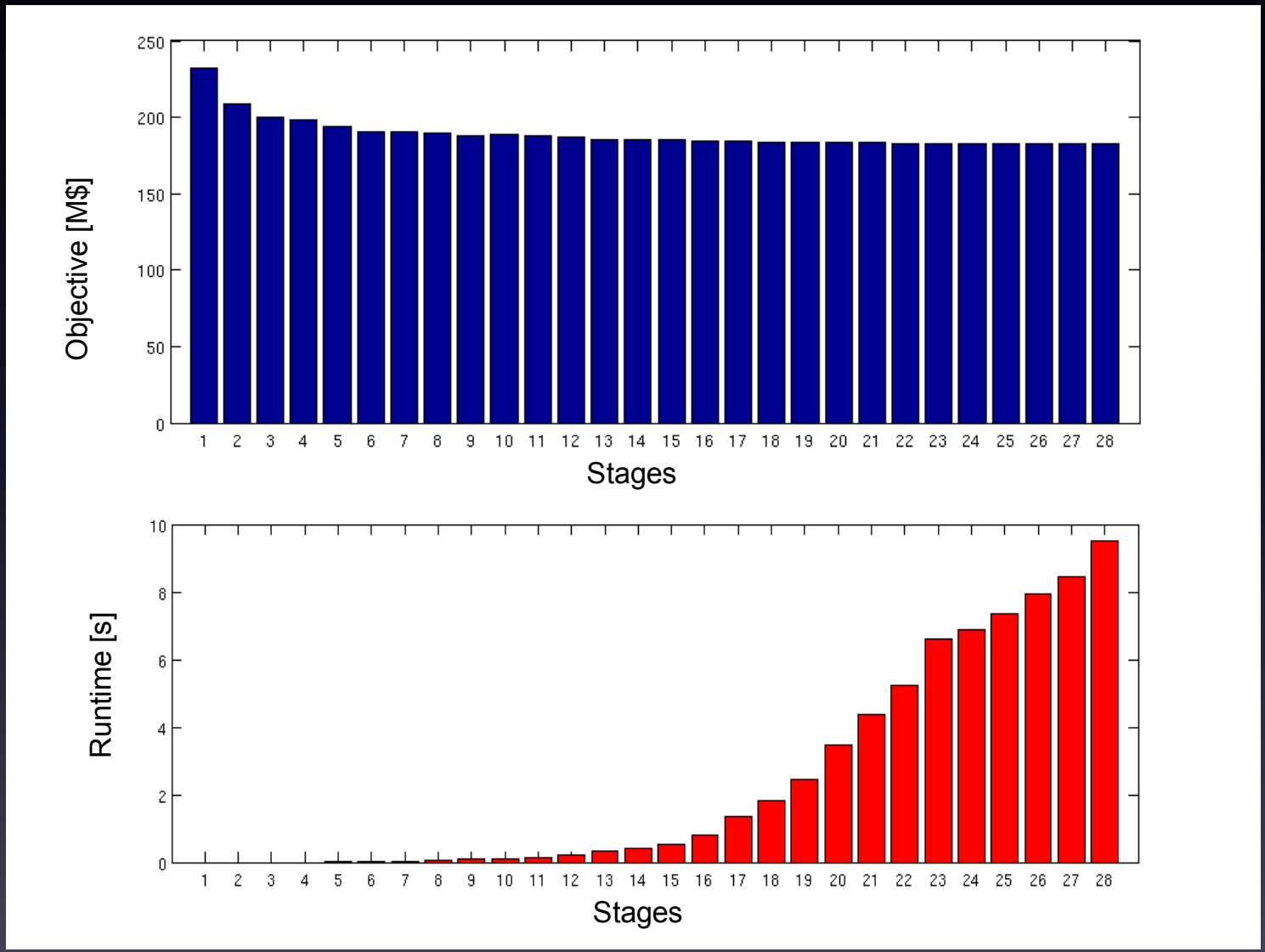

Objective [M$]
250
0
1
2
3
4
5
6
7
8
9
10
Stages


Variation [%]
600
0
1
2
3
4
5
6
7
8
9
10
Stages

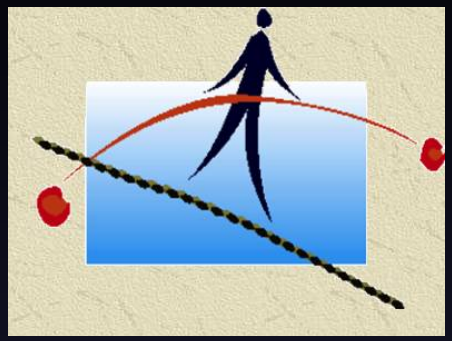

Runtime [s]
25'000
0
x 10^4
1
2
3
4
5
6
7
8
9
10
Stages


SAA2
SAA3
SAA4
SAA6
SAA11
SAA35
SAA1200

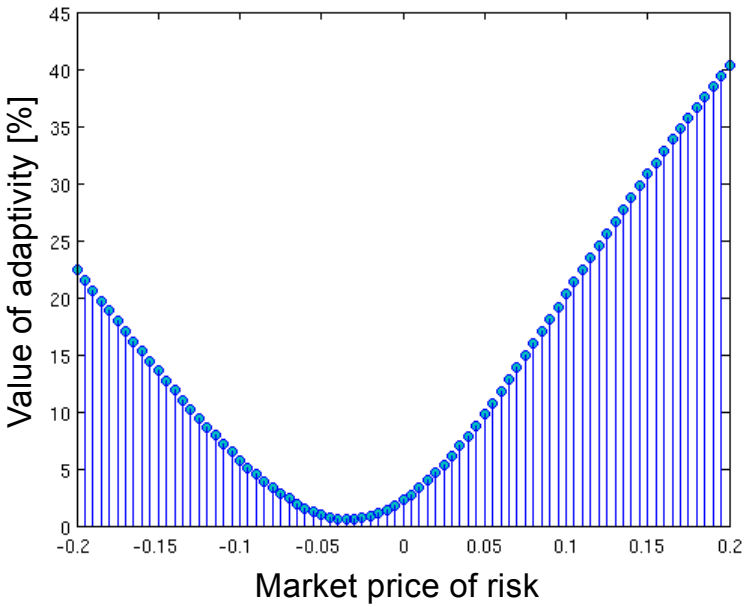
LDR Results

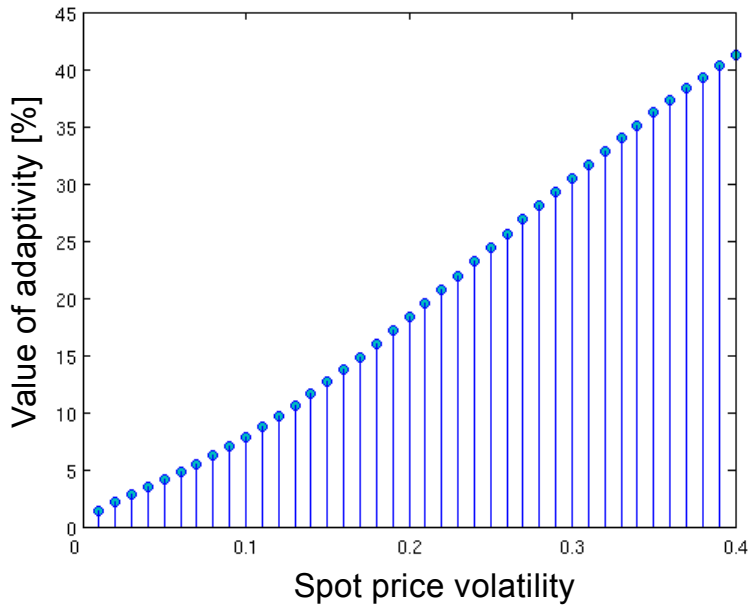

Objective [M$]
250
200
150
100
50
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28
Stages
Runtime [s]
10
8
6
4
2
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28
Stages

Value of Adaptivity



Ability to react to changing market conditions $\Rightarrow$ risk reduction!





Bibliography

Daniel Kuhn
Aggregation and discretization in multistage stochastic programming.
Mathematical Programming A 113, 1 (2008), 61–94.

Daniel Kuhn, Wolfram Wiesemann, and Angleos Georghiou
Primal and dual linear decision rules in stochastic and robust optimization.
Mathematical Programming A, in press (2010).

Paula Rocha and Daniel Kuhn
Multistage stochastic portfolio optimisation in deregulated electricity markets using linear decision rules.
Available on *Optimization Online*, submitted for publication (2010).