Stochastic Generation Dispatch – Formulation as an Optimization and Equilibrium Model

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1 Introduction

Several European countries have implemented special support schemes for renewable energy sources in order to reduce domestic emissions of carbon dioxide in the energy sector. Especially in northern Europe, wind energy became the dominating renewable energy source due to the natural conditions. However, the special characteristics of wind energy put limits on the response to market signals and significantly affects electricity markets as well as existing infrastructure.

On the one hand, renewable energy generation especially wind generation are characterized by high capital and low fuel or operational costs. Hence, wind generation is placed in the beginning of the merit order and should be dispatched first in the short run. On the other hand, generation of wind generation strongly depends on meteorological conditions and hence cannot be dispatched in a controlled manner. This results in an uncertainty on realized wind generation, which can be only partly reduced through appropriate wind forecasts. However, uncertainty has always been present in electrical power systems, in the form of possible unit failures or errors in demand prediction. In the last years, electricity production from wind has increased significantly and thus the problems associated with this form of electricity generation regarding the uncertainty about wind output and its variability. Thus, wind energy and its special characteristics have to be taken into account when planning and operating electrical power systems. In the following sections a modeling approach is described which investigates the impact of uncertain renewable generation on the dispatch of power plants. Uncertainty about wind generation is described by a scenario tree.

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2 Mathematical Formulation

2.1 Nomenclature

Sets	n, k	...	nodes
	n'	...	swing bus
	u	...	generation units (by fuel)
	l	...	power lines
	s, ss	...	scenarios
	$A(s)$	...	set of ancestor scenario (i.e., previous scenario)
Parameters	a_n	...	intercept of inverse demand function
	b_n	...	slope of inverse demand function
	c_u^m	...	marginal production costs of generation
	c_u^r	...	ramp-up costs
	cap_l^{max}	...	maximum thermal capacity of power lines
	$g_{s,n}^{wind}$	...	wind generation (exogenous and obligatory feed-in)
	$prob(s)$	...	probability of scenario
	$H_{l,k}$	...	branch susceptance matrix
	$B_{n,k}$	...	node susceptance matrix
Variables	$d_{s,n}$	...	electricity consumption
	$g_{s,n,u}$	...	electricity generation
	$g_{s,n,u}^{up}$	...	electricity generation ramp-up
	$\delta_{s,n}$	...	phase angle (decision variable of ISO)
	$\lambda_{s,n}$	...	dual for power balance constraint
	$\beta_{s,n,u}$	...	dual for maximum generation capacity constraint
	$\eta_{s,n,u}$	...	dual for ramp-up constraint
	$\overline{\mu}_{s,l}$	...	dual for line capacity constraint (positive direction)
	$\underline{\mu}_{s,l}$	...	dual for line capacity constraint (negative direction)
	γ_s	...	dual for swing bus constraint

2.2 Electricity Market Model (ELMOD)

We start from the welfare optimization problem of the Independent System Operator (ISO). The ISO maximizes the sum of consumer surplus derived from an inverse demand curve ($a_n - b_n \cdot d_{s,n}$) and producer rent, based on the merit order of the power generation units (Equation 1). Due to characteristics of electricity (grid-bounded, non-storable), thermal and renewable generation have to be equal to demand and netinput from the transmssion network (Equation 2). Renewable wind generation is directly accounted in Equation 2 to reflect firstly zero marginal generation costs, secondly the priority or obligatory feed-in of the generation type, and thirdly the non-dispatchable character of the wind generation. Thermal generation is restricted by the installed capacity (Equation 5) as well as the maximum generation increase between two time periods (so called Ramping constraint, Equation 4). The ISO must also consider the complexities of the alternate current (AC) grid. This is implemented through a simplification, called the direct current load flow (DCLF) approach. Load flows are determined by the phase angle $\delta_{s,n}$ between nodes and the technical network characteristics $H_{l,n}$. The resulting load flow in line l is specified as $\sum_n H_{l,n}\delta_{s,n}$ and restricted by thermal transmission limits of transmission lines (Equation 5 and 6). In order to solve the optimisation problem, the phase angle has to be fixed in an arbitrary node, called "slack node" or "swing bus" (Equation 7).

A scenario tree is implemented to represent uncertainties on wind generation. The scenario tree is characterized by scenario nodes s and realization probability of scenario node s $prob_s$. Futhermore, the relationship of scenario nodes s is saved in the ancestor matrix A_s defining the ancestor of scenario node s. Wind generation values are defined in parameter $g_{n,s}^{wind}$ for all considered scenario nodes. Hence, the ISO has perfect information as all scenarios are specified, but imperfect foresight due to various possible wind generation scenarios. In each scenario node, the generation dispatch and demand is optimized subject to technical and market clearing constraints. An intertemporal restriction is given by ramping constraint (Equation 4) linking different time periods and thus scenario nodes.

$$\max_{g,g^{up},d,\delta} \sum_{s,n} prob_s \cdot \left[(a_n \cdot d_{s,n} - \tfrac{1}{2} b_n \cdot d_{s,n}^2) - \sum_u c_u^m \cdot g_{s,n,u} - \sum_u c_u^r \cdot g_{s,n,u}^{up} \right] \tag{1}$$

$$d_{s,n} - \sum_u g_{s,n,u} - g_{s,n}^{wind} - \sum_k B_{n,k}\delta_{s,k} = 0 \qquad \lambda_{s,n} \text{ (free)} \qquad \forall\ s,n \tag{2}$$

$$g_{s,n,u} \leq g_{n,u}^{max} \qquad \beta_{s,n,u} \geq 0 \qquad \forall\ s,n,u \tag{3}$$

$$g_{s,n,u} - g_{A(s),n,u} - g_{s,n,u}^{up} \leq 0 \qquad \eta_{s,n,u} \geq 0 \qquad \forall\ s,n,u \tag{4}$$

$$\sum_n H_{l,n}\delta_{s,n} \leq cap_l^{max} \qquad \overline{\mu}_{s,l} \geq 0 \qquad \forall\ s,l \tag{5}$$

$$-\sum_n H_{l,n}\delta_{s,n} \leq cap_l^{max} \qquad \underline{\mu}_{s,l} \geq 0 \qquad \forall\ s,l \tag{6}$$

$$\delta_{s,n'} = 0 \qquad \gamma_s \text{ (free)} \qquad \forall\ s \tag{7}$$

2.3 Karush-Kuhn-Tucker conditions

By deriving the Karush-Kuhn-Tucker conditions of the welfare maximization problem of the ISO, combined with the technical and operational constraints, we obtain a Mixed Complementarity Problem (MCP).

$$-prob_s \cdot c_u^m + \lambda_{s,n} - \beta_{s,n,u} - \eta_{n,u,s} + \sum_{ss \in A(s)} \eta_{n,u,ss} \leq 0 \quad \perp \quad g_{s,n,u} \geq 0 \tag{8}$$

$$prob_s \cdot (a_n - b_n \cdot d_{s,n}) - \lambda_{s,n} \leq 0 \quad \perp \quad d_{s,n} \geq 0 \tag{9}$$

$$-prob_s \cdot c_u^r + \eta_{n,u,s} \leq 0 \quad \perp \quad g_{s,n,u}^{up} \geq 0 \tag{10}$$

$$\sum_k B_{k,n} \cdot \lambda_{s,nn} - \sum_l H_{l,n} \cdot \overline{\mu}_{s,l} + \sum_l H_{l,n} \cdot \underline{\mu}_{s,l} - \gamma_s = 0 \quad \perp \quad \delta_{s,n} \text{ (free)} \tag{11}$$

$$d_{s,n} - \sum_u g_{s,n,u} - g_{s,n}^{wind} - \sum_k B_{n,k}\delta_{s,k} = 0 \quad \perp \quad \lambda_{s,n} \text{ (free)} \tag{12}$$

$$g_{s,n,u} - g_{n,u}^{max} \leq 0 \quad \perp \quad \beta_{s,n,u} \geq 0 \tag{13}$$

$$g_{s,n,u} - g_{A(s),n,u} - g_{s,n,u}^{up} \leq 0 \quad \perp \quad \eta_{s,n,u} \geq 0 \tag{14}$$

$$\sum_n H_{l,n}\delta_{s,n} - cap_l^{max} \leq 0 \quad \perp \quad \overline{\mu}_{s,l} \geq 0 \tag{15}$$

$$-\sum_n H_{l,n}\delta_{s,n} - cap_l^{max} \leq 0 \quad \perp \quad \underline{\mu}_{s,l} \geq 0 \tag{16}$$

$$\delta_{s,n'} = 0 \quad \perp \quad \gamma_s \text{ (free)} \tag{17}$$

3 Test Cases

3.1 Three-Node Network

The three-node network and relevant data is based on Gabriel and Leuthold (2010).

3.2 Fifteen-Node Network

The fifteen-node network representing a stylized grid of the Western European market is based on Neuhoff *et al.* (2005). Relevant data is taken from Neuhoff *et al.* (2005) and Gabriel and Leuthold (2010). The underlying network including the naming of nodes and lines is displayed in Figure 1. The scenario tree and the stochastic wind generation is shown in Figure 2 (wind generation relative to total wind generation capacity).

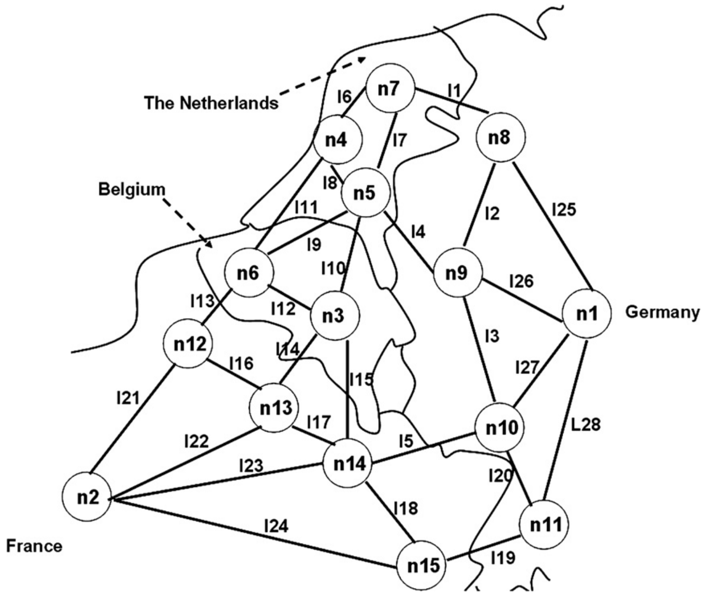

Figure 1: Stylized network of the Western European grid. Source: Neuhoff *et al.* (2005)

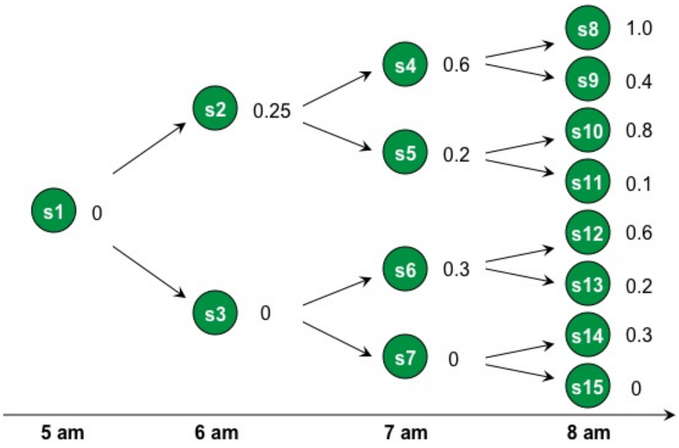

Figure 2: Scenario tree – wind generation relative to maximum wind generation capacity

4 Literature on Power System Analysis and Electricity Market Modeling

Power System Analysis

Technical Focus

- Wood and Wollenberg (1996)
- Bergen and Vittal (2006)
- Harris (2006)

Economic Focus

- Schweppe, Caramanis, Tabors, and Bohn (1988)
- Stoft (2002)
- Shahidehpour and Li (2002)
- Kirschen and Strbac (2004)
- Harris (2006)

Electricity Market Modeling

- Ventosa, Baillo, Ramos, and Rivier (2005)
- Stigler and Todem (2005)
- Leuthold, Weigt, and von Hirschhausen (2010)

Uncertainty in Electricity Market Modeling

- Wallace and Fleten (2003)
- Weber (2005)

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