TECHNISCHE UNIVERSITÄT DRESDEN

DIW BERLIN

Introduction to Electricity Network Modelling

PhD Winterschool, Oppdal
March 2011

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Agenda

1. Introduction to Electricity Markets
2. The Electricity Market Model (ELMOD)
3. Congestion Management
4. Exercise: 3-Node Network
5. Introducing Wind Power
6. Exercise: Stochastic Multi-Period European Network
7. Outlook and further developments

Literature

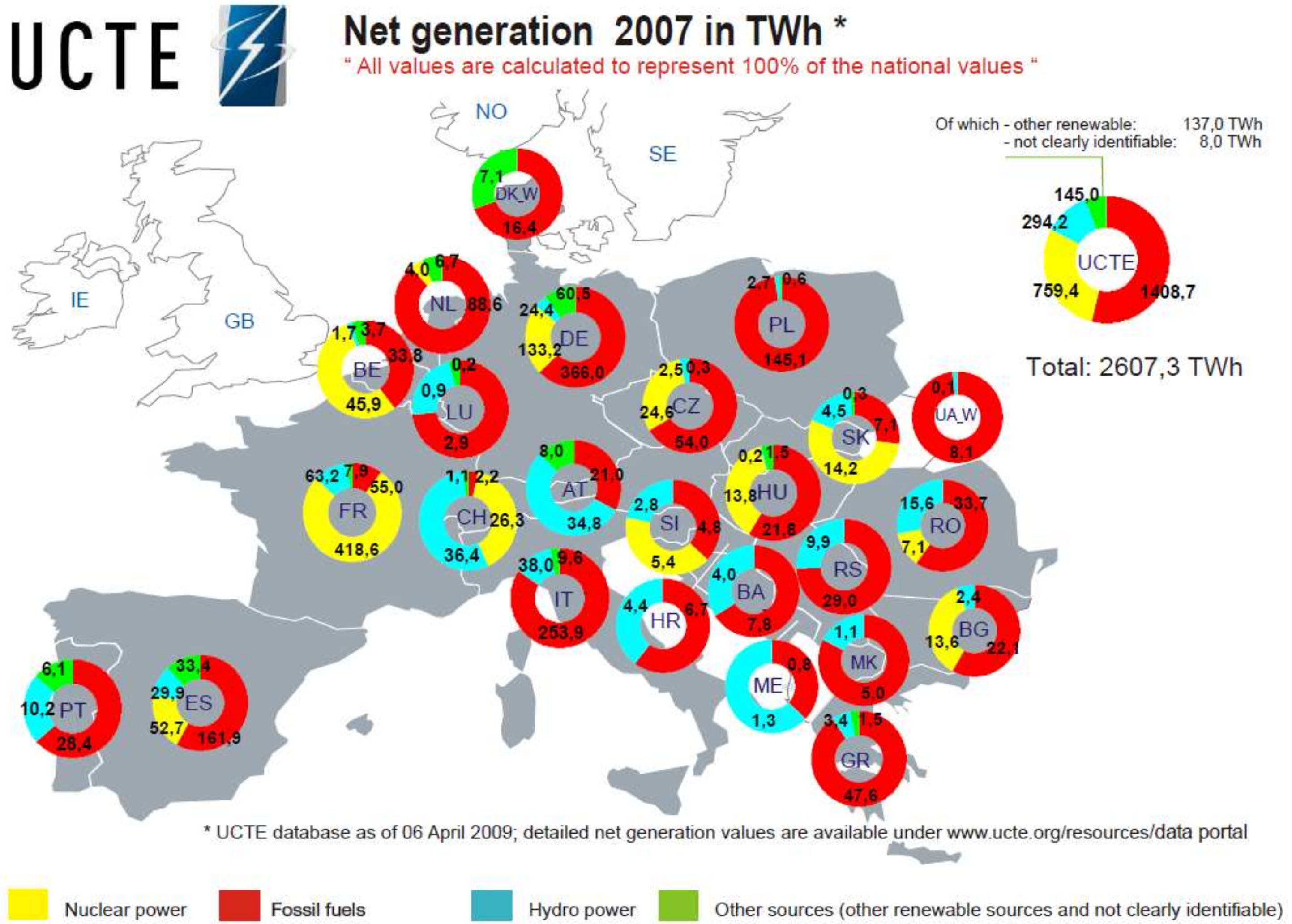
Electricity

- Non storable
- Grid-bound
- High fix cost ratio
- Economies of scale in generation and transmission
- Daily and seasonal demand patterns
- Power flows according to physical laws (Kirchhoff)

Value added chain

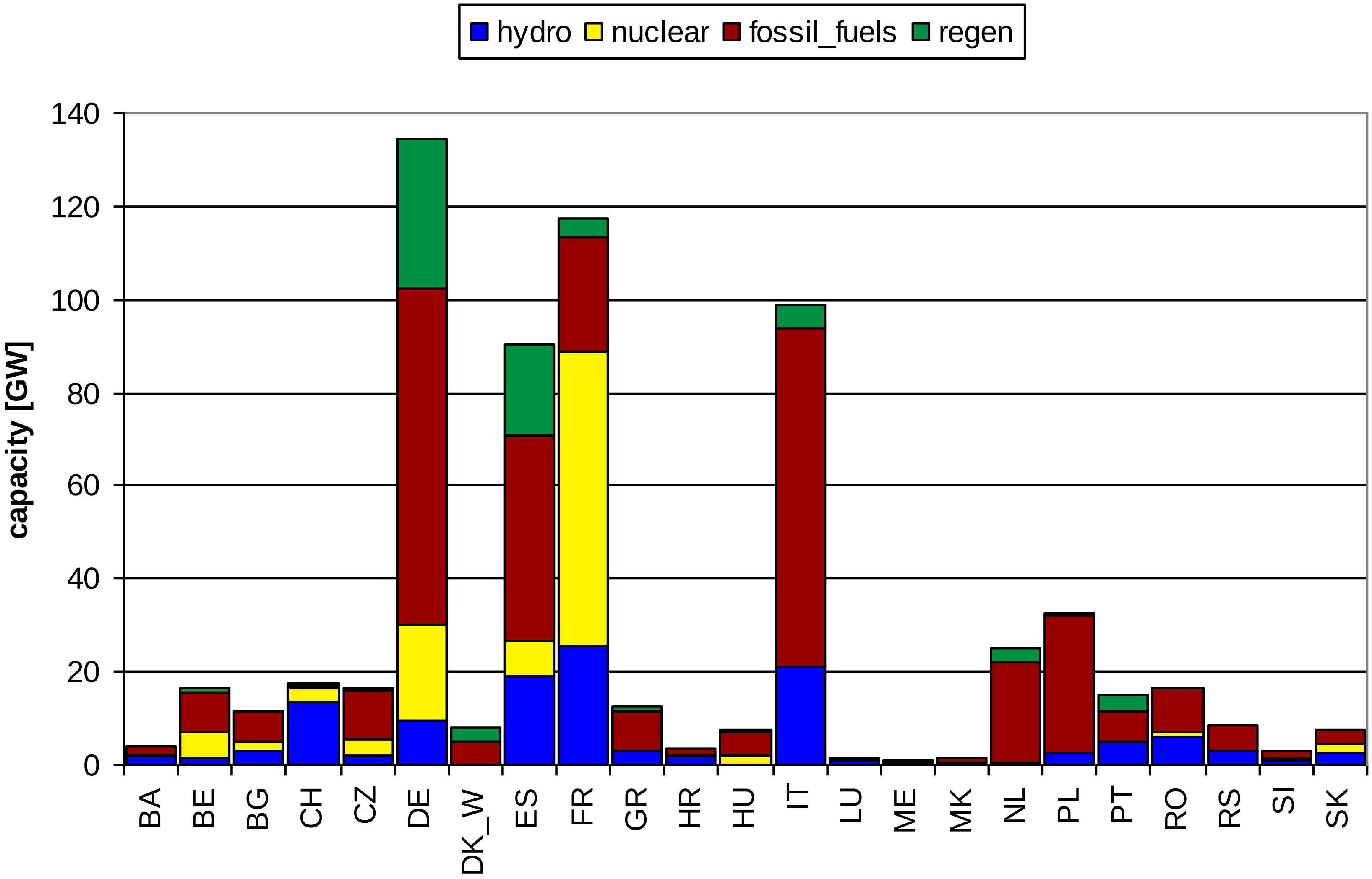
1. Generation
2. Transmission/Distribution
3. Supply

Electricity Generation



Source: ENTSO-E

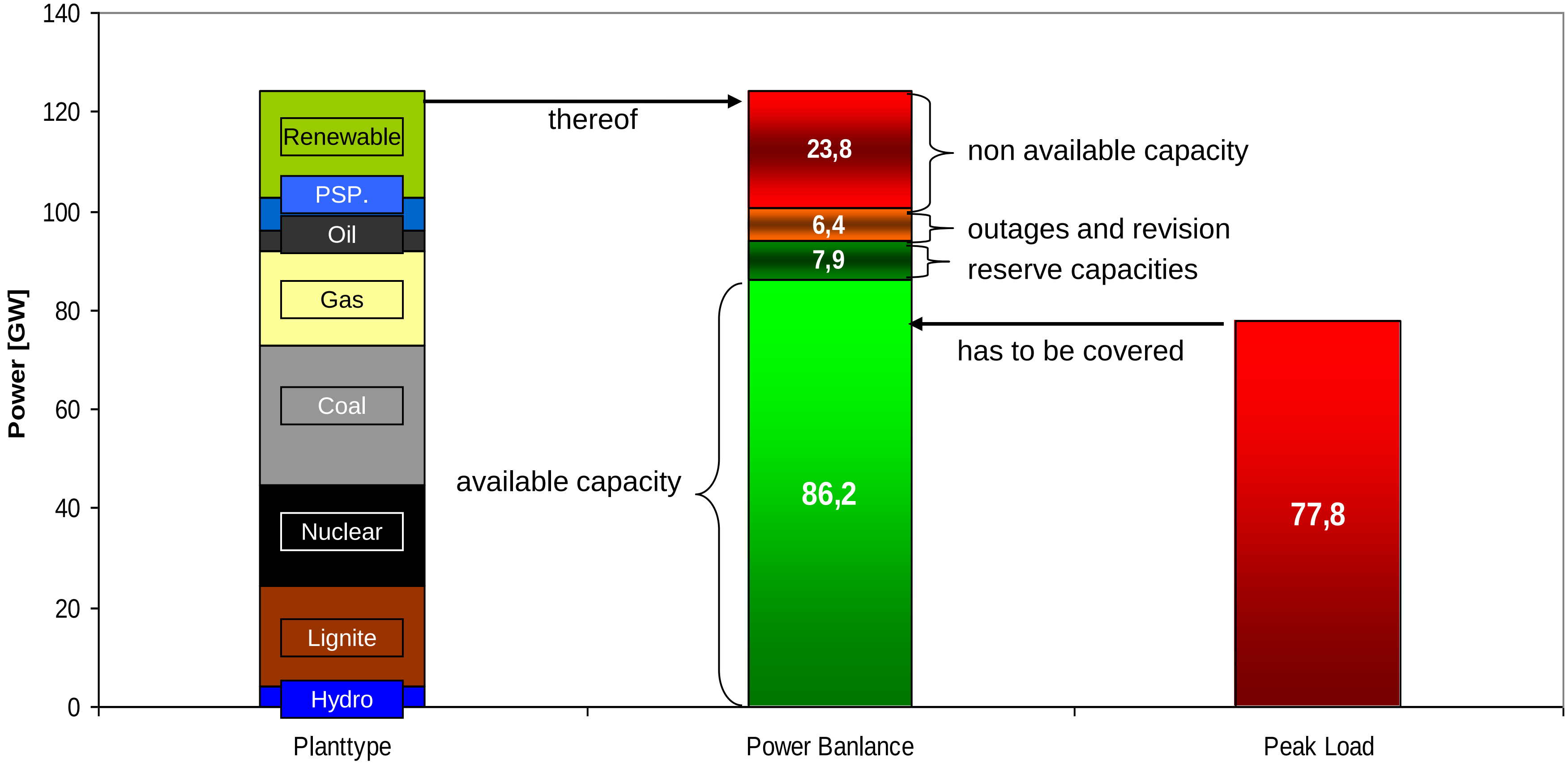
Electricity Generation Capacities



Source: ENTSO-E

Plant Capacity and Peak Load in Germany 2006

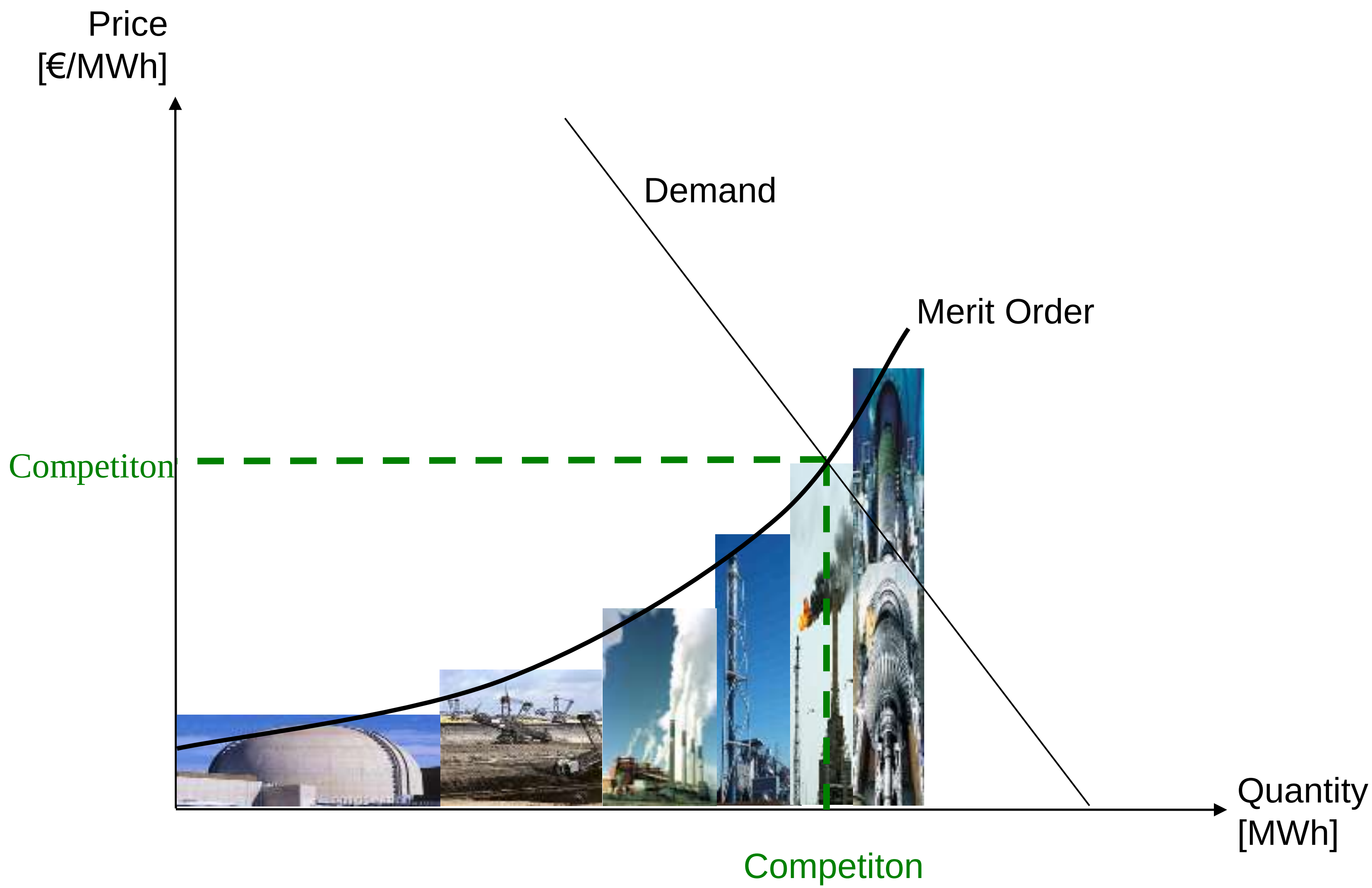
Sufficient capacity to supply Germany and still export:



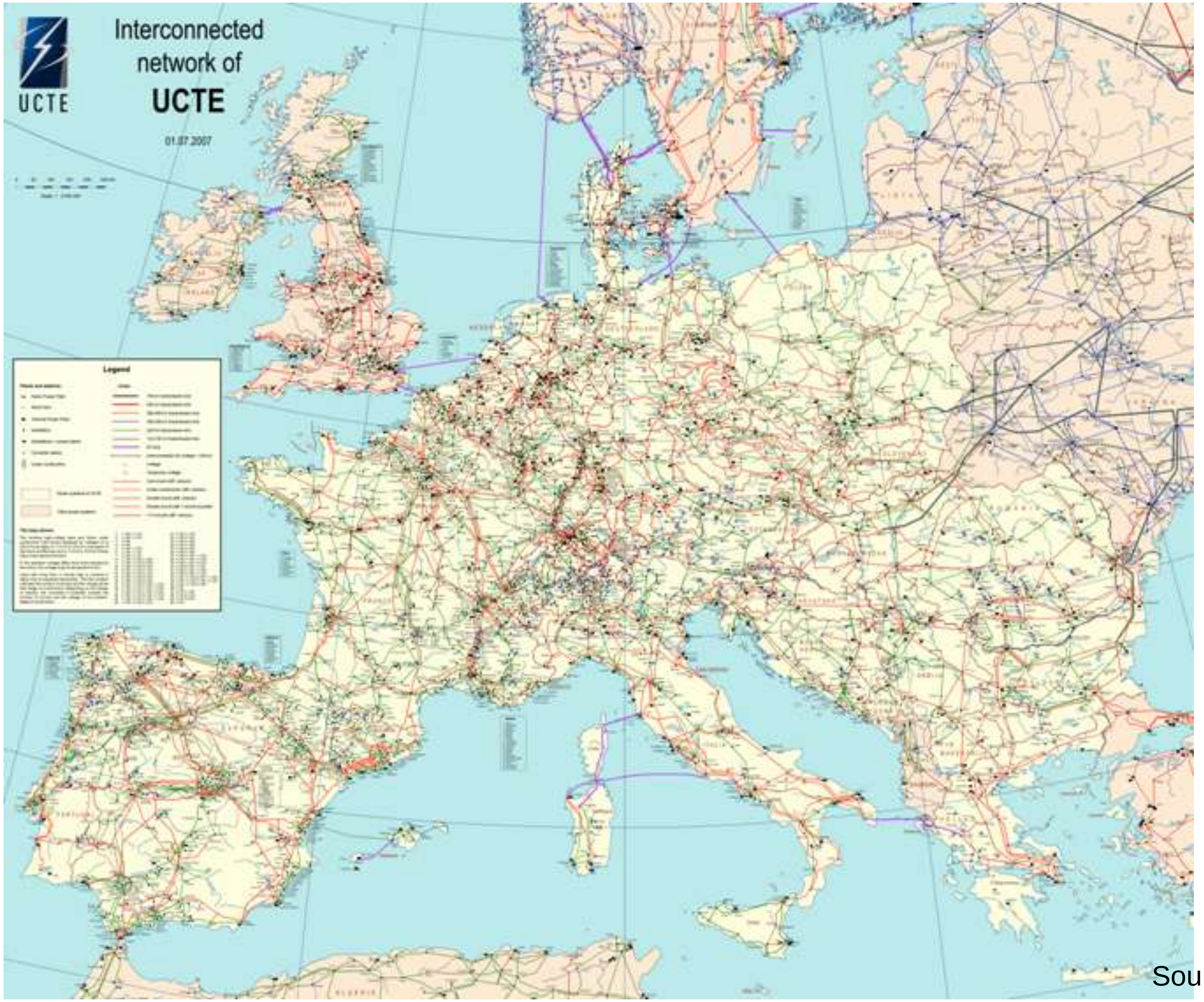
At time of peak load an export surplus of 2.1GW occured

Source: VDN 2006

The Merit-Order Cost Curve and Pricing under Competition



European High Voltage Network

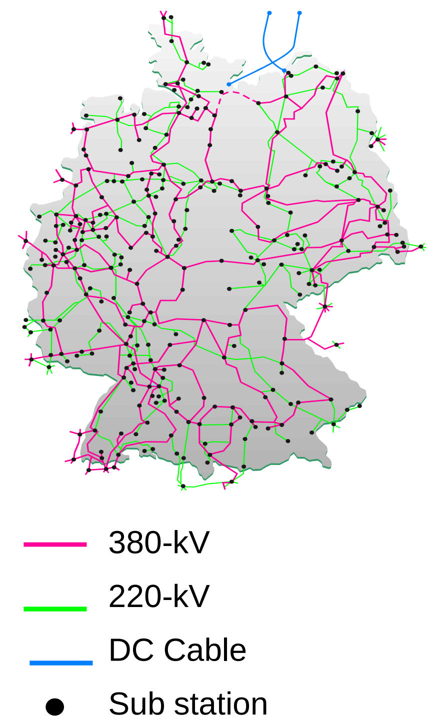


Source: ENTSO-E

4 Voltage Levels

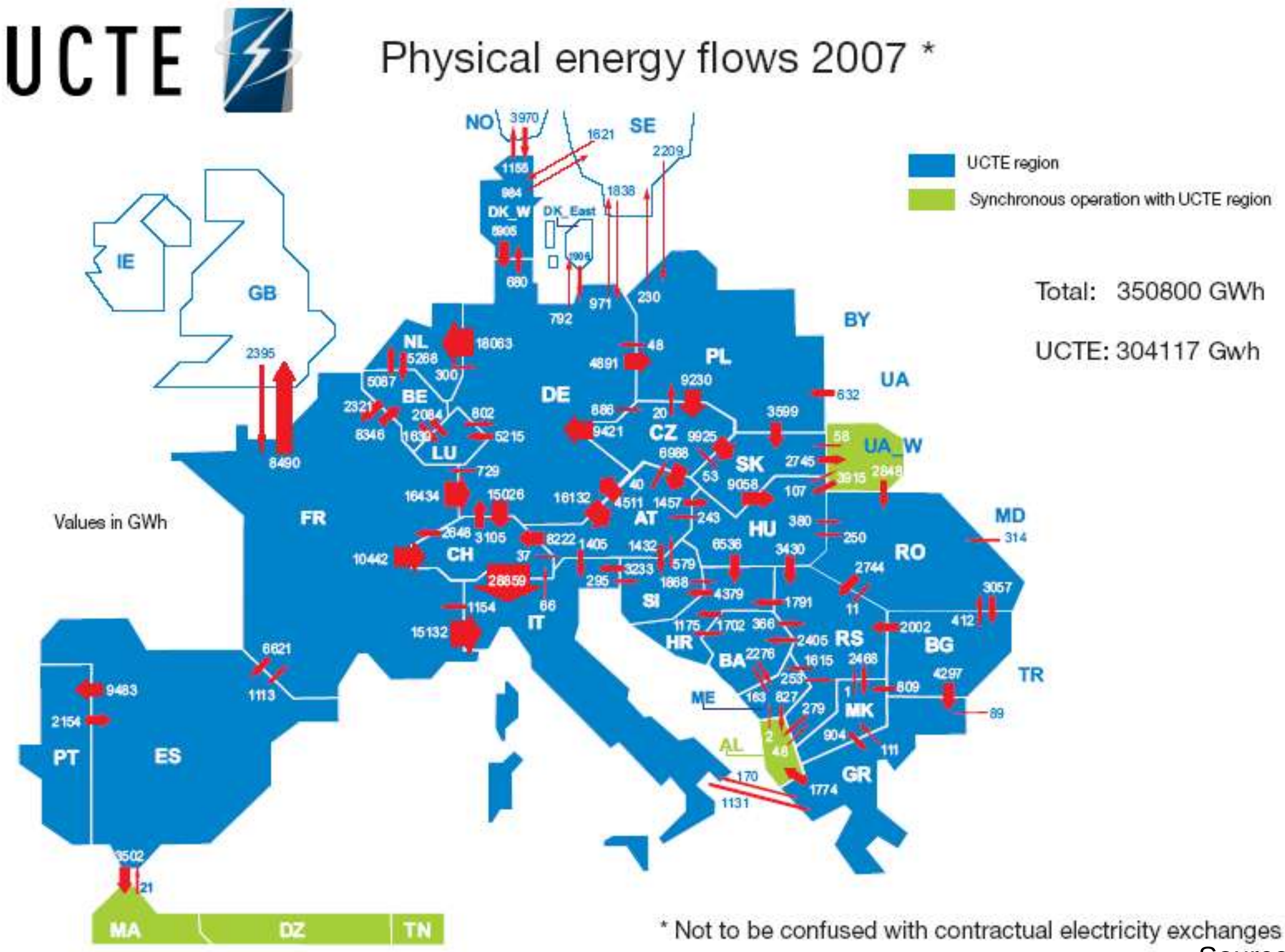
- **German network operators maintain 1.6 mio km of lines and 500 000 transformer stations**

Transmission	Voltage Level	Coverage	Consumer
Extra High Voltage	**220 … 380 kV**	**national**	**Regional suppliers, large industry, imports/exports**
Distribution	**Voltage Level**	**Coverage**	**Consumer**
High Voltage	**36 … 110 kV**	**regional**	**Local suppliers, industry**
Medium Voltage	**1 … 36 kV**	**regional**	**Industry, large commercial**
Low Voltage	**0,4 … 1 kV**	**local**	**Households, Agriculture, Commercial**



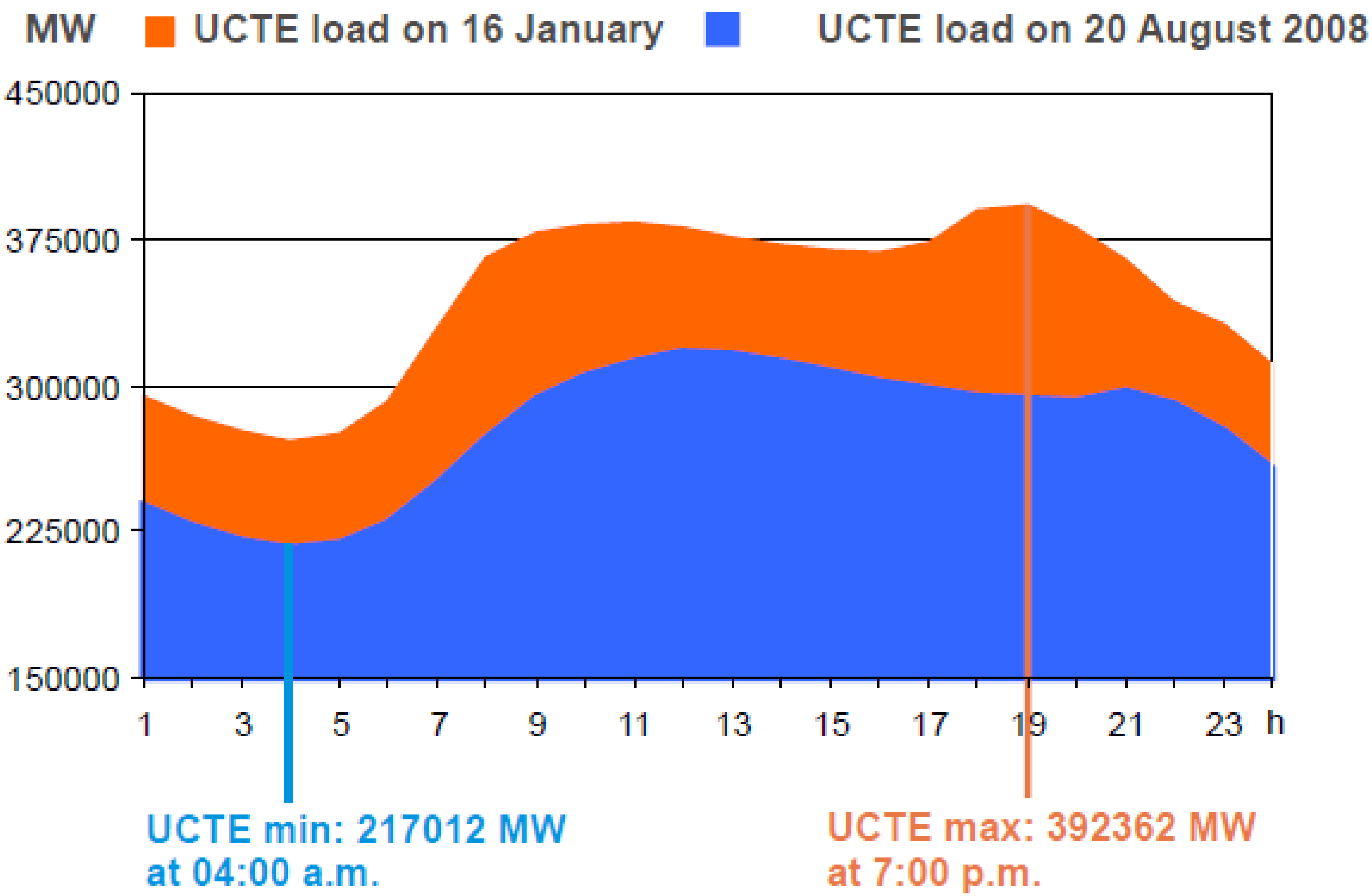
Source: VDN

Physical Electricity Exchange



Source: ENTSO-E

Electricity Demand



Source: ENTSO-E

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Introduction

- **Electricity markets are in a process of restructuring**
- **Economic modeling of electricity markets not possible without accounting for technical constraints**

- **Model-based research of electricity markets very common, e.g. in the US (Hogan, Hobbs, UC Berkeley, ...)**
- **Economic-engineering model-based research for Germany and Europe available rather limited**

↓

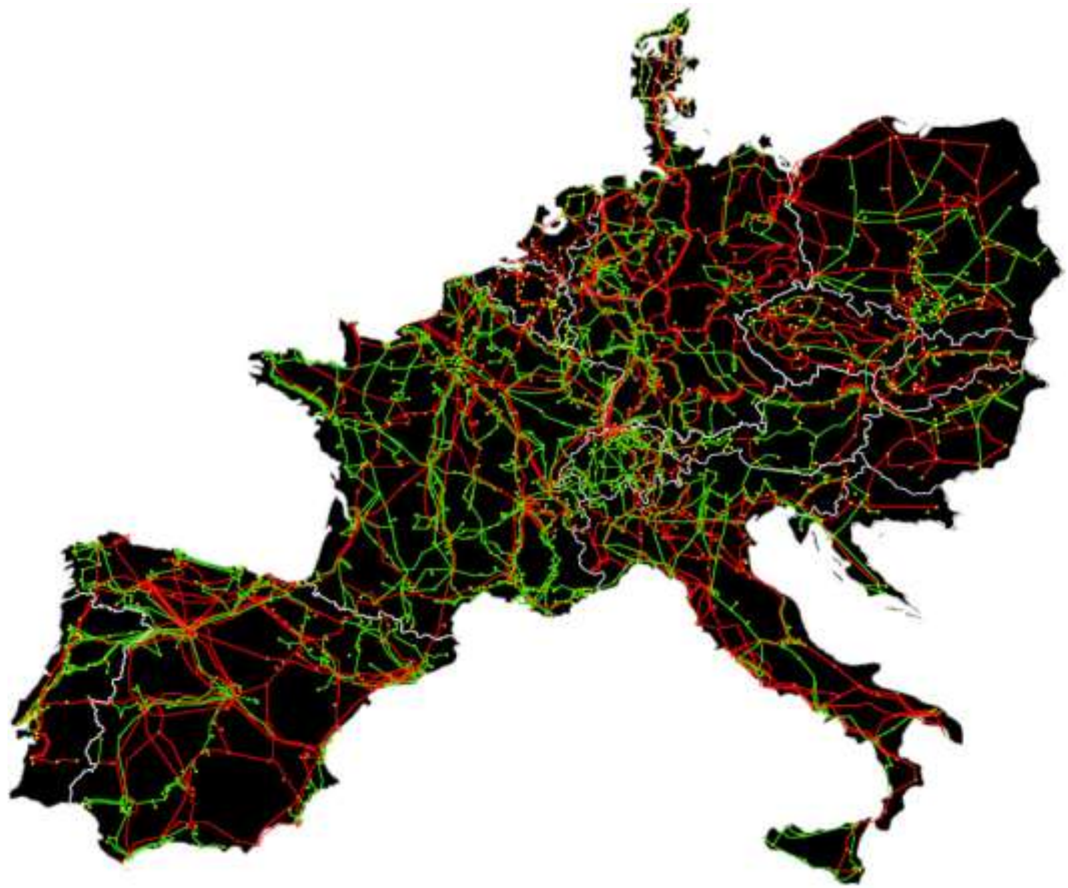
- **Development of ELMOD: Engineering-Economic Approach**

Scope of the Model

Physical model (included countries): ENTSO-E

Portugal, Spain, France, Netherlands, Belgium, Luxembourg, Denmark, Germany, Switzerland, Austria, Italy, Poland, Hungary, Czech Republic, Slovenia and Slovakia …

Nodes:	2120	(substations)
Lines:	3143	
thereof:	106	150kV
	1887	220kV
	1150	380kV



Market Assumptions and Data

- **Market:**
  - No strategic players → Perfect competition
  - Perfect market bidding (marginal cost bids, no market power)
  - Independent SO optimizes generation dispatch and network usage simultaneously
- **Node demand:**
  - Linear inverse demand function constructed using
    - a reference demand,
    - a reference price, and
    - a point demand elasticity
  - Reference demands are based on ENTSO-E data and distributed to system nodes according to regional population and/or gross domestic product
  - Reference prices are based on the spot prices of the national energy exchange
- **Wind input:**
  - Given as external parameter based on wind distributions derived from historic data
- **Reference: Leuthold et al. (2010)**

Model Formulation

Sets	n, k ...	nodes
	n' ...	swing bus
	u ...	generation units (by fuel)
	l ...	power lines
Parameters	a_n ...	intercept of inverse demand function
	b_n ...	slope of inverse demand function
	c_u^m ...	marginal production costs of generation
	cap_l^{max} ...	maximum thermal capacity of power lines
	$H_{l,k}$ ...	branch susceptance matrix
	$B_{n,k}$ ...	node susceptance matrix
Variables	d_n ...	electricity consumption
	$g_{n,u}$ ...	electricity generation
	δ_n ...	phase angle (decision variable of ISO)
	λ_n ...	dual for power balance constraint
	$\beta_{n,u}$ ...	dual for maximum generation capacity constraint
	$\overline{\mu}_l$ ...	dual for line capacity constraint (positive direction)
	$\underline{\mu}_l$ ...	dual for line capacity constraint (negative direction)
	γ ...	dual for swing bus constraint

Model Formulation
Objective Function and Constraints

Given: generation capacities, network, demand function, wind

Decide about: generation, demand

max (Social Welfare)

subject to:

demand	=	generation + netinput
generation	<=	installed capacity
ABS(loadflow)	<=	thermal limit

Model Formulation
Objective Function and Constraints

$$\max_{g,d,\delta} \sum_n (a_n \cdot d_n - \tfrac{1}{2} b_n \cdot d_n^2) - \sum_u c_u^m \cdot g_{n,u} \qquad (1)$$

$$d_n - \sum_u g_{n,u} - \sum_k B_{n,k}\delta_k = 0 \qquad \lambda_n \text{ (free)} \qquad \forall\ n \qquad (2)$$

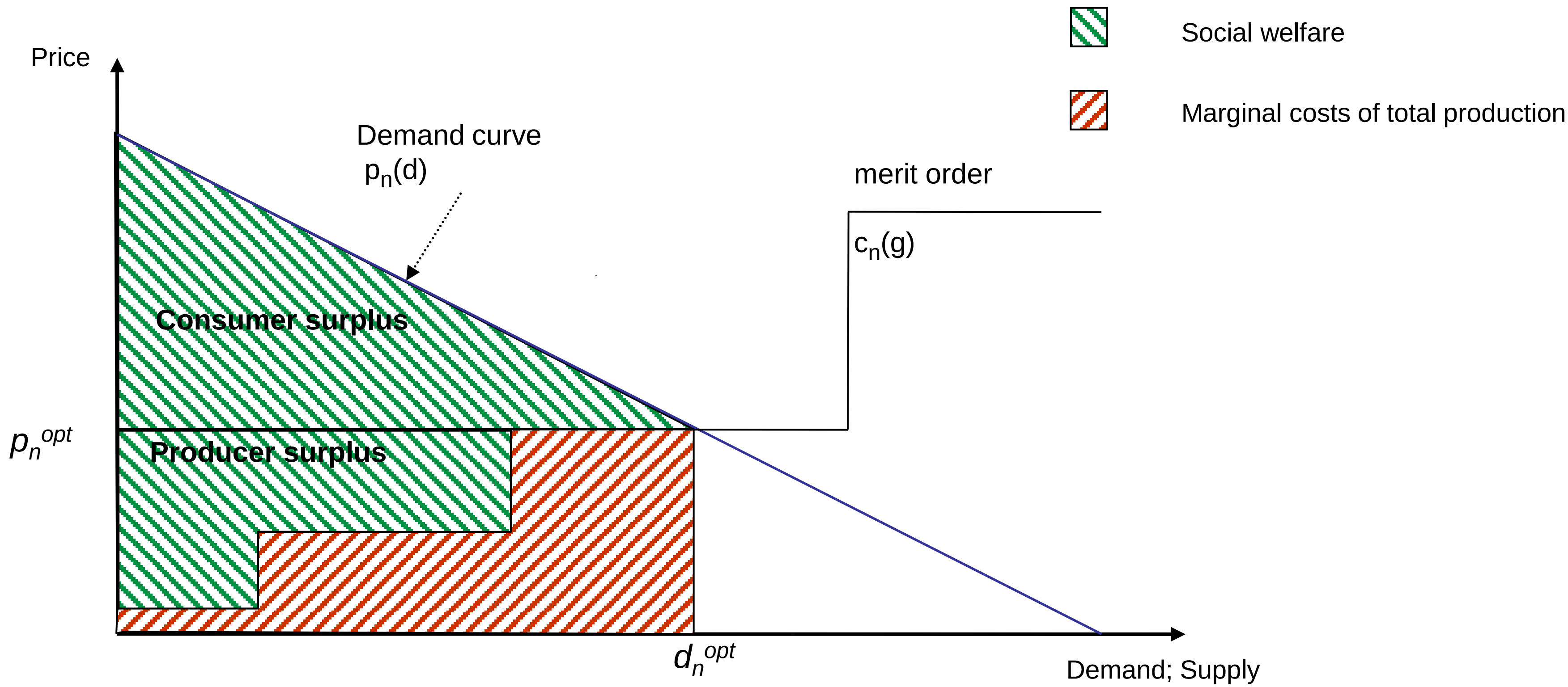
$$g_{n,u} \leq g_{n,u}^{max} \qquad \beta_{n,u} \geq 0 \qquad \forall\ n,u \qquad (3)$$

$$\sum_n H_{l,n}\delta_n \leq cap_l^{max} \qquad \overline{\mu}_l \geq 0 \qquad \forall\ l \qquad (4)$$

$$-\sum_n H_{l,n}\delta_n \leq cap_l^{max} \qquad \underline{\mu}_l \geq 0 \qquad \forall\ l \qquad (5)$$

$$\delta_{n'} = 0 \qquad \gamma \text{ (free)} \qquad (6)$$

Objective
Welfare Maximization



$$\max_{g,d,\delta} \sum_n \left(a_n \cdot d_n - \tfrac{1}{2} b_n \cdot d_n^2\right) - \sum_u c_u^m \cdot g_{n,u} \qquad (1)$$

Market Clearing Constraint or Nodal Energy Balance

- **Main characteristics of electricity**
  - **Non storable**
  - **Grid-bounded**

    → **Supply has to be equal to demand**

    → **Exchange between system nodes through transmission network**

$$\max_{g,d,\delta} \sum_n (a_n \cdot d_n - \tfrac{1}{2} b_n \cdot d_n^2) - \sum_u c_u^m \cdot g_{n,u} \qquad (1)$$

$$d_n - \sum_u g_{n,u} - \sum_k B_{n,k}\delta_k = 0 \qquad \lambda_n \text{ (free)} \qquad \forall\, n \qquad (2)$$

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$$\delta_{n'} = 0 \qquad \gamma \text{ (free)} \qquad (6)$$

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Technical Constraints: Generation

- **Generation capacity can be classified into**
  - **Maximum generation capacity**
  - **Minimum generation capacity (→ not relevant here)**

$$\max_{g,d,\delta} \sum_n (a_n \cdot d_n - \tfrac{1}{2} b_n \cdot d_n^2) - \sum_u c_u^m \cdot g_{n,u} \tag{1}$$

$$d_n - \sum_u g_{n,u} - \sum_k B_{n,k}\delta_k = 0 \qquad \lambda_n \text{ (free)} \qquad \forall\, n \tag{2}$$

$$g_{n,u} \leq g_{n,u}^{max} \qquad \beta_{n,u} \geq 0 \qquad \forall\, n, u \tag{3}$$

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$$\delta_{n'} = 0 \qquad \gamma \text{ (free)} \tag{6}$$

Technical Constraints: Load Flow
Transition to DC-Load Flow

Assumptions

1. Neglecting reactive power flows
2. Small voltage angles
3. Standardization of node voltages to respective voltage level

Power flow P on line i from node k to node m

b_{km} Series susceptance of line i from node k to m

Θ_{km} Phase angle of voltages U_k and U_m

$$P_{km} = b_{km} \cdot \Theta_{km}$$

Losses P_L on line i from node k to node m

r_{km} Series resistance of the line

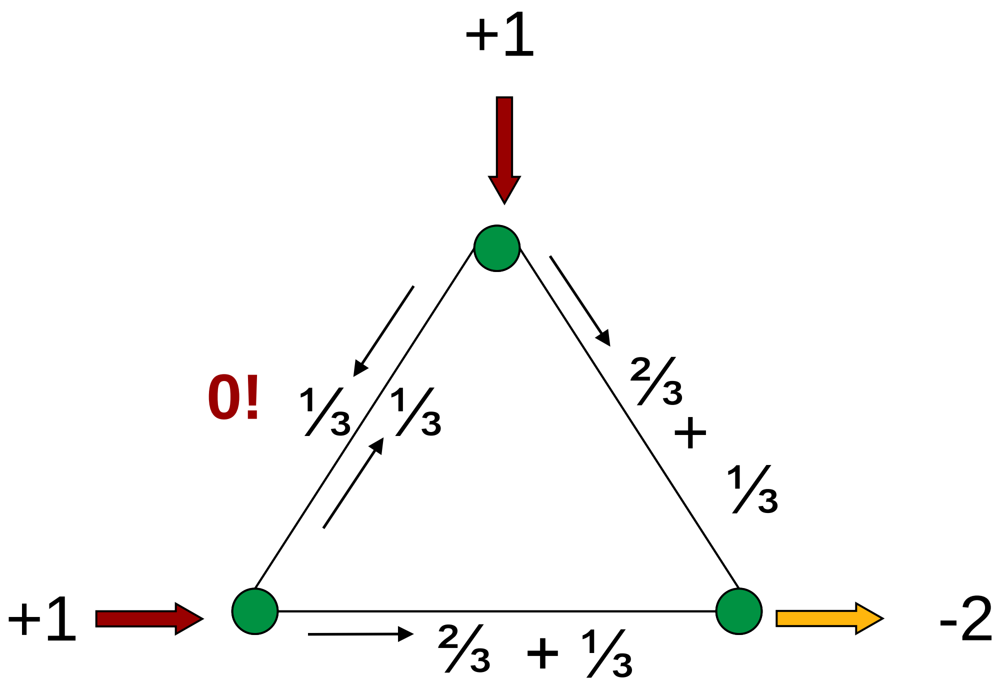
$$P_{L\,km} = r_{km} \cdot P_{km}^2$$

Technical Constraints: Load Flow Summary

$$\max_{g,d,\delta} \sum_{n} (a_n \cdot d_n - \tfrac{1}{2} b_n \cdot d_n^2) - \sum_{u} c_u^m \cdot g_{n,u} \quad (1)$$

$d_n - \sum_{u} g_{n,u} - \sum_{k} B_{n,k} \delta_k = 0$	λ_n (free)	$\forall\ n$	(2)
$g_{n,u} \leq g_{n,u}^{max}$	$\beta_{n,u} \geq 0$	$\forall\ n, u$	(3)
$\sum_{n} H_{l,n} \delta_n \leq cap_l^{max}$	$\overline{\mu}_l \geq 0$	$\forall\ l$	(4)
$-\sum_{n} H_{l,n} \delta_n \leq cap_l^{max}$	$\underline{\mu}_l \geq 0$	$\forall\ l$	(5)
$\delta_{n'} = 0$	γ (free)		(6)

Technical Constraints: Load Flow
3-Node Example



- **According to the characteristics of the transmission lines, the flow over a meshed network is distributed following Kirchoff‘s and Ohm‘s Law**

Model Formulation as an Optimization Problem

$$\max_{g,d,\delta} \sum_n (a_n \cdot d_n - \tfrac{1}{2} b_n \cdot d_n^2) - \sum_u c_u^m \cdot g_{n,u} \quad (1)$$

$$d_n - \sum_u g_{n,u} - \sum_k B_{n,k}\delta_k = 0 \qquad \lambda_n \text{ (free)} \qquad \forall\, n \quad (2)$$

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$$\delta_{n'} = 0 \qquad \gamma \text{ (free)} \quad (6)$$

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Lagrangian Function

$$
\begin{aligned}
L = & -\left(\sum_n (a_n \cdot d_n - \tfrac{1}{2} b_n \cdot d_n^2) - \sum_u c_u^m \cdot g_{n,u}\right) \\
& + \sum_n \lambda_n \cdot \left(d_n - \sum_u g_{n,u} - \sum_k B_{n,k} \delta_k\right) \\
& + \sum_{n,u} \beta_{n,u} \cdot (g_{n,u} - g_{n,u}^{max}) \\
& + \sum_l \overline{\mu}_l \cdot \left(\sum_n H_{l,n} \delta_n - cap_l^{max}\right) \\
& + \sum_l \underline{\mu}_l \cdot \left(-\sum_n H_{l,n} \delta_n - cap_l^{max}\right) \\
& + \delta_{n'} \cdot \gamma
\end{aligned}
$$

Karush-Kuhn-Tucker Conditions

$$c_u^m - \lambda_n + \beta_{n,u} \geq 0 \quad \perp \quad g_{n,u} \geq 0$$

$$-a_n + b_n \cdot d_n + \lambda_n \geq 0 \quad \perp \quad d_n \geq 0$$

$$-\sum_k B_{k,n} \cdot \lambda_k + \sum_n H_{l,n} \cdot \overline{\mu}_l - \sum_n H_{l,n} \cdot \underline{\mu}_l + \gamma = 0 \quad \perp \quad \delta_n \text{ (free)}$$

$$d_n - \sum_u g_{n,u} - \sum_k B_{n,k} \delta_k = 0 \quad \perp \quad \lambda_n \text{ (free)}$$

$$-\sum_n H_{l,n} \cdot \delta_n + cap_l^{max} \geq 0 \quad \perp \quad \overline{\mu}_l \geq 0$$

$$+\sum_n H_{l,n} \delta_n + cap_l^{max} \geq 0 \quad \perp \quad \underline{\mu}_l \geq 0$$

$$-g_{n,u} + g_{n,u}^{max} \geq 0 \quad \perp \quad \beta_{n,u} \geq 0$$

$$\delta_{n'} = 0 \quad \perp \quad \gamma \text{ (free)}$$

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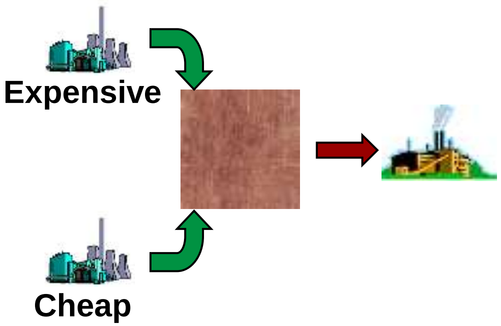
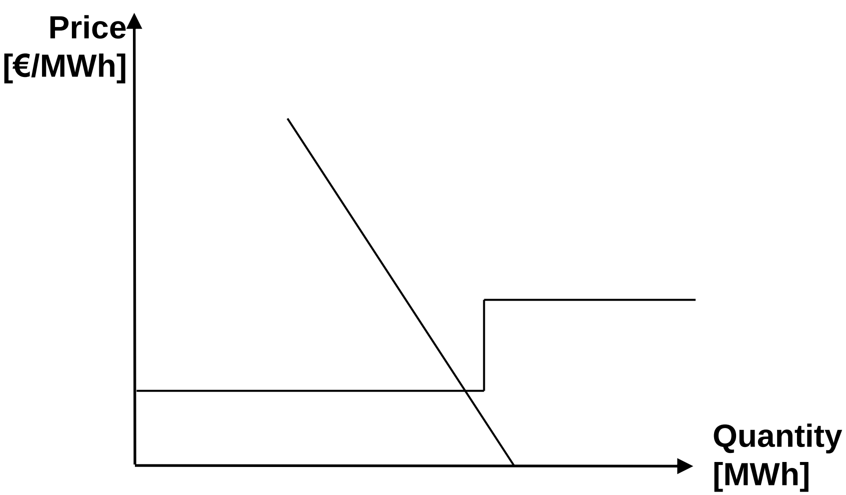
Literature

Problem: Power Flows follow Physics…

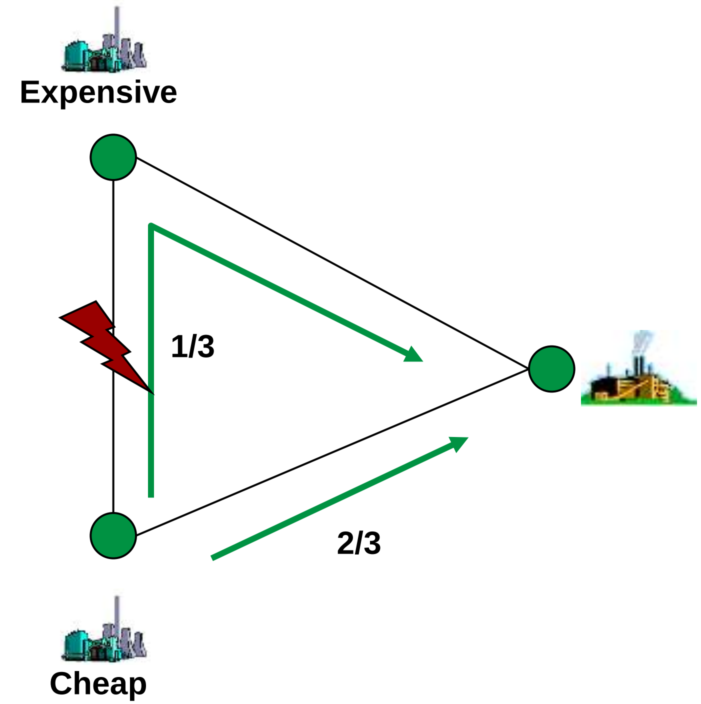
Typical market approach:
Copper Plate



Classical market clearing:



Power Flow Realization



The TSO has to ensure a reliable system operation even in case of congestion → congestion management

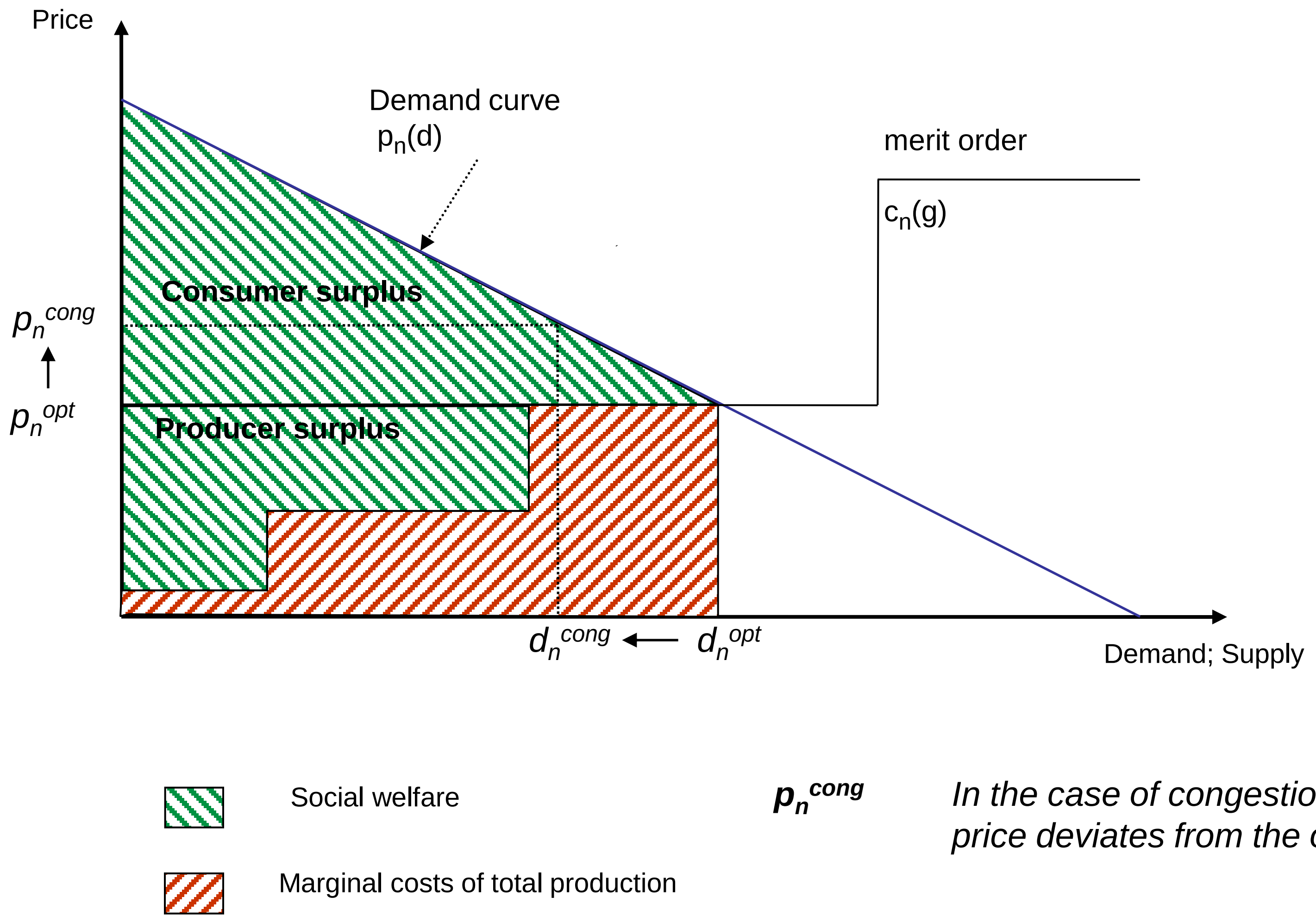
The Theory of Nodal Pricing

- **Nodal Pricing (often also referred to as Locational Marginal Pricing (LMP)):**
  - **there is a separate price for energy for each node in the network**
  - **containing cost of generation, losses and transmission („implicit auction“)**

- **Nodal Prices result from the cost:**
  - **for the supply of an additional MW(h) energy**
  - **at a specific node in the grid**
  - **while using the available least-cost generation unit(s)**
  - **subject to network constraints**

Nodal Price = Marginal Cost of Generation + Cost of Congestion + Cost of Marginal Losses

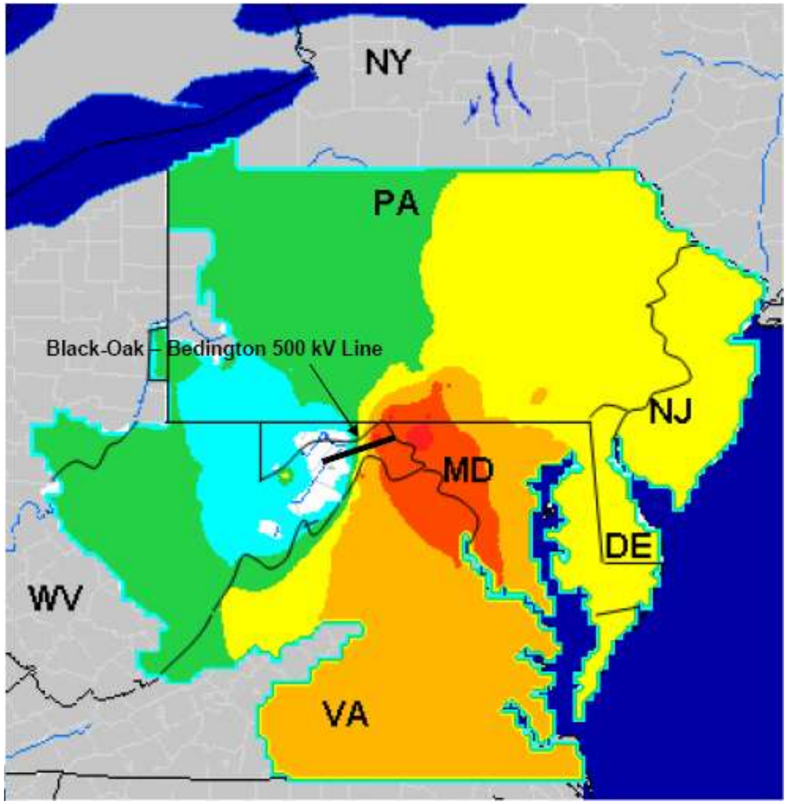
Impact on Objective Function



p_n^{cong} *In the case of congestion the nodal price deviates from the optimum*

The Realisation of Nodal Pricing
PJM (2005)

- **PJM (Pennsylvania, New Jersey, Maryland):**
- **biggest Independent System Operator (ISO) in the world**
- **134 GW peak load**
- **165 GW generation capacities**
- **728 TWh annual consumption**
- **56000 miles transmission lines**
- **164000 square miles territory**
- **including 13 states**
- **19% of US GDP produced in PJM**



Locational Price Distribution

- Source: Ott, 2005

Nodal vs Zonal Pricing

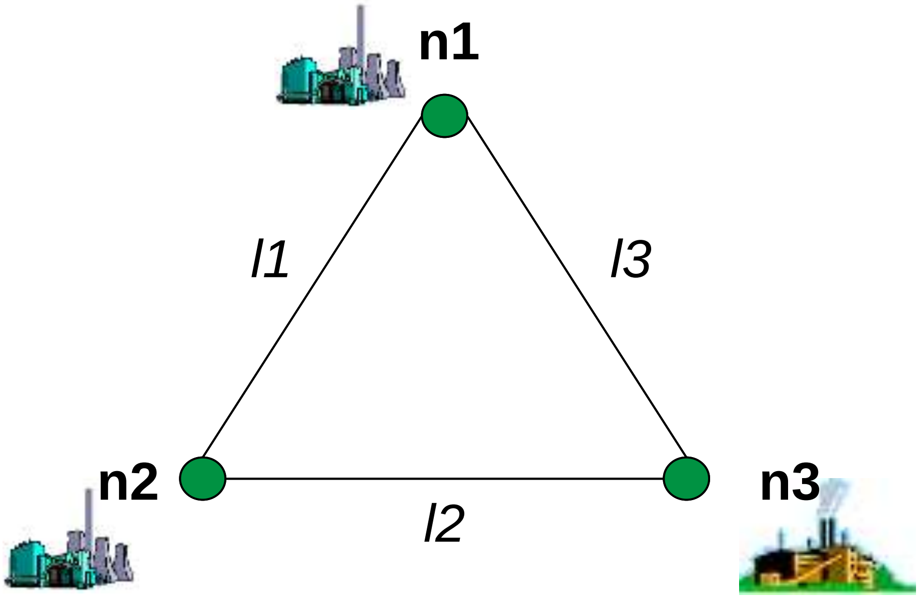
- **Nodal Pricing not applied in Europe**

- **European countries use zonal pricing**
  - **Price zones fixed and equal to country (e.g. Germany, Belgium, France)**
  - **Price zones fixed, but several zones within a country (e.g. Italy, Norway)**
  - **Price zones flexible according to network congestion → Nodal Pricing**

- **Implementation of zonal pricing in ELMOD**
  - **Additional restriction which ensures equality of prices with a price zone**
  - **a(n) + b(n)*q(n) =e= p(z) forall nodes n in zone z**

Agenda

Exercise
3-Node Network



	l1	l2	l3
cap^{max}_l	10	10	10
$resistance_l$	0.1	0.1	0.1
$reactance_l$	1	1	1

	n1	n2	n3
a_n	1	1	10
b_n	1	1	1
$gen^{max}_{n,u1}$	10 MWh	--	--
$gen^{max}_{n,u2}$	--	10 MWh	--
$gen^{max}_{n,u3}$	--	10 MWh	--
$c_{n,u1}$	2 €/MWh	--	--
$c_{n,u2}$	--	1 €/MWh	--
$c_{n,u3}$	--	3 €/MWh	--

Source: Gabriel & Leuthold (2010)

Exercise
3-Node Network

OPEN GAMS

OPEN OWS_3N_elmod.gms

Exercise
3-Node Network

- **Adjust the capacity of transmission lines!**
- **Analyze the impact on model results (prices, demand, generation)!**

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
cap^{max}_{l1}	**3**	**2**	10	10	**2**	**2**
cap^{max}_{l2}	10	10	**6**	**5**	**6**	**5**
cap^{max}_{l3}	10	10	10	10	10	10

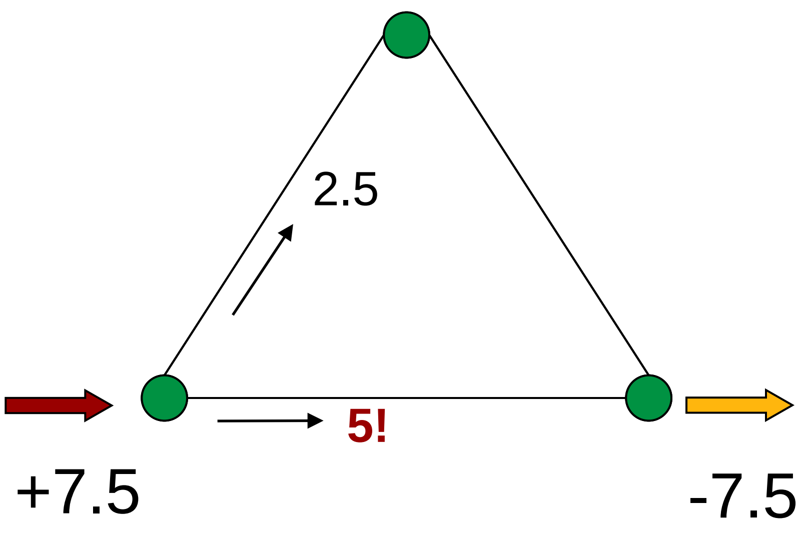
Exercise
3-Node Network

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
cap^{max}_{l1}	**3**	**2**	10	10	**2**	**2**
cap^{max}_{l2}	10	10	**6**	**5**	**6**	**5**
cap^{max}_{l3}	10	10	10	10	10	10

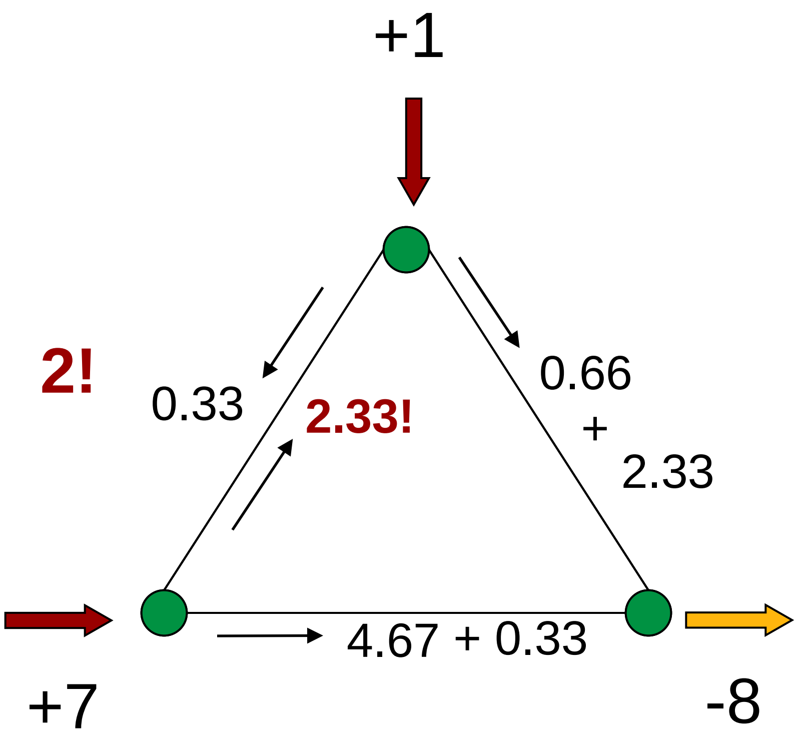
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
$flow_{l1}$	**3**	**2**	3	2.5	**2**	**2**
$flow_{l2}$	-6	-5.25	**-6**	**-5**	-5.25	**-5**
$flow_{l3}$	-3	-3.25	3	-2.5	-3.25	-3
$cons_{n3}$	9	8.5	9	7.5	8.5	8
$price_{n3}$	1	1.5	1	2.5	1.5	2

Exercise
3-Node Network

Case 4



Case 6



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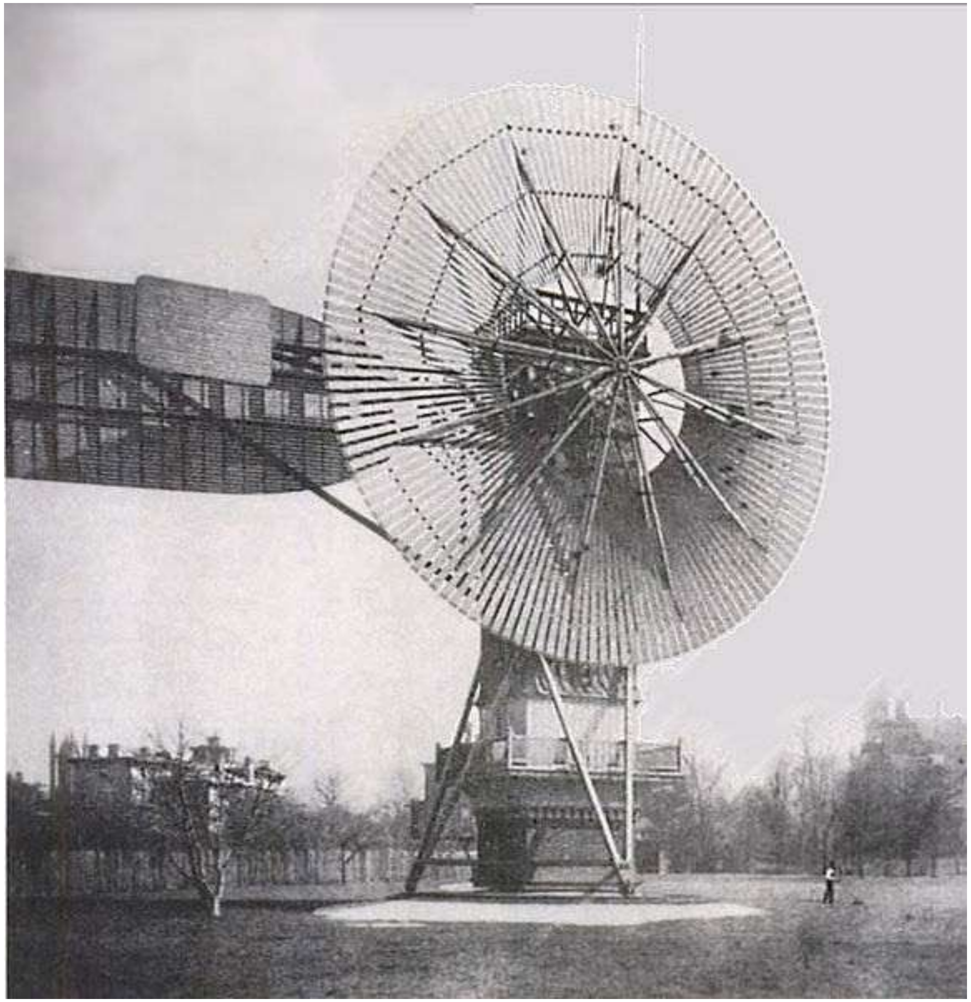
Literature

Wind mills in medieval times

14th century windmill; http://en.wikipedia.org/wiki/History_of_wind_power

Charles F. Brush's windmill (built in 1887)



12kW, 17 meter diameter rotor; http://en.wikipedia.org/wiki/History_of_wind_power

Research wind turbines in the US (built in 1981)

NASA/DOE, 7.5 MW; http://en.wikipedia.org/wiki/History_of_wind_power

Probability distribution of wind speed

Weibull distribution (two parameters)

- **Probability distribution function**

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right), \quad x \geq 0$$

x...wind speed

β...shape parameter

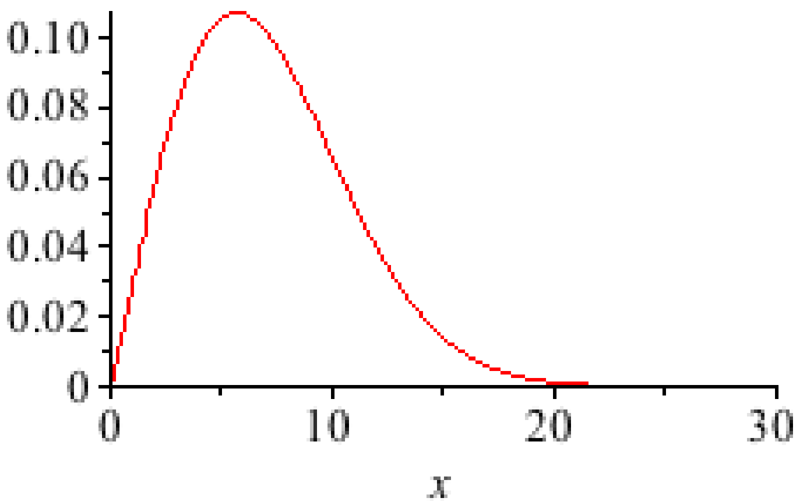
η...scale parameter

- **Cumulative distribution function**

$$F(x) = 1 - \exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right), \quad x \geq 0$$

- **Mean**

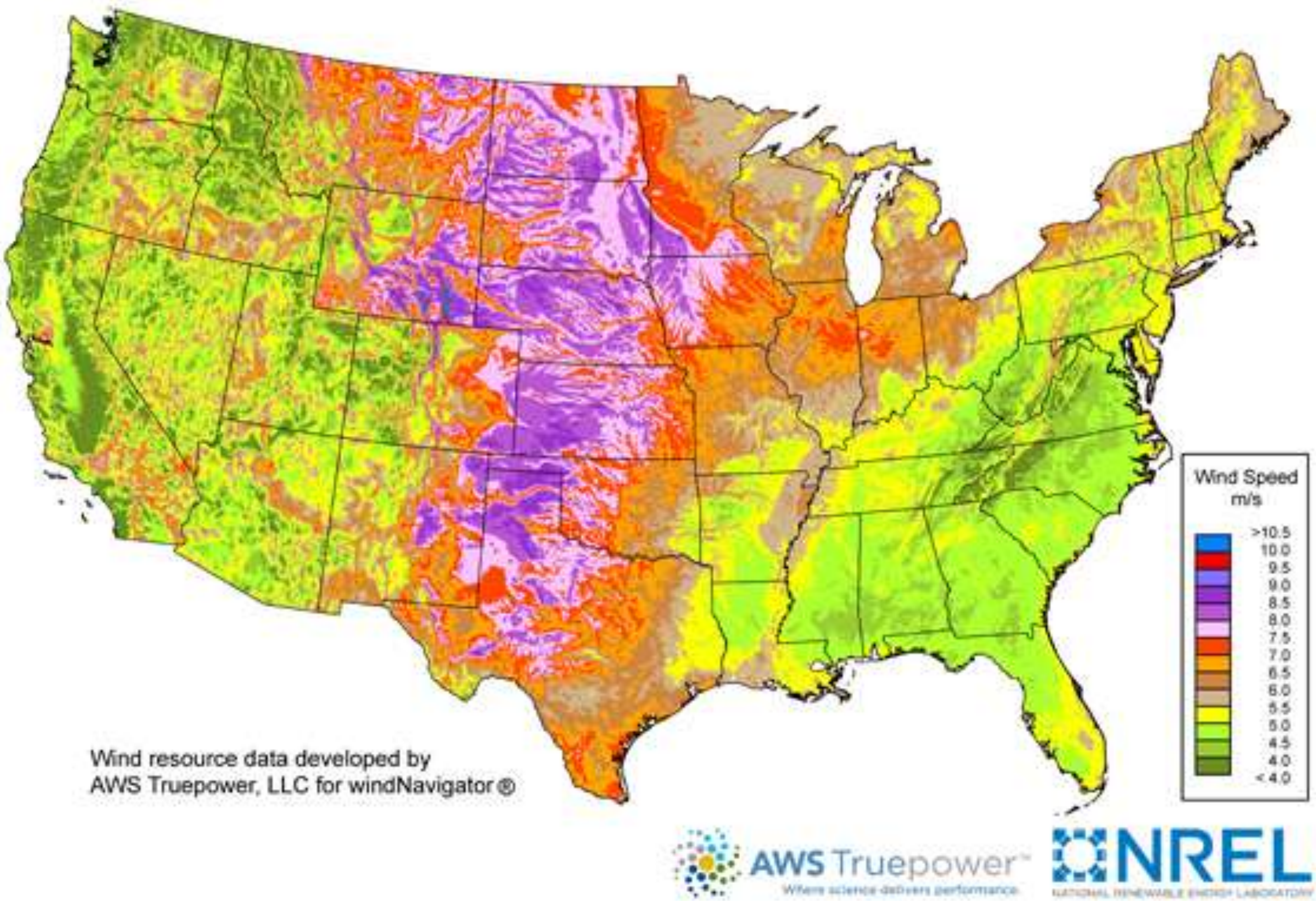
$$mean = \eta \cdot \Gamma\left(\frac{1}{\beta} + 1\right)$$



Probability distribution function
β = 2, η = 8

- **The Weibull distribution is commonly used for wind speed probability using a shape parameter β = 2 for Europe and North America**

Average wind speed for the USA



Source: http://www.windpoweringamerica.gov/wind_maps.asp

Power of wind

- **Newtons second law of motion:**

$$P = \frac{1}{2}\rho x^{3}\pi r^{2}$$

P...power of wind

ρ...density of dry air

x...wind speed

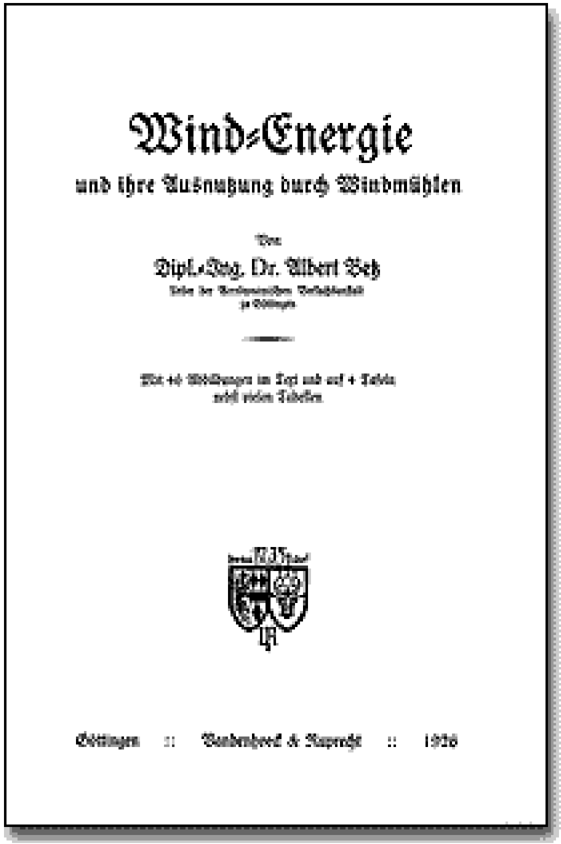
r...radius of the rotor

- **Betz law:**

  Formulated by German physicist Albert Betz in 1919

  Published „Wind-Energie“ in 1926

  „you can only convert less than 16/27 (or 59%) of the kinetic energy in the wind to mechanical energy using a wind turbine“


Wind-Energie
und ihre Ausnutzung durch Windmühlen
Von
Dipl.-Ing. Dr. Albert Betz
Göttingen :: Vandenhoeck & Ruprecht :: 1926

Danish Wind Industry Association – http://guidedtour.windpower.org/

Power Density Function

- **Distribution of wind power:**

  Probability of wind speed x **power of wind**

$$\frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}\exp\left(-\left(\frac{x}{\eta}\right)^{\beta}\right) \quad \times \quad \frac{1}{2}\rho x^{3}\pi r^{2}$$

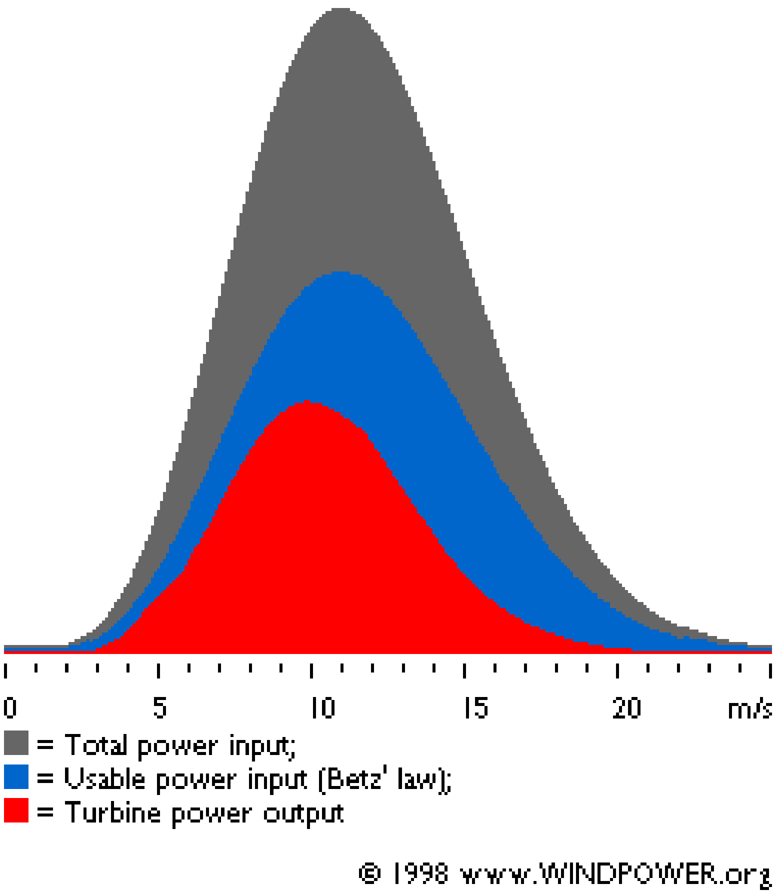
- **Important message:**

  Bulk of wind energy is found to the right of the mean of wind speed!

- **Further consideration:**

  ***Cut-in* and *cut-out* wind speed:**

  wind turbines cannot operate outside of a certain wind speed band (3-25 m/s)



Danish Wind Industry Association – http://guidedtour.windpower.org/

Some References on Electricity Data, Wind, etc.

- **References for Electricity Data**
  - **European Network of Transmission System Operators for Electricity**
    - **https://www.entsoe.eu/resources/data-portal/**
  - **EUROSTAT**
    - **http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/**

- **References for Wind Power Generation**
  - **Danish Wind Industry Association**
    - **http://guidedtour.windpower.org**
    - **http://www.talentfactory.dk/**
  - **US Department of Energy**
    - **http://www.windpoweringamerica.gov/**
    - **http://www.eere.energy.gov/**

Agenda

1. Introduction to Electricity Markets
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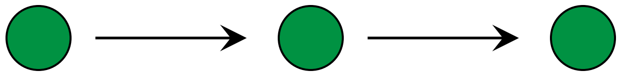
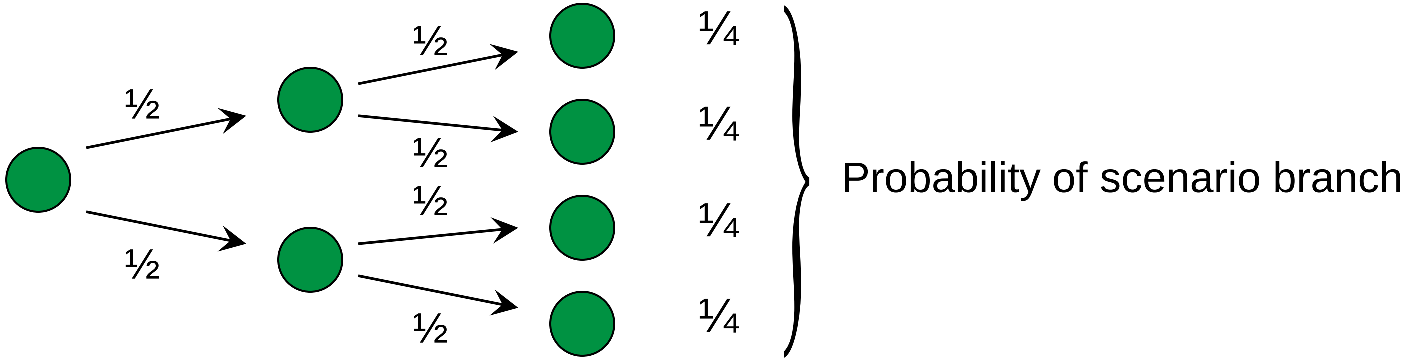
Literature

Extension to a multi-period-model

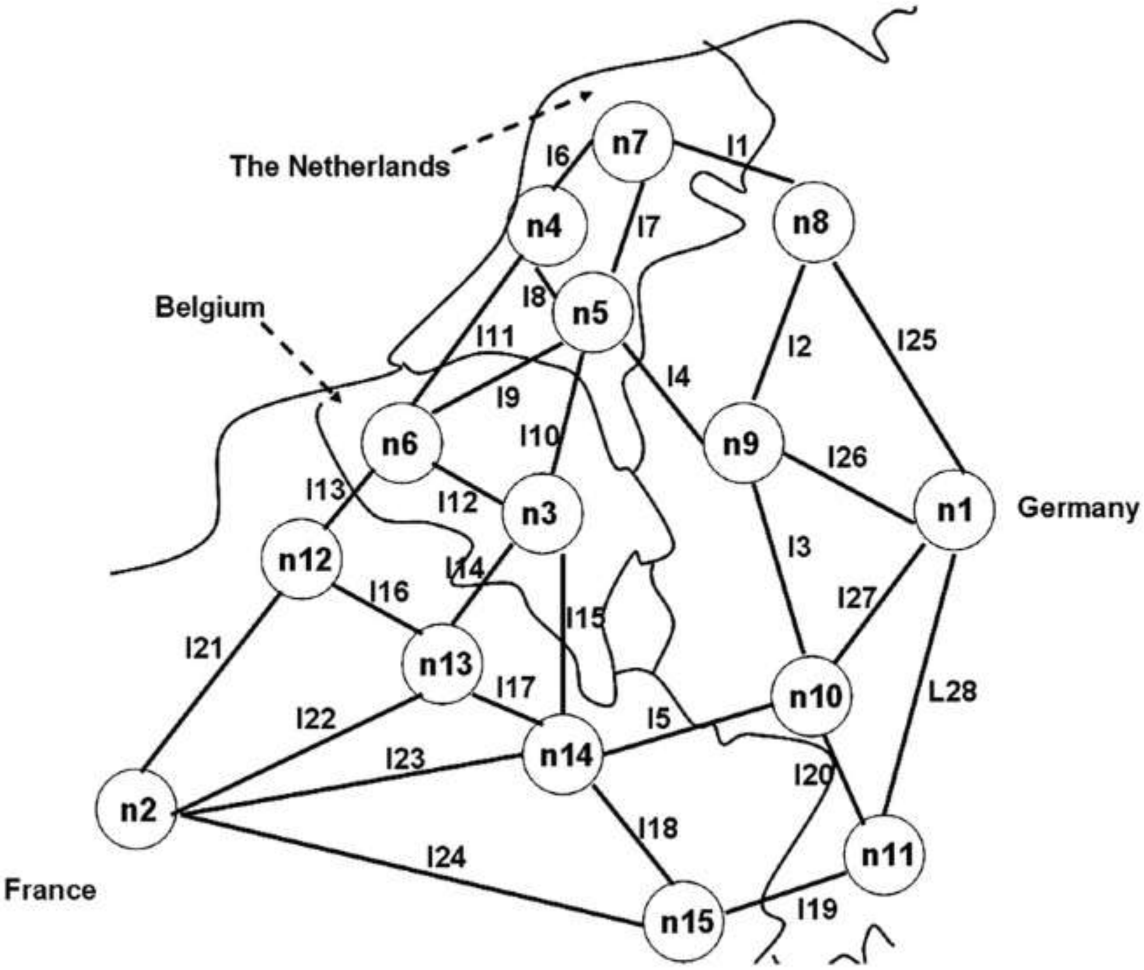
- **Additional characteristic of the model:**
  - **Ramp-up costs of power generation units**
- **Time-varying factors:**
  - **Demand (load curve)**
  - **Wind input**
- **Deterministic vs. stochastic optimization**
  - **Deterministic: future values of time-varying factors are known with certainty**



  - **Stochastic: scenarios of future values are known with respective probabilities**

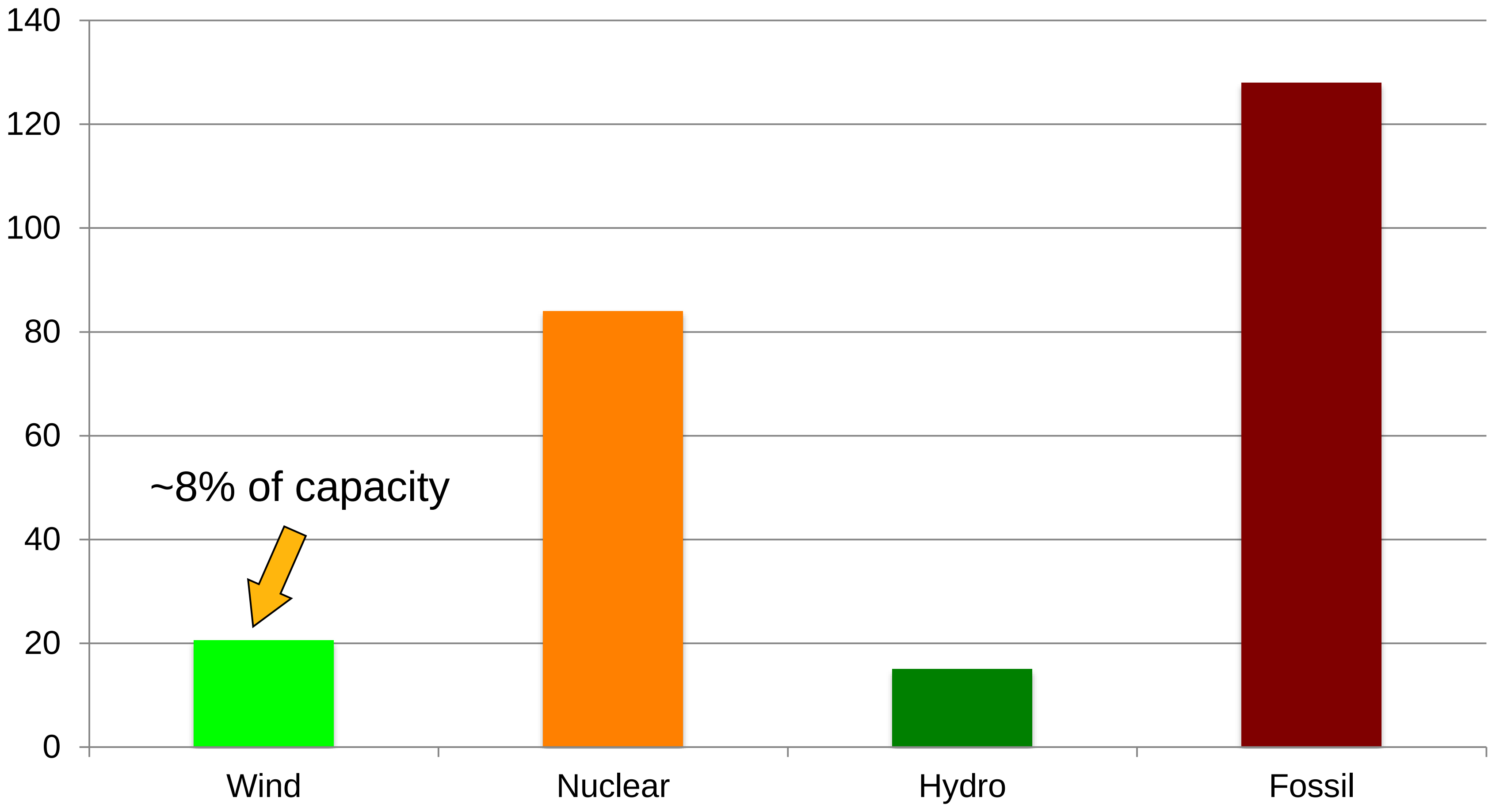


European Grid Representation (15 nodes)



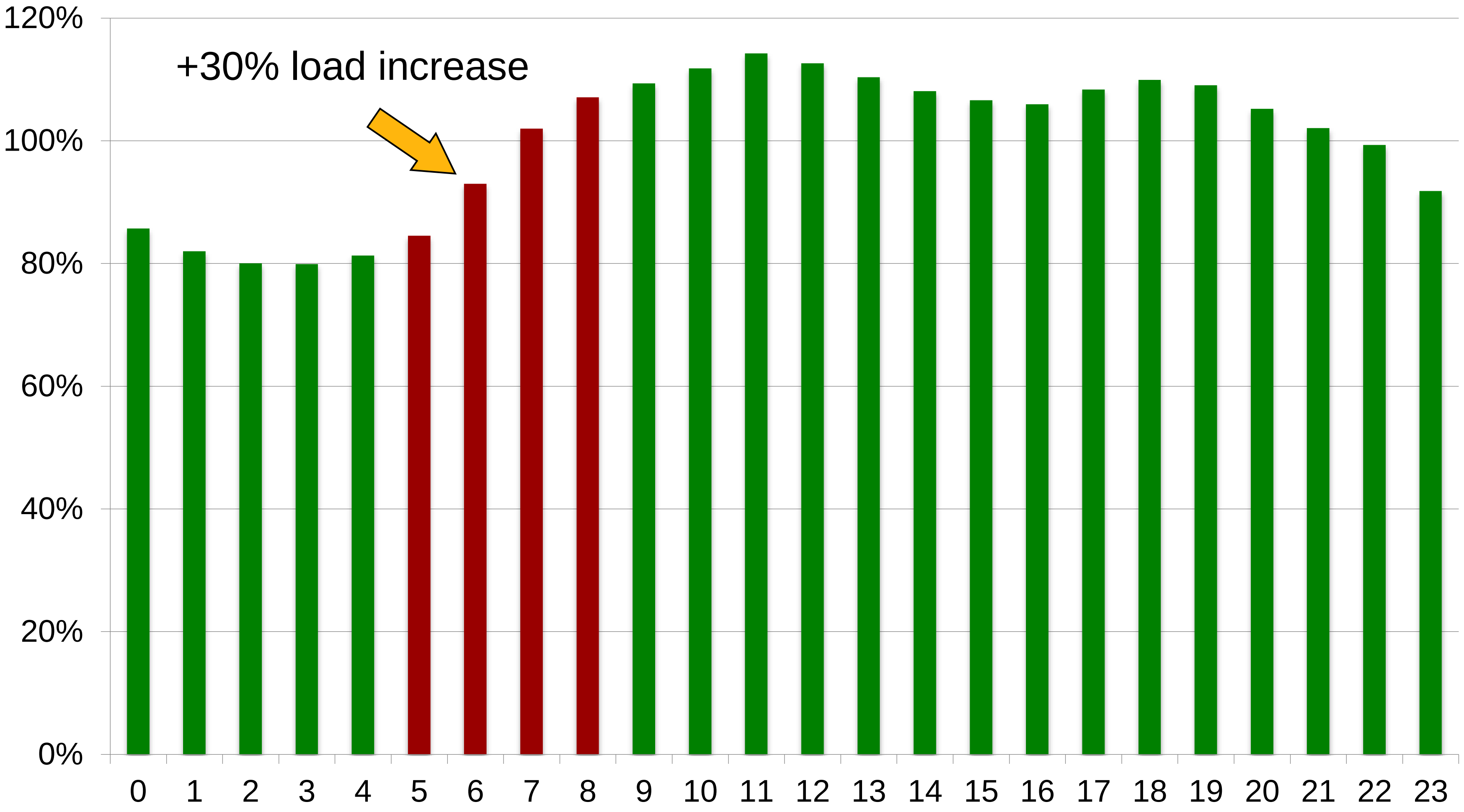
From Gabriel and Leuthold (2010), based on Neuhoff et al. (2005)

Maximum generation capacity



Maximum generation capacity in 15-node European grid example by fuel/unit

Typical load curve



Relative to daily average demand

GAMS Exercise: 15 Node European Network

- **Focus: time period 5 am – 9 am**
  - **Determine the optimal ramp-up decisions from the point of the ISO**
- **Assumptions:**
  - **Load curve exogenously given (deterministic)**
  - **Ramp-up at no cost in first period**
  - **Wind power must be fed into the grid**
  - **No wind generation at 5 am**
  - **Wind generation jumps discretely at the full hour**
  - **Wind generation relative to total capacity identical at each node**

Notation of the multi-period model (I)

Sets	n, k	... nodes
	n'	... swing bus
	u	... generation units (by fuel)
	l	... power lines
	s, ss	... scenarios
	$A(s)$	... set of ancestor scenario (i.e., previous scenario)
Parameters	a_n	... intercept of inverse demand function
	b_n	... slope of inverse demand function
	c_u^m	... marginal production costs of generation
	c_u^r	... ramp-up costs
	cap_l^{max}	... maximum thermal capacity of power lines
	$g_{s,n}^{wind}$	... wind generation (exogenous and obligatory feed-in)
	$prob(s)$	... probability of scenario
	$H_{l,k}$	... branch susceptance matrix
	$B_{n,k}$	... node susceptance matrix

Notation of the multi-period model (II)

Variables

$d_{s,n}$	...	electricity consumption
$g_{s,n,u}$	...	electricity generation
$g^{up}_{s,n,u}$	...	electricity generation ramp-up
$\delta_{s,n}$	...	phase angle (decision variable of ISO)
$\lambda_{s,n}$	...	dual for power balance constraint
$\beta_{s,n,u}$	...	dual for maximum generation capacity constraint
$\eta_{s,n,u}$	...	dual for ramp-up constraint
$\overline{\mu}_{s,l}$	...	dual for line capacity constraint (positive direction)
$\underline{\mu}_{s,l}$	...	dual for line capacity constraint (negative direction)
γ_s	...	dual for swing bus constraint

A stochastic multi-period welfare optimization problem

$$\max_{g,g^{up},d,\delta} \sum_{s,n} prob_s \cdot \left[(a_n \cdot d_{s,n} - \frac{1}{2} b_n \cdot d_{s,n}^2) - \sum_u c_u^m \cdot g_{s,n,u} - \sum_u c_u^r \cdot g_{s,n,u}^{up} \right] \qquad (1)$$

$$d_{s,n} - \sum_u g_{s,n,u} - g_{s,n}^{wind} + \sum_k B_{n,k}\delta_{s,k} = 0 \qquad \lambda_{s,n} \text{ (free)} \qquad \forall\ s,n \qquad (2)$$

$$g_{s,n,u} \leq g_{n,u}^{max} \qquad \beta_{s,n,u} \geq 0 \qquad \forall\ s,n,u \qquad (3)$$

$$g_{s,n,u} - g_{A(s),n,u} - g_{s,n,u}^{up} \leq 0 \qquad \eta_{s,n,u} \geq 0 \qquad \forall\ s,n,u \qquad (4)$$

$$\sum_n H_{l,n}\delta_{s,n} \leq cap_l^{max} \qquad \overline{\mu}_{s,l} \geq 0 \qquad \forall\ s,l \qquad (5)$$

$$-\sum_n H_{l,n}\delta_{s,n} \leq cap_l^{max} \qquad \underline{\mu}_{s,l} \geq 0 \qquad \forall\ s,l \qquad (6)$$

$$\delta_{s,n'} = 0 \qquad \gamma_s \text{ (free)} \qquad \forall\ s \qquad (7)$$

Karush-Kuhn-Tucker conditions

$$-prob_s \cdot c_u^m + \lambda_{s,n} - \beta_{s,n,u} - \eta_{n,u,s} + \sum_{ss \in A(s)} \eta_{n,u,ss} \leq 0 \quad \perp \quad g_{s,n,u} \geq 0 \qquad (8)$$

$$prob_s \cdot (a_n - b_n \cdot d_{s,n}) - \lambda_{s,n} \leq 0 \quad \perp \quad d_{s,n} \geq 0 \qquad (9)$$

$$-prob_s \cdot c_u^r + \eta_{n,u,s} \leq 0 \quad \perp \quad g_{s,n,u}^{up} \geq 0 \qquad (10)$$

$$\sum_{nn} B_{nn,n} \cdot \lambda_{s,nn} - \sum_l H_{l,n} \cdot \overline{\mu}_{s,l} + \sum_l H_{l,n} \cdot \underline{\mu}_{s,l} - \gamma_s = 0 \quad \perp \quad \delta_{s,n} \text{ (free)} \qquad (11)$$

$$d_{s,n} - \sum_u g_{s,n,u} - g_{s,n}^{wind} + \sum_k B_{n,k} \delta_{s,k} = 0 \quad \perp \quad \lambda_{s,n} \text{ (free)} \qquad (12)$$

$$g_{s,n,u} - g_{n,u}^{max} \leq 0 \quad \perp \quad \beta_{s,n,u} \geq 0 \qquad (13)$$

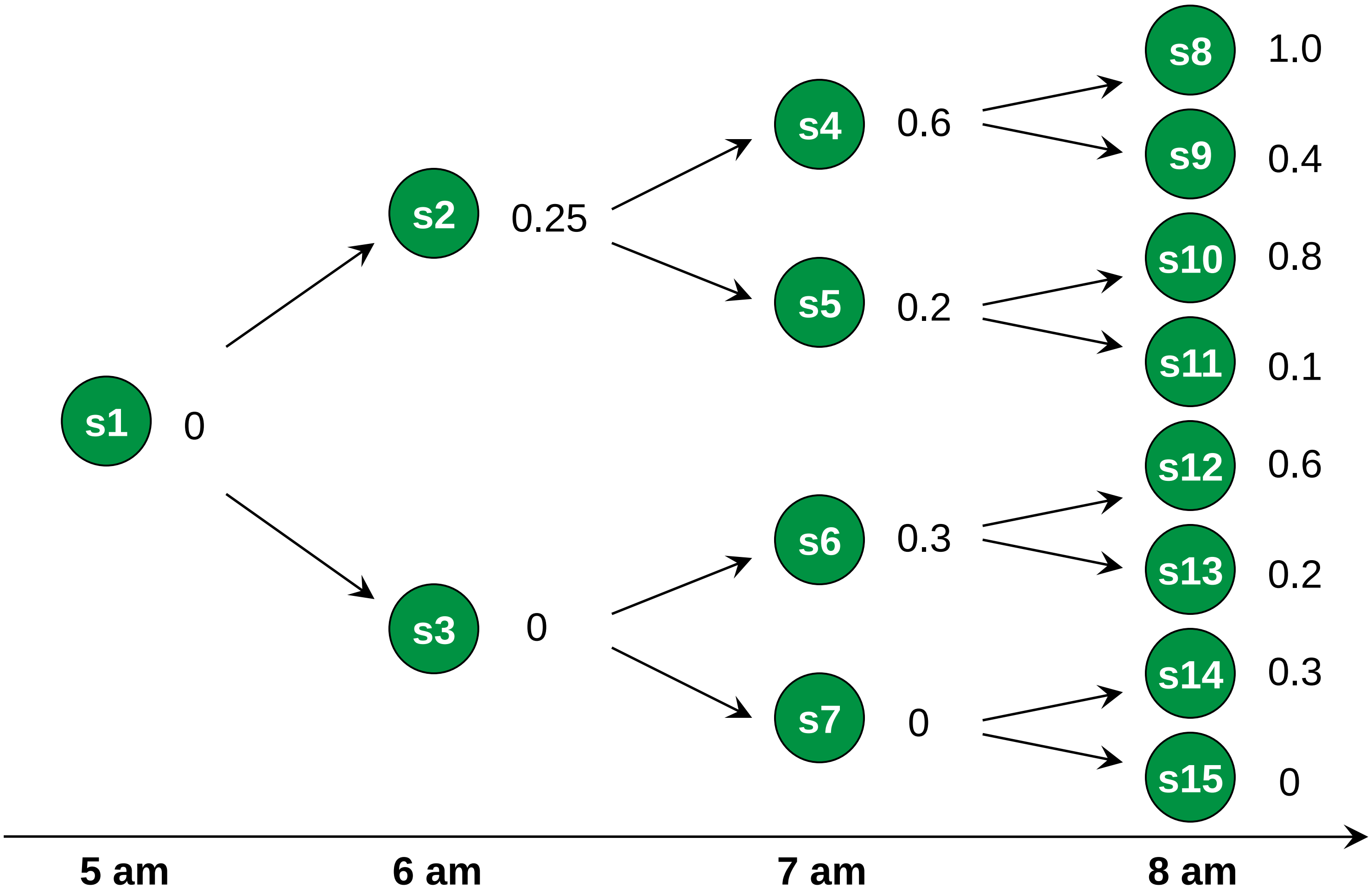
$$g_{s,n,u} - g_{A(s),n,u} - g_{s,n,u}^{up} \leq 0 \quad \perp \quad \eta_{s,n,u} \geq 0 \qquad (14)$$

$$\sum_n H_{l,n} \delta_{s,n} - cap_l^{max} \leq 0 \quad \perp \quad \overline{\mu}_{s,l} \geq 0 \qquad (15)$$

$$-\sum_n H_{l,n} \delta_{s,n} - cap_l^{max} \leq 0 \quad \perp \quad \underline{\mu}_{s,l} \geq 0 \qquad (16)$$

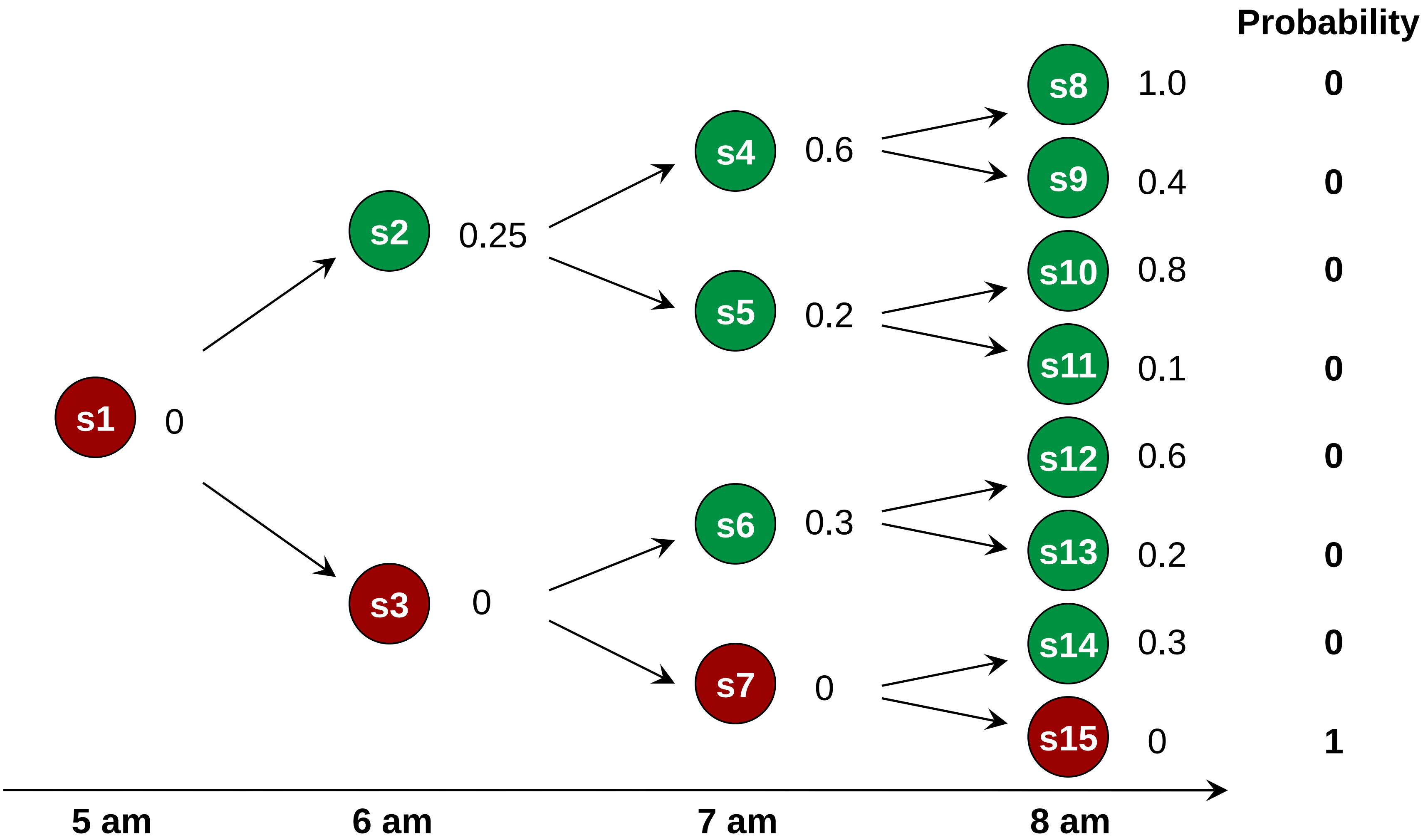
$$\delta_{s,n'} = 0 \quad \perp \quad \gamma_s \text{ (free)} \qquad (17)$$

Scenario tree – stochastic wind power generation



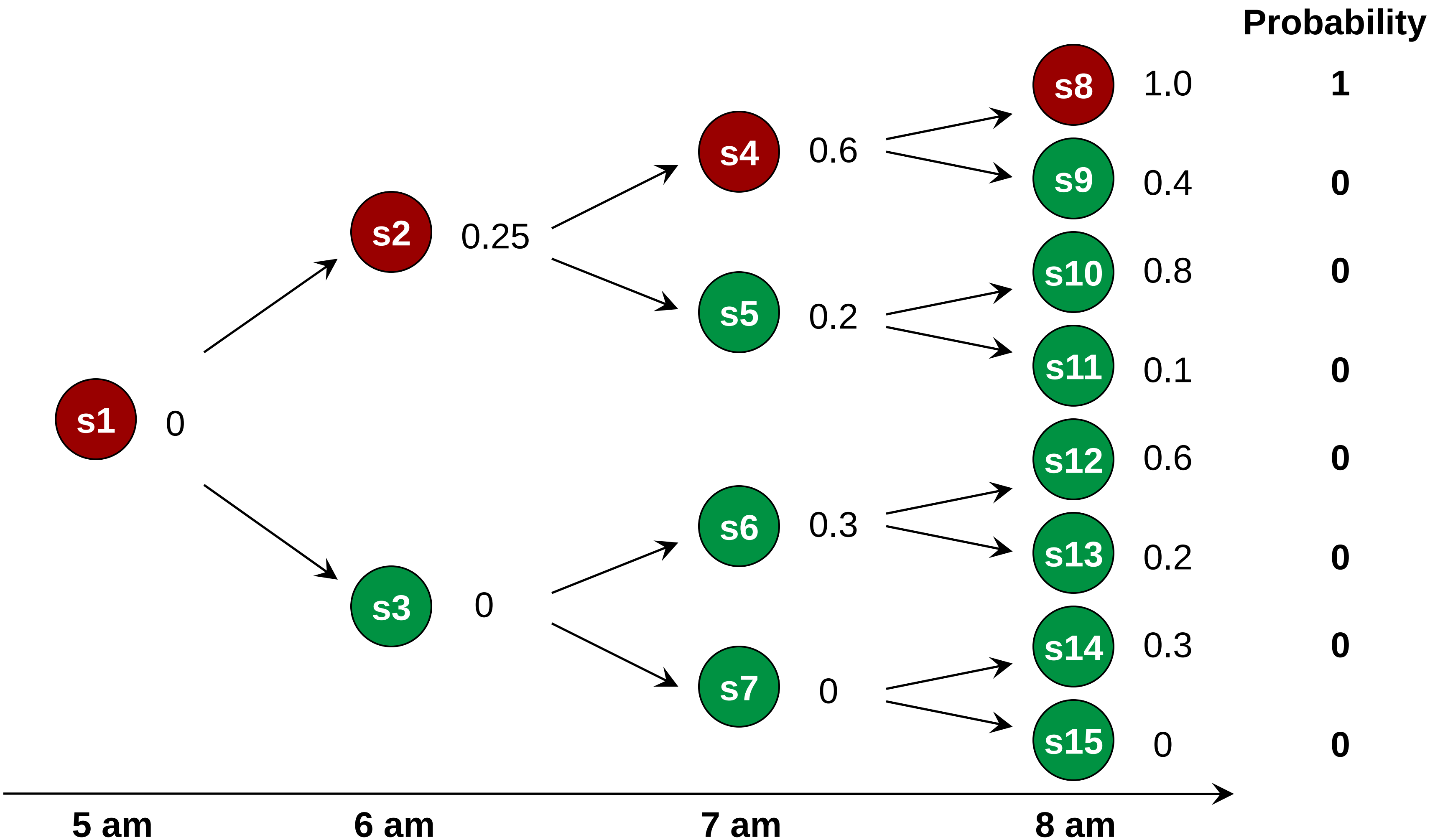
Scenario tree and respective wind power relative to maximum capacity

Deterministic optimization – no wind power generation



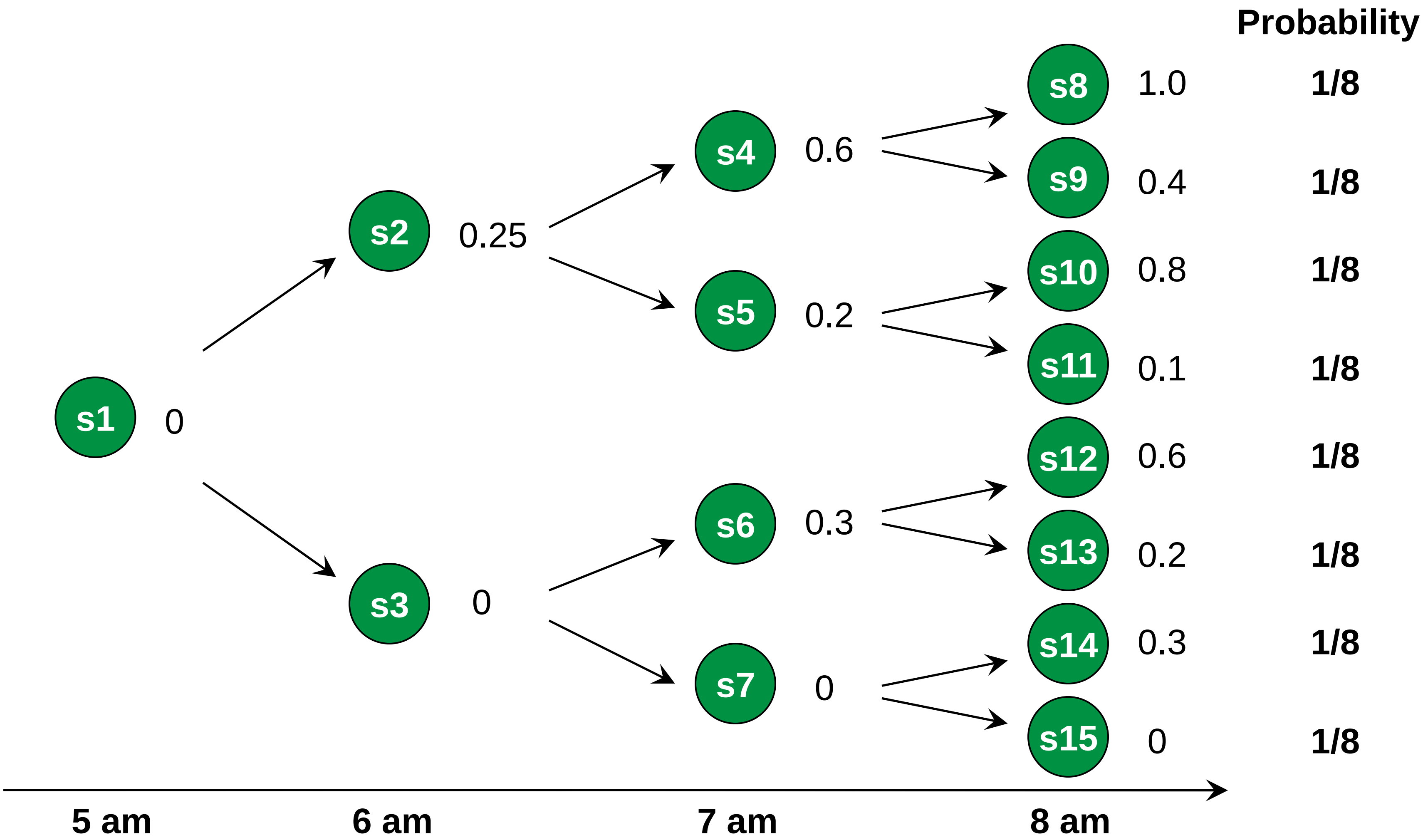
Scenario tree and respective wind power relative to maximum capacity

Deterministic optimization – full wind power at 9am



Scenario tree and respective wind power relative to maximum capacity

Stochastic Optimization – uniform probability

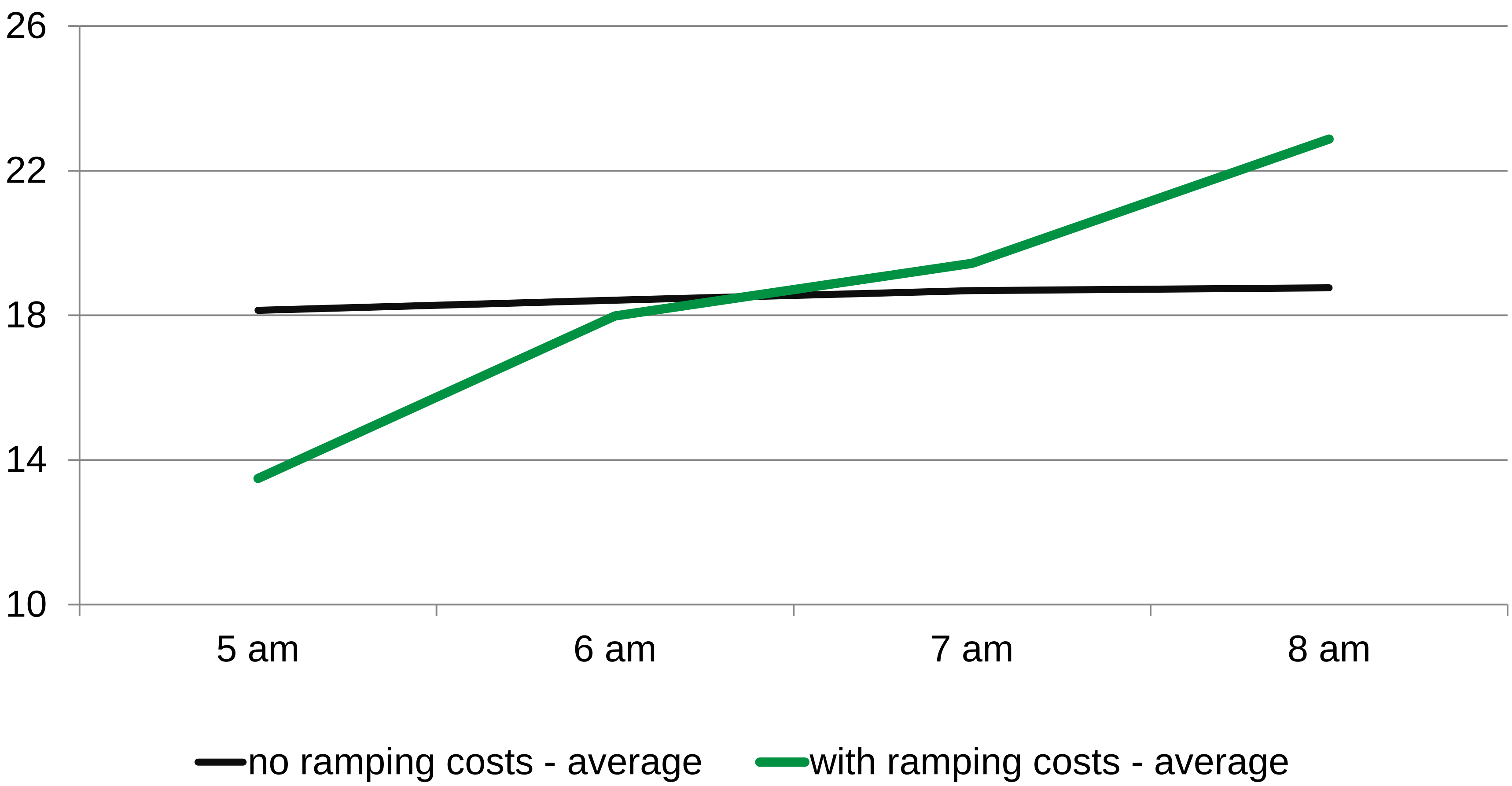


Scenario tree and respective wind power relative to maximum capacity

How to compare scenario simulation results?

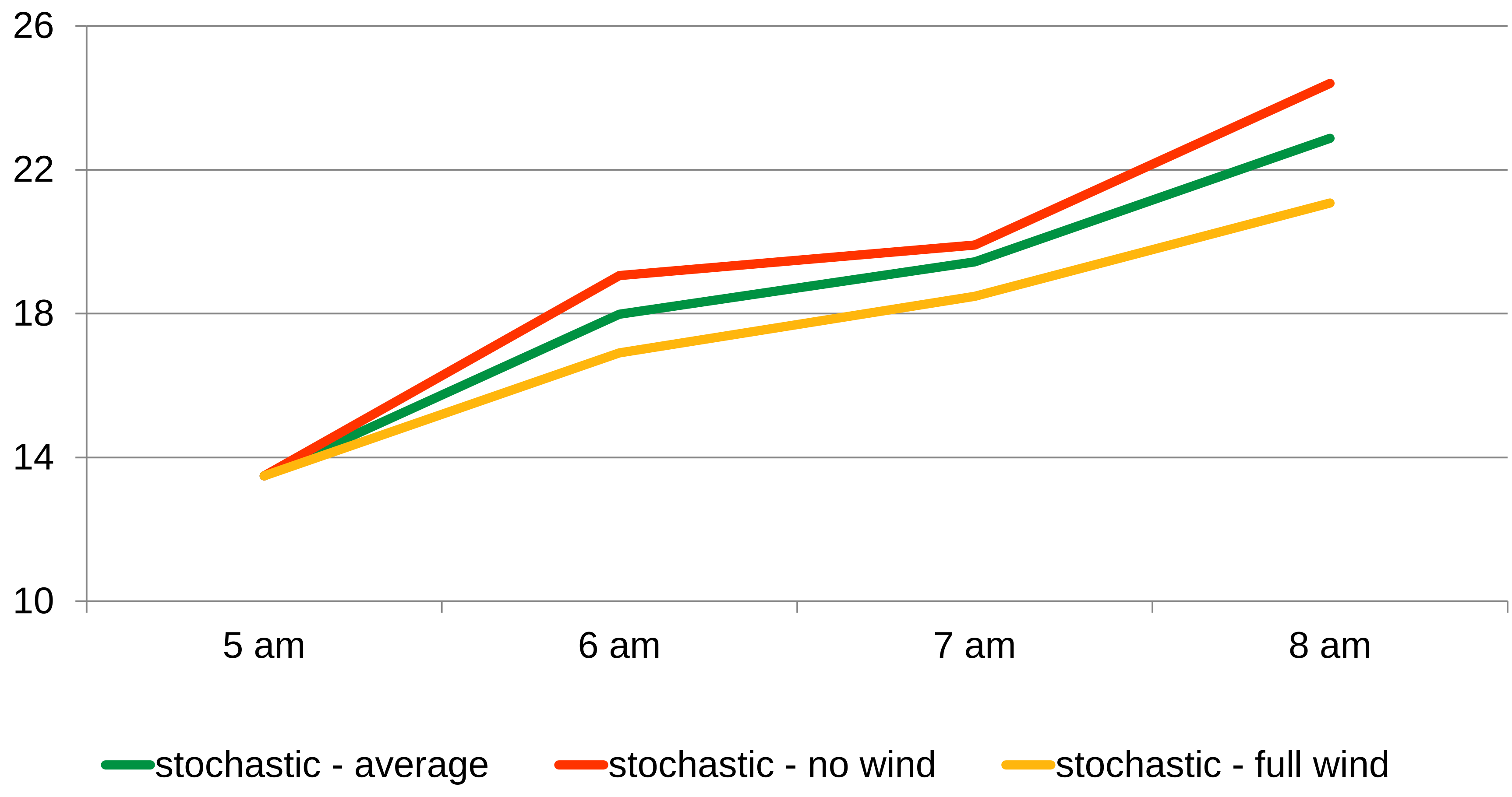
- **Objective value of optimization problem: welfare**
  - **Difficult to grasp this value intuitively**
- **Final demand or wholesale prices**
  - **The model is built on locational marginal prices, so there are no „prices“ similar to the prices observed in the real world**
- **Dual variables (shadow prices) to the energy balance constraint (λ)**
  - **Which nodes to compare?**
  - **How to weight results from different nodes?**
- **Consumption-weighted average of energy balance constraint duals**
  - **This is only an index and may hide big variations in welfare/shadow prices**

Results – stochastic model with vs. without ramping costs



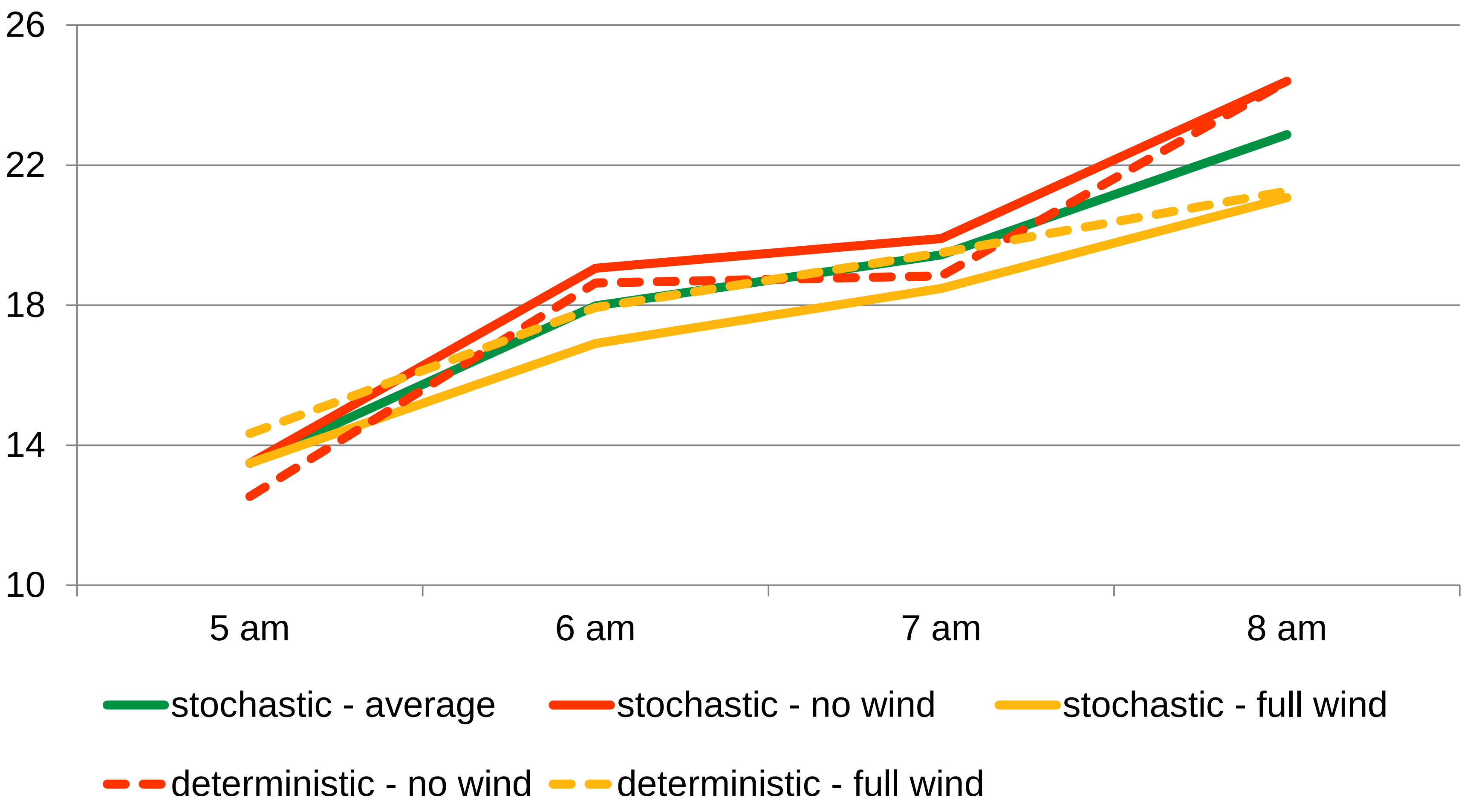
Consumption-weighted energy balance constraint dual (interpreted as price in €/MWh)

Results – variation within stochastic optimization



Consumption-weighted energy balance constraint dual (interpreted as price in €/MWh)

Results – deterministic vs. stochastic optimization



Consumption-weighted energy balance constraint dual (interpreted as price in €/MWh)

Conclusions: stochastic vs. deterministic optimization

- Ramp-up costs lead to lower costs at the beginning of the time horizon, as power plants are ramped up earlier
  - Watch out: there is a bias in this model due to zero ramp-up costs in the first period by assumption
- This effect is stronger in a deterministic no-wind scenario
- Higher wind input reduces prices (shadow prices to energy balance)
- Uncertainty leads to hedging by the ISO
- Prices converge in last period of deterministic vs. stochastic optimization

Exercise: Stochastic Multi-Period European Network

OPEN GAMS

OPEN OWS_EUR_elmod.gms

Possible Projects

- **Nice and easy start**
  - **Variation of ramping-costs**
  - **Variation of probabilities/wind power generation of scenarios**
  - **Analysing the impact of stochasticity on market results (determinisitic vs. Stochastic model setup, EVPI)**
- **Investment analysis (policy evaluation)**
  - **Expansion of wind generation capacity**
  - **Investment in new power lines**
- **Model horizon and data**
  - **Extension of observation period**
  - **Extension of scenario tree**
  - **Norwegian grid representation**
- **Model developments**
  - **Implementation of endogenous pumped-hydro storage dispatch**
  - **Implementation of zonal pricing**

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A better representation of ramping

- **In our example, ramping costs are...**
  - **Proportional to the level of ramped-up generation**
  - **Not related to the duration of down-time (i.e., cold-start vs. hot start)**

- **A more realistic representation would be possible using binary variables**
  - **Introduce a variable to indicate whether unit u is running in period t**
  - **Associate costs with this binary variable in the objective function**
  - **May introduce further technical/operational constraints**

    such as minimum up-time requirement after ramping

- **Mathematically, this leads to a Mixed Integer Problem (MIP)**
  - **More sophisticated and complex, considerably longer run-time of computation**

    # binary variables = # time steps x **# units** x **# nodes ...**

Market Power

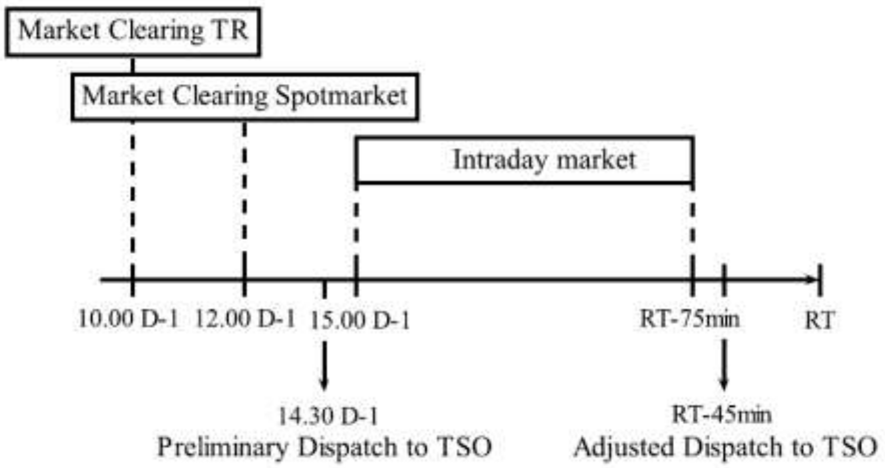
- **In our example, we assumed perfect competition as well as welfare-optimal dispatch and congestion management**

- **One could consider Cournot market power...**
  - **Simultaneous-move game by all generators**
  - **Easily applicable in a Mixed Complementarity Problem (MCP) framework by adding the conjectural variation in the KKT's of the suppliers**

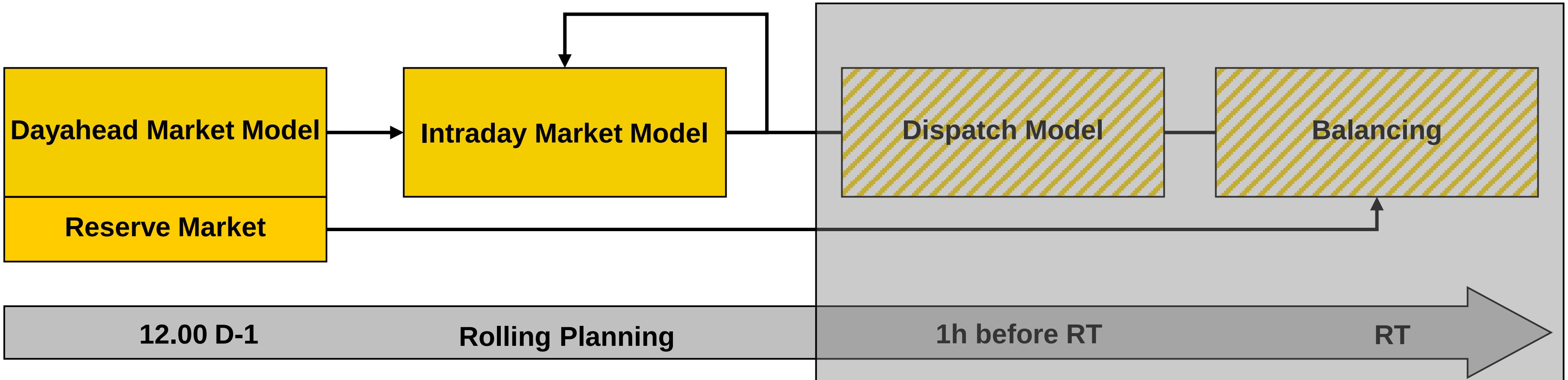
- **One could consider Stackelberg market power...**
  - **Sequential-move game: a Stackelberg leader optimizes under the constraint of an equilibrium in the market**
  - **Mathematically leads to a Mathematical Problem under Equilibrium Constraints (MPEC)**

Daily German Electricity Markets

- 12.00: Dayahead market (Spotmarket)
  - Central auction at EEX
  - Clearing for 24h of following day
- 14.30: Preliminary dispatch timetable
  - § 5 (1) StromNZV
- 15:00: Start of intraday market
  - Bilateral or standardized (EEX)
  - Closure of market RT-75min
- RT-45min: Final dispatch timetables
  - § 5 (2) StromNZV
  - Management of network constraints
- RT: Balancing of unexpected deviations



Modelling Approach



- 24h
- UC & dispatch of power plants

- Time variable (24h)
- UC & dispatch given restrictions of dayahead and previous intraday market

- Redispatch

- Hourly
- Dispatch of reserve capacity

- Initial wind forecast for delivery day

- Arrival of new wind forecasts

- Arrival of new wind forecasts

- Realization of wind generation

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Dayahead Market Model

Given: wind forecast, (past power plant plans)

Decide about: plant status, generation, reserve provision, storage use

min (Generation Cost + Startup Cost)

subject to:

Generation	=	Demand
Reserve Capacity	=	Reserve Demand
Generation	<=	Installed Capacity
Generation	>=	Minimum Generation (if online)
Offline Time	>=	Minimum Offline Time
Online Time	>=	Minimum Online Time

+ Storage restrictions, Wind Shedding

Intraday Market Model

Given: new wind forecast, current plant status, reserve capacities

Decide about: plant status, generation, reserve provision, storage use

min (Generation Cost + Startup Cost)

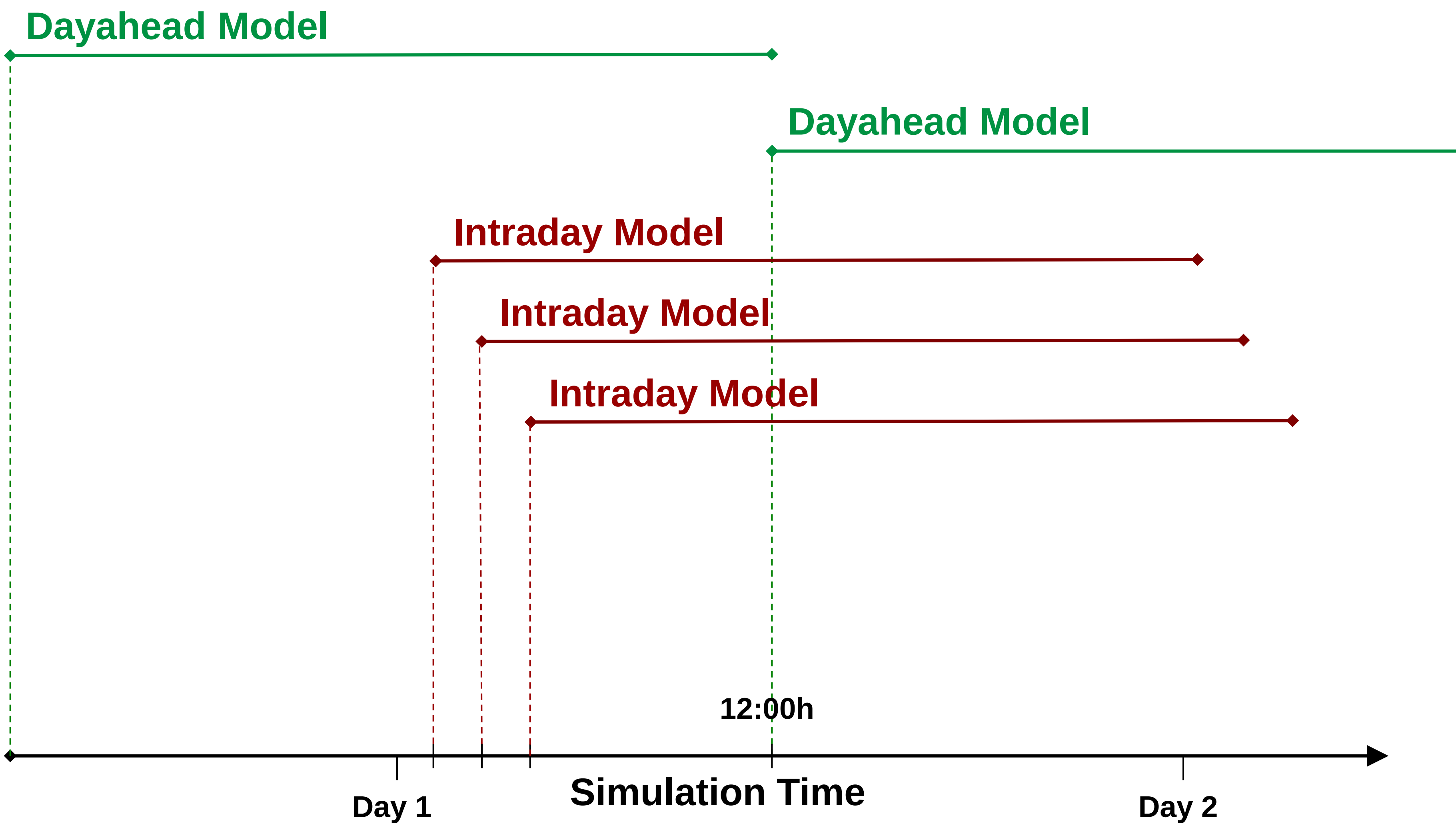
subject to:

Generation	=	Demand
Generation	<=	Installed Capacity (net of reserve)
Generation	>=	Minimum Generation (net of reserve)
Offline Time	>=	Minimum Offline Time
Online Time	>=	Minimum Online Time

+ Storage restrictions, Wind Shedding

+ Running requirements given by previous decisions (reserve, minimum times)

Rolling Planning



Intraday model run in hourly steps

Thank you very much
for your attention!
Any questions or comments?

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Literature

- **S.A. Gabriel and F.U. Leuthold. Solving discretely-constrained MPEC problems with applications in electric power markets. *Energy Economics*, 32(1):3–14, 2010.**
- **F.U. Leuthold, H. Weigt, and C. von Hirschhausen. A Large-Scale Spatial Optimization Model of the European Electricity Market. *Networks and Spatial Economics*, 2010.**
- **K. Neuhoff, J. Barquin, M.G. Boots, A. Ehrenmann, B.F. Hobbs, F.A. Rijkers, and M. Vázquez. Network-constrained cournot models of liberalized electricity markets: the devil is in the details. *Energy Economics*, 27(3):495 – 525, 2005.**
- **F.C. Schweppe, M.C. Caramanis, R.D. Tabors, and R. E. Bohn: *Spot Pricing of Electricity*. Kluwer, Boston, 1988.**
- **H. Stigler and C. Todem. Optimization of the Austrian Electricity Sector (Control Zone of VERBUND APG) under the Constraints of Network Capacities by Nodal Pricing. *Central European Journal of Operations Research*, 13:105–125, 2005.**