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Stochastic Pooling Problem for Natural Gas Production Network Design and Operation Under Uncertainty

Xiang Li
Dept. of Chemical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139

Emre Armagan
Dept. of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139

Asgeir Tomasgard
Dept. of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim 7491, Norway

Paul I. Barton
Dept. of Chemical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139

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Product quality and uncertainty are two important issues in the design and operation of natural gas production networks. This paper presents a stochastic pooling problem optimization formulation to address these two issues, where the qualities of the flows in the system are described with a pooling model and the uncertainty in the system is handled with a multi-scenario, two-stage stochastic recourse approach. In addition, multi-objective problems are handled via a hierarchical optimization approach. The advantages of the proposed formulation are demonstrated with case studies involving an example system based on Haverly's pooling problem and a real industrial system. The stochastic pooling problem is a potentially large-scale nonconvex Mixed-Integer Nonlinear Program (MINLP), and a rigorous decomposition method developed recently is used to solve this problem. A computational study demonstrates the advantage of the decomposition method over a state-of-the-art branch-and-reduce global optimizer, BARON. © 2010 American Institute of Chemical Engineers *AIChE J,* 00: 000–000, 2010
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Introduction

Natural gas is a vital component of the world's energy supply; as of 2007, it contributed around a fifth of global energy demand.[1] The importance of natural gas as a fossil fuel has been increasing in recent years because of different factors. First of all, natural gas is a less carbon-intense fuel than oil or coal, that is, its combustion produces less greenhouse emissions; it also produces relatively lower sulfur, NOx, and particulates emissions on combustion. In addition, as it is easy, cheap, and clean to convert into hydrogen, natural gas is considered to be one of the

Correspondence concerning this article should be addressed to P. I. Barton at pib@mit.edu.

most important elements in the transition to a hydrogen economy.

Raw natural gas consists primarily of methane (CH_4; usually 70–90%); the remaining components include varying amounts of heavier gaseous hydrocarbons such as ethane (C_2H_6), propane (C_3H_8), butane (C_4H_{10}), and so forth, and acid gases including carbon dioxide (CO_2) and hydrogen sulfide (H_2S), as well as nitrogen (N_2), helium (He), and water vapor.[2] Therefore, gas quality, as determined by the compositions of these components, plays an important role in natural gas production systems. Although the quality of gas produced by different reservoirs can vary over large ranges, the products produced by a natural gas production system must satisfy strict quality specifications because these products are usually sent to customers with little further processing. This is different from oil production systems in which the quality of the crude oil is usually of little concern, because it will be further processed into different products in downstream refineries. Therefore, in the design and operation of natural gas production systems, the qualities of the gas flows must be tracked throughout the entire system.

The development of natural gas production infrastructure involves large investments, and the infrastructure often remains in operation over the entire life span of the project (which can be several decades). Also, the natural gas industry has large turnovers and volumes. Hence, even small fractional performance gains made in the design and operation of natural gas production systems can translate into significant increases in profits. It is therefore not surprising that mathematical programming has been widely applied to the integrated design and operation problem arising from infrastructure development and long-term planning for natural gas production. When some information about the system is uncertain, before the decisions are implemented, the uncertainty should be described by the model and addressed in the optimization for profitability or even feasibility of the solution. One of the most important sources of uncertainty is the quality of the reservoirs, which can be known exactly only after gas wells into the reservoirs have been developed. Other important sources of uncertainty include the capacity of reservoirs, customer demands on the system, prices of the products, and so forth.

This article provides a general stochastic pooling problem formulation to address the qualities of gas flows and different sources of uncertainty in infrastructure development and long-term planning for natural gas production. This is the first study in the literature, to the best of our knowledge, of the integrated design and operation of natural gas production systems through a stochastic pooling framework.

The remaining part of the article is organized as follows. The previous relevant work is reviewed. Then, the stochastic pooling problem for the generalized pooling system is stated. Next, the deterministic and stochastic pooling models of the system are developed, and several economic objectives as well as a hierarchical multiobjective optimization approach are discussed. After that, a rigorous decomposition method is briefly described for solving the stochastic pooling problem. Case studies involving one example system and an industrial natural gas production system are presented to demonstrate the advantages of the proposed formulations and the decomposition method. The article ends with conclusions and discussions on future work.

Literature Review

This section gives a brief review of previous work on infrastructure development and long-term planning for oil and natural gas production systems, as well as the relevant approaches to address product quality and uncertainty in the problem. The work on oil production systems is reviewed because the modeling and solution strategies for oil and natural gas production problems are very similar, and only a little work on natural gas production systems has been published.

Although the application of mathematical programming in the oil and natural gas industry dates back to middle of the last century,[3] the application to infrastructure development and long-term planning during that period has been limited by the available computing capability. To ease the computational burden, the design problem and the operation problem were solved separately, for example, the design problem solved with fixed operation profile through linear programming (LP),[4] or the operation problem solved with fixed design through LP.[5] Efforts to solve the design and operation problem simultaneously emerged no later than the beginning of 1970s. Flanigan[6] presented a nonlinear programming (NLP) model for the design and operation of natural gas pipeline systems and proposed a specialized, derivative-based optimization algorithm to solve the problem. Bohannon,[7] Sullivan,[8] and Nygreen et al.[9] developed different mixed-integer linear programming (MILP) models for infrastructure development and production planning with different levels of details of the systems. Murray and Edgar[10] solved a well location problem with a simplified reservoir model via a MILP formulation and an approximating NLP formulation, respectively. McFarland et al.[11] used a simple tank model for reservoir dynamics to formulate an optimal control problem and solved it with a generalized reduced gradient method. Iyer et al.[12] approximated the nonlinear oil reservoir model with disjunctive linear models and solved the resulting large MILP problem with an iterative aggregation/disaggregation approach. Later on van den Heever and Grossmann[13] modeled the same problem using a nonlinear reservoir model and solved the resulting mixed-integer NLP (MINLP) problem with a Lagrangean decomposition heuristic, and complex economic objectives are incorporated in the same problem formulation in van den Heever et al.[14] Lin and Floudas[15] presented a continuous-time model framework for well platform planning problems and the resulting MINLP formulation is better than the one based on a discrete-time model.

The product quality, however, is not addressed in the work mentioned above. One reason for this is that most of the above work focuses on oil production systems, where the quality of the crude oil is not a key quantity to be controlled in the production system (as mentioned in the last section). Actually, oil product qualities are tracked in the downstream refineries where crude oils are mixed, separated, and reacted to produce intermediate quality streams that are then blended to produce the final products. This blending problem, usually called the pooling problem, has been systematically studied for decades.[16] The pooling problem is inherently nonconvex because of the bilinear terms in the model for tracking the qualities of the flows in the system, and it is considered a difficult optimization problem to solve reliably. (Note that a blending problem may include more complicated nonconvex terms, e.g., when a complex emission model is included,[17] but such complex problems are outside the scope of this article.) The early research work on the pooling problem obtains local solutions with different local optimization methods, such as guessing the qualities and solving the resulting problem recursively until convergence,[18,19] successive LP (SLP),[20] and generalized benders decomposition (GBD).[21] Later on, deterministic global optimization methods, which can guarantee convergence to the global optimum, have been proposed to solve pooling problems, such as the GOP method,[22,23] and the extensively studied branch-and-bound (BB) methods.[24–26] Tomasgard et al.[27] and Rømo et al.[28] approximated equations with bilinear terms as linear equations with integer variables, so the resulting formulation becomes a MILP. Gounaris et al.[29] proposed to relax the bilinear terms in the pooling problem with piecewise-linear relaxations and solve the problem via MILP subproblems. Also, reformulation-linearization techniques are presented for efficient solution of bilinear programming problems,[30,31] and Tawarmalani and Sahinidis[32] discussed different formulations of the pooling problem. Although most of the studied pooling problems contain simple pooling systems where no connections between different pools exist, more complicated pooling systems have been handled,[33,34] and the integrated design and operation of complicated pooling systems has also been addressed.[35,36]

With the advancement of computing hardware and mathematical programming techniques, more and more attention has been paid to include uncertainty in optimization formulations explicitly. Sahinidis[37] summarized the various approaches for optimization under uncertainty, among which stochastic programming with recourse[38] naturally fits and has been adopted in the integrated design and operation of oil and natural gas production systems (e.g., Ref. 27, 39). In a typical stochastic programming formulation with recourse, the possible realizations of uncertainty are approximated by a limited number of representative realizations (scenarios), and the decisions are made sequentially over two or more time periods according to the outcome of uncertain variables over these periods. Jonsbraten[40] classified uncertainty in planning problems into two categories: (1) project exogenous uncertainty, where optimization decisions do not affect the resolution of uncertainty or change the scenario tree in the stochastic recourse formulation, for example, the prices of the products, the customer demands; (2) project endogenous uncertainty, where optimization decisions do affect the resolution of uncertainty and changes the scenario tree in the stochastic recourse formulation, for example, capacity and quality of reservoirs. Generally, project endogenous uncertainty is more difficult to handle (for problems with more than two stages), because it requires the incorporation of non-anticipativity constraints to link the decisions (in the second or a later stage) for different scenarios.[41] A typical application of stochastic programming to a natural gas production system design can be found in Goel et al.,[42] and to an oil and gas production system design in Tarhan et al.[43] A more detailed survey of recent work on infrastructure development and production planning under uncertainty can be found in Goel and Grossmann.[44]

With explicit modeling of product quality and uncertainty, the formulation for integrated design and operation of natural gas production systems becomes a nonconvex MINLP problem, where the integer variables are at least from the development decisions and the nonconvex functions are at least from the bilinear terms to model the qualities. MINLP problems are typically solved with BB strategies, such as branch-and-reduce,[44] SMIN-αBB, and GMIN-αBB,[45] or decomposition strategies, such as outer approximation,[46–48] and GBD.[49] A detailed survey of MINLP solution techniques can be found in the review article by Grossmann.[50]

Problem Statement

Figure 1 illustrates the generalized pooling system addressed in this article. In this system, there are n sources (labeled from 1 to n) that supply the materials or intermediate products into the system, r pools (labeled from 1 to r) where the different materials or intermediate products are mixed or blended, and m product terminals (labeled from 1 to m) that supply the (same or different) final products. For a natural gas production system, the sources can be gas fields or individual wells in the gas fields, the pools can be production platforms, riser platforms or simple mixing and splitting units, the product terminals can be liquefied natural gas (LNG) plants that produce LNG for long distance transportation or dry gas terminals supplying end customers with pipelines directly.

Different from most of the pooling systems studied in the literature, this system not only allows connections between a source and a product terminal, a source and a pool, or a pool and a product terminal but also allows connections between two pools.[51] Connections between two sources or between two product terminals, which are also possible in real systems, are not considered explicitly in this article, but these connections can be modeled by introducing additional (virtual) pools.

Figures 1b–d give more details on the flows entering or leaving the sources, pools, and product terminals. Figure 1b shows that a flow coming out of source i may either go to a pool j (denoted by $f^{SP}_{i,j}$), or go to a product terminal k (denoted by $f^{ST}_{i,k}$). Note that as the component compositions of source i are parameters, there is no need to model the individual component flows explicitly for a flow coming out of source i (which avoids introducing more bilinear terms). Figure 1c shows that a flow entering a pool j may come from a source i or come from another pool j^+ (whose flow of component w is denoted by $f^{PP}_{j+,j,w}$). Also, a flow leaving pool j may go to another pool j^- (whose flow of component w is denoted by $f^{PP}_{j,j-,w}$) or go to a product terminal k (whose flow of component w is denoted by $f^{PT}_{j,k,w}$). The subscript $w \in \{1,\ldots,l\}$ indicates the different component. Figure 1d shows that a flow entering a product terminal k may come from a pool j or a source i. All the symbols used in this article are summarized in Table 1.

The stochastic pooling problem is stated as follows:

"Determine the optimal network design decisions and the operating flows for the pooling system that maximize the

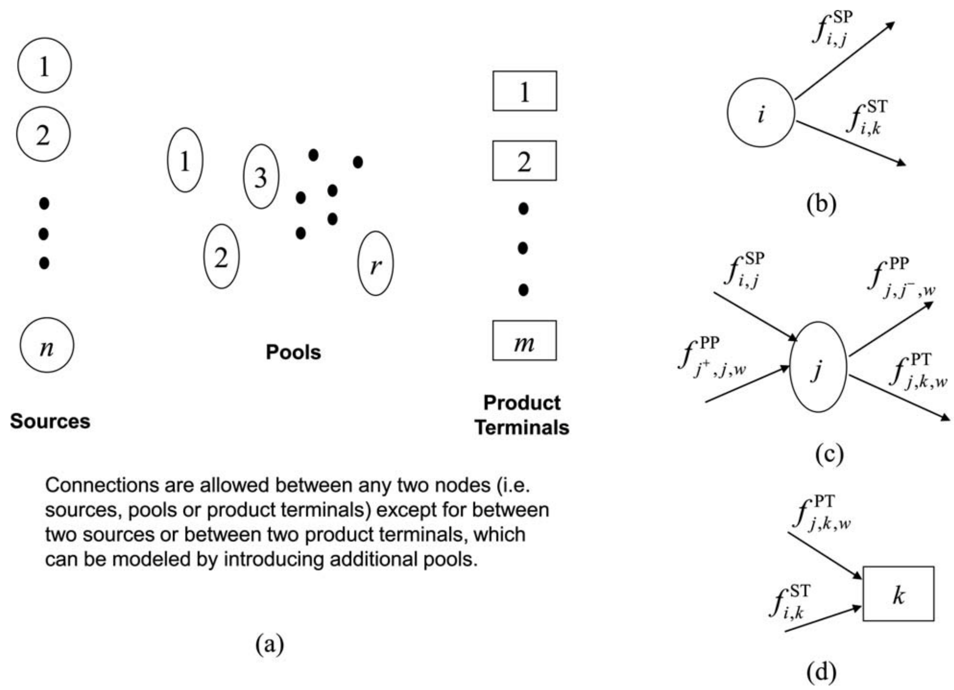

Figure 1. The generalized pooling system.

(a) The general diagram. (b) The flows at source i. (c) The flows at pool j. (d) The flows at terminal k.

profitability of developing and operating the system, while satisfying the product-specific constraints for all the uncertainty scenarios addressed.''

Notice that the profitability of developing and operating a pooling system can be evaluated according to different criteria, which lead to different objective functions in the problem formulation (and this will be discussed in more details later). Also, the problem does not address an infinite number of uncertainty scenarios that might occur, but addresses a limited number of representative scenarios that can be selected by scenario generation techniques or according to industrial experience. A stochastic programming formulation typically optimizes the expected value of an objective function over the addressed scenarios. If the formulation optimizes the objective function for the worst-case scenario or the weighted sum of the expected value and the variance of the objective function, it becomes a robust optimization formulation. The pros and cons of robust optimization formulation compared with the stochastic programming formulation have been well discussed in literature (e.g., Ref. 37, 52). As this article focuses on the advantage of addressing the uncertainty instead of comparing the different approaches to address uncertainty, the robust optimization formulation is not implemented and compared in the case studies.

Two types of formulations can be used to model the generalized pooling system shown in Figure 1. One is to formulate the mass balance equations with total flows and component compositions, and the other is to express the mass balances with individual component flows. As has been well recognized,[33] these two formulations have their own pros and cons, respectively. In general, the first formulation will lead to more bilinear terms if the total number of mixing flows entering the pools are more than the total number of splitting flows leaving the pools; the second formulation will lead to more bilinear terms otherwise. In this article, the second formulation is used because natural gas production systems are usually convergent from sources to product terminals.

Model for the Deterministic Pooling Problem

Here, the deterministic pooling problem refers to the operational problem for an existing pooling system, where the sources, pools, and product terminals and the connections between them are already constructed and the parameters of the model are assumed to be known exactly. This section gives the model for the deterministic pooling problem, which is the basis of the model for the stochastic pooling problem (that is developed in the next section). The model is built by modeling mass balances and constraints at sources, pools, and product terminals, respectively.

Model for the sources

The total flow coming out of a source i is subject to a lower bound Z_i^{LB} (which is due to non-negative flow or other system requirements) and an upper bound Z_i^{UB} (which is due to the source capacity or other system requirements), so:

$$Z_i^{\mathrm{LB}} \le \sum_{j\in\Theta_i^{\mathrm{SP}}} f_{i,j}^{\mathrm{SP}} + \sum_{k\in\Theta_i^{\mathrm{ST}}} f_{i,k}^{\mathrm{ST}} \le Z_i^{\mathrm{UB}}, \qquad \forall i \in \{1,\ldots,n\}, \quad (1)$$

where Θ_i^{SP} is an index set containing the indices of the pools that connect to source i and Θ_k^{ST} is an index set containing the indices of the product terminals that connect to source i. Also, each flow coming out of source i is subject to lower and upper

Table 1. List of Symbols

Symbol	Type	Description
b	Parameter	Number of scenarios
c	Subscript	Index for objectives in multiobjective optimization
d	Subscript	Index for objectives in multiobjective optimization
f	Variable	Flow rate
h	Subscript	Index for scenarios, $h \in \{1,\ldots,b\}$
i	Subscript	Index for sources, $i \in \{1,\ldots,n\}$
j	Subscript	Index for pools, $j \in \{1,\ldots,r\}$
j^+	Subscript	Index for the pools whose outlet flows enter a particular pool
j^-	Subscript	Index for the pools whose inlet flows are from a particular pool
k	Subscript	Index for product terminals, $k \in \{1,\ldots,m\}$
l	Parameter	Number of quality components
m	Parameter	Number of product terminals
n	Parameter	Number of sources
p	Parameter	Probability of scenario
r	Parameter	Number of pools
s	Variable	Ratio of a flow leaving a pool to the total flow entering the pool
t	Parameter	year
w	Subscript	Index of quality, $w \in \{1,\ldots,l\}$
y	Binary variable	Decision on source, pool, product terminal or pipeline investment
(AC)	Superscript	Indicator of annualized capital cost
C	Parameter	Economic coefficient with cost and price information
(Cap)	Superscript	Indicator of capital cost
D	Parameter	Demand at product terminal
F	Parameter	Bound on flow rate
L	Parameter	Life span of the system
LB	Superscript	Indicator of lower bound
N	Parameter	Number of objectives in multiobjective optimization
(OC)	Superscript	Indicator of annual cost and price information related to operation
P	Superscript	Indicator of pool
PP	Superscript	Indicator of flow from pool to pool
PP+	Superscript	Indicator of flow from pool entering a particular pool
PP−	Superscript	Indicator of flow from a particular pool entering a pool
PT	Superscript	Indicator of flow from pool to product terminal
S	Superscript	Indicator of source related quantity
SP	Superscript	Indicator of flow from source to pool
ST	Superscript	Indicator of flow from source to product terminal
T	Superscript	Indicator of product terminal
U	Parameter	Quality of materials at source
UB	Superscript	Indicator of upper bound
V	Parameter	Quality bound at product terminal
Z	Parameter	Source outlet flow bound
α	Parameter	Discount rate for calculating net present value
β	Parameter	Internal rate of return
σ	Variable	A function of internal rate of return to be optimized
Φ^{-1}	Function	Inverse cumulative distribution function
Π	Set	Index set for sources and pools connected to a product terminal
Θ	Set	Index set for pools and product terminals connected to a source
Ω	Set	Index set for sources, pools and terminals connected to a pool

bounds (that are due to the pipeline capacity, non-negative flow, or other system requirements) as:

$$F_{i,j}^{\mathrm{SP,LB}} \le f_{i,j}^{\mathrm{SP}} \le F_{i,j}^{\mathrm{SP,UB}}, \qquad \forall i \in \{1,\ldots,n\},\ \forall j \in \Theta_i^{\mathrm{SP}}, \quad (2)$$

$$F_{i,k}^{\mathrm{ST,LB}} \le f_{i,k}^{\mathrm{ST}} \le F_{i,k}^{\mathrm{ST,UB}}, \qquad \forall i \in \{1,\ldots,n\},\ \forall k \in \Theta_i^{\mathrm{ST}}, \quad (3)$$

where $F_{i,k}^{\mathrm{ST,LB}}$, $F_{i,k}^{\mathrm{ST,UB}}$, $F_{i,j}^{\mathrm{SP,LB}}$, and $F_{i,j}^{\mathrm{SP,UB}}$ denote the corresponding lower and upper bounds, respectively.

Model for the pools

Fractional variables $s_{j,j-}^{\mathrm{PP}}$ and $s_{j,k}^{\mathrm{PT}}$ are introduced to model the mass balances at pool j. They denote the ratio of the flow from pool j to pool j^- to the total flow entering pool j, and the ratio of the flow from pool j to product terminal k to the total flow entering pool j, respectively. Then each individual component flow of each outlet flow can be written as:

$$f_{j,k,w}^{\mathrm{PT}} = s_{j,k}^{\mathrm{PT}} \left(\sum_{i \in \Omega_j^{\mathrm{SP}}} f_{i,j}^{\mathrm{SP}} U_{i,w} + \sum_{j^+ \in \Omega_j^{\mathrm{PP+}}} f_{j^+,j,w}^{\mathrm{PP}} \right),$$
$$\forall j \in \{1,\ldots,r\},\ \forall k \in \Omega_j^{\mathrm{PT}},\ \forall w \in \{1,\ldots,l\}, \quad (4)$$

$$f_{j,j^-,w}^{\mathrm{PP}} = s_{j,j^-}^{\mathrm{PP}} \left(\sum_{i \in \Omega_j^{\mathrm{SP}}} f_{i,j}^{\mathrm{SP}} U_{i,w} + \sum_{j^+ \in \Omega_j^{\mathrm{PP+}}} f_{j^+,j,w}^{\mathrm{PP}} \right),$$
$$\forall j \in \{1,\ldots,r\},\ \forall j^- \in \Omega_j^{\mathrm{PP-}},\ \forall w \in \{1,\ldots,l\}, \quad (5)$$

where parameter $U_{i,w}$ denotes the fraction of component w in the flow from source i, Ω_j^{SP} is an index set containing the indices of the sources where an inlet flow to pool j can come from, $\Omega_j^{\mathrm{PP+}}$ is an index set containing the indices of the pools where an inlet flow to pool j can come from, $\Omega_j^{\mathrm{PP-}}$ is an index set containing the indices of the pools where an outlet flow from pool j can go to, and Ω_j^{PT} is an index set containing the indices of the product terminals where an outlet flow from pool j can go to. According to their definition, the split fraction variables for pool j should be non-negative and their sum should be unity because of mass balance at pool j, so:

$$\sum_{j^- \in \Omega_j^{\mathrm{PP-}}} s_{j,j^-}^{\mathrm{PP}} + \sum_{k \in \Omega_j^{\mathrm{PT}}} s_{j,k}^{\mathrm{PT}} = 1, \quad s_{j,j^-}^{\mathrm{PP}}, s_{j,k}^{\mathrm{PT}} \ge 0,$$
$$\forall j \in \{1,\ldots,r\},\ j^- \in \Omega_j^{\mathrm{PP-}},\ k \in \Omega_j^{\mathrm{PT}}. \quad (6)$$

Also, each flow is subject to upper and lower bounds due to non-negative flow, pipeline capacity and other system requirements, and the individual component flows in it are non-negative. So,

$$F_{j,j^-}^{\mathrm{PP,LB}} \le \sum_{w \in \{1,\ldots,l\}} f_{j,j^-,w}^{\mathrm{PP}} \le F_{j,j^-}^{\mathrm{PP,UB}}, \quad \forall j \in \{1,\ldots,r\}, \quad \forall j^- \in \Omega_j^{\mathrm{PP-}}, \quad (7)$$

$$F_{j,k}^{\mathrm{PT,LB}} \le \sum_{w \in \{1,\ldots,l\}} f_{j,k,w}^{\mathrm{PT}} \le F_{j,k}^{\mathrm{PT,UB}}, \quad \forall j \in \{1,\ldots,r\}, \quad \forall k \in \Omega_j^{\mathrm{PT}}, \quad (8)$$

$$f^{PP}_{j,j^-,w}, f^{PT}_{j,k,w} \geq 0, \quad \forall j \in \{1,\ldots,r\}, \quad \forall j^- \in \Omega^{PP-}_j, \quad \forall k \in \Omega^{PT}_j, \tag{9}$$

where $F^{PP,LB}_{j,j-}$, $F^{PP,UB}_{j,j-}$, $F^{PT,LB}_{j,k}$, and $F^{PT,UB}_{j,k}$ denote the corresponding lower and upper bounds, respectively.

Model for the product terminals

The total flow coming into a product terminal k to satisfy customer demand is subject to lower and upper bounds D^{LB}_k and D^{UB}_k, which are related to the minimum supply required by contract and the maximum possible demand from the market or the plant capacity, respectively. (Note they do not refer to the range of an uncertain demand.) Therefore,

$$D^{LB}_k \leq \sum_{j\in\Pi^{PT}_k} \sum_{w\in\{1,\ldots,l\}} f^{PT}_{j,k,w} + \sum_{i\in\Pi^{ST}_k} f^{ST}_{i,k} \leq D^{UB}_k, \quad \forall k \in \{1,\ldots,m\}, \tag{10}$$

where Π^{PT}_k is an index set containing the indices of the pools where an inlet flow to product terminal k can come from, and Π^{ST}_k is an index set containing the indices of the sources where an inlet flow to product terminal k can come from. Also, the product flow entering product terminal k is the final product, and it is subject to quality requirements imposed by contracts, technological limitations, or laws. The quality requirements are usually ranges for the percentages of specific components permitted in the product. Define $V^{LB}_{k,w}$ and $V^{UB}_{k,w}$ as the lower and upper bounds, respectively, on the fraction of component w in the final product at product terminal k. Then, the quality constraints can be written as:

$$\begin{aligned}\left(\sum_{j\in\Pi^{PT}_k} \sum_{w\in\{1,\ldots,l\}} f^{PT}_{j,k,w} + \sum_{i\in\Pi^{ST}_k} f^{ST}_{i,k}\right) V^{UB}_{k,w} &\geq \sum_{j\in\Pi^{PT}_k} f^{PT}_{j,k,w} \\ + \sum_{i\in\Pi^{ST}_k} f^{ST}_{i,k} U_{i,w} &\geq \left(\sum_{j\in\Pi^{PT}_k} \sum_{w\in\{1,\ldots,l\}} f^{PT}_{j,k,w} + \sum_{i\in\Pi^{ST}_k} f^{ST}_{i,k}\right) V^{LB}_{k,w}, \\ &\forall k \in \{1,\ldots,m\}, \ \forall w \in \{1,\ldots,l\}. \end{aligned} \tag{11}$$

According to the discussion in this section, Eq. 1–11 constitute the model for the deterministic pooling problem.

Model for the Stochastic Pooling Problem

This section discusses the model for the stochastic pooling problem for the design and operation of natural gas systems under uncertainty. We will restrict our discussion to a two-stage stochastic recourse formulation,[38] where the first-stage decisions develop the sources, pools and product terminals, and the pipelines between them while the second-stage decisions plan the operation of the system. The two-stage stochastic recourse formulation is basically a bilevel optimization formulation whose inner optimization problems mimic the second-stage planning process. As has been widely recognized,[38] due to special structure, two-stage stochastic programs can be naturally reformulated into an equivalent single-level optimization problem. So, this article addresses the single-level optimization formulation of two-stage recourse directly.

The model for the stochastic pooling problem differs from that for the deterministic pooling problem in two aspects. One is that the existence of sources, pools, product terminals, and their connections in the system can be decided by the optimization problem, and the other is that the optimal flows can be different for each different realization of the uncertainty in the system. As has been done for the deterministic model, the stochastic model will be constructed by modeling mass balances and constraints at sources, pools, and product terminals, respectively.

Model for the sources

According to Eq. 1–3 for the deterministic model, the constraints on the flows leaving the sources in the context of the stochastic model can be written as:

$$y^S_i Z^{LB}_{i,h} \leq \sum_{j\in\Theta^{SP}_i} f^{SP}_{i,j,h} + \sum_{k\in\Theta^{ST}_i} f^{ST}_{i,k,h} \leq y^S_i Z^{UB}_{i,h}, \tag{12}$$

$$y^{SP}_{i,j} F^{SP,LB}_{i,j} \leq f^{SP}_{i,j,h} \leq y^{SP}_{i,j} F^{SP,UB}_{i,j}, \tag{13}$$

$$y^{ST}_{i,k} F^{ST,LB}_{i,k} \leq f^{ST}_{i,k,h} \leq y^{ST}_{i,k} F^{ST,UB}_{i,k}, \quad y^S_i, y^{SP}_{i,j}, y^{ST}_{i,k} \in \{0,1\}, \tag{14}$$

$$\forall i \in \{1,\ldots,n\}, \ \forall j \in \Theta^{SP}_i, \ \forall k \in \Theta^{ST}_i, \ \forall h \in \{1,\ldots,b\},$$

where the new subscript $h \in \{1,\ldots b\}$ is used in the stochastic model for all the variables and parameters whose values may be different in the b different uncertainty scenarios. y^S_i is a binary decision variable to determine whether source i is developed or not. If $y^S_i = 1$, then source i will be developed and Eq. 12 will become the source capacity constraint; otherwise source i will not be developed and Eq. 12 forces the total flow from source i to be zero. Similarly, in Eqs. 13 and 14, $y^{SP}_{i,j}$ or $y^{ST}_{i,k}$ is a binary variable to determine whether the pipeline between source i and pool j or the pipeline between source i and product terminal k is developed. Each of these two binary variables will enforce the bounds on the flow if the corresponding pipeline is developed or force the flow to be zero otherwise.

Because of topological restrictions on the sources and the pipelines connecting to them, the above decision variables are subject to the following constraints:

$$y^S_i \geq y^{SP}_{i,j}, \tag{15}$$

$$y^S_i \geq y^{ST}_{i,k}, \tag{16}$$

$$\forall i \in \{1,\ldots,n\}, \ \forall j \in \Theta^{SP}_i, \ \forall k \in \Theta^{ST}_i,$$

where Eq. 15 means that the pipeline between a source and a pool can be developed only when the source is developed, and Eq. 16 means that the pipeline between a source and a product terminal can be developed only when the source is developed.

Model for the pools

According to Eqs. 4–8 for the deterministic model, the mass balances and the constraints at the pools for the stochastic model can be written as:

$$f^{\rm PT}_{j,k,w,h} = s^{\rm PT}_{j,k,h}\left(\sum_{i\in\Omega^{\rm SP}_j} f^{\rm SP}_{i,j,h}U_{i,w,h} + \sum_{j^+\in\Omega^{\rm PP+}_j} f^{\rm PP}_{j^+,j,w,h}\right), \tag{17}$$

$$f^{\rm PP}_{j,j^-,w,h} = s^{\rm PP}_{j,j^-,h}\left(\sum_{i\in\Omega^{\rm SP}_j} f^{\rm SP}_{i,j,h}U_{i,w,h} + \sum_{j^+\in\Omega^{\rm PP+}_j} f^{\rm PP}_{j^+,j,w,h}\right), \tag{18}$$

$$\sum_{j^-\in\Omega^{\rm PP-}_j} s^{\rm PP}_{j,j^-,h} + \sum_{k\in\Omega^{\rm PT}_j} s^{\rm PT}_{j,k,h} = 1, \quad s^{\rm PP}_{j,j^-,h}, s^{\rm PT}_{j,k,h} \geq 0, \tag{19}$$

$$y^{\rm PP}_{j,j^-}F^{\rm PP,LB}_{j,j^-} \leq \sum_{w\in\{1,\ldots,l\}} f^{\rm PP}_{j,j^-,w,h} \leq y^{\rm PP}_{j,j^-}F^{\rm PP,UB}_{j,j^-}, \tag{20}$$

$$y^{\rm PT}_{j,k}F^{\rm PT,LB}_{j,k} \leq \sum_{w\in\{1,\ldots,l\}} f^{\rm PT}_{j,k,w,h} \leq y^{\rm PT}_{j,k}F^{\rm PT,UB}_{j,k}, \tag{21}$$

$$f^{\rm PP}_{j,j^-,w,h}, f^{\rm PT}_{j,k,w,h} \geq 0, \tag{22}$$

$$y^{\rm P}_j, y^{\rm PP}_{j,j^-}, y^{\rm PT}_{j,k} \in \{0,1\},$$

$$\forall j \in \{1,\ldots,r\},\ \forall j^- \in \Omega^{\rm PP-}_j,\ \forall k \in \Omega^{\rm PT}_j,\ \forall w \in \{1,\ldots,l\},\ \forall h \in \{1,\ldots,b\}$$

where the binary decision variable $y^{\rm P}_j$ determines whether pool j is developed or not, $y^{\rm PP}_{j,j-}$ determines whether the pipeline between pool j and a downstream pool j^- is developed or not, and $y^{\rm PT}_{j,k}$ determines whether the pipeline between pool j and product terminal k is developed or not.

Because of topological restrictions on the pools and the pipelines connecting to them, the above decision variables are subject to the following constraints:

$$y^{\rm P}_j \geq y^{\rm SP}_{i,j}, \tag{23}$$

$$y^{\rm P}_j \geq y^{\rm PP}_{j,j^-}, \tag{24}$$

$$y^{\rm P}_{j^-} \geq y^{\rm PP}_{j,j^-}, \tag{25}$$

$$y^{\rm P}_j \geq y^{\rm PT}_{j,k}, \tag{26}$$

$$\forall j \in \{1,\ldots,r\},\ \forall j^- \in \Omega^{\rm PP-}_j,\ \forall k \in \Omega^{\rm PT}_j,$$

which mean that each of the pipelines connecting to a pool can be developed only when that pool is developed.

Model for the product terminals

According to Eqs. 10–11 for the deterministic model, the constraints on the flows entering the product terminals for the stochastic model can be written as:

$$y^{\rm T}_k D^{\rm LB}_k \leq \sum_{j\in\Pi^{\rm PT}_k}\sum_{w\in\{1,\ldots,l\}} f^{\rm PT}_{j,k,w,h} + \sum_{i\in\Pi^{\rm ST}_k} f^{\rm ST}_{i,k,h} \leq y^{\rm T}_k D^{\rm UB}_{k,h}, \tag{27}$$

$$\left(\sum_{j\in\Pi^{\rm PT}_k}\sum_{w\in\{1,\ldots,l\}} f^{\rm PT}_{j,k,w,h} + \sum_{i\in\Pi^{\rm ST}_k} f^{\rm ST}_{i,k,h}\right)V^{\rm UB}_{k,w} \geq \sum_{j\in\Pi^{\rm PT}_k} f^{\rm PT}_{j,k,w,h}$$
$$+ \sum_{i\in\Pi^{\rm ST}_k} f^{\rm ST}_{i,k,h}U_{i,w,h} \geq \left(\sum_{j\in\Pi^{\rm PT}_k}\sum_{w\in\{1,\ldots,l\}} f^{\rm PT}_{j,k,w,h} + \sum_{i\in\Pi^{\rm ST}_k} f^{\rm ST}_{i,k,h}\right)V^{\rm LB}_{k,w},$$
$$\forall k \in \{1,\ldots,m\},\ \forall w \in \{1,\ldots,l\},\ \forall h \in \{1,\ldots,b\}. \tag{28}$$

The binary variable $y^{\rm T}_k$ is to determine whether a product terminal k is developed or not, and the following topology constraints must hold:

$$y^{\rm T}_k \geq y^{\rm ST}_{i,k}, \tag{29}$$

$$y^{\rm T}_k \geq y^{\rm PT}_{j,k}, \tag{30}$$

$$\forall i \in \Pi^{\rm ST}_k,\ \forall j \in \Pi^{\rm PT}_k,\ \forall k \in \{1,\ldots,m\},$$

which mean that each of the pipelines connecting to a product terminal can be developed only when that product terminal is developed.

According to the discussion in this section, Eqs. 12–30 constitute the model for the stochastic pooling problem. Note that all the integer variables in the model are the first-stage decisions of the two-state stochastic program, which determine the development of sources, pools, product terminals, and the pipelines between them; all the continuous variables are the second-stage decisions, which determine the long-term operation of the system.

Objectives for the Stochastic Pooling Problem

Annualized profit

Annualized profit is the difference between the annual net income of the operation and the annualized capital cost of the facilities developed in the system, so the expected value of annualized profit can be maximized as:

$$\max \quad -C^{\rm (AC)} + \sum_{h\in\{1,\ldots,b\}} p_h C^{\rm (OC)}_h, \tag{31}$$

where $C^{\rm (AC)}$ denotes the annualized capital cost and

$$C^{\rm (AC)} = \sum_{i\in\{1,\ldots,n\}} C^{\rm S,(AC)}_i y^{\rm S}_i + \sum_{j\in\{1,\ldots,r\}} C^{\rm P,(AC)}_j y^{\rm P}_j$$
$$+ \sum_{k\in\{1,\ldots,m\}} C^{\rm T,(AC)}_k y^{\rm T}_k + \sum_{i\in\{1,\ldots,n\}}\sum_{j\in\Theta^{\rm SP}_i} C^{\rm SP,(AC)}_{i,j} y^{\rm SP}_{i,j}$$
$$+ \sum_{i\in\{1,\ldots,n\}}\sum_{k\in\Theta^{\rm ST}_i} C^{\rm ST,(AC)}_{i,k} y^{\rm ST}_{i,k} + \sum_{j\in\{1,\ldots,r\}}\sum_{j^-\in\Omega^{\rm PP-}_j} C^{\rm PP,(AC)}_{j,j^-} y^{\rm PP}_{j,j^-}$$
$$+ \sum_{j\in\{1,\ldots,r\}}\sum_{k\in\Omega^{\rm PT}_j} C^{\rm PT,(AC)}_{j,k} y^{\rm PT}_{j,k}, \tag{32}$$

where $C^{\rm S,(AC)}_i$, $C^{\rm P,(AC)}_j$, and $C^{\rm T,(AC)}_k$ are the annualized investment costs of source i, pool j, and product terminal k, respectively, and $C^{\rm SP,(AC)}_{i,j}$, $C^{\rm ST,(AC)}_{i,k}$, $C^{\rm PP,(AC)}_{j,j-}$, and $C^{\rm PT,(AC)}_{j,k}$ are the annualized pipeline investment costs connecting different nodes in the network.

Also, $C_h^{(OC)}$ denotes the annual net income from operating the production system, which depends on the scenario of uncertainty realization h with probability p_h, and:

$$C_h^{(OC)} = \sum_{i\in\{1,\ldots,n\}} -C_i^{S,(OC)}\left(\sum_{j\in\Theta_i^{SP}} f_{i,j,h}^{SP} + \sum_{k\in\Theta_i^{ST}} f_{i,k,h}^{ST}\right) + \sum_{k\in\{1,\ldots,m\}} C_{k,h}^{T,(OC)}\left(\sum_{j\in\Pi_k^{PT}}\sum_{w\in\{1,\ldots,l\}} f_{j,k,w,h}^{PT} + \sum_{i\in\Pi_k^{ST}} f_{i,k,h}^{ST}\right), \quad (33)$$

where $C_i^{S,(OC)}$ denotes the annual cost related to the operation of source i per unit of gas produced, and $C_{k,h}^{T,(OC)}$ denotes the annual revenue related to the operation of product terminal k in scenario h per unit of gas produced. Thus, the second term of the objective (31) is actually the expected annual net income of the project. Operating costs incurred by the pipelines and pools are not considered in the article, but they can be incorporated in the objective function easily.

When maximizing the expected annualized profit, the stochastic pooling problem to be solved is

$$\text{obj} = (31)$$
$$\text{s.t. } (32-33) \text{ and } (12-30).$$

Net present value

Net present value is the sum of the discounted values of all the cash flows at the present. Assume the annual discount rate for the calculation is α and the system life span is L years, then the expected net present value of the project over the system life-span can be maximized as:

$$\max \quad -C^{(Cap)} + \left(\sum_{t\in\{1,\ldots,L\}} \frac{1}{(1+\alpha)^t}\right)\left(\sum_{h\in\{1,\ldots,b\}} p_h C_h^{(OC)}\right), \quad (34)$$

where $C^{(Cap)}$ denotes the total capital cost and

$$C^{(Cap)} = \sum_{i\in\{1,\ldots,n\}} -C_i^{S,(Cap)} y_i^S + \sum_{j\in\{1,\ldots,r\}} C_j^{P,(Cap)} y_i^P + \sum_{k\in\{1,\ldots,m\}} C_k^{T,(Cap)} y_k^T + \sum_{i\in\{1,\ldots,n\}}\sum_{j\in\Theta_i^{SP}} C_{i,j}^{SP,(Cap)} y_{i,j}^{SP} + \sum_{i\in\{1,\ldots,n\}}\sum_{k\in\Theta_i^{ST}} C_{i,j}^{ST,(Cap)} y_{i,k}^{ST} + \sum_{j\in\{1,\ldots,r\}}\sum_{j^-\in\Omega_j^{PP}} C_{i,j^-}^{PP,(Cap)} y_{i,j^-}^{PP} + \sum_{j\in\{1,\ldots,r\}}\sum_{k\in\Omega_j^{PT}} C_{j,k}^{PT,(Cap)} y_{j,k}^{PT}, \quad (35)$$

where $C_i^{S,(Cap)}$, $C_j^{P,(Cap)}$, and $C_k^{T,(Cap)}$ are the investment costs of source i, pool j, and product terminal k, respectively, and $C_{i,j}^{SP,(Cap)}$, $C_{i,k}^{ST,(Cap)}$, $C_{j,j-}^{PP,(Cap)}$, and $C_{j,k}^{PT,(Cap)}$ are the investment costs of the pipelines connecting different units in the network. Notice that, in Eq. 34, each gas flow is assumed to be same for each year in the system life-span.

When maximizing the expected net present value, the stochastic pooling problem to be solved is

$$\text{obj} = \text{Equation}(34)$$
$$\text{s.t. Equations}(33), (35) \text{ and } (12-30).$$

Addressing multiple objectives

A natural gas production system design and operation problem may have more than one objective to optimize. For example, in addition to maximize the profitability of the project, one may want to exploit sour fields preferentially over sweet fields when the quality constraints can be satisfied anyway. In these cases, the hierarchical multiobjective optimization approach presented by Selot et al.[53] applies.

Assume there are N objectives to maximize, which are z_1, z_2, … z_N, and their priorities are ranked as 1st, 2nd, … Nth respectively. Then, the problem is optimized N times with the hierarchical optimization strategy; for the cth optimization ($c \in \{1,\ldots N\}$), the objective is

$$\max \quad z_c, \quad (36)$$

and if $c > 1$, the problem has the following additional constraints:

$$z_d \geq z_{d,c-1}^*, \quad \forall d \in \{1,\ldots,c\}, \quad (37)$$

which ensure the other objectives are no worse than their values obtained in the $(c-1)$th optimization. $z_{d,c-1}^*$ denotes the value of z_d at the optimum of the $(c-1)$th optimization.

Decomposition Method

The stochastic pooling problem is a potentially large-scale nonconvex MINLP problem. The solution time for this problem with BB type methods increases dramatically with the number of scenarios (which will be demonstrated in the case studies later). This section briefly introduces a novel decomposition method recently developed by Li et al. (submittted). The computational advantage of the decomposition method for the stochastic pooling problem will be demonstrated through the case study results in the next section.

Overview of the decomposition method

The decomposition method is developed based on the framework of concepts presented by Geoffrion for the design of large-scale mathematical programming techniques.[54] This framework includes two groups of concepts: problem manipulations and solution strategies. Problem manipulations, such as convexification, projection, and dualization, are devices for restating a given problem in an alternative form more amenable to solution. The result is often what is referred to as a master problem. Solution strategies, such as relaxation and restriction, reduce the master problem to a related sequence of simpler subproblems.

In the novel decomposition approach, the bilinear terms in the stochastic pooling problem are replaced by their

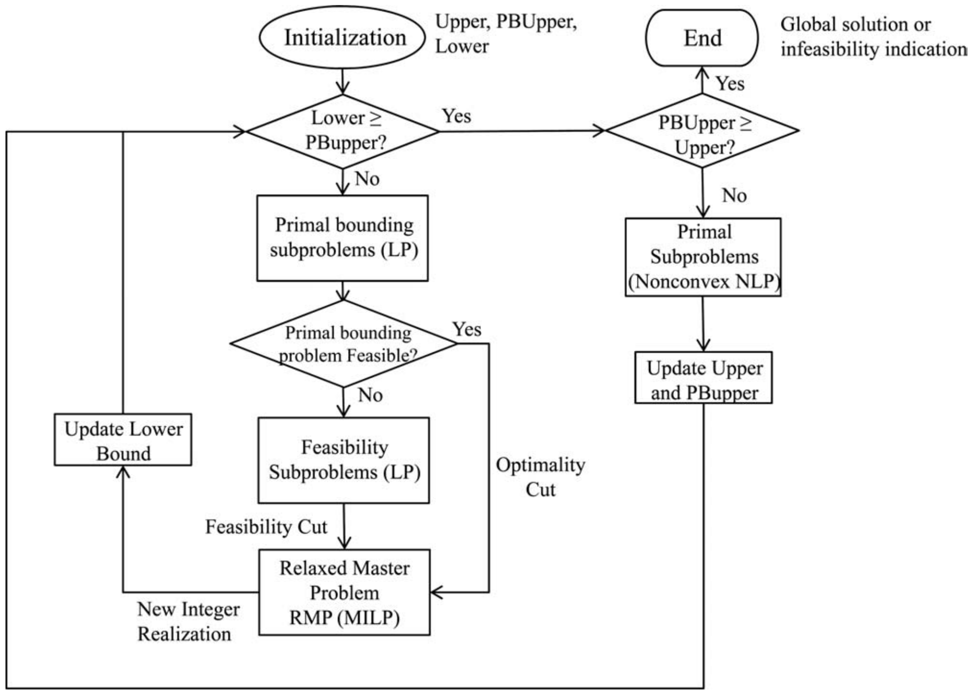

Figure 2. Flowchart for the decomposition algorithm.

Upper, Upper bound on the original problem; PBUpper, Upper bound on the lower bounding problem/master problem; Lower, Lower bound on the lower bounding problem/master problem

convex and concave envelopes,[55] so the lower bounding problem is a potentially large-scale MILP. The lower bounding problem is equivalently transformed into a master problem by projection and dualization, as in Benders decomposition.[56] This master problem is solved by solving a sequence of "primal bounding problems" and "relaxed master problems." The primal bounding problem is constructed by restricting the integer variables to specific values in the lower bound problem, whose solution yields a valid upper bound on the optimal objective value of the lower bounding problem. The primal bounding problem is a potentially large-scale LP, but it can be further decomposed into LP subproblems for each scenario. When the primal bounding problem is infeasible, a corresponding "feasibility problem" is solved, which yields valid information for the algorithm to proceed. The feasibility problem is also a potentially large-scale LP, and it can be decomposed into LP subproblems for each scenario as well. The relaxed master problem is constructed by relaxing the master problem with finite number of constraints (cuts), and the solution of the relaxed master problem yields a valid lower bound on the optimal objective of the master problem. The relaxed master problem is a MILP whose size is independent of the number of scenarios.

On the other hand, a restriction of the stochastic pooling problem, called the Primal Problem, is constructed by restricting the integer variables to specific values in the stochastic pooling problem. The primal problem is a potentially large-scale nonconvex NLP, which can be further decomposed into NLP subproblems for each scenario.

The decomposition algorithm is implemented by solving the aforementioned subproblems iteratively, as illustrated by the diagram in Figure 2. The algorithm terminates finitely with an ε-optimal solution or an indication of the infeasibility of the problem. This proof and other details of the algorithm can be found in Li et al. (submitted).

Reformulation-linearization technique for tighter relaxation

The lower bounding problem is a convex relaxation of the stochastic pooling problem. According to the idea of the reformulation-linearization technique,[30,31] additional redundant constraints can be integrated into the stochastic pooling problem formulation for a tighter convex relaxation. Tawarmalani and Sahinidis[32] gave a thorough discussion on the reformulation-linearization technique for the classical pooling problem (where no connections between pools are allowed). They proved that a so-called pq-formulation, which includes proper redundant constraints, yields a convex relaxation that is as good as the Lagrangian relaxation for the original pooling problem. Here, the pq-formulation is extended for the stochastic pooling problem and the generalized pooling system, that is, the following redundant constraints are integrated in the formulation:

$$\sum_{j^- \in \Omega_j^{PP-}} s_{j,j^-,h}^{PP} f_{i,j,h}^{SP} + \sum_{k \in \Omega_j^{PT}} s_{j,k,h}^{PT} f_{i,j,h}^{SP} = f_{i,j,h}^{SP},$$
$$\forall j \in \{1,\ldots,r\},\ \forall i \in \Omega_j^{SP},\ \forall h \in \{1,\ldots,b\}, \quad (38)$$

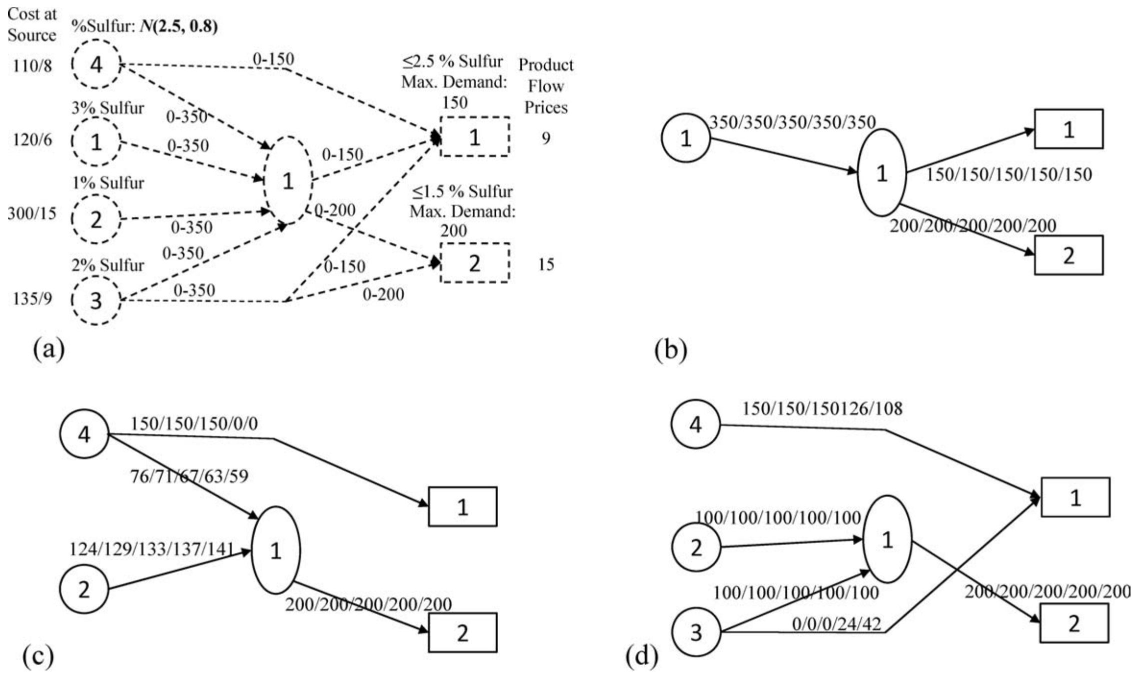

Figure 3. Case study A and the different system design results.

(a) The problem superstructure and parameters. Data along pipelines are flow bounds. Cost at sources denotes Annualized investment cost of source/material flow cost. Annualized investment cost of pool, product terminal or pipe is 10. (b) Design Result with Formulation 1. Data along pipelines show flows in scenario 1 – 5 in order. (c) Design Result with Formulation 2. Data along pipelines show flows in scenario 1 – 5 in order. (d) Design Results with Formulation 3a. Data along pipelines show flows in scenario 1 – 5 in order.

$$\sum_{j^- \in \Omega_j^{\mathrm{PP}-}} s^{\mathrm{PP}}_{j,j^-,h} f^{\mathrm{PP}}_{j^+,j,w,h} + \sum_{k \in \Omega_j^{\mathrm{PT}}} s^{\mathrm{PT}}_{j,k,h} f^{\mathrm{PP}}_{j^+,j,w,h} = f^{\mathrm{PP}}_{j^+,j,w,h}, \quad . \quad \forall j \in \{1,\ldots,r\},\ \forall w \in \{1,\ldots,l\},\ \forall j^+ \in \Omega_j^{\mathrm{PP}+},\ \forall h \in \{1,\ldots,b\} \tag{39}$$

Notice that those redundant constraints are generated by multiplying both sides of Eq. 19 by $f^{\mathrm{SP}}_{i,j,h}$ and $f^{\mathrm{PP}}_{j+,j,w,h}$, respectively.

Case Studies

Formulations and implementation

The advantages of the stochastic pooling problem formulation over other formulations will be demonstrated in the case studies. The following formulations are compared for each case study:

Formulation 1. A deterministic formulation that does not address uncertainty in the formulation explicitly (i.e., the total number of scenarios addressed is $b = 1$) and does not have any quality constraints. It has one of the economic objectives developed in the last section.

This deterministic formulation does not address other potential scenarios with recourse. However, once the system has been developed and the uncertainty realized, an operational problem can be solved to improve the long-term operational plan obtained at the design stage, according to the known uncertainty realization. The formulation of this operational problem can be derived from the deterministic formulation solved at the system design stage with the integer decision variables fixed according to the existing system design and the scenario fixed according to the realized uncertainty scenario. Note that this operational problem does not have quality constraints either.

It is important to highlight Formulation 1 (which does not track gas qualities in system) can lead to infeasible design and operation, because otherwise we do not need to model the gas qualities with bilinear terms and the problem formulation becomes a MILP, which is much easier to solve than Formulation 2 and 3 (which are nonconvex MINLPs).

Formulation 2. A deterministic formulation that does not address uncertainty in the formulation explicitly (i.e., the total number of scenarios addressed is $b = 1$) but has quality constraints on the final products at the product terminals. It has one of the economic objectives developed in the last section.

Again an operational problem can be solved once the system is developed and the uncertainty realized, according to the realized uncertainty scenario. The formulation of this operational problem can be developed using the same approach explained for Formulation 1, but in this case there are quality constraints on the final products so the operating variables may be adjusted to different values from those in the deterministic design.

Formulation 3a. A two-stage stochastic recourse formulation with quality constraints on the final products at the product terminals. It has one of the economic objectives developed in the last section.

This formulation provides solutions for all the operational problems for each uncertainty realization as part of the design.

Formulation 3b. A two-stage stochastic recourse formulation with quality constraints on the final products at the product terminals. It has one of the economic objectives developed in the last section as well as additional objectives with lower priorities. So, it is solved via the hierarchical multiobjective optimization approach described in the last section.

This formulation provides solutions for all the operational problems for each uncertainty realization as part of the design.

Formulation 1 leads to an MILP problem, and the other three formulations lead to nonconvex MINLP problems. For all the nonconvex MINLP problems, additional redundant constraints (38–39) are integrated to yield better convex relaxations. All the case study problems are solved on GAMS 22.8.1[59] with a computer-allocated single 2.83-GHz CPU and running Linux kernel. The problems are solved with BARON 8.1.5,[44] a state-of-the-art branch-and-reduce global optimizer, which uses SNOPT 7.2.4[58] as local NLP solver and CPLEX 11.1.1 as local LP solver. The problems are also solved with the decomposition algorithm that uses BARON 8.1.5[59] for solving nonconvex NLP subproblems and CPLEX 11.1.1[60] for solving LP and MILP subproblems. The relative termination criterion for all the case study problems is 10^{-2}.

Case study A (stochastic Haverly pooling problem)

This example is inspired by Haverly's pooling problem.[18] Figure 3a shows the superstructure and the parameters of the system in case study A. All the potential facilities are shown in dashed lines, which include four sources, one pool, two product terminals, and several pipelines between them. The product quality in this problem refers to the sulfur percentage in the product flow. The quality requirements at the product terminals and the qualities of the source flows are shown Figure 3a, where the quality of Source 4 (i.e., the sulfur percentage in the flow from Source 4) is uncertain and it obeys a normal distribution with a mean of 2.5 and a standard deviation of 0.8. The costs of the flows from the sources, the prices of the product flows, the annualized investment cost of each facility, the maximum demands on the product terminals, and the bounds on the flows in each of the pipelines are also shown in Figure 3a. In addition, the capacity of each source can be deemed as unlimited.

A naive sampling rule is used here to generate scenarios for the normal distribution. If the normal distribution has mean μ and standard deviation σ and the total number of scenarios to be sampled is b, then scenarios are only sampled in the range $[-3\sigma + \mu, 3\sigma + \mu]$ and the h th scenario is

$$x_h = -3\sigma + \mu + 3\sigma/b + (h-1)6\sigma/b.$$

This is illustrated in Figure 4. Also, the probability of each scenario is

$$P(x = x_h) = \begin{cases} \Phi^{-1}(-3\sigma + \mu + 6\sigma/b), & \text{if } h = 1, \\ \Phi^{-1}(-3\sigma + \mu + 6\sigma h/b) - \Phi^{-1}(-3\sigma + \mu + 6\sigma(h-1)/b), & \text{if } 1 < h < b, \\ 1 - \Phi^{-1}(-3\sigma + \mu + 6\sigma(b-1)/b), & \text{if } h = b, \end{cases}$$

where Φ^{-1} denotes the inverse cumulative distribution function of the normal distribution.

According to this sampling rule, five scenarios for the uncertain Source 4 quality realizations are addressed here. In the deterministic formulations, the mean value of Source 4 quality is used as the deterministic Source 4 quality, whereas in the stochastic formulations the Source 4 qualities in the five scenarios are addressed explicitly.

Formulations 1, 2, and 3a are compared for this problem, and the objective of all these formulations is to maximize the annualized profit. Figures 3b–d show the system design using the three formulations. As Formulation 1 does not consider the quality constraints at the product terminals, the system designed with this formulation Source 1, which supplies the cheapest flow for the two product terminals. However, this design is unreasonable concerning the quality constraints at the product terminals, because the quality of the flow from Source 1 violates both of the constraints. Formulation 2 includes the quality constraints, so the system designed by this formulation has Source 2 and Source 4 instead of Source 1 to supply the flows. The drawback of this design is that the Source 4 quality is uncertain and this source cannot be used to supply Terminal 1 when its sulfur percentage is greater than 2.5. When considering this uncertainty explicitly in Formulation 3a, another design is obtained for the system where Source 3 is developed as well, which can be used to compensate the high sulfur percentage of Source 4 for supplying Terminal 1 when needed.

The advantages of the stochastic formulation, Formulation 3a, over the other two deterministic formulations can be further recognized with the results in Table 2. Table 2 summarizes the design and operation results for case study A with the three formulations. For each formulation, the annualized capital cost is calculated according to each designed system and each result is shown in the table; the annualized profit for each scenario with this formulation is calculated and the averages over the five scenarios are shown in the table. Table 2 also shows whether product quality upper bound is satisfied at each product terminal for each scenario. It can be found that with Formulation 1, the product qualities at both product terminals violate the bounds, so the high profit calculated by this formulation is meaningless for the real problem. Both Formulation 2 and Formulation 3a observe the product quality constraints, but Formulation 3a achieves better average annualized profits. Note that it is important to highlight Formulation 1 can lead to infeasible design and operation, because otherwise we do not need to model the gas qualities with bilinear terms and the problem formulation is a MILP, which is much easier to solve than Formulation 2 and 3 (which are nonconvex MINLPs).

Next, the computational efficiencies of BARON and the decomposition method for case study A are compared by solving the stochastic pooling problem Formulation 3a with different numbers of scenarios. It is assumed that the demands at the two product terminals are uncertain as well, which obey normal distributions with means of 180 and 200,

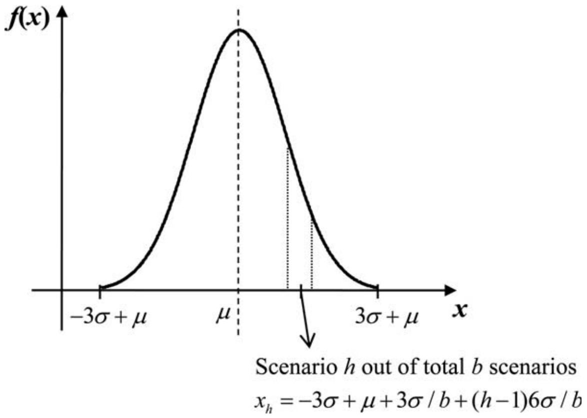

Figure 4. Scenario generation for normal distribution.

and standard deviations of 10 and 10, respectively. Also, the three uncertain parameters are assumed to be independent. 1, 2, 3, 4, and 5 scenarios are generated for each uncertain parameter in the way described before, which lead to problems with 1, 8, 27, 64, and 125 scenarios. These five problems are solved with both BARON and the decomposition method, and the solver times are displayed in Table 3. It can been seen that the decomposition method is much faster than BARON when 3 or more scenarios are addressed for each uncertain parameters (i.e., 27 or more scenarios for the problem), although it is slower than BARON when the number of scenarios is unrealistically small for a stochastic formulation. Also, the solver time with the decomposition method increases moderately with the number of scenarios, whereas the time with BARON increases dramatically with the number of scenarios.

Case study B

This case study is motivated by a real industrial system, the Sarawak gas production system (SGPS), which is located in the South China Sea off the coast of the state of Sarawak in East Malaysia. The details and the modeling issues for this system can be found in Selot et al.[53] and Selot.[61] The system model used in this case study is the one presented by Selot et al.,[53] whose structure and parameters are changed from the real system because of the confidentiality of industrial data. The gas processing inside the LNG plants, for example, removal of CO_2 and heavy hydrocarbons, is not considered in the model. Also, the water removal at production platforms is not modeled, because water constitutes a tiny proportion of each gas flow, and it is negligible for the calculation of other gas components of interest. In addition, three major revisions of the model are made to simplify the problem:

1. The complex production sharing contracts are not addressed in the model. The gas from any field may go to any LNG plant (if it is physically possible).
2. It is assumed that any desired flow rate in a particular pipeline (within the pipeline capacity) can be achieved by (adding) a compressor at an upstream platform, so the pressure-flow relationships in the pipelines are not included in the model.
3. Only the quality bounds on CO_2 are considered. So only two individual component flows (i.e., CO_2 flow and non-CO_2 flow) are modeled for each pipeline.

Figure 5 shows the superstructure of the system. The units and the connecting pipelines with solid lines in Figure 5 represent the existing part of the system, which has eight gas fields (D35, BY, SC, E11, F6, F23SW, F23, and BN) as sources, four platforms (BYP, E11P, F23P, and E11R-A) and one plant slugcatcher (SC-1) as pools, and one LNG plant (LNG1) as product terminal. Because of expansion of the market, more gas fields, platforms, pipelines need to be developed to feed gas to two potential LNG plants. The potential units and the connecting pipelines of the new part of the system are shown in dashed lines in the figure, including seven gas fields (B11, HL, SE, M3, M4, M1, and JN) as sources, five platforms (B11P, M3P, M1P, E11R-B, and E11R-C), and one pipeline connection (T) and two plant slugcatchers (SC-2 and SC-3) as pools, and two LNG plants (LNG2 and LNG3) as product terminals. The gas platform B11P is designated to locate at the gas field B11, which should at least serve gas from B11. This means B11 must be developed if B11P is developed and vice versa. The same relationship exists between M3 and M3P, M1 and M1P, SC-2 and LNG2, and SC-3 and LNG3 (where SC-2 or SC-3 is part of plant LNG2 or LNG3) as well. Such relationship is enforced by additional topology constraints to the model. Figure 5 also displays the investment costs of the potential units and pipelines in the new part of the system used in the case study, which are estimated according to industrial data and literature.[62,63] The cost of the gas produced from each gas field is assumed to be zero. The new part of system to be developed has a life span of 25 years.

The parameter values for this problem are shown in Figure 6a. (Note the CO_2 specifications labeled by LNG2 and LNG3

Table 2. Case Study A Design and Operation Results with Different Formulations

	Average Annualized Profit	Satisfaction of Product Quality* Bound at Each Product Terminal for the Five Scenarios		Annualized Capital Cost
		Terminal 1	Terminal 2	
Formulation 1	2070†	N/N/N/N/N‡	N/N/N/N/N	180
Formulation 2	104	Y/Y/Y/Y/Y	Y/Y/Y/Y/Y	480
Formulation 3a	118	Y/Y/Y/Y/Y	Y/Y/Y/Y/Y	625

*Product quality means the percentage of sulfur in product flow.
†This profit cannot be achieved in reality because of the violation of the quality bounds!
‡"Y" or "N" indicates whether the product quality upper bound is satisfied for each of the five scenarios.

Table 3. Case Study A Computational Results With Three Uncertain Parameters

Number of Scenarios	1	8	27	64	125
Number of variables	16/13*	16/97	16/325	16/769	16/1501
Time with BARON (s)	0.1	2.8	199.9	9011.8	–†
Time with decomposition method‡ (s)	0.9	4.0	17.1	30.5	77.4

*Number of integer variables/number of continuous variables.
†No solution is returned after 100,000 s.
‡This is the total time for solving all the subproblems in the local solvers.

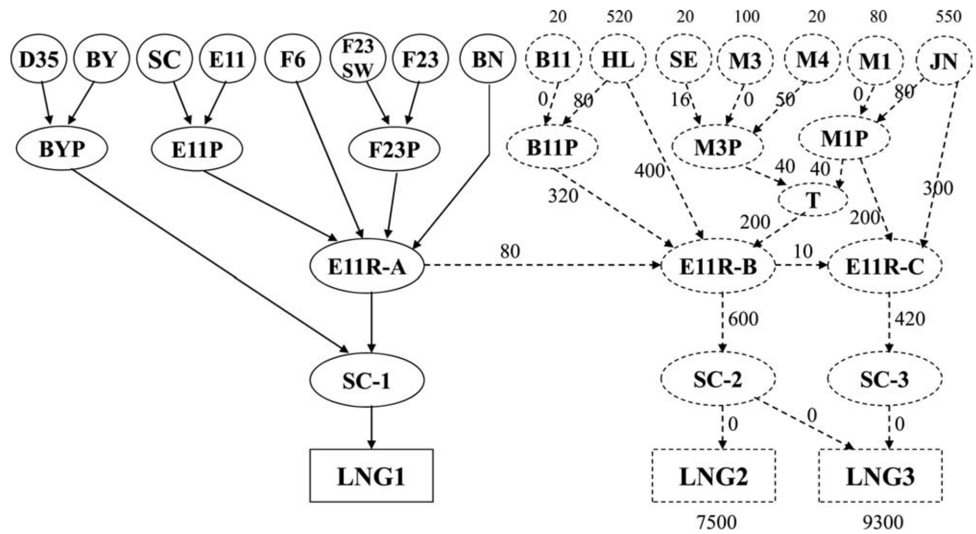

Figure 5. The superstructure and economic information for case study B.

Labeled data are costs of gas fields, LNG plants and pipelines (unit: Million \$). Costs of pools (Million \$): (a) BYP, E11P, F23P, B11P, M3P, M1P, E11R-A, E11R-B, E11R-C: 500 (b) T, SC-1, SC-2, SC-3: 0 Price of natual gas: 5364.17\$/Mmol (1 Mmol = 10^6 mol)

are for the gas flows entering the LNG plants and they are due to the CO_2 separation capacity at the LNG plants.) The uncertainty in this design problem comes from the quality of gas field M1 (i.e., the CO_2 mole percentage of gas from M1), which obeys a normal distribution with a mean of 3.34 mol % and a standard deviation of 0.6 mol %. Five scenarios of the uncertain M1 quality are selected according to the sampling rule described before. In the deterministic formulations, the mean quality is used as the deterministic quality, whereas in the stochastic formulations, the five M1 qualities in the five scenarios are addressed explicitly.

Formulations 1, 2, 3a, and 3b are compared for this problem. All these formulations have an economic objective to maximize the net present value with a discount rate of 12%. Formulation 3b has an additional objective with a lower priority, which is to maximize the total flow rate of the individual flows of CO_2 entering the LNG plants. This additional objective is introduced into the formulation to exploit the gas in the sour fields as much as possible (as long as the quality constraints are satisfied at the LNG plants). The hierarchical optimization approach discussed in the previous section is applied for solving this multiobjective optimization problem.

Figures 6b–d show the three system designs using the four formulations (Formulations 3a and 3b lead to the same system). As Formulation 1 does not consider the quality constraints at the LNG plants, the new part of the system designed with this formulation contains gas fields M1 and JN for the lowest investment cost. However, this design is infeasible for some scenarios considering the quality constraints at the LNG plants, because the quality of gas field JN severely violates the quality upper bounds, and the quality of gas field M1 violates the bounds as well in some scenarios. Formulation 2 observes the quality constraints, so the new part of the system designed by this formulation has gas fields B11, M3, and M1 instead. The blending of the gases from these fields can satisfy the quality constraints at the LNG plants in the deterministic case. The drawback of this design is that the quality of M1 may be so high that M1 cannot supply as much gas for blending to final products as expected by the deterministic formulation; in this situation, gas field M1 will be of little use and the investment in it is not profitable. When considering the quality uncertainty explicitly in Formulations 3a and 3b, the designed system is different from the one designed with Formulation 2, where gas fields HL, SE, and M4 are developed instead of B11. Although these gas fields are more expensive to develop than B11, they can serve gas flows with much better qualities, so M1 can still supply substantial amount of gas for blending final products when its quality is much worse than the mean quality value.

The advantage of the two stochastic formulations over the two deterministic formulations can be further recognized with Table 4, which summarizes the design and operation results of the case study B problem with different formulations. For each formulation, the total capital cost is calculated according to each designed system and shown in the table; the net present value of each scenario is calculated and the average over the five scenarios is shown in the table. As the operating cost is not included in all the formulations, all the net present values shown in Table 4 will be higher than the real ones. Table 4 also shows whether the product quality upper bound is satisfied at the LNG plants for each scenario. It can be found that with Formulation 1, the product quality at either of the two new LNG plants violates the bound for four of the five scenarios, so the net present value calculated by this formulation is meaningless for the real problem. Formulations 2, 3a, and 3b observe the product quality constraints, but Formulations 3a and 3b achieve better average net present values than the one achieved by Formulation 2 (with a large improvement of \$3.2 billion). Table 5 compares the CO_2 percentage at each LNG plant in each scenario with Formulation 3a and that with Formulation 3b. It can be inferred that Formulation 3b achieves a better operation than Formulation 3a does in the sense that it prefers to exploit more gas from sour fields while observing all the

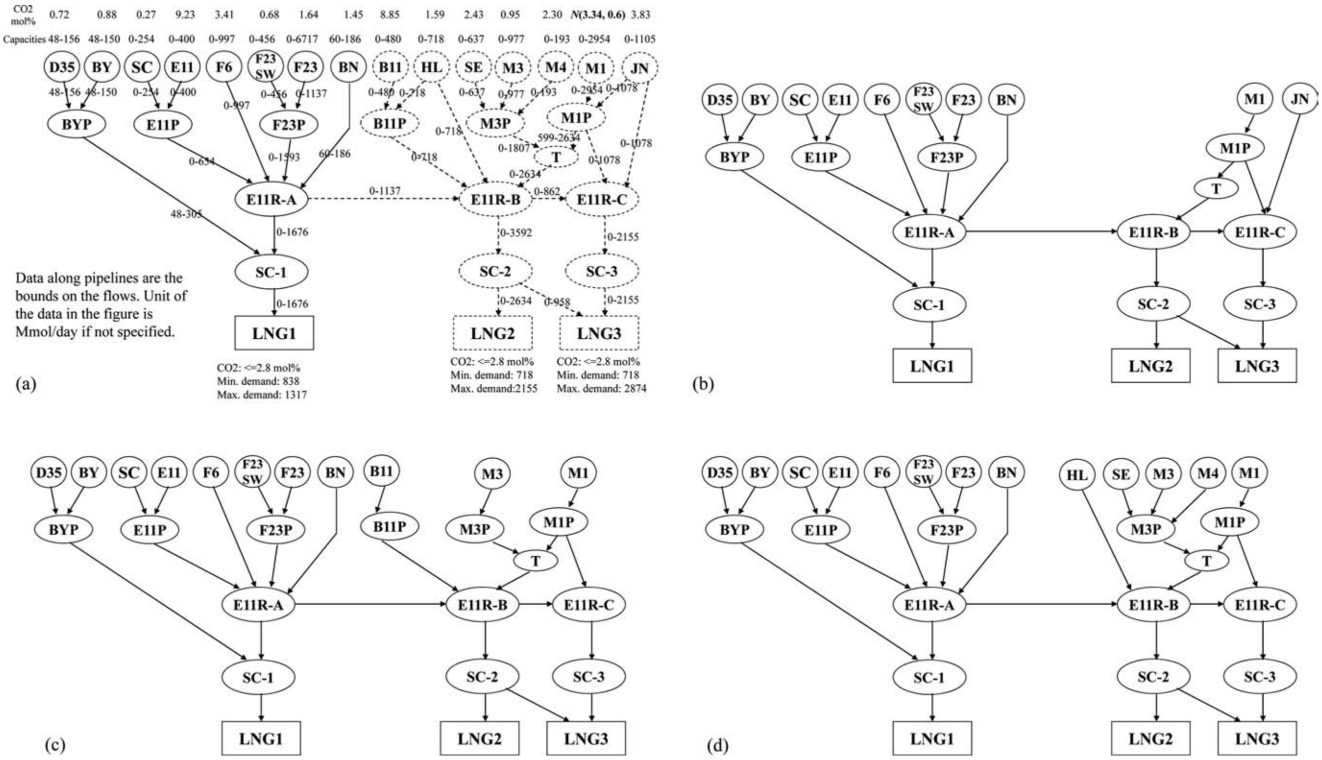

Figure 6. Case study B and the different design results.

(a) The problem superstructure and parameters. (b) System design with Formulation 1. (c) System design with Formulation 2. (d) System design with Formulations 3a and 3b.

quality constraints (because the CO_2 percentage with Formulation 3b is higher than or equal to that with Formulation 3a for each scenario and each LNG plant).

Again, the computational efficiencies of BARON and the decomposition method for case study B are compared, by solving the stochastic pooling problem Formulation 3a with different numbers of scenarios. It is assumed that there are four independent uncertain parameters in the system, which are the qualities of gas fields M1, JN, and the maximum demands at LNG plants 2 and 3, which obey normal distributions with means 5.04 mol %, 2.63 mol %, 1736 Mmol/day, and 2275 Mmol/day, and standard deviations 1 mol %, 0.4 mol %, 144 Mmol/day, and 239 Mmol/day, respectively. 1, 2, 3, 4, and 5 scenarios are generated for each uncertain parameter in the way described before, which lead to problems with 1, 16, 81, 256, and 625 scenarios. These five problems are solved with both BARON and the decomposition method, and the solver times are displayed in Table 6. Although the decompostion method is slower than BARON when the number of scenarios is 1 (i.e., a deterministic formulation is solved), it is much faster when more scenarios are addressed, and the solver time with the decomposition method increases moderately with the number of scenarios. On the other hand, BARON cannot obtain a solution for the problem within 100,000 seconds when 3 or more scenarios are addressed for each uncertain parameters (i.e., 81 or more scenarios for the problem).

Conclusions and Future Work

A stochastic pooling problem optimization formulation for the integrated design and operation of natural gas production systems is presented in this article. This formulation is developed based on combining a generalized pooling model with a two-stage stochastic recourse approach. Economic objectives, such as annualized profit and net present value, can be

Table 4. Case Study B Design and Operation Results with Different Formulations

	Average Net Present Value (Billion $)	Satisfaction of Product Quality* Bound at Each LNG Plant for the Five Scenarios			Capital Cost (Billion $)
		LNG 1	LNG 2	LNG 3	
Formulation 1	33.1†	Y/Y/Y/Y/N‡	Y/Y/N/N/N	Y/N/N/N/N	20.8
Formulation 2	29.0	Y/Y/Y/Y/Y	Y/Y/Y/Y/Y	Y/Y/Y/Y/Y	21.1
Formulation 3a	32.2	Y/Y/Y/Y/Y	Y/Y/Y/Y/Y	Y/Y/Y/Y/Y	21.6
Formulation 3b	32.2	Y/Y/Y/Y/Y	Y/Y/Y/Y/Y	Y/Y/Y/Y/Y	21.6

*Product quality means the percentage of CO_2 in the product.
†This net present value cannot be achieved in reality because of the violation of the quality bound!
‡"Y" or "N" indicates whether the quality upper bound is satisfied or not for each of the five scenarios.

Table 5. Case Study B Gas Product Qualities with Formulations 3a and 3b

	CO_2 Mole Percentage in the Gas Flow Entering Each LNG Plant in Each of the Five Scenarios		
	LNG1	LNG2	LNG3
Formulation 3a	2.8/2.6/1.7/1.2/1.2	2.4/2.4/2.4/2.1/1.8	2.2/2.5/2.7/2.8/2.8
Formulation 3b	2.8/2.8/2.8/2.4/1.2	2.4/2.8/2.5/2.1/1.8	2.2/2.7/2.8/2.8/2.8

incorporated in the stochastic pooling framework, and a problem with multiple objectives can be handled via a hierarchical optimization approach. The case study results for a simple system and a real industrial system demonstrate the advantages of the proposed stochastic formulation over the traditional deterministic formulations by observing quality constraints and addressing uncertainty in the system. The advantage of having multiple objectives over having single objective in the optimization is also shown in the case studies.

The stochastic pooling problem is a nonconvex MINLP problem, whose scale depends on the number of scenarios addressed. The state-of-the-art branch-and-reduce global optimization solver, BARON, can solve the stochastic pooling problem with a small number of scenarios within reasonable time. When the number of scenarios increases (due to more uncertain parameters included, larger uncertainty regions or finer uncertainty representation), however, the run time with BARON increases dramatically and can be prohibitively large in practice. Therefore, a rigorous decomposition method developed recently is used for solving the stochastic pooling problem. The computational study results show that the solver time with the decomposition method increases moderately with the number of scenarios, and an industrial problem with 38 binary variables and 36,251 continuous variables can be solved to guaranteed global optimality within 14 min of solver time. This indicates the potential of the stochastic pooling problem formulation and the decomposition method for real industrial systems with larger scales and larger numbers of uncertainty scenarios.

In the decomposition method, a large number of decomposed subproblems can be solved in parallel without exchanging any information between them. Therefore, exploitation of a parallel computing architecture can significantly reduce the run time, and this will be an interesting avenue for future work. Also, the stochastic pooling problem formulation can be extended to include more than two stages. The multistage stochastic pooling problem formulation can be naturally applied to the multistage design, which is a commonly applied strategy for natural gas production systems.

Table 6. Case Study B Computational Results with Four Uncertain Parameters

Number of Scenarios	1	16	81	256	625
Number of Variables	38/59*	38/929	38/4699	38/14849	38/36251
Time With BARON (s)	1.6	3894.1	–†	–	–
Time With Decomposition Method‡ (s)	4.4	19.5	98.4	376.4	792.6

*Number of integer variables/number of continuous variables.
†No solution is returned after 100000 seconds.
‡This is the total time for solving all the subproblems in the local solvers.

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Literature Cited

1. International Energy Agency. World Energy Outlook 2009. 2009.
2. Speight JG. *Natural Gas: A Basic Handbook*. Houston, Texas: Gulf Publishing Company, 2007.
3. Charnes A, Cooper WW, Mellon B. Blending aviation gasolines — a study in programming interdependent activities in an integrated oil company. *Econometrica*. 1952;20:135–159.
4. Aronofsky JS, Williams AC. The use of linear programming and mathematical models in underground oil production. *Manage Sci*. 1962;8:394–407.
5. Lee AS, Aronofsky JS. A linear programming model for scheduling crude oil production. *J Petrol Tech*. 1958;10:51–54.
6. Flanigan O. Constrained derivatives in natural gas pipeline system optimization. *J Petrol Tech*. 1972;24:549–556.
7. Bohannon J. A linear programming model for optimum development of multi-reservoir pipeline systems. *J Petrol Tech*. 1970;22:1429–1436.
8. Sullivan J. A computer model for planning the development of an offshore gas field. *J Petrol Tech*. 1982;34:1555–1564.
9. Nygreen B, Christiansen M, Haugen K, Bjørkvoll T, Kristiansen Ø. Modeling Norwegian petroleum production and transportation. *Ann Oper Res*. 1998;82:251–267.
10. Murray JE III, Edgar TF. Optimal scheduling of production and compression in gas fields. *J Petrol Tech*. 1978;30:109–116.
11. McFarland JW, Lasdon L, Loose V. Development planning and management of petroleum reservoirs using tank models and nonlinear programming. *Oper Res*. 1984;32:270–289.
12. Iyer RR, Grossmann IE. Optimal planning and scheduling of offshore oil field infrastructure investment and operations. *Ind Eng Chem Res*. 1998;37:1380–1397.
13. van den Heever SA, Grossmann IE. A Lagrangean decomposition heuristic for the design and planning of offshore hydrocarbon field infrastructure with complex economic objectives. *Ind Eng Chem Res*. 2001;40:2857–2875.
14. van den Heever SA, Grossmann IE, Vasantharajan S, Edwards K. Integrating complex economic objectives with the design and planning of offshore oilfield infrasturctures. *Comput Chem Eng*. 2000; 24:1049–1055.
15. Lin X, Floudas CA. A novel continuous-time modeling and optimization framework for well platform planning problems. *Optim Eng*. 2003;4:65–95.
16. Misener R, Floudas CA. Advances for the pooling problem: Modeling, global optimization, and computational studies. *Appl Comput Math*. 2009;8:3–22.
17. Furman KC, Androulakis IP. A novel MINLP-based representation of the original complex model for predicting gasoline emissions. *Comput Chem Eng*. 2008;32:2857–2876.
18. Haverly CA. Studies of the behaviour of recursion for the pooling problem. *ACM SIGMAP Bull*. 1978;25:29–32.
19. Haverly CA. Behaviour of recursion model - more studies. *ACM SIGMAP Bull*. 1979;26:22–28.
20. Lasdon LS, Waren AD, Sarkar S, Palacios F. Solving the pooling problem using generalized reduced gradient and successive linear programming algorithms. *ACM SIGMAP Bull*. 1979;27:9–15.
21. Floudas CA, Aggarwal A. A decomposition strategy for optimum search in the pooling problem. *ORSA J Comput*. 1990;2:225–235.
22. Visweswaran V, Floudas CA. New properties and computational improvement of the GOP algorithm for problems with quadratic

objective functions and constraints. *J Global Optim.* 1993;3:439–462.
23. Androulakis IP, Visweswaran V, Floudas C. Distributed decomposition-based approaches in global optimization. In: Floudas CA, Pardalos PM. *State of the Art in Global Optimization: Computational Methods and Applications.* Dordrecht, The Netherlands: Kluwer Academic Publishers, 1996.
24. Foulds LR, Haugland D, Jornsten K. A bilinear approach to the pooling problem. *Optimization.* 1992;24:165–180.
25. Ben-Tal A, Eiger G, Gershovitz V. Global minimization by reducing the duality gap. *Math Prog.* 1994;63:193–212.
26. Adhya N, Tawarmalani M, Sahinidis NV. A Lagrangian approach to the pooling problem. *Ind Eng Chem Res.* 1999;38:1956–1972.
27. Tomasgard A, Rømo F, Fodstad M, Midthun K. *Optimization models for the natural gas value chain.* In: Hasle G, Lie K, Quak E. *Geometric Modelling, Numerical Simulation, and Optimization: Applied Mathematics at SINTEF.* Berlin: Springer, 2007.
28. Rømo F, Tomasgard A, Hellemo L, Fodstad M, Eidesen BH, Pedersen B. Optimizing the Norwegian natural gas production and transport. *Interfaces.* 2009;39:46–56.
29. Gounaris CE, Misener R, Floudas CA. Computational comparison of piecewise-linear relaxations for pooling problems. *Ind Eng Chem Res.* 2009;48:5742–5766.
30. Liberti L, Pantelides CC. An exact reformulation algorithm for large nonconvex NLPs involving bilinear terms. *J Global Optim.* 2006;36:161–189.
31. Sherali HD, Alameddine A. A new reformulation-linearization technique for bilinear programming problems. *J Global Optim.* 1992;2:379–410.
32. Tawarmalani M, Sahinidis NV. Convexification and Global Optimization in Continuous and Mixed-Integer Nonlinear Programming. Dordrecht, The Netherlands: Kluwer Academic Publishers, 2002.
33. Quesada I, Grossmann IE. Global optimization of bilinear process networks with multicomponent flows. *Comput Chem Eng.* 1995; 19:1219–1242.
34. Furman KC, Jia Z, Ierapetritou MG. A robust event-based continuous time formulation for tank transfer scheduling. *Ind Eng Chem Res.* 2007;46:9126–9136.
35. Meyer CA, Floudas CA. Global optimization of a combinatorially complex generalized pooling problem. *AIChE J.* 2006;52:1027–1037.
36. Karuppiah R, Grossmann IE. Global optimization of multiscenario mixed integer nonlinear programming models arising in the synthesis of integrated water networks under uncertainty. *Comput Chem Eng.* 2008;32:145–160.
37. Sahinidis NV. Optimization under uncertainty: state-of-the-art and opportunities. *Comput Chem Eng.* 2004;28:971–983.
38. Birge JR, Louveaux F. *Introduction to Stochastic Programming.* New York: Springer, 1997.
39. Midthun KT. Optimization models for liberalized natural gas markets. Ph.D. thesis, Norwegian University of Science and Technology, 2007.
40. Jonsbraten TW. Optimization Models for Petroleum Field Exploitation. Ph.D. thesis, Stavanger College, 1998.
41. Goel V, Grossmann IE. A stochastic programming approach to planning of offshore gas field developments under uncertainty in reserves. *Comput Chem Eng.* 2004;28:1409–1429.
42. Goel V, Grossmann IE, El-Bakry AS, Mulkay EL. A novel branch and bound algorithm for optimal development of gas fields under uncertainty in reserves. *Comput Chem Eng.* 2006;30:1076–1092.
43. Tarhan B, Grossmann IE, Goel V. Stochastic programming approach for the planning of offshore oil or gas field infrastructure under decision-dependent uncertainty. *Ind Eng Chem Res.* 2009;48: 3078–3097.
44. Tawarmalani M, Sahinidis NV. A polyhedral branch-and-cut approach to global optimization. *Math Prog.* 2005;103:225–249.
45. Adjiman CS, Androulakis IP, Floudas CA. Global optimization of mixed-integer nonlinear problems. *AIChE J.* 2000;46:1769–1797.
46. Duran M, Grossmann IE. An outer-approximation algorithm for a class of mixed nonlinear programs. *Math Prog.* 1986;66:327–349.
47. Fletcher R, Leyffer S. Solving mixed integer nonlinear programs by outer approximation. *Math Prog.* 1994;66:327–349.
48. Kesavan P, Allgor RJ, Gatzke EP, Barton PI. Outer approximation algorithms for separable nonconvex mixed-integer nonlinear programs. *Math Prog, Series A.* 2004;100:517–535.
49. Geoffrion AM. Generalized Benders decomposition. *J Optim Theor Appl.* 1972;10:237–260.
50. Grossmann IE. Review of nonlinear mixed-integer and disjunctive programming techniques. *Optim Eng.* 2002;3:227–252.
51. Audet C, Brimberg J, Hansen P, Digabel SL, Mladenović N. Pooling problem: alternative formulations and solution methods. *Manag Sci.* 2004;50:761–776.
52. Mulvey JM, Vanderbei RJ, Zenios SA. Robust optimization of large-scale systems. *Oper Res.* 1995;43:264–281.
53. Selot A, Kuok LK, Robinson M, Mason TL, Barton PI. A short-term operational planning model for natural gas production systems. *AIChE J.* 2008;54:495–515.
54. Geoffrion AM. Elements of large-scale mathematical programming: Part I: Concepts. *Manag Sci.* 1970;16:652–675.
55. McCormick GP. Computability of global solutions to factorable nonconvex programs: Part I. Convex underestimating problems. *Math Prog.* 1976;10:147–175.
56. Benders JF. Partitioning procedures for solving mixed-variables programming problems. *Numer Math.* 1962;4:238–252.
57. GAMS. General Algebraic and Modeling System. Available at http://www.gams.com/ Accessed on September 24, 2010.
58. Gill PE, Murray W, Saunders MA. SNOPT: an SQP algorithm for large-scale constrained optimization. *SIAM Rev.* 2005;47:99–131.
59. Sahinidis NV. BARON: a general purpose global optimization software package. *J Global Optim.* 1996;8:201–205.
60. IBM. IBM ILOG CPLEX: High-performance mathematical programming engine. Available at http://www-01.ibm.com/software/integration/optimization/cplex/ Accessed on September 24, 2010.
61. Selot A. Short-term supply chain management in upstream natural gas system. Ph.D. thesis, Massachusetts Institute of Technology, 2009.
62. Seddon D. *Gas Usage & Value.* Tulsa, Oklahoma: PennWell, 2006.
63. Energy Information Administration. Energy Information Administration Webpage. Available at: http://www.eia.doe.gov Accessed on September 24, 2010.

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