A stochastic capacity expansion and equilibrium model for the global natural gas market

Ruud Egging, Winterschool, 10 March 2011

Outline

- Natural gas, market, players and roles
  - GAMS 1
- Capacity expansions
  - GAMS 2
- BREAK 15 min
- Uncertainty
  - GAMS 3
- Stoch MCP
- Benders for Stoch MCP
- BREAK / GAMS 4

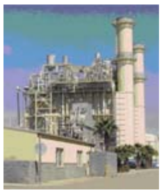
Main sources

- Egging 2010, PhD Thesis, http://drum.lib.umd.edu/handle/1903/11188
- IEA 2005 Energy Statistics Manual, www.iea.org/textbase/nppdf/free/2005/statistics_manual.pdf
- BP Statistical Review of World Energy 2010, http://bp.com/statisticalreview
- Gabriel, Fuller, 2010. A Benders Decomposition Method for Solving Stochastic Complementarity Problems with an Application in Energy. Computational Economics 35
- (more on natural gas: www.naturalgas.org)

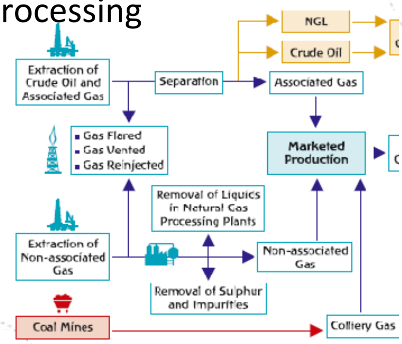
Natural Gas…



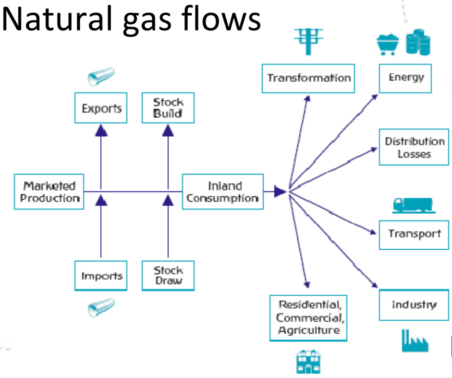





Natural gas production & processing

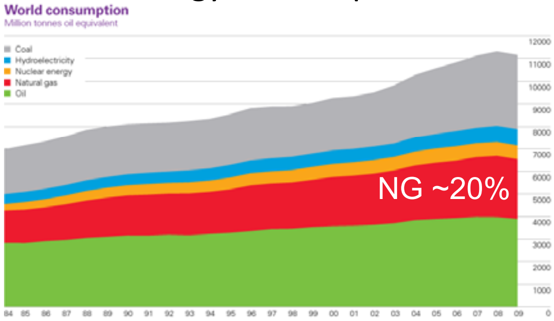


Natural gas flows



Source: IEA 2005, fig. 3.3, p 58

World energy consumption



World primary energy consumption fell by 1.1% in 2009, the first decline since 1982. Consumption was weaker than average in all regions. While oil remains the leading fuel (accounting for 34.8% of global primary energy consumption), it continues to lose market share. Coal's share of global energy consumption was the highest since 1970.

BP 2010

Developments and challenges

- Globally rising demand
- Locally depleting reserves
- Increasing number of players involved
- Increasing complexity of market and trade relations
- Increasing uncertainties and risks
- Interplay with environmental issues
- Conflicting objectives within and between countries

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In sum

- Complexity
- Uncertainty

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A Stochastic Global Gas Market Model

- Multi-period stochastic MCP with endogenous expansions for transport and storage
- 98% of worldwide consumption and production
- Market power
- Pipeline and LNG supply chains
- Contracts
- Demand seasonality
- Decision variables: various operating levels and capacity expansions

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Production

- State-owned vs. privately owned
  - ExxonMobil, RD Shell, …
  - GazProm, Qatargas, …
- Vertical/Horizontal integration
  - Oil, electricity,..
  - Trade, storage, …
- Representation:
  - Geographical regions
  - Produce and sell gas

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Trade

- Varying ownership and integration
  - GazExport , Pepco, RWE, GDF SUEZ, Washington Gas
- Representation:
  - Geographical regions
  - Purchase gas from producers
  - Sell gas to marketers
  - Storage and transportation services
  - Market power

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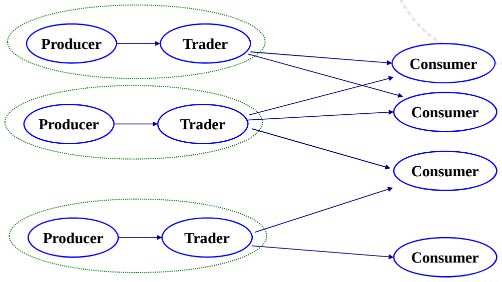
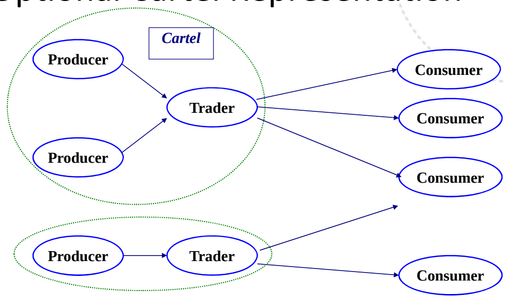
Standard Trader Representation



Optional Cartel Representation



Liquefaction and regasification

Figure 10: LNG supply chain. Source: Panhandle Energy[68]

- -260F: 1 m^3 LNG ~ 600 m^3 NG
- Cheaper or only option
- Liquefiers
  - E.g., GDF Suez, Qatargas
- Regasifiers
  - E.g., GDF Suez, Cheniere Energy

Storage Operators

- Ownership
  - Access regimes
- Roles:
  - Demand seasonality, strategic storage, speculation, …
- Representation: regulated service provider

Injection
April
May
June
July
August
September

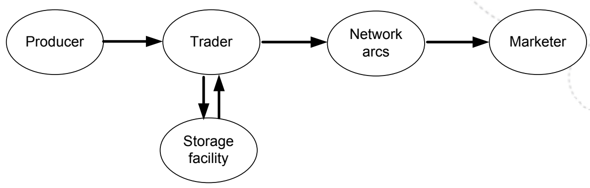
Extraction
October
November
December
January
February
March

Transport network

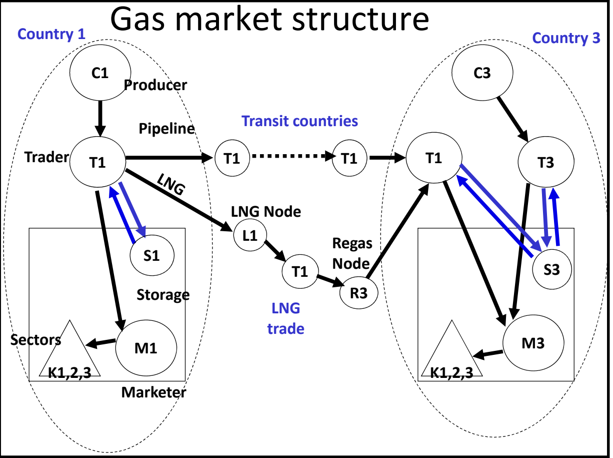
- International/domestic
- Ownership
- Access regimes

- Representation
  - Trans System Operator
  - All transportation arcs
  - Regulated service provider
  - Regulated fee + congestion charge

Supply chain



Gas market structure



Time to play

The assignment at the end of Monday's lecture

Oligopoly

Supplier: $\max \left(\pi\left(\sum_i q_i\right) q_i - c_i(q_i) - \tau^{pipe} q_i \right)$

s.t. $q_i \le cap_i^{prod} \quad (\lambda^{prod})$

Inverse demand: $\pi\left(\sum_i q_i\right)_i = \left(INT - SLP \sum_i q_i\right)$

Pipeline operator: $\max \left(\tau^{pipe} f^{pipe}\right)$

s.t. $f^{pipe} \le cap^{pipe} \quad (\lambda^{pipe})$

Market clearing for pipeline capacity:

$$f^{pipe} = \sum_i q_i \quad (\tau^{pipe})$$

Solution oligopoly

Supplier: $0 \le q_i \quad \perp -\left(INT - SLP \sum_i q_i - SLP q_i\right) + \frac{\partial c(q_i)}{q_i} + \lambda^{prod} + \tau^{pipe} \ge 0$

$0 \le \lambda^{prod} \perp cap_i^{prod} - q_i \ge 0$

Pipeline operator: $0 \le f^{pipe} \perp \lambda^{pipe} - \tau^{pipe} \ge 0$

$0 \le \lambda^{pipe} \perp cap^{pipe} - f^{pipe} \ge 0$

Market clearing: $f^{pipe} - \sum_i q_i = 0, \tau^{pipe}$ *free in sign*

Oligopoly in GAMS

- GAMS File Thu_1_...gms
- Walk-through
- Fill out the stationarity conditions (see slide 22)
- ~15 minutes

Oligopoly in GAMS

```
set I /1*3/;
alias (i,ii);
parameter int /10/, slp /1.0/
          cost_a / 1/, cost_b /0.5/
          cap_prod / 5/, cap_arc /5/;
positive variable q(i), flow, tau_arc, lam_prod(i), lam_arc;
equation stat_prod,eq_cap_prod, stat_arc_cap, eq_cap_arc, mcc_arc;

stat_prod(i) ..          -(int-slp*sum(ii,q(ii))-slp*q(i))
                         +cost_a+2*cost_b*q(i)
                         +lam_prod(i)
                         +tau_arc                =G= 0;
eq_cap_prod(i) ..        cap_prod - q(i)         =G= 0;
stat_arc_cap ..          lam_arc - tau_arc       =G= 0;
eq_cap_arc ..            cap_arc - flow          =G= 0;
mcc_arc ..               flow - sum(i,q(i))      =E= 0;
model oligop /   stat_prod.q,
                 eq_cap_prod.lam_prod
                 stat_arc_cap.flow
                 eq_cap_arc.lam_arc
                 mcc_arc.tau_arc/;
solve oligop using mcp;
```

Some assumptions

- Perfect foresight…? Perfect information!
- Risk-neutrality
- Open-loop: all decisions for all stages at start
  - Vs. closed-loop: 'feedback strategies': at every stage former decisions and outcomes taken into account when choosing a course of action
  - Assumptions open-loop more restrictive but generally mathematically tractable

Investments in MCP - outline

- Agent with perfect foresight
- Decide on sales in year y $SALES_y$
  capacity expansions Δ_y
- Selling price π_y discount rate γ_y (both exogenous)
- Convex cost curve $c_y(SALES_y)$
- Initial capacity $\overline{CAP}$
- Investment costs per unit b_y
- Upper bounds on expansions $\bar{\Delta}_y$

Investments – Formulation

$$\max_{SALES_y, \Delta^L_{lm}} \sum_{y \in Y} \gamma_y \left\{ \pi_y SALES_y - c_y(SALES_y) - b_y \Delta_y \right\}$$

$$s.t. \quad SALES_y \le \overline{CAP} + \sum_{y' < y} \Delta_{y'} \quad \forall y$$

$$\Delta_y \le \bar{\Delta}_y \quad \forall y$$

$$SALES_y \ge 0 \quad \forall y$$

$$\Delta_y \ge 0 \quad \forall y$$

Investments - KKT

$$0 \le \gamma_y \left(-\pi^L_y + \frac{\partial c_y(SALES_y)}{\partial SALES_y} \right) + \alpha_y \quad \perp \quad SALES_y \ge 0 \quad \forall y$$

$$0 \le \gamma_y b_y - \sum_{y' > y} \alpha_{y'} + \rho_y \quad \perp \quad \Delta_y \ge 0 \quad \forall y$$

$$0 \le \overline{CAP} + \sum_{y' < y} \Delta_{y'} - SALES_y \quad \perp \quad \alpha_y \ge 0 \quad \forall y$$

$$0 \le \bar{\Delta}_y - \Delta_y \quad \perp \quad \rho_y \ge 0 \quad \forall y$$

GAMS

- Code up the model on slide 28, for a two-period model, monopoly player, current capacity 5, investment costs 2/unit, Inverse Demand curve 20-q
- When you're done, time for a break
- ~25 minutes, **including** break

Capacity expansion

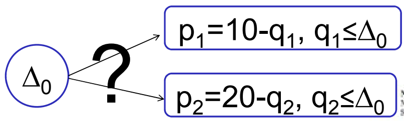
```
positive variable q, expans, lam;
equations         stat_q, stat_expans, eq_cap;

stat_q  ..        (2*q-20) + lam          =G= 0 ;
stat_expans ..    2 - lam                 =G= 0 ;
eq_cap ..         5 + expans - q          =G= 0 ;

model expans_det /  stat_q.q,
                    stat_expans.expans,
                    eq_cap.lam
/;
solve expans_det using mcp
;
```

Demand Uncertainty - Simple Stochastic Investment Problem

- Producer q: c(q)=0. Current Cap = 5
- Two periods
  - Period 1: Cap expansion I: 0 ≤ I ≤ 10, @ $2/unit
  - Period 2: 50%-50%: p=10-q OR p=20-q
- Invest how much to maximize exp profit?



Demand Uncertainty - Simple Stochastic Investment Problem

$$\max_{\Delta_0, q_1, q_2} \left\{ \tfrac{1}{2}(10 - q_1)q_1 + \tfrac{1}{2}(20 - q_2)q_2 - 2\Delta_0 \right\}$$

$$s.t.$$

$$0 \le q_1 \le 5 + \Delta_0$$

$$0 \le q_2 \le 5 + \Delta_0$$

Stochastic Investments - KKT

Minimization!

$$0 \le q_1 \perp \tfrac{1}{2}(2q_1 - 10) + \alpha_1 \ge 0$$

$$0 \le q_2 \perp \tfrac{1}{2}(2q_2 - 20) + \alpha_2 \ge 0$$

$$0 \le \Delta_0 \perp 2 - \alpha_1 - \alpha_2 \ge 0$$

$$0 \le \alpha_1 \perp 5 + \Delta_0 - q_1 \ge 0$$

$$0 \le \alpha_2 \perp 5 + \Delta_0 - q_2 \ge 0$$

GAMS!

- Code up the model on slide 33, using a set of scenarios
- ~5 minutes (see former exercise!)

GAMS – type it all up

```
positive variable q1,q2, del0, lam1, lam2;
equations         stat_q1, stat_q2, stat_del0, eq_cap1, eq_cap2;

stat_q1 ..        0.5*(2*q1-10) +lam1      =G= 0 ;
stat_q2 ..        0.5*(2*q2-20) +lam2      =G= 0 ;
stat_del0 ..      2 - lam1 - lam2          =G= 0 ;
eq_cap1 ..        5 +del0 -q1              =G= 0 ;
eq_cap2 ..        5 +del0 -q2              =G= 0 ;

model expans_sto /        stat_q1.q1,
                          stat_q2.q2,
                          stat_del0.del0,
                          eq_cap1.lam1,
                          eq_cap2.lam2

/;
```


GAMS, using sets and variables

```
positive variable q(m), expans, lam(m)
;
equations         stat_q, stat_expans, eq_cap
;
stat_q(m)  ..     prob(m)*(2*slp(m)*q(m)-int(m)) + lam(m) =G= 0 ;
stat_expans ..    cinv - sum(m,lam(m))         =G= 0 ;
eq_cap(m) ..      cap + expans - q(m)          =G= 0 ;

model expans_det / stat_q.q,
                   stat_expans.expans,
                   eq_cap.lam
/;
solve expans_det using mcp
```

Stochastic Natural Gas Market Model

- Extensive-form stochastic MCP
- Consider multiple futures when making capacity expansion decisions
- Additional assumption: risk-neutrality
- Maximize *expected* profits
- Include all considered futures and assign probabilities

Producer – Opt problem

$$\max_{q^{P\to T}_{pndm}} \sum_{n,d,m} p_m \gamma_m d_d \left(\pi^{p}_{ndm} q^{P\to T}_{pndm} - c^{P}_{pndm}\left(q^{P\to T}_{pndm} \right) \right)$$

$$q^{P\to T}_{pndm} \leq CAP^{P}_{pnm} \qquad \left(\alpha^{p}_{pndm} \right)$$

Producer - KKT

$$0 \leq q^{P\to T}_{pndm} \perp \quad p_m \gamma_m d_d \frac{\partial c^{P}_{pndm}(\cdot)}{\partial q^{P\to T}_{pndm}} + \alpha^{P}_{pndm} - p_m \gamma_m d_d \pi^{P}_{ndm}$$

$$0 \leq \alpha^{P}_{pndm} \perp \quad CAP^{P}_{pnm} - q^{P\to T}_{pndm} \geq 0$$

Arc operator – Opt problem

$$\max \sum_{am} p_m \gamma_m \left(\sum_{d} \left(d_d \tau^{A}_{adm} q^{A\to T}_{adm} \right) - c^{\Delta A}_{a} \Delta^{A}_{am} \right)$$

$$q^{A\to T}_{adm} \leq CAP^{A}_{am} + \sum_{m' \in pred(m)} \Delta^{A}_{am'} \qquad \left(\alpha^{A}_{adm} \right)$$

$$\Delta^{A}_{am} \leq \bar{\Delta}^{A}_{am} \qquad \left(\rho^{A}_{am} \right)$$

Arc operator - KKT

$$0 \leq q^{A\to T}_{adm} \perp \alpha^{A}_{adm} - p_m \gamma_m d_d \tau^{A}_{adm} \geq 0$$

$$0 \leq \Delta^{A}_{am} \perp p_m \gamma_m c^{\Delta A}_{am} + \rho^{A}_{am} - \sum_{d, m' \in succ(m)} \alpha^{A}_{adm'} \geq 0$$

$$0 \leq \alpha^{A}_{adm} \perp CAP^{A}_{am} + \sum_{m' \in pred(m)} \Delta^{A}_{am'} - q^{A\to T}_{adm} \geq 0$$

$$0 \leq \rho^{A}_{am} \perp \bar{\Delta}^{A}_{am} - \Delta^{A}_{am} \geq 0$$

Et cetera…

Players in the model

• Producer	V
• Arc network operator	V
• Trader	See Thesis
• Storage Operator	See Thesis
• Market clearing conditions,	next

Market clearing conditions

$$\pi^{P}_{ndm}\ free,\quad \sum_{p} q^{P\rightarrow T}_{pndm} - \sum_{t} q^{T\leftarrow P}_{tndm} = 0$$

$$\tau^{A}_{adm}\ free,\quad q^{A\rightarrow T}_{adm} - \sum_{t} f^{A}_{tadm} = 0$$

$$\pi^{TS}_{ndm}\ free,\quad \sum_{t} q^{T\rightarrow S}_{tnd_1m} - \sum_{s} q^{S\leftarrow T}_{snd_1m} = 0$$

$$\pi^{W}_{ndm}\ free,\quad \pi^{W}_{ndm} - \left(INT^{W}_{ndm} - SLP^{W}_{ndm} \left(\sum_{t} q^{T\rightarrow W}_{tndm} + \sum_{s} q^{S\rightarrow W}_{sndm} \right) \right) = 0$$

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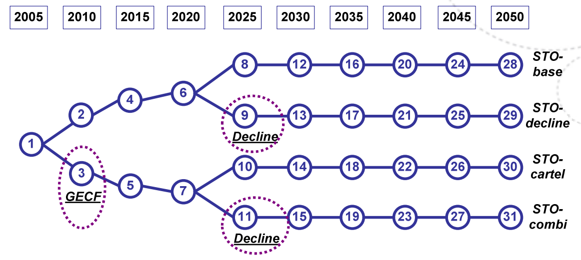
Scalability issues - Scenario tree with four scenarios



Capacity additions in 2005: four futures; 2010-2020: two futures

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Stochastic gas market model - Remarks

- Two uncertain events is not so much
- Doubling of model size, calc time: 5-10 times as big
- When uncertainty is in far future, the impact is largely 'discounted away'
- Hedging affects timing and sizes, but many results 'close to averages'
- Some detailed developments and results differ due to interplay of timing, hedging and game-theoretic approach

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Benders Decomposition Outline

- Some variables make the problem *hard*.
  - Fixing these: remaining problem easy
- Decompose problem in two parts
  - Difficult variables → Master Problem (MP)
  - Remaining variables → Sub Problems (SP)
- Iteratively MP and SP are solved
  - MP: 'fixing values'
  - SP: feasible solution + info to improve MP

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Orig problem: block structure

			$-Q_1$				>=	-CAP
Δ_1				$-Q_2$			>=	-CAP
Δ_1	$+\Delta_2$				$-Q_3$		>=	-CAP
Δ_1	$+\Delta_2$	$+\Delta_3$				$-Q_4$	>=	-CAP

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Orig problem: complicating var

			$-Q_1$				>=	-CAP
Δ_1				$-Q_2$			>=	-CAP
Δ_1	$+\Delta_2$				$-Q_3$		>=	-CAP
Δ_1	$+\Delta_2$	$+\Delta_3$				$-Q_4$	>=	-CAP

Complicating variables

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Sub problems in BD

$-Q_1$				>=	$-CAP$
	$-Q_2$			>=	$-CAP - \Delta_1$
		$-Q_3$		>=	$-CAP - \Delta_1 - \Delta_2$
			$-Q_4$	>=	$-CAP - \Delta_1 - \Delta_2 - \Delta_3$

Subproblems separate by block

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Benders Decomposition Loop

- INIT:
  - Convergence Gap = +INF
- WHILE (ConvGap >Treshold) DO:
  - Solve MP
    - Find suggested values for fixing variables
  - Solve SPs
    - Fix vars according to last MP Soln
    - Collect info to update MP
  - Update ConvGap
  - Pass info from SPs to MP by Adding a *Benders Cut*

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BD for Simple Stochastic Investment Problem

- Define MP: $\min_{I,\alpha} 2I + \alpha, \ s.t.\ 0 \le I \le 10$
- Define SP(1): $\min_{q_1} \ z_1 = \frac{1}{2}(q_1 - 10)q_1, \ s.t.\ q_1 \le 5 + I, \ I = I^k \ (\lambda_1)$
- Define SP(2): $\min_{q2} \ z_2 = \frac{1}{2}(q_2 - 20)q_2, \ s.t.\ q_2 \le 5 + I, I = I^k \ (\lambda_2)$

- INIT LB=-INF, UB=+INF
- MP: I=0, α=-inf
- SP: (q_1, z_1, λ_1) =(5,-12½, 0)
  (q_2, z_2, λ_2) =(5,-37½,-5)
- Add Cut to MP

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Benders Cuts

- Every cut limits MP solution space, cutting off infeasible solutions
- Uses
  - Dual Prices (λ_1, λ_2) of the SPs
  - the fixed MP solution I^k
- Provide linear approximation how aggregate SP objective would change from current fixed solution
- Approximation… ⇨ overestimate

$$\alpha \ge z_1^k + z_2^k + \sum_{i=1}^{2} \lambda_i^k \left(I - I^k \right) \ \Rightarrow \ \alpha \ge -50-5I$$

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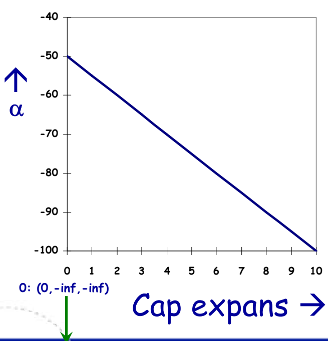
Adding First Benders Cut



#	Z	α	I
0	-inf	-inf	0

#: (I,α,z)

Cap expans →

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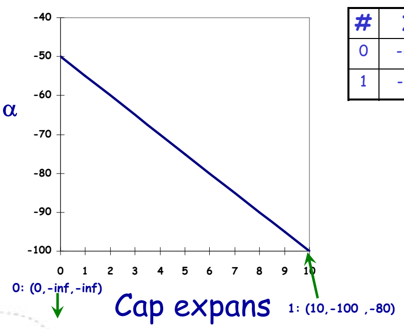
Second MP solution



#	Z	α	I
0	-inf	-inf	0
1	-80	-100	10

Cap expans

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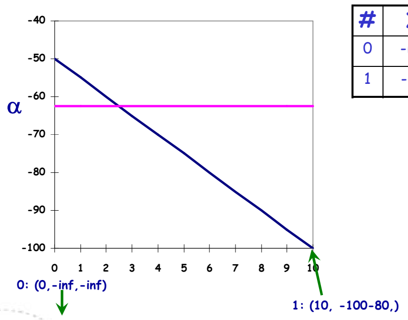
Second iteration SP solutions

- SP(1,2): $\min_{q_1} z_1 = \frac{1}{2}(q_1 - 10)q_1, \quad s.t.\ q_1 \le 5+10$
- SP(2,2): $\min_{q2} z_2 = \frac{1}{2}(q_2 - 20)q_2, \quad s.t.\ q_2 \le 5+10$

- $(q_1, z_1, \lambda_1) = (5, -12½, 0)$
- $(q_2, z_2, \lambda_2) = (10, -50\ \ , 0)$

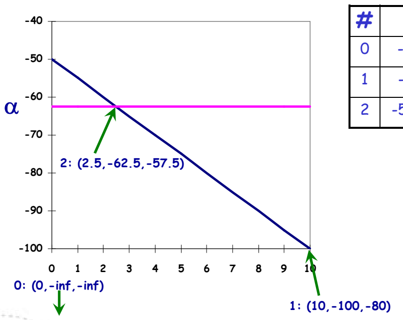
- Add Cut to MP: $\alpha \geq -62½$

Adding Second Benders Cut



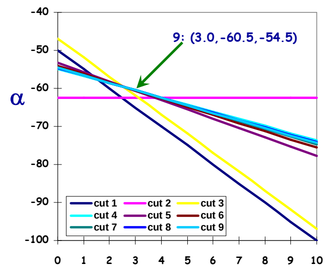
#	Z	α	I
0	-inf	-inf	0
1	-80	-100	10

Third MP solution



#	Z	α	I
0	-inf	-inf	0
1	-80	-100	10
2	-57.5	-62.5	2.5

And so on…



#	Z	α	I
0	-inf	-inf	0
1	-80	-100	10
2	-57.5	-62.5	2.5
3	-56.250	-62.500	3.125
4	-54.563	-59.689	2.563
5	-54.527	-60.215	2.844
6	-54.510	-60.480	2.985
7	-54.501	-60.611	3.055
8	-54.500	-60.540	3.020
9	-54.500	-60.506	3.003

Capacity expansion

Approach

- BD and cuts originally for optimization
- BD for Stoch MCP (Gabriel & Fuller, 2010)
  - general, two-stage, electricity market (stylized)
  - small number players, many scenarios
  - market power in SP: MP are LCP, SP are LCP
  - G&F derive alternative cut to be used for imperfectly competitive lower level problems

Benders Decomposition

- Benders (1962)
- Originally Mixed Integer Problems
- Fuller and Chung (2007): VIs
- Gabriel and Fuller (2008): Stoch MCP
  - two-stage
  - small #players
  - many scenarios

Notation

The following symbols are used in the cuts, for iteration *it*:

$\Delta_{am}^{A,it}$	MP solution value for arc capacity expansions in (mcm/d)
$\Delta_{sm}^{S,it}$	MP solution value for storage working gas capacity expansions
$\lambda_{adm}^{A,it}$	SP solution value for dual price of arc capacity constraint (7.8.
$\lambda_{sm}^{S,it}$	SP solution value for dual price of storage capacity constraint (
Z_{it}^{SP}	The probability-weighted discounted sum of SP obj

Traditional Benders Optimality Cuts

In iteration *it+1* the it^{th} cut is added to the MP. The set of cuts is written as:

$$\alpha + \sum_m p_m \gamma_m \left(\sum_{a,d} \lambda_{adm}^{A,it} \sum_{m' \in succ(m)} \Delta_{am}^{A} + \sum_s \lambda_{sm}^{S,it} \sum_{m' \in succ(m)} \Delta_{sm}^{S} \right) \quad \forall it \qquad (\theta^{it})$$
$$\geq Z_{it}^{SP} + \sum_m p_m \gamma_m \left(\sum_{a,d} \lambda_{adm}^{A,it} \sum_{m' \in succ(m)} \Delta_{am}^{A,it} + \sum_s \lambda_{sm}^{S,it} \sum_{m' \in succ(m)} \Delta_{sm}^{S,it} \right)$$

Cut according to Gabriel and Fuller (2010)

$$0 \leq \theta \perp \underbrace{\left(X_{\lambda}^{it}\right)^T \left(\bar{A}\Delta - \bar{b}\right) + \left(X_{\varphi}^{it}\right)^T \left(\hat{A}\Delta - \hat{b}\right)}_{\text{First stage variables and approximated impact}} + \underbrace{\left(X_{F(s)}^{it}\right)^T F^{-1} \left(X_{F(s)}^{it} \theta\right)}_{\text{Approximated impact second stage variables}} + \alpha \geq 0$$

Cut APPLIED

$$0 \leq \theta^{it} \perp \quad \alpha + \sum_m p_m \gamma_m \left(\begin{array}{l} \sum_{p,d} \lambda_{pdm}^{P,it} \left(\overline{CAP}_{pm}^{P} \right) \\ + \sum_{a,d} \lambda_{adm}^{A,it} \left(\overline{CAP}_{am}^{A} + \sum_{m' \in pred(m)} \Delta_{am}^{A} \right) \\ + \sum_{s} \lambda_{sm}^{S,it} \left(\overline{CAP}_{sm}^{S} + \sum_{m' \in pred(m)} \Delta_{sm}^{S} \right) \\ - \sum_{t,n,d,it'} d_d \left(INT_{ndm}^{W} - SLP_{ndm}^{W} \left(\begin{array}{l} \sum_{t' \in T} Q_{t'nd}^{T \to W,it} \\ + \delta_{tnm}^{T} Q_{tnd}^{T \to W,it} \end{array} \right) \right) \theta^{it'} Q_{t,n,d,m}^{T \to W,it'} \\ + \sum_{p,d,it'} d_d \left(\frac{\partial c_{pm}^{P} (Q_{pdm}^{P \to T,it})}{Q_{pdm}^{P \to T,it}} \right) \theta^{it'} Q_{pdm}^{P \to T,it'} \end{array} \right) \geq 0$$

Implementation

- Multi-stage
- GAMS 22, 2GB RAM, dual core 2x1.2 GHz

Convergence analysis

	A	B
Model periods	4	6
Scenarios	4	4
Scenario nodes	11	19
Num capacity expansion	339	763
Total num variables	27,221	47,373
Full MCP calc time (seconds)	263	1,005
VI-MCP Net calc time (seconds)	267	2,036
Num iterations	46	188

Convergence analysis - continued

	C (*)	D (*)	E (*)
Model periods	**6**	**8**	**8**
Scenarios	**8**	**4**	**8**
Scenario nodes	**31**	**27**	**47**
Num expansion var	1,187	1,187	2,035
Total num variables	77,177	67,525	117,481
Full MCP calc time	13,853	3,005	18,679
Num iterations	316	325	179
feasible MP calc time &	502	550	333
infeasible MP calc time %	122	96	301
feasible SP calc time &	4,934	4,576	4,373
infeasible SP calc time %	7	0	6
Num infeasible MP	14	8	7
Num infeasible SP	1	0	1

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	A	B	C (*)	D (*)	E (*)
Model periods	**4**	**6**	**6**	**8**	**8**
Scenarios	**4**	**4**	**8**	**4**	**8**
Scenario nodes	**11**	**19**	**31**	**27**	**47**
Num expansion var	339	763	1,187	1,187	2,035
Total num variables	27,221	47,373	77,177	67,525	117,481
Full MCP calc time	263	1,005	13,853	3,005	18,679
VI-MCP Net calc time ^	267	2,036	5,572	5,222	5,013
Num iterations	46	188	316	325	179
VI-MCP Gross calc time	521	13,684	52,272	51,207	32,502
feasible MP calc time &	4	129	502	550	333
infeasible MP calc time %	4	60	122	96	301
feasible SP calc time &	259	1,847	4,934	4,576	4,373
infeasible SP calc time %	0	0	7	0	6
Num infeasible MP	7	18	14	8	7
Num infeasible SP	0	0	1	0	1
Convergence criterion	Expans	Expans	MP infeas	MP infeas	MP infeas

Benders in GAMS

- Same problem with monopoly supplier facing a high and a low demand scenario

GAMS small BD

- Note: numerical deviations, but no complications yet…

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Conclusions

- Making use of Benders cuts from (Gabriel and Fuller, 2010) allows easier derivation MP and SP
- GAMS needs much time for model generation (unless it would be possible to keep models in memory) Implementations should use software that allows efficient file processing and model generation
- Large potential to reduce solution times of large-scale stochastic MCP (considering parallel processing)
- Numerical complications (!!!)

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Benders for Stoch MCP…?

Speculation:

- Fewer time periods
- Smaller first-stage problem

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GAMS

Package:

- AssignmentsOppdal.docx
- Thu_4_... Gms
- Open it.
  - Two data files, the main file, 'cournot parameter'
- Dan & Fritz!
- Optional: maximum expansion constraint (derive and adjust KKT and implement in GAMS)
- Until dinner… or when your are done

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