Stochastics in planning-the use of what-if analysis

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Why do we model?

- Complexity
- Uncertainty
- Often both ...

- And by model I mean any model we formulate on paper or just in our minds – to make an argument or draw a conclusion.
The simple question

• The world is obviously stochastic –
• but does it matter (to us)?

• Can we model and advise well based on deterministic thinking?
  – When we argue
  – When we calculate
  – When we teach
Outline

• Fashion apparel production
• Upstream logistics in petroleum production
• An example from network design
• Oil to Nepal
• A scheduling example
• The relationship to options
Basic setting


Example

You have two lots of land that you can develop

<table>
<thead>
<tr>
<th></th>
<th>Developing the land</th>
<th>Building the plant now</th>
<th>Building the plant later</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lot 1</td>
<td>600</td>
<td>200</td>
<td>220</td>
</tr>
<tr>
<td>Lot2</td>
<td>100</td>
<td>600</td>
<td>660</td>
</tr>
</tbody>
</table>

Plants produce one unit of a good sold at price $p$, presently unknown
Nine possible solutions

No 9 is to do nothing
Sensitivity analysis

Sample (or construct) possible futures, and solve for each of these possible futures. Compare (combine) solutions to find an overall solution.

- \( p < 700 \): Do nothing
- \( 700 < p < 800 \): Delayed construction is never used
- \( P > 800 \): No other scenario solutions are possible
- Flexibility has no value
Assume $p$ can take on two different values
$\Pr(p=210) = \Pr(p=1250) = 0.5$

Expected value $p = 730$

Deterministic problem: Use $p = 730$ and get Decision with profit $730 - 700 = 30$

Expected value of scenario solutions:

- Decision 4: $-700 + 0.5 \times 210 + 0.5 \times 1250 = 30$
- Decision 9: 0
- Decision 7: $-1500 + 0.5 \times 2 \times 210 + 0.5 \times 2 \times 1250 = -40$
<table>
<thead>
<tr>
<th>Decision</th>
<th>Invest p=210</th>
<th>Income if p=1250</th>
<th>Expected profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>600</td>
<td>1030</td>
<td>-85</td>
</tr>
<tr>
<td>2</td>
<td>800</td>
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<tr>
<td>3</td>
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<td>590</td>
<td>195</td>
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</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Expected profit for all scenarios

Delayed construction used whenever profitable
The IQ of hindsight

• After the fact one of the scenario solutions will turn out to be best
• The stochastic programming solution is hardly ever best
• Be careful with how decision makers are evaluated
Safety Personnel Example

- Three types of personnel A, B and C
- One safety related role
- One shift needs (10,2,3) personnel with two trained for safety role
- Training costs (20,15,10)
- If personnel are missing they must be replaced. Extra cost if they are needed in safety organization (100,250,250)
Who to train for a shift?
Scenarios for missing personnel

- Nobody is missing
- Two of type A are missing
- One of type A and one of type B
- One of type A and one of type C
- Two of type B
- Two of type C

All equally likely
Main question: How many to train?

Textbook method: Scenario analysis:
- Nobody is missing
  - Train two of type C at a cost of 20
- Two of type A are missing
  - Train two of type C at a cost of 20
- One of type A and one of type B
  - Train two of type C at a cost of 20
- One of type A and one of type C
  - Train two of type B at a cost of 30
- Two of type B are missing:
  - Train two of type C at a cost of 20
- Two of type C are missing:
  - Train two of type B at a cost of 30
Expected cost of scenario solutions:

- Two B's: $30 + \frac{1}{6}(0+0+250+0+500+0) = 155$
- Two C's: $20 + \frac{1}{6}(0+0+0+2/3 \times 250+0+(1/3 \times 500+2/3 \times 250)) = 103$

But what about?

- Two A's: Expected cost = 53.
- Two A's and a C: Total cost = 51.5
- Three C's and a B: Total cost = 45

The latter is cheaper and without risk
Scenario solutions

Cost 155

Non-scenario solution

Cost 103

Cost 45
What did we see?

- The scenario solutions were not particularly good
- There was another solution which was cheaper and without risk
- The tradeoff between profit and risk was “false” due to the chosen method.
- Flexibility was not valued
In the language of options …

- An option is the possibility to observe the outcome of a random variable, and do something about it (if you so wish)
- An option normally comes at a cost
- It provides flexibility
- And flexibility has value
A deterministic model …

• will never suggest to train more people than you need,

• because it “knows” that there will be no surprises – it “knows” the need.

• If you run your model with several different needs (scenarios)
  – You will get differently training patterns, but not a single one will contain extra people!

• You will not get any options!
Options

• … and the union of many solutions without options will also be without options.

• So, if flexibility is important, you need another tool, what-if analysis (scenario analysis, sensitivity analysis) will not do.
What are options?

• More than buffers and slack
• Options can be implicit in the solution
  – The option of doing it differently
• They are in general simply different!
Some examples

- Let planes cross at airports
- Limit the number of different planes at an airport
- Do not mix crews
- Do not be too clever at optimizing the schedule!
Just for mathematics?

• No, applies to any structured approach to (strategic) planning
• Scenario thinking does not deliver! It is not the right way of thinking.
• Difficulties of uncertainty do not go away just by not thinking about them or pretending they do not exist.
Using what-if in real life

A MIP model has been made for the Norwegian Petroleum Directorate to support the development of oil and gas fields in addition to the transportation infrastructure in the North Sea. The model takes more than one hour to run. 50 runs have been made with different levels of demand, prices, and field sizes.
• In 80% of all cases, the gas pipe from A to B has a diameter of 24”. In the other cases it is larger. Hence, we can conclude that 24” is a lower bound.

• In all cases, field C is developed between 2015 and 2025. Hence, we can safely assume that we have found a time interval in which the field is developed. Choosing 2020 is clearly a good approximation.
• These conclusions may be good or correct.

• The arguments leading to the conclusions are false.
A small case from network design

• Number of commodities: 2

• Capacity of truck: 2

• Demand either 0, 2 or 4 with equal probabilities of occurring
Deterministic Solution
– Planning using averages

2 Trucks
Stochastic Solution
- Taking uncertainty into account

It is always beneficial to have commodities share paths – even if they are positively correlated.
Expected Overflow

Period t
Four commodities

Truck routes in stochastic solution

Truck routes in deterministic solution
Counting Paths

The commodities:

- 6 Paths
- 10 Paths

Flow of freight can be shifted from one path to another.
Insurance

• Negative correlations amount to insurance.
• Always look for them – they are very helpful
• You may have to make sure you can utilize the negative correlations
Other Findings

• We obtain hub-and-spoke systems due to uncertainty and uncertainty alone!
  – Not because we enforced it, but because it is the best for our problems.

• Deterministic models would not give any of these properties
  – Many paths
  – Shared paths
  – Consolidation for hedging
Robustness and flexibility

- Many peculiar definitions around
- For us: We wish a robust schedule in light of random demand by having flexibility in commodity routing.

- **Robustness**: The ability to withstand shocks (like a tree)
- **Flexibility**: The ability to adjust to shocks (like grass)
- These are *modeling* choices
Examples

• Bus schedules should be robust as seen by customers
• Customers care about service robustness, but probably not robustness in truck routes. But maybe drivers care?
• Nobody (?) cares about robustness in commodity itineraries, so flexibility may be modeled to achieve robustness at higher levels.
Examples …

• In production we may accept both, and simply go by minimal cost.

• In our network design case:
  – Services are given and hence robust
  – Truck routes are modeled as robust
  – Itineraries are modeled as flexible
Product variety and hedging in fashion apparel production

• Product variety and some initial production decisions must be determined before the major fashion trends are known
• How do we hedge against this uncertainty?
• Will assumptions of normal distributions do
A model

• A multi-item newsboy model
• Markowitz-like portfolio setup
• Demand is two-level:
  – The state-of-the-world – this year’s trend
  – Given trend, conditional distributions are approximately normal
Two competing SKUs
Does it matter?

Yes, it matters a lot. A portfolio should be built to meet the uncertainty. If you miss the form of the distribution, you can be very far off a good choice of collection. Extra information not very useful.
Upstream logistics in offshore petroleum production

- Norwegian Sea – very harsh climate
- Shortage costs of 15 million GBP per day
- 10+ fields, 3 vessels, one onshore base
- Main uncertainty: Bad weather prohibiting pickups and deliveries.
- How to think when planning
  - Tools
  - Information
Uncertainty

• How to make a model for this vehicle routing problem?

• The basic facts:
  – Nice weather: this is an easy problem
  – Bad weather: this is easy too
  – Potentially bad weather: The real issue: How to run a system when the weather **might** turn bad?
Organizational issues

• The float of information in an organization
  – randomness is often lost between departments

• Will result in reduced randomness and hence reduced value of flexibility.
Board rooms

• Leaders want **clear advise** and explanations

• Results in **randomness being suppressed** and hence flexibility being **undervalued**
Discount rates

• Many companies use very high **discount rates** to express that only very profitable projects will be accepted

• Investments in flexibility normally have costs early and potential income late

• Results in **reduced option values** and incorrect investments in accepted projects
Subjective issues

• **Over confidence in own estimates**
  – reduces option values

• **Hindsight learning**
  – we have problems seeing that other scenarios could have happened
  – bad learning
Flexibility is often important, but …

• Disregarded by methods
• Lost in organizations
• Undervalued by individuals
The way forward

- Study more problems to understand better how uncertainty affects decision problems
- Use the understanding to solve practical problems in a practical way.
- Understand the planning process itself and how it is affected by uncertainty