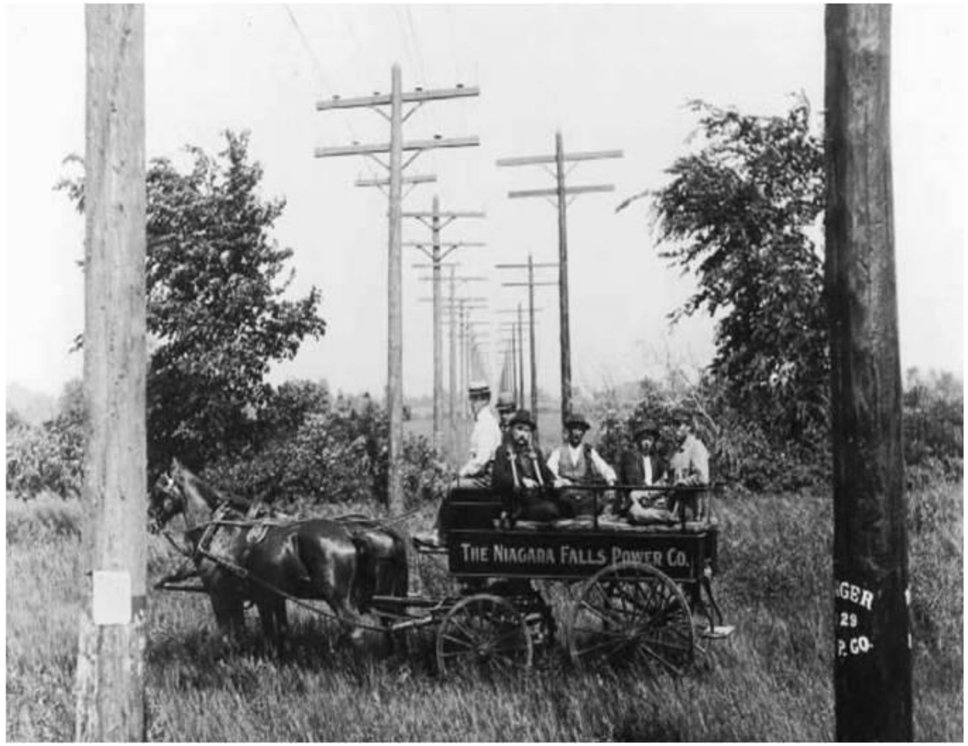
Math Background



Antonio J. Conejo
Univ. Castilla – La Mancha
2011

Contents

— Complementarity

— Stochastic programming

Complementarity

— Optimization problem

— OPcOP

—MPEC

—Equilibrium

—EPEC

References

- R. W. Cottle, J.-S. Pang, and R. E. Stone. *The Linear Complementarity Problem. Society for Industrial and Applied Mathematics (SIAM), Philadelphia,* 2009.
- Z.-Q. Luo, J.-S. Pang, and D. Ralph. *Mathematical Programs with Equilibrium Constraints. Cambridge University Press, Cambridge, UK, 2008.*
- S. A. Gabriel, A. J. Conejo, J. D. Fuller, B. F. Hobbs, C. Ruiz *Complementarity Modeling in Energy Markets Springer, New York, Fall 2011.*

Complementarity
(Optimization Problem)

Optimization Problem (OP)
Objective function (minimize or maximize)
subject to:
Equality constraints Inequality constraints Bound on variables

Complementarity (MPEC)

Mathematical Program with Equilibrium Constraints (MPEC)

Objective function (minimize or maximize)

subject to:

Constraining optimization problem 1

•
•
•

Constraining optimization problem n

Complementarity (MPEC-KKT)

Mathematical Program with Equilibrium Constraints (MPEC)

Objective function (minimize or maximize)

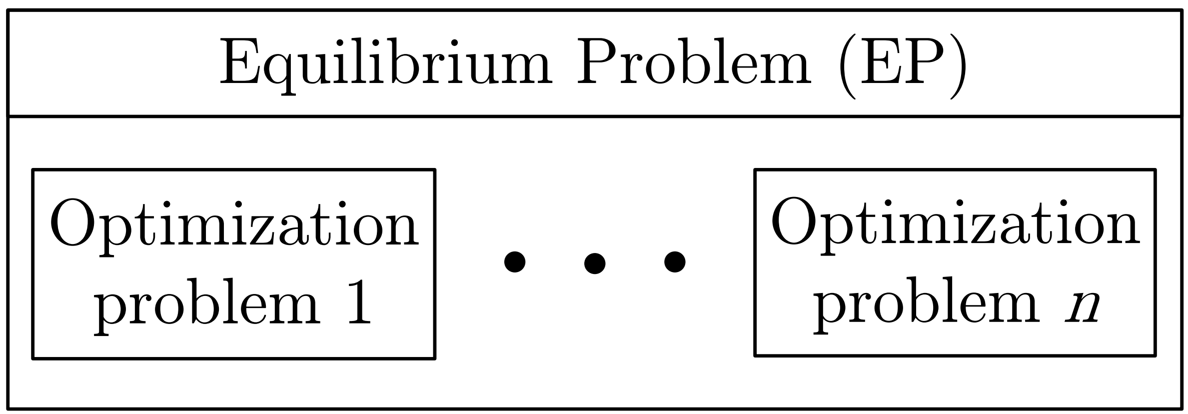
subject to:

KKT conditions of constraining problem 1

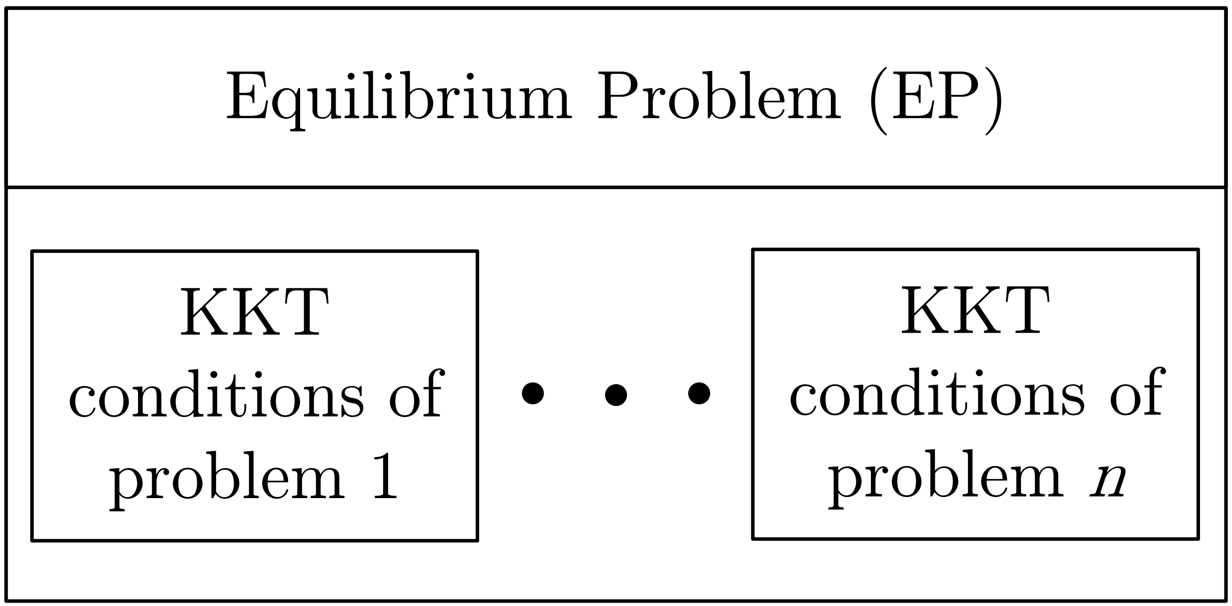
•
•
•

KKT conditions of constraining problem n

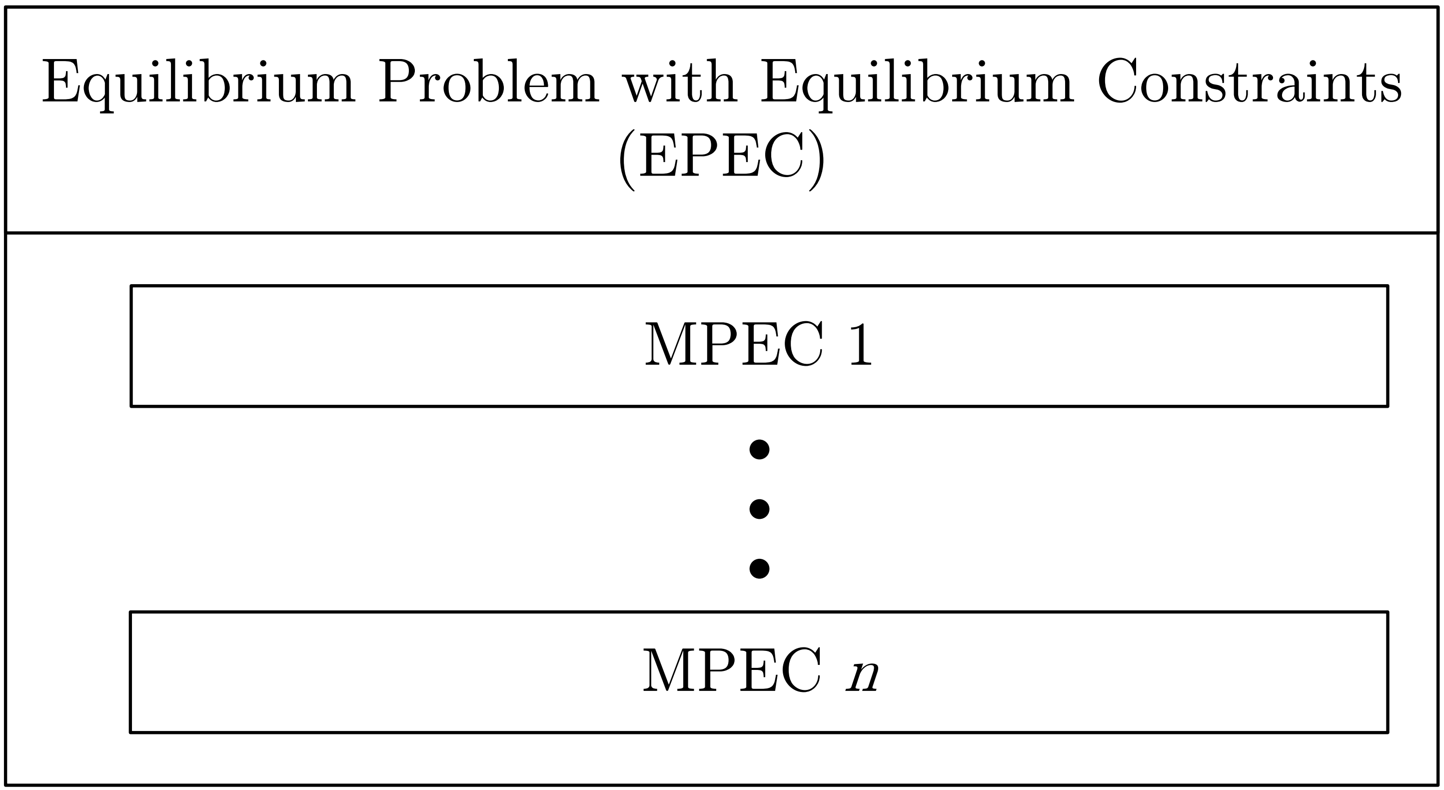
Complementarity (Equilibrium)



Complementarity (Equilibrium-KKT)



Complementarity
(EPEC)



Stochastic Programming

- References
- Two-stage stochastic programming
- Decision framework
- Decision tree
- Scenario formulation
- Node formulation
- EVPI
- VSS

References

— J. R. Birge and F. Louveaux. "Introduction to Stochastic Programming". Springer, New York, New York, 1997.

— J. L. Higle. Tutorials in Operations Research, INFORMS 2005. *Chapter 2:* "Stochastic Programming: Optimization When Uncertainty Matters". INFORMS, Hanover, Maryland, 2005.

— A. J. Conejo, M. Carrión, J. M. Morales, "Decision Making Under Uncertainty in Electricity Markets" International Series in Operations Research & Management Science, Springer, New York. 2010.

Two-stage stochastic programming

Expectation

CVaR

$$
\begin{aligned}
\text{minimize}_{x,y} \quad & O_{\lambda} f(x, y, \lambda) \\
\text{subject to} \quad & h(x, y, \lambda) = 0 \\
& g(x, y, \lambda) \leq 0
\end{aligned}
$$

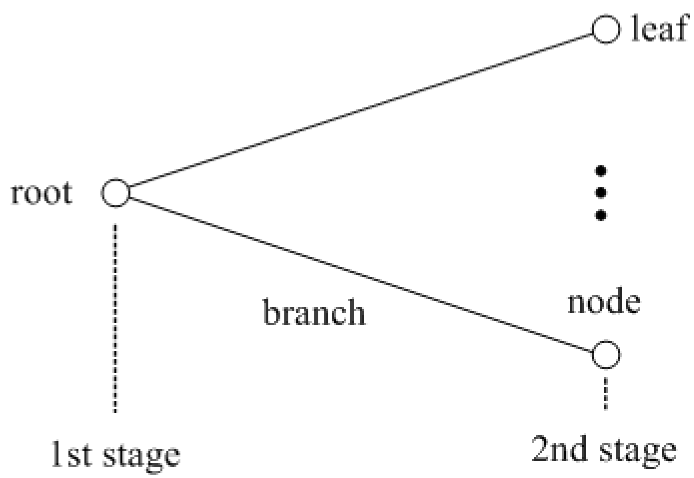
Decision framework

— Decisions *x* are made

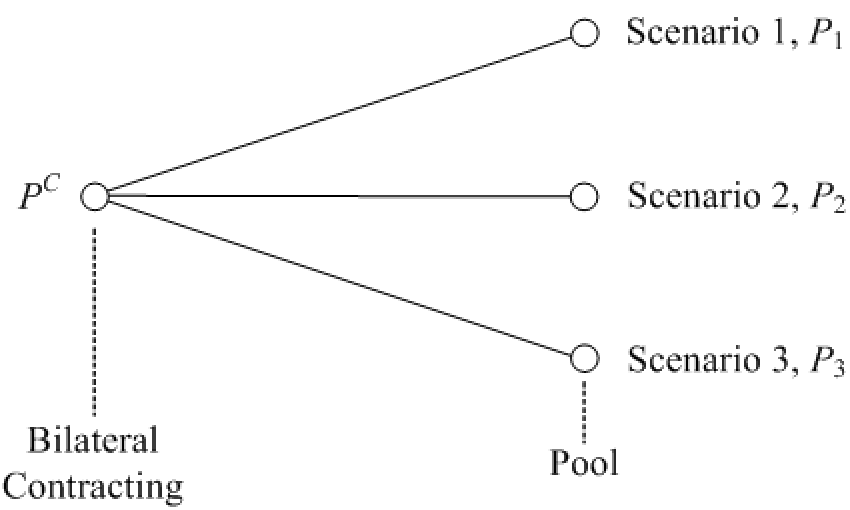
— Stochastic vector *λ* realizes in a scenario

— Given *x*, decisions *y(x,λ)* are made for each realization of *λ*

Decision tree



Decision tree



Scenario formulation

$$\lambda = \begin{cases} \lambda_1 & \alpha_1 & \text{high} \\ \lambda_2 & \alpha_2 & \text{average} \\ \lambda_3 & \alpha_3 & \text{low} \end{cases}$$

Realization

Probability

Scenario formulation

$$\underset{x_1, x_2, x_3, y_1, y_2, y_3}{\text{minimize}} \quad Z^S = \sum_{i=1}^{3} \alpha_i f(x_i, y_i, \lambda_i)$$

$$\text{subject to} \quad h(x_i, y_i, \lambda_i) = 0 \qquad i = 1,2,3$$

$$g(x_i, y_i, \lambda_i) \leq 0 \qquad i = 1,2,3$$

$$x_1 = x_2 = x_3$$

Non-anticipativity constraint

$$x_1 = x_2 = x_3$$

Node formulation

$$\text{minimize}_{x,y_1,y_2,y_3} \quad Z^S = \sum_{i=1}^{3} \alpha_i f(x, y_i, \lambda_i)$$

$$\text{subject to} \quad h(x, y_i, \lambda_i) = 0 \qquad i = 1,2,3$$

$$g(x, y_i, \lambda_i) \le 0 \qquad i = 1,2,3$$

Non-anticipativity constraints are implicit!

EVPI

EXPECTED

VALUE of the

PERFECT

INFORMATION

Measure of the value of “perfect” information

EVPI

$$\text{minimize}_{x_1,x_2,x_3,y_1,y_2,y_3} \quad Z^P = \sum_{i=1}^{3} \alpha_i f(x_i, y_i, \lambda_i)$$

$$\text{subject to} \quad f(x_i, y_i, \lambda_i) = 0 \qquad i = 1,2,3$$

$$g(x_i, y_i, \lambda_i) \le 0 \qquad i = 1,2,3$$

No anticipativity constraints!

EVPI

$$EVPI = Z^{S} - Z^{P}$$

EVPI is non-negative

VSS
(only expectation)

VALUE of the

STOCHASTIC

SOLUTION

Measure of the relevance (gain) of using a stochastic approach

VSS

$$\text{maximize}_{x,y} \quad f\left(x, y, \lambda^{avg}\right)$$

$$\text{subject to} \quad h\left(x, y, \lambda^{avg}\right) = 0$$

$$g\left(x, y, \lambda^{avg}\right) \leq 0$$

Solution x^D

VSS

$$Z^D = \sum_{i=1}^{3} \alpha_i f\left(x^D, y_i, \lambda_i\right)$$

VSS

$$\text{VSS} = Z^{D} - Z^{S}$$

VSS is non-negative