Transmission Expansion Planning: A Mixed-Integer LP Approach

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***Abstract*—This paper presents a mixed-integer LP approach to the solution of the long-term transmission expansion planning problem. In general, this problem is large-scale, mixed-integer, nonlinear, and nonconvex. We derive a mixed-integer linear formulation that considers losses and guarantees convergence to optimality using existing optimization software. The proposed model is applied to Garver's 6-bus system, the IEEE Reliability Test System, and a realistic Brazilian system. Simulation results show the accuracy as well as the efficiency of the proposed solution technique.**

***Index Terms*—Linearized power flow, mixed-integer linear programming, power loss modeling, transmission expansion planning.**

I. Nomenclature

The main mathematical symbols used throughout this paper are classified below for quick reference.

Constants

$\mathrm{b_{srk}}$	Susceptance of line $(\mathrm{s,r,k})$.
$\mathrm{g_{srk}}$	Conductance of line $(\mathrm{s,r,k})$.
$\mathrm{K_{srk}}$	Investment cost of building line $(\mathrm{s,r,k})$.
L	Number of blocks of the piecewise linearization of power losses.
$\mathrm{M_{sr}}$	Large enough positive constant.
$\mathrm{p_{Gi}^{max}}$	Upper bound on the power output of the ith generating unit.
$\mathrm{p_{srk}^{max}}$	Maximum capacity of the line $(\mathrm{s,r,k})$.
$\alpha_{\mathrm{sr}}(\ell)$	Slope of the ℓth block of the voltage angle for the corridor $(\mathrm{s,r})$.
$\Delta\delta_{\mathrm{sr}}$	Upper bound of the angle blocks of corridor $(\mathrm{s,r})$.
λ_{Gi}	Cost of the energy produced by the ith generating unit.
σ	Weighting factor to make comparable investment and operation costs.

Variables

$\mathrm{f_{srk}}$	Lossless power flow in line $(\mathrm{s,r,k})$ at node s.
$\mathrm{p_{Ds}}$	Total power consumed at node s.
$\mathrm{p_{Gi}}$	Total power produced by the ith unit.
$\mathrm{p_s}$	Power injection at node s.
$\mathrm{p_{srk}}$	Power injected in line $(\mathrm{s,r,k})$ computed at node s.
$\mathrm{q_{srk}}$	Power losses in line $(\mathrm{s,r,k})$.
$\mathrm{w_{srk}}$	Binary variable: $\mathrm{w_{srk}} = 1$ if line $(\mathrm{s,r,k})$ is built; $\mathrm{w_{srk}} = 0$ if not.
δ_{n}	Voltage angle at node n.
$\delta_{\mathrm{sr}}(\ell)$	Variable used in the linearization of the power losses in corridor $(\mathrm{s,r})$; it represents the ℓth angle block relative to this corridor.

Sets

E	Set of all transmission lines, prospective and existing.
E^+	Set of prospective transmission lines.
I	Set of indices of the generating units.
N	Set of all buses (or nodes).
Ω_{s}	Set of lines connected to bus s.
Ψ_{s}	Set of generators located at bus s.

Subscripts

Ds	Indicates demand s.
Gi	Indicates generator i.

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II. Introduction

In its general form, the long term transmission expansion problem is a stochastic decision problem that consists of determining the time, the location, and the type of the transmission lines to be built. This problem arises either in centralized or competitive frameworks. In a centralized environment, an electric utility owning both generation and transmission assets should perform jointly its generation and transmission expansion planning. In a deregulated environment, the transmission provider is often a single regulated entity whose main task is to operate and expand the transmission network with the target of maximizing energy trade opportunities. This is the case in most Western European countries, and particularly, in mainland Spain [1]. In the formulation and analyses that follow, we adopt this view: the network provider is a single regulated entity with the task of operating and expanding the transmission network so that energy trade opportunities are maximized. It should be emphasized that transmission expansion planning is a highly complex, multiobjective decision making task which overall complexity cannot be embodied into a single mathematical programming tool. However, such a tool provides most valuable information for the decision-maker to carry out the most appropriate decisions. In any case, a set of physical and possibly budget constraints should be satisfied. This set of constraints includes, among others, the dynamic constraints on the investment and operating variables, as well as the nonlinear and nonconvex static transmission constraints. Because of its inherent complexity and the lack of adequate computational tools, attempts to solve the transmission expansion problem have been made based on two generic simplified models pertaining to whether the stochastic and dynamic aspects of the problem are considered or not [2]–[6]. In practice, a further simplification is needed, and it consists of assuming a given voltage level for both static and dynamic approaches. Notwithstanding, finding a solution tool that guarantees optimality, for either instance of the transmission expansion problem, remains an open issue because of the combinatorial nature of the problem, which renders its solution difficult for medium size problems [7]. For this reason, some researchers have applied various heuristic techniques to obtain a good feasible solution. Among those techniques are simulated annealing, neural networks, game theory, and genetic algorithms [2], [7]–[9]. Linear programming and dynamic programming were used in [10]–[12]. The application of mixed-integer programming techniques using Benders decomposition combined with some heuristics can be found in [3], [4], and [13]. However, these results hinge on a modification of the natural formulation of the original problem. A more rigorous decomposition scheme that is suitable for Benders decomposition was reported in [5] and [14]. In these references, line losses are not considered and nonlinearities derive from the product of continuous and discrete variables.

We set a mixed-integer formulation for the static transmission expansion problem that takes into account transmission power losses. No stochastic aspects are treated. The proposed model may be solved to optimality by using existing optimization packages such as CPLEX 7.5 [15].

The contributions of this paper are twofold:

- a revisited MILP formulation of the transmission expansion planning problem that presents an efficient computational behavior when using conventional MILP solvers, therefore not requiring a Benders partitioning procedure;
- a rigorous modeling of power transmission losses through the used of linear expressions.

The remaining sections of this paper are outlined as follows. Section III presents a basic formulation of the transmission expansion problem, which arises naturally as a mixed-integer, nonlinear mathematical program. Using an approximation of the power flow equations up to quadratic terms, a recast of the problem into a mixed-integer linear program is obtained in Section IV. This allows the solution of the transmission expansion problem using available mixed-integer linear programming (MILP) software. Section V provides some numerical results of the proposed model using Garver's 6-bus system [11], the IEEE Reliability Test System [16], and a realistic Brazilian system [14]. Finally, some concluding remarks are drawn in Section VI.

III. Formulation

In this section, a formulation of the transmission expansion planning problem is presented. Note that the proposed model is an improved version of the one suggested in [5], [14], or in [17], since line losses are taken into account

$$\underset{\mathrm{w_{srk},p_s,p_{srk},\delta_s,f_{srk},p_{Gi},q_{srk}}}{\text{Minimize}} \sum_{\forall(\mathrm{s,r,k})\in E^+} \mathrm{K_{srk}w_{srk}} + \sigma \sum_{\forall \mathrm{i}\in I} \lambda_{\mathrm{Gi}}\mathrm{p_{Gi}} \tag{1}$$

subject to

$$\sum_{\mathrm{i}\in\Psi_\mathrm{s}} \mathrm{p_{Gi}} - \mathrm{p_s} = \mathrm{p_{Ds}}; \forall \mathrm{s} \in N \tag{2}$$

$$\mathrm{p_s} \equiv \sum_{\forall(\mathrm{r,k})\in\Omega_\mathrm{s}} \mathrm{p_{srk}} = \sum_{\forall(\mathrm{r,k})\in\Omega_\mathrm{s}} \left[\mathrm{f_{srk}} + \frac{1}{2}\mathrm{q_{srk}}\right]; \forall \mathrm{s} \in N \tag{3}$$

$$\mathrm{f_{srk}} = -\,\mathrm{b_{srk}w_{srk}}\sin\left(\delta_\mathrm{s} - \delta_\mathrm{r}\right); \quad \forall(\mathrm{s,r,k}) \in E \tag{4}$$

$$\mathrm{q_{srk}} = 2\mathrm{g_{srk}w_{srk}}\left[1 - \cos\left(\delta_\mathrm{s} - \delta_\mathrm{r}\right)\right]; \quad \forall(\mathrm{s,r,k}) \in E \tag{5}$$

$$\max\left\{\mathrm{p_{srk}};\mathrm{p_{rsk}}\right\} \le \mathrm{p_{srk}^{max}}; \forall(\mathrm{s,r,k}) \in E \tag{6}$$

$$0 \le \mathrm{p_{Gi}} \le \mathrm{p_{Gi}^{max}}; \forall \mathrm{i} \in I \tag{7}$$

$$\mathrm{w_{srk}} = 1; \forall(\mathrm{s,r,k}) \in E \backslash E^+ \tag{8}$$

$$\mathrm{w_{srk}} \in \{0,1\}; \forall(\mathrm{s,r,k}) \in E \tag{9}$$

where $\mathrm{p_{srk}^{max}} = \mathrm{p_{rsk}^{max}}$ denotes the transmission capacity of the line $(\mathrm{s,r,k})$ – here s designates the sending node and r, the receiving one.

The objective function in (1) represents the sum of the investment cost of new lines and the operating cost of the generating units. As it is commonly assumed in the technical literature (see, for instance, [5], [13], or [14]), a single load scenario is considered, typically corresponding to the highest load demand forecasted for the considered planning horizon. It should be noted that the operating cost of the generating units have to be estimated by the network provider. Weighting factor σ is used to make comparable the investment and operating costs.

The constraints (2) enforce the power balance at every node. The constraints (3) state that the power injection at node s is the summation, over all lines connected to that node, of a lossless line flow component $\mathrm{f_{srk}}$ and a loss component $\mathrm{q_{srk}}$. For each line, $(\mathrm{s,r,k})$, these two components are formulated as the product of the binary variable $\mathrm{w_{srk}}$ and a sinusoidal function of the difference of the angles prevailing at the sending and receiving ends of the line, as shown in (4) and (5), respectively. This multiplication of a binary variable and a continuous function is a natural modeling of the fact that the power injected in a line is nil if that line is not physically connected to the network, or equivalently, if $\mathrm{w_{srk}} = 0$. The constraints (6) enforce the line flow limits. The constraints (7) are operating constraints that specify that if a generator is dispatched, its power output must be within a certain range represented by a minimum output and a maximum output. Note that, for the static long-term expansion problem, it is practical to assume that the lower bound on the power output of each generating unit is zero, thereby neglecting the effects of unit commitment and decommitment. This assumption is consistent with previously published works and will also be applied throughout this paper. Finally, the integrality of the investment decision variables $\mathrm{w_{srk}}$ is stated in (9), and the fact that the existing lines have already been built is enforced in (8). Note that a predetermined decision variable

— $w_{srk} = 1$ for all $(s, r, k) \in E \backslash E^{+}$ — has been assigned to every existing line throughout this paper, for the sakes of clarity and conciseness of presentation. These *a priori* known variables may certainly be dropped, if necessary, to achieve a more optimized implementation that would systemically avoid the extra overhead required handling them.

IV. Linear Model

We propose a more accurate linear approximation of the transmission static expansion problem by transformation of (1)–(9) into a mixed-integer linear mathematical program [18]. The derivation of the new formulation is presented in two stages.

First, we assume that conditions for normal power system operations hold; that is, for any feasible solution of the transmission expansion problem, the difference of the angles prevailing at both ends of every existing line, or every built prospective line, is sufficiently small (typically within 20°). This is a rather practical decision to render the system far from any dynamic instability or other security limits under normal operating conditions. Then, based on this assumption, we obtain a linear approximation for the cosine function (power losses) and the sine functions (lossless power flow). Note that we incur no loss of generality, as higher values of the aforementioned angle differences can be modeled with more linearization segments in order to achieve a finer approximation of the sinusoidal functions [19].

Second, considering the previous approximation, we transform the model in such a way that the products of discrete and continuous variables are eliminated. Observe, however, that the transformation in this step introduces no further approximation in the original model subsequent to the linearization in the first step.

In this section, all of the expressions presented apply to every transmission line. Therefore, the indication $\forall (s, r, k) \in E$ will be explicitly omitted, except where needed for clarity.

A. Linearization of the Sinusoidal Functions Around the Normal Operating Point

Suppose that $(s, r, k) \in E$. For normal operation, under the flat voltage assumption, the real power injection in the line (s, r, k) computed at bus s, p_{srk}, and the real power injection in the line (s, r, k) computed at bus r, p_{rsk}, are given by

$$\begin{aligned} p_{srk} =& - b_{srk} w_{srk} \sin(\delta_s - \delta_r) \\ &+ g_{srk} w_{srk} [1 - \cos(\delta_s - \delta_r)] \end{aligned} \tag{10}$$

$$\begin{aligned} p_{rsk} =& b_{srk} w_{srk} \sin(\delta_s - \delta_r) \\ &+ g_{srk} w_{srk} [1 - \cos(\delta_s - \delta_r)] \end{aligned} \tag{11}$$

where $y_{rsk} = g_{rsk} + jb_{rsk}$, y_{rsk} being the admittance of the line (s, r, k). The power losses in the line (s, r, k), q_{srk}, can be obtained as follows:

$$\begin{aligned} q_{srk} =& p_{srk} + p_{rsk} \\ =& 2 g_{srk} w_{srk} [1 - \cos(\delta_s - \delta_r)] \\ \approx& g_{srk} w_{srk} (\delta_s - \delta_r)^2 \end{aligned} \tag{12}$$

where the last equality follows from a second order approximation of the cosine function, which has proven to be a good approximation of the power losses in the line (s, r, k) under normal operation.

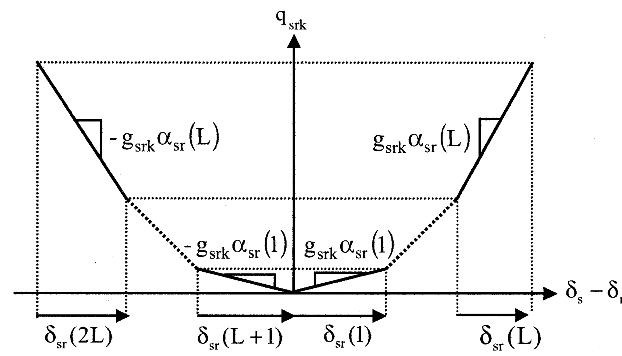

Fig. 1. Modeling a piecewise linear per-line loss function.

A linear approximation of q_{srk} can be obtained using $2L$ piecewise linear blocks as shown in Fig. 1. However, the same results may be achieved with only $L + 2$ variables per corridor, where L variables are those needed to model the domain restriction of the function q_{srk} to the positive orthant, and the two remaining variables are necessary to obtain a linear equivalent to the absolute function, as will be explained next.

In order to reduce the linearization to the positive orthant only, we introduce the following definitions:

$$q_{srk} = g_{srk} w_{srk} \sum_{\ell=1}^{L} \alpha_{sr}(\ell) \delta_{sr}(\ell) \tag{13}$$

$$|\delta_s - \delta_r| = \sum_{\ell=1}^{L} \delta_{sr}(\ell) \tag{14}$$

$$\delta_{sr}(\ell) \geq 0; \quad \ell = 1, \ldots, L \tag{15}$$

$$\delta_{sr}(\ell) \leq \Delta\delta_{sr} + (1 - w_{srk}) M_{sr}; \ \ell = 1, \ldots, L \tag{16}$$

where $\alpha_{sr}(\ell)$ and $\delta_{sr}(\ell)$ denote, respectively, the slope and value of the ℓth block of angle in corridor (s, r). Constraints (13) are the linear approximations of lines losses. Constraints (14) state that the absolute value of the difference of the angles between two nodes is equal to the sum of the values in each block of the discretization. Constraints (15) and (16) set the upper and lower limits of the contribution of each angle block to the total angle difference for the line (s, r, k). This contribution is nonnegative, which is expressed in (15). The upper bound of the angle blocks is modeled in (16) using the decision variable w_{srk}. If the line is built ($w_{srk} = 1$), then the angle blocks are bounded above by $\Delta\delta_{sr}$; otherwise, for a sufficiently large positive constant M_{sr}, these constraints are nonbinding. Both groups of constraints, (15) and (16), implicitly enforce the adjacency of the angle blocks if operating costs are considered in the objective function. Alternatively, losses can be penalized in the objective function so as to enforce the adjacency condition.

The following expression of the slope of the blocks of angles $\alpha_{sr}(\ell)$ can be used in the linear approximation of the power loss function for all lines:

$$\alpha_{sr}(\ell) = \frac{(\ell\Delta\delta_{sr})^2 - [(\ell\Delta\delta_{sr}) - \Delta\delta_{sr}]^2}{\Delta\delta_{sr}} = (2\ell - 1)\Delta\delta_{sr}. \tag{17}$$

A linear expression of the absolute value in (14) is needed, which is obtained by means of the following substitution [18]

$$|\delta_s - \delta_r| = \delta_{sr}^+ + \delta_{sr}^- \tag{18}$$

$$\delta_s - \delta_r = \delta_{sr}^+ - \delta_{sr}^- \tag{19}$$

$$\delta_{sr}^+ \geq 0, \quad \delta_{sr}^- \geq 0. \tag{20}$$

Using (13) and a second order approximation for $\sin(\delta_s - \delta_r)$, the real power injection in the line (s, r, k) computed at bus s, p_{srk}, and at bus r, p_{rsk}, can be cast as follows:

$$\begin{aligned} p_{srk} =& f_{srk} + \frac{1}{2} q_{srk} \\ \approx& -b_{srk} w_{srk} \left(\delta_{sr}^+ - \delta_{sr}^-\right) \\ &+ \frac{1}{2} g_{srk} w_{srk} \sum_{\ell=1}^{L} \alpha_{sr}(\ell)\, \delta_{sr}(\ell) \end{aligned} \tag{21}$$

$$\begin{aligned} p_{rsk} =& -f_{srk} + \frac{1}{2} q_{srk} \\ \approx& b_{srk} w_{srk} \left(\delta_{sr}^+ - \delta_{sr}^-\right) \\ &+ \frac{1}{2} g_{srk} w_{srk} \sum_{\ell=1}^{L} \alpha_{sr}(\ell)\, \delta_{sr}(\ell). \end{aligned} \tag{22}$$

In the same vein, the power injected into a node $s \in N$, p_s, can be written as

$$\begin{aligned} p_s \equiv& \sum_{\forall(r,k)\in\Omega_s} \left[f_{srk} + \frac{1}{2} q_{srk} \right] \\ \approx& \sum_{\forall(r,k)\in\Omega_s} w_{srk} \left[-b_{srk}\left(\delta_{sr}^+ - \delta_{sr}^-\right) + \frac{1}{2} g_{srk} \sum_{\ell=1}^{L} \alpha_{sr}(\ell)\, \delta_{sr}(\ell) \right] \end{aligned} \tag{23}$$

which is a nonlinear expression due to the multiplication of continuous and discrete variables.

B. Elimination of Nonlinearities Induced by the Product of Discrete and Continuous Variables

To get rid of the nonlinearities found in the expression of the lossless flow f_{srk} the following constraints are included in the model:

$$-w_{srk} p_{srk}^{\max} \leq f_{srk} \leq w_{srk} p_{srk}^{\max} \tag{24}$$

$$-(1 - w_{srk}) M_{sr} \leq \frac{f_{srk}}{b_{srk}} + \left(\delta_{sr}^+ - \delta_{sr}^-\right) \leq (1 - w_{srk}) M_{sr} \tag{25}$$

where the disjunctive parameter M_{sr} must be selected so as not to limit unnecessarily the voltage angle difference between two nodes that are not connected at all; that is, if none of the prospective lines, if any, between two nodes is selected. In other words, M_{sr} must be assigned a sufficiently large positive constant that provides enough degree of freedom to the voltage angle difference between every unconnected node of the network. However, setting the disjunctive parameter rightly to a large value should be made without overlooking the finite machine precision arithmetic that would otherwise render the algorithm numerically unstable and the results unreliable. A detailed discussion on the selection of this parameter is given in [5] and [14].

Equations (24) and (25) are explained below. Suppose that line k in corridor (s, r) is built ($w_{srk} = 1$); then the lossless flow f_{srk} is limited by the maximum capacity of the line according to (24), and its value is obtained from (25), as the latter inequality constraints are converted into an equality. If the line is not built ($w_{srk} = 0$), then (24) produces $f_{srk} = 0$, and (25) is nonbinding for a convenient selection of M_{sr}, which does not limit the angle difference unnecessarily.

Similarly, the nonlinearities can be removed from the expression of line losses q_{srk}. The following constraints are added to the model:

$$0 \leq q_{srk} \leq w_{srk} p_{srk}^{\max} \tag{26}$$

$$0 \leq \frac{q_{srk}}{g_{srk}} + \sum_{\ell=1}^{L} \alpha_{sr}(\ell)\, \delta_{sr}(\ell) \leq (1 - w_{srk}) M_{sr}^2. \tag{27}$$

The logic of (26) and (27) can be analogously explained. If the line is built ($w_{srk} = 1$), then line losses are positive and limited by the maximum capacity of the line according to (26), and its value is obtained from (27) (a constraint whose bounds are both zero). If the line is not built, then (26) yields $q_{srk} = 0$, and (27) ensures that the square of the angle difference is positive and free from its right-side constraint.

In practice, the addition of the losses does not influence the selection of the disjunctive parameter; therefore, the conclusions drawn in previous papers [5] and [14] about this parameter are still valid in this work.

To conclude the linearization of the initial model, the "max" function in (6) is reformulated as

$$\begin{aligned} p_{srk} &\leq w_{srk} p_{srk}^{\max} \\ p_{rsk} &\leq w_{srk} p_{srk}^{\max}. \end{aligned} \tag{28}$$

Finally, the proposed linear model of the expansion planning problem comprises the objective function (1), and the constraints (2), (3), (7)–(9), (14)–(16), (18)–(20), and (24)–(28). For the sake of clarity and quick reference, the complete model is written below

$$\underset{\substack{w_{srk}, f_{srk}, p_{Gi}, q_{srk}, \\ \delta_s, \delta_{sr}(\ell), \delta_{sr}^+, \delta_{sr}^-}}{\text{Minimize}} \sum_{\forall(s,r,k)\in E^+} K_{srk} w_{srk} + \sigma \sum_{\forall i\in I} \lambda_{Gi} p_{Gi} \tag{29}$$

subject to

$$\sum_{i\in\Psi_s} p_{Gi} - \sum_{\forall(r,k)\in\Omega_s} \left[f_{srk} + \frac{1}{2} q_{srk} \right] = p_{Ds};\ \forall s \in N \tag{30}$$

$$-w_{srk} p_{srk}^{\max} \leq f_{srk} \leq w_{srk} p_{srk}^{\max};\ \forall (s,r,k) \in E \tag{31}$$

$$-(1 - w_{srk}) M_{sr} \leq \frac{f_{srk}}{b_{srk}} + \left(\delta_{sr}^+ - \delta_{sr}^-\right) \leq (1 - w_{srk}) M_{sr};\ \forall (s,r,k) \in E \tag{32}$$

$$0 \leq q_{srk} \leq w_{srk} p_{srk}^{\max};\ \forall (s,r,k) \in E \tag{33}$$

$$0 \leq -\frac{q_{srk}}{g_{srk}} + \sum_{\ell=1}^{L} \alpha_{sr}(\ell)\, \delta_{sr}(\ell) \leq (1 - w_{srk}) M_{sr}^2;\ \forall (s,r,k) \in E \tag{34}$$

$$\delta_{sr}^+ + \delta_{sr}^- = \sum_{\ell=1}^{L} \delta_{sr}(\ell);\ \forall (s,r) \in E \tag{35}$$

$$\delta_s - \delta_r = \delta_{sr}^+ - \delta_{sr}^-;\ \forall (s,r) \in E \tag{36}$$

$$f_{srk} + \frac{1}{2} q_{srk} \leq p_{srk}^{\max};\ \forall (s,r,k) \in E \tag{37}$$

TABLE I
LINE DATA FOR GARVER'S EXAMPLE

Corridor	Resistance (per unit)	Reactance (per unit)	Investment Cost [US$]	Capacity [MW]
1-2	0.10	0.40	40	100
1-3	0.09	0.38	38	100
1-4	0.15	0.60	60	80
1-5	0.05	0.20	20	100
1-6	0.17	0.68	68	70
2-3	0.05	0.20	20	100
2-4	0.10	0.40	40	100
2-5	0.08	0.31	31	100
2-6	0.08	0.30	30	100
3-4	0.15	0.59	59	82
3-5	0.05	0.20	20	100
3-6	0.12	0.48	48	100
4-5	0.16	0.63	63	75
4-6	0.08	0.30	30	100
5-6	0.15	0.61	61	78

TABLE II
BUS DATA FOR GARVER'S EXAMPLE

Bus	p_{Gi}^{max} [MW]	λ_{Gi} [US$/MWh]	p_{Di} [MW]
1	150	10	80
2			240
3	360	20	40
4			160
5			240
6	600	30	

TABLE III
SOLUTIONS FOR GARVER'S 6-BUS EXAMPLE

Corridor	Number of lines built	
	No losses	Losses
2-6	0	2
3-5	1	1
4-6	3	2
Investment Cost ($)	110	140

$$-\mathrm{f}_{\mathrm{srk}} + \frac{1}{2}\mathrm{q}_{\mathrm{srk}} \leq \mathrm{p}_{\mathrm{srk}}^{\max}; \forall(\mathrm{s,r,k}) \in E \tag{38}$$

$$0 \leq \mathrm{p}_{\mathrm{Gi}} \leq \mathrm{p}_{\mathrm{Gi}}^{\max}; \forall i \in I \tag{39}$$

$$\mathrm{w}_{\mathrm{srk}}{=}1; \forall(\mathrm{s,r,k}) \in E\backslash E^{+} \tag{40}$$

$$\mathrm{w}_{\mathrm{srk}} \in \{0,1\}; \forall(\mathrm{s,r,k}) \in E \tag{41}$$

$$\delta_{\mathrm{s}}{=}0, \mathrm{s} \text{ :reference bus} \tag{42}$$

$$\delta_{\mathrm{sr}}^{+} \geq 0, \delta_{\mathrm{sr}}^{-} \geq 0; \forall(\mathrm{s,r}) \in E \tag{43}$$

$$\delta_{\mathrm{sr}}(\ell) \geq 0; \forall(\mathrm{s,r}) \in E; \ell = 1, \ldots, \mathrm{L} \tag{44}$$

$$\delta_{\mathrm{sr}}(\ell) \leq \Delta\delta_{\mathrm{sr}} + (1 - \mathrm{w}_{\mathrm{srk}})\,\mathrm{M}_{\mathrm{sr}}; \forall(\mathrm{s,r,k}) \in E; \ell = 1, \ldots, \mathrm{L}. \tag{45}$$

Note that this linear version of the transmission expansion model requires three sets of additional continuous variables $\delta_{\mathrm{sr}}(\ell)$, δ_{sr}^{+}, δ_{sr}^{-}.

V. CASE STUDIES

The proposed model has been applied to Garver's 6-bus system [11], to the IEEE Reliability Test System [16] and to a realistic Brazilian system. The cases have been implemented on a SGI R12000, 400-MHz-based processor with 500 MB of RAM using CPLEX 7.5 under GAMS [15].

Garver's example is a network with six buses and six installed lines. The data of every corridor are presented in Table I (obtained from [7]). The maximum number of lines (prospective plus installed) per corridor is 3. We have added operation costs, associated with the generating units, to the objective function. Cost functions of all three generating units are assumed linear as specified in Table II. In this table, the generation capacity and the demand at every node are also provided.

The solutions obtained for this system neglecting and taking into account losses, respectively, are compared in Table III.

The solution obtained neglecting losses is the one reported in [2]–[6], and [11]. However, Table III clearly shows that taking into account losses results in a different expansion strategy. It should be noted that not taking into account losses results in under investment. This originates today's savings but future unplanned and typically expensive investment adjustment. Under reasonably correct forecasts for load and other uncertain variables, expansion plans that require unplanned investments constitute more expensive, and therefore, inferior decisions than those plans that do not require such investment adjustments.

Considering the IEEE Reliability Test System with 24 nodes, 35 corridors (see Table IV), and 32 generating units, we have made the following assumptions.

1) All of the lines in any corridor have the same characteristics.
2) The investment cost of every line is proportional to its reactance.
3) The operating costs of the generating units are those provided in [16].

Two cases have been considered, namely, case A and case B. In either case, the number of blocks used for the linearization of the line losses is 4, and the expansion strategies obtained with and without network losses are tabulated for comparison. The purpose of this comparison is to point out the impact of modeling losses in the optimal expansion plan. As pointed out in the previous example, neglecting losses results in today savings but tomorrow investment adjustment. These unplanned investment adjustments typically overshadow the initial savings. The significant differences in investment optimal plans with and without losses are apparent from Table V. In case A, the initial network has no preinstalled lines, and it is possible to build just one line per corridor. The number of binary variables in this case is 35, which corresponds to the number of corridors.

In case B, the initial network topology is the one provided in [16], and a maximum of three lines per corridor is allowed. Therefore, the number of binary variables is 70; that is, twice as much as the number of corridors. Moreover, to allow investment decisions, the maximum capacity of each line has been reduced to one third of the capacity given in [16].

The total load in both cases is 2850 MW, which corresponds to the Tuesday of week 51 from 5 to 6 P.M.

TABLE IV
LINE DATA FOR THE IEEE RTS SYSTEM

Corridor	Resistance (per unit)	Reactance (per unit)	Investment Cost [10^6 US $]	Capacity [MW]
1-2	0.0026	0.0139	7.04	175
1-3	0.0546	0.2112	106.92	175
1-5	0.0218	0.0845	42.78	175
2-4	0.0328	0.1267	64.14	175
2-6	0.0497	0.1920	97.20	175
3-9	0.0308	0.1190	60.24	175
3-24	0.0023	0.0839	42.47	400
4-9	0.0268	0.1037	52.50	175
5-10	0.0228	0.0883	44.70	175
6-8	0.0159	0.0614	31.08	175
6-10	0.0139	0.0605	30.63	175
7-8	0.0159	0.0614	31.08	175
8-9	0.0427	0.1651	83.58	175
8-10	0.0427	0.1651	83.58	175
9-11	0.0023	0.0839	42.47	400
9-12	0.0023	0.0839	42.47	400
10-11	0.0023	0.0839	42.47	400
10-12	0.0023	0.0839	42.47	400
11-13	0.0061	0.0476	24.10	500
11-14	0.0054	0.0418	21.16	500
12-13	0.0061	0.0476	24.10	500
12-23	0.0124	0.0966	48.90	500
13-23	0.0111	0.0865	43.79	500
14-16	0.0050	0.0489	19.70	500
15-16	0.0022	0.0173	8.76	500
15-21	0.0063	0.049	24.81	500
15-24	0.0067	0.0519	26.27	500
16-17	0.0033	0.0259	13.11	500
16-19	0.0030	0.0231	11.70	500
17-18	0.0018	0.0144	7.29	500
17-22	0.0135	0.1053	53.31	500
18-21	0.0033	0.0259	13.11	500
19-20	0.0051	0.0396	20.05	500
20-23	0.0028	0.0216	10.93	500
21-22	0.0087	0.0678	34.32	500

Table V illustrates the solution obtained for both cases in terms of the number of lines built, investment cost, and solution CPU times. For both cases, A and B, the set of lines built when losses are considered is different from the corresponding set when losses are neglected. The above results in higher investment cost for the case in which losses are considered. Solution times are larger with losses than without losses; they are, however, reasonably small.

Table VI further illustrates the solution obtained for both cases when considering losses. This table shows the total number of lines per corridor, the total line flows (including power losses) and the losses for anyone of the lines in every corridor. In case A, the total losses are 80 MW, approximately 3% of the total load, while in case B, the losses represent 1% (31 MW) of the total load.

The proposed technique has also been applied to the realistic Brazilian system used in [14], [20], and reported in [21]. This system, considered standard for transmission expansion planning studies, includes 46 buses, 47 existing transmission corridors, and 32 new corridors. The number of existing lines is 62 and the number of lines that can be built is 175.

TABLE V
SOLUTIONS FOR THE IEEE RTS SYSTEM

Corridor	Case A		Case B	
	No losses	Losses	No losses	Losses
	# of lines built			
	18	18	6	7
1-2			1	1
1-3			1	1
1-5	1	1	1	1
2-4	1		1	1
2-6			1	1
3-9			1	1
3-24	1	1	1	1
4-9		1	1	1
5-10			1	1
6-8		1	1	1
6-10	1	1	2	2
7-8	1	1	3	2
8-9			1	1
8-10			1	1
9-11	1		1	1
9-12		1	1	1
10-11	1		1	2
10-12		1	2	1
11-13	1		2	2
11-14	1		1	1
12-13		1	1	1
12-23		1	1	1
13-23			1	1
14-16	1	1	1	2
15-16	1	1	1	1
15-21			1	1
15-24	1	1	1	1
16-17	1	1	1	2
16-19	1		1	1
17-18	1	1	1	1
17-22			1	1
18-21	1	1	1	1
19-20	1	1	1	1
20-23	1	1	2	2
21-22			1	1
Investment Cost [10^6 US$]	472.22	507.70	170.29	172.02
CPU Time [s]	1.50	9.88	13.02	35.26

The expansion strategy for this network, neglecting losses, can be found in [14], [20]. In particular, new lines built include two lines in corridors 5–6 and 20–21, and one line in corridors 13–20, 20–23, 42–43, and 46–6. To study the impact of losses, we have applied the procedure proposed in the present paper, which produced an optimal solution in approximately 45 min of CPU time. The following observations are made. Differences with the no-loss case include building 1 line in corridor 18–20, and not building the line in corridor 13–20. Losses constitute about 2% of the total energy produced. Additional investment cost for this case represents about 8% of the investment cost for the no-loss case.

VI. CONCLUSIONS

This paper proposes a linear mixed-integer formulation for the transmission network static expansion planning problem that takes into account line losses. This problem is of high interest

TABLE VI
NUMBER OF LINES, LINE FLOWS, AND LOSSES FOR THE RTS SYSTEM

Corridor	Case A			Case B		
	# lines built	p_{srk} [MW]	$q_{srk}/2$ [MW]	# lines built	p_{srk} [MW]	$q_{srk}/2$ [MW]
1-2				1	-6.92	0.01
1-3				1	28.04	0.40
1-5	1	73.52	1.26	1	54.87	0.32
2-4				1	48.92	0.42
2-6				1	34.64	0.45
3-9				1	-20.78	0.17
3-24	1	-180.00	0.71	1	-131.99	0.17
4-9	1	-74.00	1.63	1	-25.93	0.18
5-10				1	-16.76	0.10
6-8	1	0.37	0.00	1	9.58	0.04
6-10	1	-136.37	1.49	2	-55.93	0.20
7-8	1	175.00*	2.19	2	58.33*	0.24
8-9				1	-22.23	0.25
8-10				1	-23.58	0.27
9-11				1	-112.16	0.15
9-12	1	-252.25	1.00	1	-132.99*	0.17
10-11				2	-109.31	0.14
10-12	1	-334.36	1.31	1	-130.13	0.17
11-13				2	-147.69	0.65
11-14				1	-36.27	0.14
12-13	1	-265.78	3.67	1	-111.66	0.49
12-23	1	-325.45	8.83	1	-152.15	1.37
13-23				1	-108.85	0.88
14-16	1	-194.00	2.12	2	-115.28	0.41
15-16	1	-305.16	1.45	1	-70.45	0.11
15-21				1	-165.16*	0.75
15-24	1	186.81	2.70	1	133.61	0.64
16-17	1	-451.29	3.24	2	-131.75	0.31
16-19				1	15.62	0.03
17-18	1	-457.77	1.78	1	-129.34	0.17
17-22				1	-135.42	1.33
18-21	1	-394.34	2.83	1	-62.67	0.15
19-20	1	-181.00	2.03	1	-165.45*	0.61
20-23	1	-313.06	1.92	2	-147.33	0.30
21-22				1	-159.90	1.01

(* indicates congested line)

for both centralized and competitive electric energy systems. Losses are represented without using additional binary variables. It should be noted that modeling losses constitutes a more accurate representation of the network functioning that may result in different expansion plans than those obtained if losses are neglected. Neglecting losses results in today's savings but tomorrow's investment adjustments that typically overshadow those initial savings. The proposed mixed-integer linear formulation is accurate, allows reaching the optimal solution, and is flexible enough to build new networks and to reinforce existing ones. Results from different case studies show the accuracy and efficiency of the proposed approach.

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