Transmission Expansion Planning in Electricity Markets

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***Abstract*—This paper presents a mixed-integer linear programming (LP) formulation for the long-term transmission expansion planning problem in a competitive pool-based electricity market. To achieve optimal expansion planning while modeling market functioning, we define a number of scenarios based on the future demand in the system and we simulate the maximization of the aggregate social welfare. Investment and operating costs, transmission losses and generator offers, and demand bids are considered. We propose to use a set of metrics to rate the effect of the expansion on the generators, demands, and the system as a whole. The proposed model is applied to the Garver six-bus system and to the IEEE 24-bus Reliability Test System. Simulation results can be interpreted in economic terms based on the values of the metrics obtained for different scenarios, parameters, and topologies.**

***Index Terms*—Electricity market, mixed-integer linear programming, social welfare, transmission expansion planning.**

NOMENCLATURE

THE mathematical symbols used throughout this paper are classified below as follows.

A. Constants

b_{srk}	Susceptance of line k in corridor (s,r).
g_{srk}	Conductance of line k in corridor (s,r).
K_{srk}	Investment cost of constructing line k in corridor (s,r).
L	Number of blocks of the piecewise linearization of power losses.
M	Large enough positive constant.
N_d	Number of blocks of the dth demand in all scenarios.
N_i	Number of blocks of the ith generating unit in all scenarios.
$\bar{p}^c_{D_{dh}}$	Size of the hth block of the dth demand in scenario c.
$p^{\max}_{G_i}$	Upper bound of the power output of the ith generating unit.
$\bar{p}_{G_{ib}}$	Size of the bth block of the ith generating unit in all scenarios.
$p^{\max}_{srk}$	Maximum capacity of line k in corridor (s,r).
w^c	Weight of scenario c.
$\alpha_{sr}(\ell)$	Slope of the ℓth block of the linearization of the voltage angle for corridor (s,r).
$\Delta\delta_{sr}$	Upper bound of the angle blocks of corridor (s,r).
$\lambda^c_{D_{dh}}$	Price bid by the hth block of the dth demand in scenario c.
$\lambda^c_{G_{ib}}$	Price offered by the bth block of the ith generating unit in scenario c.
σ	Weighting factor to make investment and operational costs comparable.

B. Scenario-dependent variables

f^c_{srk}	Lossless power flow in line k of corridor (s,r) in scenario c.
$p^c_{D_d}$	Total power consumed by the dth demand in scenario c.
$p^c_{D_{dh}}$	Power consumed by the hth block of the dth demand in scenario c.
$p^c_{G_i}$	Total power produced by the ith generating unit in scenario c.
$p^c_{G_{ib}}$	Power produced by the bth block of the ith generating unit in scenario c.
p^c_s	Power injection at bus s in scenario c.
p^c_{srk}	Power injection in line k of corridor (s,r) computed at bus s in scenario c.
q^c_{srk}	Power losses in line k of corridor (s,r) in scenario c.
δ^c_s	Voltage angle at bus s in scenario c.
$\delta^c_{sr}(\ell)$	Variable used in the linearization of the power losses in corridor (s,r) : ℓth angle block relative to this corridor in scenario c.
$\delta^{c+}_{sr}, \delta^{c-}_{sr}$	Auxiliary variables used in the linearization of the power losses in corridor (s,r) in scenario c.
λ^c_s	Nodal price at bus s in scenario c.

C. Global variables

w_{srk}	Binary variable that equals 1 if line k from corridor (s,r) is built and equals 0 otherwise.

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D. Sets

Ψ_D^s	Set of all demands located at bus s.
Ψ_G^s	Set of all generators located at bus s.
Ψ_L^s	Set of all lines connected to bus s.
Ω_C	Set of all scenarios of the period of study.
Ω_d	Set of indices of the blocks of the dth demand.
Ω_D	Set of indices of the demands.
Ω_i	Set of indices of the blocks of the ith generating unit.
Ω_G	Set of indices of the generating units.
Ω_L	Set of all possible transmission lines, prospective and existing.
Ω_{L+}	Set of all prospective transmission lines.
Ω_N	Set of all network buses.

E. Metrics

$\mu_1, \ldots, \mu_4$	Metrics to assess the impact of new transmission lines in the network.

I. Introduction

A. Background

In the past, both the operating and the planning aspects of the electric network were in the hands of centralized entities organized in vertically-integrated structures. Nowadays, the unbundling of the electricity business has raised new challenges that the restructured industry must face. The restructuring of the electricity industry has resulted in the advent of new players, such as brokers, marketers, and independent power producers. A salient characteristic of the new structure is the decentralized decision making. One critical outcome of the large number of players and the increasing number of transactions is a more frequent stressing of the transmission grid due to the creation of congestion situations.

Network expansion is by its very nature a very complex multiperiod and multi-objective optimization problem [1]. Its nonlinear nature and the inherent uncertainty of future developments constitute major complications. Its solution is therefore difficult, even in centralized environments. In the past, under the vertically integrated structure, the construction of new transmission facilities has been associated with the addition of new generating resources and their integration into the existing network. This was done under the strong control exerted by the regulators over virtually every aspect of the regulated utility activities. Under the new paradigm, the economic signals that result from the daily operations of the electric markets (prices, congestion metrics, etc.) need to be considered together with the economics of investment in new facilities in an environment of regulatory and legislative uncertainty and with the operational control of the facilities being vested in hands different than the ownership.

B. Literature Review

There are many mathematical models traditionally available to solve the transmission expansion problem from the cost minimization standpoint. The solution techniques proposed can be classified as mathematical optimization methods, such as: linear programming [2], [3], mixed-integer linear programming [4], [5], Benders decomposition [6], and dynamic programming methods [7]; heuristic methods based on sigmoid functions [8], intelligent systems, such as genetic algorithms [9], simulated annealing [10]; and others, such as game theory models [11]–[13]. With the introduction of pool-based markets and bilateral contracting, new transmission expansion models propose social welfare maximization instead, [14], [15].

C. Aim and Contributions

In this paper, we present a mixed-integer formulation for the transmission expansion problem in pool-based electric energy markets. To achieve that, we model the network topology, energy losses, generator offers, and demand bids, and we set an appropriate timeframe. Different scenarios are considered to account for different levels of the demand. They are weighted in the objective function to consider the number of hours in which the demand has reached a certain level. Later, we calculate the generator and consumer surpluses and, consequently, the social surplus. We assume that the planning of the network corresponds to a single transmission entity, although the functioning of our pool-based market model allows for independent power generators and demands that can be in the hands of different companies.

The novel contributions of this paper are threefold:

1) modeling the behavior of the demand through demand-side bidding;
2) a scenario-weighted formulation;
3) use of appropriate metrics to measure the welfare of the network planner, the generators, and the demands.

D. Paper Organization

The remaining sections of this paper are organized as follows: Section II presents the formulation of the transmission expansion problem in competitive markets as a mixed-integer problem, Section III defines the metrics used to analyze transmission expansion solutions, Section IV presents numerical results using both the Garver six-bus system and the IEEE 24-bus Reliability Test System, and Section V provides concluding remarks.

II. Formulation

In this section, we formulate the transmission expansion planning problem. This model is an extended version of the one presented in [5], where we have expanded the formulation to account for market-driven generation offers and demand bids. The general formulation of the model is as follows:

$$\text{Maximize} \left\{ \sigma \left[\sum_{\forall c \in \Omega_C} w^c \left(\sum_{\forall d \in \Omega_D} \sum_{\forall h \in \Omega_d} \lambda_{D_{dh}}^c p_{D_{dh}}^c - \sum_{\forall i \in \Omega_G} \sum_{\forall b \in \Omega_i} \lambda_{G_{ib}}^c p_{G_{ib}}^c \right) \right] - \sum_{\forall (s,r,k) \in \Omega_{L+}} K_{srk} w_{srk} \right\} \quad (1)$$

subject to

$$\sum_{i\in\Psi_G^s} p_{G_i}^c - p_s^c = \sum_{d\in\Psi_D^s} p_{D_d}^c \quad \forall s\in\Omega_N,\quad \forall c\in\Omega_C : \lambda_s^c \quad (2)$$

$$p_s^c = \sum_{\forall(r,k)\in\Psi_L^s} p_{srk}^c = \sum_{\forall(r,k)\in\Psi_L^s}\left[f_{srk}^c + \frac{1}{2}q_{srk}^c\right] \quad \forall s\in\Omega_N,\quad \forall c\in\Omega_C \quad (3)$$

$$f_{srk}^c = -b_{srk}w_{srk}\sin(\delta_s^c - \delta_r^c) \quad \forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (4)$$

$$q_{srk}^c = 2g_{srk}w_{srk}\left[1-\cos(\delta_s^c-\delta_r^c)\right], \quad \forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (5)$$

$$|p_{srk}^c| \le p_{srk}^{\max}\forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (6)$$

$$0\le p_{G_{ib}}^c \le \bar{p}_{G_{ib}} \quad \forall b\in\Omega_i;\quad \forall i\in\Omega_G;\quad \forall c\in\Omega_C \quad (7)$$

$$0\le p_{G_i}^c \le p_{G_i}^{\max}\forall i\in\Omega_G;\quad \forall c\in\Omega_C \quad (8)$$

$$\sum_{\forall b\in\Omega_i} p_{G_{ib}}^c = p_{G_i}^c;\forall i\in\Omega_G;\quad \forall c\in\Omega_C \quad (9)$$

$$0\le p_{D_{dh}}^c \le \bar{p}_{D_{dh}}^c \quad \forall d\in\Omega_D,\quad \forall h\in\Omega_d;\quad \forall c\in\Omega_C \quad (10)$$

$$\sum_{\forall h\in\Omega_d} p_{D_{dh}}^c = p_{D_d}^c\forall d\in\Omega_D;\quad \forall c\in\Omega_C \quad (11)$$

$$w_{srk}=1;\quad \forall(s,r,k)\in\Omega_L\backslash\Omega_{L+} \quad (12)$$

$$w_{srk}\in\{0,1\};\quad \forall(s,r,k)\in\Omega_L \quad (13)$$

where λ_s^c is the nodal price at bus s in scenario c calculated as the dual variable of the power balance constraint (2). Its value is computed once the mixed integer problem has been solved, fixing the binary variables to their respective optimal values and solving the resulting continuous linear programming problem.

The objective function in (1) represents the scenario-weighted social welfare, where the welfare is expressed as the aggregate demand utility bid function minus the aggregate generator offer function, minus the investment cost in new lines.[1] To evaluate expression (1), it is necessary to add up all the offers/bids provided by the generators and the demands and to use a set of weights corresponding to the periods in which these bids take place. Moreover, as the proposed planning procedure considers a single period, the weighting factor σ is used to make the investment and operating costs comparable. It simulates the ratio between the annualized investment costs and the scenario-weighted operating costs. The analysis of the results obtained by changing this parameter is explained at the end of the case studies section.

The constraints in (2) enforce the power balance at every node. The constraints in (3) imply that the power injection at node s is the summation, over all lines connected to that node, of a lossless line flow component and a loss component. For each line, these two components are the product of a binary variable and a sinusoidal function of the difference of the angles prevailing at the sending and receiving ends of the line, as shown in (4) and (5), respectively. This multiplication of a binary variable and a continuous function is the consequence of the fact that the power through a line is zero if that line is not physically connected to the network, or equivalently, if $w_{srk}=0$. The constraints in (6) enforce the line flow limits. The constraints in (7) and (10) define the size of the blocks of the generators (demands) per scenario. The constraints in (8) are the operating constraints that specify that, if a generator is dispatched, its power output must be within a certain range represented by a minimum output and a maximum output. The constraints in (9) and (11) define the power produced (consumed) by any generator (demand) as the summation of its corresponding production (consumption) blocks. Finally, the fact that the existing lines have already been built is enforced in (12) and the binary investment decision variables are defined in (13).

[1]Note that the aggregate social welfare is also equal to the demands' surplus plus the generators' surplus plus the merchandising surplus (total payments from the demands minus total payments to the generators) minus the investment cost in new lines.

The transmission expansion problem formulation presented in (1)–(13) can be expressed as a mixed-integer linear programming problem. For completeness, the mixed-integer formulation of the model is provided in the following:

Maximize

$$\left\{\sigma\left[\sum_{\forall c\in\Omega_C} w^c\left(\sum_{\forall d\in\Omega_D}\sum_{\forall h\in\Omega_d}\lambda_{D_{dh}}^c p_{D_{dh}}^c - \sum_{\forall i\in\Omega_G}\sum_{\forall b\in\Omega_i}\lambda_{G_{ib}}^c p_{G_{ib}}^c\right)\right] - \sum_{\forall(s,r,k)\in\Omega_{L+}} K_{srk}w_{srk}\right\} \quad (14)$$

subject to

$$\sum_{i\in\Psi_G^s} p_{G_i}^c - \sum_{\forall(r,k)\in\Psi_L^s}\left[f_{srk}^c+\frac{1}{2}q_{srk}^c\right] = \sum_{d\in\Psi_D^s} p_{D_d}^c,\quad \forall s\in\Omega_N,\quad \forall c\in\Omega_C : \lambda_s^c \quad (15)$$

$$-w_{srk}p_{srk}^{\max} \le f_{srk}^c \le w_{srk}p_{srk}^{\max} \quad \forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (16)$$

$$-(1-w_{srk})M \le \frac{f_{srk}^c}{b_{srk}^c} + \left(\delta_{sr}^{c+}-\delta_{sr}^{c-}\right) \le (1-w_{srk})M \quad \forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (17)$$

$$0\le q_{srk}^c \le w_{srk}p_{srk}^{\max},\quad \forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (18)$$

$$0\le -\frac{q_{srk}^c}{g_{srk}} + \sum_{\ell=1}^{L}\alpha_{sr}(\ell)\delta_{sr}^c(\ell) \le (1-w_{srk})M^2 \quad \forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (19)$$

$$\delta_{sr}^{c+}+\delta_{sr}^{c-} = \sum_{\ell=1}^{L}\delta_{sr}^c(\ell),\quad \forall(s,r)\in\Omega_L,\quad \forall c\in\Omega_C \quad (20)$$

$$\delta_s^c-\delta_r^c = \delta_{sr}^{c+}-\delta_{sr}^{c-},\quad \forall(s,r)\in\Omega_L,\forall c\in\Omega_C \quad (21)$$

$$f_{srk}^c+\frac{1}{2}q_{srk}^c \le p_{srk}^{\max},\quad \forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (22)$$

$$-f_{srk}^c+\frac{1}{2}q_{srk}^c \le p_{srk}^{\max},\quad \forall(s,r,k)\in\Omega_L,\quad \forall c\in\Omega_C \quad (23)$$

$$0\le p_{G_{ib}}^c \le \bar{p}_{G_{ib}};\quad \forall b\in\Omega_i,\quad \forall i\in\Omega_G,\quad \forall c\in\Omega_C \quad (24)$$

$$0 \le p^c_{G_i} \le p^{\max}_{G_i}, \quad \forall i \in \Omega_G, \quad \forall c \in \Omega_C \tag{25}$$

$$\sum_{\forall b \in \Omega_i} p^c_{G_{ib}} = p^c_{G_i}, \quad \forall i \in \Omega_G, \quad \forall c \in \Omega_C \tag{26}$$

$$0 \le p^c_{D_{dh}} \le \bar{p}^c_{D_{dh}}, \quad \forall d \in \Omega_D, \quad \forall h \in \Omega_d, \quad \forall c \in \Omega_C \tag{27}$$

$$\sum_{\forall h \in \Omega_d} p^c_{D_{dh}} = p^c_{D_d}, \quad \forall d \in \Omega_D, \quad \forall c \in \Omega_C \tag{28}$$

$$w_{srk} = 1, \quad \forall (s,r,k) \in \Omega_L \backslash \Omega_{L+} \tag{29}$$

$$w_{srk} \in \{0,1\}, \quad \forall (s,r,k) \in \Omega_L \tag{30}$$

$$\delta^c_s = 0, \quad s: \text{reference bus}, \quad \forall c \in \Omega_C \tag{31}$$

$$\delta^{c+}_{sr} \ge 0, \quad \delta^{c-}_{sr} \ge 0, \quad \forall (s,r) \in \Omega_L, \quad \forall c \in \Omega_C \tag{32}$$

$$\delta^c_{sr}(\ell) \ge 0, \quad \forall (s,r) \in \Omega_L, \quad \ell = 1,\ldots,L, \forall c \in \Omega_C \tag{33}$$

$$\delta^c_{sr}(\ell) \le \Delta\delta_{sr} + (1 - w_{srk})M, \quad \forall (s,r,k) \in \Omega_L, \quad \ell = 1,\ldots,L, \forall c \in \Omega_C \tag{34}$$

recall that λ^c_s is the nodal price at bus s in scenario c calculated as the dual variable of the power balance constraint (15). The linear expression of the losses needs two additional sets of continuous variables: δ^{c+}_{sr} and δ^{c-}_{sr}, whose derivation is provided in [16]. Further details of this mixed-integer linear programming formulation can be found in [5], [17], and [18]. This second formulation of the problem can be solved by any of the many commercially available programs that deal with mixed-integer linear problems; in particular, we have used the CPLEX optimizer within GAMS.

III. Metrics for Transmission Investment

In order to analyze and compare future investments in transmission, we need to define a set of metrics that show the welfare obtained by the different agents of the market: generators, demands, and the transmission entity.

The metric that shows the change in the aggregate social welfare as a result of adding new lines with respect to the investment cost in new lines is given by the following parameter:

$$\mu_1 = \frac{\mathrm{SW}^* - \mathrm{SW}^0}{\sum_{\forall (s,r,k) \in \Omega_{L+}} K_{srk} w_{srk}} \tag{35}$$

where SW^* is the optimal aggregate social welfare, i.e., the first term of (1), and SW^0 is the aggregate social welfare if transmission expansion is not considered. The calculation of SW^* results from the solution of problem (14)–(34). SW^0 is obtained if problem (14)–(34) is solved with the additional constraint that no lines can be built, hence calculating the social welfare in the situation without transmission expansion. From this definition, metric μ_1 should be greater than one to justify the investment in new lines.

The previous ratio can be a useful metric for the entire system, but it could be objected by generators or demands that do not see a real improvement for themselves with the new transmission lines added. To account for that, we define two other metrics: one for the generators, one for the demands. Furthermore, we define an additional metric that considers the effect of new lines on the merchandising surplus.

The metric available to the generators is the change in the generator surplus with respect to the investment cost in new lines. It is defined as

$$\mu_2 = \frac{\mathrm{GS}^* - \mathrm{GS}^0}{\sum_{\forall (s,r,k) \in \Omega_{L+}} K_{srk} w_{srk}} \tag{36}$$

where GS^* is the aggregate generator surplus defined as the aggregate revenue minus the total costs of the generators, and GS^0 is the aggregate generator surplus if transmission expansion is not considered. This metric illustrates how the generators could benefit from the investment in new lines. The value of μ_2 should be greater than the share of the cost that the generators will have to pay for, because otherwise, the generators will most likely be against the construction of those lines; e.g., if the generators are responsible for 50% of the cost of the new lines, they would like to obtain, at least, a value of $\mu_2 = 0.5$. If $\mu_2 < 0.5$, the construction of lines will imply a loss for the generators. Also note that values of $\mu_2 < 0$ are possible in systems with a high degree of congestion, because the construction of new lines may connect inexpensive isolated generators with the demand, which will cause prices to fall and, hence, generators profits to decrease. However, how the increase in social welfare or generators surplus or consumer surplus due to the construction of new lines is allocated to generators or demands is outside the scope of this paper.

In addition, both single-generator metrics and generating-company metrics could be calculated, taking into account only the surpluses of the relevant units. These metrics could be useful to compare the effect of the new lines for different generating companies.

Likewise, the demands can measure the increment of their surplus with respect to the investment cost in new lines. This metric is defined as

$$\mu_3 = \frac{\mathrm{CS}^* - \mathrm{CS}^0}{\sum_{\forall (s,r,k) \in \Omega_{L+}} K_{srk} w_{srk}} \tag{37}$$

where CS^* is the aggregate demand surplus defined as the aggregate demand utility function minus the total payment of the demands, and CS^0 is the aggregate demand surplus if transmission expansion is not considered. This metric shows how the demands could benefit from the investment in new lines. Also, both single-consumer and consuming company metrics could be calculated, taking into account only the surpluses of the relevant demands. These metrics could be useful to compare the effect of the new lines for different consuming companies.

Finally, the ratio of the change in the merchandising surplus with respect to the investment cost in new lines is defined as

$$\mu_4 = \frac{\mathrm{MS}^* - \mathrm{MS}^0}{\sum_{\forall (s,r,k) \in \Omega_{L+}} K_{srk} w_{srk}} \tag{38}$$

where MS^* is the aggregate merchandising surplus, and MS^0 is the aggregate merchandising surplus if transmission expansion is not considered.

Note that, as previously stated, SW^*, GS^*, CS^*, and MS^* are results obtained from solving problem (14)–(34). Also note that SW^0, GS^0, CS^0, and MS^0 are easily obtained solving the same problem with the additional constraint that no lines can be built.

IV. Case Studies

A. Introduction

The methodology proposed has been successfully applied to two case studies. The first case study analyzes the Garver six-bus system [2], and the second case implemented is based on the IEEE RTS 24-bus system [19]. The market structure of the systems considered consists of a number of generating units and a number of loads; both generating units and loads submit offers/bids to the market trying to attain their respective maximum profits. All lines are built by a central entity, the network planner.

In the case studies presented next, a linear approximation of the generators cost function is used; this linear approximation consists of a set of "blocks" for each of which the marginal cost is considered constant. Moreover, we consider that each unit submits to the market one block of offered energy and the price selected is the marginal cost corresponding to that block. We also assume that the bids of the demands express their actual utility functions. Given the fact that all internal data to the agents are confidential, our metrics can only be computed in terms of "declared" surpluses. However, each generator and each consumer can calculate its own metrics based on "real" surpluses. If offers and bids do not match marginal costs and utilities, only the "declared" surpluses can be used to compute the metrics used to allocate transmission expansion costs. Regarding bidding data, we use reasonably sized generator units and cost data closely matched with actual values observed in actual markets. Demand data are obtained from realistic cases of the day-ahead electricity market of mainland Spain [20] to get a reasonable number of scenarios based on demand patterns corresponding to year 2004.

A different number of scenarios are considered in the case studies to describe the behavior of the demand. Each of the scenarios represents a significant number of hours during one typical year of operation of the network. The different scenarios are weighted in the objective function in order to correctly consider their relative relevance; hence, the scenarios that represent situations that take place very often are given higher weights, and conversely, scenarios that represent market situations that take place only seldom, are assigned lower weights. The main difference among the considered scenarios is the amount of demand. For each demand block, its size is different in every scenario. The weight of each scenario has been calculated dividing the amount of hours represented by the specific scenario by the total number of hours considered. For example, if scenario number 1 represents the first three hours of every day (i.e., from midnight until 3:00 A.M.) for a whole year, then the weighting factor for this scenario is: $365 \times 3/365 \times 24 = 1/8$.

We consider a time horizon of one year, that is, a "target year." For this "target year," we estimate the demand, the generation offers, and the demand bids. Therefore, our model represents a "static transmission expansion planning" problem, since it considers a "target year" for which the net social welfare is maximized.[2]

We have made the following assumptions in order to estimate the weighting factor σ in (1). We have assumed that a line built today will be operative for at least 25 years; thus, a 25 year investment return period has been considered. Also, a 10% discount rate is assumed. With these two values in mind, the value of the capital recovery factor can be calculated as $r(1+r^t)/[(1+r^t)-1]$, where r is the discount rate and t the number of years; this formula provides a capital recovery factor value of 0.10. This value means that, for the next 25 years, the investment cost in new lines is yearly repaid at a rate of approximately 10% of the total initial investment. This is also known as the annualized cost.

[2]For a detailed explanation of these concepts, please refer to [21]. Also, [22] and [23] define some of the basic concepts regarding "static transmission expansion planning."

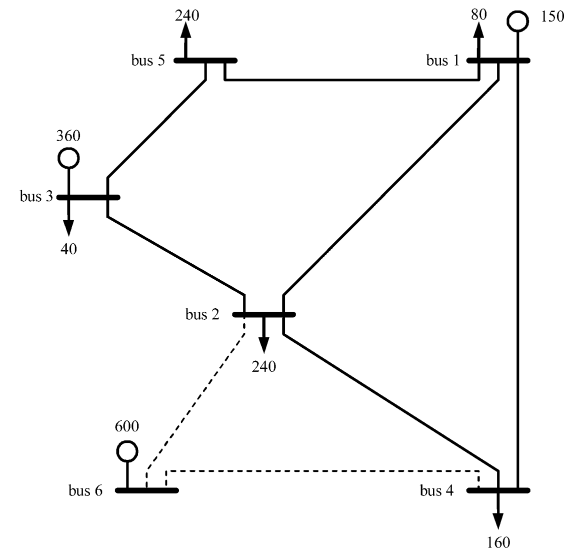

Fig. 1. Diagram for the Garver six-bus system used in the first case study.

In the case studies presented in this paper, the word corridor is always used to describe all the lines that connect the same pair of nodes, whereas the word "line" is simply used to describe each one of the lines within a corridor. Hence, subindexes "s, r, k" are used to refer to the particular line "k" that is placed inside the corridor that connects nodes "s" and "r." For example, in Fig. 2, the corridor between nodes 1 and 5 comprises four lines, and the corridor between nodes 1 and 2 contains just one line.

B. First Case Study

The system shown in Fig. 1 contains five nodes and six lines connecting them; moreover, a sixth node is considered, at which some generation is placed. This node is not initially connected to the other five nodes, but lines to connect it to the system could be built if necessary. The constraints imposed in the formulation of the problem (14)–(34) allow for the addition of lines in new or already existing corridors, up to a total maximum of three lines per corridor. The market structure of the system considered consists of ten generating units and five loads. Table I provides line data. The first two columns provide the nodes of origin and destination of the lines, the third and fourth columns show the electric parameters of the lines, and the fifth column shows, in p.u., the capacity of the line. The cost value is shown in the sixth column for all lines, where the annualized cost is 10% of that value. Finally, in the last column, the number of lines already built for every possible connection between nodes is shown.

Four different scenarios are considered to describe the behavior of the demand. The four scenarios considered can be regarded as: low demand, medium-low demand, medium-high

TABLE I
LINE STRUCTURE FOR THE FIRST CASE STUDY

From	To	R [pu]	X [pu]	Limit [pu]	Cost [$M]	Already Built
1	2	0.10	0.40	1.00	40	1
1	3	0.09	0.38	1.00	38	0
1	4	0.15	0.60	0.80	60	1
1	5	0.05	0.20	1.00	20	1
1	6	0.17	0.68	0.70	68	0
2	3	0.05	0.20	1.00	20	1
2	4	0.10	0.40	1.00	40	1
2	5	0.08	0.31	1.00	31	0
2	6	0.08	0.30	1.00	30	0
3	4	0.15	0.59	0.82	59	0
3	5	0.05	0.20	1.00	20	1
3	6	0.12	0.48	1.00	48	0
4	5	0.16	0.63	0.75	63	0
4	6	0.08	0.30	1.00	30	0
5	6	0.15	0.61	0.78	61	0

TABLE II
CHARACTERISTICS OF THE DIFFERENT SCENARIOS. FIRST CASE STUDY

Scenario	Weight	Demand coefficient
1	0.4120	0.47
2	0.3297	0.85
3	0.1592	1.20
4	0.0991	1.70

TABLE III
GENERATORS AND DEMANDS LOCATION. FIRST CASE STUDY

	Generators			Demands		
Node	Name	MW offer	Offer price [$/MWh]	Name	MW bid	Bid price [$/MWh]
1	G1	150	10	D1	80	30, 28, 26, 24, 20
2	-	-	-	D2	240	34, 32, 30, 28, 25
3	G2	120	20	D3	40	20, 16, 14, 12, 10
	G3	120	22			
	G4	120	25			
4	-	-	-	D4	160	30, 27, 24, 21, 17
5	-	-	-	D5	240	34, 30, 26, 24, 18
6	G5	100	8	-	-	-
	G6	100	12			
	G7	100	15			
	G8	100	17			
	G9	100	19			
	G10	100	21			

demand, and high demand, respectively. Table II presents the weights and relative demands for the four scenarios considered. Scenario 1 has a weight of 0.412; hence, it represents 41.2% of the hours of a year; its demand coefficient is 0.47, which means that demand for that scenario is 47% of the preselected reference demand level. Table III provides the location of generators and demands in the network along with other relevant information. For the generators, the maximum power production and the offer price is shown. For the demands, the total amount of energy bid in the market is shown.

This total amount of energy demanded is divided in five blocks of equal size with the five prices also shown in the table. Note that, using this information about the bids made by the demands, their utility functions could be constructed.

From Table III, it is clear that most of the generation is located at the initially isolated node 6; hence, transmission expansion plans will probably tend to construct lines connecting node 6 to the rest of the system.

The solution obtained is described next: three new lines are proposed to be built, having a total annualized cost of $9M. Out of the three new lines built, two of them connect node 6 with node 2 and the remaining one connects node 6 to node 4; this solution allows the energy produced at node 6 to flow to all the consumers in the system.

The solution obtained is analyzed below from the viewpoint of the generators: the generators annual cost is $53.9M; on the other hand, their total yearly revenue is $79.4M, hence obtaining a total $25.5M annual profit. These results mean that 32% of the generators income becomes profit.

From the consumer viewpoint, the solution obtained implies total annual payments for energy of $88.5M, and demand utility of $116.5M, which implies a consumer surplus of $28.0M. These results mean that the utility that consumers obtain is 31% higher than their payments; i.e., they obtain a 31% profit.

Finally, from the merchandising surplus viewpoint, a total surplus of $9.1M is obtained; as already stated, this surplus derives from the fact that the market price is not uniform for the whole system, and the demands have to pay more for energy than the generators receive for producing it. Of course, this is a consequence of locational marginal pricing.

Summing up all the surpluses, and subtracting the amount needed to built new lines, the annual social welfare results in a total $53.6M. Moreover, if the new lines were built by the generators, the $9M invested in new lines would represent 11.3% of the generators revenue, or 35.3% of their profit.

Table IV(a)–(c) shows relevant data that provide insights to characterize the features of the optimal solution found. Note that the social welfare values in Table IV(c) do not include the annualized investment cost in new lines, since that cost cannot be split per scenario.

From Table IV(a)–(c), it can be shown that scenario 4 (that only represents 10% of the hours; see Table II) is very important, because it provides high prices and high merchandising surplus with almost the same amount of energy produced, as compared to scenario 3. This means that in scenario 4, the system is under congestion and this fact forces the prices up. Also note that the losses, in percentage, are very similar for the four scenarios considered. Table IV(c) shows that the producer and merchandising surpluses increase as the demand increases, due to higher average nodal prices well above the generators offer prices, and higher levels of congestion, respectively. On the contrary, the consumer surplus first increases, since the average nodal prices are well below the demands bid prices, but decreases later, since higher levels of congestion imply that prices are much closer to the demands bid prices.

Table V shows how several generation and transmission limits are reached for each of the scenarios with the proposed solution (recall from Table II that scenario 4 is the scenario with the highest demand).

As previously stated, the system is highly congested in scenario 4, for which four lines and five generators are at maximum capacity.

TABLE IV
(a) SOLUTION FOR THE FIRST CASE STUDY. PRODUCTION, CONSUMPTION AND LOSSES. (b) SOLUTION FOR THE FIRST CASE STUDY. PRICES. (c) SOLUTION FOR THE FIRST CASE STUDY. SURPLUSES AND SOCIAL WELFARE

Scenario	Generated Energy [MWh]	Consumed Energy [MWh]	Energy Losses [MWh] [%]
1	362.6	342.2	20.4 (5.6%)
2	551.3	517.6	33.7 (6.1%)
3	637.6	600.1	37.5 (5.9%)
4	650.0	611.2	38.8 (6.0%)

(a)

Scenario	Maximum Nodal Price [$/MWh]	Minimum Nodal Price [$/MWh]	Average Nodal Price [$/MWh]
1	17.9	15.0	16.60
2	22.1	17.0	20.14
3	26.0	17.0	23.08
4	30.0	17.0	25.33

(b)

Scenario	Producer surplus [$M]	Consumer Surplus [$M]	Merchandising Surplus [$M]	Social Welfare [$M]
1	16.8	29.3	1.8	47.9
2	27.2	33.1	6.9	67.2
3	35.2	23.0	18.4	76.6
4	39.7	13.4	31.5	84.6
all	**25.5**	**28.0**	**9.1**	**62.5**

(c)

TABLE V
LIMITS REACHED FOR THE SOLUTION OBTAINED

Scenario	Generators at maximum capacity	Lines at maximum capacity
1	3	0
2	4	3
3	5	3
4	5	4

TABLE VI
METRICS FOR THE FIRST CASE STUDY

Parameter	μ_1	μ_2	μ_3	μ_4
Value	2.84	0.51	1.91	0.42

The metrics presented in Section IV have been calculated for this case study. Table VI provides the results.

From Table VI, note that for each dollar invested in new lines, a total $2.84 is obtained as social welfare for the participants in the market. It can be concluded that this investment will be very profitable for the system. Particularly, the increase in social welfare is divided as follows: $0.51 goes to the generators, $1.91 goes to the consumers, and $0.42 goes to the network operator.

A total computing time of 1.46 s is needed to solve the above problem under a Linux-based server with four Xeon processors clocking at 1.60 GHz and 2 GB of RAM. The software used is CPLEX under GAMS [24].

C. Second Case Study

The second case study is based on a system similar to the one described in [19] and with the topology shown in Fig. 2. Note that the original parallel lines considered in [19] are initially

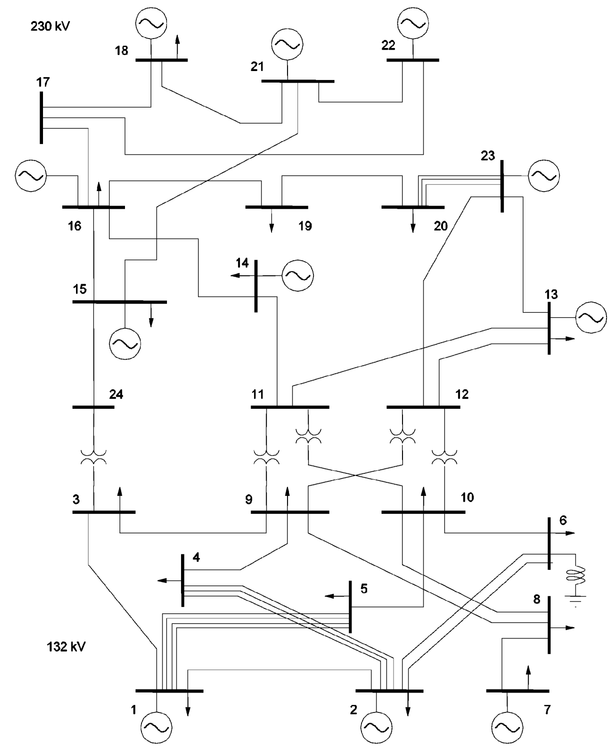

Fig. 2. Second case study: diagram for the 24-bus system with the addition of the eight new lines proposed.

disregarded in this case study. The case study presents the following characteristics: the system considered contains 24 nodes and 34 lines connecting them; in this case, for the sake of simplicity, new lines can only be constructed in parallel to existing lines, up to a total number of four, i.e., only new lines for old corridors are allowed. The market structure of the system considered consists of 11 generating units and 17 loads.

Table VII provides the location of generators and demands in the network along with other relevant information. For the generators, the maximum power production and the energy offers are shown. Note that in this case study, four blocks are used to represent the offers of the generators; for simplicity, each of these four blocks represents 25% of the capacity of each generator. For each demand, the total amount of the energy bid in the market is provided; this total amount of energy demanded is divided into three blocks of equal size with the three prices also provided in the table.

From Table VII, it is clear that both generation and consumption are evenly distributed throughout the network.

One hundred different scenarios are considered in this case study to describe the behavior of the demand. For the sake of simplicity, all the scenarios have the same weight: 1%. The actual procedure used to obtain the different scenarios used for this case study is described next: firstly, based on real data, we obtain a demand forecast for each and every hour of the target year; then, we sort these demand values in decreasing order; next, we divide the previously formed decreasing curve into 100 different portions, one for each scenario; finally, we calculate for each of these portions the average amount of energy demanded; demand coefficients can now be calculated for each scenario dividing its average demand by the reference level of demand. In this way,

TABLE VII
GENERATORS AND DEMANDS LOCATION. SECOND CASE STUDY

	Generators			Demands		
Node	Name	MW offer	Offer price [$/MWh]	Name	MW bid	Bid price [$/MWh]
1	G1	250	15.0, 18.8, 22.5, 26.3	D1	62	39.0, 36.4, 33.8
2	G2	250	13.0, 16.3, 19.5, 22.8	D2	70	44.2, 41.6, 39.0
3	-	-		D3	48	26.0, 20.8, 18.2
4	-	-		D4	125	39.0, 36.4, 33.8
5	-	-		D5	187	44.2, 41.6, 39.0
6	-	-		D6	62	39.0, 36.4, 33.8
7	G3	220	15.0, 18.8, 22.5, 26.3	D7	187	44.2, 41.6, 39.0
8	-	-		D8	187	26.0, 20.8, 18.2
9	-	-		D9	125	39.0, 36.4, 33.8
10	-	-		D10	187	44.2, 41.6, 39.0
11	-	-		-	-	
12	-	-		-	-	
13	G4	100	15.0, 18.8, 22.5, 26.3	D11	62	39.0, 36.4, 33.8
14	G5	100	14.0, 17.5, 21.0, 24.5	D12	70	44.2, 41.6, 39.0
15	G6	100	16.0, 20.0, 24.0, 28.0	D13	31	26.0, 20.8, 18.2
16	G7	100	15.0, 18.8, 22.5, 26.3	D14	125	39.0, 36.4, 33.8
17	-	-		-	-	
18	G8	100	13.0, 16.3, 19.5, 22.8	D15	148	44.2, 41.6, 39.0
19	-	-		D16	62	39.0, 36.4, 33.8
20	-	-		D17	226	44.2, 41.6, 39.0
21	G9	100	14.0, 17.5, 21.0, 24.5	-	-	
22	G10	300	15.0, 18.8, 22.5, 26.3	-	-	
23	G11	200	15.0, 18.8, 22.5, 26.3	-	-	
24	-	-		-	-	

we obtain 100 different scenarios, all of which are based on real data and that represent in detail the demand in the system. In the case study presented here, the maximum demand coefficient is 2.20. The energy demanded at each node for each scenario is calculated multiplying the MW demand provided in Table VII by the coefficient associated to the scenario.

The cost of construction of new lines is as follows: we assume for simplicity that, on average, lines connecting two nodes in the lower half of the system have an annualized cost of $2M per line, lines connecting two nodes in the upper part of the system have an annualized cost of $4M per line, and lines connecting the upper and the lower parts, i.e., new lines between 3–24, 9–11, 9–12, 10–11, and 10–12, are not allowed due to their high costs. Note that the annualized cost values are equivalent to 10% of the total construction costs.

The solution obtained for the case study presented is described next. Eight new lines are proposed to be built: three lines connect nodes 1 and 5; two lines are built between nodes 2 and 4; two lines between nodes 20 and 23; and finally one line is built between nodes 2 and 6, thus increasing the transportation capacity from the large generators at nodes 1, 2, and 23. The total annualized cost is $20M.

Summing up the surpluses obtained by the generators, the consumers and the network planner and subtracting the amount needed to built new lines, the annual social welfare results in a total $200.51M.

Table VIII(a) and (b) shows the optimal solution found for selected scenarios.

From Table VIII(a)–(c), it can be observed that the last scenarios are important, because of their high prices and high merchandising surplus. This means that system congestions make prices increase. Also note that the losses are very similar in all the scenarios considered. Table VIII(c) shows that the producer and merchandising surpluses increase as the demand increases, due to higher average nodal prices and higher levels of congestion, respectively. Note that for very high demand, merchandising surplus is slightly reduced, due to very high saturation with nodal prices reaching their maximum values (see Table VIII(b) allowing cheaper generators to enter the auction. However, the consumer surplus first increases, but decreases later with some slight oscillation, due to the combined effects of congestion and demand bidding (see the two different demand

TABLE VIII
(a) SOLUTION FOR THE SECOND CASE STUDY. PRODUCTION, CONSUMPTION, AND LOSSES. (b) SOLUTION FOR THE SECOND CASE STUDY. PRICES. (c) SOLUTION FOR THE SECOND CASE STUDY. SURPLUSES AND SOCIAL WELFARE

Scenario	Generated Energy [MWh]	Consumed Energy [MWh]	Energy Losses [MWh] [%]
1	453.58	441.01	12.57 (2.8%)
10	740.49	724.13	16.36 (2.2%)
20	971.15	952.88	18.27 (1.9%)
30	1148.02	1128.17	19.85 (1.7%)
40	1259.26	1239.23	20.03 (1.6%)
50	1316.12	1296.23	19.89 (1.5%)
60	1343.91	1324.46	19.45 (1.4%)
70	1366.76	1346.55	20.21 (1.5%)
80	1392.82	1372.73	20.09 (1.4%)
90	1419.64	1399.53	20.10 (1.4%)
100	1444.96	1425.08	19.88 (1.4%)

(a)

Scenario	Maximum Nodal Price [$/MWh]	Minimum Nodal Price [$/MWh]	Average Nodal Price [$/MWh]
1	18.6	14.7	17.2
10	33.8	14.0	21.0
20	39.0	15.8	25.0
30	39.0	17.2	28.0
40	44.2	17.5	30.1
50	44.2	17.5	31.8
60	44.2	17.5	33.0
70	44.2	17.5	33.8
80	44.2	17.5	34.2
90	44.2	17.5	34.3
100	44.2	17.5	34.6

(b)

Scenario	Producer surplus [$M]	Consumer Surplus [$M]	Merchandising Surplus [$M]	Social Welfare [$M]
1	8.33	72.11	0.96	81.40
10	19.03	99.84	15.08	133.95
20	39.81	104.72	30.44	174.97
30	53.92	91.83	58.68	204.43
40	70.25	92.64	62.42	225.31
50	113.35	64.62	61.86	239.83
60	123.45	57.88	67.55	248.89
70	129.41	54.76	72.58	256.74
80	136.63	50.51	75.75	262.90
90	139.26	53.94	75.30	268.51
100	154.10	48.02	70.61	272.74
all	**90.05**	**73.97**	**56.49**	**220.51**

(c)

TABLE IX
OVERALL METRICS FOR THE SECOND CASE STUDY. GENERATORS AND DEMANDS EVENLY DISTRIBUTED

Parameter	μ_1	μ_2	μ_3	μ_4
Value	1.49	0.73	0.29	0.47

TABLE X
METRICS FOR THE SECOND CASE STUDY. GENERATORS AND DEMANDS NOT EVENLY DISTRIBUTED

Parameter	μ_1	μ_2	μ_3	μ_4
Value	1.19	0.21	0.69	0.29

bidding prices in Table VII). Recall that the social welfare values in Table VIII(c) do not include the annualized investment cost in new lines, since that cost cannot be split per scenario.

Table IX provides the metrics defined in Section III for this case study.

For this case study, for each dollar invested in new lines, a total $1.49 is recovered as net social welfare; particularly, generators increase their profits $0.73 for each dollar invested in new lines; similarly, consumers benefit by earning $0.29 more and the system operator by earning $0.47 more.

For this second case study, a total computing time of 44 min is needed to solve the problem.

D. Non-Homogeneous Generation and Demand Distribution

A new case study is constructed based on the previous one by changing the location of some generators and demands. For this system, the upper portion can be considered a generation area (area above nodes 11-12-24 in Fig. 2, all included) and the rest can be considered mainly as a consumption area.

The changes made to obtain the new system are described next. Demand at D2 is increased by 300 MW and at D6 by 400 MW; demand is decreased at D15 by 300 MW and at D17 by 400 MW. Generation at G1 is decreased by 200 MW, at G2 by 200 MW, and at G3 by 200 MW. Finally, generation at G8 is increased by 100 MW, at G9 by 300 MW, and at G10 by 200 MW.

The solution obtained for this new case study is described next: only two new lines are proposed to be built, having a total annualized cost of $8M. These new lines connect nodes 12 and 23 and nodes 20 and 23, respectively. Note that these lines are built to help the transfer of energy generated at nodes 18, 21, 22, and 23 to the consumption nodes in the lower part of the system. Table X shows the values for the metrics defined in Section III.

In this case, note that, for each dollar invested in new lines, $1.19 are obtained as net social welfare. This implies that in the modified case study, new lines produce less benefit. Considering that the natural tendency for a system like this should be to invest in new lines to strengthen the interconnection between its two areas, the solution obtained is counter-intuitive. That is, although the modified system is in a greater need of interconnecting lines to serve the demands, the solution of the problem shows that not many lines are constructed. The explanation is that the modified system has a generation and demand pattern very unevenly distributed; therefore, connecting generation and demand is more expensive; hence, it is difficult to obtain the net social welfare that pays for ambitious network expansion plans.

For this modified case study, the total computing time is 14.6 min.

E. Analysis of the Effect of the σ Parameter

In order to establish the importance of the weighting parameter σ from (1), some variations on the second case study (see Section V-C) are performed. These tests prove that the election of this weighting parameter is important, as small variations have a significant impact on the number of lines to be constructed. Initially, the value of the weighting factor is set to 1000/8760.[4] For a first test, the value of σ is increased to 2000/8760. Under these conditions, the simulation results change dramatically, and no new lines are proposed. In other words, for $\sigma = 2000/8760$, no social welfare increase can be attained by constructing new lines. On the other hand, a second test is run reducing the value of σ to 500/8760; in this test, the number of new lines proposed increases from 8 to 13, but the costs are reduced from $20M to $17M.[3] In this case, the lines to be built are the same eight lines of the base case plus another five lines: one more line connecting nodes 2 and 4, two more lines connecting nodes 2 and 6, one line between nodes 17 and 22, and one line between nodes 18 and 21.

From an economic point of view, the increment of the σ parameter σ can be seen as an increase in the cost associated to the construction of new lines, and conversely, a decrease in its value can be regarded as a decrease in line construction costs. Also, this analysis allows for the heuristic determination of a priority list for line construction, i.e., if σ is initially set to a high value and then it is smoothly decreased, prospective lines are selected, one at a time, in an orderly manner, according to their economic importance. In other words, there must be a value of σ for which only one line is proposed to be built, this line can be regarded as the most profitable line, and hence, the first line that should be funded.

V. CONCLUSION

We have presented a mixed-integer formulation of the transmission expansion planning problem in pool-based electricity markets. We have set the problem considering investment costs and weighting the aggregate social welfare according to the different levels of demand scenarios that can occur. Different economic indicators are used to appraise the quality of the transmission expansion plans proposed by the algorithm. Results from different case studies show that changes in the topology, scenarios, and the weighting factor relating investment and operating costs have appropriate economic interpretations for the generators, demands, and the network planner.

APPENDIX

In this Appendix we present a multi-year formulation of the problem presented in the paper. This multi-year model has not been used in the simulations and is presented only to illustrate to the readers about its complexity and difficulties.

[3]Note that in this test, lines are considered to be 50% cheaper to build because parameter σ has been divided by 2.

[4]Note that 8760 is the number of hours in a year and 1000 is a scaling factor.

In the multi-year model, we have considered that the scenarios are now defined per year, and consequently, the variables related to scenarios are now defined per scenario per year. Note that we have dropped the weighting factor σ and we have included a new constraint (52) to account for the fact that one line can only be built once in the entire multi-year period of study. The general formulation of the multi-year model is as follows:

$$\text{Maximize} \left\{ \sum_{y \in Y} \left[\sum_{\forall c(y) \in \Omega_{C(y)}} w^{c(y)} \times \left(\sum_{\forall d \in \Omega_D} \sum_{\forall h \in \Omega_d} \frac{\lambda_{D_{dh}}^{c(y)} p_{D_{dh}}^{c(y)}}{(1+\tau)^{y-y_0}} - \sum_{\forall i \in \Omega_G} \sum_{\forall b \in \Omega_i} \frac{\lambda_{G_{ib}}^{c(y)} p_{G_{ib}}^{c(y)}}{(1+\tau)^{y-y_0}} \right) \right] - \sum_{y \in Y} \sum_{\forall (s,r,k) \in \Omega_{L+}} \frac{K_{srk} w_{srk}^{y}}{(1+\tau)^{y-y_0}} \right\} \tag{39}$$

subject to

$$\sum_{i \in \Psi_G^s} p_{G_i}^{c(y)} - p_s^{c(y)} = \sum_{d \in \Psi_D^s} p_{D_d}^{c(y)} \quad \forall s \in \Omega_N, \quad \forall c(y) \in \Omega_{C(y)} \forall y \in Y : \lambda_s^{c(y)} \tag{40}$$

$$p_s^{c(y)} = \sum_{\forall (r,k) \in \Psi_L^s} p_{srk}^{c(y)} = \sum_{\forall (r,k) \in \Psi_L^s} \left[f_{srk}^{c(y)} + \frac{1}{2} q_{srk}^{c(y)} \right] \quad \forall s \in \Omega_N, \quad \forall c(y) \in \Omega_{C(y)}, \quad \forall y \in Y \tag{41}$$

$$f_{srk}^{c(y)} = -b_{srk} w_{srk}^{y} \sin\left(\delta_s^{c(y)} - \delta_r^{c(y)}\right) \quad \forall (s,r,k) \in \Omega_L, \quad \forall c(y) \in \Omega_{C(y)}, \quad \forall y \in Y \tag{42}$$

$$q_{srk}^{c(y)} = 2 g_{srk} w_{srk}^{y} \left[1 - \cos\left(\delta_s^{c(y)} - \delta_r^{c(y)}\right) \right] \quad \forall (s,r,k) \in \Omega_L \forall c(y) \in \Omega_{C(y)}, \quad \forall y \in Y \tag{43}$$

$$\left| p_{srk}^{c(y)} \right| \le p_{srk}^{\max} \quad \forall (s,r,k) \in \Omega_L, \quad \forall c(y) \in \Omega_{C(y)}, \quad \forall y \in Y \tag{44}$$

$$0 \le p_{G_{ib}}^{c(y)} \le \bar{p}_{G_{ib}} \quad \forall b \in \Omega_i, \quad \forall i \in \Omega_G, \quad \forall c(y) \in \Omega_{C(y)}, \quad \forall y \in Y \tag{45}$$

$$0 \le p_{G_i}^{c(y)} \le p_{G_i}^{\max} \quad \forall i \in \Omega_G, \quad \forall c(y) \in \Omega_{C(y)}, \quad \forall y \in Y \tag{46}$$

$$\sum_{\forall b \in \Omega_i} p_{G_{ib}}^{c(y)} = p_{G_i}^{c(y)} \quad \forall i \in \Omega_G, \quad \forall c(y) \in \Omega_{C(y)}, \quad \forall y \in Y \tag{47}$$

$$0 \le p_{D_{dh}}^{c(y)} \le \bar{p}_{D_{dh}}^{c(y)} \quad \forall d \in \Omega_D, \quad \forall h \in \Omega_d, \quad \forall c(y) \in \Omega_{C(y)} \forall y \in Y \tag{48}$$

$$\sum_{\forall h \in \Omega_d} p_{D_{dh}}^{c(y)} = p_{D_d}^{c(y)} \quad \forall d \in \Omega_D, \quad \forall c(y) \in \Omega_{C(y)}, \quad \forall y \in Y \tag{49}$$

$$w_{srk}^{y} = 1, \quad \forall (s,r,k) \in \Omega_L \backslash \Omega_{L^-}, \quad \forall y \in Y \tag{50}$$

$$w_{srk}^{y} \in \{0,1\}, \quad \forall (s,r,k) \in \Omega_L, \quad \forall y \in Y \tag{51}$$

$$\sum_{y \in Y} w_{srk}^{y} \le 1 \tag{52}$$

where

y	current year of the period of study;
y_0	base year;
τ	discount rate;
w_{srk}^{y}	binary variable: $w_{srk}^{y} = 1$ if line k from corridor (s,r) is built in year y of the study period; $w_{srk}^{y} = 0$ if not;
Y	set of all years of the period of study.

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