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On the Impact of Forward Markets on Investments in Oligopolistic Markets with Reference to Electricity

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Allaz and Vila made the seminal contribution that forward contracts mitigate market power on the spot market. This result is widely quoted and elaborated in studies of restructured power markets, where generators can potentially exploit the special characteristics of this industry in order to extract higher prices. Allaz-Vila established their result under the assumption that the production capacities of the players are infinite. We show that the Allaz-Vila result does not hold when capacities are endogenous and constraining generation. Specifically, a forward market can enhance or mitigate market power when capacities are endogenous and demand is unknown at the time of investment. We also show that forward markets do not mitigate market power when capacities are endogenous and demand is known at the time of investment. Our results complement other work that shows that forward markets systematically enhance market power in some symmetric capacity-constrained markets.

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1. Introduction

Market power, contracts, and resource adequacy are three major themes of the literature on the restructuring of the electricity industry. This paper discusses an aspect of the impact of long-term contracts on investments in an electricity market subject to market power. The exercise of market power in spot markets probably constitutes the bulk of the literature in the restructured electricity industry. The analysis of investment questions has been less developed for some time but is gaining a lot of attention today. von der Fehr and Harbord (1997) were probably the first to offer a realistic treatment of investments in generation capacity under imperfect competition. Gabszewicz and Poddar (1997) offer a general treatment of investments in oligopolies, and Reynolds and Wilson (2000) is also a useful reference. The bulk of the literature is more recent however with papers from Zoettl (2008) Boom and Bueller (2007), Grimm and Zoettl (2006), de Frutos et al. (2008). These models assume symmetric players and concentrate on symmetric or asymmetric equilibria. They differ in their assumptions on competition in the spot and investment markets and, accordingly, arrive at different, sometimes contradictory, results. Joskow and Tirole (2007), with a treatment of price sensitive and price insensitive consumers, is somewhat apart in that literature. Murphy and Smeers (2005) probably offer the sole treatment of asymmetric equilibria for asymmetric players confronted with a full load duration curve. They consider a stylized Cournot competition model at both the investment and operations stage. We adopt a related framework here and treat a problem of asymmetric equilibrium of games with asymmetric players.

This paper deals with the relationship between contracts and market power in investments. The interest in contracts in the restructured electricity sector and their impact on market power probably goes back to Green (1992) (see Henney 1994 for the context). Newbery (1998) and Green and Newbery (1992) developed the idea. Wolfram (1999) and Wolak (2000) elaborated on the market power mitigation effect of forward contracts, which is also confirmed more recently in Bushnell et al. (2008). This literature resulted in a general consensus that long forward positions decrease market power (e.g., Newbery 2002, Twoney et al. 2005, and Joskow 2008). Harvey and Hogan (2000) asked why parties would enter forward markets that decrease their market power. The seminal paper of Allaz and Vila (1993) has often been invoked to provide some explanation. They developed a two-stage game in which players take positions in the forward market in the first stage and act on the spot market in the second stage. Assuming homogenous Cournot competitors and affine demand functions and cost curves, they showed that competing generators have an incentive to enter forward positions for purely strategic reasons. This creates a prisoner's dilemma game. This result added to the interest in forward contracts and an extensive literature developed looking at variants of their paper. Allaz and Vila were also the first ones to question the generality of their result; they noted that slightly different but equally plausible assumptions about the game change their conclusions. Different authors later confirmed the instability of this result. Newbery (1998) and Green (1999) elaborated on this idea in a supply function equilibrium context. Gans et al. (1998) adopted the original model of Allaz and Vila and reaffirmed their results. However, as in Newbery (1998), they noted that contracts could be used to restrict entry, leading to higher prices in the long run. Liski and Montero (2006) posed the problem as an infinitely repeated game and obtained different results. Laboratory experiments with students conducted by Le Coq and Orzen (2006) partially supported the Allaz-Vila findings. The general conclusion is that the Allaz-Vila result does not always hold. Notwithstanding these conflicting results, the analytical attractiveness of the pure Cournot model of Allaz-Vila continues to generate interest in academic discussions of market power in restructured electricity systems. The work of Bushnell et al. (2008) suggests that the Cournot assumption should not be dismissed too quickly for representing restructured electricity market with contracts. Policy oriented papers, even though advocating the benefits of contracts to reduce market power, often take the position that requirements to write contracts should be imposed and the contracts should be regulated.

As argued in Oren (2005), financial contracts can also play a more direct role in increasing investment. Oren's proposal has generated a whole stream of new thinking on contracts for inducing investments. Cramton and Stoft (2006) summarize these developments and they are also discussed in Joskow (2007).

Our paper is organized around the following thought experiment. Assume an Allaz-Vila world where Cournot players spontaneously enter forward contracts that mitigate their market power. Because market power reduces investments and financial contracts reduce market power, the natural inference is that financial contracts expand investment. We present results that show that this reasoning is not correct. The effect of financial contracts is ambiguous. This argument has another side: the capability of financial contracts to mitigate market power under the Allaz-Vila assumptions is overstated when investments are needed. The market power mitigation that exists when capacity is slack disappears when it is tight. Kamat and Oren (2004) uncover a related result in a congestion management problem. They reproduce the Allaz-Vila results when the transmission constraints are not binding but also find that the market mitigation property of the Allaz-Vila model disappears when constraints are binding.

We contribute to this literature in different ways. First, in contrast to most of the work examining the Allaz-Vila results, we retain their original model except for the minimal differences necessary to capture some basic features of the power sector. Specifically, we do not assume homogeneous players because different technologies compete in the power sector. We keep the Allaz-Vila linear cost curve but introduce a capacity constraint in order to cast the model in an investment context. We then show that this slight extension destroys all its convexity properties. The resulting model is an extension of an equilibrium problem subject to equilibrium constraints (EPEC). Obtaining positive qualitative results for EPECs is very rare. The model we present is a three-stage game, a type of problem that is barely mentioned in the literature (see Sauma and Oren 2006 for an example of such a model in a network investment context). Our third and main contribution is a negative result. The prisoner's-dilemma argument that drives the Allaz-Vila results disappears because the investment game gives the players an instrument to neutralize it. Specifically, we show that in a model with deterministic demand, forward contracts no longer modify the equilibrium, and hence market power, provided this equilibrium continues to exist. The result is not as clear cut with uncertain demand where forward contracts can enhance or mitigate market power when investments are possible.

We believe that these results can be important in practice. Specifically, the observed variety of situations obtained with a simplified model shows that we know very little if anything about the properties of endogenous financial contracts for mitigating market power in a realistic environment where investments have to be decided. The analysis also reveals the analytical complexity of the problem: introducing capacities transforms a model with an analytical solution into a combinatorial problem that is probably not solvable in a realistic context.

We first present the general framework of our models in §2. In order to offer some intuition to our results, §3 gives a small example illustrating the ambiguity of the impact of contracts when investments in new capacities are possible. The example is not self-contained in the sense that it calls upon phenomena that are discussed in detail in the course of the paper. However, it clearly shows the forces at work that make uncertain the alleged beneficial effects of contracts on market power in electricity. We next derive the properties of our model. We then summarize the discussion by presenting an intuitive explanation of the results using the experience we garnered from our numerical experiments and the form of some of the equations that are central to the qualitative results.

2. The Models

We assume two generating companies are in competition, each specializing in one particular technology. Alternatively, both generators can specialize in the same technology. Following much of the economic literature, we assume that there is no existing generation system. Each company first invests in new capacity and then competes on the spot market given its capacity. We thus represent a merchant system. The two main models considered in the paper differ in that one has a forward market and the other does not. In the deterministic case we also include the open-loop game, where capacity and operating decisions occur simultaneously and there is no forward market.

With a forward market, the spot-market equilibrium is found given the capacities and forward positions. The forward-market equilibrium is found given the capacities, taking into account the ensuing spot equilibrium. Capacity decisions are made knowing their impact on the forward and spot equilibria. The model without a forward market has a spot-market equilibrium that is a function of the capacities, and the capacity decisions are made knowing their effect on the spot market.

We define our notation assuming uncertainty and then present any alterations necessary for certain demand. We model uncertain demand with an inverted demand function of the form

$$p=\xi-q \tag{1}$$

where p and q, respectively, denote the price and quantity, and ξ is a random intercept with density $f(\xi)$ defined over (L, U).

In the deterministic case $L=U$, and we use ξ as the intercept.

We index two players by i, using $-i$ to indicate the player that is not i. The economic characteristics of the technologies determine the investment and operating cost parameters k_i and ν_i for $i=1,2$, where k_i is measured in € or \$/Mw and ν_i is measured in € or \$/Mwh; Stoft (2002) discusses these units.

We assume that the competing companies behave like Cournot players in all of the markets (spot, forward, and capacity): they exert market power by setting quantities (energy delivered, forward positions, capacities invested). These quantity variables for $i=1,2$ are

x_i, the capacity invested by firm i;

y_i, the forward position of firm i; and

$z_i(\xi)$, the energy delivered by firm i when the demand realization is ξ.

We choose this notation as a mnemonic device with the order of the letters the same as the order of the decisions. When demand is deterministic, we use z_i.

2.1. The Spot Game Optimizations and Market Equilibrium

Let x_i and y_i be, respectively, the capacity and forward positions of player i when it enters the spot market. When demand is uncertain, we assume that the demand function (1) is revealed after the investments are made and forward positions are taken. For each realization of ξ, the two companies compete as Cournot players on the spot market. Rewriting $z_i(\xi)$ as z_i for the sake of convenience, this implies that each company i takes the production z_{-i} of the other player as given and solves

$$\max_{z_i} \quad (\xi-z_i-z_{-i})(z_i-y_i)-\nu_i z_i \tag{2}$$

$$\text{s.t.} \quad 0\leqslant z_i \qquad (\omega_i) \tag{3}$$

$$0\leqslant x_i-z_i \quad (\lambda_i). \tag{4}$$

In this formulation, after selecting a forward position y_i at the established forward price, the incentive of the generator to manipulate the market by restricting z_i is limited to the residual market z_i-y_i. Let ω_i and λ_i be the dual variables of the constraints $z_i\geqslant 0$ and $x_i-z_i\geqslant 0$, respectively. Solving the problems of both generators simultaneously, we obtain the equilibrium conditions of the Cournot spot market.

$$\begin{aligned} &0\leqslant \xi-2z_i-z_{-i}-\nu_i+y_i-\lambda_i+\omega_i\perp z_i\geqslant 0, \quad i=1,2\\ &0\leqslant x_i-z_i\perp\lambda_i\geqslant 0, \quad 0\leqslant z_i\perp\omega_i\geqslant 0, \quad i=1,2.\end{aligned} \tag{5}$$

The equilibrium on the spot market is a parametric complementarity problem. With the z_i and z_{-i} satisfying condition (5), the profit from its spot-market operations is

$$(\xi-z_i-z_{-i}-\nu_i)(z_i-y_i). \tag{6}$$

2.2. The Forward Game Optimizations and Market Equilibrium

Let y_i be the position taken by agent i in the forward market. We invoke the usual no-arbitrage assumption of finance theory, that is, y_i is sold at the expected price using some risk neutral probability of the spot prices. Thus, we reinterpret the distribution $f(\xi)$ of the parameter ξ as a risk neutral probability resulting from the trading of the forward positions. The expected forward price is

$$\int_L^U(\xi-z_i-z_{-i})f(\xi)\,d\xi. \tag{7}$$

Given the position y_{-i} of player $-i$, the profit of player i on both the spot and forward markets is then

$$\begin{aligned} &y_i\int_L^U(\xi-z_i-z_{-i})f(\xi)\,d\xi\\ &\quad+\int_L^U(\xi-z_i-z_{-i})(z_i-y_i)f(\xi)\,d\xi\\ &\quad=\int_L^U(\xi-z_i-z_{-i})z_i f(\xi)\,d\xi.\end{aligned} \tag{8}$$

This formula invokes y_i and y_{-i} implicitly since z_i and z_{-i} are parameterized by y_i and y_{-i}. The Cournot problem on the forward market is then defined as follows. Given y_{-i}, generator i solves

$$\max_{y_i}\int_L^U(\xi-z_i-z_{-i})z_i f(\xi)\,d\xi \tag{9}$$

where z_i and z_{-i} are the solution of (5). Note that in (7), (8), and (9) and subsequent expressions, by removing the integrals, we have the deterministic case.

Equilibrium problems, like this equilibrium in y, subject to equilibrium constraints (EPEC), here relation (5), belong to the class of generalized Nash games and suffer from problems of existence and uniqueness of pure strategy equilibria.

Since the solution to the spot equilibrium conditions (5) is unique, there exist unique (nondifferentiable) functions

$$z_i(y_i, y_{-i}; \xi) \quad \text{and} \quad z_{-i}(y_i, y_{-i}; \xi) \tag{10}$$

that solve (5). Replacing (10) in (9) we obtain the reformulation of (9)

$$g(x_i, x_{-i}) = \max_{y_i} \int_L^U [\xi - z_i(y_i, y_{-i}; \xi) - z_{-i}(y_i, y_{-i}; \xi) - \nu_i] \cdot z_i(y_i, y_{-i}; \xi) f(\xi)\, d\xi \tag{11}$$

which is a standard Nash equilibrium and not an EPEC. This problem is unconstrained in y_i as generators can take long or short positions in forward markets. As is usually the case, the convexity properties of the second stage (here spot market) problem of an EPEC are lost when moving to the first stage (here the forward market) of the EPEC, and we cannot guarantee that the game has a pure strategy solution.

2.3. The Capacity Game Optimizations

The profit function of generator i in the forward market depends on the capacities x_i, $i = 1, 2$. We define the net profit (after accounting for capital charges) of company i to be

$$p_i(x_i, x_{-i}) = g(x_i, x_{-i}) - k_i x_i. \tag{12}$$

In the Cournot model both players simultaneously solve

$$\max_{x_i \geqslant 0}\ p_i(x_i, x_{-i}). \tag{13}$$

We are ultimately interested in the impact of a forward market on this game.

3. Simple Examples

Although we develop our results for the continuous case, we can illustrate what happens using a probability distribution with discrete outcomes. Here we make assertions about strategic moves that we justify in subsequent sections. Formulas relative to these examples are given in online Appendix A. The online appendices are placed in an electronic companion to this paper and can be found at http://or.journal.informs.org.

Assume a distribution of ξ with two-outcomes, 200 and 140 of equal probability. With no forward market, we use costs where both players are at capacity for just the first outcome. The costs reported in Table 1 are chosen for illustrative purposes only and do not match any real costs.

Table 1. Cost structure.

	Capacity	Operating
Player i	5	4
Player $-i$	10	3

What happens with the addition of a forward market depends on the relative size of demand with the second outcome. If the increase in production induced by the forward market in the second outcome stays below capacity for both players, then the players find the Allaz-Vila equilibrium for this outcome, and total capacity does not change because the marginal values of capacity do not change for the demand outcomes. Total capacity is 121 without and with a forward market in our example, and total production with the lower demand outcome is 97.7 and 117.2 without and with a forward market, respectively.

The existence of capacity allows one player to play Allaz-Vila and to prevent the other from doing so. This leads to two more equilibria, each with one of the players at capacity in the second outcome and the other below capacity. In the next sections we show that players can sell in the forward market so that they always generate at capacity in the spot market, blocking the other player from the forward market. This strategic use of the contract market is available to both players, including the player with higher short-run operating costs. Also, an inefficient generator can increase production while possibly reducing total generation, because the marginal values of generation capacities change.

First, we show a total increase of capacity and production. Recall that capacity and production in the first outcome is 121, and production is 97.7 with the second outcome. With a forward market, when player i is at capacity in both outcomes, total capacity is 124.6 and production with the second outcome is 104.6. The equilibrium with player $-i$ at capacity in both outcomes has a total capacity of 121.5, and production with the second outcome is 101.5. Here, both capacity and production increase as in Allaz-Vila. The player at capacity for both steps has a positive marginal value for capacity in the second step, leading to an increase in its capacity. The other player reduces its capacity less than the increase by the first, resulting in a total increase in capacity. The reason for the positive marginal value of capacity with the second outcome is that the player sees how its increase in capacity decreases the equilibrium quantity produced by the other player in the spot game unlike before. Thus, its marginal revenue is higher and the marginal value of capacity increases from 0 to a positive number.

We now show that capacity can increase or decrease. Our demand distribution is as indicated in Table 2. Results are given in Table 3.

Table 2. Demand distribution with three possibilities.

	Outcome 1	Outcome 2	Outcome 3
Demand	200	180	150
Probability	0.4	0.3	0.3

Without a forward market, player $-i$ is at capacity for the first two outcomes and player i is at capacity for only the first. With a forward market we have two equilibria with one player at capacity in all three outcomes and the other at capacity for the first outcome only. We term an equilibrium where one player is at capacity for all outcomes a corner equilibrium.

After adding a forward market, capacity increases with player $-i$ (the more efficient short-run generator) moving to the corner equilibrium and decreases when player i (the less efficient short-run generator) goes to the corner solution. In both cases profits increase with the addition of a forward market. The explanation for total capacity being higher when player $-i$ is at capacity for all three outcomes is the same as in the example with two demand outcomes.

When player i (the less efficient short-run operator) is at the corner solution, even though the capacity of player i increases, the reaction of player $-i$ is to decrease capacity by a greater amount than i increases capacity. The reason for the cutback is that the value of capacity for player $-i$ takes a discrete drop to 0 in outcome 2 with the addition of forward markets because player $-i$ is no longer at capacity in that outcome. Without forward markets, in outcome 2 player $-i$ sees that an increase in its capacity decreases i's production in the operating equilibrium. However, with forward markets and with i no longer decreasing production in response to an increase by player $-i$, player $-i$ experiences a lower marginal revenue. Because the value of capacity is lower with the change in the marginal revenue calculation and because player i increases its capacity, player $-i$ has a reaction that leads to the total decrease in capacity.

4. Formulae for the Equilibrium Solutions at Each Stage

The equilibrium model with a capacity game and forward and spot markets is a three-stage game and is beyond what is generally handled using mathematical programming, a problem more complex than an EPEC. Our first objective is to negate the claim that the forward market always mitigates market power. A second objective is to investigate the practice of replacing the complex (and currently intractable) three-stage model by the easier (but still complex) two-stage model. These objectives allow us to introduce simplifying assumptions as we need them.

4.1. The Spot Market

An equilibrium of the spot market always exists, and under our assumptions it is also unique. The equilibrium spot is characterized by which constraints are binding. The following cases can occur: Neither player constrained:

$$\text{(i)} \qquad 0 < z_i(\xi) < x_i \quad i = 1, 2. \tag{14.1}$$

One constrained:

$$\text{(ii)} \qquad 0 < z_i(\xi) < x_i \qquad 0 < z_{-i}(\xi) = x_{-i}. \tag{14.2}$$

Both constrained:

$$\text{(iii)} \qquad 0 < z_i(\xi) = x_i \quad i = 1, 2. \tag{14.3}$$

One does not produce anything:

$$\text{(iv)} \qquad 0 = z_i(\xi) \leqslant x_i \qquad 0 < z_{-i}(\xi) = x_{-i}. \tag{14.4}$$

Neither produces anything:

$$\text{(v)} \qquad 0 = z_i(\xi) \leqslant x_i \quad i = 1, 2. \tag{14.5}$$

For our objective it is sufficient to consider only equilibria for which $z_i > 0$, $i = 1, 2$ and limit ourselves to the first three cases, (14.1), (14.2), and (14.3) because these are the cases with a functioning duopoly. This implies that we simplify the complementarity relations (5) to

$$\xi - 2z_i - z_{-i} - \nu_i + y_i + \lambda_i = 0 \quad i = 1, 2, \tag{15.1}$$

$$0 \leqslant x_i - z_i \perp \lambda_i \geqslant 0 \quad i = 1, 2. \tag{15.2}$$

The value of ξ determines which of the inequalities (14.1), (14.2), and (14.3) holds. Define $\alpha_i(x, y)$ and $\alpha_{-i}(x, y)$ to be the smallest values of ξ such that

$$\begin{aligned} z_{-i}(\xi) = x_{-i} \quad &\text{and} \quad z_i(\xi) < x_i \quad \text{for } \xi = \alpha_{-i}(x, y), \\ z_{-i}(\xi) = x_{-i} \quad &\text{and} \quad z_i(\xi) = x_i \quad \text{for } \xi = \alpha_i(x, y). \end{aligned} \tag{16}$$

The definition implies

$$\alpha_{-i}(x, y) < \alpha_i(x, y). \tag{17}$$

The definitions (16) apply in the model without forward markets by setting $y = 0$. Note that one cannot assess exante whether $i = 1$ or 2 in (17) solely from the data.

4.1.1. The Spot Market with Forward Positions. We successively consider the first three cases in relations (14.1), (14.2), and (14.3).

Case 1. When capacity is not binding, we solve for $z_i^*(y)$ in

$$0 = \xi - 2z_i - z_{-i} - \nu_i + y_i \quad i = 1, 2 \tag{18}$$

and find

$$z_i^*(y) = \tfrac{1}{3}\big[\xi - 2(\nu_i - y_i) + (\nu_{-i} - y_{-i})\big]. \tag{19}$$

The profit in the spot market is

$$\begin{aligned} &\tfrac{1}{3}\big(3\xi - \xi + 2\nu_i - 2y_i - \nu_{-i} + y_{-i} - \xi + 2\nu_{-i} \\ &\qquad\qquad - 2y_{-i} - \nu_i + y_i - 3\nu_i\big) \end{aligned} \tag{20}$$

$$\begin{aligned} &\cdot \tfrac{1}{3}[\xi - 2(\nu_i - y_i) + \nu_i - y_{-i}] \\ &= \tfrac{1}{9}(\xi - y_i - y_{-i} - 2\nu_i + \nu_{-i})(\xi - 2\nu_i + 2y_i + \nu_{-i} - y_{-i}) \end{aligned} \tag{21}$$

Table 3. Capacity and costs without and with a forward market.

	No forward	With forward, $-i$ at the corner	With forward, i at the corner
Capacity i	58.69	57.58	64.07
Capacity $-i$	66.11	68.33	55.53
Total capacity	124.81	125.92	119.61
Profits i	2,860.42	2,643.93	3,489.66
Profits $-i$	2,848.64	3,191.74	2,687.12
Total profits	5,709.06	5,835.67	6,176.78

and the market clearing price is

$$p(\xi)=\tfrac{1}{3}\big[\xi+(\nu_i-y_i)+(\nu_{-i}-y_{-i})\big]. \tag{22}$$

The profit of player $-i$ is found by interchanging i and $-i$. This is the case studied by Allaz (1992) and Allaz and Vila (1993). Appendix A.2.1 presents an adapted version of that result. Specifically, we show that the corresponding positions on the forward market in a fully unconstrained case are given by

$$\begin{aligned} y_i&=\tfrac{1}{5}[E(\xi)-3\nu_i+2\nu_{-i}],\\ y_{-i}&=\tfrac{1}{5}[E(\xi)-3\nu_{-i}+2\nu_i], \end{aligned} \tag{23}$$

where

$$E(\xi)=\int_L^U \xi f(\xi)\,d\xi.$$

Case 2. For $z_{-i}=x_{-i}$ and $z_i<x_i$, we find z_i by solving (15.1) for player i

$$\xi-2z_i-x_{-i}-\nu_i+y_i=0$$

or

$$z_i=\frac{\xi-x_{-i}-\nu_i+y_i}{2}. \tag{24}$$

The profit for player i, including the revenue from the forward positions, is:

$$\begin{aligned} &\tfrac{1}{4}(\xi-x_{-i}-\nu_i-y_i)(\xi-x_{-i}-\nu_i+y_i)\\ &\quad=\tfrac{1}{4}[(\xi-x_{-i}-\nu_i)^2-y_i^2]. \end{aligned} \tag{25}$$

The profit for player $-i$ is:

$$\begin{aligned} &(\xi-z_i-x_{-i}-\nu_{-i})x_{-i}\\ &\quad=\left(\xi-\frac{\xi-x_{-i}-\nu_i+y_i}{2}-x_{-i}-\nu_{-i}\right)x_{-i}\\ &\quad=\tfrac{1}{2}(\xi-x_{-i}-2\nu_{-i}+\nu_i-y_i)x_i. \end{aligned} \tag{26}$$

Case 3. For $z_i=x_i$ and $i=1,2$, the profit is

$$(\xi-x_i-x_{-i}-\nu_i)x_i.$$

The three cases apply with deterministic and uncertain demand. With uncertain demand, the values of α come from (16) and (17), where the profit functions switch from Case 1 to Case 2 and from Case 2 to Case 3. Since $\alpha_{-i}(x,y)$ is where the spot-market solution (19) equals capacity for $-i$, we have

$$x_{-i}=\tfrac{1}{3}[\xi-2(\nu_{-i}-y_{-i})+(\nu_i-y_i)]$$

or

$$\alpha_{-i}(x,y)=3x_{-i}+2(\nu_{-i}-y_{-i})-(\nu-y_i). \tag{27}$$

Similarly,

$$\alpha_i(x,y)=2x_i+x_{-i}+\nu_i-y_i. \tag{28}$$

4.2. The Forward Market

When there is a forward market with uncertain demand, using the expressions in § 4.1.1, we define the profit function of both agents i and $-i$ using the relation $\alpha_{-i}(x,y)<\alpha_i(x,y)$ or $\alpha_{-i}(x)<\alpha_i(x)$. Let $p_i(x,y)$ and $p_{-i}(x,y)$ be the profit functions of generators i and $-i$, respectively,

$$\begin{aligned} p_i(x,y)={}&\tfrac{1}{9}\int_L^{\alpha_{-i}(x,y)}(\xi-y_i-y_{-i}-2\nu_i+\nu_{-i})\\ &\qquad(\xi+2y_i-y_{-i}-2\nu_i+\nu_{-i})f(\xi)\,d\xi\\ &+\tfrac{1}{4}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)}\big[(\xi-x_{-i}-\nu_i)^2-y_i^2\big]f(\xi)\,d\xi\\ &+\int_{\alpha_i(x,y)}^{U}(\xi-x_i-x_{-i}-\nu_i)x_if(\xi)\,d\xi-k_ix_i, \end{aligned} \tag{29}$$

$$\begin{aligned} p_{-i}(x,y)={}&\tfrac{1}{9}\int_L^{\alpha_{-i}(x,y)}(\xi-y_i-y_{-i}+\nu_i-2\nu_{-i})\\ &\qquad(\xi-y_i+2y_{-i}+\nu_i-2\nu_{-i})f(\xi)\,d\xi\\ &+\tfrac{1}{2}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)}(\xi-x_{-i}+\nu_i-2\nu_{-i}-y_i)x_{-i}f(\xi)\,d\xi\\ &+\int_{\alpha_i(x,y)}^{U}(\xi-x_i-x_{-i}-\nu_{-i})x_{-i}f(\xi)\,d\xi-k_{-i}x_{-i}. \end{aligned} \tag{30}$$

These functions are differentiable with respect to y_i as long as $\alpha_{-i}>L$ and $\alpha_i<U$. Under these conditions, the equilibrium on the forward market, if it exists, is obtained by solving

$$\frac{\partial p_i(x,y)}{\partial y_i}=\frac{\partial p_{-i}(x,y)}{\partial y_{-i}}=0. \tag{31}$$

Existence and uniqueness of the forward equilibrium also require

$$\frac{\partial^2 p_i(x,y)}{\partial y_i^2} < 0 \quad \text{and} \quad \frac{\partial^2 p_{-i}(x,y)}{\partial y_{-i}^2} < 0.$$

If the equilibrium exists and is unique, these relations define forward positions $y_i(x)$ and $y_{-i}(x)$ for both agents i and $-i$. When $\alpha_{-i} = L$, we have

$$\frac{\partial p_i(x,y)}{\partial y_i} = 0, \qquad \frac{\partial p^-_{-i}(x,y)}{\partial y_{-i}} \geqslant 0. \tag{32}$$

with the second term being the left derivative at the point where the profit function is nondifferentiable.

If there were useful qualitative properties of these models, we would need to show that the equilibrium exists for these properties to be useful. Because the mathematics provides no useful properties from a policy perspective, showing existence has little value. Thus, we do not try to prove existence.

4.3. The Capacity Game Objective Functions

The profit function for the capacity game with a forward market is obtained after replacing the y_i by the equilibrium solution $y(x)$ on the forward market. This can be stated as

$$p_i(x) = p_i[x, y(x)] \quad i = 1, 2. \tag{33}$$

Without a forward market the objective functions in the capacity game are obtained by setting y_i and y_{-i} to zero in (29) and (30). This leads to

$$\begin{aligned} p_i(x,0) = &\int_L^{\alpha_{-i}(x)} \tfrac{1}{9}(\xi - 2\nu_i + \nu_{-i})(\xi - 2\nu_i + \nu_{-i}) f(\xi)\, d\xi \\ &+ \int_{\alpha_{-i}(x)}^{\alpha_i(x)} \tfrac{1}{4}(\xi - x_{-i} - \nu_i)^2 f(\xi)\, d\xi \\ &+ \int_{\alpha_i(x)}^{U} (\xi - x_i - x_{-i} - \nu_i) x_i f(\xi)\, d\xi - k_i x_i, \end{aligned} \tag{34}$$

and

$$\begin{aligned} p_{-i}(x,0) = &\int_L^{\alpha_{-i}(x)} \tfrac{1}{9}(\xi - 2\nu_{-i} + \nu_i)(\xi - 2\nu_{-i} + \nu_i) f(\xi)\, d\xi \\ &+ \int_{\alpha_{-i}(x)}^{\alpha_i(x)} \tfrac{1}{2}(\xi - x_{-i} - 2\nu_{-i} + \nu_i) x_{-i} f(\xi)\, d\xi \\ &+ \int_{\alpha_i(x)}^{U} (\xi - x_i - x_{-i} - \nu_{-i}) x_{-i} f(\xi)\, d\xi - k_{-i} x_{-i}. \end{aligned} \tag{35}$$

5. The Deterministic Case

When demand is deterministic, the Allaz-Vila result no longer applies in a model with capacity decisions. Appendix A.1 has the results in full detail. Here we provide an overview and an explanation of the results. Consider a solution to the capacity game in any of the three games with $z_i < x_i$. Player i can increase its profits by reducing x_i. Thus, $z_i = x_i$ at the equilibrium of all three games. The equilibrium conditions in the open-loop game for $i = 1, 2$ are

$$\xi - 2x_i^* - x_{-i}^* - \nu_i = \lambda_i^* = k_i \tag{36}$$

where λ_i^* is the dual variable of the capacity constraint and x_i^* is the equilibrium quantity. Let superscript c denote the solution to the closed-loop game without a forward market. Because $z_i^c = x_i^c$, for $i = 1, 2$,

$$\xi - 2x_i^c - x_{-i}^c - \nu_i = \lambda_i^c = k_i. \tag{37}$$

Thus, both games have the same solution. They are outcome equivalent with $x_i^c = x_i^*$ and $\lambda_i^c = \lambda_i^*$. This result is consistent with Murphy and Smeers (2005), with the load curve in that paper having one step. Repeating the same arguments for the futures game, if the equilibrium exists, it has the same capacity, production, and prices as the open-loop game. Thus, the capacity stage negates the Allaz-Vila effect.

6. Necessary Equilibrium Conditions with Uncertain Demand

Multistage games do not necessarily have pure strategy equilibria or may have several of them. We first analyze the necessary conditions that equilibria should satisfy and discuss why they do not always lead to a pure strategy equilibrium.

6.1. The Necessary Conditions of the Equilibrium Without a Forward Market

Setting $y_i = y_{-i} = 0$ in relations (27) and (28) we obtain

$$\alpha_{-i}(x) = 3x_{-i} + 2\nu_{-i} - \nu_i \tag{38}$$

$$\alpha_i(x) = 2x_i + x_{-i} + \nu_i. \tag{39}$$

The profit functions (34) and (35) are clearly differentiable when $L < \alpha_{-i}(x)$ (and $\alpha_i(x) < U$, which holds). We first assume that the inequality holds and then check whether it does. We refer to this case as an interior solution. Alternatively, $L = \alpha_{-i}(x)$ defines a corner solution where the profit function is not differentiable. Note that the corner solution has right and left derivatives.

Taking the partial derivatives of (34) and (35) and noting that the terms resulting from the derivatives of the limits of integration cancel, we get

$$\frac{\partial p_i}{\partial x_i} = \int_{\alpha_i}^{U} (\xi - 2x_i - x_{-i} - \nu_i) f(\xi)\, d\xi - k_i = 0 \tag{40}$$

$$\frac{\partial p_{-i}}{\partial x_{-i}} = \frac{1}{2}\int_{\alpha_{-i}}^{\alpha_i} (\xi - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\xi) + \int_{\alpha_i}^{U} (\xi - x_i - 2x_{-i} - \nu_{-i}) f(\xi)\, d\xi - k_{-i} = 0. \quad (41)$$

The integrand in (40) is the classic Cournot formula for demands above α_i. The value of the integrand is the marginal value of capacity at each possible demand and the integral equals the cost of capacity. For the demands in the first integral in (41), $z_i < x_i$, and an increase in x_{-i} leads to a decrease in z_i, leading to an expression different from the Cournot condition.

Relation (40) can be rewritten as

$$\int_{\alpha_i(x)}^{U} (\xi - \alpha_i) f(\xi) = k_i.$$

It is an equation in α_i from which we infer an equivalent relation

$$\alpha_i(x) = 2x_i + x_{-i} + \nu_i = \overline{\alpha}_i.$$

An equilibrium must satisfy this relation with $\overline{\alpha}_i < U$ for capacity to have a positive value. The second-order condition of (40) is

$$\frac{\partial^2 p_i}{\partial x_i^2} = \int_{\alpha_i}^{U} (-2) f(\xi)\, d\xi - (\alpha_i - \alpha_i)\frac{\partial \alpha_i}{\partial x_i} = -2\int_{\alpha_i}^{U} f(\xi) < 0. \quad (42)$$

Consider now the second-order condition $\partial^2 p_{-i}/\partial x_{-i}^2$. We have

$$\begin{aligned}\frac{\partial^2 p_{-i}}{\partial x_{-i}^2} &= \tfrac{1}{2}\int_{\alpha_{-i}}^{\alpha_i} (-2) f(\xi)\, d\xi + \int_{\alpha_i}^{U} (-2) f(\xi)\, d\xi \\ &\quad + \frac{1}{2}(\alpha_i - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\alpha_i)\frac{\partial \alpha_i}{\partial x_{-i}} \\ &\quad - \frac{1}{2}(\alpha_{-i} - 2x_{-i} + \nu_i - 2\nu_{-i}) f(\alpha_{-i})\frac{\partial \alpha_{-i}}{\partial x_{-i}} \\ &\quad - (\alpha_i - x_i - 2x_{-i}\grave{\text{U}} - \nu_{-i}) f(\alpha_i)\frac{\partial \alpha_i}{\partial x_{-i}}.\end{aligned}$$

The last three terms can be written after replacing α_i, α_{-i}, $\partial\alpha_i/\partial x_{-i}$, and $\partial\alpha_{-i}/\partial x_{-i}$ with their values

$$\begin{aligned}& f(\alpha_i)\left(x_i - \frac{x_{-i}}{2} + \nu_i - \nu_{-i}\right) \\ &\quad - \frac{3}{2} f(\alpha_{-i}) x_{-i} - f(\alpha_i)(x_i - x_{-i} + \nu_i - \nu_{-i}) \\ &= \frac{x_{-i}}{2}(f(\alpha_i) - 3f(\alpha_{-i})).\end{aligned}$$

To sum up, we have

$$\frac{\partial^2 p_{-i}}{\partial x_{-i}^2} = -\int_{\alpha_{-i}}^{\alpha_i} f(\xi)\, d\xi - 2\int_{\alpha_i}^{U} f(\xi)\, d\xi - \frac{x_{-i}}{2}(3f(\alpha_{-i}) - f(\alpha_i)). \quad (43)$$

The sign of this expression is generally indeterminate, making it impossible to guarantee the existence and uniqueness of an equilibrium. This result is in line with Murphy and Smeers (2005). In contrast, the expression is always negative in the special case of a uniform or exponential distribution of ξ, and the capacity game has an equilibrium. The examples in online Appendix A4 assume a uniform distribution.

6.2. Necessary Equilibrium Conditions with a Forward Market

We first consider the equilibrium conditions of the forward game and then turn to the capacity game.

6.2.1. First Order Conditions of the Forward Game with an Interior Solution. Let x be given. Assuming again an interior solution ($L < \alpha_{-i}(x)$) to guarantee the differentiability of the profit functions, the necessary conditions of the futures game are

$$\frac{\partial p_i}{\partial y_i} = \frac{\partial p_{-i}}{\partial y_{-i}} = 0 \quad (44)$$

where from (29) and (30)

$$\frac{\partial p_i}{\partial y_i} = \frac{1}{9}\int_{L}^{\alpha_{-i}(x,y)} (\xi - 4y_i - y_{-i} - 2\nu_i + \nu_{-i}) f(\xi)\, d\xi - \frac{y_i}{2}\int_{\alpha_{-i}(x,y)}^{\alpha_i(x,y)} f(\xi)\, d\xi \quad (45)$$

and

$$\frac{\partial p_{-i}}{\partial y_{-i}} = \frac{1}{9}\int_{L}^{\alpha_{-i}(x,y)} (\xi - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) f(\xi)\, d\xi. \quad (46)$$

The right-hand side of (46) represents the marginal profit of player $-i$. The integral includes all ξ with neither player at capacity. Since player $-i$ reaches capacity first, Equation (45) has two terms on the right. The first term is the equivalent of the integral of player $-i$ and the second includes the demand levels where player $-i$ is at capacity and its production does not respond to increased production by i. Since y_i is set before ξ is known and always leads to an increase in production by i, this term represents the expected marginal losses for player i from overproduction induced by its forward position. Let

$$\psi_{-i}(\xi, x, y) = \tfrac{1}{9}(\xi - y_i - 4y_{-i} + \nu_i - 2\nu_{-i}) \quad \text{for } \xi \in [L, \alpha_{-i}(x,y)] \quad (47)$$

and

$$\psi_i(\xi, x, y) = \begin{cases} \dfrac{1}{9}(\xi - 4y_i - y_{-i} - 2\nu_i + \nu_{-i}) & \text{for } \xi \in [L, \alpha_{-i}(x,y)] \\ -\dfrac{y_i}{2} & \text{for } \xi \in [\alpha_{-i}(x,y), \alpha_i(x,y)]. \end{cases} \quad (48)$$

Relation (44) can be restated as

$$\Psi_i(x,y)=\int_L^{\alpha_i(x,y)}\psi_i(\xi,x,y)f(\xi)\,d\xi=0, \tag{49}$$

$$\Psi_{-i}(x,y)=\int_L^{\alpha_{-i}(x,y)}\psi_{-i}(\xi,x,y)f(\xi)\,d\xi=0. \tag{50}$$

Solving these relations together with

$$\alpha_{-i}(x,y)=3x_{-i}+2(\nu_{-i}-y_{-i})-(\nu_i-y_i) \tag{51}$$

$$\alpha_i(x,y)=2x_i+x_{-i}+\nu_i-y_i \tag{52}$$

gives a candidate equilibrium on the forward market, provided that (51) and (52) satisfy $L<\alpha_{-i}(x,y)$ and $\alpha_i(x,y)<U$, extending the notion of an interior solution to the case of a forward market.

Note that for $y_i=0$, $\alpha_{-i}(x,y)=L$ always satisfies relations (49)–(52). This is the corner solution with a forward market. Nondifferentiability of the profit function occurs only at a corner solution.

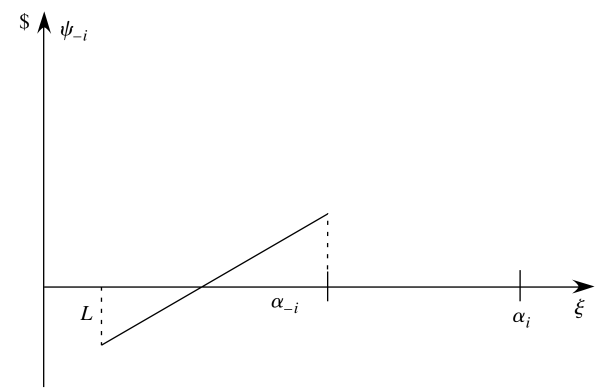
In Appendix A.2.2. we present the second-order conditions for both corner and interior equilibria and give analytical expressions for the reaction curves of the players (Appendix A.2.3). For the second-order conditions we find that the sign of $\partial^2 p_i/\partial y_i^2$ is indeterminate with an interior solution as well as with a corner solution. Thus, in contrast to the infinite capacity model of Allaz-Vila, as restated in Appendix 2.1, the equilibrium does not necessarily exist in the forward market game.

6.2.2. A Reaction Curve Analysis. We complete the forward-market analysis by exploring the structure of the reaction curves of the two agents in the forward market. This analysis assumes that the first-order conditions suffice to determine the optimal behavior of an agent given the action of the other. We illustrate why the results are indeterminate with graphs. In the Appendix A.2.3. we present the mathematical analysis, and show that the existence of the equilibrium is not guaranteed because the slopes of the reaction functions do not necessarily fall in the range of $(-1,0)$. The results in the appendix illustrate how the properties for an equilibrium hold in the standard Allaz-Vila case, where capacities are infinite, and fail when they are finite. In these equations, if we let $\alpha=\infty$, and set to zero all terms except the integrals from L to ∞, we have the reaction functions with infinite capacity, that is reaction functions with the no-capacity game. In this case the slopes then fall in the range of $(-1,0)$ and the game of the forward market is well behaved.

Plotting ψ_i and ψ_{-i} in (47) and (48), we see the marginal contribution to profit at each ξ. We perturb the variables to see how the profit in the forward game changes, beginning with ψ_{-i}.

In Figure 1 as ξ increases, the contribution to profit increases linearly and then stops once capacity is reached, when z_{-i} is equal to x_{-i}. Without a capacity constraint the line would continue indefinitely.

Figure 1. Marginal contribution in the spot and forward markets of y_{-i} as a function of ξ at the equilibrium solution as seen in the forward market game.



We now look at the effect of increasing y_i on ψ_{-i} in Figure 2. Increasing y_i for $\xi<\alpha_{-i}$ decreases ψ_{-i}. Since α_{-i} is increasing, the direction in the change in profit is dependent on which area is larger, the decreasing area ranging over the ξ or the increasing area associated with the change in α_{-i}. Hence, the outcome is ambiguous. Plotting ψ_i we get Figure 3.

Note that between the αs the contribution is negative because of the second integral in (45), unlike Figure 1. In this range increasing y_i leads to increased production by player i even though player $-i$ does not decrease production in response to this increase. Increasing x_i increases α_i and hence adds to the negative area. The impact of an increase in y_{-i} can be seen in Figure 4.

Increasing y_{-i} decreases ψ_i in $[L,\alpha_{-i}(x,y)]$ and decreases $\alpha_{-i}(x,y)$. It does not modify $\alpha_i(x,y)$. The effect is unambiguous in that the marginal contribution to capital costs decreases. This implies that player i sees its marginal profit becoming negative as a result of an increase of y_{-i}. It reacts by decreasing y_i. We are, however, unable to determine by how much. Again, note that in the forward market game without capacity limits, the negative ψs between the αs do not exist. These graphs show that the boundaries of

Figure 2. The effect of increasing y_i on ψ_{-i}.

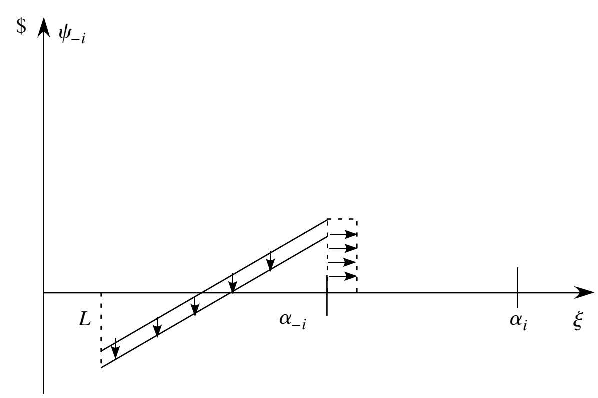
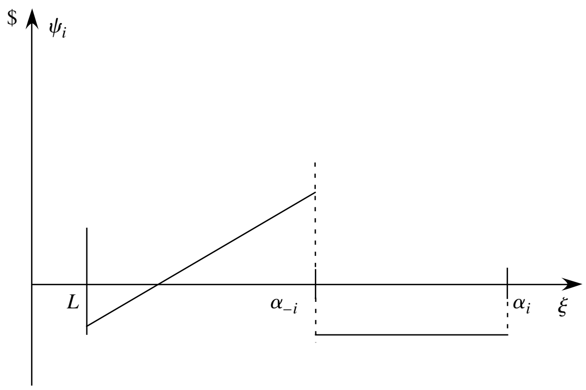


Figure 3. Marginal contribution in the spot and forward markets of y_i on ψ_i, as a function of ξ at the equilibrium solution as seen in the forward market game.



the integrals, the αs, change the character of the results of the forward market and create the possibility for capacity to increase or decrease through the addition of a futures market.

6.3. Necessary Equilibrium Conditions on the Capacity Game

Assume that the forward game has a unique equilibrium and let $y(x)$ be the corresponding futures positions of the players. We want to explore whether there is an equilibrium in the capacity game.

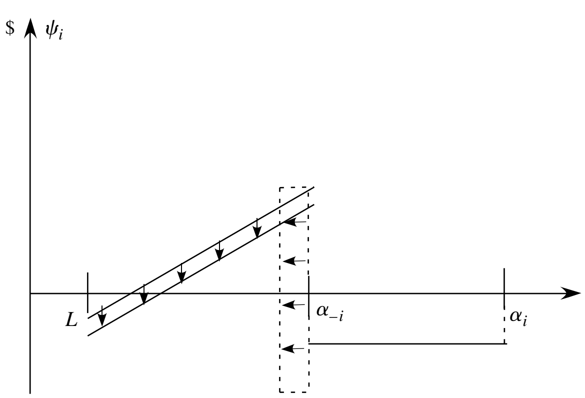
Let $p_i[x, y(x)]$ be the profit accruing to generator i on the capacity game after taking the optimal forward position $y(x)$. The equilibrium on the capacity market must satisfy

$$\frac{dp_i}{dx_i}=0 \tag{53}$$

or

$$\frac{\partial p_i}{\partial x_i}+\frac{\partial p_i}{\partial y_i}\frac{\partial y_i}{\partial x_i}+\frac{\partial p_i}{\partial y_{-i}}\frac{\partial y_{-i}}{\partial x_i}=0. \tag{54}$$

Figure 4. Effect of an increase in y_{-i} on the marginal contribution of y_i.



Taking into account that $\partial p_i/\partial y_i = 0$ at the equilibrium on the forward market, we obtain

$$\frac{\partial p_i}{\partial x_i}+\frac{\partial p_i}{\partial y_{-i}}\frac{\partial y_i}{\partial x_i}=0. \tag{55}$$

Similarly $dp_{-i}/dx_{-i}=0$ implies

$$\frac{\partial p_{-i}}{\partial x_{-i}}+\frac{\partial p_{-i}}{\partial y_i}\frac{\partial y_i}{\partial x_{-i}}=0. \tag{56}$$

To meet the necessary equilibrium conditions on the capacity market, we compute

(i) $\partial p_i/\partial x_i$ and $\partial p_{-i}/\partial x_{-i}$;
(ii) $\partial p_i/\partial y_{-i}$ and $\partial p_{-i}/\partial y_i$; and
(iii) $\partial y_i/\partial x_{-i}$ and $\partial y_{-i}/\partial x_i$.

The formulae for the interior solution are given in Appendix A.2.4. Because of nondifferentiability, special attention must be given to the case where $L=\alpha_{-i}(x,y)$ (the corner solution). We discuss this case next.

6.3.1. The Forward Market at a Corner Equilibrium. As can be seen from (25), this equilibrium is characterized by

$$y_i=0; \qquad y_{-i}\geqslant \tfrac{1}{2}[3x_{-i}-\nu_i+2\nu_{-i}-L].$$

We immediately obtain

$$\frac{\partial y_i}{\partial x_{-i}}=0; \qquad \frac{\partial y_{-i}}{\partial x_i}=0.$$

The necessary equilibrium conditions on the capacity market reduce to

$$\frac{\partial p_i}{\partial x_i}=\frac{\partial p_{-i}}{\partial x_{-i}}=0 \tag{57}$$

which looks similar to the equilibrium solutions obtained when there is no forward market. The equilibrium is not the same though, because player $-i$ has a nonzero position on the forward market. The equilibrium condition for player i can be stated as

$$\int_{\alpha_i}^{U}(\xi-2x_i-x_{-i}-\nu_i)f(\xi)\,d\xi-k_i=0 \tag{58}$$

where

$$\alpha_i=2x_i+x_{-i}+\nu_i, \tag{59}$$

since $y_i=0$. These conditions are again equivalent to

$$\alpha_i(x)=2x_i+x_{-i}+\nu_i=\bar{\alpha}_i \tag{60}$$

where $\bar{\alpha}_i$ is a solution of

$$\int_{\alpha}^{U}(\xi-\alpha)f(\xi)=k_i$$

that must satisfy $\bar{\alpha}_i\leqslant U$. This equilibrium condition is identical to the one with no forward market.

The equilibrium conditions of x_{-i} are different and are

$$\tfrac{1}{2}\int_L^{\alpha_i}(\xi-2x_{-i}+\nu_i-2\nu_{-i})f(\xi)\,d\xi,$$
$$+\int_{\alpha_i}^{U}(\xi-x_i-2x_{-i}-\nu_{-i})f(\xi)\,d\xi-k_{-i}=0 \tag{61}$$

or

$$\tfrac{1}{2}\int_L^{\bar{\alpha}_i}\xi f(\xi)\,d\xi+\int_{\bar{\alpha}_i}^{U}\xi f(\xi)\,d\xi,$$
$$=k_{-i}+\tfrac{1}{2}(-2x_{-i}+\nu_i-2\nu_{-i})(\bar{\alpha}_i-L),$$
$$-(x_i+2x_{-i}-\nu_{-i})(U-\bar{\alpha}_{-i}) \tag{62}$$

which, because $\bar{\alpha}_i$ is known, is a linear expression in x_i and x_{-i}. The capacity equilibrium for the forward-market corner equilibrium is found by solving these equations. The second order conditions are

$$\frac{\partial^2 p_i}{\partial x_i^2}=-2\int_{\alpha_i}^{U}f(\xi)\,d\xi<0$$
$$\begin{aligned}\frac{\partial^2 p_{-i}}{\partial x_{-i}^2}&=-\frac{1}{2}2\int_L^{\alpha_i}f(\xi)+\frac{1}{2}(\alpha_i-2x_{-i}+\nu_i-2\nu_{-i})f(\alpha)\\&\quad-2\int_{\alpha_i}^{U}f(\xi)\,d\xi-(\alpha_i-x_i-2x_{-i}-\nu_{-i})f(\alpha)\\&=-\int_L^{\alpha_i}f(\xi)\,d\xi-2\int_{\alpha}^{U}f(\xi)\\&\quad+\left(-\frac{\alpha_i}{2}+x_i+x_{-i}+\frac{\nu_i}{2}\right)f(\alpha_i)\\&=-\int_L^{\alpha_i}f(\xi)\,d\xi-2\int_{\alpha_i}^{U}f(\xi)\,d\xi+\frac{x_i}{2}f(\alpha_i),\end{aligned} \tag{63}$$

which are indeterminate in sign. Again, we cannot show exante the existence of an equilibrium solution.

6.3.2. Assessing the Effect of the Addition of a Forward Market at the Corner Solution. Say (x_i,x_{-i}) are the capacities from the game without a forward market. From (58), given x_{-i}, x_i is also the optimal capacity in the model with a forward market because $y_i=0$, and the solution without a futures market is a point on player i's reaction curve. We can use this point to see how x_i responds as player x_{-i} changes its capacity from the no-futures equilibrium with the addition of a futures market. We show that the reaction function of player i is well behaved and has a slope between 0 and -1. However, the direction of change in player $-i$'s capacity is indeterminate with uncertain demand, and no qualitative conclusion can be made about total capacity changes resulting from adding forward markets.

We first analyze the sign of the derivative of $-i$'s profit function at the corner solution. The left side of (61) evaluated with the capacities from the equilibrium without a forward market is the derivative of the profit function of player $-i$ and gives the direction of change in capacity. The equilibrium condition for x_{-i} in the capacity market with no forward market, (41), differs from (62) only in the lower limit of the first integral, which changes from α_{-i} to L. Subtracting (41) from the left side of (62), leads to the following expression for the rate of change in player $-i$'s profit, starting at the no-futures equilibrium,

$$\frac{\partial p_{-i}}{\partial x_{-i}}=\frac{1}{2}\int_L^{\alpha_{-i}}(\xi-2x_{-i}+\nu_i-2\nu_{-i})f(\xi)\,d\xi. \tag{64}$$

The sign of this integral determines the direction of change in x_{-i} at the capacities from the no futures equilibrium. At $\xi=\alpha_{-i}$, we have

$$\xi=\alpha_{-i}=3x_{-i}+2\nu_{-i}-\nu_i. \tag{65}$$

Rearranging terms, we have

$$\xi-2x_{-i}-2\nu_{-i}+\nu_i=x_{-i}>0. \tag{66}$$

For ξ close to α_{-i} the term in the integral is positive and can be negative for small ξ. Thus, we cannot determine the sign of $\partial p_{-i}/\partial x_{-i}$ in general. Yet, we can determine the response of x_i to a change in x_{-i}.

Taking the derivative of (58) with respect to x_i, we get

$$\int_{\alpha_i}^{U}\left(-2-\frac{dx_{-i}}{dx_i}\right)f(\xi)\,d\xi$$
$$-(\alpha_i-2x_i-x_{-i}-\nu_i)\frac{\partial\alpha_i}{\partial x_{-i}}=0 \tag{67}$$

or because of (59)

$$\left(-2-\frac{dx_{-i}}{dx_i}\right)\int_{\alpha_i}^{U}f(\xi)\,d\xi=0 \tag{68}$$

and

$$\frac{dx_i}{dx_{-i}}=-\frac{1}{2}. \tag{69}$$

Thus, with the inclusion of forward markets, if (64) is positive (negative), total capacity increases (decreases). In the numerical experiments in Appendix 4, we generate cases with (64) having either sign.

The reaction-curve analysis shows the indeterminacy of the capacitated futures game. The necessary equilibrium conditions for the capacity game with an interior solution on the forward market are in the appendix. They do not lead to any general property as with the corner solution.

7. Explanation

With discrete distributions we have provided numerical examples with no change, an increase, and a decrease in capacity. When the distribution of the intercept is continuous, the most realistic case, there are no meaningful qualitative results on the direction of change in capacity and numerical examples in Appendix 4 show that capacity can increase or decrease.

7.1. Anticipating the Response of the Other Player and the Allaz-Vila Result

These results are different from the Allaz-Vila results because of the underlying assumptions and knowledge the players have about the actions of the other players. In multistage Cournot games each player presumes the other player does not react to its actions in the current game, but the player sees the equilibrium shift in the subsequent game. In the capacity game player i sees how the futures and operating decisions of player $-i$ change in response to its capacity decisions. Yet, it presumes its capacity decisions do not affect player $-i$'s capacity decisions. When in the spot game, each player assumes that the other player does not respond to changes in its production.

What player i sees as the response of the other player determines the optimality conditions in each game. We include in the equilibrium conditions for the spot market the response of $-i$ to i's actions, the rate of change in z_{-i} as a function of a change in z_i. The player i optimization condition in the spot game is as follows:

$$\xi - 2z_i - z_{-i} - z_i \frac{\partial z_{-i}}{\partial z_i} - \nu_i = 0. \tag{70}$$

This is the Cournot equilibrium condition with an extra term that captures the changes by player $-i$ in response to a change by player i. Setting the partial derivative to 0, we have the Cournot equilibrium condition,

$$\xi - 2z_i - z_{-i} - \nu_i = 0, \tag{71}$$

for each player in the spot market. The coefficient 2 expresses the extent to which the player exercises market power. As the 2 decreases to 1, production increases and excess profits decrease and we reach the competitive equilibrium. In the forward game, player i has information on how $-i$ responds in the spot game from an increase in y_i that induces an increase in z_i. When player i increases z_i by ϵ through increasing y_i, it sees the spot equilibrium shift and player $-i$ decrease z_{-i} by $-1/2\epsilon$. The partial derivative in (70) is $-1/2$. We get the following equilibrium condition from moves in the forward market.

$$\xi - \tfrac{3}{2} z_i - z_{-i} - \nu_i = 0. \tag{72}$$

The 2 in (71) then becomes $\frac{3}{2}$ in (72), and i increases production. Since both players see the same behavior on the part of the other in the spot game, the equilibrium in the forward market in the Allaz-Vila game is the solution of the pair of Equations (72) for both players. Their analysis is that the ability to see the effect on the spot equilibrium of futures decisions leads them to anticipate a variation of $-\frac{1}{2}$.

As both players presume no response by the other player in the futures game, they reduce their total profits by effectively playing a prisoner's dilemma game. A capacity game allows the players to see the others' responses in the futures game and the effect on profits. This is why our results differ from theirs.

7.2. The Deterministic Case

The deterministic game highlights the role of anticipation of other players' actions. The optimality condition in the capacity game requires that the marginal value of capacity equal the cost of capacity for both players. Thus, the spot equilibrium condition for both players is

$$\xi - 2z_i - z_{-i} - z_i \frac{\partial z_{-i}}{\partial z_i} - \nu_i - \lambda_i = 0 \tag{73}$$

with $\lambda_i = k_i$. By complementary slackness, if $\lambda_i > 0$, we must have $z_i = x_i$. Thus, when z_{-i} increases in (73), λ_i must decrease to 0 before $z_i < x_i$. Thus, for a small change in z_{-i}, z_i remains unchanged for $i = 1, 2$. Thus, the partial derivative term in (73) is 0 and the equilibrium condition becomes

$$\xi - 2z_i - z_{-i} - \nu_i - \lambda_i = 0 \tag{74}$$

for both players. That is, the 2 remains the 2 in the Cournot condition and we have the original Cournot solution.

7.3. The Case with Discrete Probabilities

Assume two possible two load outcomes. Let both players be at capacity with the first demand outcome and below capacity with the second. Assume that the Allaz-Vila solution has both players operating below their capacities with the second demand outcome. Then we can extend (74) to outcome 1, high demand. With p the probability of outcome 1, the capacity equilibrium condition becomes

$$p(\xi^1 - 2x_i - x_{-i} - \nu_i) - \lambda_i = 0 \tag{75}$$

with $\lambda_i = k_i$ for both players. That is, a futures market has no impact on capacities.

Now assume that adding a futures market leads to a corner solution for player $-i$. Letting $z_i^2 < x_i$ be the operating level at the second outcome with $z_{-i}^2 = x_{-i}$, in the forward game, we need

$$\xi^2 - \frac{3}{2} x_{-i} - z_i^2 - \nu_{-i} \geqslant 0. \tag{76}$$

When this is true, the marginal value of adding capacity for player $-i$ at the second demand outcome is positive. The equilibrium condition for player i remains (75). For player $-i$ it becomes

$$p(\xi^1 - 2x_{-i} - x_i - \nu_{-i}) + (1-p)\left(\xi^2 - \frac{3}{2} x_{-i} - z_i^2 - \nu_{-i}\right) - k_{-i} = 0. \tag{77}$$

From the optimality condition for the capacity game (75), the reaction of x_i to a change in x_{-i} is $-1/2$. The optimality condition of i in the spot and futures games with the second outcome is

$$(1-p)(\xi^2 - 2z_i^2 - x_{-i} - \nu_i) = 0 \tag{78}$$

and the change in z_i to a change in x_{-i} is also $-1/2$. Thus, the slopes of the reaction curves for i are both $-1/2$ and between -1 and 0, and an increase in capacity by player $-i$ leads to higher total capacity and increased production. Here a player adds an outcome where it produces at capacity, and the marginal value of capacity increases, increasing total capacity. Capacity never decreases with two outcomes.

7.4. Equilibrium Conditions with Three Possible Demand Outcomes

Without a futures market, let player i be at capacity for outcome 1 and player $-i$ be at capacity for outcomes 1 and 2. In outcome 3 without a forward market in the spot market we have the equilibrium conditions

$$\xi^3 - 2z_i^3 - z_{-i}^3 - \nu_i = 0. \tag{79}$$

With a forward market and both players below capacity, the equilibrium condition is

$$\xi^3 - \tfrac{3}{2}z_i^3 - z_{-i}^3 - \nu_i = 0, \tag{80}$$

the Allaz-Vila equilibrium condition. With $-i$ at capacity and i below capacity in outcome 3

$$\xi^3 - \tfrac{3}{2}x_{-i} - z_i^3 - \nu_{-i} \geqslant 0 \tag{81}$$

must hold for the solution to be an equilibrium. As in the two-outcome case, capacity increases as long as the futures position of player i does not lead this player to reach capacity in outcome 2.

Now let us reverse the situations of i and $-i$ and examine the corner solution with player i at capacity for all outcomes and player $-i$ at capacity only in outcome 1. Before adding a forward market, the equilibrium condition in the capacity game for player $-i$ is

$$p^1(\xi^1 - 2x_{-i} - z_i^1 - \nu_{-i}) + p^2\big(\xi^2 - \tfrac{3}{2}x_{-i} - z_i^2 - \nu_{-i}\big) - k_{-i} = 0. \tag{82}$$

With the forward market added and player i at the corner, the equilibrium condition for player $-i$ is

$$p^1(\xi^1 - 2x_{-i} - z_i^1 - \nu_{-i}) - k_{-i} = 0. \tag{83}$$

We no longer have the contribution to the cost of capacity from the second outcome, which leads to a discrete change in the reaction curve. The numerical results are that player $-i$ has a drop in capacity greater than the increase in i's capacity. That leads to the overall decline in capacity.

The distinction between interior and corner solutions is not significant in the discrete-distribution case, and our results are broadly applicable. For example, in our numerical example we could add a fourth outcome where the intercept is 50. Neither player would operate at capacity for this outcome with or without forward markets, and the results in the capacity game would remain the same.

7.5. The Continuous Case

The difference between the discrete case and the continuous case is that in the continuous case, the αs shift continuously. In the discrete case, the outcomes where the players operate at capacity are fixed until a discrete change occurs. The changes determine whether total capacity increases or decreases. With corner solutions in the continuous case we can analyze patterns and do not address marginal shifts in the αs. If $-i$ reaches the corner, total capacity increases and if i reaches the corner, capacity decreases. In other words the corner case with a continuous probability distribution behaves like the case with a discrete distribution. (See the Appendix 4 for numerical examples.)

With an interior solution, how the αs shift and the probability weights on those outcomes creates an extra layer of ambiguity. In the graphs of the reaction functions, a move in capacity increases profits for ξ where the firm operates at capacity but the set of outcomes at capacity decreases. If we could freeze the αs in those graphs, the net increase or decrease in areas would be unambiguous. What happens depends on where the probability mass in the demand distribution is relative to the αs. Also, the values of the capacity and operating costs affect which player reaches capacity first and the operational response to a futures market. Thus, we cannot predict the effect of a futures market.

What makes the continuous case different from having a discrete distribution of outcomes is that when taking the total derivatives as we have done, the equations have added terms for what happens on the boundary as the αs change, and we know even less as to how the results behave.

8. Conclusions

The common wisdom is that incumbent generation companies have market power and will eventually exercise it. Besides offering hedging possibilities, forward contracts are almost universally seen as good instruments to mitigate market power. Following the seminal contribution of Allaz (1992) and Allaz and Vila (1993), a whole stream of literature argues that position. The beneficial properties of long-term contracts have been established under ideal situations, either exogenously given as in the early electricity literature, or endogenously determined in a market with infinite capacities. We show that endogenously limiting capacities can destroy the ability of forward contracts to mitigate market power. We then prove that they have an undetermined effect when demand is unknown at the time the investment and forward positions are taken and they have no effect with deterministic demand.

Furthermore, we give examples with multiple equilibria. Having multiple equilibria is destabilizing because each player can try to reach the equilibrium that is more advantageous to it, leading to outcomes that are quite different from the equilibria when both players act to achieve their desired market positions. These equilibria can involve corner solutions where one player operates at capacity for all possible demand outcomes and the other is driven out of the futures market. Here the forward market does not lead to a prisoner's dilemma game. Instead, it rewards aggressive behavior, unlike an interior solution where each player maximizes its profits and lets the other operate profitably in the forward market.

Although there is no load curve in our model, the results are broadly applicable to pricing at the peak, when markets are most susceptible to market power. Given the high levels of demand at or near the peak, corner solutions can create opportunities for a player to keep other players out of the forward market and potentially limit capacity to levels below what would be case without a forward market.

Our results also show the conceptual difficulties of making broad conclusions about complicated markets using simple models. This serves as a caution when generalizing theoretical results in modeling abstractions as the basis for forming government policy.

9. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

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