Introduction to Bilevel Models and Energy Systems

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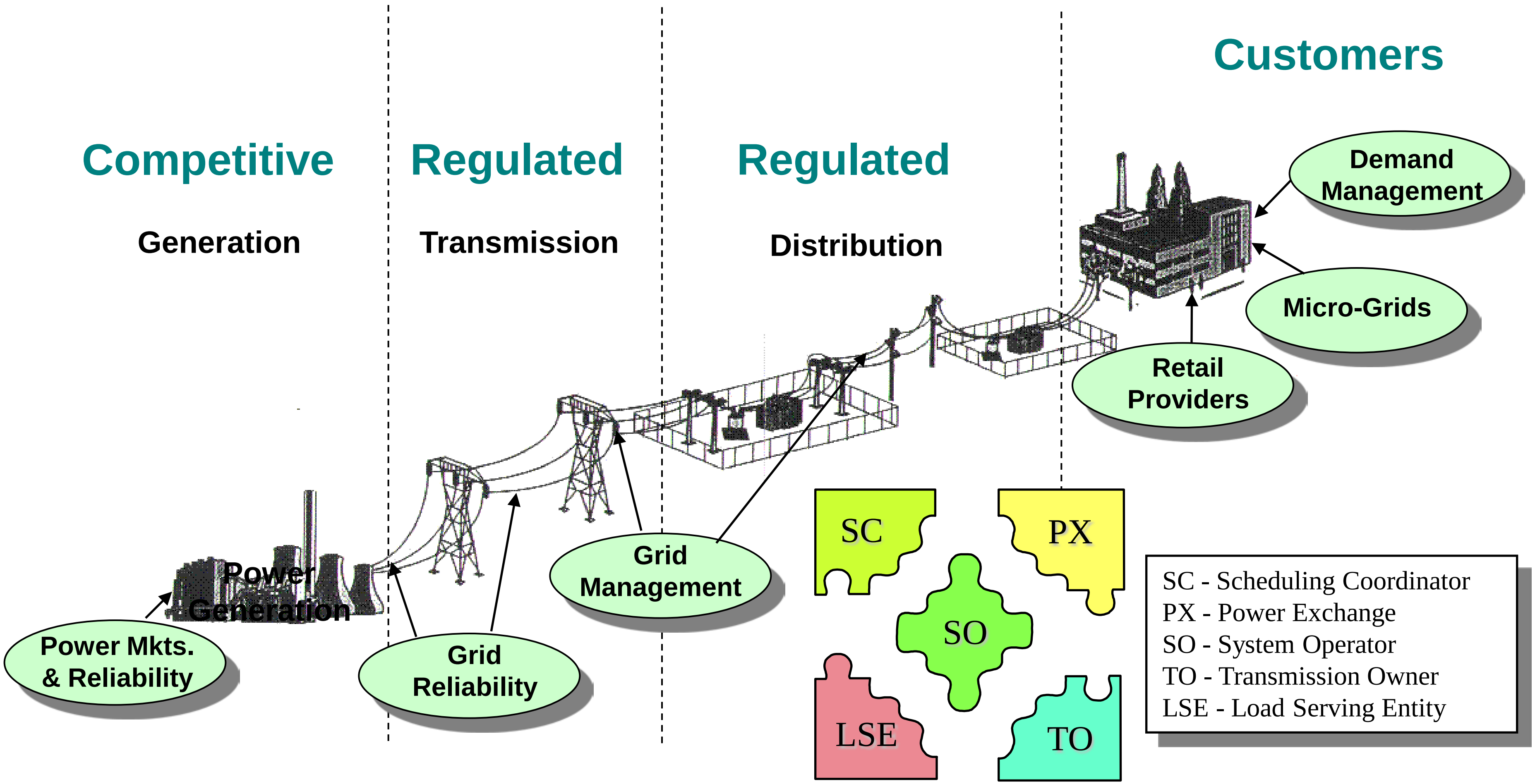
Objectives for the deregulated power market

- Overall short run and long run efficiency through
  - Competition on the supply and demand side
  - Efficient pricing of transmission
- Short run:
  - Demand functions are given
  - Optimize the use of existing facilities in generation and transmission/distribution
- Long run:
  - Incentives for location of production and consumption
  - Optimal expansion of grid

Power market players



Source: S. Oren

Outline

- Part I
  - Characteristics of transmission pricing
- Part II
  - Optimal power flow and physical equilibrium
- Part III
  - Bilevel structures in market power modeling

Part I

Characteristics of transmission pricing

Why is the grid so important?

- The grid integrates geographically dispersed markets
- The grid affects the formation of prices
  - The technology for transmitting electricity presents some special challenges to the competitive markets model
    - Electricity is very costly to store
    - Supply must equal demand at every instant in time
    - Severe capacity constraints
- Short run relevant costs for transmission
  - Losses
  - Ancillary services, reactive power
  - Congestion cost
    - The opportunity cost that results from out-of-merit order dispatch, i.e. the cost of not being able to dispatch the cheapest generators first

A market for transmission of power?

- The conditions for a well-functioning market are not fulfilled
  - Decreasing cost per unit (natural monopoly)
  - Externalities
    - ”Loop flow” (parallel flow, Kirchhoff’s laws)
    - Ancillary services
- Congestion requires coordination
  - Why not accomplish this through the market?
  - A central unit must at least coordinate information
    - The opportunity cost of out-of-merit order dispatch is determined when the market is cleared
    - Congestion cost influences market clearing
  - Market power and strategic bids

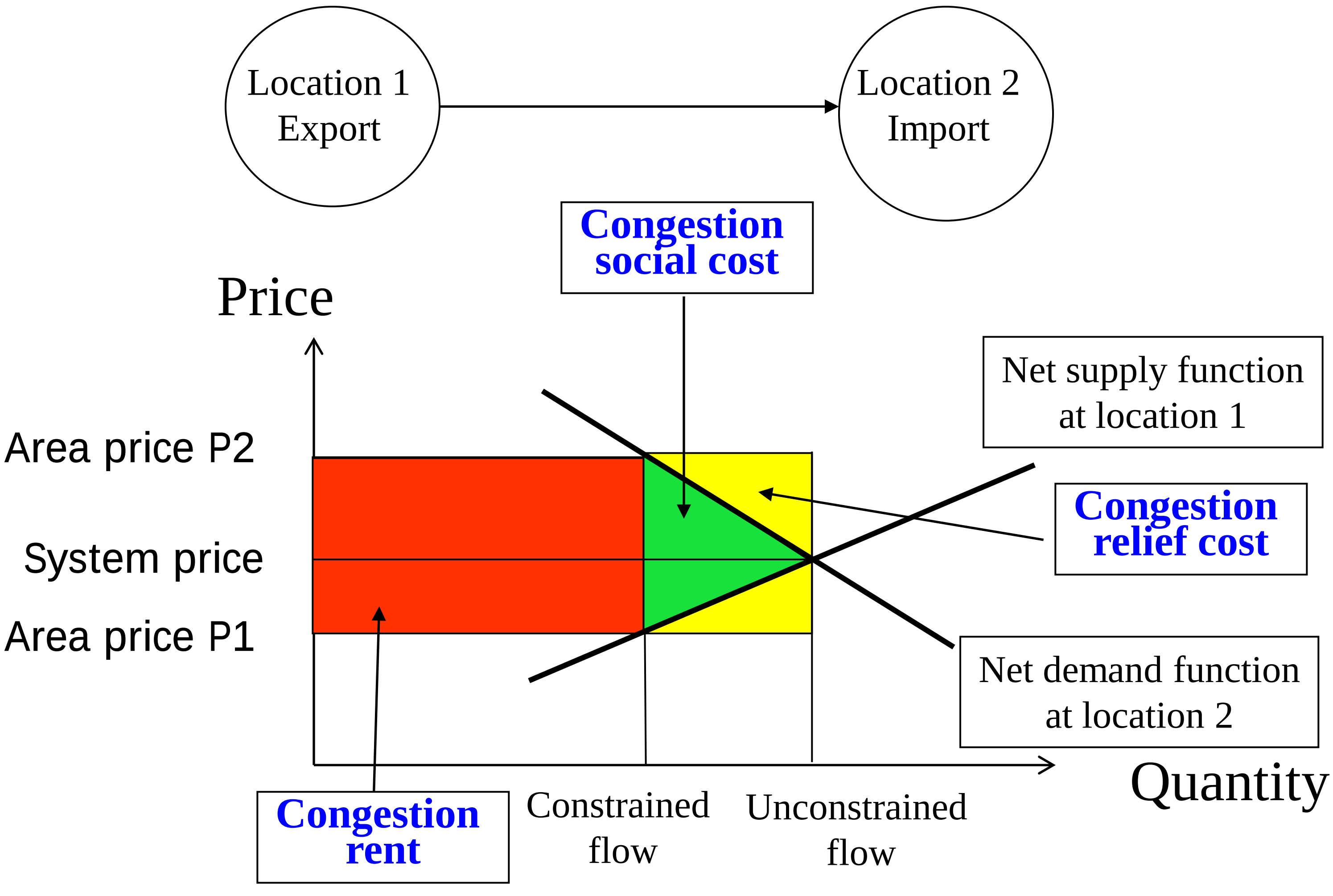
Theoretical benchmark – optimal power flow

- Computing optimal (economic) power flow:
- In general for alternating current systems:
  - Power flow is computed by a non-linear equation system
  - The optimization problem is non-convex
- However, in normal operation
    - Constant voltage levels, small angle differences, real power, no losses
  - Power flow equations are approximated by linear function
  - With reasonable objective functions, the optimal power flow problem becomes convex
  - Necessary assumptions for a market mechanism to replicate the solution that maximizes social surplus

Congestion management

- Objective
  - Optimal economic dispatch
    - Max social welfare (consumer benefit – production cost)
    - S.t. thermal and security constraints
  - Gives the value of power in every node
    - Benchmark
- Alternative methods to realize optimal dispatch
  - Nodal prices, Flowgate prices, Optimal redispatch…
- Provide price signals
  - For efficient use of the transmission system
  - For transmission, generation and load upgrades

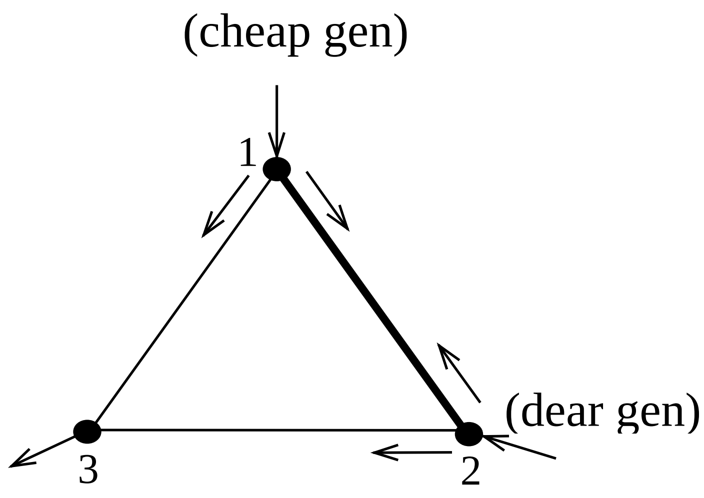
Measures of congestion cost



"DC" approximation

Given a base load:

⇒ Routing of an incremental injection is fixed



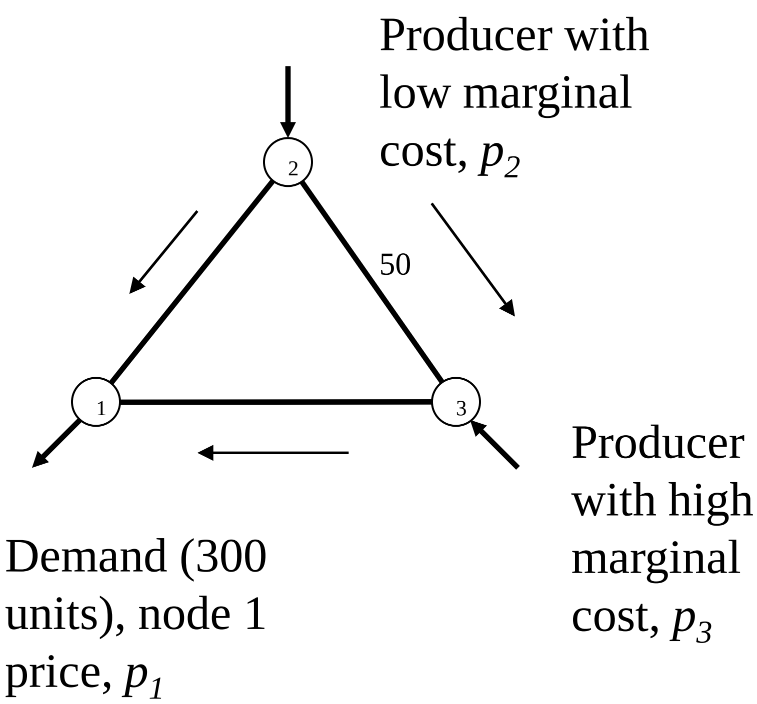
If injection into node 1 increases:

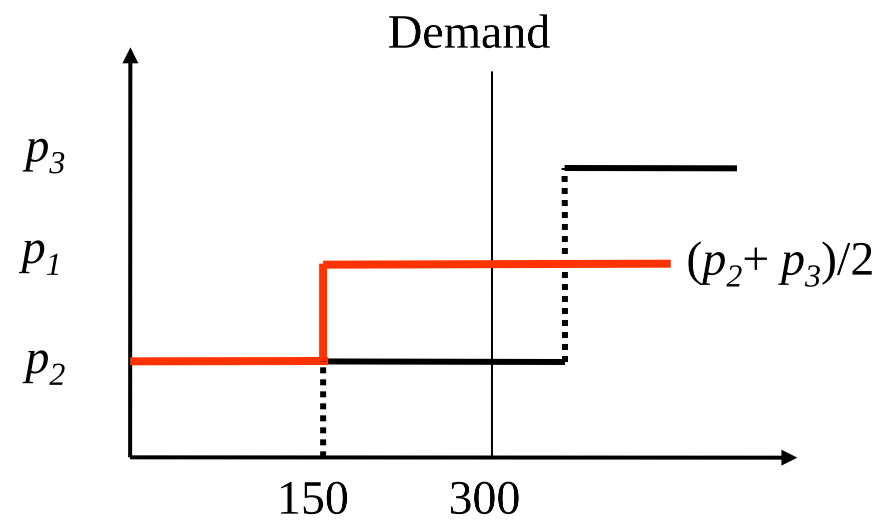
⇒ the power flows over paths 1-3 and 1-2-3 increase in given proportions (given by the "load factors")

⇒ if line 1-2 is congested in base load, injection into node 1 cannot increase without increasing injection into node 2 as well (thereby generating counter flows to relieve the constraint)

⇒ since an increase in injections at any node of the grid affects all transmission lines, a transmission constraint is a network problem rather than a link problem

Optimal nodal prices and parallel flows





Optimal power flow and optimal nodal prices

- A single limitation can induce price differences throughout the network
- There may be flow from high price to low price
  - Will result in negative grid revenue for the line in question
- A line that is not congested may generate a positive grid revenue
- A new line may result in lower social surplus
  - Even if excluding investment cost
- There are infinitely many market equilibria
  - Defined as a set of prices and quantities such that capacity restrictions etc. are fulfilled

Optimal nodal prices

- Optimal power flow

Maximize welfare

Consumers’ willingness to pay – production costs

s.t.

power flow equations (p)

thermal capacity constraints (μ)

Shadow prices

- Both p and μ may be used as references for price mechanisms

Alternative methods

Nodal Pricing and Approximations

1) Schweppe, F. C., M. C. Caramanis, R. D. Tabors, & R. E. Bohn (1988), *Spot Pricing of Electricity*, Kluwer Academic Publishers.

2) Hogan, W. W. (1992), “Contract Networks for Electric Power Transmission”, *Journal of Regulatory Economics*, 4, 211-242.

3) Wu, F., P. Varaiya, P. Spiller & S. Oren (1996), ”Folk Theorems on Transmission Access: Proofs and Counterexamples”, *Journal of Regulatory Economics*, 10, 5-23.

4) Stoft, S. (1997), “Zones: Simple or Complex?”, *The Electricity Journal*, Jan/Feb, 24-31.

5) Bjørndal, M. & K. Jørnsten (2001), “Zonal Pricing in a Deregulated Electricity Market”, *The Energy Journal*, 22, 51-73.

6) Ehrenmann, A. & Y. Smeers (2005), “Inefficiencies in European Congestion Management Proposals”, *Utilities Policy*, 13, 135-152.

Alternative methods

Explicit Congestion Pricing

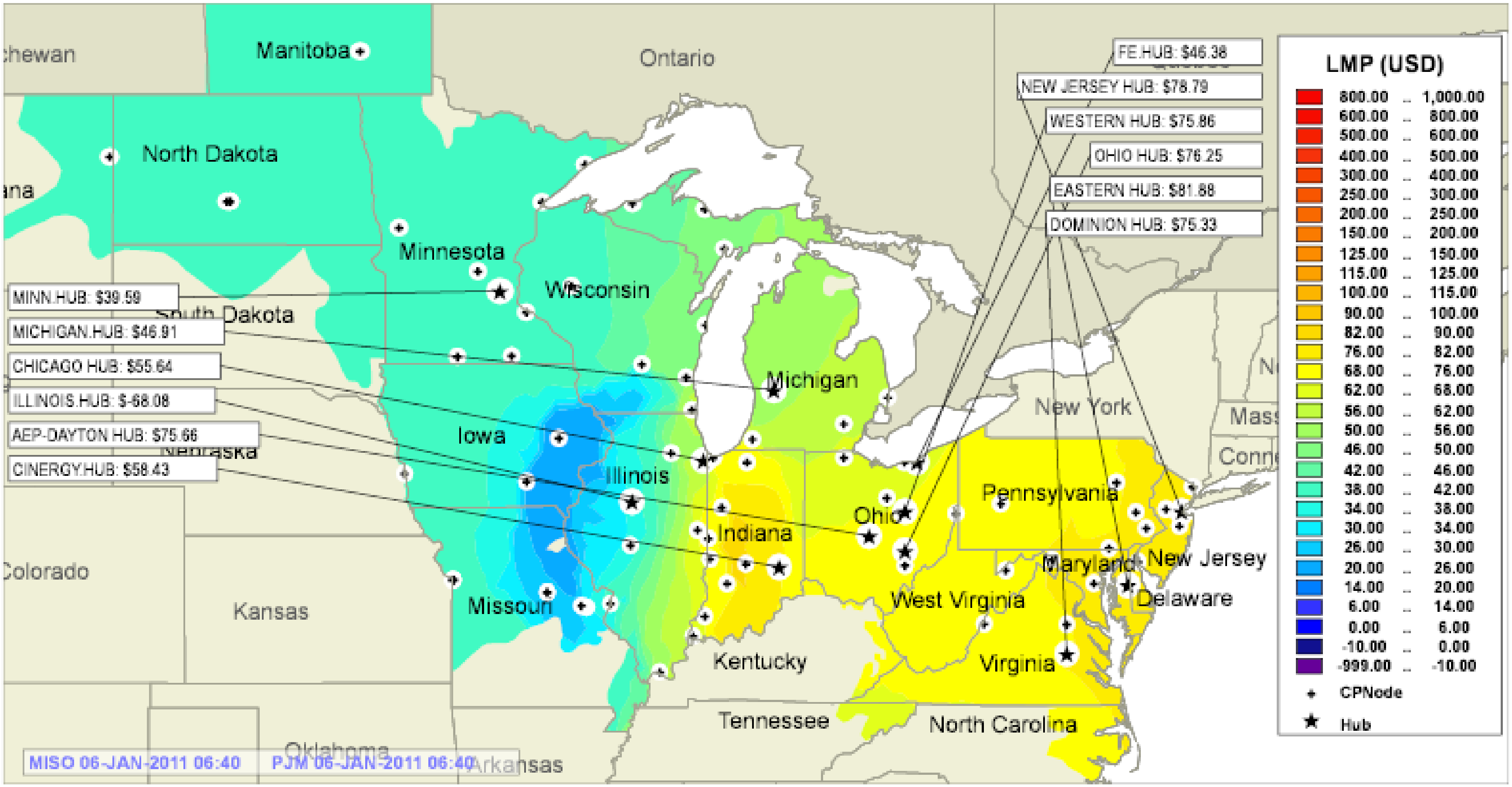
- Chao, H.-P., & S. Peck (1996), “A Market Mechanism for Electric Power Transmission,” *Journal of Regulatory Economics*, 10, 25-59.
- Stoft, S. (1998), “Congestion Pricing with Fewer Prices than Zones,” *The Electricity Journal* (May), 23-31.

Iterative Approaches

- Wu, F., & P. Varaiya (1995), “Coordinated Multilateral Trades for Electric Power Networks: Theory and Implementation,” Department of Electrical Engineering and Computer Sciences, University of California.
- Glavitch, H., & F. Alvarado (1997), “Management of Multiple Congested Conditions in Unbundled Operation of a Power System,” *IEEE Transactions on Power Systems*, 13, 374-380.

Summary: Congestion management methods

- Coordination by prices
  - Nodal prices / Zonal prices
  - Chao-Peck price / Flowgate prices
- Coordination through constraints
  - Coordinated Multilateral Trade Model
- Countertrading / Redispatching
- Methods may be consistent with optimal power flows and optimal nodal prices
  - Different strengths and weaknesses
  - Differ with respect to allocation of social surplus



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Midwest ISO Market data is based on Eastern Standard Time (EST) while PJM Market data is based on Eastern Prevailing Time.

PJM – 51 mill people/max load 145 000 MW/730 TWh/650 members/8700 nodes

Europe 2007

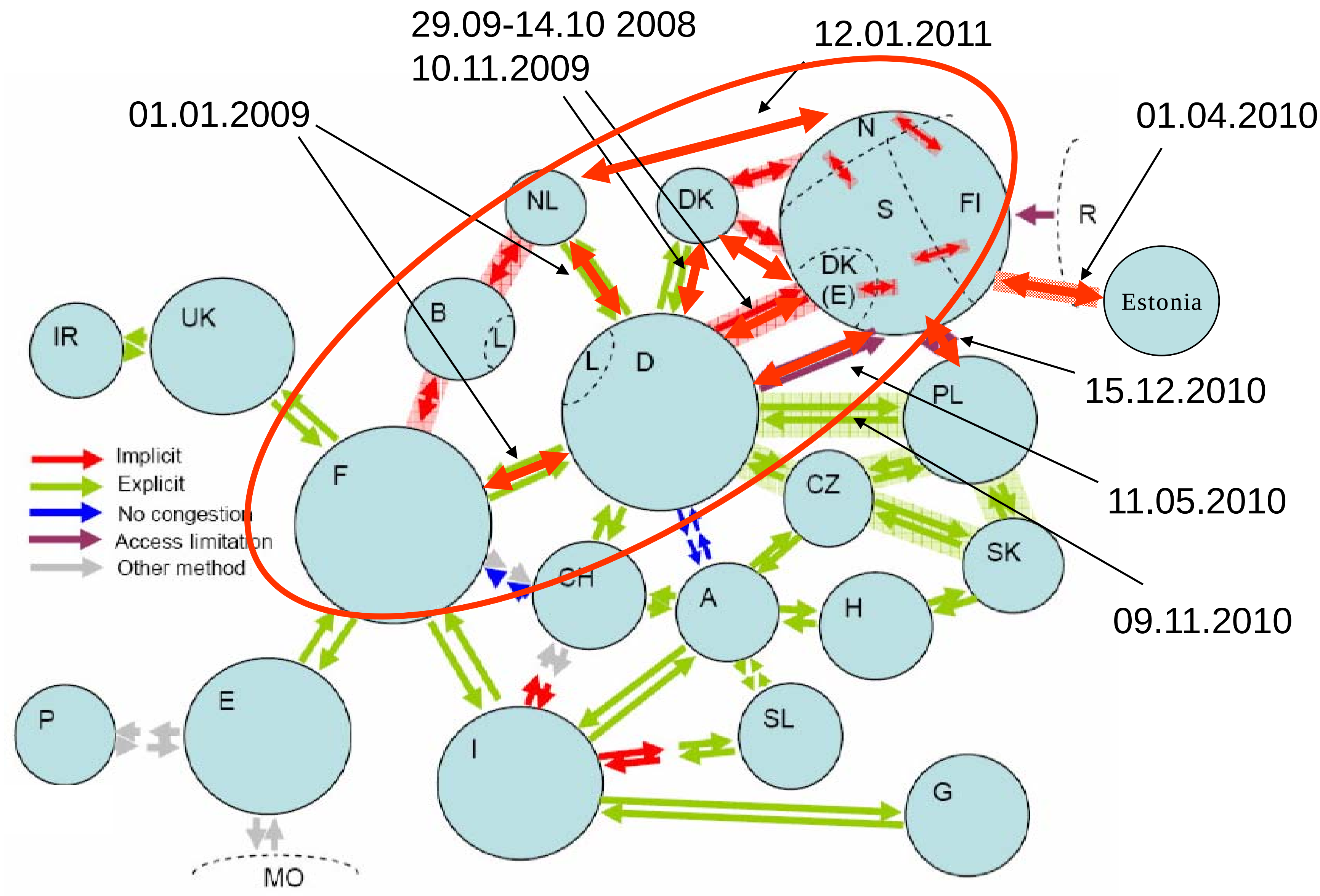
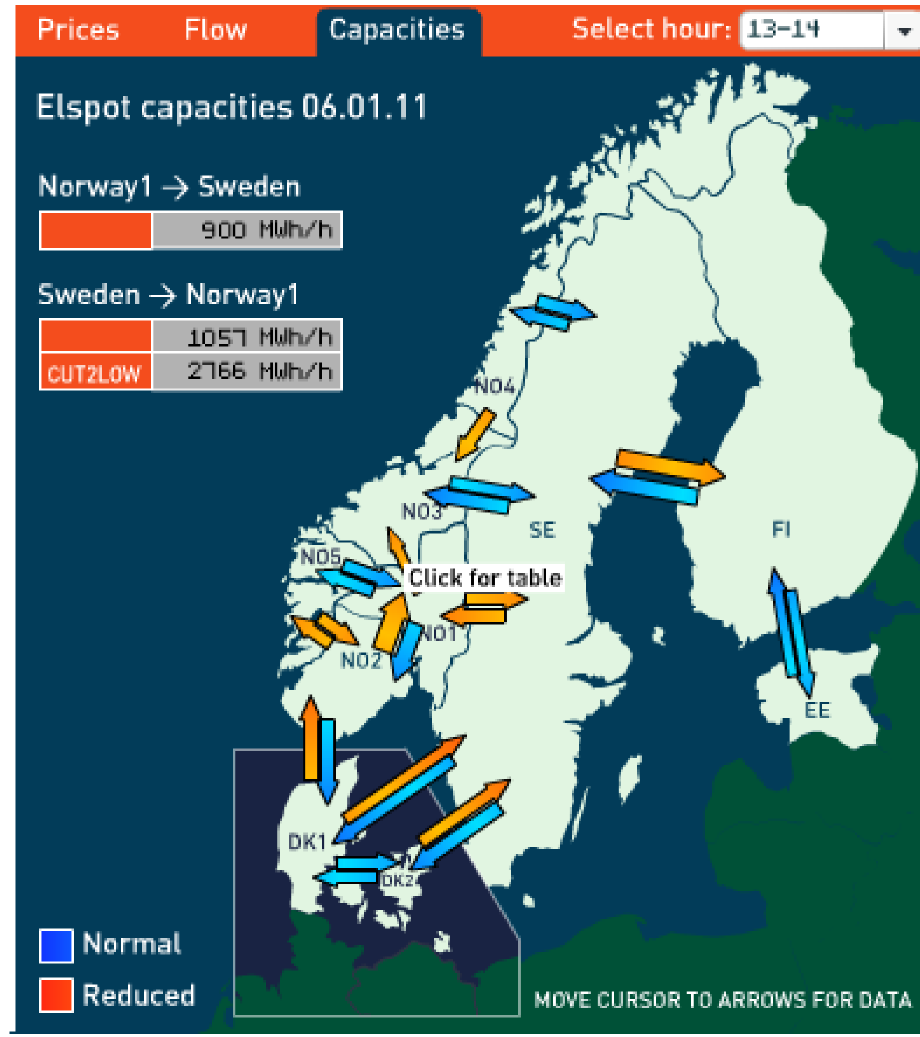


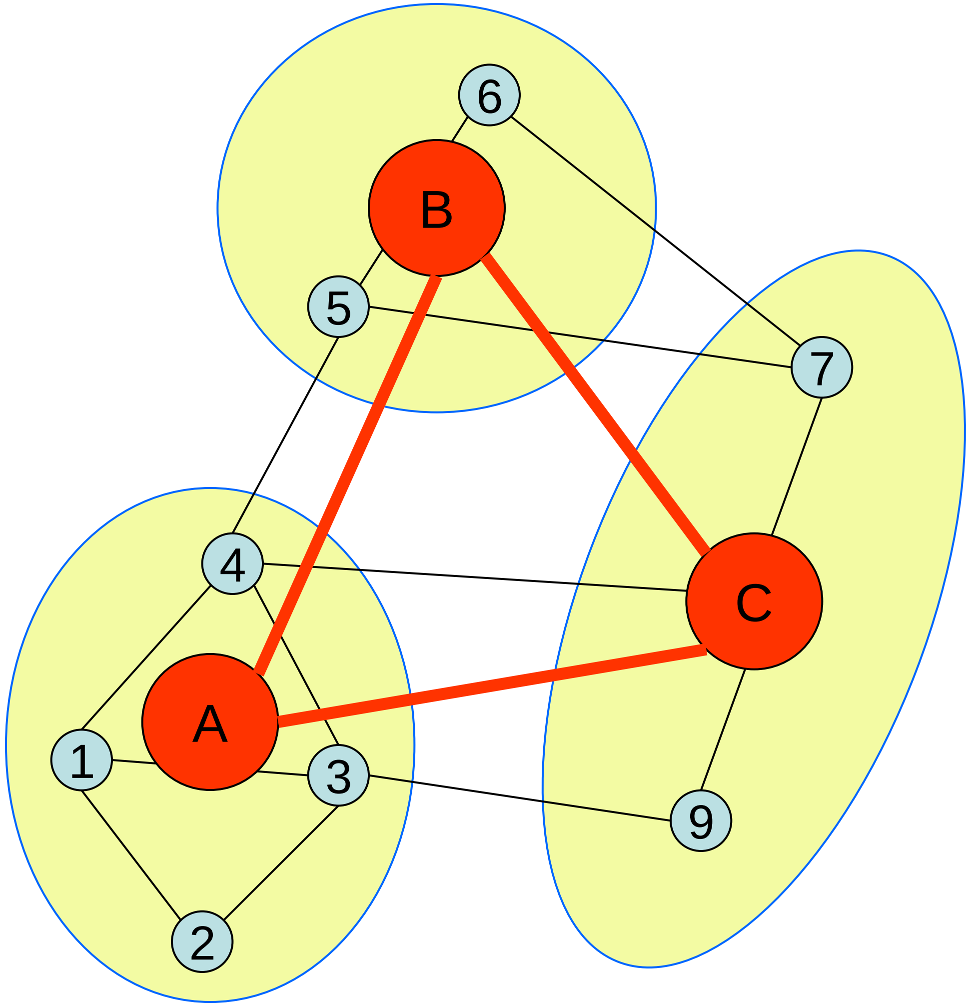
Figure 4.1 – Day-ahead transmission capacity allocations across Europe (updated June 2007)

Nord Pool


DET NORDISKE TRANSMISSIONSNET
The Transmission Grid in the Nordic Countries
Nordel
ISLAND
NORGE
SVERIGE
FINLAND
DANMARK
EESTI
ROSSIJA
POLSKA


Prices
Flow
Capacities
Select hour:
13-14
Elspot capacities 06.01.11
Norway1 → Sweden
900 MWh/h
Sweden → Norway1
1057 MWh/h
CUT2LOW
2766 MWh/h
NO4
NO3
SE
FI
NO5
Click for table
NO1
NO2
EE
DK1
DK2
Normal
Reduced
MOVE CURSOR TO ARROWS FOR DATA

What is zonal pricing?



True network
- ”All” nodes included
- ”All” lines represented

Economic aggregation
- ”All” nodes included
- ”All” lines represented
- Zones with uniform prices

Physical aggregation
- Aggregate nodes
- Aggregate lines

Simplifications under NPS Market Splitting

Optimal Power Flow - AC/”DC”

I: Theoretical benchmark:
“DC” is an approximation of the full alternate current (AC) power flows

↓

Optimal Zonal Prices

II: Require the same prices in several nodes:
A restriction / More constrained model

↓

Aggregated Nodes (Location of bids unknown)

III: Intra-zonal constraints are not taken into account:
Relaxation / Less restrictive model

↓

Aggregated Lines

IV: Capacities are added on aggregated lines:
Relaxation / Less restrictive model

↓

Without Loop Flow

V: Characteristics of electrical power flows are not considered:
Relaxation / Less restrictive model

↓

Heuristic for Determining Aggregated Capacity

VI: Restrictions added in order to obtain feasible solution in the original problem

↓

Heuristic for Market Splitting

VII: The old trading system, SAPRI, computes prices from sequentially splitting the system in two parts
SESAM is optimization based and solves this approximation

Market Coupling: …and then some…

International implementations T. Krause, 2005

	Main characteristics	Auctioning	International Implementation Examples
Nodal Pricing	Requires a centralized dispatch, often implemented in pool-based markets, high degree of centralization, FTR market for hedging	Implicit	PJM, New England, New York, Singapore, Ireland, upcoming market design of Texas and California
Zonal Pricing	May be implemented using a centralized dispatch (Australia) or using market splitting (Nordel), in any case zones defined a priori, Cfds for hedging possible	Implicit	Australia, Nord Pool
Uniform Pricing	Congestion not taken into account in the day-ahead phase, redispatch or countertrading for congestion relief	na	Finland, Sweden, former England and Wales Pool
Explicit Auctioning	Decentralized auctioning of transmission capacity	Explicit	Some European interconnections

Part II

Optimal power flow and physical equilibrium

Constrained optimal dispatch (AC)

(1)	max	$\sum_i [B_i(S_i^d) - C_i(S_i^s)]$		Maximizes social surplus from active and reactive power
(2)	s.t.	$S_i = S_i^s - S_i^d$	$\forall i$	Define net injections to every node
(3)		$S_i = V_i \cdot I_i^*$	$\forall i$	Relate complex power ($S = P + jQ$) to (complex) voltage and currents (I^* is conjugate of the complex current I)
(4)		$S_{ik} = V_i \cdot I_{ik}^*$	$\forall ik$	
(5)		$\lvert S_{ik} \rvert \le C_{ik}$	$\forall ik$	Capacity constraint on line *ik*, stated as a limit on the magnitude of apparent power
(6)		$I_i = \sum_{k \neq i} I_{ik}$	$\forall i$	Kirchhoff’s junction rule
(7)		$I_{ik} = Y_{ik}(V_i - V_k)$	$\forall ik$	Ohm’s law with Kirchhoff’s loop rule incorporated

“Behavioral” assumption

- The electric current follows the path of least resistance
  - Given node currents I_i and r_{ik} being the resistance of line ik, the optimal line currents I_{ik} are obtained by solving

$$\min \quad \tfrac{1}{2}\sum r_{ik} I_{ik}^2$$

$$\text{s.t.} \quad I_i = \sum_{k \neq i} I_{ik} \qquad \forall i$$

  - With dual variables V_i, the Lagrangean is $\Phi = \tfrac{1}{2}\sum_{ik} r_{ik} I_{ik}^2 + \sum_i (I_i - \sum_{k \neq i} I_{ik}) \cdot V_i$
  - With first order conditions

$$\frac{\partial \Phi}{\partial I_{ik}} = r_{ik} I_{ik} - V_i + V_k = 0 \qquad \forall ik \qquad \text{or} \qquad I_{ik} = \frac{V_i - V_k}{r_{ik}} = Y_{ik}(V_i - V_k) \quad \forall ik$$

$$\frac{\partial \Phi}{\partial V_i} = I_i - \sum_{k \neq i} I_{ik} = 0 \qquad \forall i\,.$$

} Correspond to (6) and (7) in the optimal dispatch problem

Bilevel program formulation

P1 $\max_{P_i^s, P_i^d, I_i} \sum_i [B_i(P_i^d) - C_i(P_i^s)]$

s.t. $P_i = P_i^s - P_i^d \quad \forall i$

$P_i = V_i I_i \quad \forall i$

$P_{ik} = V_i I_{ik} \quad \forall ik$

$P_{ik} \le C_{ik} \quad \forall ik$

and given $I_i \ \forall i$, I_{ik} is implicitly defined by,

P2 min $\frac{1}{2} \sum r_{ik} I_{ik}^2$

s.t. $I_i = \sum_{k \neq i} I_{ik} \quad \forall i$

which provides also the dual variables V_i.

P1 sets node currents and the ”agents”, i.e. the electrons respond to this by following the path of least resistance given by P2

P1-P2 fits into the framework of bilevel programs (Kolstad 1985)

Hence, the optimal dispatch problem can be seen as a bilevel program consisting of

P1:
The upper level program, which is the social maximization problem

and

P2:
The lower level program or behavioral program, which determines flows

Implications

- The optimal dispatch problem is similar to economic models like Stackelberg leader-follower games or principal-agent problems
  - the leader/principal solves an upper level program taking into account that the follower/agent acts in his own self-interest, solving a lower level program
- Interpreting the optimal dispatch problem within the bilevel programming framework draws attention to the differences between an electrical network and economic transportation models like for instance the *spatial price equilibrium* model of Enke or Samuelson

Investment paradoxes

- Investments in the grid may lead to a degradation in network performance
  - even without considering investment costs
- Similar for
  - Traffic equilibrium problems (Braess' paradox)
  - Communication / Computer networks
- Occur because of the non-cooperative structure of certain networks, where the term non-cooperative emphasizes that the networks are
  - operated according to a decentralized control paradigm
  - control decisions are made by each user independently
  - according to the user's own individual performance objective

System optimum versus user-equilibrium

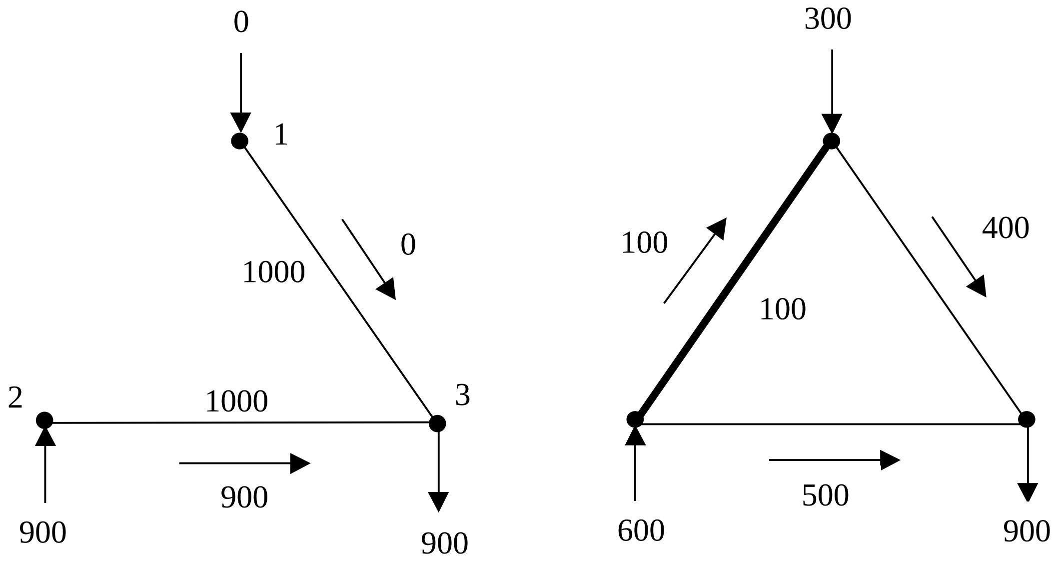
- Braess' Paradox
  - In user-equilibrium each driver takes the shortest path, without paying attention to the effect this has on the other users (eventually including himself)
- Computer Networks
  - Routing protocols
- Electric Networks
  - The economic equilibrium model include physical equilibrium constraints, electrons behave “non-cooperatively” and power cannot be routed
  - The optimal dispatch problem can be seen as a bilevel program where the Karush-Kuhn-Tucker conditions give the power flow equations
    - Kirchhoff's junction rule
    - Kirchhoff's loop rule

$\Rightarrow$ The optimal dispatch problem is similar to

- Stackelberg Problems
- Principal-Agent Problems

Example - reduced effective capacity

- The new line leads to reduced capacity between the low cost producer in node 2 and the consumer in node 3
- Still, the new line collects congestion rent (defined by the merchandizing surplus).

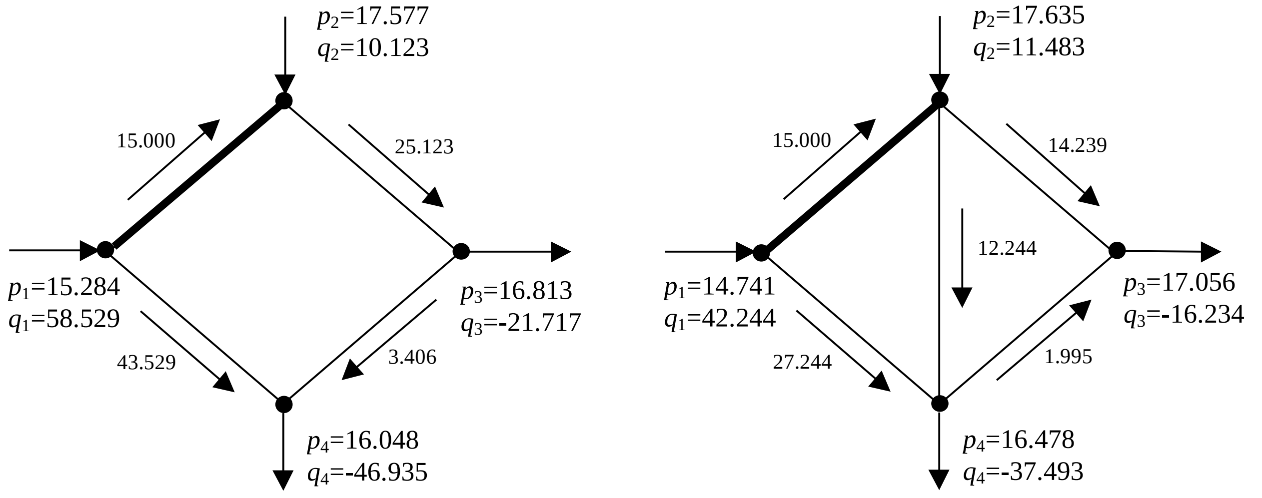


Part A: Initial Trades

Part B: Trades With the New Line

Example - reduced social surplus

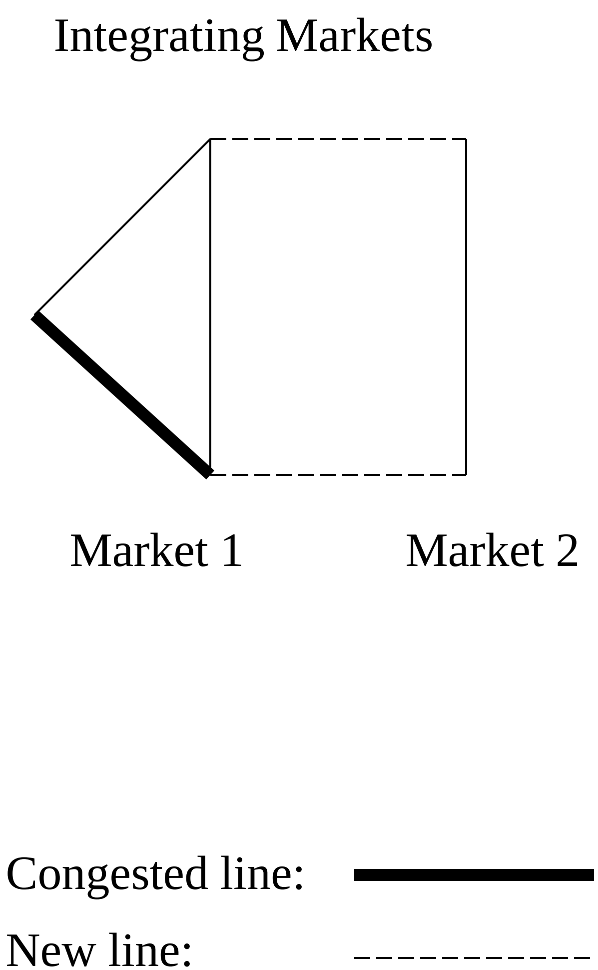
- Elastic supply and demand functions
- New line leads to
  - Reduction in consumption/production
  - Increase in grid revenue



Part A: No Line between Nodes 2 and 4
Social Surplus: 2878.526
Grid Revenue: 45.848

Part B: New Line between Nodes 2 and 4
Social Surplus: 2852.660
Grid Revenue: 69.444

Example - Market integration



- Unconstrained dispatch
  - No transmission constraints
  - Positive effect of integration
- Constraint that is internal to market 1
  - Integrating markets lead to lower social surplus
    - 3000.433 versus 2988.241
  - Grid revenue increases
    - 67.139 versus 72.535

Part III

Bilevel structures in market power modeling

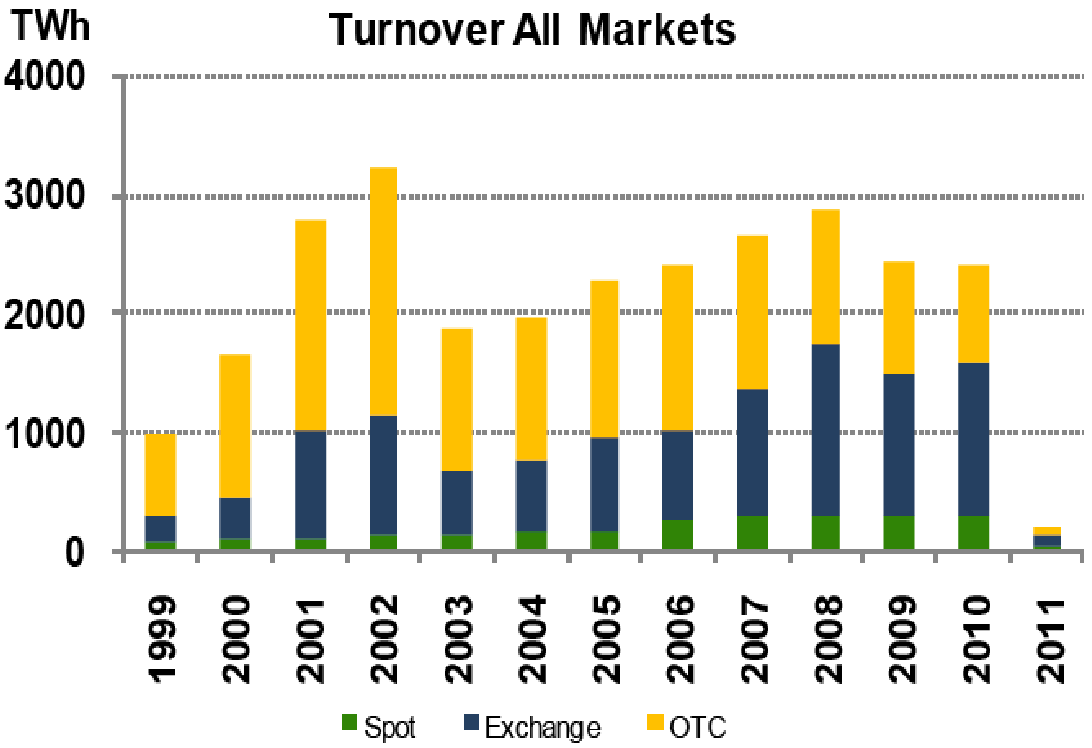
Strategic bidding

- Bids deviate from marginal cost curves
- Norway
  - 99% of production capacity is hydro
  - Payable marginal cost for hydro production ≈ 0
  - Storage capacity opens for inter-temporal dispositions
    - ”Water-value”
    - Alternative value of water
    - Inter-temporal optimization
    - MUST and SHOULD be taken into account

$\Rightarrow$ Bid curves based on beliefs / judgments

Strategic bidding

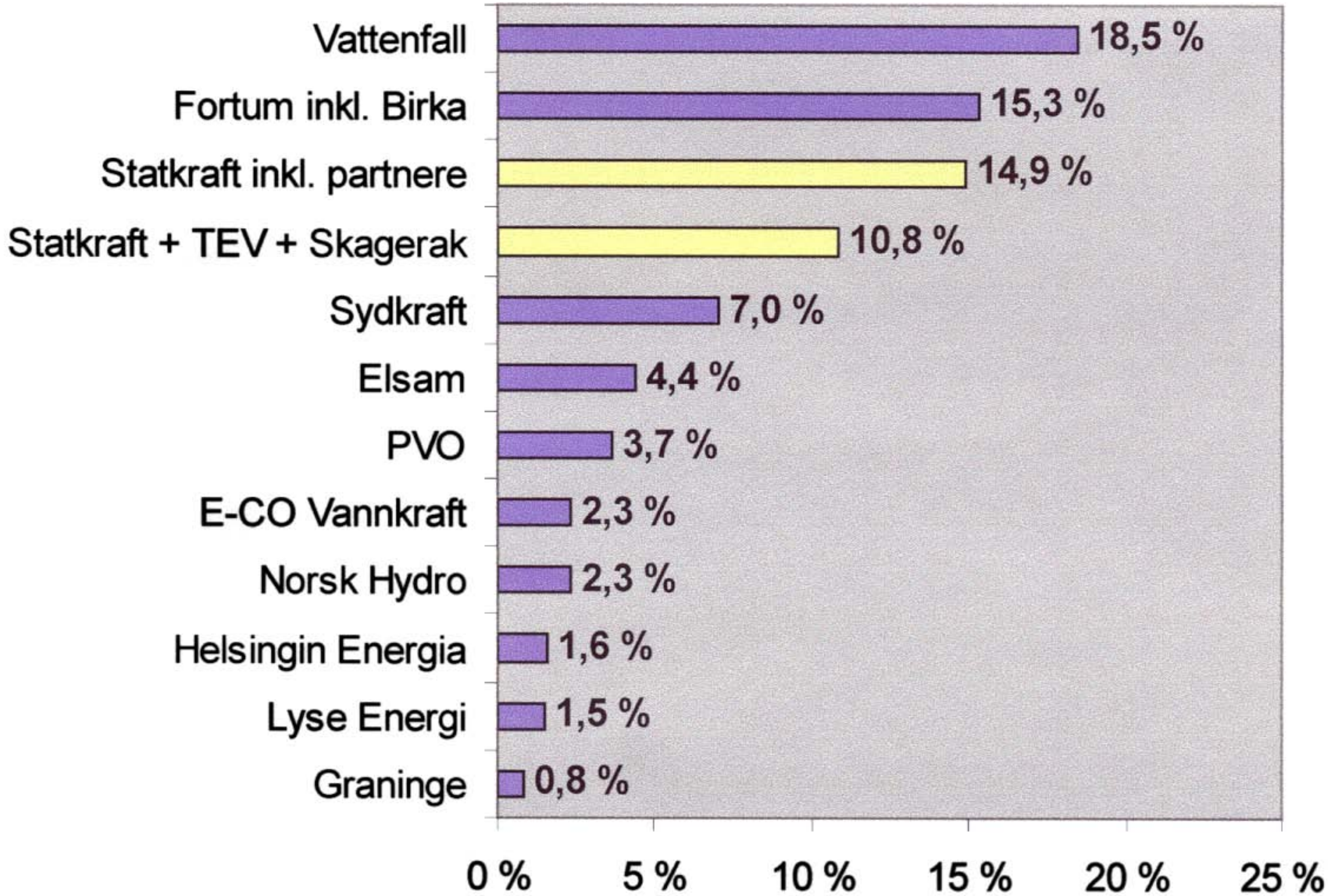
- In relation to what?
  - Bottlenecks
  - Reservoirs – water- values
  - Financial market positions
- What is the relevant market?
  - Regional
  - Physical
  - Financial



Total consumption ≈ 400 TWh

Market shares Nordic power market

Source: Statkraft



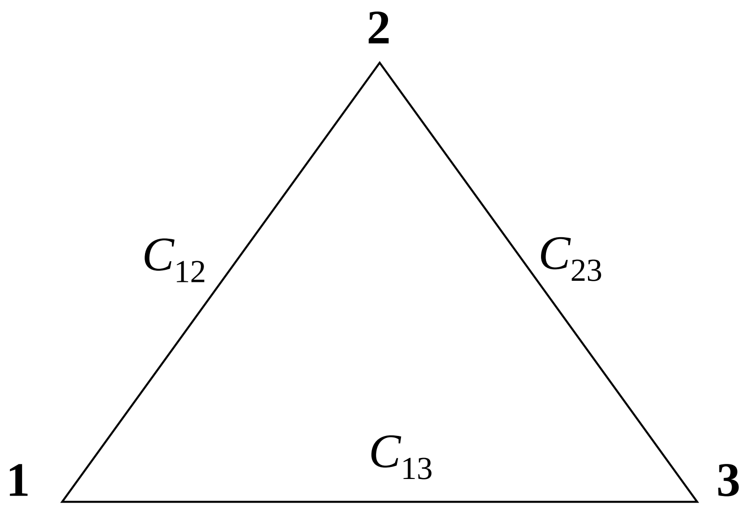
Statkraft's acquisition of Agder Energi

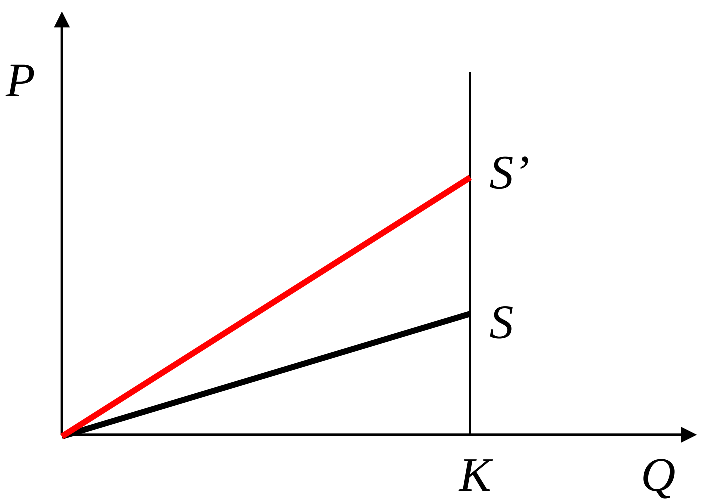
- Evaluated by the competition authorities
- Analysis of the effects of congestion on market power
  - Relevant market?
- Model
  - Single interconnecting line
  - Two time periods
- Equilibrium characterized by prices and the relationships between them
  - Intuitive
  - This intuition doesn't work in meshed structure networks

Simplified electricity market

- Static
  - A single period
- Market
  - Linear supply; slope + capacity
  - Linear demand
- Three node triangular network
  - A single capacitated line
- Market clearing
  - System Operator – Optimal Power Flow

Market power and bottlenecks





A single period
Full information

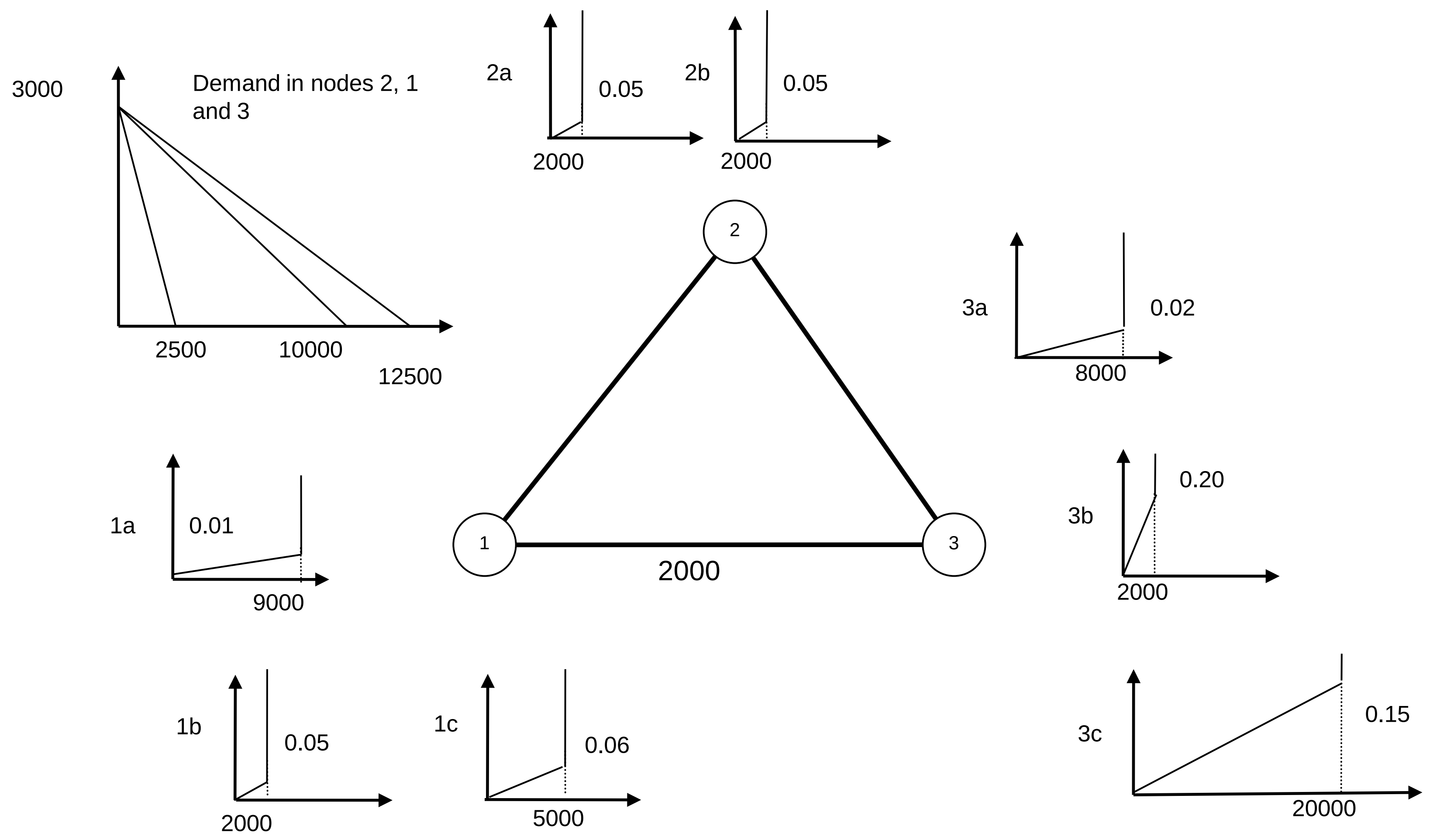
Capacity constraint:
$C_{13} = C_{31} = 2000$ MW

Problem for producer:

Max $\Pi(S, S')$
s.t. OPF(S', D, C)

$\Rightarrow$ Bilevel program

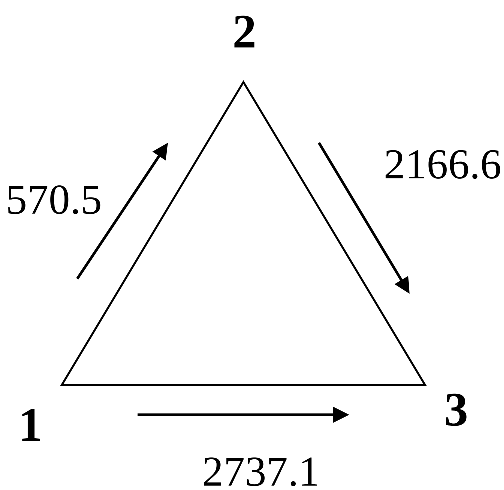
Demand and cost parameters



Example 1: Strategic Player in Node 1

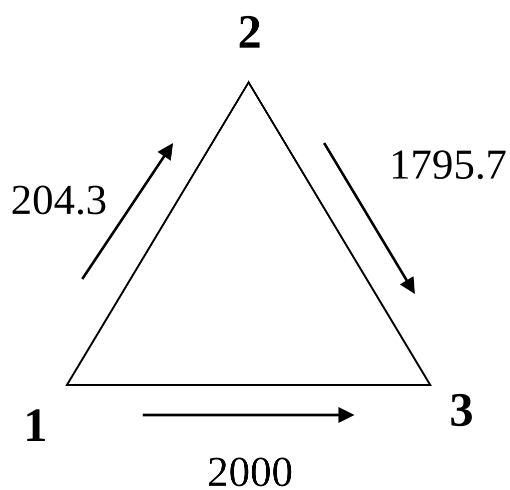
Limited transmission capacity: $C_{13} = 2000$

Unconstrained power flow (S)



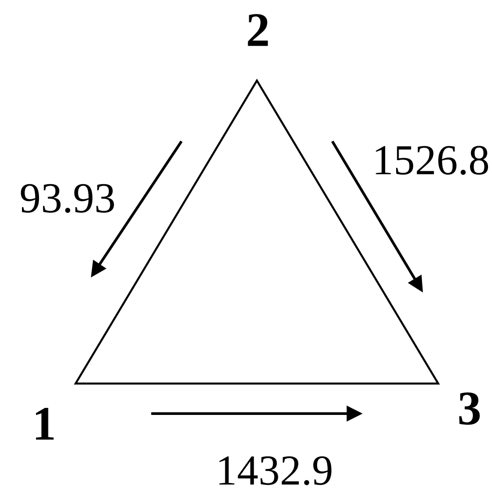
Price: 115.38
$\Pi_1 = 633\ 461$
$Q_1 = 9000$

Optimal power flow (S)



$P_1 = 87.17$
$P_2 = 109.70$
$P_3 = 132.22$
$\Pi_1 = 379\ 962$
$Q_1 = 8717$

Max $\Pi_1(S')$

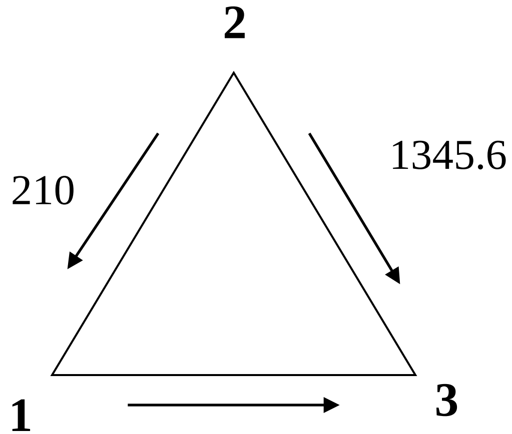


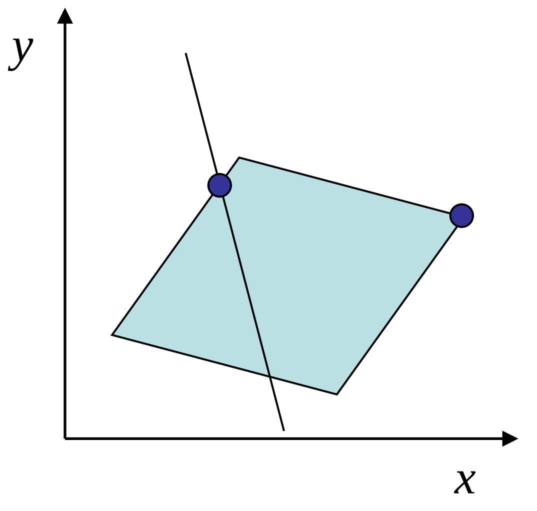
Price: 144.91
$\Pi_1 = 725\ 941$
$Q_1 = 6441$

Example 1: ”Irrelevant Constraints”

Limited transmission capacities: C_{13} = 2000 og C_{12} = 210

Max $\Pi_1(S')$





Principal:
Max y
$x \in X$
s.t.
Agent:
Min $y \mid x$

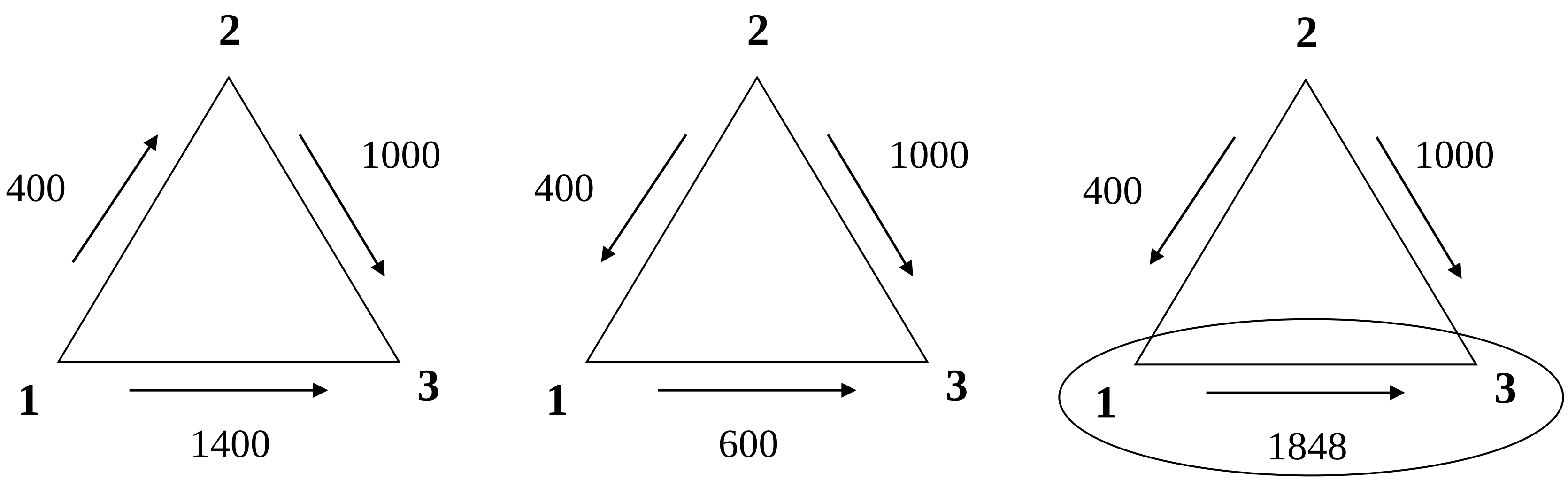
P_1 = 205.05
P_2 = 99.32
P_3 = 152.18
Π_1 = 872 897
Q_1 = 4825

⇒ Seemingly irrelevant constraints can influence on the solution

Example 2

Transmission capacity: $C_{12} = 400$, $C_{13} = 2000$, $C_{23} = 1000$

Optimal power flow (S)	Max $\Pi_1(S')$	Without loop Flow (S')

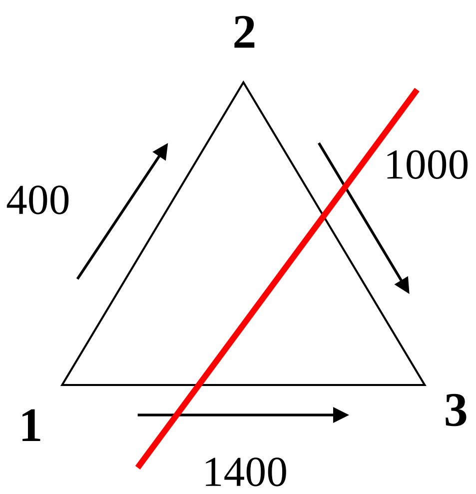


$P_1 = 84.29$	$P_1 = 223.64$ ↔	$P_1 = 146.61$
$P_2 = 75.92$	$P_2 = 95.51$	$P_2 = 95.51$
$P_3 = 153.42$	$P_3 = 183.16$ ↔	$P_3 = 146.61$
$\Pi_1 = 355\,204$	$\Pi_1 = 764\,091$	$\Pi_1 = 743\,018$
$Q_1 = 8429$	$Q_1 = 3727$	$Q_1 = 3736$

Example 2: Zonal Pricing

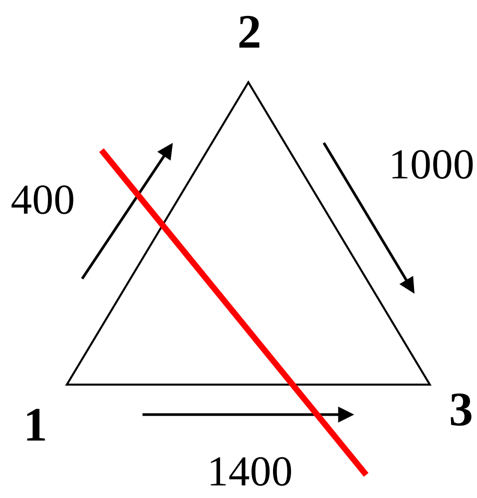
Transmission capacity: C_{12} = 400, C_{13} = 2000, C_{23} = 1000

Optimal zonal prices (*S*)
(Zone Allocation 1)



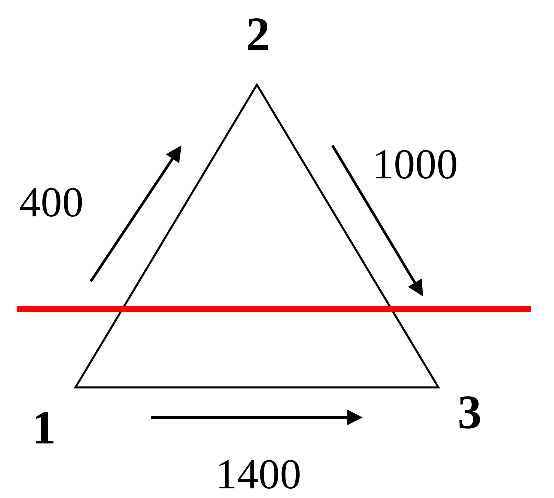
P_1 = 84.29
P_2 = 84.29
P_3 = 153.42
Π_1 = 355 204
Q_1 = 8429

Optimal zonal prices (*S*)
(Zone Allocation 2)



P_1 = 84.29
P_2 = 153.42
P_3 = 153.42
Π_1 = 355 204
Q_1 = 8429

Optimal zonal prices (*S*)
(Zone Allocation 3)

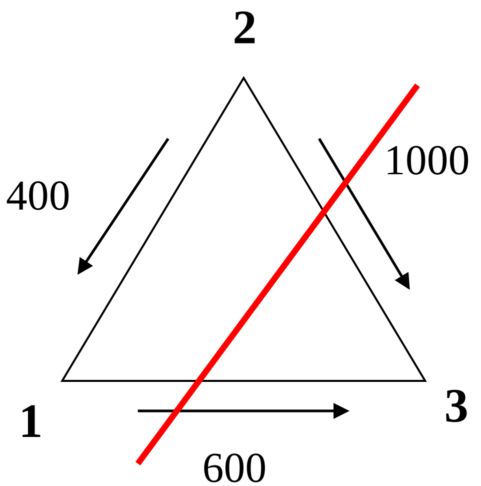


P_1 = 75.92
P_2 = 153.92
P_3 = 153.92
Π_1 = 926 089
Q_1 = 8260

Example 2: Zonal Pricing – Max Profit

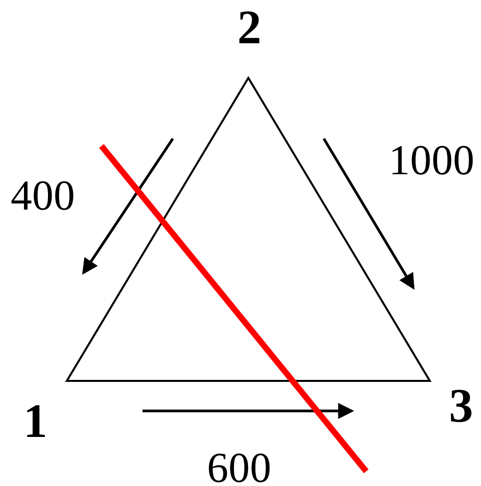
Transmission capacity: C_{12} = 400, C_{13} = 2000, C_{23} = 1000

Max $\Pi_1(S')$
(Zone Allocation 1)



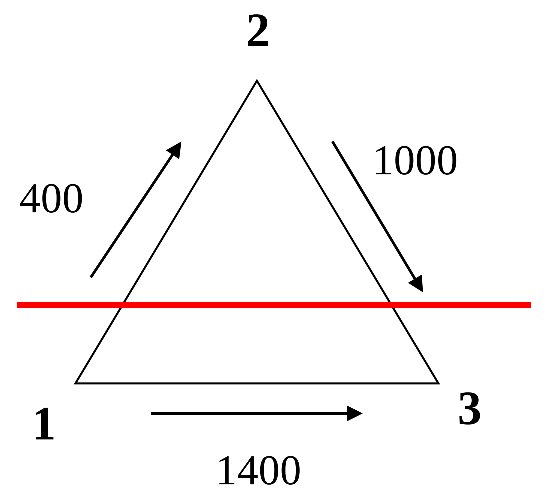
P_1 = 223.64
P_2 = 223.64
P_3 = 183.16
Π_1 = 764 091
Q_1 = 3727

Max $\Pi_1(S')$
(Zone Allocation 2)



P_1 = 223.64
P_2 = 183.16
P_3 = 183.16
Π_1 = 764 091
Q_1 = 3727

Max $\Pi_1(S')$
(Zone Allocation 3)

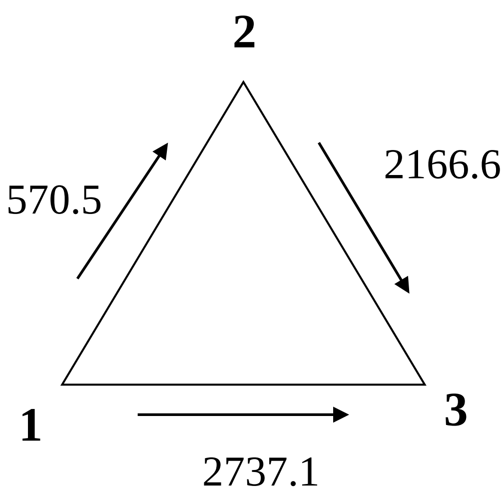


P_1 = 153.42
P_2 = 75.92
P_3 = 153.42
Π_1 = 975 759
Q_1 = 9000

Example 3: Effect of Size

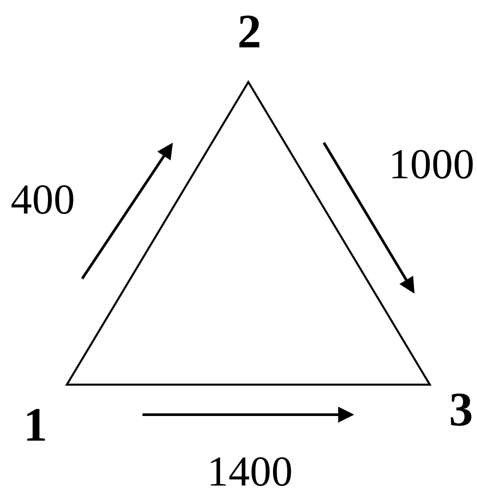
Characteristics large / small producers:
Capacity 9000 / 2000, Slope of S 0.01 / 0.05

Unconstrained power flow (S)



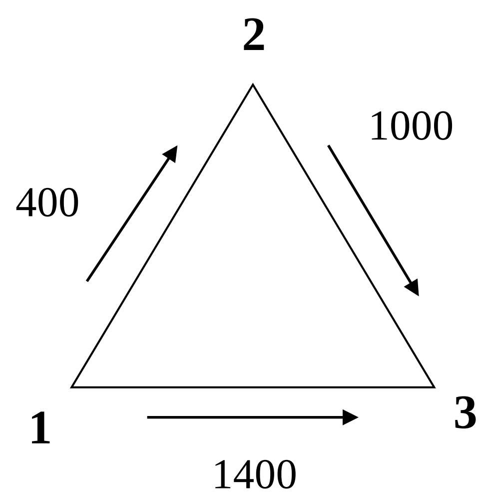
Price: 115.38
Π_2 = 130 770
Q_2 = 2000

Optimal power flow (S)



P_1 = 84.29
P_2 = 75.92
P_3 = 153.42
Π_2 = 71 041
Q_2 = 1686

Max $\Pi_1(S')$



P_1 = 85.89
P_2 = 75.92
P_3 = 153.42
Π_2 = 72 509 (347 272)
Q_2 = 1494

Lessons to be learned

- Locational electricity prices may look ”strange” to those unfamiliar with power flow models
- There are several sources for bilevel structures of optimization / equilibria problems
  - Physical laws
  - Market power / strategic bidding
- Solutions are sensitive to assumptions