A stochastic generalized nash model for natural gas transport systems

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Outline

• Optimization models versus equilibriums
• Spatial equilibrium models
• A stochastic complementarity model
Mathematical programming

• Maximization or minimization of a real function by choosing values of variables from within an allowed set

\[ \max_x f(x) \quad s.t. \quad x \in X \]

• A large number of problem classes:
  – Linear programming
  – Nonlinear programming
  – Integer programming
  – Stochastic programming, etc.
Optimality conditions

- Optimization problem with inequalities and equalities
  \[ \max_x f(x) \]
  \[ g(x) \leq 0 \]
  \[ h(x) = 0 \]

- Given that we have a convex problem the first-order conditions of optimality (KKT) is sufficient
  - Convex feasible set
  - Concave objective function for maximization / convex objective function for minimization

\[ \Delta f(x) + \Delta g(x)^T \lambda + \Delta h(x)^T \mu = 0 \]
\[ 0 \geq g(x) \perp \lambda \geq 0 \]
\[ 0 = h(x) \]
Complementarity problems

- Linear system of equations
- Nonlinear system of equations
- Linear complementarity problem
- Nonlinear complementarity problem
- Nonlinear program
- Finite-dimensional system of variational inequalities

- All of these problems can be generalized to Mixed complementarity problems
The linear complementarity problem

- Problem statement
- Find a vector $x$ such that

$$x \geq 0$$

$$q + Mx \geq 0$$

$$x^T(q + Mx) = 0$$

- For a given vector $q$ and matrix $M$
- Denoted: $\text{LCP}(q, M)$
Nonlinear complementarity problem (NCP)

• Find a vector $x$ such that:

\[ x \geq 0 \]
\[ F(x) \geq 0 \]
\[ x^T F(x) = 0 \]

• Applications:
  – General equilibrium theory of economics, policy design and analysis, game theory, mechanics, etc.
Mixed Complementarity Problem

• Many practical applications generate problems where some of the variables are nonnegative, others are bounded and others are free
  – To accommodate this flexibility, the MCP is used
• Find a vector $x \in [l, u]$ such that

\[
\begin{align*}
  x_i &= l_i \quad \text{and} \quad F_i(x) > 0, \\
  x_i &= u_i \quad \text{and} \quad F_i(x) < 0, \\
  x_i &\in [l_i, u_i] \quad \text{and} \quad F_i(x) = 0.
\end{align*}
\]
Mixed Complementarity Problem

• Example (KKT for an optimization problem):

\[ \Delta \phi(x) + \Delta g(x)^T \lambda + \Delta h(x)^T \mu = 0 \]
\[ 0 \geq g(x) \perp \lambda \geq 0 \]
\[ 0 = h(x) \]

• Complementary pairs of variables:
  – Economics: the price of a commodity and excess supply
  – Contact mechanics: the contact force between two variables and the distance between them

• MCP appear in study of equilibrium problems
  – Numerous applications (economics, engineering and chemistry)
Terminology

- A feasible $x$ satisfies the inequalities
  - If $z$ strictly satisfies the inequalities, it is called strictly feasible

- The set of feasible vectors is called its feasible region and is denoted $\text{FEA}(q,M)$

- A vector $x$ satisfying the complementarity condition is called complementary

- The CP is then to find a $x$ that is both feasible and complementary
Complementarity Problems versus Optimization

- Optimization problems (via KKT-conditions)
- Game theory problems (for instance Nash-Cournot games)
- Many other problems in engineering and economics

• Theorems and algorithms developed for CP can be applied to a large number of applications
• CPs can include problems where dual prices (Lagrangean multipliers) appear in the primal formulation
Equilibrium Problems

• Equilibrium is a stable situation in which forces cancel one another
  – Economics: supply equals demand
  – Chemistry: the forward rate and reverse rate of reaction is equal
  – Physics: all forces acting on an object are balanced
  – Game theory: Nash equilibrium (situation where no player has an incentive to deviate from his strategy unilaterally)

\[ \forall i, \quad f_i(q_i^*, q_{-i}^*) \geq f_i(q_i, q_{-i}^*) \]

• Formulation of equilibria:
  – Normally formulated as MCP or more generally as a Variational Inequality (VI)
  – The VI is a unifying methodology for the study of equilibrium systems
Spatial equilibrium models

• Consider a network with
  – A set of suppliers (I), supplies $a_i$
  – A set of markets (J), demands $b_j$
  – Transport costs of $c_{ij}$

• Want to find a transportation schedule which minimizes the cost of supplying all markets:

\[
\min \sum_{i,j} c_{ij} x_{ij}
\]

\[s.t. \sum_j x_{i,j} \leq a_i\]

\[\sum_i x_{i,j} \geq b_j\]

\[x \geq 0\]
Example (cont.)

• Can be interpreted as a market equilibrium problem
  – The dual multiplier for the supply constraint represents the price in the supply markets \( (w_i) \)
  – The dual multiplier for the demand constraint represents the price in the demand markets \( (p_j) \)

• We can then formulate the equilibrium conditions in the following way (LCP):

\[
\sum_{j} x_{ij} \leq a_i, \ w_i \geq 0, \ w_i \left( a_i - \sum_{j} x_{ij} \right) = 0
\]

\[
\sum_{i} x_{ij} \geq b_j, \ p_j \geq 0, \ p_j \left( b_j - \sum_{i} x_{ij} \right) = 0
\]

\[
w_i + c_{ij} \geq p_j, \ x_{ij} \geq 0, \ x_{ij} \left( w_i + c_{ij} - p_j \right) = 0
\]
Example (cont.)

• So far we have assumed constant demand and supply
  – Now suppose that demand and supplies are price responsive
    • All markets are perfectly competitive
• An associated optimization problem can be used to compute the equilibrium prices and quantities
• Here, formulated as an NCP
Example (cont.)

• We would then get the following equilibrium conditions:

\[
\sum_{j} x_{ij} \leq a_i, \quad w_i(a_i) \geq 0, \quad w_i(a_i) \left( a_i - \sum_{j} x_{ij} \right) = 0
\]

\[
\sum_{i} x_{ij} \geq b_j, \quad p_j(b_j) \geq 0, \quad p_j(b_j) \left( \sum_{i} x_{ij} - b_j \right) = 0
\]

\[
w_i(a_i) + c_{ij} \geq p_j(b_j), \quad x_{ij} \geq 0, \quad x_{ij} \left( w_i(a_i) + c_{ij} - p_j(b_j) \right) = 0
\]
CP or optimization problem

- So far the spatial equilibrium model could have been formulated as an optimization problem

- What if we introduce competition in a spatial equilibrium model?
Liberalized network industry

• Telecoms, energy, natural gas, railway, mail, ...

• Vertical separation
  – upstream market: network infrastructure (natural monopoly characteristics)
  – downstream market: sales to customers

• Many actors in downstream market:
  – use network infrastructure (lease, buy access, ...)
  – provide products, value-added services, ...
  – compete for customers
  – often also subsidiary of upstream actor
    (separated former monopolist)

• Downstream actors act egoistically:
  make decisions which are best for them – and not for the industry (or social surplus etc.)
Formulating and solving CPs

• Several articles explains how GAMS can be used to formulate CP
  
  
  
  – *Complementarity Problems in GAMS and the PATH solver*, Ferris and Munson, 1998

• Solvers: PATH and MILES

• In addition: AMPL with KNITRO
Example from the North sea gas pipeline network

A stochastic complementarity model

• Decision structure
• Notation & model
• Generalized Nash Equilibrium
• Case results
The North-Sea Case

• Production nodes (with gas fields), transportation nodes, market nodes

• Roles: large producers, smaller producers (modelled as a competitive fringe), Gassco (independent system operator).
Representation of transport network

![Diagram of transport network]
Transportation market structure

• Primary market
  – Large producers have capacity booking rights
    • Booking rights are more than 2 times the overall capacity
    • Conflicts resolved using Capacity Allocation Key
    • Tariff is fixed
• Secondary markets
  – ISO releases any available capacity
  – Bilateral trades of capacity between players
  – Price is negotiated
  – A competitive fringe clears the secondary market
The decision structure

Period 1

Big producers
- book capacity

Period 2

Big producers
- production level
- sale in the market nodes
- delivery in TOP contracts
- trade of transportation capacity

ISO
- routing
- trade of transportation capacity

Simultaneous

Simultaneous
Large producers decision problem

- Each large producer faces a two-stage stochastic program with recourse.

- This is still a one level game because the contingent strategy is laid at the time of booking and not changed as a result of the other players bookings.

- Stochastic parameters:
  - Spot price
  - TOP volumes
Purpose of the analysis

• Investigate the effect of different objectives for the ISO
  – Max flow, max value and max social surplus

• Analyze the effect of stochasticity
Price in the secondary market

- Demand from a competitive fringe in each field g
  - Comes from the profit maximization of the competitive fringe:

\[
\prod_{gs} = \max \sum_{m \in \mathcal{M}} (p_{ms} \cdot x_{gms} - t_{gms} \cdot x_{gms}) - W_g \left( \sum_{m \in \mathcal{M}} x_{gms} \right)
\]

- The first order condition of optimality is:

\[
\frac{\delta \prod_{gs}}{\delta x_{gms}} = p_{ms} - t_{gms} - \frac{\delta W_g (\sum_{m \in \mathcal{M}} x_{gms})}{\delta x_{gms}} = 0, \quad g \in \mathcal{G}, \ m \in \mathcal{M}, \ s \in \mathcal{S}
\]

- We assume that the cost function \((W())\) is quadratic. The inverse demand function can then be formulated as:

\[
t_{gms} = p_{ms} - c_g \sum_{m' \in \mathcal{M}} x_{gms}, \quad g \in \mathcal{G}, \ m \in \mathcal{M}, \ s \in \mathcal{S}
\]
Large producers objective function

\[ \Pi_l = \max \left( - \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} T_{gm} b_{gm} + \sum_{s \in \mathcal{S}} \phi_s \left[ \sum_{m \in \mathcal{M}} \left( p_{ms} q_{lms} + P_{lm} v_{lms} \right) \right] + \sum_{s \in \mathcal{S}} \phi_s \left[ \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} h_{lgms} \left( p_{ms} - c_g \left( \sum_{m' \in \mathcal{M}} \left( z_{gm's} + \sum_{l' \in \mathcal{L}} h_{lg'm's} \right) \right) \right) \right] - \sum_{s \in \mathcal{S}} \phi_s \left[ \sum_{g \in \mathcal{G}} C_{lg}(d_{lgs}) \right] \right) \]

- Income from the spot markets and delivery in the TOP contracts
- Cost of booking in the primary market
- Surplus from trade in the secondary market for transportation capacity
- Production cost
Large producers constraints

\[ b_{lgm} \leq B_{lgm}, \quad g \in \mathcal{G}, \ m \in \mathcal{M}, \]
\[ d_{lgs} = \sum_{m \in \mathcal{M}} \left( b_{lgm} - h_{lgms} \right), \quad g \in \mathcal{G}, \ s \in \mathcal{S}, \]
\[ q_{lms} + v_{lms} = \sum_{g \in \mathcal{G}} \left( b_{lgm} - h_{lgms} \right), \quad m \in \mathcal{M}, \ s \in \mathcal{S}, \]
\[ h_{lgms} \leq b_{lgm}, \quad g \in \mathcal{G}, \ m \in \mathcal{M}, \ s \in \mathcal{S}, \]
\[ z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0, \quad g \in \mathcal{G}, \ m \in \mathcal{M}, \ s \in \mathcal{S}. \]
ISO - objective function (1)

- Maximize flow (MF):

\[
\max \sum_{m \in M} \sum_{i \in I(m)} f_{ims}
\]
ISO – objective function (2)

- Maximize value of flow (MVF):

\[
\max \sum_{m \in M} \sum_{i \in I(m)} p_{ms} \left( f_{ims} - \sum_{l \in L} v_{lms} \right)
\]
ISO - objective function (3)

- Maximize social surplus (MSS):

$$\max \sum_{m \in M} \sum_{i \in \mathcal{I}(m)} p_{ms} \left( f_{ims} - \sum_{l \in \mathcal{L}} v_{lms} \right) + \sum_{m \in M} \sum_{l \in \mathcal{L}} P_{lm} v_{lms}$$

$$- \frac{1}{2} \sum_{g \in G} MC_g \left( \sum_{i \in \mathcal{O}(g)} f_{gi} \right)^2$$

The slope in the linear, aggregated supply function is given as:

$$MC_g = \frac{1}{\sum_{l \in \mathcal{L}_g} \frac{1}{2c_{lg}}}$$
ISO – some of the constraints

• Conservation of mass for the field nodes

\[
\sum_{j \in \mathcal{O}(g)} f_{gjs} = \sum_{m \in \mathcal{M}} \left( z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \quad g \in \mathcal{G}, \ s \in \mathcal{S}
\]

• Conservation of mass for the junction nodes

\[
\sum_{g \in \mathcal{I}(j)} f_{gjs} = \sum_{m \in \mathcal{O}(j)} f_{jms}, \quad j \in \mathcal{J}, \ s \in \mathcal{S}
\]

• Conservation of mass for the market nodes

\[
\sum_{n \in \mathcal{I}(m)} f_{nms} = \sum_{g \in \mathcal{G}} \left( z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \quad m \in \mathcal{M}, \ s \in \mathcal{S}
\]
ISO

- Positive price in the secondary market

\[ p_m - c_g \left( \sum_{m' \in \mathcal{M}'} \left( z_{g_m's} + \sum_{l \in \mathcal{L}} h_{l,g_m's} \right) \right) \geq 0, \quad g \in \mathcal{G}, \quad m \in \mathcal{M} \]
Benchmark

• The ISO schedules production, routing and sale in order to maximize the social surplus of all the players in the network

• Objective function:

$$\max \sum_{m \in \mathcal{M}} \sum_{l \in \mathcal{L}} (p_{ms} q_{lms} + P_{lms} v_{lms}) - \sum_{g \in \mathcal{G}} \sum_{l \in \mathcal{L}} C_{lg} d_{lgs}^2$$

• Constraints:

$$\sum_{l \in \mathcal{L}} d_{lgs} = \sum_{j \in \partial(g)} f_{gjs}, \quad g \in \mathcal{G}, \quad s \in \mathcal{S}$$

$$\sum_{g \in \mathcal{G}} f_{gjs} = \sum_{m \in \mathcal{M}} f_{jms}, \quad j \in \mathcal{J}, \quad s \in \mathcal{S}$$

$$\sum_{l \in \mathcal{L}} (q_{lms} + v_{lms}) = \sum_{j \in \mathcal{I}(m)} f_{jms}, \quad m \in \mathcal{M}, \quad s \in \mathcal{S}$$
Generalized Nash Equilibrium games

A generalized Nash equilibrium (GNE) is defined as a point \( x^* \in X \) that simultaneously optimizes all the players individual decision problems so that: \( x_l^* \in K_l(x_{-l}^*), \ l \in \bar{L} \) and \( \Pi_l(x^*) \geq \Pi_l(x_l, x_{-l}^*), \ x_l \in K_l(x_{-l}^*), \ l \in \bar{L} \) where \( \Pi : R^{\alpha \beta} \rightarrow R \) is the objective function of player \( l \).
Common constraints

- The common constraints in our model
  - Defined as constraints where decision variables from more than one player appear

\[
\begin{align*}
  z_{gms} + \sum_{l \in \mathcal{L}} h_{l gms} & \geq 0, \quad \tau_{gms} \\
  \sum_{j \in \mathcal{O}(g)} f_{j g s} & = \sum_{m \in \mathcal{M}} \left( z_{gms} + \sum_{l \in \mathcal{L}} b_{l g m} \right), \quad \nu_{g j s} \\
  \sum_{n \in \mathcal{I}(m)} f_{n m s} & = \sum_{g \in \mathcal{G}} \left( z_{gms} + \sum_{l \in \mathcal{L}} b_{l g m} \right), \quad \nu_{j m s} \\
  p_m - c_g \left( \sum_{m' \in \mathcal{M}'} \left( z_{g m' s} + \sum_{l \in \mathcal{L}} h_{l g m' s} \right) \right) & \geq 0, \quad \chi_{gms}
\end{align*}
\]
(Quasi) Variational inequalities

Following the lines of the discussion in Harker (1991), we define $F_l(x^*) = \nabla x_l \Pi_l(x_l^*, x_{-l}^*)$ and $F(x^*) = (F_0(x^*)^T, \ldots, F_{|L|}(x^*)^T)^T$. Then the GNE may be expressed as the Quasi Variational Inequality $QVI(F, K(x)):$

$$F(x^*)^T(x - x^*) \geq 0, \quad x \in K(x^*), \quad (1)$$

where $K(x) = \prod_{l \in L} K_l(x_{-l}).$
The VI solution

• Theorems 4-6 from Harker (1991)

If $F$ is a continuous function in the $VI(F, K)$ then the $VI$ solutions are the only points in the solution set of the $QVI(F, K(x))$ at which the optimal dual variables $\lambda^* \in R^p \beta$ for the common constraints are such that $\lambda_0^* = \lambda_j^*, \ j \in \overline{L}$. The theorems also state that any strictly interior solution (for the common constraints) of the $QVI(F, K(x))$ is a solution to the $VI(F, K)$. Further, if $F$ is strictly monotone there is a unique solution to the $VI$ over $X$, Facchinei and Pang (2003), Theorem 2.3.3.
## The complementarity program

<table>
<thead>
<tr>
<th>Large producer $n$</th>
<th>Independent System Operator (ISO)</th>
<th>Competitive fringe</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time 0</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Booking capacity</td>
<td>– Time 0</td>
<td>– Time 0</td>
</tr>
<tr>
<td>– Price in spot market is unknown</td>
<td>– No decision</td>
<td>– No decision</td>
</tr>
<tr>
<td>– (contingent production and capacity decisions)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Time 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– Price in spot market is known</td>
<td>– Time 1</td>
<td>– Time 1</td>
</tr>
<tr>
<td>– Production decision implemented</td>
<td>– Routing decision</td>
<td>– Production decision</td>
</tr>
<tr>
<td>– Sell surplus capacity</td>
<td>– Sell spare capacity</td>
<td>– Buy capacity from ISO and/or large producers</td>
</tr>
<tr>
<td>– Buy addtional capacity</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**KKT- conditions for $n$ two-stage stochastic programs**

**KKT - conditions for $s$ deterministic optimization problems (this is a wait and see problem)**

**1st order optimality conditions (wait and see)**
Network used in the analysis

- 2 production nodes
- 1 junction node
- 2 market nodes
- 2 large producers ($L_1$ and $L_2$) are present in both production nodes
- 1 competitive fringe in each production node

- Model solved by Path (to find VI solution)
  - Dual variables for the common constraints are the same for all players
## Case 1: Results

<table>
<thead>
<tr>
<th></th>
<th>Max social surplus (MSS)</th>
<th>Max value (MVF)</th>
<th>Max flow (MF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive fringe $g_1$ (NOK)</td>
<td>258.49</td>
<td>194.79</td>
<td>222.22</td>
</tr>
<tr>
<td>Competitive fringe $g_2$ (NOK)</td>
<td>704.17</td>
<td>704.17</td>
<td>704.17</td>
</tr>
<tr>
<td>Producer 1 (NOK)</td>
<td>3085.89</td>
<td>2595.33</td>
<td>3129.77</td>
</tr>
<tr>
<td>Producer 2 (NOK)</td>
<td>2435.35</td>
<td>2282.92</td>
<td>2410.73</td>
</tr>
<tr>
<td>ISO profit (NOK)</td>
<td>678.15</td>
<td>763.51</td>
<td>638.5</td>
</tr>
<tr>
<td>Social surplus (NOK)</td>
<td>7162.05</td>
<td>6540.72</td>
<td>7105.39</td>
</tr>
<tr>
<td>Flow ($m^3$)</td>
<td>80.32</td>
<td>88.85</td>
<td>76.35</td>
</tr>
<tr>
<td>Value of flow (NOK)</td>
<td>11668.39</td>
<td>12870.88</td>
<td>11207.20</td>
</tr>
</tbody>
</table>

- Benchmark 7220,43
- Difficult to interpret the max flow solution because of VI solution
- MSS gives the largest total surplus in the network
- MV gives the largest value of flow
Case 1: Changed weighting for MF

- Corresponds to a change in currency from \((1/100)\) NOK to EUR

<table>
<thead>
<tr>
<th></th>
<th>Max flow (MF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive fringe (g_1) (NOK)</td>
<td>174.18</td>
</tr>
<tr>
<td>Competitive fringe (g_2) (NOK)</td>
<td>704.17</td>
</tr>
<tr>
<td>Producer 1 (NOK)</td>
<td>2328.40</td>
</tr>
<tr>
<td>Producer 2 (NOK)</td>
<td>2097.78</td>
</tr>
<tr>
<td>ISO profit (NOK)</td>
<td>785.86</td>
</tr>
<tr>
<td>Social surplus (NOK)</td>
<td>6090.39</td>
</tr>
<tr>
<td>Flow ((Sm^3))</td>
<td>91.09</td>
</tr>
<tr>
<td>Value of flow (NOK)</td>
<td>13191.20</td>
</tr>
</tbody>
</table>
Case 3: The effect of stochasticity

- What is the difference between a stochastic and a deterministic setting?
  - Cost of uncertainty
  - Wait-and-see solution (WSS)
  - Expected value of perfect information (EVPI)
  - Value of stochastic solution (VSS)

- 15 scenarios
Case 3: stochastic solution and WSS

<table>
<thead>
<tr>
<th></th>
<th>Booking limit = $+\infty$</th>
<th>Wait-and-see solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max social surplus</td>
<td>Max value</td>
</tr>
<tr>
<td>Competitive fringe $g_1$</td>
<td>441.91</td>
<td>420.63</td>
</tr>
<tr>
<td>Competitive fringe $g_2$</td>
<td>621.50</td>
<td>637.82</td>
</tr>
<tr>
<td>Producer 1 (NOK)</td>
<td>3180.73</td>
<td>3275.30</td>
</tr>
<tr>
<td>Producer 2 (NOK)</td>
<td>2927.52</td>
<td>2985.92</td>
</tr>
<tr>
<td>ISO profit (NOK)</td>
<td>1505.52</td>
<td>1155.55</td>
</tr>
<tr>
<td>Social surplus (NOK)</td>
<td>8677.18</td>
<td>8475.30</td>
</tr>
<tr>
<td>Flow ($m^3$)</td>
<td>94.43</td>
<td>99.84</td>
</tr>
<tr>
<td>Value of flow (NOK)</td>
<td>14522.37</td>
<td>15040.02</td>
</tr>
</tbody>
</table>

- The EVPI is large for the MSS formulation
- Benchmark is 9008.59
  - In the WSS solution the distance to the benchmark is only 1.1%
Case 3: Expected result of using the expected value solution

<table>
<thead>
<tr>
<th></th>
<th>Max social surplus</th>
<th>Max value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competitive fringe $g_1$ (NOK)</td>
<td>420.73</td>
<td>420.63</td>
</tr>
<tr>
<td>Competitive fringe $g_2$ (NOK)</td>
<td>613.84</td>
<td>696.77</td>
</tr>
<tr>
<td>Producer 1 (NOK)</td>
<td>3262.36</td>
<td>3216.85</td>
</tr>
<tr>
<td>Producer 2 (NOK)</td>
<td>2994.11</td>
<td>2892.63</td>
</tr>
<tr>
<td>ISO profit (NOK)</td>
<td>1235.14</td>
<td>1236.56</td>
</tr>
<tr>
<td>Social surplus (NOK)</td>
<td>8526.18</td>
<td>8463.44</td>
</tr>
<tr>
<td>Flow ($m^3$)</td>
<td>95.84</td>
<td>99.84</td>
</tr>
<tr>
<td>Value of flow (NOK)</td>
<td>14587.55</td>
<td>15040.02</td>
</tr>
</tbody>
</table>

- We first solve a deterministic problem where the stochastic parameters were replaced with their expected values (EVP)
- We then fixed the first stage decisions from the EVP solution and solved the stochastic problem
- For the MSS formulation: the stochastic first stage solution is 1.77% better than the EVP first stage solution (in the stochastic problem)
Conclusions

• Both the MSS and MVF have meaningful interpretation when finding a VI solution. The max flow formulation only when we scale down the other players objectives!

• The inclusion of stochasticity leads to inefficiencies in the network
  – Both social surplus and surplus for the large producers are affected
  – The flow is higher in the stochastic setting than in the wait and see solution where booking rights are exercises just before production takes place.