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A stochastic generalized nash model for natural gas transport systems

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Outline

- Optimization models versus equilibriums
- Spatial equilibrium models
- A stochastic complementarity model

Mathematical programming

- Maximization or minimization of a real function by choosing values of variables from within an allowed set

$$\max_{x} f(x)$$
$$s.t. \;\; x \in X$$

- A large number of problem classes:
  - Linear programming
  - Nonlinear programming
  - Integer programming
  - Stochastic programming, etc.

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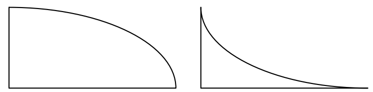
Optimality conditions

- Optimization problem with inequalities and equalities

$$\max_x f(x)$$
$$g(x) \leq 0$$
$$h(x) = 0$$

- Given that we have a convex problem the first-order conditions of optimality (KKT) is sufficient
  - Convex feasible set
  - Concave objective function for maximization / convex objective function for minimization



$$\Delta f(x) + \Delta g(x)^T \lambda + \Delta h(x)^T \mu = 0$$
$$0 \geq g(x) \ \perp \ \lambda \geq 0$$
$$0 = h(x)$$

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Complementarity problems

- Linear system of equations
- Nonlinear system of equations
- Linear complementarity problem
- Nonlinear complementarity problem
- Nonlinear program
- Finite-dimensional system of variational inequalities

- All of these problems can be generalized to Mixed complementarity problems

The linear complementarity problem

- Problem statement
- Find a vector x such that

$$
\begin{aligned}
x &\geq 0 \\
q + Mx &\geq 0 \\
x^T (q + Mx) &= 0
\end{aligned}
$$

- For a given vector q and matrix M
- Denoted: LCP(q,M)

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Nonlinear complementarity problem (NCP)

- Find a vector x such that:

$$x \geq 0$$
$$F(x) \geq 0$$
$$x^T F(x) = 0$$

- Applications:
  - General equilibrium theory of economics, policy design and analysis, game theory, mechanics, etc.

Mixed Complementarity Problem

- Many practical applications generate problems where some of the variables are nonnegative, others are bounded and others are free
  - To accommodate this flexibility, the MCP is used
- Find a vector $x \in [l, u]$ such that

$$x_i = l_i \quad \textit{and} \quad F_i(x) > 0,$$
$$x_i = u_i \quad \textit{and} \quad F_i(x) < 0,$$
$$x_i \in \langle l_i, u_i \rangle \quad \textit{and} \quad F_i(x) = 0.$$

Mixed Complementarity Problem

- Example (KKT for an optimization problem):

$$\Delta\phi(x) + \Delta g(x)^T \lambda + \Delta h(x)^T \mu = 0$$
$$0 \geq g(x) \perp \lambda \geq 0$$
$$0 = h(x)$$

- Complementary pairs of variables:
  - Economics: the price of a commodity and excess supply
  - Contact mechanics: the contact force between two variables and the distance between them
- MCP appear in study of equilibrium problems
  - Numerous applications (economics, engineering and chemistry)

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Terminology

- A feasible x satisfies the inequalities
  - If z strictly satisfies the inequalities, it is called strictly feasible
- The set of feasible vectors is called its feasible region and is denoted FEA(q,M)
- A vector x satisfying the complementarity condition is called complementary
- The CP is then to find a x that is both feasible and complementary

Complementarity Problems versus Optimization

  - Optimization problems (via KKT-conditions)
  - Game theory problems (for instance Nash-Cournot games)
  - Many other problems in engineering and economics
- Theorems and algorithms developed for CP can be applied to a large number of applications
- CPs can include problems where dual prices (Lagrangean multipliers) appear in the primal formulation

Equilibrium Problems

- Equilibrium is a stable situation in which forces cancel one another
  - Economics: supply equals demand
  - Chemistry: the forward rate and reverse rate of reaction is equal
  - Physics: all forces acting on an object are balanced
  - Game theory: Nash equilibrium (situation where no player has an incentive to deviate from his strategy unilaterally)

$$\forall i, \; f_i\left(q_i^*, q_{-i}^*\right) \geq f_i\left(q_i, q_{-i}^*\right)$$

- Formulation of equilibria:
  - Normally formulated as MCP or more generally as a Variational Inequality (VI)
  - The VI is a unifying methodology for the study of equilibrium systems

Spatial equilibrium models

- Consider at network with
  - A set of suppliers (I), supplies a_i
  - A set of markets (J), demands b_j
  - Transport costs of c_{ij}
- Want to find a transportation schedule which minimizes the cost of supplying all markets:

$$\begin{aligned} \min \sum_{ij} c_{ij} x_{ij} & \\ s.t. \quad \sum_j x_{ij} &\le a_i \\ \sum_i x_{ij} &\ge b_j \\ x &\ge 0 \end{aligned}$$

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Example (cont.)

- Can be interpreted as a market equilibrium problem
  - The dual multiplier for the supply constraint represents the price in the supply markets (w_i)
  - The dual multiplier for the demand constraint represents the price in the demand markets (p_j)
- We can then formulate the equilibrium conditions in the following way (LCP):

$$\sum_j x_{ij} \leq a_i,\ \ w_i \geq 0,\ \ w_i\left(a_i - \sum_j x_{ij}\right) = 0$$

$$\sum_i x_{ij} \geq b_j,\ \ p_j \geq 0,\ \ p_j\left(b_j - \sum_i x_{ij}\right) = 0$$

$$w_i + c_{ij} \geq p_j,\ \ x_{ij} \geq 0,\ \ x_{ij}\left(w_i + c_{ij} - p_j\right) = 0$$

Example (cont.)

- So far we have assumed constant demand and supply
  - Now suppose that demand and supplies are price responsive
    - All markets are perfectly competitive
- An associated optimization problem can be used to compute the equilibrium prices and quantities
- Here, formulated as an NCP

Example (cont.)

- We would then get the following equilibrium conditions:

$$\sum_j x_{ij} \leq a_i,\ \ w_i(a_i) \geq 0,\ \ w_i(a_i)\left(a_i - \sum_j x_{ij}\right) = 0$$

$$\sum_i x_{ij} \geq b_j,\ \ p_j\left(b_j\right) \geq 0,\ \ p_j\left(b_j\right)\left(\sum_i x_{ij} - b_j\right) = 0$$

$$w_i(a_i) + c_{ij} \geq p_j\left(b_j\right),\ \ x_{ij} \geq 0,\ \ x_{ij}\left(w_i(a_i) + c_{ij} - p_j\left(b_j\right)\right) = 0$$

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CP or optimization problem

- So far the spatial equilibrium model could have been formulated as an optimization problem
- What if we introduce competition in a spatial equilibrium model?

Liberalized network industry

- Telecoms, energy, natural gas, railway, mail, ...
- Vertical separation
  - upstream market: network infrastructure (natural monopoly characteristics)
  - downstream market: sales to customers
- Many actors in downstream market:
  - use network infrastructure (lease, buy access, ...)
  - provide products, value-added services, ...
  - compete for customers
  - often also subsidiary of upstream actor (separated former monopolist)
- Downstream actors act egoistically:
  make decisions which are best for them – and not for the industry (or social surplus etc.)

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Formulating and solving CPs

- Several articles explains how GAMS can be used to formulate CP
  - *Extension of GAMS for complementarity problems arising in applied economic analysis*, Journal of Economic Dynamics and Control, Rutherford, 1995
  - *Traffic Modeling and Variational Inequalities using GAMS*, Dirkse and Ferris, 1997
  - *Complementarity Problems in GAMS and the PATH solver*, Ferris and Munson, 1998
- Solvers: PATH and MILES
- In addition: AMPL with KNITRO

Example from the North sea gas pipeline network

A stochastic complementarity model

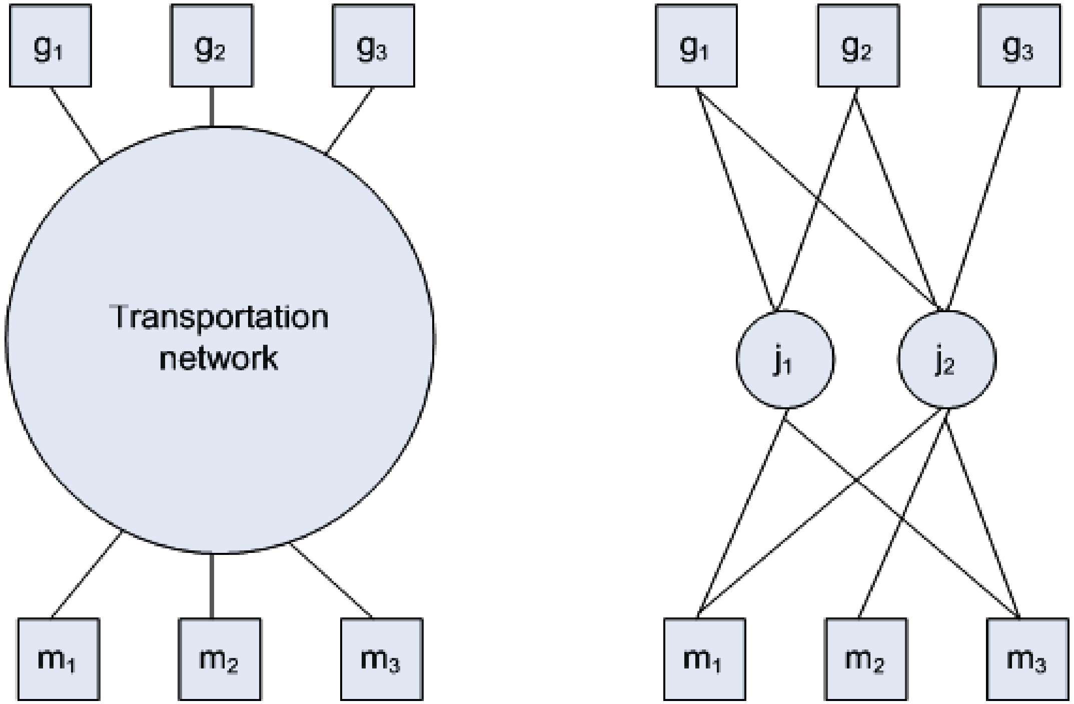
- Decision structure
- Notation & model
- Generalized Nash Equilibrium
- Case results

The North-Sea Case

- Production nodes (with gas fields), transportation nodes, market nodes

- Roles: large producers, smaller producers (modelled as a competitive fringe), Gassco (independent system operator).

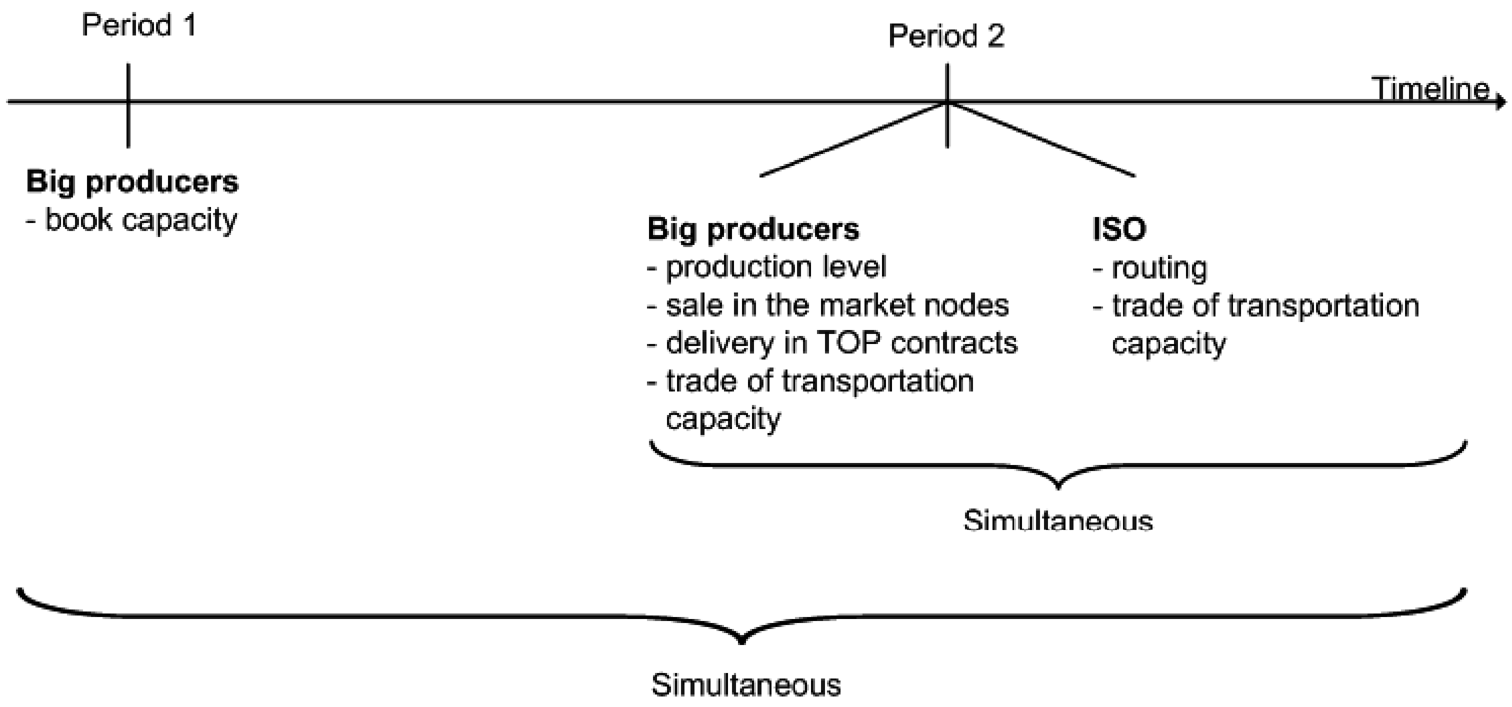
Representation of transport network



Transportation market structure

- Primary market
  - Large producers have capacity booking rights
    - Booking rights are more than 2 times the overall capacity
    - Conflicts resolved using Capacity Allocation Key
    - Tariff is fixed
- Secondary markets
  - ISO releases any available capacity
  - Bilateral trades of capacity between players
  - Price is negotiated
  - A competitive fringe clears the secondary market

The decision structure

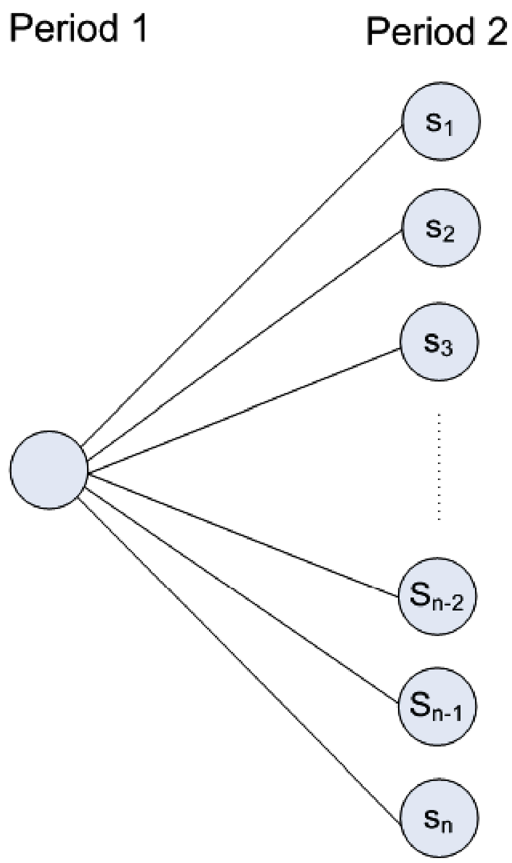


Large producers decision problem

- Each large producer faces a two-stage stochastic program with recourse

- This is still a one level game because the contingent strategy is laid at the time of booking and not changed as a result of the other players bookings.

- Stochastic parameters:
  - Spot price
  - TOP volumes



Purpose of the analysis

- Investigate the effect of different objectives for the ISO
  - Max flow, max value and max social surplus

- Analyze the effect of stochasticity

Price in the secondary market

- Demand from a competitive fringe in each field g
  - Comes from the profit maximization of the competitive fringe:

$$\Pi_{gs} = \max \sum_{m \in \mathcal{M}} (p_{ms} \cdot x_{gms} - t_{gms} \cdot x_{gms}) - W_g \left(\sum_{m \in \mathcal{M}} x_{gms} \right)$$

- The first order condition of optimality is:

$$\frac{\delta \Pi_{gs}}{\delta x_{gms}} = p_{ms} - t_{gms} - \frac{\delta W_g \left(\sum_{m \in \mathcal{M}} x_{gms} \right)}{\delta x_{gms}} = 0, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}$$

- We assume that the cost function (W()) is quadratic. The inverse demand function can then be formulated as:

$$t_{gms} = p_{ms} - c_g \sum_{m' \in \mathcal{M}} x_{gm's}, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}$$

Large producers objective function

Income from the spot markets and delivery in the TOP contracts

Cost of booking in the primary market

$$\Pi_l = \max - \sum_{g\in\mathcal{G}} \sum_{m\in\mathcal{M}} T_{gm} b_{lgm} + \sum_{s\in\mathcal{S}} \phi_s \left[\sum_{m\in\mathcal{M}} (p_{ms} q_{lms} + P_{lm} v_{lms}) \right]$$
$$+ \sum_{s\in\mathcal{S}} \phi_s \left[\sum_{g\in\mathcal{G}} \sum_{m\in\mathcal{M}} h_{lgms} \left(p_{ms} - c_g \left(\sum_{m'\in\mathcal{M}} \left(z_{gm's} + \sum_{l'\in\mathcal{L}} h_{l'gm's} \right)\right)\right)\right]$$
$$- \sum_{s\in\mathcal{S}} \phi_s \left[\sum_{g\in\mathcal{G}} C_{lg}(d_{lgs}) \right]$$

Surplus from trade in the secondary market for transportation capacity

Production cost

Large producers constraints

$$
\begin{aligned}
& b_{lgm} \le B_{lgm}, \quad g \in \mathcal{G},\ m \in \mathcal{M}, \\
& d_{lgs} = \sum_{m \in \mathcal{M}} \left(b_{lgm} - h_{lgms} \right), \quad g \in \mathcal{G},\ s \in \mathcal{S}, \\
& q_{lms} + v_{lms} = \sum_{g \in \mathcal{G}} \left(b_{lgm} - h_{lgms} \right), \quad m \in \mathcal{M},\ s \in \mathcal{S}, \\
& h_{lgms} \le b_{lgm}, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}, \\
& z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \ge 0, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}.
\end{aligned}
$$

ISO - objective function (1)

- Maximize flow (MF):

$$\max \sum_{m \in \mathcal{M}} \sum_{i \in \mathcal{I}(m)} f_{ims}$$

ISO – objective function (2)

- Maximize value of flow (MVF):

$$\max \sum_{m \in \mathcal{M}} \sum_{i \in \mathcal{I}(m)} p_{ms} \left(f_{ims} - \sum_{l \in \mathcal{L}} v_{lms} \right)$$

ISO - objective function (3)

- Maximize social surplus (MSS):

$$\max \sum_{m\in\mathcal{M}} \sum_{i\in\mathcal{I}(m)} p_{ms} \left(f_{ims} - \sum_{l\in\mathcal{L}} v_{lms} \right) + \sum_{m\in\mathcal{M}} \sum_{l\in\mathcal{L}} P_{lm} v_{lms}$$
$$-\frac{1}{2} \sum_{g\in\mathcal{G}} MC_g \left(\sum_{i\in\mathcal{O}(g)} f_{gi} \right)^2$$

The slope in the linear, aggregated supply function is given as:

$$MC_g = \frac{1}{\sum_{l\in\widetilde{\mathcal{L}}_g} \frac{1}{2c_{lg}}}$$

ISO – some of the constraints

- Conservation of mass for the field nodes

$$\sum_{j \in \mathcal{O}(g)} f_{gjs} = \sum_{m \in \mathcal{M}} \left(z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \quad g \in \mathcal{G},\ s \in \mathcal{S}$$

- Conservation of mass for the junction nodes

$$\sum_{g \in \mathcal{I}(j)} f_{gjs} = \sum_{m \in \mathcal{O}(j)} f_{jms}, \quad j \in \mathcal{J},\ s \in \mathcal{S}$$

- Conservation of mass for the market nodes

$$\sum_{n \in \mathcal{I}(m)} f_{nms} = \sum_{g \in \mathcal{G}} \left(z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \quad m \in \mathcal{M},\ s \in \mathcal{S}$$

ISO

- Positive price in the secondary market

$$p_m - c_g \left(\sum_{m' \in \mathcal{M}'} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right) \geq 0, \quad g \in \mathcal{G}, \; m \in \mathcal{M}$$

Benchmark

- The ISO schedules production, routing and sale in order to maximize the social surplus of all the players in the network
- Objective function:

$$\max \sum_{m\in\mathcal{M}} \sum_{l\in\widetilde{\mathcal{L}}} (p_{ms}q_{lms} + P_{lms}v_{lms}) - \sum_{g\in\mathcal{G}} \sum_{l\in\widetilde{\mathcal{L}}} \frac{1}{2} MC_{lg} d_{lgs}^2$$

- Constraints:

$$\sum_{l\in\widetilde{\mathcal{L}}} d_{lgs} = \sum_{j\in\mathcal{O}(g)} f_{gjs}, \quad g \in \mathcal{G}, \quad s \in \mathcal{S}$$

$$\sum_{g\in\mathcal{G}} f_{gjs} = \sum_{m\in\mathcal{M}} f_{jms}, \quad j \in \mathcal{J}, s \in \mathcal{S}$$

$$\sum_{l\in\widetilde{\mathcal{L}}} (q_{lms} + v_{lms}) = \sum_{j\in\mathcal{I}(m)} f_{jms}, \quad m \in \mathcal{M}, \quad s \in \mathcal{S}$$

Generalized Nash Equilibrium games

A generalized Nash equilibrium (GNE) is defined as a point $x^* \in X$ that simultaneously optimizes all the players individual decision problems so that: $x_l^* \in K_l(x^*_{-l}),\ l \in \bar{L}$ and $\Pi_l(x^*) \geq \Pi_l(x_l, x^*_{-l}),\ x_l \in K_l(x^*_{-l}), l \in \bar{L}$ where $\Pi : R^{\alpha\beta} \longrightarrow R$ is the objective function of player l.

Common constraints

- The common constraints in our model
  - Defined as constraints where decision variables from more than one player appear

$$z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0 \qquad \tau_{gms}$$

$$\sum_{j \in \mathcal{O}(g)} f_{gjs} = \sum_{m \in \mathcal{M}} \left(z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \qquad u_{gjs}$$

$$\sum_{n \in \mathcal{I}(m)} f_{nms} = \sum_{g \in \mathcal{G}} \left(z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \qquad u_{jms}$$

$$p_m - c_g \left(\sum_{m' \in \mathcal{M}'} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right) \geq 0, \quad \chi_{gms}$$

(Quasi) Variational inequalities

Following the lines of the discussion in Harker (1991), we define $F_l(x^*) = \nabla_{x_l} \Pi_l(x_l^*, x_{-l}^*)$ and $F(x^*) = (F_0(x^*)^T, \ldots, F_{|L|}(x^*)^T)^T$ Then the GNE may be expressed as the Quasi Variational Inequality $QVI(F, K(x))$:

$$F(x^*)^T(x - x^*) \geq 0, \quad x \in K(x^*), \qquad (1)$$

where $K(x) = \prod_{l \in \bar{L}} K_l(x_{-l})$.

The VI solution

- Theorems 4-6 from Harker (1991)

If F is a continuous function in the $VI(F, K)$ then the VI solutions are the only points in the solution set of the $QVI(F, K(x))$ at which the optimal dual variables $\lambda^* \in R^{p\beta}$ for the common constraints are such that $\lambda_0^* = \lambda_j^*,\ j \in \overline{L}$. The theorems also state that any strictly interior solution (for the common constraints) of the $QVI(F, K(x))$ is a solution to the $VI(F, K)$. Further, if F is strictly monotone there is a unique solution to the VI over X, Facchinei and Pang (2003), Theorem 2.3.3.

The complementarity program

Large producer *n*

- Time 0
  - Booking capacity
  - Price in spot market is unknown
  - (contingent production and capacity decisions)
  -
- Time 1
  - Price in spot market is known
  - Production decison imlemented
  - Sell surplus capacity
  - Buy addtional capacity

Independent System Operator (ISO)

Time 0
  - No decision

- Time 1
  - Routing decision
  - Sell spare capacity

Competitive fringe

- Time 0
  - No decision
- Time 1
  - Production decision
  - Buy capacity from ISO and/or large producers

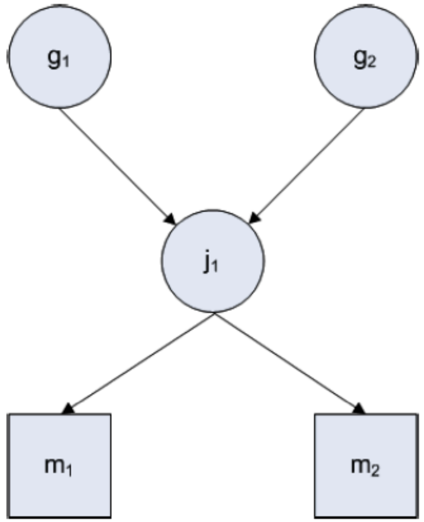
KKT- conditions for *n* two-stage stochastic programs

KKT - conditions for *s* deterministic optimization problems (this is a wait and see problem)

1st order optimality conditions (wait and see)

Network used in the analysis

- 2 production nodes
- 1 junction node
- 2 market nodes
- 2 large producers (L_1 and L_2) are present in both production nodes
- 1 competitive fringe in each production node



- Model solved by Path (to find VI solution)
  – Dual variables for the common constraints

are the same for all players

Case 1: Results

	Max social surplus (MSS)	Max value (MVF)	Max flow (MF)
Competitive fringe g_1 (NOK)	258.49	194.79	222.22
Competitive fringe g_2 (NOK)	704.17	704.17	704.17
Producer 1 (NOK)	3085.89	2595.33	3129.77
Producer 2 (NOK)	2435.35	2282.92	2410.73
ISO profit (NOK)	678.15	763.51	638.5
Social surplus (NOK)	7162.05	6540.72	7105.39
Flow (Sm^3)	80.32	88.85	76.35
Value of flow (NOK)	11668.39	12870.88	11207.20

- Benchmark 7220,43
- Difficult to interpret the max flow solution because of VI solution
- MSS gives the largest total surplus in the network
- MV gives the largest value of flow

Case 1: Changed weighting for MF

- Corresponds to a change in currency from (1/100) NOK to EUR

	Max flow (MF)
Competitive fringe g_1 (NOK)	174.18
Competitive fringe g_2 (NOK)	704.17
Producer 1 (NOK)	2328.40
Producer 2 (NOK)	2097.78
ISO profit (NOK)	785.86
Social surplus (NOK)	6090.39
Flow (Sm^3)	91.09
Value of flow (NOK)	13191.20

Case 3: The effect of stochasticity

- What is the difference between a stochastic and a deterministic setting?
  - Cost of uncertainty
  - Wait-and-see solution (WSS)
  - Expected value of perfect information (EVPI)
  - Value of stochastic solution (VSS)

  - 15 scenarios

Case 3: stochastic solution and WSS

	Booking limit $= +\infty$		Wait-and-see solution	
	Max social surplus	Max value	Max social surplus	Max value
Competitive fringe g_1	441.91	420.63	453.98	581.19
Competitive fringe g_2	621.50	637.82	726.52	786.71
Producer 1 (NOK)	3180.73	3275.30	3666.62	3462.53
Producer 2 (NOK)	2927.52	2985.92	3303.10	3124.28
ISO profit (NOK)	1505.52	1155.55	758.84	765.28
Social surplus (NOK)	8677.18	8475.30	8909.06	8719.99
Flow (Sm^3)	94.43	99.84	84.93	96.89
Value of flow (NOK)	14522.37	15040.02	13660.29	14830.25

- The EVPI is large for the MSS formulation
- Benchmark is 9008,59
  - In the WSS solution the distance to the benchmark is only 1.1%

Case 3: Expected result of using the expected value solution

	Max social surplus	Max value
Competitive fringe g_1 (NOK)	420.73	420.63
Competitive fringe g_2 (NOK)	613.84	696.77
Producer 1 (NOK)	3262.36	3216.85
Producer 2 (NOK)	2994.11	2892.63
ISO profit (NOK)	1235.14	1236.56
Social surplus (NOK)	8526.18	8463.44
Flow (Sm^3)	95.84	99.84
Value of flow (NOK)	14587.55	15040.02

- We first solve a deterministic problem where the stochastic parameters were replaced with their expected values (EVP)
- We then fixed the first stage decisions from the EVP solution and solved the stochastic problem
- For the MSS formulation: the stochastic first stage solution is 1,77% better than the EVP first stage solution (in the stochastic problem)

Conclusions

- Both the MSS and MVF have meaningful interpretation when finding a VI solution. The max flow formulation only when we scale down the other players objectives!

- The inclusion of stochasticity leads to inefficiencies in the network
  - Both social surplus and surplus for the large producers are affected
  - The flow is higher in the stochastic setting than in the wait and see solution where booking rights are exercises just before production takes place.