Paper IV

Kjetil T. Midthun, Mette Bjørndal, Asgeir Tomasgard and Yves Smeers:

Capacity booking in a Transportation Network with stochastic demand and a secondary market for Transportation Capacity

Chapter 5

Capacity booking in a Transportation Network with stochastic demand and a secondary market for Transportation Capacity

Abstract:

We present an equilibrium model for transport booking in a gas transportation network. The booking regime is similar to the regime implemented in the North-Sea. The model looks at the challenges faced by the network operator in regulating such a system. There are some privileged players in the network, with access to a primary market for transportation capacity. The demand for capacity is stochastic when the booking in the primary market is done. There is also an open secondary market for transportation capacity where all players participate including a competitive fringe. We consider different objective functions for the network operator, and the difference between setting fixed capacities and modeling the pressure constraints in a sub-sea pipeline-network. This is modelled as a Generalized Nash Equilibrium using a stochastic complementarity problem.

5.1 Introduction

We study booking of transportation capacity in a natural gas network with several large players and a competitive fringe. The offshore pipeline system in the North-Sea provides a case for our analysis, but the model and results are interesting for natural gas transportation in general. There are two booking stages in the transport capacity market. In the first stage the large producers book capacity within their predefined capacity rights. In the second stage there is a redistribution of capacity in a bilateral secondary market, where also the competitive fringe participates. Here the network operator can sell remaining capacity in the system, and capacity bought in the first-stage primary market can be sold by the producers.

The purpose of the paper is to develop a model that can be used to analyze how different objective functions for the system operator affect the efficiency of the transportation system. We also investigate the effect of using different model representations of the physical properties of the transport network. Another interesting topic is how stochasticity in the price for natural gas influences our results. The model is based on Generalized Nash Equilibrium and is represented as a stochastic complementarity problem. To our knowledge this is the first time the booking system for natural gas transportation is studied using this approach.

The network operator influences the efficiency in the network through the routing. The routing decisions will also determine the capacity sold in the secondary market. This is different from the role of the network operator compared to the articles studying electricity networks, by for instance Yao et al. (2004) and Hu et al. (2004) where the network operator choose the production from each producer in order to maximize social surplus. In the North-Sea, the network operator acts as a neutral third party.

We formulate the model as a mixed complementarity problem, see for example Ferris & Pang (1997) and Facchinei & Pang (2003). A path-breaking paper for the use of complementarity problems modelling economic equilibrium was Lemke & Howson (1964). In the energy sector there are numerous examples of papers using complementarity problems to model and solve economic equilibria. Gabriel, Zhuang & Kiet (2005) presents a linear complementarity equilibrium model for the North American natural gas market. Gabriel, Kiet & Zhuang (2005) presents a multi-seasonal, multiyear mixed nonlinear complementarity problem of natural gas markets. Smeers (2003*a*) and Smeers (2003*b*) discuss the deregulation of the electricity markets and the organization of regional electricity transmission. In Jing-Yuan & Smeers (1999) spatial oligopolistic electricity models are given and Generalized Nash Equilibria are found in a system with Cournot generators and regulated transmission prices. Yao et al. (2006) presents a model of two-settlement electricity markets using an Equilibrium Problems with Equilibrium Constraints (EPEC). Hu et al. (2004) model strategic bidding by generators to an ISO that is maximizing social surplus. The loop flow is taken into consideration and shown to be important for the results. The model is an EPEC solved as an All-KKT system in PATH. Hobbs (2001) presents Cournot models of bilateral power markets.

In Section 5.2 we discuss the background for this article, as well as the assumptions we have made. The model formulation as a stochastic Mixed Complementarity Problem is presented in Section 5.3. A more detailed description of the equilibrium conditions is given in Appendix 5.A. The properties of the model are discussed in Section 5.4. We then move on to some numerical examples in Section 5.5. Finally, the conclusions are given in Section 5.6.

5.2 Problem description and assumptions

We present here the ideas and motivation for our case analysis, the assumptions we have made and the reason for introducing them.

System in the North-Sea

We study a system with field nodes, each with a set of large producers in addition to a competitive fridge. The producers deliver natural gas into a transportation network passing through junction nodes and ending in market nodes. The market for capacity in this network is managed by an independent system operator (ISO) named Gassco. The producers book transport capacity from field to market and can not determine the actual routing of the gas through the network. The routing is the responsibility of the ISO. The image on the left in Figure 5.1 illustrates the point-to-point perspective of the producers. The transportation network can be considered as a black box for the producers. The system operator operate the network taking into account the details in the network, as illustrated in the image on the right in Figure 5.1.

At the Norwegian Continental Shelf (NCS) capacity distribution is done in a primary market, and the remaining capacity after this initial distribution is handled through a secondary market. In the secondary market, both transactions of capacity facilitated by the ISO and bilateral transactions between shippers are included. The secondary market is open to all qualified shippers. Only the large producers book capacity in the primary market limited by predefined capacity rights. This booking right depends on their need to transport induced by the TOP contracts. The actual demand for capacity due to the TOP contracts is uncertain until delivery. In sum, the available capacity in the primary market is actually larger than the total capacity in the network. If a conflict arises with respect to over-booking, a capacity allocation key is used to resolve these matters. We have not explicitly modeled this rule in this paper.

In addition to the long term contracts for gas in the markets nodes, there are short-term markets where gas may be sold. In this article we have assumed that the producers may act strategically in the transport capacity market, but that they are price-takers in the spot markets in the market nodes. This is reasonable as Norway's overall production is around 15% of the European consumption of natural gas. The main market hubs are in UK, Germany, France and Belgium. In the market hubs there are large buyers of natural gas who distribute the gas further to either the suppliers or end-customers. In our model, the analysis ends at the market hubs. For details regarding the liberalization of the European gas market see European Union (1998) and European Union (2003), and for details on the Norwegian case, see Austvik (2003).

The purpose of our model is mainly to analyze the effect different objectives of the ISO will have on the operation of the system. The price the ISO can take is regulated and fixed both in the primary and in the secondary market, so its decision variables are only routing and secondary market sales of available capacity. If we represent the ISO with a feasibility problem, the corresponding game will have an infinite amount of equilibria. For each choice of secondary market sales from the ISO, a solution satisfying the large producers' equilibrium conditions can be found. Hence we focus on the following alternatives: max flow, max value of flow and max social surplus. In the following we assume that the ISO does not have economic interests in the routing, and acts as a benevolent central planner.

We also investigate how the representation of the physical networks as well as the booking rights in the primary markets will influence the efficiency of the network.

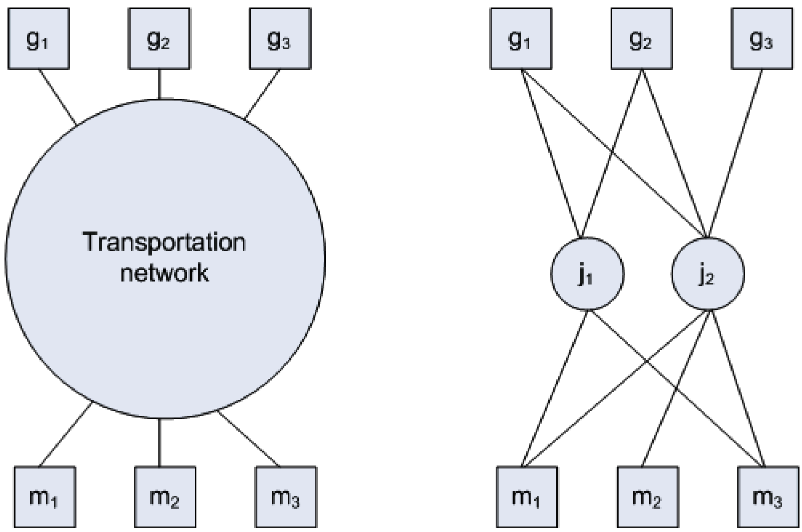

Figure 5.1: The field nodes are denoted by g, junction nodes by j and market nodes by m. The gas flows from top to bottom.

Second-stage decision structure

Our model is a one level game where each of the producers decison problem is a stochastic two-stage program with recourse (Kall & Wallace 1994). The stochastic elements are the spot price in the markets and the quantity in the

TOP-contracts. The uncertainty is modeled with scenarios (see Figure 5.3). The decisions in the two stages are illustrated in Figure 5.2.

In the secondary market (in the second stage) we assume that the large producers and the competitive fringe make simultaneous volume decisions in a Cournot manner. Each of the large producers recognizes that they will influence the price for transportation capacity, but make independent volume decisions. The players in the competitive fringe are price takers in the capacity market. Their reaction function is expressed as their demand for transportation capacity at a given transportation price. This demand is positive as long as the market price for natural gas in a market hub is higher than the marginal production cost for the competitive fringe in a field node plus the transportation price from that node to the market.

Further, we assume that the ISO's decisions are made simultaneously with the producers. Hence, the ISO is a Cournot player whose volume decisions cannot be manipulated by other players strategically. An alternative would be to model this as a multi-leader one-follower Stackelberg game (Yao 2006) with the ISO as a follower. A common way of modeling this follower situation, when the ISO has a convex optimization problem, is by including the KKT-conditions for the ISO's routing and capacity release in the other players' optimization problem. They will then act strategically because they anticipate the ISO's reaction to their own volume decisions. In this case each player solves a mathematical program with equilibrium constraints (MPEC, Luo et al. (1996)) and the resulting game over all the players become an EPEC. In our approach we stay within the framework of Mixed Complementarity Problems as all decisions are simultaneous, and a common way of modeling this is to merge all the players KKT-conditions into a large complementarity system. We think that the setting with simultaneous decisions is closer to the reality of the Norwegian continental shelf. Firstly, the players are not supposed to act strategically, for example in terms of influencing the ISO in the transportation market. Secondly, the other players never know or get information about the ISO's routing decisions. This is confidential information, and so are the booking requests, sales and production volumes of the other players.

First-stage decision structure

In the first stage each of the large producers decides on a booking volume. This booking decision is based on maximizing the excepted revenue for the second stage where production and transportation strategies are made as well as trades in the secondary market for transportation capacity.

We assume that each player makes his first-stage decisions and his second-stage decisions simultaneously. In practice this means that the second-stage decisions will depend on the outcomes of the stochastic variables, but the contingent strategy covering all possible outcomes is made before the player observes the other players booking. Each producer's optimization problem is then a stochastic two-stage program with recourse, given the other players fixed decisions. The overall problem is still a Mixed Complementarity Problem, often called a Stochastic Mixed Complementarity Problem because of the stochastic variables and two-stage structure.

If, on the other hand the booking decisions had been used strategically by the players, we would need to include the second-stage equilibrium over all the players as a part of the booking problem in the first stage for each player. Normally this is done by including the KKT-conditions from the second stage equilibrium in each player's first-stage optimization problem. In such a setting each player's problem would be a stochastic MPEC, where the second-stage equilibrium conditions for each scenario is part of the first-stage optimization problem and parameterized on the first stage decisions (Patriksson & Wynter 1999).

When the first- and second-stage decisions are made simultaneously we model the situation where either a player does not know the other players' booking decisions when he makes his second-stage decisions, or he has this booking information but does not let it influence his second stage decisions. In the Norwegian regime with a confidential booking process, we feel that this is a sound model. Then the only information revealed (or acted on) between the first and second stage is the uncertainty that is resolved. This is a one level game as the scenarios are independent of the first-stage decisions.

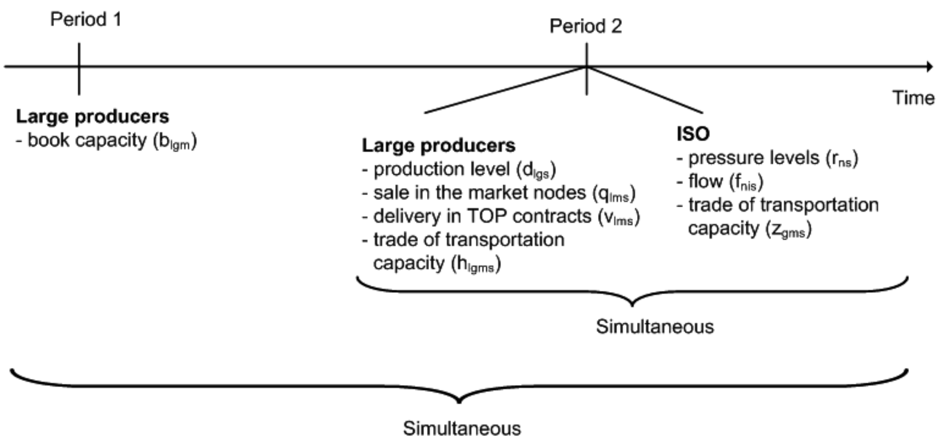

Figure 5.2: The sequencing of decisions.

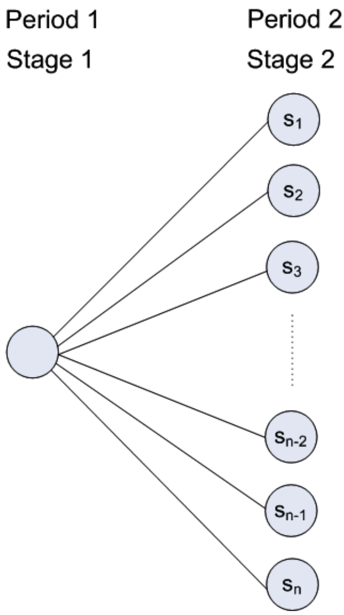

Figure 5.3: The scenario structure in the large producers' decision problem

5.3 Model

We start by introducing the notation. We then move on to a discussion of the price of transportation capacity in the secondary market. The networks we present are connected graphs.

Notation

Sets

$\mathcal{N}$	The set of all nodes in the network.
$\mathcal{G}$	The set of field nodes in the network.
$\mathcal{J}$	The set of junction nodes in the network.
$\mathcal{M}$	The set of market nodes in the network.
$\mathcal{I}(n)$	The set of nodes with pipelines going into node n (predecessor nodes).
$\mathcal{O}(n)$	The set of nodes with pipelines going out of node n (successor nodes).
$\mathcal{L}$	The set of large producers in the network.
$\widetilde{\mathcal{L}}_g$	The set of all producers in field g (including the competitive fringe).
$\mathcal{S}$	The set of scenarios.

Indexes

n	Used for nodes in general.
g	Index for field nodes.
j	Index for junction nodes.
m	Index for market nodes.
s	Scenario index.
l	The index used for producers.

Constants

$\overline{R}_n$	The maximum pressure in node n.
$\underline{R}_n$	The minimum pressure in node n.
K_{ij}	The Weymouth constant for the pipeline going from i to j.
B_{lgm}	Booking limit for producer l from field g to market m.
P_{lm}	Price in the long term contracts for producer l in market m.
T_{gm}	Per unit tariff for transportation between field g and market m.
MC_g	Aggregated marginal cost parameter in field g.
$\overline{C}_{ni}$	Capacity in the pipeline between node n and i.
c_g	Parameter in the cost function for the competitive fringe in field g.
c_{lg}	Parameter in the cost function for producer l in field g.

Decision variables

b_{lgm}	Booking in the primary market by producer l between field g and market m.
d_{lgs}	Production in field g by producer l in scenario s.
q_{lms}	Spot sale in market m by producer l in scenario s.
h_{lgms}	Capacity between g and m traded by producer l in the secondary market in scenario s.
f_{nis}	The flow between node n and i in scenario s.
r_{ns}	The pressure in node n in scenario s.
z_{gms}	Capacity sold by the ISO in the secondary market between field g and market m in scenario s.
t_{gms}	Price of transportation capacity between field g and market m in scenarios s.
x_{gms}	Quantity produced in field g and sold in market m in scenario s by the competitive fringe.

Stochastic variables and probabilities

v_{lms}	Nomination in long-term contracts in market m.
p_{ms}	Spot price in market m.
ϕ_s	Probability of a given scenario.

Functions

$C_{lg}(d)$	The cost function for producer l in field g.
$W_g(y)$	Cost function for the competitive fringe in field g.

Price of capacity in the secondary market

The price in the secondary market in a node is given by a demand function from the competitive fringe in that node. We assume that the competitive fringes in the different field nodes are independent. The competitive fringe's demand function for transportation capacity between field g and market m in scenario s is then found from the profit maximization problem for the competitive fringe in field g:

$$\Pi_{gs} = \max \sum_{m \in \mathcal{M}} (p_{ms} \cdot x_{gms} - t_{gms} \cdot x_{gms}) - W_g \left(\sum_{m \in \mathcal{M}} x_{gms} \right), \tag{5.1}$$

where x_{gms} is the quantity traded in spot market m by the competitive fringe in field g in scenario s, t_{gms} is the price of transportation capacity between g and m in the secondary market in scenario s. W_g is the cost function in field g. In order to find the demand function for the competitive fringe, the first order condition for optimality is used:

$$\frac{\partial \Pi_{gs}}{\partial x_{gms}} = p_{ms} - t_{gms} - \frac{\partial W_g \left(\sum_{m \in \mathcal{M}} x_{gms} \right)}{\partial x_{gms}} = 0, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}. \tag{5.2}$$

In this article, we assume that W_g is a quadratic function. For ease of presentation, we will in the following assume that the cost function for the competitive fringe is:

$$W_g \left(\sum_{m \in \mathcal{M}} x_{gms} \right) = \frac{1}{2} c_g \cdot \left(\sum_{m \in \mathcal{M}} x_{gms} \right)^2 \tag{5.3}$$

where c_g is the cost parameter for the competitive fringe in field g. Nevertheless, all results are valid for general quadratic cost functions (and most for a general cost function).

We model this implicitly in the large producers' problem as an elastic demand function. The inverse demand function is given as:

$$t_{gms} = p_{ms} - c_g \cdot \sum_{m' \in \mathcal{M}} x_{gm's}, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}. \tag{5.4}$$

The volume bought by the competitive fringe, x_{gms} is given as the sum of capacities sold by the ISO, z_{gms}, and the large producers, h_{lgms}. The h_{lgms} variable is positive when the large producers sell capacity, and negative if the large producers buy capacity. We then have the following relation between x_{gms}, z_{gms} and h_{lgms}:

$$x_{gms} = z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms}, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}, \tag{5.5}$$

which leads to the following expression for the price in the secondary market:

$$t_{gms} = p_{ms} - c_g \cdot \left(\sum_{m' \in \mathcal{M}} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right), \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}. \tag{5.6}$$

Since we only allow flow in one direction in our network, we need to make sure that x_{gms} cannot be negative.

$$z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}, \tag{5.7}$$

where h_{lgms} is the secondary market trades of producer l of capacity from g to m. The inclusion of this constraint means that the decision space for each producer depends on the other participants decisions (the other producers and the ISO).

The large producers

The income for the large producers ($\mathcal{L}$) in the network comes from deliveries in the long term contracts, sales in the spot markets and sales in the secondary market for transportation capacity. The cost for the producers come from the per unit tariff paid in the primary market for transportation capacity (which we assume is independent of the large producers' decisions), the cost of production and from purchasing additional transportation capacity in the secondary market. The objective function for producer l can be formulated as:

$$\begin{aligned}
\Pi_l = \max &- \sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} T_{gm} b_{lgm} + \sum_{s \in \mathcal{S}} \phi_s \left[\sum_{m \in \mathcal{M}} (p_{ms} q_{lms} + P_{lm} v_{lms}) \right] \\
&+ \sum_{s \in \mathcal{S}} \phi_s \left[\sum_{g \in \mathcal{G}} \sum_{m \in \mathcal{M}} h_{lgms} \cdot \left(p_{ms} - c_g \cdot \left(\sum_{m' \in \mathcal{M}} \left(z_{gm's} + \sum_{l' \in \mathcal{L}} h_{l'gm's} \right) \right) \right) \right] \\
&- \sum_{s \in \mathcal{S}} \phi_s \left[\sum_{g \in \mathcal{G}} C_{lg}(d_{lgs}) \right],
\end{aligned} \tag{5.8}$$

where b_{lgm} is the booking in the primary market, T_{gm} is the tariff in the primary market, ϕ_s is the probability of scenario s, p_{ms} is price in the spot market, q_{lms}

is volume sold in the spot market, P_{lm} is the price in the take-or-pay contracts, v_{lms} is the volume in the take-or-pay contracts, h_{lgms} is the capacity traded in the secondary market (positive when the producer sell capacity, negative when he buys), the price in the secondary market is given by (5.6), C_{lg} is the cost function for the producer and d_{lgs} is the production. z_{gms} is the capacity sold by the ISO in the secondary market.

The booking constraint in the primary market is given as:

$$b_{lgm} \leq B_{lgm}, \quad g \in \mathcal{G},\ m \in \mathcal{M}, \tag{5.9}$$

where B_{lgm} is the fixed upper limit on booking for the producer. For the second stage the following constraints are needed:

$$d_{lgs} = \sum_{m \in \mathcal{M}} \left(b_{lgm} - h_{lgms} \right), \quad g \in \mathcal{G},\ s \in \mathcal{S}, \tag{5.10}$$

$$q_{lms} + v_{lms} = \sum_{g \in \mathcal{G}} \left(b_{lgm} - h_{lgms} \right), \quad m \in \mathcal{M},\ s \in \mathcal{S}, \tag{5.11}$$

$$h_{lgms} \leq b_{lgm}, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}, \tag{5.12}$$

$$z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0, \quad g \in \mathcal{G},\ m \in \mathcal{M},\ s \in \mathcal{S}. \tag{5.13}$$

Constraint (5.10) make sure that the producer has booked enough capacity for the production in field g. Constraint (5.11) make sure that the producer has booked enough capacity for the total sale in market m. The two constraints also make sure that the producer utilizes all the booked capacity. Constraint (5.12) makes sure that the producer cannot sell more capacity than he has booked in the primary market, and constraint (5.13) ensures that the producers cannot buy more capacity than the ISO sells.

Independent system operator

We present three different objective function alternatives for the ISO:

- maximize flow (MF):

$$\Pi_s^{MF} = \max \sum_{m \in \mathcal{M}} \sum_{i \in \mathcal{I}(m)} f_{ims}, \tag{5.14}$$

The network operator will always choose z_{gms} in order to maximize the flow under the constraint that all prices (for field-market combinations) must be positive (see Equation (5.26)). With this objective, the system operator will be indifferent with regards to prices in the market nodes and cost functions in the field nodes.

- maximize value of flow (MVF):

$$\Pi_s^{MVF} = \max \sum_{m\in\mathcal{M}} \sum_{i\in\mathcal{I}(m)} p_{ms} \cdot \left(f_{ims} - \sum_{l\in\mathcal{L}} v_{lms} \right), \tag{5.15}$$

The strength of this formulation, MVF, compared with the MF formulation is that the ISO now routes the gas according to value. The weakness is that he has no incentive to route according to marginal cost in the fields.

If we assume that the network operator has full information regarding the cost functions of the participants, the ISO can take both value of flow and cost structure in the fields into account by maximizing social surplus.

- maximize social surplus (MSS):

$$\begin{aligned}\Pi_s^{MSS} = &\max \sum_{m\in\mathcal{M}} \sum_{i\in\mathcal{I}(m)} p_{ms} \cdot \left(f_{ims} - \sum_{l\in\mathcal{L}} v_{lms} \right) + \sum_{m\in\mathcal{M}} \sum_{l\in\mathcal{L}} P_{lm} v_{lms} \\ &- \frac{1}{2} \sum_{g\in\mathcal{G}} MC_g \cdot \left(\sum_{i\in\mathcal{O}(g)} f_{gi} \right)^2 . \end{aligned} \tag{5.16}$$

MC_g is the slope of the linear aggregated supply function for field g:

$$MC_g \cdot \sum_{i\in\mathcal{O}(g)} f_{gi}. \tag{5.17}$$

The supply function is found by assuming that all producers have a cost function of the form:

$$W_g = c_{lg} d_{lg}^2, \tag{5.18}$$

and that no production capacities exist. Under these assumptions, the aggregate supply function is linear. MC_g is found in the following manner:

$$MC_g = \frac{1}{\sum_{l\in\widetilde{\mathcal{L}}_g} \frac{1}{2c_{lg}}}, \tag{5.19}$$

where $\widetilde{\mathcal{L}}_g$ is the set of producers $\mathcal{L}$ and the competitive fringe in field node g. The aggregated supply function is found by horizontal summation of the individual supply functions.

Between the production facilities and the market-hubs there is a transportation network. The gas molecules are transported from nodes with high pressure to nodes with lower pressure through pipelines. The design parameters of the pipelines (length, diameter, roughness) as well as external variables (temperature) decide how much gas is transported for a given pressure difference. The relation between pressure in the nodes and flow in the pipelines are determined based on the Weymouth equation, see for instance Menon (2005). For a discussion of system effects on capacity related to pressure constraints see Midthun et al. (2006).We have chosen to linearize this expression with the formulation used in Tomasgard et al. (2007):

$$f_{ij} \leq K_{ij} \frac{RI_i}{\sqrt{RI_i^2 - RO_j}^2} r_i - K_{ij} \frac{RO_j}{\sqrt{RI_i^2 - RO_j^2}} r_j. \tag{5.20}$$

About 20 of these constraints that are approximating the Weymouth constraint are used for each pipeline in order to linearize the flow around pairs of pressure in, RI_i, and pressure out, RO_j. Here f_{ij} is the flow from node i to j and r_n is the pressure in node n.

In addition, constraints on the pressure level in each node must satisfied:

$$\underline{R}_n \leq r_{ns} \leq \overline{R}_n \quad n \in \mathcal{N}, \ s \in \mathcal{S}, \tag{5.21}$$

where $\underline{R}_n$ is the smallest allowed pressure in node n, and $\overline{R}_n$ is the largest allowed pressure in node n.

In the numerical analysis, we will also look at an alternative formulation with fixed capacities. In this case the pressure constraints and the Weymouth equation are replaced with the following formulation:

$$f_{nis} \leq \overline{C}_{ni}, \ n \in \mathcal{N}, \ i \in \mathcal{O}(n). \tag{5.22}$$

In the following we will refer to this formulation as Independent Static Flow (ISF), while the Weymouth formulation is referred to as WF. It is non-trivial to determine appropriate values for the ISF capacities. See Midthun et al. (2006) for a discussion. In this paper we solve an optimization model (with WF formulation) where the objective is to maximize the throughput in the network. The ISF capacities are then set equal to the resulting flow pattern in this model. The WF formulation is a relaxation of this ISF formulation, but it also represents the real system more precisely as it includes the flexibility of moving bottlenecks by adjusting pressures. The ISF formulation is more restricted but any increase in its capacities will allow a solution which is infeasible in the WF formulation.

The system operator must make sure that the mass is conserved in the network. We assume that each field is connected to a junction node, and that each market is connected to a junction node. In addition for ease of notation, we assume that no junction nodes are connected to each other. The mass balance equations are given as:

$$\sum_{j \in \mathcal{O}(g)} f_{gjs} = \sum_{m \in \mathcal{M}} \left(z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \quad g \in \mathcal{G}, \ s \in \mathcal{S}, \tag{5.23}$$

where $\mathcal{O}(g)$ is the set of nodes connected to a pipeline leaving from field g. In the junction nodes, the mass balance can be formulated as:

$$\sum_{g \in \mathcal{I}(j)} f_{gjs} = \sum_{m \in \mathcal{O}(j)} f_{jms}, \quad j \in \mathcal{J}, \ s \in \mathcal{S}, \tag{5.24}$$

where $\mathcal{I}(j)$ is the set of nodes connected to a pipeline entering node j. Finally, a constraint for the mass conservation in the market nodes must be included:

$$\sum_{n \in \mathcal{I}(m)} f_{nms} = \sum_{g \in \mathcal{G}} \left(z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \quad m \in \mathcal{M}, \ s \in \mathcal{S}. \tag{5.25}$$

The following constraint is included in the model with maximum flow and maximum value in order to ensure that the price in the secondary market is positive:

$$p_m - c_g \cdot \left(\sum_{m' \in \mathcal{M}'} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right) \geq 0, \quad g \in \mathcal{G}, \ m \in \mathcal{M}. \tag{5.26}$$

Alternatively, we could have introduced a constraint that ensured that the ISO income was positive in total (or for all field-market combinations).

Benchmark

In Chapter 5.5 we benchmark our solutions with an optimization model where an independent operator schedules production, routing and sale in order to maximize the social surplus of all the players in the network. The closer the equilibrium in our game gets to the benchmark solution, the better the strategy is with respect to maximizing the social surplus. The mathematical formulation of the benchmark model is given below.

$$\Pi_s^{BM} = \max \sum_{m\in\mathcal{M}} \sum_{l\in\tilde{\mathcal{L}}} (p_{ms}q_{lms} + P_{lms}v_{lms}) - \sum_{g\in\mathcal{G}} \sum_{l\in\tilde{\mathcal{L}}} \frac{1}{2} MC_{lg} d_{lgs}^2, \tag{5.27}$$

where MC_{lg} is the slope of the linear supply function of producer l in field g.

In addition, we need constraints (5.20) and (5.21) from the network operator problem presented in section 5.3. The mass balance is taken care of by:

$$\sum_{l\in\tilde{\mathcal{L}}} d_{lgs} = \sum_{j\in\mathcal{O}(g)} f_{gjs}, \quad g \in \mathcal{G},\ s \in \mathcal{S}, \tag{5.28}$$

$$\sum_{g\in\mathcal{G}} f_{gjs} = \sum_{m\in\mathcal{M}} f_{jms}, \quad j \in \mathcal{J}, s \in \mathcal{S} \tag{5.29}$$

$$\sum_{l\in\tilde{\mathcal{L}}} (q_{lms} + v_{lms}) = \sum_{j\in\mathcal{I}(m)} f_{jms}, \quad m \in \mathcal{M},\ s \in \mathcal{S}. \tag{5.30}$$

5.4 Model properties

Our model is a General Nash Equilibrium game where the feasible regions of the players depend on the other players' decisions. Let $X_l \in R^\alpha$ be the strategy set of player l with decision variables $x_l = (x_{l1}, \ldots, x_{l\alpha})$. We have $|L|$ producers and 1 ISO, constituting the set of players, $\bar{L}$. Define $\beta = |L| + 1$. The set $X = \prod_{l\in\bar{L}} X_l$ is the full Cartesian product of the strategy sets of individual players and $x = (x_1^T, \ldots, x_\beta^T)^T$ (in the case that no common constraints existed, it would be the strategy set of the game). Also define the vector x_{-l} of all players' decisions except player l's and correspondingly $X_{-l} = \prod_{j\in\bar{L}|j\neq l} X_j$.

We will define more formally the dependence between the players through the common constraints and define the point to set mapping $K_l : X_{-l} \Rightarrow X_l$ representing player l's feasible region, given the actions of the other players. $K_l(x_{-l}) \subseteq X_l,\ x \in X$.

Then a generalized Nash equilibrium (GNE) is defined as a point $x^* \in X$ that simultaneously optimizes all the players individual decision problems so that: $x_l^* \in K_l(x_{-l}^*),\ l \in \bar{L}$ and $\Pi_l(x^*) \geq \Pi_l(x_l, x_{-l}^*),\ x_l \in K_l(x_{-l}^*), l \in \bar{L}$ where $\Pi : R^{\alpha\beta} \to R$ is the objective function of player l. That is, the Generalized Nash Equilibrium is reached when no player has incentive to change his strategy given that the other players do not change their strategy.

Pioneering results on the existence of GNE are presented in the papers of Debreu (1952) (social equilibrium) and Arrow & Debreu (1954) (abstract economy) that generalized the results of Nash (1950). Rosen (1965) is an early paper concerning not only existence but also investigating uniqueness of solutions for a restricted class of problems. lchiishi (1983) gave more general results concerning the existence of such GNE.

It is well known that Nash equilibria (with independent player strategy sets) can be viewed as Variational Inequalities (VI), see Lions & Stampacchia (1967) for a nice overview. An early reference formulating the generalized Nash equilibrium as a Quasi Variational Inequality (QVI) is Bensoussan (1974). See for example Ferris & Pang (1997) or Facchinei & Pang (2003) for more on the relationships between complementarity problems and Variational Inequalities. This means that in addition to existence and uniqueness proofs following the Arrow/Debreu/Rosen tradition, also the theory of VI may be used to analyze this, see Harker & Pang (1990), Harker (1991) and Pang & Fukushima (2005) for good overviews of this direction of analysis.

Following the lines of the discussion in Harker (1991), we define $F_l(x^*) = \nabla_{x_l}\Pi_l(x_l^*, x_{-l}^*)$ and $F(x^*) = (F_0(x^*)^T, \ldots, F_{|L|}(x^*)^T)^T$ Then the GNE may be expressed as the Quasi Variational Inequality $QVI(F, K(x))$:

$$F(x^*)^T(x - x^*) \geq 0, \quad x \in K(x^*), \tag{5.31}$$

where $K(x) = \prod_{l \in \bar{L}} K_l(x_{-l})$.

It may here be noted that a standard Nash equilibrium may be expressed as a VI(F,K):

$$F(x^*)^T(x - x^*) \geq 0, x \in X. \tag{5.32}$$

In our case, the x vector consist of the following variables: $x = (b, h, d, q, f, r, z)$. Theorem 5.2 from Chan & Pang (1982) (Theorem 2 in Harker (1991)) give conditions for existence of a solution. We use notation in accordance with what we defined above:

Theorem 5.4.1. *Let F and K be a point-to-point mapping and point-to-set mapping respectively from $R^{\alpha\beta}$ into itself. Suppose that there exists a nonempty compact set X such that*

1. *$K(x) \subseteq X, \ x \in X,$*
2. *F is continuous on X,*
3. *K is a nonempty, continuous, closed and convex valued mapping on X.*

Then there exists at least one solution to the $QVI(F, K(x))$ in (5.31).

For our problem this is satisfied by the definitions of F and K. F consists of continuous, linear expressions since our objective functions are quadratic (see Equations (5.8) and (5.14)-(5.16)). The mapping in our model is defined by Equations (5.13), (5.23) and (5.25)-(5.26). Since all these equations are linear, the conditions in Theorem 5.4.1 are satisfied. We then know that our Generalized Nash Game has at least one solution.

Common constraints

We define common constraints as constraints where decision variables for more than one player appear. In our model, all the common constraints are continuous, linear functions (see Equations (5.33)-(5.36)) and satisfy the necessary constraint qualifications (LICQ). We can therefore apply Theorems 4-6 from Harker (1991) directly. These theorems state that if F is a continuous function in the $VI(F, X)$ then the VI solutions are the only points in the solution set of the $QVI(F, K(x))$ at which the optimal dual variables $\lambda^* \in R^{p\beta}$ for the common constraints are such that $\lambda_0^* = \lambda_j^*,\ j \in \overline{L}$. The theorems also state that any strictly interior solution (for the common constraints) of the $QVI(F, K(x))$ is a solution to the $VI(F, X)$ as described in (5.32). In general there will be several GNE in the game, but only the VI solutions will have a common positive value of an additional unit of a common resource (if the resource is depleted), or a zero value of a common resource for all players (if not used in full). Further, if F is strictly monotone there is a unique solution to the VI over X, Facchinei & Pang (2003), Theorem 2.3.3. This means that if an interior x^* is known, the only other GNE may be found at the boundary of the common constraints, and they will not have equal λ's for the common constraints.

We have focused on the VI solution in this article. A discussion of the common constraints and the implication of requiring equal shadow prices are given in the next sections. In our model we have the following common constraints (dual variables belonging to each constraint are given to the right):

$$z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0 \qquad \tau_{gms}, \tag{5.33}$$

$$\sum_{j \in \mathcal{O}(g)} f_{gjs} = \sum_{m \in \mathcal{M}} \left(z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \qquad u_{gs}, \tag{5.34}$$

$$\sum_{n \in \mathcal{I}(m)} f_{nms} = \sum_{g \in \mathcal{G}} \left(z_{gms} + \sum_{l \in \mathcal{L}} b_{lgm} \right), \qquad u_{ms}, \tag{5.35}$$

$$p_m - c_g \cdot \left(\sum_{m' \in \mathcal{M}'} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right) \geq 0, \qquad \chi_{gms}. \tag{5.36}$$

Constraint (5.33) gives the balance between capacity sold by the system operator and capacity traded by the large producers. If this constraint is not binding, the large producers buy less capacity than the ISO sells. If the constraint is binding, the large producers are buying all capacity sold by the ISO. For the producers, the shadow price τ_{gms} then gives the value of an additional unit of capacity bought. For the ISO, the shadow price gives the value of selling one additional unit of capacity and thus increasing the flow in the network with one unit. Constraints (5.34) and (5.35) specify that the booked capacity in the network must be equal to the actual flow in the pipelines. Constraint (5.34) gives the balance for each field node, and constraint (5.35) gives the balance for each market node. For the producers, the shadow price u_{gs} gives the value of booking one additional unit of capacity out of field g in the primary market. For the ISO, the shadow price gives the value of increasing the difference between the flow out of field g and the capacity sold, z_{gms}. Since the flow variable is part of the objective function for the ISO, the shadow price gives the value for the ISO of increasing the flow out of the field. The same argument is valid for the shadow price u_{ms}. Constraint (5.36) ensures that the price in the secondary market is positive. The price depends on the volumes sold by the ISO and the large producers. For both the producers and the ISO, the shadow price χ_{gms} gives the value of selling one additional unit of transportation capacity.

For the MVF and MSS formulation, we advocate that the *VI* solution to the GNE game is the important one. In this case the ISO will have made routing decisions which make sure that all players' marginal value of an additional transportation unit is equal. In the system we have described, the tariff is fixed and may not be changed in order to give specific incentives to the players. Hence it is clear that the ISO has a lot of influence through the routing decisions, and such a fair routing policy is preferable. For the MF formulation however, the VI solution depends on the conversion of 1 Sm^3 to NOK, since we relate objective functions that are not commensurable with respect to the units. Since the marginal values are given in different units, it may not make sense to require equality in the equilibrium solution. The equilibrium solution will change if we change the currency (from NOK to for instance Dollars or Euros).

We have not been able to prove that the F function is strictly monotone, and the equilibriums we present in the numerical examples may therefore not be unique.

5.5 Numerical examples

We consider the network illustrated in Figure 5.4. There are two large producers, each present in both g_1 and g_2. In addition, there is a competitive fringe in g_1 and g_2.

In the following sections, we use our model to analyze several cases. We start with a deterministic setting in which we look at the different ISO objective function alternatives and the difference between the WF formulation and the ISF formulation. We then introduce stochasticity to our model to see how it influences the efficiency in the network.

Our model is designed for a situation where both a primary market and a

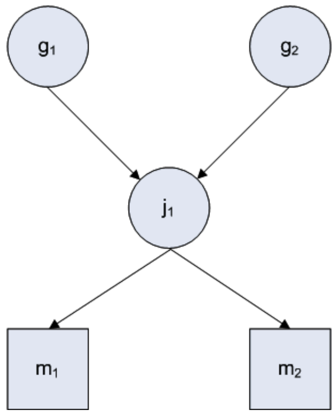

Figure 5.4: The network used in the numerical examples.

secondary market is used to allocate capacity in the network. The ISO influences the efficiency of the network through routing decisions and capacity distribution in the secondary market, while the large producers influence the efficiency by booking in the primary market and trading in the secondary market. In the North-Sea today, the booking in the primary market is limited by predefined booking limits and in case of overbooking a capacity allocation key is used to distribute the scarce capacity. In our model we resemble this capacity allocation key by requiring equal marginal value for all players in our common constraints. Because of this allocation rule, we can use unlimited booking rights in the primary market. In reality, the total booking rights in the North-Sea is twice the real capacity.

In each case we solve the stochastic MPC by formulating the equilibrium conditions for the problem. The equilibrium conditions consist of the aggregated KKT-conditions for all players (see Appendix 5.A). In order to find an equilibrium, we enter the KKT-conditions to the complementarity problem solver PATH (Dirkse & Ferris 1995). As we discussed in Section 5.4, we focus on the VI solution to the problem. All prices and costs are given in $\frac{1}{100}NOK$. Since we have inelastic demand functions in the market nodes, the social surplus will be identical with the producer surplus in our network (which is an interesting setting from a Norwegian perspective).

Node/pipeline	$\overline{R}$	$\underline{R}$	K_{ij}	$\overline{C}_{ij}$
g_1	180	170		
g_2	185	170		
j_1	170	130		
m_1	130	115		
m_2	130	100		
g_1-j_1			0.5	38.39
g_2-j_1			0.6	52.71
j_1-m_1			0.4	46.11
j_1-m_2			0.35	44.99

Table 5.1: The design parameters for the network

Case 1: The different ISO objective function alternatives (WF formulation)

We start by illustrating the difference in the objective function alternatives we have presented for the ISO. The parameters in the cost functions for the large producers (see Equation (5.18)) are given as c_{lg}: $c_{11} = \frac{5}{2}, c_{12} = 6, c_{21} = 3, c_{22} = 5$, and for the competitive fringe in field g (see Equation (5.3)): $c_1 = 10, c_2 = 12$. The network parameters are given in Table 5.1. The prices in the two markets are given as: $p_{m_1} = 130$ and $p_{m_2} = 160$. The tariff in the primary market is 10 for each field market combination.

When we solve the benchmark model (see Section 5.3), we get a total surplus for all the players of 7220.43. This corresponds to the maximal achievable surplus in the network. The results from the optimization with the three different objective functions for the ISO is given in Table 5.2.

As we can see from the results, the model version where the ISO maximizes social surplus (MSS) gives the highest total surplus in the network. The total surplus is only 0.8% lower than the benchmark solution. The total social surplus obtained in the MVF and MF models are, respectively, 9.41% and 1.59% smaller than the benchmark solution. We also see that the value of flow is largest in the MVF formulation, while the social surplus has decreased. The reason for the decrease in social surplus is that the production costs have increased more than the income from the spot market. The reason is that the VI solution requires equal marginal values for all players in the common constraints. Since the ISO only considers the income from the flow in the network (and not the production costs), the ISO has a large marginal value of flow and therefore forces inefficient production decisions from the producers.

The equilibrium for the MF model is, as discussed in Section 5.4, difficult to

	Max social surplus (MSS)	Max value (MVF)	Max flow (MF)
Competitive fringe g_1 (NOK)	258.49	194.79	222.22
Competitive fringe g_2 (NOK)	704.17	704.17	704.17
Producer 1 (NOK)	3085.89	2595.33	3129.77
Producer 2 (NOK)	2435.35	2282.92	2410.73
ISO profit (NOK)	678.15	763.51	638.5
Social surplus (NOK)	7162.05	6540.72	7105.39
Flow (Sm^3)	80.32	88.85	76.35
Value of flow (NOK)	11668.39	12870.88	11207.20

Table 5.2: Results from the different ISO objective functions. WF formulation.

interpret since the units are different in the objective functions for the ISO and the large producers. If we change the currency (corresponds to changing the weighting of the flow for the ISO), the equilibrium also changes. By using a currency of $\frac{1}{100}NOK$ we put the emphasis on the large producers, and since the social surplus corresponds to producer surplus in our models, we get a solution close to the benchmark. In Table 5.3 we see the results from changing the currency from $\frac{1}{100}NOK$ to EUR (this is done by changing the weighting of the flow for the ISO, so the units are comparable with the results in Table 5.2). While the flow in the MF formulation was the lowest among the three alternatives in Table 5.2, it has increased to the maximum possible flow in the network in Table 5.3.

In the MSS and the MVF formulation, the change of currency will not affect the solutions, and in the remaining examples we will therefore focus on the MSS and the MVF formulations.

Case 2: ISF versus WF formulation

In this example we look at the difference between using the WF formulation and the ISF formulation (see Section 5.3 for a discussion of how the ISF capacities are determined). We use the same parameters as in the previous example (Section 5.5). Every flow pattern obtained with the ISF formulation is feasible within the WF formulation. In the ISF formulation the capacity in the network is therefore more restricted than in the WF formulation (the reason for including the ISF formulation is that it is a common approach for economic analysis in gas networks).

The results from this optimization is shown in Table 5.4. We see the same pattern in these results as we saw for the WF formulation: the MSS formulation

	Max flow (MF)
Competitive fringe g_1 (NOK)	174.18
Competitive fringe g_2 (NOK)	704.17
Producer 1 (NOK)	2328.40
Producer 2 (NOK)	2097.78
ISO profit (NOK)	785.86
Social surplus (NOK)	6090.39
Flow (Sm^3)	91.09
Value of flow (NOK)	13191.20

Table 5.3: Results from the MF formulation with a larger weight on the ISO objective function.

gives the highest social surplus in the network. Compared with the WF formulation, the total surplus is reduced with 2.58% for the MSS formulation and 6.85% for the MVF formulation.

The importance of using the WF formulation depends on the network structure, the uncertainty in prices and the volume uncertainty in the TOP-contracts. Large fluctuations (as is common in natural gas prices) give more value to flexibility and therefore the WF formulation will improve the efficiency in the network. The correlation between prices is also important. High correlation may result in less difference between the ISF and the WF formulation (since the flexibility in the network is less important in this case).

	Max social surplus	Max value
Competitive fringe g_1 (NOK)	174.42	174.42
Competitive fringe g_2 (NOK)	707.18	707.18
Producer 1 (NOK)	2599.82	2329.55
Producer 2 (NOK)	2904.06	2098.80
ISO profit (NOK)	591.71	782.96
Social surplus (NOK)	6977.19	6092.91
Flow (Sm^3)	71.97	91.10
Value of flow (NOK)	10705.93	13192.28

Table 5.4: Results from the different ISO objective functions. ISF formulation.

Node/pipeline	$\overline{R}$	$\underline{R}$	K_{ij}
g_1	190	170	
g_2	185	170	
j_1	170	130	
m_1	130	100	
m_2	130	90	
g_1-j_1			0.5
g_2-j_1			0.6
j_1-m_1			0.4
j_1-m_2			0.35

Table 5.5: The design parameters for the network

Case 3: The effect of stochasticity

In this example we look at the effect of stochasticity in our model. We use the network parameters in Table 5.5, and the following cost parameters for the large producers c_{lg}: $c_{11} = 3,\ c_{12} = 4,\ c_{21} = 4,\ c_{22} = \frac{7}{2}$, and for the competitive fringe in field g: $c_1 = 9,\ c_2 = 9$. The tariffs in the primary market are put at 10 for all field-market combinations.

The effects of stochasticity are largest when the price is volatile, and the correlation between the market prices is low, or negative. If volatility is low, or correlation is very high, the optimal booking in the first stage varies less between the scenarios. When the optimal booking in the first stage is similar in all scenarios, the effect of stochasticity is reduced.

We have chosen to use negative correlation and uniformly distributed prices between 75 and 225. Table 5.6 shows the results from the optimization. The benchmark solution in this case is 9008.59. We see that the total expected social surplus in the network has been reduced with 3.68% and 5.92% for the MSS and MV formulation, respectively, compared to the benchmark solution. The reason for these results is the capacity allocation we have chosen (focus on the VI solution), and the fact that all booked capacity must be used. In a stochastic setting, the capacity allocation in the primary market is done such that the marginal unit goes to the player that has the largest expected marginal value. When prices are very volatile, this means that the large producers in some scenarios have more capacity than they ideally would have wanted to have.

We have also looked at the wait-and-see solution (Madansky 1960) and expected result of using the expected value solution (Birge & Loveaux 1997). In the wait-and-see solution (WSS), the 15 scenarios are solved independently and we then find the expected value over the 15 scenarios. That is, we assume that

	Booking limit = $+\infty$		Wait-and-see solution	
	Max social surplus	Max value	Max social surplus	Max value
Competitive fringe g_1	441.91	420.63	453.98	581.19
Competitive fringe g_2	621.50	637.82	726.52	786.71
Producer 1 (NOK)	3180.73	3275.30	3666.62	3462.53
Producer 2 (NOK)	2927.52	2985.92	3303.10	3124.28
ISO profit (NOK)	1505.52	1155.55	758.84	765.28
Social surplus (NOK)	8677.18	8475.30	8909.06	8719.99
Flow (Sm^3)	94.43	99.84	84.93	96.89
Value of flow (NOK)	14522.37	15040.02	13660.29	14830.25

Table 5.6: Results from the model with stochasticity. Columns 2-3 shows the result with unlimited booking for each producer, and each field-market combination, and columns 4-5 shows the wait-and-see solution with unlimited booking.

the large producers somehow get perfect information of the future before they make their decisions in the first stage. The difference between the WSS solution and the solution from the stochastic model is the expected value of perfect information (EVPI). EVPI tells us how much each player would have been willing to pay for knowing the outcome in the second stage. The results from this test (columns 4-5 in Table 5.6) shows that the total surplus in the network has increased drastically in the WSS solution. The total expected social surplus is now only 1.1% lower than the benchmark solution for the MSS formulation, and 3.2% for the MVF formulation.

In order to find the the expected result of using the expected value solution (EEV), we first solve a deterministic problem where the stochastic variables are represented with their expected values (EVP). We then use the booking decisions from the EVP in the stochastic problem. The results from the EEV formulation is shown in Table 5.7. For the MSS formulation, we see that the stochastic solution is 1.77% higher than the EEV solution. The differences are small for the MVF formulation.

The situation without a primary market

We have also tested the model without a primary market (booking limits equal to zero), and found that the pricing mechanism in the secondary market was inefficient in this case. Since the price of capacity is based only on one producer's marginal cost (the competitive fringe), we found equilibria with a large distance

	Max social surplus	Max value
Competitive fringe g_1 (NOK)	420.73	420.63
Competitive fringe g_2 (NOK)	613.84	696.77
Producer 1 (NOK)	3262.36	3216.85
Producer 2 (NOK)	2994.11	2892.63
ISO profit (NOK)	1235.14	1236.56
Social surplus (NOK)	8526.18	8463.44
Flow (Sm^3)	95.84	99.84
Value of flow (NOK)	14587.55	15040.02

Table 5.7: Results form the EEV formulation.

to the benchmark solution. For each of the large producers, a decision to increase production will lead to an increase in production cost in addition to an increase in price of transportation capacity (when h_{lgms} is increased, the price of capacity increase). It may therefore be beneficial for the large producer to decrease the production even if the marginal production cost is lower than the marginal revenue.

In order to represent a situation without a primary market, a different market clearing mechanism in the secondary market is needed. As illustrated in the numerical examples in this section, the market clearing mechanism we have chosen works well in the presence of a primary market. The design and tests of new clearing mechanisms is an interesting topic for future research.

5.6 Conclusions

We have presented a stochastic MCP model based on Generalized Nash Equilibrium for analyzing a capacity distribution system with two stages: a primary market where only privileged players can participate and an open secondary market. This system is based on the existing capacity distribution system in the North-Sea. We have compared the results from our model with a benchmark model where a central planner with full information maximizes social surplus in the network. We have shown that there exists at least one equilibrium solution (the VI solution) to our models.

We found that the MSS formulation for the ISO lead to a higher total social surplus in the network than the alternatives. In the deterministic setting we found a difference of 0.8% between the benchmark solution and the MSS solution. The formulation requires that the system operator has full information regarding the cost structure of the producers in the fields.

An alternative that we have considered in this paper is to maximize value of flow to the market nodes. In this case we only need to assume that the ISO knows the market prices of natural gas. In the deterministic case, the distance to the benchmark solution was 9.41% for the MVF formulation. The social surplus for the MVF formulation was 8.6% lower than the social surplus in the MSS formulation for the deterministic case, and 2.3% lower in the stochastic case. The results from the WF formulation were highly dependent on the chosen weighting in the objective functions.

Secondly, we found that stochasticity is important for our results. The booking rights lead to suboptimal solutions in some of the scenarios when prices are uncertain. The WSS solution indicated a high value of perfect information (social surplus increased with 2.67% for the MSS formulation). The EEV solution illustrated that there was a value of solving the stochastic problem (social surplus increased with 1.77% for the MSS formulation).

Finally we found that modelling the pressure constraints in the network is important. In this article we have set the fixed capacities such that the total throughput of the system is maximized. We still found that the flexibility in the WF formulation was valuable. In our example, we found that the WF formulation gave an increase of 2.65 % for the MSS formulation.

Given that the value of the flow in the pipelines in the North-Sea in 2006 was approximately 130 billion NOK, the relatively low percentage differences we have shown in this paper still amounts to a substantial amount of money.

Possible future extensions of the model are other market clearing mechanisms in the secondary market, inclusion of elastic demand functions in the spot markets for natural gas, the possibility for the large producers to hold back capacity in the secondary market and strategic behavior in the primary market.

Appendix

5.A The equilibrium conditions

In this section we give the equilibrium conditions for our model. Shadow prices for constraints are introduced directly in the Lagrangian function. The matching of shadow prices with constraints can also be seen from the KKT-conditions. We distinguish two types of shadow prices: those that are unrestricted in sign (URS) and those that are restricted in sign. For the shadow prices that are restricted in sign, we use the following notation for the complementarity condition with the belonging constraint: $G(x) - a \leq 0 \perp \varpi \geq 0$. The complementarity condition states that either $G(x) - a$ or ϖ must be equal to zero.

The large producers

The KKT-conditions for producer l is found through the Lagrangian function:

$$
\begin{aligned}
L_l = & -\sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} T_{gm}b_{lgm} + \gamma_{lgm}\left(B_{lgm} - b_{lgm}\right) \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[\sum_{m\in\mathcal{M}}\left(p_{ms}q_{lms} + P_{lm}v_{lms}\right)\right] \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[\sum_{g\in\mathcal{G}}\sum_{m\in\mathcal{M}} h_{lgms}\left(p_{ms} - c_g\left(\sum_{m'\in\mathcal{M}}\left(z_{gm's} + \sum_{l'\in\mathcal{L}} h_{l'gm's}\right)\right)\right)\right] \\
& - \sum_{s\in\mathcal{S}}\phi_s\left[\sum_{g\in\mathcal{G}} C_{lg}(d_{lgs})\right] \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[\mu_{1lgs}\left(\sum_{m\in\mathcal{M}}\left(b_{lgm} - h_{lgms}\right) - d_{lgs}\right)\right] \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[\mu_{2lms}\left(\sum_{g\in\mathcal{G}}\left(b_{lgm} - h_{lgms}\right) - q_{lms} - v_{lms}\right)\right] \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[\alpha_{lgms}\left(b_{lgm} - h_{lgms}\right)\right] \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[\tau_{gms}\left(z_{gms} + \sum_{l\in\mathcal{L}} h_{lgms}\right)\right] \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[u_{gs}\left(\sum_{m\in\mathcal{M}}\left(\sum_{l\in\mathcal{L}} b_{lgm} + z_{gms}\right) - \sum_{j\in\mathcal{O}_(g)} f_{gjs}\right)\right] \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[u_{ms}\left(\sum_{g\in\mathcal{G}}\left(\sum_{l\in\mathcal{L}} b_{lgm} + z_{gms}\right) - \sum_{j\in\mathcal{I}(m)} f_{jms}\right)\right] \\
& + \sum_{s\in\mathcal{S}}\phi_s\left[\chi_{gms}\left(p_m - c_g\left(\sum_{m'\in\mathcal{M}}\left(z_{gm's} + \sum_{l\in\mathcal{L}} h_{lgm's}\right)\right)\right)\right].
\end{aligned}
$$

Finding the derivative of the Lagrangian function with respect to the decision variables we get the KKT-conditions for optimality:

$$\frac{\partial L_l}{\partial b_{lgm}} = -T_{gm} - \gamma_{lgm}$$
$$+ \sum_{s \in \mathcal{S}} \phi_s \left(\mu_{1lgs} + \mu_{2lms} + \alpha_{lgms} + u_{gs} + u_{ms} \right) \leq 0 \;\; \perp \;\; b_{lgm} \geq 0, \tag{5.37}$$
$$\frac{\partial L_l}{\partial \gamma_{lgm}} = B_{lgm} - b_{lgm} \geq 0 \;\; \perp \;\; \gamma_{lgm} \geq 0, \tag{5.38}$$
$$\frac{\partial L_l}{\partial q_{lms}} = p_{ms} - \mu_{2lms} \leq 0 \;\; \perp \;\; q_{lms} \geq 0, \tag{5.39}$$
$$\frac{\partial L_l}{\partial d_{lgs}} = -\frac{\partial C_{lg}}{\partial d_{lgs}} - \mu_{1lgs} \leq 0 \;\; \perp \;\; d_{lgs} \geq 0, \tag{5.40}$$
$$\frac{\partial L_l}{\partial h_{lgms}} = p_{ms} - c_g \sum_{m' \in \mathcal{M}} z_{gm's} - c_g \sum_{l' \in \mathcal{L}} \sum_{m' \in \mathcal{M}} h_{l'gm's} - c_g \sum_{m' \in \mathcal{M}} h_{lgm's}$$
$$- c_g \sum_{m' \in \mathcal{M}'} \chi_{gm's} - \mu_{1lgs} - \mu_{2lms} - \alpha_{lgms} + \tau_{gms} = 0, \quad h_{lgms} \;\; URS, \tag{5.41}$$
$$\frac{\partial L_l}{\partial \mu_{1lgs}} = \sum_{m \in \mathcal{M}} \left(b_{lgm} - h_{lgms} \right) - d_{lgs} = 0, \quad \mu_{1lgs} \;\; URS, \tag{5.42}$$
$$\frac{\partial L_l}{\partial \mu_{2lms}} = \sum_{g \in \mathcal{G}} \left(b_{lgm} - h_{lgms} \right) - q_{lms} - v_{lms} = 0, \quad \mu_{2lms} \;\; URS, \tag{5.43}$$
$$\frac{\partial L_l}{\partial \alpha_{lgms}} = b_{lgm} - h_{lgms} \geq 0, \;\; \perp \;\; \alpha_{lgms} \geq 0, \tag{5.44}$$
$$\frac{\partial L_l}{\partial \tau_{gms}} = z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0, \;\; \perp \;\; \tau_{gms} \geq 0, \tag{5.45}$$
$$\frac{\partial L}{\partial u_{gs}} = \sum_{m \in \mathcal{M}} \left(\sum_{l \in \mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j \in \mathcal{O}(g)} f_{gjs} = 0, \quad u_{gs} \;\; URS, \tag{5.46}$$
$$\frac{\partial L}{\partial u_{ms}} = \sum_{g \in \mathcal{G}} \left(\sum_{l \in \mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j \in \mathcal{I}(m)} f_{jms} = 0, \quad u_{ms} \;\; URS, \tag{5.47}$$
$$\frac{\partial L}{\partial \chi_{gms}} = p_m - c_g \left(\sum_{m' \in \mathcal{M}} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right) \geq 0, \;\; \perp \;\; \chi_{gms} \geq 0. \tag{5.48}$$

The network operator

For the network operator, we present the KKT-conditions for the three different objective function alternatives. First the maximize flow objective.

Maximize flow The Lagrangian function for the system operator can be formulated as [1]:

$$
\begin{aligned}
L_s = & \sum_{s\in\mathcal{S}} \phi_s \left[\sum_{m\in\mathcal{M}} \sum_{i\in\mathcal{I}(m)} f_{ims} + \eta_{nils}\left(K^1_{nil} r_{ns} - K^2_{nil} r_{is} - f_{nis}\right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{gs} \left(\sum_{m\in\mathcal{M}} \left(\sum_{l\in\mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j\in\mathcal{O}(g)} f_{gjs} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\chi_{gms} \left(p_m - c_g \left(\sum_{m'\in\mathcal{M}} \left(z_{gm's} + \sum_{l\in\mathcal{L}} h_{lgm's} \right) \right) \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{js} \left(\sum_{m\in\mathcal{O}(j)} f_{jms} - \sum_{g\in\mathcal{I}(j)} f_{gjs} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{ms} \left(\sum_{g\in\mathcal{G}} \left(\sum_{l\in\mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j\in\mathcal{I}(m)} f_{jms} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\omega_{1ns}\left(\overline{R}_n - r_{ns}\right) + \omega_{2ns}\left(r_{ns} - \underline{R}_n\right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\tau_{gms} \left(z_{gms} + \sum_{l\in\mathcal{L}} h_{lgms} \right) \right].
\end{aligned}
$$

KKT-conditions The KKT-conditions:

[1] We have simplified the Weymouth equation such that K^1_{nil} and K^2_{nil} represents the constants in the expression

$$\frac{\partial L}{\partial f_{gjs}} = -\eta_{gjs} - u_{gs} - u_{js} \leq 0 \ \perp \ f_{gjs} \geq 0, \tag{5.49}$$

$$\frac{\partial L}{\partial f_{jms}} = 1 - \eta_{jms} + u_{js} - u_{ms} \leq 0 \ \perp \ f_{jms} \geq 0, \tag{5.50}$$

$$\frac{\partial L}{\partial z_{gms}} = -c_g \sum_{m' in \mathcal{M}'} \chi_{gm's} + u_{gs} + u_{ms} + \tau_{gms} \leq 0 \ \perp \ z_{gms} \geq 0, \tag{5.51}$$

$$\frac{\partial L}{\partial r_{gs}} = \sum_{j \in \mathcal{O}(g)} \left(\sum_{l \in \mathcal{L}} \eta_{gjls} r_{gs} K^1_{gjl} \right) - \omega_{1gs} + \omega_{2gs} \leq 0 \ \perp \ r_{gs} \geq 0, \tag{5.52}$$

$$\frac{\partial L}{\partial r_{ms}} = \sum_{j \in \mathcal{I}(m)} \left(-\sum_{l \in \mathcal{L}} \eta_{jmls} r_{ms} K^2_{jml} \right) - \omega_{1ms} + \omega_{2ms} \leq 0 \ \perp \ r_{ms} \geq 0, \tag{5.53}$$

$$\frac{\partial L}{\partial r_{js}} = \sum_{g \in \mathcal{I}(j)} \left(-\sum_{l \in \mathcal{L}} \eta_{gjls} r_{js} K^1_{gjl} \right)$$

$$+ \sum_{m \in \mathcal{O}(j)} \left(\sum_{l \in \mathcal{L}} \eta_{jmls} r_{js} K^2_{jml} \right) - \omega_{1js} + \omega_{2js} \leq 0 \ \perp \ r_{js} \geq 0, \tag{5.54}$$

$$\frac{\partial L}{\partial u_{gs}} = \sum_{m \in \mathcal{M}} \left(\sum_{l \in \mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j \in \mathcal{O}(g)} f_{gjs} = 0, \ \ u_{gs} \ URS, \tag{5.55}$$

$$\frac{\partial L}{\partial u_{js}} = \sum_{m \in \mathcal{O}(j)} f_{jms} - \sum_{g \in \mathcal{I}(j)} f_{gjs} = 0, \ \ u_{js} \ URS, \tag{5.56}$$

$$\frac{\partial L}{\partial u_{ms}} = \sum_{g \in \mathcal{G}} \left(\sum_{l \in \mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j \in \mathcal{I}(m)} f_{jms} = 0, \ \ u_{ms} \ URS, \tag{5.57}$$

$$\frac{\partial L}{\partial \chi_{gms}} = p_m - c_g \left(\sum_{m' \in \mathcal{M}} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right) \geq 0, \ \ \perp \ \chi_{gms} \geq 0 \tag{5.58}$$

$$\frac{\partial L}{\partial \omega_{1ns}} = \overline{R}_n - r_{ns} \geq 0 \ \perp \ \omega_{1ns} \geq 0, \tag{5.59}$$

$$\frac{\partial L}{\partial \omega_{2ns}} = r_{ns} - \underline{R}_n \geq 0 \ \perp \ \omega_{2ns} \geq 0, \tag{5.60}$$

$$\frac{\partial L}{\partial \eta_{nis}} = K_{ni} \sqrt{r^2_{ns} - r^2_{is}} - f_{nis} \geq 0 \ \ \eta_{nis} \geq 0, \tag{5.61}$$

$$\frac{\partial L_l}{\partial \tau_{gms}} = z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0, \ \ \perp \tau_{gms} \ \ \geq 0. \tag{5.62}$$

Maximize value The Lagrangian function for the system operator can be formulated as:

$$
\begin{aligned}
L = & \sum_{s\in\mathcal{S}} \phi_s \left[\sum_{m\in\mathcal{M}} \sum_{i\in\mathcal{I}(m)} p_{ms} \left(f_{ims} - \sum_{l\in\mathcal{L}} v_{lms} \right) + \eta_{nils} \left(K^1_{nil} r_{ns} - K^2_{nil} r_{is} - f_{nis} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{gs} \left(\sum_{m\in\mathcal{M}} \left(\sum_{l\in\mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j\in\mathcal{O}(g)} f_{gjs} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\chi_{gms} \left(p_m - c_g \left(\sum_{m'\in\mathcal{M}} \left(z_{gm's} + \sum_{l\in\mathcal{L}} h_{lgm's} \right) \right) \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{js} \left(\sum_{m\in\mathcal{O}(j)} f_{jms} - \sum_{g\in\mathcal{I}(j)} f_{gjs} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{ms} \left(\sum_{g\in\mathcal{G}} \left(\sum_{l\in\mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j\in\mathcal{I}(m)} f_{jms} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\omega_{1ns} \left(\overline{R}_n - r_{ns} \right) + \omega_{2ns} \left(r_{ns} - \underline{R}_n \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\tau_{gms} \left(z_{gms} + \sum_{l\in\mathcal{L}} h_{lgms} \right) \right].
\end{aligned}
$$

KKT-conditions The KKT-conditions:

$$\frac{\partial L}{\partial f_{gjs}} = -\eta_{gjs} - u_{gs} - u_{js} \leq 0 \;\perp\; f_{gjs} \geq 0, \tag{5.63}$$

$$\frac{\partial L}{\partial f_{jms}} = p_{ms} - \eta_{jms} + u_{js} - u_{ms} \leq 0 \;\perp\; f_{jms} \geq 0, \tag{5.64}$$

$$\frac{\partial L}{\partial z_{gms}} = -c_g \sum_{m' \in \mathcal{M}'} \chi_{gms} + u_{gs} + u_{ms} + \tau_{gms} \leq 0 \;\perp\; z_{gms} \geq 0, \tag{5.65}$$

$$\frac{\partial L}{\partial r_{gs}} = \sum_{j \in \mathcal{O}(g)} \left(\sum_{l \in \mathcal{L}} \eta_{gjls} r_{gs} K^1_{gjl} \right) - \omega_{1gs} + \omega_{2gs} \leq 0 \;\perp\; r_{gs} \geq 0, \tag{5.66}$$

$$\frac{\partial L}{\partial r_{ms}} = \sum_{j \in \mathcal{I}(m)} \left(-\sum_{l \in \mathcal{L}} \eta_{jmls} r_{ms} K^2_{jml} \right) - \omega_{1ms} + \omega_{2ms} \leq 0 \;\perp\; r_{ms} \geq 0, \tag{5.67}$$

$$\frac{\partial L}{\partial r_{js}} = \sum_{g \in \mathcal{I}(j)} \left(-\sum_{l \in \mathcal{L}} \eta_{gjls} r_{js} K^1_{gjl} \right)$$
$$+ \sum_{m \in \mathcal{O}(j)} \left(\sum_{l \in \mathcal{L}} \eta_{jmls} r_{js} K^2_{jml} \right) - \omega_{1js} + \omega_{2js} \leq 0, \;\perp\; r_{js} \geq 0, \tag{5.68}$$

$$\frac{\partial L}{\partial u_{gs}} = \sum_{m \in \mathcal{M}} \left(\sum_{l \in \mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j \in \mathcal{O}(g)} f_{gjs} = 0 \;\; u_{gs} \;\, URS, \tag{5.69}$$

$$\frac{\partial L}{\partial u_{js}} = \sum_{m \in \mathcal{O}(j)} f_{jms} - \sum_{g \in \mathcal{I}(j)} f_{gjs} = 0, \;\; u_{js} \;\, URS, \tag{5.70}$$

$$\frac{\partial L}{\partial u_{ms}} = \sum_{g \in \mathcal{G}} \left(\sum_{l \in \mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j \in \mathcal{I}(m)} f_{jms} = 0 \;\; u_{ms} \;\, URS, \tag{5.71}$$

$$\frac{\partial L}{\partial \chi_{gms}} = p_m - c_g \left(\sum_{m' \in \mathcal{M}} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right) \geq 0, \;\perp\; \chi_{gms} \geq 0 \tag{5.72}$$

$$\frac{\partial L}{\partial \omega_{1ns}} = \overline{R}_n - r_{ns} \geq 0 \;\perp\; \omega_{1ns} \geq 0, \tag{5.73}$$

$$\frac{\partial L}{\partial \omega_{2ns}} = r_{ns} - \underline{R}_n \geq 0 \;\perp\; \omega_{2ns} \geq 0, \tag{5.74}$$

$$\frac{\partial L}{\partial \eta_{nis}} = K_{ni}\sqrt{r^2_{ns} - r^2_{is}} - f_{nis} \geq 0 \;\; \eta_{nis} \geq 0, \tag{5.75}$$

$$\frac{\partial L_l}{\partial \tau_{gms}} = z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0, \;\perp \tau_{gms} \;\; \geq 0. \tag{5.76}$$

Maximize social surplus The Lagrangian function for the system operator can be formulated as:

$$
\begin{aligned}
L = & \sum_{s\in\mathcal{S}} \phi_s \left[\sum_{m\in\mathcal{M}} \sum_{i\in\mathcal{I}(m)} p_{ms} \left(f_{ims} - \sum_{l\in\mathcal{L}} v_{lms} \right) + \sum_{m\in\mathcal{M}} \sum_{l\in\mathcal{L}} P_{lm} v_{lms} \right] \\
& - \sum_{s\in\mathcal{S}} \phi_s \left[\sum_{g\in\mathcal{G}} \frac{1}{2} MC_g \left(\sum_{j\in\mathcal{O}(g)} f_{gjs} \right)^2 \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\eta_{nils} \left(K^1_{nil} r_{ns} - K^2_{nil} r_{is} - f_{nis} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{gs} \left(\sum_{m\in\mathcal{M}} \left(\sum_{l\in\mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j\in\mathcal{O}} (g) f_{gjs} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\chi_{gms} \left(p_m - c_g \left(\sum_{m'\in\mathcal{M}} \left(z_{gm's} + \sum_{l\in\mathcal{L}} h_{lgm's} \right) \right) \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{js} \left(\sum_{m\in\mathcal{O}(j)} f_{jms} - \sum_{g\in\mathcal{I}(j)} f_{gjs} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[u_{ms} \left(\sum_{g\in\mathcal{G}} \left(\sum_{l\in\mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j\in\mathcal{I}(m)} f_{jms} \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\omega_{1ns} \left(\overline{R}_n - r_{ns} \right) + \omega_{2ns} \left(r_{ns} - \underline{R}_n \right) \right] \\
& + \sum_{s\in\mathcal{S}} \phi_s \left[\tau_{gms} \left(z_{gms} + \sum_{l\in\mathcal{L}} h_{lgms} \right) \right].
\end{aligned}
$$

$$\frac{\partial L}{\partial f_{gjs}} = -MC_g \sum_{j' \in \mathcal{O}(g)} f_{gjs} + \eta_{gjs} - u_{gs} - u_{js} \leq 0 \ \perp \ f_{gjs} \geq 0, \tag{5.77}$$

$$\frac{\partial L}{\partial f_{jms}} = p_{ms} - \eta_{jms} + u_{js} - u_{ms} + \tau_{gms} \leq 0 \ \perp \ f_{jms} \geq 0, \tag{5.78}$$

$$\frac{\partial L}{\partial z_{gms}} = -c_g \sum_{m' \in \mathcal{M}'} \chi_{gms} + u_{gs} + u_{ms} \leq 0 \ \perp \ z_{gms} \geq 0, \tag{5.79}$$

$$\frac{\partial L}{\partial r_{gs}} = \sum_{j \in \mathcal{O}(g)} \left(\sum_{l \in \mathcal{L}} \eta_{gjls} r_{gs} K^1_{gjl} \right) - \omega_{1gs} + \omega_{2gs} \leq 0 \ \perp \ r_{gs} \geq 0, \tag{5.80}$$

$$\frac{\partial L}{\partial r_{ms}} = \sum_{j \in \mathcal{I}(m)} \left(-\sum_{l \in \mathcal{L}} \eta_{jmls} r_{ms} K^2_{jml} \right) - \omega_{1ms} + \omega_{2ms} \leq 0 \ \perp \ r_{ms} \geq 0, \tag{5.81}$$

$$\frac{\partial L}{\partial r_{js}} = \sum_{g \in \mathcal{I}(j)} \left(-\sum_{l \in \mathcal{L}} \eta_{gjls} r_{js} K^1_{gjl} \right)$$

$$+ \sum_{m \in \mathcal{O}(j)} \left(\sum_{l \in \mathcal{L}} \eta_{jmls} r_{js} K^2_{jml} \right) - \omega_{1js} + \omega_{2js} \leq 0, \ \perp \ r_{js} \geq 0, \tag{5.82}$$

$$\frac{\partial L}{\partial u_{gs}} = \sum_{m \in \mathcal{M}} \left(\sum_{l \in \mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j \in \mathcal{O}(g)} f_{gjs} = 0 \ \ u_{gs} \ \ URS, \tag{5.83}$$

$$\frac{\partial L}{\partial u_{js}} = \sum_{m \in \mathcal{O}(j)} f_{jms} - \sum_{g \in \mathcal{I}(j)} f_{gjs} = 0, \ \ u_{js} \ \ URS, \tag{5.84}$$

$$\frac{\partial L}{\partial u_{ms}} = \sum_{g \in \mathcal{G}} \left(\sum_{l \in \mathcal{L}} b_{lgm} + z_{gms} \right) - \sum_{j \in \mathcal{I}(m)} f_{jms} = 0 \ \ u_{ms} \ \ URS, \tag{5.85}$$

$$\frac{\partial L}{\partial \chi_{gms}} = p_m - c_g \left(\sum_{m' \in \mathcal{M}} \left(z_{gm's} + \sum_{l \in \mathcal{L}} h_{lgm's} \right) \right) \geq 0, \ \perp \ \chi_{gms} \geq 0 \tag{5.86}$$

$$\frac{\partial L}{\partial \omega_{1ns}} = \overline{R}_n - r_{ns} \geq 0 \ \perp \ \omega_{1ns} \geq 0, \tag{5.87}$$

$$\frac{\partial L}{\partial \omega_{2ns}} = r_{ns} - \underline{R}_n \geq 0 \ \perp \ \omega_{2ns} \geq 0, \tag{5.88}$$

$$\frac{\partial L}{\partial \eta_{nis}} = K_{ni} \sqrt{r^2_{ns} - r^2_{is}} - f_{nis} \geq 0 \ \ \eta_{nis} \geq 0, \tag{5.89}$$

$$\frac{\partial L_l}{\partial \tau_{gms}} = z_{gms} + \sum_{l \in \mathcal{L}} h_{lgms} \geq 0, \ \ \perp \tau_{gms} \ \ \geq 0. \tag{5.90}$$

Bibliography

Arrow, K. J. & Debreu, G. (1954), 'Existence of an equilibrium for a competitive economy', *Econometrica* **22**, 265–291.

Austvik, O. G. (2003), *Norwegian Natural Gas. Liberalization of the European Gas Market*, Europa-programmet, Oslo, Norway.

Bensoussan, A. (1974), 'Points de nash dans le cas de fonctionnelles quadratiques et jeux diffdrentiels lindaires a N personnes', *SIAM Journal on Control and Optimization* **12**.

Birge, J. R. & Loveaux, F. V. (1997), *Introduction to Stochastic Programming*, Springer.

Chan, D. & Pang, J. (1982), 'The generalized quasi-variational inequality problem', *Mathematics of Operations Research* **7**, 211–222.

Debreu, G. (1952), A social equilibrium existence theorem, *in* 'Proceedings of the National Academy of Sciences'.

Dirkse, S. P. & Ferris, M. C. (1995), 'The PATH solver: A non-monotone stabilization scheme for mixed complementarity problems', *Optimization Methods and Software* **5**, 123–156.

European Union (1998), 'Directive 98/30/EC of the european parliament and of the council'.

European Union (2003), 'Directive 2003/55/EC of the european parliament and of the council'.

Facchinei, F. & Pang, J. (2003), *Finite-Dimensional Variational Inequalities and Complementarity Problems: Volume I*, Springer series of operations research, New York.

Ferris, M. C. & Pang, J. S. (1997), 'Engineering and economic applications of complementarity problems', *SIREV* **39**, 669–713.

Gabriel, S. A., Zhuang, J. & Kiet, S. (2005), 'A large-scale complementarity model of the North American natural gas market', *Energy Economics* **27**, 639–665.

Gabriel, S., Kiet, S. & Zhuang, J. (2005), 'A mixed complementarity-based equilibrium model of natural gas markets', *Operations Research* **53**(5), 799–818.

Harker, P. T. (1991), 'Generalized nash games and quasivariational inequalities', *European Journal of Operational Research* **54**, 81–94.

Harker, P. T. & Pang, J. S. (1990), 'Finite-dimensional variational inequality and nonlinear complementarity problems: a survey of theory, algorithms and applications', *Mathematical Programming* **48**, 161–220.

Hobbs, B. F. (2001), 'Linear complementarity models of nash cournot competition in bilateral and poolco power markets', *IEEE Transaction on Power Systems* **16**(2).

Hu, X., Ralph, D., Ralph, E. K., Bardsley, P. & Ferris, M. C. (2004), Electricity generation with looped transmission networks: Bidding to an iso, Technical report, CMI EP Working Paper No. 65. Available at SSRN: http://ssrn.com/abstract=556809.

Jing-Yuan, W. & Smeers, Y. (1999), 'Spatial oligopolistic electricity models with cournot generators and regulated transmission prices', *Operations Research* **47**(1), 102–112.

Kall, P. & Wallace, S. W. (1994), *Stochastic Programming*, John Wiley & Sons, Chichester.

lchiishi, T. (1983), *Game Theory for Economic Analysis*, Academic Press, New York.

Lemke, C. E. & Howson, J. T. J. (1964), 'Equilibrium points of bimatrix games', *Journal of the Society for Industrial and Applied Mathematics* **12**, 413–423.

Lions, J. & Stampacchia, G. (1967), 'Variational inequalities', *Communications on Pure and Applied Mathematics* **20**, 493–519.

Luo, Z.-Q., Pang, J.-S. & Ralph, D. (1996), *Mathematical programs with equilibrium constraints*, Cambridge University Press.

Madansky, A. (1960), 'Inequalities for stochastic linear programming problems', *Management Science* **6**, 197–204.

Menon, S. E. (2005), *Gas pipeline hydraulics*, CRC Press, Boca Raton, USA.

Midthun, K. T., Bjørndal, M. & Tomasgard, A. (2006), Modeling optimal economic dispatch and flow externalities in natural gas networks. Working paper, NTNU, Trondheim, Norway. Submitted to international journal.

Nash, J. (1950), Equilibrium points in n-person games, *in* 'Proceedings of the national academy of sciences'.

Pang, J. S. & Fukushima, M. (2005), 'Quasi-variational inequalities, generalized nash equilibria, and multi-leader-follower games', *Computational management science* **2**.

Patriksson, M. & Wynter, L. (1999), 'Stochastic mathematical programs with equilibrium constraints', *Operations Research Letters* **25**, 159–167.

Rosen, B. (1965), 'Existence and uniqueness of equilibrium points for concave n-person games', *Econometrica* **33**, 520–534.

Smeers, Y. (2003*a*), 'Market incompleteness in regional electricity transmission. Part I: The forward market', *Networks and Spatial Economics* **3**, 151–174.

Smeers, Y. (2003*b*), 'Market incompleteness in regional electricity transmission. Part II: The forward and real time markets', *Networks and Spatial Economics* **3**, 175–196.

Tomasgard, A., Rømo, F., Fodstad, M. & Midthun, K. (2007), Optimization models for the natural gas value chain, *in* G. Hasle, K.-A. Lie & E. Quak, eds, 'Geometric Modelling, Numerical Simulation, and Optimization: Applied Mathematics at SINTEF', Springer, chapter Optimization Models for the Natural Gas Value Chain.

Yao, J. (2006), Cournot Equilibrium in Two-Settlement Electricity Markets: Formulation and Computation, PhD thesis, University of California, Berkeley, USA.

Yao, J., Adler, I. & Oren, S. (2006), Modeling and computing two-settlement oligopolistic equilibrium in a congested electricity network. Working paper, UCEI.

Yao, J., Oren, S. S. & Adler, I. (2004), Computing cournot equilibria in two settlement electricity markets with transmission constraints, *in* 'Proceedings of the 37th Hawaii International Conference on System Sciences'.